Exploring gaps in the pre-existing mathematics knowledge of entrant NC(V) Level 2 engineering students at a South African TVET college

by

Thabisa Capa

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Supervisor: Dr. E. G. Makwakwa

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DECLARATION

Name: Thabisa Capa

Student number: 60042672

Degree: Master of Education in Mathematics Education

Exploring gaps in the pre-existing mathematics knowledge of entrant Level 2 engineering students at a South African TVET college

I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

_____ _____

SIGNATURE DATE

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DEDICATION

This study is dedicated to my family;

especially, to my three boys:

Uyivile Capa

Enathi A. Capa

and

Yakhanani K. Capa.

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ABSTRACT

This study explored the gaps in the pre-existing mathematical knowledge on the achievement of NC(V) Level 2 students. The study was guided by constructivist learning theory and an error analysis framework to examine students' alternative conceptions and errors. An explanatory sequential mixed-method design was used. A diagnostic test was administered to 150 Level 2 Engineering students at a TVET college in the Ekurhuleni district. The test, based on a Grade 9 mathematics paper, covered ratios, algebraic expressions, equations, exponents, geometry, measurement, functions, and statistics. Based on the test results, 21 students were purposively selected for interviews. The findings revealed widespread conceptual and procedural knowledge errors, insufficient retention of Grade 9 content, and over-reliance on memorised procedures and calculators. Many students experienced difficulty applying rules, solving equations, and translating word problems. These difficulties negatively impacted their readiness for NC(V) Level 2 Mathematics. The study implies that TVET colleges need to develop or implement instruments to assess the readiness of students who enrol in the NC(V) Mathematics programme. Such diagnostic tools would enable lecturers to identify students who are underprepared and to implement appropriate support mechanisms early in the programme. The study recommends bridging programmes, improved instructional strategies, and formative assessments to strengthen foundational knowledge and enhance student achievement.

Keywords: pre-existing mathematics knowledge, student achievement, TVET College, NC(V) Level 2, conceptual knowledge, procedural knowledge, error analysis, alternative conceptions, constructivist theory.

CHAPTER 1: INTRODUCTION

1.1. Background and context of the study

Mathematical knowledge includes conceptual understanding, procedural fluency, and the ability to apply mathematical principles appropriately in problem-solving contexts. In mathematics education, conceptual and procedural knowledge are regarded as foundational for learners' progression into more advanced mathematical topics and related disciplines such as engineering. Mathematical knowledge supports students' problem-solving abilities, logical reasoning, critical thinking, and the application of mathematical concepts in real-life and technical contexts. As a result, mathematics is considered a key competency area for engineering students.

In South Africa, the basic education curriculum is organized into four phases: Foundation Phase (Grades R to 3), Intermediate Phase (Grades 4 to 6), Senior Phase (Grades 7 to 9), and Further Education and Training (FET) Phase (Grades 10 to 12) (Department of Basic Education (DBE, 1995). The FET Phase is offered both as part of high school education and at Technical and Vocational Education and Training (TVET) colleges, where students can enrol in a National Certificate and Vocational (NCV) programme for the FET Phase. The NC(V) Level 2 programme serves as an entry level into vocational studies and is equivalent to Grades 10–12 in the schooling system. Students entering this level are expected to possess foundational mathematical knowledge acquired in Grade 9, including conceptual and procedural understanding of topics such as algebra, ratios, equations, geometry, functions, and measurement. This pre-existing mathematical knowledge is important because NC(V) Level 2 Mathematics builds on these foundational concepts to support learning in engineering-related subjects.

TVET colleges in South Africa offer the National Certificate (Vocational) [NC(V)] programme, which provides vocational education and training in fields such as engineering and business studies. The NC(V) programme consists of Levels 2 to 4, with Level 2 regarded as the entry level and equivalent to Grade 10 in the schooling system. Students entering NC(V) Level 2 are expected to possess foundational mathematical knowledge acquired in Grade 9, including conceptual and procedural understanding of algebra, equations, geometry, ratios,

functions, and measurement. This foundational knowledge is important because NC(V) Level 2 Mathematics builds on these concepts to support engineering-related learning.

The engineering programme requires entrants to Level 2 (year 1) to have passed Mathematics in Grade 9, and students are required to pass Mathematics at each level to progress to the next level of mathematics. Students are assessed internally by lecturers and externally by DHET. The NC(V) Level 2 Mathematics syllabus covers numbers, algebra, functions, space, shapes and orientation, data handling, and financial mathematics. These content areas require both conceptual and procedural mathematical knowledge. Conceptual knowledge involves understanding mathematical principles, relationships, and meanings, while procedural knowledge refers to the ability to apply rules, formulas, and procedures accurately when solving problems. Each topic includes sub-topics. For example, numbers include types of numbers, exponents, number patterns, and surd forms, which require understanding number relationships and operational procedures. Functions include linear, parabola, hyperbola, and exponential graphs, which require interpretation of relationships between variables and graphical representations. Space, shapes, and orientation include trigonometry, area, perimeter, and volume, requiring spatial reasoning and application of geometric principles. Data handling and financial mathematics require interpretation, calculation, and application of mathematical information in practical contexts. Many of these topics build on knowledge taught in the Grade 9 curriculum.

In a study conducted by Ngoveni and Mofolo-Mbokane (2019), NC(V) Level 2 students were found to have constructed alternative conceptions in algebra, particularly involving algebraic operations, simplification, factorisation, and manipulation of expressions. These misconceptions impeded their ability to progress to the NC(V) Level 3 and 4 syllabi, which require more advanced mathematical reasoning, procedural fluency, and conceptual understanding. The lack of algebraic knowledge may further affect the application of other mathematical concepts, as algebra is foundational to many mathematics topics.

In another study conducted with NC(V) Level 2 students, Sehole (2020) found that students experienced difficulty learning algebraic functions. They did not understand that functions describe relationships and were unable to differentiate between independent variables and dependent variables when working with functions.

In the South African curriculum, students are introduced to the topic of functions in Grade 7 and are expected to master this topic in Grade 10 or the corresponding NC(V) Level 2, where

they are required to apply conceptual and procedural knowledge of variables, coordinates, equations, and graphical relationships to interpret and plot graphs.

TIMSS (2019) compared the mathematics competence of Grade 9 students in South Africa with that of international students on topics such as geometric shapes and measurement, algebra, data handling, probability, and multiplication. These topics require both conceptual and procedural mathematical knowledge, including spatial reasoning, interpretation of data, multiplicative reasoning, algebraic thinking, and application of mathematical operations. The study found that South African students performed poorly in geometric shapes and measurement, data handling, and multiplicative reasoning, indicating weaknesses in spatial visualisation, interpretation of diagrams, understanding of measurement concepts, and application of multiplication procedures. Although students performed relatively better in algebra, weaknesses were still evident in algebraic reasoning and manipulation of expressions (Reddy, 2022). South African students further experienced difficulty interpreting diagrams and geometry-related problems and were unable to reconstruct shapes based on dimensions, suggesting limited conceptual understanding of geometric relationships and spatial reasoning (TIMSS, 2019)

Similarly, Kgopane (2021) found that Grade 9 students struggled to recognise and define basic geometrical shapes, describe the properties of three-dimensional shapes, and calculate area and volume using appropriate formulae. These difficulties indicate gaps in conceptual knowledge of geometric properties and procedural knowledge required to apply formulas correctly. Grade 9 students in South Africa were also found to struggle with data handling and probability (TIMSS, 2019). Learners lacked conceptual understanding of statistical measures such as mean and were unable to reason that if none of the data values exceed ten, the mean could not equal ten. Students further experienced difficulty with ratios, rates, and multiplicative reasoning in probability tasks, suggesting weaknesses in proportional reasoning and multiplication skills. Moyo (2020) found that Grade 9 students had developed misconceptions related to fractions and were unable to place fractions correctly on a number line because they did not understand fractions as representing positions and relationships within the number system. Students also displayed procedural difficulties when dividing fractions, particularly in applying the inverse operation correctly by changing multiplication to division and inverting the second fraction.

Taking a broader view, the TIMSS (2019) found that 59% of South African Grade 9 students had not acquired basic mathematical knowledge. This suggests that the mathematical

competence of students registering for NC(V) Level 2 may be inadequate to support their learning at this level.

1.2. Problem statement

The 2019 Trends in International Mathematics and Science Study (TIMSS) found that South African students' mathematics performance was among the lowest of the 21 participating middle-income countries. Reports from TIMSS (2019), Moyo (2020), and Kgopane (2021) revealed that Grade 9 students experienced difficulties in mathematical content areas such as number systems, algebra, geometry of straight lines, two-dimensional geometry, functions, the theorem of Pythagoras, data handling, probability, and perimeter. These difficulties were associated with deficits in conceptual and procedural mathematical knowledge, including weaknesses in algebraic reasoning, geometric interpretation, proportional reasoning, spatial visualisation, application of formulas, and interpretation of statistical information. Ngoveni (2018) further found that NC(V) Level 2 mathematics students displayed alternative conceptions in algebra, particularly in simplifying expressions, manipulating variables, and applying algebraic procedures correctly, which contributed to poor performance in Mathematics Level 2. Similarly, Moyo (2020) found that Grade 9 students demonstrated misconceptions related to fractions and procedural difficulties when performing operations involving fractions, while Kgopane (2021) reported that learners struggled to recognise geometric shapes, apply measurement formulas, and reason spatially in geometry tasks. These findings suggest that many students enter TVET colleges with insufficient conceptual and procedural mathematical knowledge required to support learning in NC(V) Level 2 Mathematics and related engineering subjects. This study therefore explored the influence of pre-existing mathematical knowledge on NC(V) Level 2 students' mathematics achievement.

1.3. Aims and objectives of the study

The purpose of the study was to explore the gaps in the influence of pre-existing knowledge on NC(V) Level 2 students' mathematics achievement. The objectives were:

1. to determine the level of pre-existing mathematics knowledge of new entrants to the NC(V) Level 2 programme.
2. to assess the alternative concepts and errors displayed by students answering Grade 9 mathematics questions.
3. to determine the nature of students' pre-existing mathematical knowledge acquired up to Grade 9; and

4. To explore strategies that may improve achievement in NC(V) Level 2 Mathematics.

1.4. Research question and sub-questions

The primary research question that drove this study is:

What gaps in pre-existing mathematics knowledge do NC(V) Level 2 students have when they enter a TVET college?

To support the investigation of this primary question, four secondary questions were developed:

1. What is the pre-existing level of mathematics knowledge of new entrants to a NC(V) programme at TVET college?
2. What alternative conceptions and errors do students display when solving previous grade level mathematics?
3. What is the nature of students' pre-existing mathematical knowledge acquired up to Grade 9?
4. What strategies may improve achievement in NC(V) Level 2 Mathematics?

1.5. Rationale for the study

The engineering subjects taught in the NC(V) programme require students to apply pre-existing mathematical knowledge when learning engineering concepts and solving technical problems. This includes conceptual knowledge, such as understanding mathematical relationships and principles, as well as procedural knowledge required to apply formulas, manipulate algebraic expressions, interpret graphs, and perform calculations accurately. Weaknesses in foundational mathematical knowledge may therefore hinder students' understanding and achievement not only in Mathematics Level 2, but also in related engineering subjects. These concerns prompted the current study to investigate the nature and influence of students' pre-existing mathematical knowledge upon entering NC(V) Level 2.

The DHET has set the minimum performance rate for TVET colleges in South Africa at 50% for NC(V) Level 2 Mathematics (Makole, 2015). The DHET monitors the colleges' performance and works both with lectures directly, through proof of portfolio of assessment (POA), and with the college, through auditing and requiring reporting on findings related to pass rate (Makole 2015).

Students enrolled in NC(V) Level 2 Mathematics at South African TVET colleges have historically performed poorly. This is illustrated in Table 1 below, which shows the results of Level 2 NC(V) Mathematics students 2021 -2023 years at a South African TVET college.

Table 1 – Performance of NC(V) Level 2 Mathematics students at a South African college for 2021-2023 (DHET NC(V) results analysis)

NC(V) RESULTS 2021 TO 2023 analysis										
Engineering subjects that's need calculations indicating the level										
Subject name	**Absent**	**Fail**	**Pass**	**Pass condoned**	**Pass distinction**	**Results pending**	**Number of students written**	**Pass % 2021**	**Pass% 2022**	**Pass% 2023**
Materials L2	59	6	59	0	2	0	67	72.53	65.20	48.41
Materials L3	10	5	47	0	4	0	56	88.23	77.45	77.27
Materials L4	2	1	27	0	1	0	29	95.00	92.35	90.32
Electronics L2	126	129	170	0	0	0	303	40.56	43.01	51.61
Electronics control L2	126	168	179	299	0	0	478	46.28	41.52	37.65
Electrical Systems L2	169	100	335	0	0	0	435	55	49.90	57.32
Mathematics L2	901	261	786	41	40	0	1128	39.63%	44.88	42.73
Mathematics L3	331	325	437	38	15	0	815	42.76%	41.44	40.75

The stipulated targeted performance rate set by DHET of 50% of students registered for Mathematics Level 2 was not achieved at this colleges for 3 consecutive years (2021 to 2023). Table 1 shows that the Level 2 performance rates were 39.63% in 2021, 44.88% in 2022, and 42.73% in 2023. The pass rate did not reach 50% in any of these years. To conclude, the college did not achieve the required minimum requirement performance rate of 50% for NC(V) Level 2 in the period 2021 to 2023.

Previous studies on NC(V) Level 2 Mathematics have examined the mathematical difficulties and misconceptions experienced by students in specific content areas. For example, Dolley (2015) investigated factors associated with students' difficulties in Mathematics NC(V), while Ngoveni (2018) examined mathematical challenges linked to students' poor performance in NC(V) Level 2 Mathematics. Ngoveni and Mofolo-Mbokane (2019) explored students' misconceptions in algebra, particularly difficulties related to algebraic operations and manipulation of expressions. Similarly, Sehole (2020) investigated difficulties experienced by NC(V) Level 2 students in learning functions, including challenges in understanding relationships between variables and interpreting graphs. Collectively, these studies highlighted

conceptual and procedural mathematical knowledge difficulties experienced by NC(V) students in selected mathematics topics. However, limited attention has been given to the broader nature of the pre-existing mathematical knowledge that students bring from Grade 9 into NC(V) Level 2 Mathematics. In particular, there is limited research examining how foundational conceptual and procedural mathematical knowledge acquired in Grade 9 may influence students' achievement in NC(V) Level 2 Mathematics.

1.6. Significance of the study

The policy legislation Further Education and Training Colleges(FETC) Act of 2006 'Formal Further Education and Training College Programmes at Levels 2 on the National Qualifications Framework (NQF)' states that anyone who has completed Grade 9 may apply to the NC(V) programme, however, TVET colleges include students who have completed Grades 10, 11, and 12 (Ngoveni, 2018). Thus, students in Mathematics classes may have a range of backgrounds and levels of competency (Ngoveni, 2018), which poses a challenge to lecturers (Mashongoane, 2015).

The findings of this study are anticipated to assist mathematics lecturers in understanding the level and profile of the mathematics knowledge of students at entry to NC(V) Level 2. This can assist them with targeting content areas and designing lesson plans to access and build on students' pre-existing knowledge. Targeting students entering Level 2, which is the first year of the three-year NC(V) programme, is strategic as it supports intervention at the start of the programme, which could benefit students as they progress through the programme.

1.7. Structure of the dissertation

This dissertation is presented in five chapters, as follows:

Chapter 1: Introduction. This chapter has provided the context to the study by describing the background to the study, the problem to which the study responds, the aim and research questions that drove the study, and its significance to the field.

Chapter 2: Literature Review. This chapter discusses the literature related to key issues encountered in the learning of NC(V) Level 2 mathematics. The theoretical framework that guided the study is presented in this chapter.

Chapter 3: Methodology. This chapter presents the methods used to conduct the study, including the research design, sample selection, data collection instruments, and methods of data collection and analysis. The ethical aspects of the study are also considered.

Chapter 4: Data analysis. This chapter presents the results of the data analysis. The findings of the study are used to answer the research questions.

Chapter 5: Conclusion and Recommendations. This chapter summarises the study. The findings are discussed together with their implications and the limitation of the study. Furthermore, the chapter draws conclusions, upon which recommendations are made.

CHAPTER 2: LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1. Introduction

A literature review involves exploring, locating, and summarising studies conducted by other researchers (Creswell, 2014). Khanyile (2016) indicates that a literature review presents what has been studied and discovered about the topic in specific research. The literature review relates the relevance of findings to the existing body of knowledge and enables researchers to position their study within the context.

This chapter examines literature relevant to mathematics curriculum and difficulties experienced by grade 9 and NC(V) students. This chapter discusses prior knowledge, comparison of knowledge covered in grade 9 and NC(V) level 2 mathematics, challenges experienced by Grade 9 and NC(V) Level 2 students in mathematics, knowledge acquisition from Grade 9, and possible ways to improve mathematics at Level 2. The conceptual framework informing this study is constructivism, which describes how learners acquire knowledge, and Movshovitz-Hadar et al. (1987)'s error analysis.

2.2. Prior knowledge and pre-existing knowledge

Prior knowledge, as defined by Sweller (2019), refers to the information that a student has learned before engaging with a new concept. This knowledge encompasses the information students have gained over time through a range of experiences, including personal experiences (such as self-study), social interactions, and broader cultural contexts (Tophat, 2021). It plays an integral role in learning, acting as a springboard for students to connect topics and serving as links that can be reconstructed into new and improved knowledge (Van Lloyd Diaz, 2017). Prior knowledge provides the foundation upon which new learning is built, supporting students to comprehend and grasp new concepts (Geofrey, 2021).

In this study, pre-existing knowledge is defined more narrowly than prior knowledge as the formal mathematics knowledge acquired in earlier grades of schooling—particularly, the knowledge learnt through Grade 9,which students are expected to draw on in the NC(V) Level 2 programme (Rittle-Johnson & Schneider, 2015; Ningsih & Retnowati, 2020). While the term *prior knowledge* can include both informal and formal learning, *pre-existing knowledge* here refers specifically to mathematics knowledge from previous schooling.

Scholars often distinguish between conceptual and procedural dimensions of mathematics knowledge (Ningsih & Retnowati, 2020). Conceptual knowledge consists of specific concepts and facts, while procedural knowledge involves understanding different procedures and knowing when and how to apply them (Hakim & Yasmadi, 2021; Ningsih & Retnowati, 2020). Conceptual knowledge develops through relationships between previously acquired and new concepts, while procedural knowledge is the understanding of how to perform particular tasks or skills, including the steps or procedures necessary for completion (Rittle-Johnson & Schneider, 2015; Hakim & Yasmadi, 2021; NCTM, 2000). Procedural knowledge plays a vital role in helping students understand the processes of analysing and solving problems (Hakim & Yasmadi, 2021).

Building effective mathematical knowledge relies on conceptual knowledge, which enables students to recall and apply mathematical concepts when solving problems. Having a strong base of conceptual knowledge makes it easier for learners to link new ideas with pre-existing ones. For example, understanding the concept of ratio as a multiplicative relationship allows students to extend this understanding to proportions and percentages (Hakim & Nurlaelah, 2018). To gain conceptual understanding, students need to engage in investigation and exploration of mathematical objects, making use of images and representations (Hakim & Nurlaelah, 2018). Teachers play a crucial role in supporting conceptual development by using classroom resources such as textbooks, supplementary materials, and creative teaching methods (Sianturi et al., 2019). Students are more likely to develop alternative conceptions, or conceptual errors, when they lack sufficient conceptual understanding (Sianturi et al., 2019). When teachers understand students' level of conceptual knowledge, teaching and learning participation is enhanced (Sianturi et al., 2019).

Procedural knowledge, on the other hand, develops through practice and is influenced by previous experiences and expertise (Sprague et al., 2021). The procedural knowledge that students have acquired forms a foundation from which teachers can further expand learning by introducing different procedures to ensure understanding. Teachers can evaluate students' procedural knowledge by presenting problem-solving tasks. If students encounter difficulties or make errors during problem-solving, teachers can reinforce and extend students' procedural knowledge through targeted practice, ensuring that students consolidate correct procedures and avoid repeating mistakes (Sprague et al., 2021).

In mathematics, conceptual and procedural knowledge are interdependent: growth in one supports growth in the other (Hakim & Yasmadi, 2021). Research shows that students with strong conceptual knowledge are better able to select appropriate procedures and apply them flexibly in problem-solving, leading to higher accuracy and deeper understanding (Schneider, 2015). Similarly, students with strong procedural knowledge can demonstrate correct problem-solving steps and enhance their conceptual understanding (Schneider, 2015). Therefore, the development of both conceptual and procedural knowledge is essential for improved mathematics learning outcomes.

While prior knowledge is a broad concept that includes both pre-existing knowledge from formal schooling and informal knowledge acquired outside the classroom, this study focuses specifically on *pre-existing knowledge* from students' formal mathematics education through Grade 9, with the purpose of examining how this knowledge influences achievement in NC(V) Level 2 mathematics.

2.3. The importance of pre-existing knowledge

Pre-existing knowledge plays an essential role in students' learning processes by facilitating connections between newly acquired information and previously understood concepts. As noted by Carpenter et al. (2016), such pre-existing knowledge functions as a bridge, linking pre-existing knowledge understanding to new learned knowledge. Pre-existing knowledge enables students to link and relate what they have learned in the past to the new information they are learning and is considered sufficient and lasting when it is relevant, accurate and easily accessible in the student's memory (Striky, 2021). Sufficient pre-existing knowledge enables students to make connections and build upon their pre-existing knowledge during the teaching and learning process, leading to positive outcomes (Diaz, 2017). Furthermore, students with sufficient pre-existing knowledge are more likely to constructively build upon their knowledge and are less likely to have difficulties when learning new information (Geofrey, 2021). They also tend to integrate new information more successfully, demonstrate a more positive attitude towards learning, and achieve higher than those who lack the necessary pre-existing knowledge (Jordan et al, 2020).

When pre-existing knowledge is used effectively, it supports learners to achieve various outcomes. On the other hand, pre-existing mathematics knowledge that is insufficient, or inaccurate can hinder students' learning, possibly leading to inaccurate formation of new concepts (David, 2017) as it prevents learners from linking their pre-existing understanding to

new knowledge (Arbiyah, 2020). Students with less pre-existing knowledge may require additional support or different instructional techniques compared to those with more pre-existing knowledge (Sprague, 2021). Insufficient pre-existing knowledge results in students struggling to engage with new information productively, negatively impacting their learning outcomes.

When students enter the classroom, they each bring a unique set of pre-existing knowledge, including their conceptual understanding, skills, and beliefs (Ghanat & Davis, 2019). Their unique set of pre-existing knowledge influences how they process and integrate new information, and each student may understand the presentation of new information differently and link it to their pre-existing knowledge based on their understanding (Yang et al., 2018). Since new knowledge and skills build upon pre-existing knowledge, understanding what students know and can do before starting a new topic helps teachers craft instructional activities that build students' strengths and address their weaknesses (Pascarella & Terenzini, 2005).

Thus, it is good practice for teachers to assess their students' pre-existing knowledge before they introduce a new topic (Delucchi, 2014). Assessing students' pre-existing knowledge helps teachers identify knowledge gaps and inaccurate constructions of concepts, enabling them to adapt their teaching to meet students' needs (Fatih, 2020). By assessing pre-existing knowledge, teachers become aware of the diversity of backgrounds in their classrooms and are better able bridge between students' pre-existing knowledge and new information. This enables teachers to address any errors in students' pre-existing knowledge before introducing new topics. Various tools can be used to assess pre-existing knowledge, including pre-tests, self or peer assessment, concept mapping, brainstorming, problem solving (Blackburn, 2017). The importance of assessing pre-existing knowledge assists assessors in determining the level of students' pre-existing knowledge before introducing new information. This provides a conceptual foundation that facilitates the connection and development of new concepts. Numerous studies have demonstrated that pre-existing knowledge provides formative data to integrate the connection of learners' knowledge. (Blackburn, 2017).

2.4. Comparison of knowledge covered in Grade 9 and NC(V) Level 2 Mathematics curriculum

The knowledge taught in the National Certificate Vocational (NC(V)) Level 2 programme builds on the knowledge taught in Grade 9 mathematics. The Grade 9 Mathematics curriculum focuses on the following content areas: numbers, operations, relationships; patterns, functions, algebra; space and shape (geometry); measurement; and data handling (Department of Basic Education (DBE), 2011). Each of these content areas equip students with specific skills and knowledge for solving problems. Similarly, the subject guidelines for NC(V) Level 2 Mathematics focus on content areas such as numbers, functions and algebra; space, shape and measurements; data handling; and financial mathematics (DHET, 2011).

The following sections compare the topics covered in the Grade 9 and NC(V) Level 2 mathematics curricula by content area.

2.4.1 Comparison of content on numbers, operations, and relationships

In the Grade 9 curriculum, the topic 'numbers, operations and relationships' is divided into several sub-topics: whole numbers, integers, exponents, common fractions, and decimal fractions. Students learn specific concepts under each sub-topic. For example, for the sub-topic 'whole numbers', students learn about the different types of numbers—such as natural numbers, whole numbers, integers, rational numbers, and irrational numbers. For the sub-topic 'common fractions', students learn to perform calculations using all four operations (addition, subtraction, multiplication, division) with fractions. They also learn techniques like converting mixed numbers to common fractions and simplifying fractions using multiples and factors. In the subtopic 'decimal numbers', students learn to estimate the number of decimal places in a result by using place value. They also perform calculations using all four operations with integers, as well as numbers related to squares, cubes, square roots, and cube roots of integers. In the subtopic 'exponents', students learn to solve problems involving numbers in exponential form, including scientific notation. They also learn to calculate numbers using exponential laws. For example: $a^m \times a^n = a^{m+n}$

$$a^m \div a^n = a^{m-n}$$

$$(a^m) = a^{mn}$$

$$a^{-m} = \frac{1}{a^m} \quad or\ a^0 = 1$$

For this content area, Grade 9 students are expected to acquire the following content skills: (1) understand number representations in different ways and transition between representations; (2) recognize and apply the properties of formulations with different number systems; and (3) solve a variety of problems using an extended number range and perform multiple operations accurately and fluently.

In the NC(V) Level 2 mathematics curriculum, the content area 'numbers' is more advanced. Students learn about surd form, which was not covered in the Grade 9 curriculum. Students learn to simplify surd form by breaking square roots into integer and surd form (e.g., $\sqrt{45} = 3\sqrt{5}$). They also learn a technique for rationalizing the denominator by multiplying the denominator by its conjugate to get the rational number. The knowledge taught in Grade 9 Mathematics assists students at the NC(V) Level 2, as they need to understand number properties and identify them within a builder set. At NC(V) Level 2, students learn to convert recurring decimals to common fractions and how to apply the BODMAS rule for calculations. They also develop the skills to use scientific calculators correctly to solve expressions involving addition, subtraction, multiplication, division, squares, cubes, square roots, and cube roots. Additionally, they learn to estimate and approximate physical quantities such as length, time, mass, and temperature to solve problems in practical situations. Overall, the NC(V) Level 2 Mathematics syllabus continues to build on the Grade 9 curriculum for this topic.

2.4.2 Comparison of content on patterns, functions, and algebra

In Grade 9, the content area 'patterns, functions, and algebra' is divided into the following sub-topics: numeric and geometric patterns; functions and relationships; algebraic expressions; algebraic equations; and graphs. Students learn various concepts under these sub-topics. For numeric and geometric patterns, students learn how to investigate and extend patterns while understanding the relationships between numbers. Students learn how to describe and justify the general rules for observing relationships between numbers using algebraic language. For functions and relationships, students develop skills to determine input and output values for patterns and relationships by using flow diagrams, tables, formulae, and equations. Students continue to learn to determine, interpret, and justify the equivalence of different descriptions of the same relationships or rules presented verbally, in flow diagrams, in tables by formulae, in equations, and in graphs on a cartesian plane.

For algebraic expressions, students acquire the skills to recognize expressions and identify the required conventions for writing expressions. Students learn to identify and classify like and unlike terms in algebraic expressions; to recognize and identify coefficients and exponents in algebraic expressions; to expand and simplify algebraic expressions by multiplying integers, monomials by monomials, binomials, and trinomials; to factorize algebraic expressions with common factors; the difference between two squares and trinomials; and to simplify algebraic fractions using factorization.

For algebraic equations, students learn about setting up equations and describing problems. They also learn problem-solving skills, such as using factorization to determine unknown variables, solving equations by inspection using additive and multiplicative inverses, using laws of exponents, and determining the numerical value of an expression by substitution.

For graphs, students learn to use tables of ordered pairs to plot points to draw graphs on a Cartesian plane, plot linear graphs from given equations, and determine the equation of any given linear graph. They also learn the importance of gradient and its effects and learn about the x and y - intercepts. They learn to analyse and interpret global graphs for problem scenarios, with a special focus on linear graphs, constant increase and decrease, and maximum and minimum.

The Grade 9 syllabus states that for this content area (patterns, functions and algebra), students are expected to acquire the following skills: (1) investigate numerical and geometric patterns to establish the relationships between variables; (2) express rules governing patterns in algebraic language or symbols; (3) develop algebraic manipulative skills that recognize the equivalence between different representations of the same relationship; (4) analyse situations in various contexts to make sense of them; and (5) represent and describe situations in algebraic language, formulae, expressions, equations, and graphs.

NC(V) Level 2 Mathematics builds upon the knowledge students gained in Grade 9 on how to plot a linear graph, as they learnt about other algebraic graphs (parabola, hyperbola and exponential) and how to identify and plot their equations. NC(V) Level 2 students learn to plot graphs using their given equations, calculate the critical points (x and y intercepts), determine their equation, and interpret the given graph. Students learn about the domain, range and asymptotes of graphs. They learn factorization by taking out a common factor, difference of two squares, trinomials, and how to simplify algebraic fractions using factorization. With

regard to patterns, at NC(V) Level 2 students learn to determine unknown terms, common differences, and sum of the terms using arithmetic formulae. Students gain an understanding of arithmetic series, the relationship between graphs, and the manipulation of algebraic expressions.

2.4.3 Comparison of content on space and shape

In Grade 9, the topic of space and shape is divided into the sub-topics geometry of 2D shapes, geometry of 3D shapes, geometry of straight lines, transformation geometry, and construction of geometric figures. Students learn various concepts under these subtopics. For geometry 2D shapes, students learn to solve geometric problems involving unknown sides and angles of triangles and quadrilaterals using known properties of triangles and quadrilaterals as well as properties of congruent and similar triangles. Students learn to investigate and establish the minimum conditions for congruent and similar triangles. They also learn to use appropriate formulae and conversions between International System of Units SI units, to solve problems and to calculate the perimeter and area of polygons and circles.

For geometry of 3D shapes, students learn how the five platonic solids are defined in terms of shape and number of faces, vertices, and edges. They also learn to recognize and describe the properties of spheres and cylinders. Additionally, students develop a technique to build 3D models using nets to create models of geometric solids such as cubes, prisms, pyramids, and cylinders. Lastly, students learn about calculating the surface area and volume of 3D objects, and the capacity of cubes, rectangular prisms, and triangular prisms using appropriate formulae.

For geometry of straight lines, students learn how to write clear descriptions of the relationships between angles formed by perpendicular lines, intersecting lines, and parallel lines cut by a transversal. Additionally, students learn to solve geometric problems using the relationships between pairs of angles.

For transformation geometry, students learn how to recognize, describe, and perform transformations with points, line segments, and simple geometric figures on a coordinate plane focusing on reflection in the x- or y-axis, translation within and across quadrants, and reflection in the line $y=x$.

For construction of geometric figures, students learn how to construct geometric figures appropriately using a compass, ruler, and protractor including bisecting angles of a triangle. Students learn to investigate the properties of geometric figures by constructing and investigating the angles in a triangle, focusing on the relationship between the exterior angle of a triangle and its interior angles. Additionally, students also learn how to construct and investigate sides, angles, and diagonals in quadrilaterals focusing on the diagonals of rectangles, squares, parallelograms, rhombi, and kites. Students also learn to calculate and solve problems using a calculator and convert between appropriate SI units including $mm^2 \rightarrow m^2 \rightarrow cm^2 \rightarrow km^2$

$mm^3 \leftrightarrow m^3 \leftrightarrow cm^3 \leftrightarrow km^3$

For the Theorem of Pythagoras, students focus on how to investigate the relationship between the lengths of the sides of a right-angled triangle to develop the Theorem of Pythagoras. They also determine whether the triangle is right-angled or not and if the lengths of all three sides of the triangle are known.

For this content area (space and shape), Grade 9 students are expected to acquire the following content knowledge: (1) the ability to display a wide range of geometric figures and solids using appropriate geometric instruments while developing an appreciation for the use of constructions to investigate the properties of geometric figures and solids; and (2) the ability to solve a variety of geometric problems by drawing on known properties of geometric figures and solids and be able to differentiate between the formulas for calculating area and volume.

In NC(V) Level 2, the knowledge gained from mathematics Grade 9 assists students to understand and be able to perform calculations as follows: (1) using formulae for measuring area, perimeter, surface area, and volume of geometric figures and solids; (2) using formulae for measuring area, perimeter, surface area, and volume of geometric figures and solids; and (3) selecting and converting between appropriate units of measurement and using the Theorem of Pythagoras to solve problems involving right-angled triangles. Students in NC(V) Level 2 gain an understanding of 2D and 3D shapes. The Grade 9 content on space and shape builds a strong foundation in mathematics problem solving skills and critical thinking that assist NC(V) Level 2 students succeed in this content, which counts for 30% of the final paper of NC(V) Level 2.

2.4.4 Comparison of content on data handling and probability

The Grade 9 content area ‘data handling and probability’ is divided into the sub-topics collecting and organizing data, representing data, and interpreting and analysing data. Students learn various concepts in this area.

For organizing and collecting data, students learn about how to collect data by posing questions relating to social, economic, and environmental issues. Students learn about distinguishing between samples and populations and suggest appropriate samples for investigation. They also learn to organize numerical data in different ways to summarize data by determining measures of central tendency and dispersion, including extremes and outliers.

For representing data, students learn how to draw a variety of graphs by hand or using technology to display and interpret data using bar graphs, double graphs, histograms, pie charts, and broken line graphs.

For probability, students learn how to determine probabilities for compound events using two-way tables and tree diagrams. They also learn how to determine the probabilities for outcomes of events and predict their relative frequency in simple experiments. Lastly, students learn to compare relative frequency with probability and explain possible differences.

For NC(V) Level 2, the knowledge learnt in Grade 9 mathematics assists students as they need to be able to define and calculate different measures of central tendency (mean, mode, and median). At this level, students also learn techniques for representing data effectively and appropriately, such as tally charts, stem-leaf plots, bar and compound graphs, histograms, pie charts, and broken line graphs. Additionally, they learn to calculate measures of dispersion: quartiles, inter-quartiles range, percentiles, and semi-interquartile range, which are new topics in NC(V) Level 2. The concept of stem-leaf plotting is first introduced to students in Grade 8, but it is not covered in the Grade 9 Mathematics curriculum. In NC(V) Level 2, stem-leaf plotting is re-introduced as a new concept, with an outcome of constructing a steam-leaf plot using three steps. Quartiles are introduced for the first time in NC(V) Level 2.

Students in NC(V) Level 2 gain an understanding of how to represent data effectively by choosing the appropriate form of organizing and interpreting data. The importance of

understanding the content area of data handling at NC(V) Level 2 is paramount to students as it carries a total of 40% in the final examination.

It is evident that Grade 9 knowledge acquisition plays a significant role in preparing students for success in the NC(V) level 2 mathematics program. The curriculum covered in Grade 9, which includes fundamental topics such as numbers, operations, relationships; patterns, functions, algebra; space and shape (geometry); measurement; and data handling (DBE, 2011) serve as the building blocks for acquiring specific skills necessary to excel in NC(V) Level 2 mathematics. The overlap in topics covered between Grade 9 and NC(V) level 2 enables students to reinforce their understanding of various mathematical concepts. This reinforcement can empower them to approach more advanced topics with confidence and ease. Consequently, students enrolled in NC(V) Level 2 will potentially have a solid foundation upon which they can build , allowing for smoother transition and greater potential for achieving success at this level.

2.5. Challenges experienced with Grade 9 Mathematics and NC(V) Level 2

Errors displayed by learners may be conceptual or procedural in nature. According to Siyepu (2015), conceptual errors show a failure to identify relationships in a problem, while procedural errors occur when learners are unable to carry out operations or procedures, even if concepts are understood. A study by Ngoveni (2018) found that poor achievement at NC(V) Level 2 was caused by several factors, such as the alternative concepts and errors displayed by students when solving mathematics algebra. The alternative concepts and errors displayed by NC(V) Level 2 students reveal that these students did not build the foundation in Grade 9 Mathematics required for NC(V) Level 2 mathematics. These alternative concepts are highlighted in the 2019 TIMSS report, which assesses the competencies of Grade 6 and 9 students across several countries. TIMSS aims to assist countries to identify areas that need improvement in the teaching and learning of mathematics and science. The 2019 TIMSS study, which involved 20 829 South African grade 9 students, found that these students achieved low scores in the areas of geometric shapes and measurements, data handling, multiplicative reasoning and higher scores in algebra.

Bowie et al. (2022) reports that South African Grade 9 students struggled to work with geometric shapes and measurements in the 2019 TIMMS. They reported that when students

were given the problem shown in Figure 1, which assessed their understanding of quadrilaterals, they were unable to arrive at the correct answer for *x*. Bowie et al. (2022) argue that as the problem required students to know only that the sum of angles in a quadrilateral is 360 degrees, their inability to solve the problem indicated that they had not developed an adequate knowledge of polygons during earlier grades.

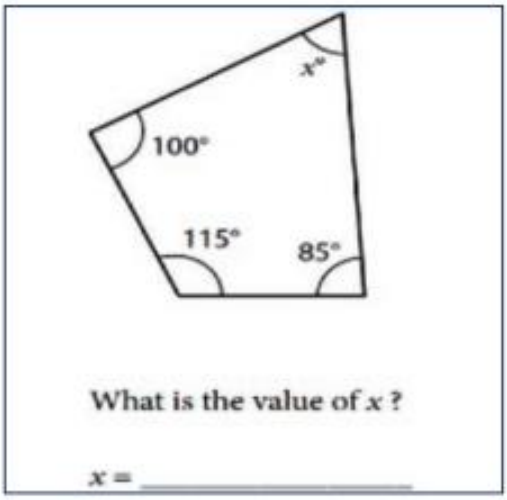


Figure 1 - (Question from the 2019 TIMMS investigating Grade 9 students' understanding of angles of a quadrilateral).

The problem shown in Figure 2, also taken from the 2019 paper, was designed to assess Grade 9 students' understanding of angles. South African Grade 9 (70%) students failed to apply the properties of alternate angles and parallel lines, resulting in incorrect answers to this question. Their poor performance might have been caused by a lack of basic knowledge of general geometry principles and a lack of the necessary procedural knowledge to apply the principles of angles correctly. Reddy (2022) recommends that teachers increase the number of classroom exercises on geometric principles.

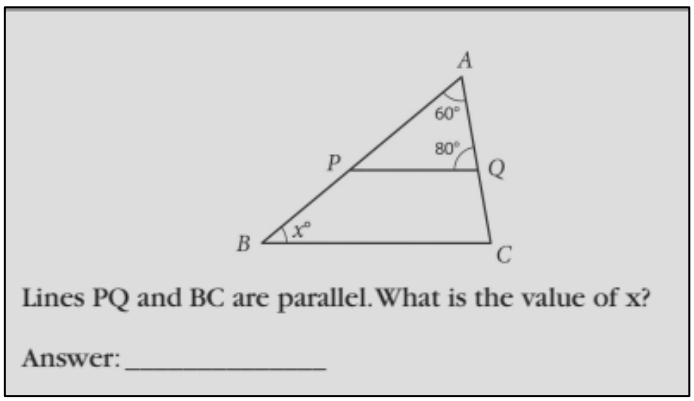


Figure 2 - (Question from the 2015 TIMMS investigating Grade 9 students' understanding of geometry of triangles).

2.5.1. Challenges with perimeter

South African Grade 9 students also struggled with the concept of perimeter (Reddy, 2022). They were given the problem shown in Figure 3, which required them to apply their knowledge of algebraic equations to solve for *x*, combining the knowledge of calculating the perimeter of 2D shapes. The South African Grade 9 (90%) students were unable to add the three sides of the triangle together and equate them to 21cm, as the total perimeter, and then solve for x. The students lacked an understanding of the formula used to calculate the perimeter of a triangle. The TIMSS (2019) report and Reddy (2022) indicate that students lacked adequate knowledge of algebraic equations and had minimal understanding of how to manipulate a formula to make x the subject of the formula.

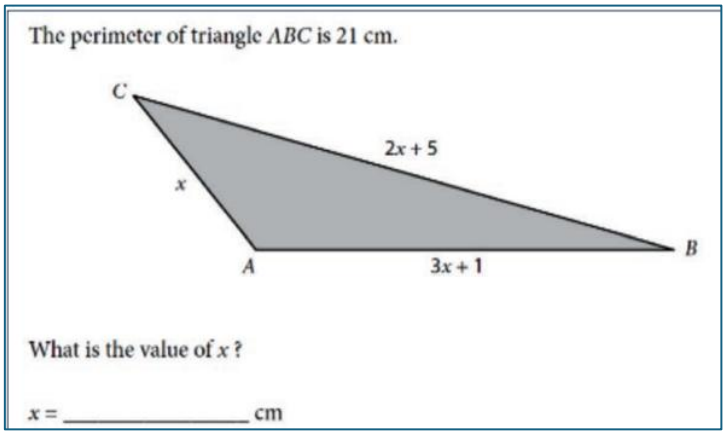


Figure 3 - (Question from the 2019 TIMMS investigating Grade 9 students' understanding of perimeter and algebra.

2.5.2. Challenges with 3D shapes in geometry

Kgopane (2021) conducted a longitudinal mixed-methods study to explore the difficulties experienced by Grade 9 Mathematics learners in understanding 3D shapes in geometry. The study included sixty Grade 9 students and five Grade 9 Mathematics teachers from two schools in the Lydenburg Circuit, in the South African province of Mpumalanga. The students performed poorly on a written test designed to assess their knowledge of 3D shape, demonstrating a lack of pre-existing knowledge of 3D shapes. They were unable to calculate the area of 3D shapes. They confused the formulae used to calculate the area of cylinders and triangles because both use similar letter such as "*h"* in their calculations. Kgopane (2021) concluded that the Grade 9 students were unable to define basic geometrical shapes correctly or calculate their volumes.

The findings of Reddy (2022) and Kgopane (2021) suggest that Grade 9 students may have inadequate knowledge to accurately differentiate between the area and volume of geometric shapes. Shapes are taught in the South African Mathematics curriculum in all grades—from Grade R up to Grade 12 with the complexity of the content increasing with each grade (CAPS 2007). Poor performance in this content area could be caused by the increase of level of difficulty when progressing from Grade R to Grade 12 content areas (Kgopane 2021).

2.5.3. Challenges with data handling and probability

As outlined earlier, the South African Grade 9 students struggled with data handling and probability content area (Reddy, 2022). Reddy (2022) argues that 89% of Grade 9 South African learners who took the 2019 TIMSS were unable to find the mean in the problem shown in Figure 4 because they did not understand the word *mean*.

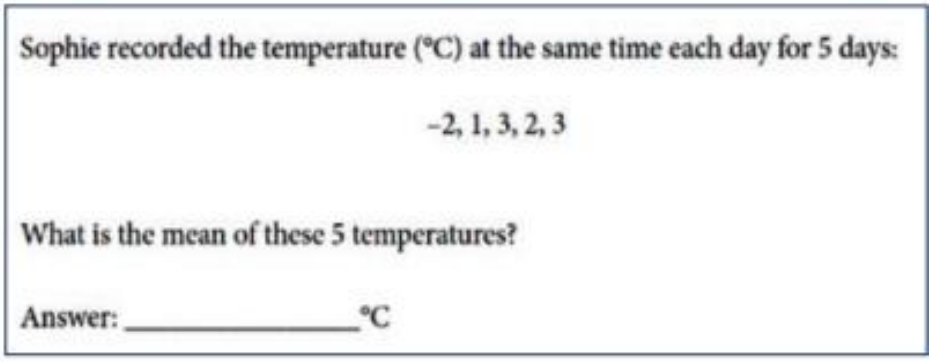


Figure 4 - (Question from the 2019 TIMMS investigating Grade 9 students' understanding of data handling).

Bowie et al. (2022) wondered if the inclusion of a negative number resulted in students being unable to find the mean. Based on the report released by TIMSS (2019), the poor performance of Grade 9 students on this question might have been caused by a lack of knowledge of how to calculate using negative integers. Students' abilities to answer the question required knowledge of adding integers; knowledge that should have been gained in earlier grades. The conceptual knowledge that was possibly lacking is that they are unable to add positive and negative integers.

2.5.4. Challenges with rates and ratios

South African Grade 9 learners experienced difficulty with rates and ratios, as revealed in the report released by TIMSS (2019). In the problem shown in Figure 5 below, 40% of Grade 9 students were unable to determine the equivalent ratio (Bowie et.al, 2022). The students may have lacked the procedural knowledge required to simplify the ratio by finding the highest

common factor. Grade 9 students learned the concept of dividing numbers from the earlier grades; they were not supposed to select the answer that is similar to the question (e.g. answer D in Figure 5). The conceptual knowledge of equivalent numbers relies on the concepts of fractions taught in earlier grades; students lacked this conceptual knowledge.

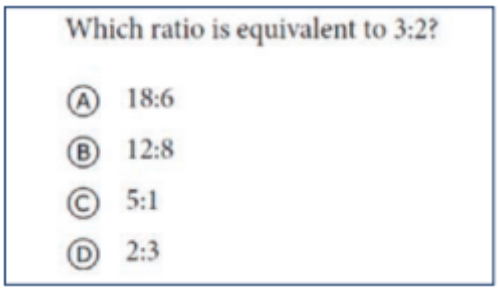


Figure 5 - (Question from the 2019 TIMMS investigating Grade 9 students' understanding of ratios)

A study by Tanudjaya and Doorman (2020) on higher-order thinking in Grade 9 secondary mathematics classrooms also reported that Grade 9 students had an inadequate understanding of the concept of ratios and rounding off. In their study, 372 Grade 9 students were given an open-ended mathematical problem in which they were asked to go to the supermarket, observe the number of customers entering the store, and ask them to donate their change. The task given the students was to demonstrate their ability to utilize proportional reasoning—using ratios, proportions, rates and percentages—to explain, draw conclusions, or make decisions about the estimated number of customers who would donate, demonstrate how such estimates were made, and round up the amount of change they were given customers. None of the students were able to complete the tasks correctly: 83% provided no answer and 17% gave wrong answers. They were unable to give the estimated number of customers in percentages and could not divide the total donations according to their ratios. Tanudjaya and Doorman (2020) concluded that these Grade 9 students had difficulty in the concept of ratios.

2.5.5. Challenges with common fractions

Common fractions is another concept area that South African Grade 9 students have been found to struggle with (Moyo, 2022). Moyo and Machaba (2020) explored the misconceptions of Grade 9 learners in the multiplication and division of fractions and analysed their misconceptions. A sample of 40 students wrote a test with 5 questions (3 on multiplication and 2 on division). The results indicated a lack of both conceptual and procedural knowledge for fractions. The students demonstrated a lack of procedural knowledge to multiply fractions, as they attempted to find the lowest common denominator. When students were given the division

problem $8 \div \frac{1}{2}$, they demonstrated an understanding of how to convert the operation sign to multiplication, but did not invert the second fraction to utilize cross multiplication, they were multiplying the numerator of other fraction by the denominator of another fraction which indicated lack of dividing the fractions. In an earlier study that examined the common misconceptions and associated errors that Grade 9 learners made when adding and subtracting common fractions, Pitsi (2016) found that participants were unable to provide a detailed definition of a common fraction and added fractions with different denominators without finding the lowest common denominator, as shown in Figure 6.

$$\frac{\mathbf{1}}{\mathbf{3}} + \frac{\mathbf{2}}{\mathbf{7}} = \frac{\mathbf{3}}{\mathbf{10}}$$

Figure 6 - (Question from Pitsi 2016 investigating Grade 9 students' understanding of common fractions)

Pitsi (2016) also found that participants were unable to compare fractions with different denominators For example, when given: $\frac{5}{8}$ and $\frac{3}{4}$, they indicated that the fraction with the bigger denominator was the bigger fraction.

These researchers have found that students had inadequate knowledge of fractions and recommended that teachers spend significant time teaching this content area to students (Moyo, 2022; Moyo & Machaba, 2020; Pitsi, 2016).

2.5.6. Challenges with probability

The TIMSS (2019) found that Grade 9 students also demonstrated difficulties working with the concept of probability. For probability, the Grade 9 CAPS curriculum outcome for students is to achieve how to determine the probabilities for compound events using two-way tables and tree diagrams, for outcomes of events, and to predict their relative frequency in simple experiments. In the 2019 TIMSS, learners were given the problem shown in Figure 7, where they were required to apply the principles of probabilities when more than one colour ball was selected.

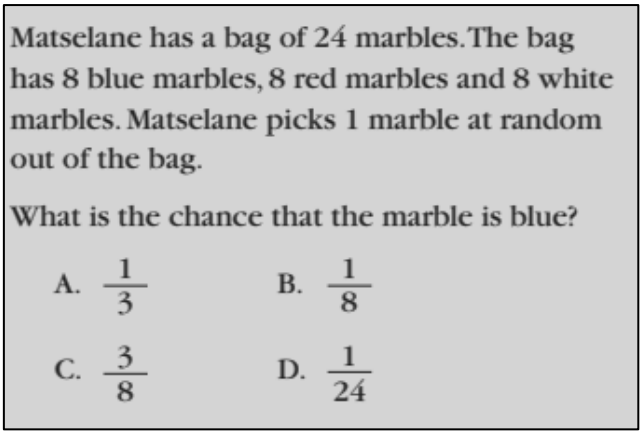


Figure 7 -(Question from the 2019 TIMMS investigating Grade 9 students' understanding of probability)

TIMSS (2019) highlighted that South African 76% learners could not select the correct answer for the question instead they appeared to select any answer from the four answers given, indicating that they lacked adequate conceptual knowledge of probability. In South Africa, probability is taught in Grades 4 to 12 and teachers must spend the given time on the topic and keep track of the timeframe in CAPS and adhere to it (Reddy, 2022).

2.5.7. Challenges with algebraic expressions

For the content area algebraic expressions, a study by Luneta (2016) analysed the errors made by South African Grade 9 students. The study found that participants were unable to apply mathematical rules to simplify algebraic expressions and confused conceptual rules. These students exhibited errors with removing brackets in the content area of distributive law as illustrated by the example in Figure 8, where all participated students multiplied only the first number in brackets by the number outside the bracket and provided an answer of 6a+4.

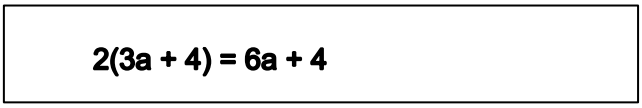


Figure 8 - -(Question from Luneta 2016 investigating Grade 9 students' understanding of algebraic equations)

Glancing at the outcomes of algebra teaching based on the CAPS documents students are expected to be able to solve equations successfully through the attainment of meaningful and conceptual learning by establishing mathematical relationships both in real-life situations and by finding the unknown. In this study, Grade 9 students lacked the procedural knowledge for removing brackets which they should have learnt in earlier grades. This concept of distributive law is introduced in earlier grades and students were expected to demonstrate knowledge of that concept. Luneta's (2016) study also found that these Grade 9 students were unable to

operate with negative signs when subtracting integers. For example, when presented with the equation shown in Figure 9 and gave an answer as per Figure 9, they indicated an idea when subtracting integers by taking the sign of the highest number and ignored the minus sign outside the second brackets.

$$(8x^2 + 3x + 4) - (5x^2 - 7x + 2) = 3x^2 - 4x + 2$$

Figure 9 - -(Question from Luneta 2016 investigating Grade 9 students' understanding of algebraic equations)

2.5.8. Challenges with algebra

Mathaba (2019) also explored Grade 9 students' misconceptions and errors in the content area of algebra. One hundred Grade 9 completed an assessment with four questions addressing different key areas.

The first question asked participants to determine the value of an unknown variable *x* using algebraic fractions. Most students' answers showed a lack of adherence to algebraic rules, and they committed careless computation errors, resulting in incorrect solutions. Mathaba (2019) also reported that students used operational signs incorrectly and lacked the necessary knowledge of algebraic manipulation to solve the equations correctly, thus by making the unknown variable the subject of the formula. These finding concur with those of Luneta (2016), who found that the concept of algebraic fractions was particularly difficult for students to comprehend.

The second question required participants to translate between graphs, tables, and equations. Mathaba (2019) reported that students struggled to distinguish between the expression $\boldsymbol{x + 2}$ and the equation $= \frac{x^2}{2}$**.** The study revealed that students were unable to draw algebraic graphs based on the functions. Brown et al. (2016) attributes such errors to a lack of vocabulary and conceptual understanding, especially when identifying independent values for graph plotting.

The third question required participants to convert word problems to equations and then solve for *x*. Most students either left this question blank or failed to generate the correct equation from the word problems, resulting in incorrect answers. Mathaba (2019) concluded that the students lacked an understanding of mathematical rules and the ability to follow all steps

sequentially. Egodawatte and Stoilescu (2015) reported that students struggled with the meaning attributed to the concept of word problems.

The fourth question required students to solve for unknown values using diagrams that included a square, a cube, and a rectangular prism. This question also tested their competence to use formulae for length, perimeter, height, and other variables. The findings revealed that students made errors such as identifying unlike terms as like terms, leading to incorrect solutions. Other researchers, including Brown et al. (2016), Hodgen et al. (2017), and Mulungye et al. (2016) report that these co-joining errors were caused by students' lack of conceptual and procedural knowledge in algebra.

Furthermore, a study conducted by Ari et al. (2021) examined the translation skills of Grade 9 students across multiple representations in algebra and investigated the relationship between these translation skills. The study involved 637 students who participated in a test designed to assess their skills in translating representation. The test questions included algebraic functions and tables. The findings of the study revealed that students' skills for translating between representations were poor, particularly when it came to translating from graphical representation to algebraic representation. De Bock et al. (2015) posited that the challenges observed in students' algebraic function representation translation could be attributed to inadequate understanding of the effect of changing parameters within mathematical equations. Ari et al. (2021) concluded that students experienced difficulty visualising the scenarios presented with algebraic equations and consequently interpreted these equations from different aspects of algebraic functions. This suggests that students experienced gaps in their ability to understand mathematical sentences and connect them with other representations.

The literature thus suggests that, with regard to algebra, Grade 9 students have experienced difficulties retaining previously acquired procedural and conceptual knowledge. This has implications for students' readiness for NC(V) Level 2 Mathematics, as the content areas covered in Grade 9 form the foundation for the knowledge taught at NC(V) Level 2. The 2019 TIMMS and other studies reviewed in this discussion have examined multiple content areas to gauge competencies acquired and retained by Grade 9 students, including space and shape, algebra, data handling, probability, functions, common fractions, and numbers.

In the content area of space and shape, Grade 9 students struggled to apply basic understandings of quadrilaterals, angles, volume, and perimeter. When it comes to common fractions, students display deficiencies in both conceptual and procedural knowledge in tasks involving the addition, subtraction, and multiplication of fractions. Moreover, in the content area of numbers, Grade 9 students lack proficiency in working with ratios. The difficulties displayed by Grade 9 students have the potential to influence their performance in NC(V) level 2 mathematics, as they demonstrate alternative conceptions across various content areas. The literature presented in this section corresponds with the research question aimed at assessing the mathematical proficiency of students upon entry into NC(V) Level 2.

2.6. Challenges experienced by students and errors in NC(V) Level 2 Mathematics

This section introduced the alternative conceptions displayed by NC(V) Level 2 mathematics students when they solved Level 2 mathematics. An alternative conception is a piece of false knowledge that may arise because of a student's prior knowledge and learning both inside and outside the classroom (Sibuyi, 2016).

Mofolo-Mbokane and Ngoveni (2019) conducted a qualitative case study on the misconceptions and errors of NC(V) Level 2 mathematics students, with a focus on algebra. A sample of 12 students was selected from a population of 150 students, and their 2016 final examination scripts were analysed, with a focus on the algebra section, which represented 25 of the 100 marks on the exam. The questions addressed like terms, factorization, simultaneous equations, and distributive law. The question on like terms was given as: $2x^2 + 3x$; its aim was to assess students' ability to add like terms. Students' responses showed a lack of procedural knowledge, as they were adding different variables as they gave an answer of $5x^2$. The researcher identified that students seem to make the mistake that the final answer in mathematics must be one term.

Analysing the findings from the above study, the expectation of NC(V) Level 2 students is that they can add like terms, as this is taught in Grade 9.

The question on factorization focused on factorizing trinomials and the difference of two squares. Only four out of the 12 students displayed knowledge of factorization, and only to a limited degree. They first grouped terms that contain common factors, taking out the common factor; they then took what was in the bracket to be a common factor and ignored the last term.

None of the 12 students were able to factorize the difference between the difference of two squares as they lacked the procedural knowledge to find the factors of the two squares. These students lack the knowledge of factorization that is taught in Grade 9.

For simultaneous equations, the 12 students were unable to use the elimination method and instead used substitution methods to deal with simultaneous equations but had difficulties in creating the third equation as they were unsure where to substitute it. They demonstrated understanding in solving simultaneous equations but struggled with procedural steps. Their difficulty creating the third equation resulted from a lack knowledge of how to manipulate an algebraic formula and make another variable the subject of the formula. NC(V) Level 2 students are expected to understand this concept as it builds on content taught in Grade 9.

The question on distributive law provided the equation $y(2y + 1) = 15$. None of the 12 were able to correctly remove the brackets and they were unable to factorize the quadratic equation after multiplying out. Most of the students appeared to ignore the *y* outside the brackets as they factorized. The students demonstrated a misconception that equations are only linear, which indicates a knowledge gap. The students lacked the procedural knowledge to multiply the bracket and find the value of *y*.

Sehole (2020) conducted a case study to explore the conceptual and procedural difficulties working with functions experienced by 26 NC(V) Level 2 Mathematics students in the province of Gauteng. The outcomes of the study highlighted that the NCV level 2 students battle to learn algebraic functions and lack an understanding that functions are about relationships (Sehole, 2020). The study found that the students were unable to comprehend or relate this relationship and lacked the ability to differentiate between input and output variables when taught functions. The topic of functions is introduced in the Grade 7 South African curriculum and students are expected to know the difference between input and output at this level.

Ngoveni (2018) conducted another study to investigate the poor performance of NC(V) Level 2 Mathematics students at a TVET college in Tshwane district in 2018. From a sample of 24 students, 12 examination scripts were analysed for alternative conceptions and 17 were interviewed to investigate their poor performance in mathematics. The researcher analysed students responses to the expression of simplification $\frac{4x^2-8xy^3}{-xy}$ to assess students' understanding of algebraic fractions. The students indicated a lack of understanding of the question, as they did not attempt to answer it. The students failed to recognize common factors;

when they did recognize a common factor, they made errors related to division (Ngobeni 2018). The students lacked conceptual knowledge of algebraic fractions, although this is part of the Grade 9 curriculum. These findings align with those Makonye and Khanyile (2015), who reported that students had difficulties recognizing common factors when completing simplification of fractions.

Ngwabe (2018) conducted a study on alternative conceptions and errors displayed by NC(V) Level 2 students when learning functions using GeoGebra software. A mixed method design was used, with a purposive sample size of 76 NC(V) Level 2 Mathematics students from the Nelson Mandela District in Gqeberha. Data was collected through pre- and post-tests, focus group interviews, and GeoGebra intervention and observations. Four questions were given to participants with the aim of analysing their alternative conceptions for functions: 2 hyperbola questions, one straight-line question, and a parabola question.

Participants were given the question shown in Figure 10 where they were asked to match the given graph with the correct equation from the list of equations. Most students selected incorrect answers. Their choices demonstrated that they did not understand the effect of constant '*a'* and the interpretation of the constant '*q'* in a hyperbola equation and showed an alternative conception by selecting an incorrect answer. The study found that the participants lacked the knowledge to correctly interpret graphs to determine their formulae (Ngwabe, 2018).

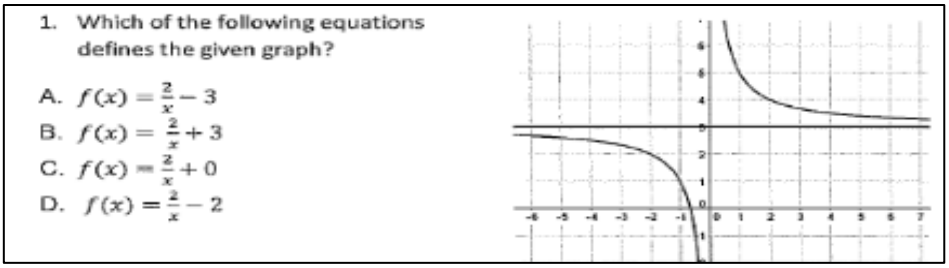


Figure 10 - (Question from Ngwabe 2018 investigating NC(V) students' understanding of Hyperbola)

Students were then given the problem shown in Figure 11, where they were expected to choose the equation defining the given graph. Students' responses displayed alternative conceptions, demonstrating an inability to recognize that a negative value of constant '*a*' resulted in a graph occupying the second and fourth quadrants. It is imperative to understand the effect of *a* and *q* in a hyperbola graph as it changes the location of the graph on a Cartesian plane: if the sign of constant *'a'* is positive, the graph occupies quadrants 1 and 3; inversely if it is negative, it occupies quadrants 2 and 4.

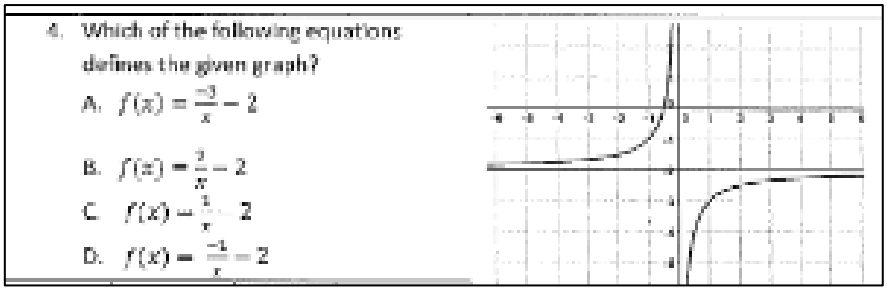


Figure 11 -**(Question from Ngwabe 2018 investigating NC(V) students' understanding of probability)**

The topic of hyperbola functions is not introduced in Grade 9: students are introduced to it in NC(V) Level 2 Mathematics for the first time; this could be the reason for their poor performance in this concept.

In Ngwabe's study (2018), participants were given the straight-line graph shown in Figure 12 below and asked to match the graph to the correct equation. The students were unable to distinguish between the effect of a negative parameter 'a' and a positive parameter 'a'. Ngwabe (2018) concluded that given students' lack of ability to interpret the graph and to determine the equation of a given graph they were unaware of the procedure for finding the equation of a straight-line graph—which is to find the gradient and substitute the answer to the original equation to find the y-intercept. The procedural knowledge related to a straight-line graph is first introduced in the Grade 9 curriculum.

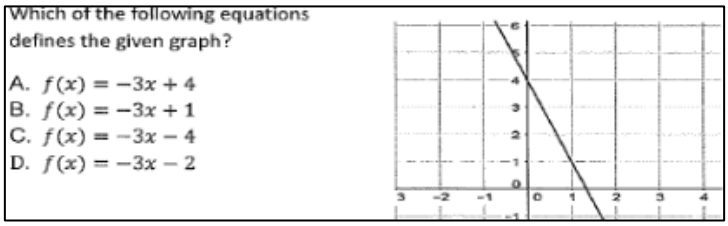


Figure 12 - **(Question from Ngwabe 2018 investigating NC(V) students' understanding of straight line)**

For parabola, participants were given the equation $y = x^2 - 9$, shown in Figure 13, and asked to match the coordinates of the turning point. They displayed alternative conceptions and errors as they were unable to identify the coordinates of the equation into graphical representation (Ngwabe, 2018).

The concept of parabola is introduced for the first time in NC(V) Level 2, which could be the reason for poor performance. Students are to be given an opportunity to learn and understand new concepts and allowed a reasonably extended period to grasp the new concepts not learned in previous grades.

6. The graphical representation of a quadratic function $y = x^2 - 9$ will have a:
A. maximum turning point $(0; 9)$
B. minimum turning point $(0, -9)$
C. maximum turning point $(0; -9)$
D. minimum turning point $(0; 9)$

Figure 13 - (Question from Ngwabe 2018 investigating NC(V) students' understanding of parabola)

The enrolment requirements for both NC(V) Level 2 Mathematics and Mathematics Literacy are the same, which is to have passed Grade 9 Mathematics. The curriculum for Level 2 Mathematics and Mathematics literacy includes the topics space, shapes, and orientation. NC(V) Level 2 Mathematics Literacy students displayed alternative conceptions and errors in this concept and there is a possibility that students will struggle in NC(V) Mathematics Level 2 with the same topic.

Hassan (2020) conducted a study on NC(V) Level 2 Mathematics Literacy students' conceptualisation of space, shapes, and orientations, with an aim to identify strategies to improve their performance. Qualitative data was collected using semi-structured interviews with both lecturers and students to understand students' performance and actions when solving a problem involving 2-dimensional shapes and the Theorem of Pythagoras.

Students were given the problem shown in Figure 14 involving space, shape, and orientation and asked to calculate: 1) the length of the gate, 2) calculate the total fencing required by Musa to erect around land, and 3) calculate the total area of Musa's land.

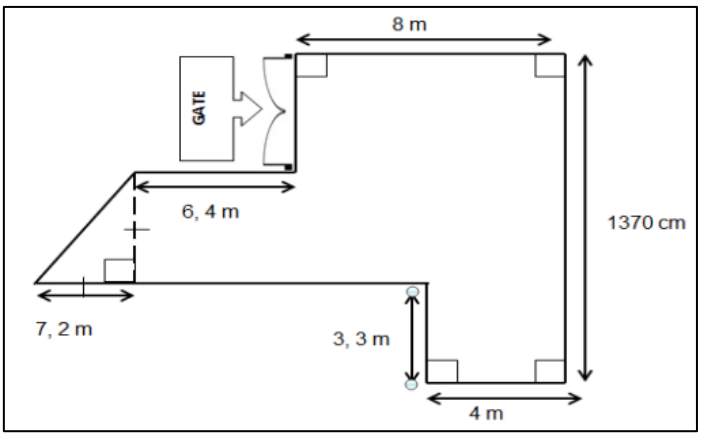


Figure 14 -(Question from Hassan 2020 investigating NC(V) students' understanding of 2D Geometric shape

Hassan (2020) reported that the students struggled to calculate the length of the gate and the perimeter of the fence using the given measurements and, instead, used a ruler to calculate. Their answers revealed that students were unaware of the requirement to calculate the

perimeter. Hassan's (2020) findings indicated that the students had inadequate knowledge of the space, shape, and orientation topics.

Lastly, students were assessed on their ability to answer an irregular shapes question, as represented in Figure 15. The question required them to calculate the length of the fence and the area of the entire shape.

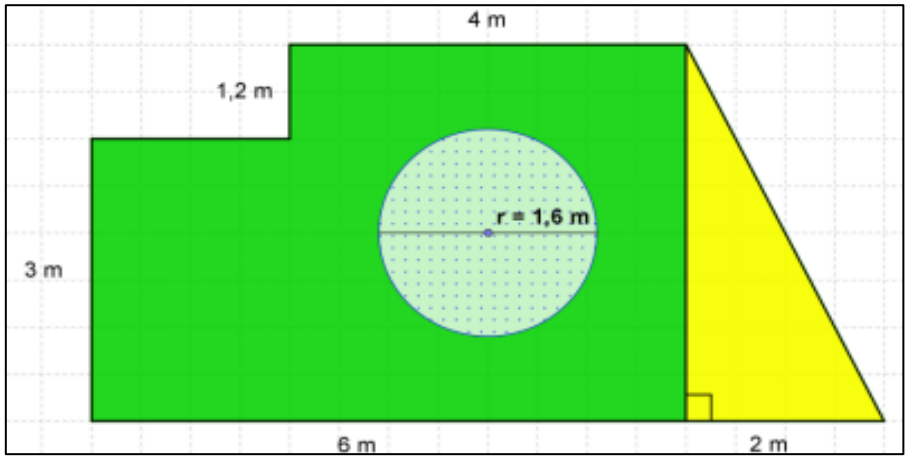


Figure 15 - (Question from Hassan 2020 investigating NC(V) students' understanding of 2D Composite shape)

Hassan's (2020) findings revealed that the students attempted to answer this question but were confused by the word 'length' and could not link it to the word 'perimeter'. Students did not understand how to calculate the perimeter and included the perpendicular line measurement as part of their calculation. Some students had an idea to calculate the length of the slant height but struggled to use the theorem of Pythagoras correctly. Students failed to calculate the area of the shape and failed to identify that the area of the circle was to be subtracted from the total area.

The difficulties experienced by NC(V) level 2 students reported in Hassan's (2020) study are the same challenges experienced by Grade 9 students, as reported in the literature reviewed in the previous section. The NC(V) Level 2 Mathematics students struggled with basic algebra, were unable to add like terms, solve simultaneous equations, or apply the distributive law. They also struggled with the concept of functions—which is used until Level 4. These concepts build on the knowledge they were expected to acquire in Grade 9. The difficulties experienced by NC(V) Level 2 Mathematics students might be influenced by having constructed inadequate knowledge in Grade 9. . This section reviews literature relevant to the research question about alternative conceptions shown by NC(V) Level 2 students when solving previous grade exams.

2.7. Approaches to addressing low achievement in Mathematics Level 2

This section presents recommendations from the literature on ways to address low achievement in Mathematics Level 2.

2.7.1 Recommendation from the literature on how to improve competence of NC(V) Level 2 Mathematics students on functions

Sehole (2020) concluded that the alternative conceptions displayed by NC(V) Level 2 Mathematics students might indicate that they did not understand the question and recommends that teachers look for such errors and use them to reflect on the question and possibly rephrase it. The researcher further concluded that Lecturer in this study shown a tendency not to pay attention to the students' incorrect responses seemed to have cemented students' misconceptions in relation to functions. Sehole (2020) recommended Teachers have to watch out for students' incorrect or alternative answers which results in incorrect knowledge being rooted in the students' cognitions (Sehole, 2020).

Dolley (2015) noted that the NCV mathematics curriculum is designed for grade eleven and twelve levels, which does not align with the admission policy that allows grade nine students to enrol. Lecturers cannot bridge this knowledge gap in the first year of the course. Dolley (2015) therefore recommends that students entering from grade nine or those lacking prior mathematics experience should take a bridging course to address deficiencies in their mathematical knowledge. Dolley (2015) mentioned that NC(V) Level 2 mathematics students lack positive attitude towards the subject therefore the recommendation that lecturers and management should commit in creating a culture that encourages positive motivation and attitude towards mathematics across the campus was mentioned. The findings revealed an evident shortage of adequate physical resources necessary to support effective teaching and learning for students (Dolley 2015). It is recommended that management invest in upgrading this facility into a modern, state-of-the-art, and student-friendly resource centre that allows students unlimited access (Dolley 2015). Additionally, students require access to physical resources, such as actual 3D shapes, to support effective teaching and learning of Mathematics Level 2 (Dolley, 2015).

Ngoveni (2018) identified students' lack of practice and lecturers' teaching strategies as key factors in poor mathematics NC(V) Level 2 performance. Ngoveni (2018) recommends that students practice mathematics on a daily basis in order to pass it. Ngoveni (2018) also

suggested that mathematical workshops be provided to capacitate lecturers to teach mathematics Level 2 content knowledge, and that TVET colleges should hold subject meetings on a regular basis across the province to allow lecturers to share their teaching strategies, and empower each other.

Muhrman (2022) revealed that many vocational students lack the motivation to learn mathematics and an important reason for this is that they do not see any relevance to the subject because they have difficulty linking it to their future profession. Muhrman (2016) recommended one way to increase vocational students' achievement in mathematics can therefore be to integrate mathematics with students' vocational subjects, this approach can give students a deeper understanding when they can see how subjects are connected and also Lecturers should create a connection between the material being taught and the students' daily lives or other subjects. The literature discussed in this section presented strategies for enhancing mathematics at NC(V) Level 2, which correspond to a key research question focused on understanding students' perspectives about improving mathematics at this level.

2.8. Conceptual framework

This study used concepts from Piaget's (1896) theory of constructivism and Movshovitz-Hadar et al.'s (1987) error analysis theory as a theoretical framework. Constructivism is a learning theory that describes how individuals build their understanding and knowledge of the world through experiences and reflection, rather than simply receiving information. According to Bruner (1990), constructivism views learners as active participants who develop understanding and knowledge by engaging with theories and reflecting on their experiences. When learners experience new theories, they reconcile with pre-existing knowledge and adapt or exclude if learners deem it irrelevant. Constructivism posits that students arrive with pre-existing knowledge to a mathematics class, what is delivered in a lesson, and the ability for students to acquire new knowledge is what the teachers need to be cognizant of when delivering the lesson. A knowledgeable teacher is required to mediate students' construction knowledge (Ay, 2017). Educators are encouraged to develop lessons that engage students in activities which critically examine and challenge their pre-existing understandings (Ay, 2017). Alternative conceptions may occur during a lesson as students incorrectly generalize prior knowledge to tackle new tasks (Ay, 2017).

Alternative conception is an occurrence that learners experience in mathematics in the quest for new knowledge not incorporated with the current ideas the learners have with an existing knowledge as alluded by Ay (2017) in their study of teaching and learning. Furthermore, Kieran (2014) considers alternative conceptions and scientific conceptions as a combination require alignment of learner's ideas about certain phenomena. Alternative conceptions experienced by students makes it difficult for them to understand if they cannot link the new ideas with existing knowledge (McDonald, 2010). Makonye and Fakude (2016) suggest that alternative conceptions are experienced by learners and are directly linked to conceptual knowledge that is built on incorrect ideas. Alternative conceptions manifest through errors (Makonye, 2016).

Luneta and Makonye (2010) further define errors as the inaccuracies that students make that enable deviations, unnecessary mistakes, minor slip-ups and blunders. Mathaba (2019) states that errors occur in the process of solving a mathematical problem algorithmically, procedurally, or by any other method. Errors are caused by carelessness, subjectivity, misapplication of rules and inaccurate calculation in solving mathematical problems (Tong & Phu-loc, 2017). Matuku (2017) argues that students fail to recall the correct steps from their long-term memory and, as a result, apply the wrong scheme in problem-solving situations resulting in a mathematical error. Based on the above interpretation by researchers, this may suggest that students in NC(V) Level 2 have some difficulties in learning mathematics, and this may contribute to low achievement in mathematics.

This study used the five categories of error defined by Movshovitz-Hadar et al. (1987) to analyse errors made by learners. They include misused data, misinterpreted language, logically invalid inference, distorted theorem. and technical errors. Misused data refers to errors that relate to some discrepancy between the data as given in a question and how the students treat them. Misinterpreted language includes mathematical errors that involve an incorrect translation of mathematical facts described in one possibly symbolic language to another possibly symbolic language. Logically invalid inference refers to errors that involve faulty reasoning rather than specific content; that is, new information invalidly drawn from a given piece of information or from a previously inferred one. Distorted theorem or definition refers to errors that result from the distortion of a specific and identifiable principle, rule, theorem or definition. Technical errors include computational errors (e.g., 7 x 8 = 54), errors extracting data from tables, and errors manipulating algebraic expressions.

This classification of errors by Movshovitz-Hadar et al. (1987) helped the researcher to identify students' weaknesses and lack of mathematical understanding in this study. Errors identified by the researcher could improve the achievement of NC(V) Level 2 Mathematics students as lecturers may attempt to correct them during their teaching and learning.

2.9. Conclusion

The minimum requirement for enrolment for NC(V) level 2 Mathematics is passing Grade 9 Mathematics. The Grade 9 curriculum includes fundamental topics such as numbers, operations, relationships; patterns, functions, algebra; space and shape (geometry); measurement; and data handling DBE (2011). The stimulation of students' pre-existing knowledge from Grade 9 can influence their ability to learn positively or negatively and the linking of pre-existing knowledge and new knowledge can influence the learning abilities and outcomes of students in Level 2. The knowledge students acquire in Grade 9 has a direct relationship to their achievement in NC(V) Level 2 Mathematics.

The achievement of NC(V) Level 2 Mathematics students is proving to be a challenge due to the gaps on pre-existing knowledge. Pre-existing knowledge serves as a bridge that can assist students in linking the existing knowledge and new knowledge. It is evident that Grade 9 knowledge acquisition plays a significant role in preparing students for success in the NC(V) Level 2 mathematics program. Ngoveni (2018), Sehole (2020), and Dolley,(2015), who conducted studies on the NC(V) Level 2, found that that NC(V) Level 2 mathematics students struggled with most of the content areas. These content areas serve as the building blocks for acquiring specific skills necessary to excel in NC(V) Level 2 mathematics.

The overlap in topics covered between Grade 9 and NC(V) Level 2 enables students to reinforce their understanding of various mathematical concepts. This reinforcement empowers them to approach more advanced topics with confidence and ease. Consequently, students enrolled in NC(V) Level 2 have a solid foundation upon which they can build, allowing for a smoother transition and greater potential for achieving success at this level.

The stronger the base of pre-existing knowledge from Grade 9, the stronger its influence on achievement in NC(V) Level 2 mathematics. NC(V) level 2 lecturers require awareness of students' pre-existing knowledge to identify these difficulties to enable them to strategize and structure their lessons to re-teach these concepts again. Sehole (2020) and Ngoveni (2018)

noted that topics that students have problems with in Grade 9 are still problematic at NC(V) Level 2 Mathematics. They will be expected to assess the level of pre-existing knowledge of the students entering TVET colleges and address it before they can build new knowledge. The content of functions and algebra at Level 2 represents 25% of the marks on the final examination; the content of space, shapes, and measurement weighs 30%; and data handling weighs 30%. If the foundation for these topics laid in Grade 9 is poor, as the literature reveals, this might influence the achievement of NC(V) Level 2 Mathematics students negatively.

Chapter 3 (Methodology) explores the methods used to conduct the study, including the research design, sample selection, data collection instruments, and methods of data collection and analysis including the ethical aspects of the study are also considered.

CHAPTER 3: METHODOLOGY

3.1. Introduction

The previous chapter reviewed literature on difficulties experienced by Grade 9 and NC(V) Level 2 Mathematics students. This chapter describes the research methodology used in this study. The chapter discusses the research paradigm, research design and approach, sampling, data collection instruments, data analysis, validity and reliability of the study, as well as ethical considerations.

3.2. Research paradigm

According to Burns and Grove (2001), a research paradigm provides a frame of reference for seeing and making sense of research. Pretorius (2024) defines a paradigm as the set of common beliefs and agreements shared by scientists about how problems should be understood and addressed and how a researcher can make meaning of discoveries and enquire about these beliefs. The importance of choosing a paradigm for a research study is to establish the foundation for the study's research and methodologies (Cresswell, 2014).

Pragmatism was used as the paradigm in this study. Pragmatism is a flexible research paradigm that chooses methods based on what best answers the research question (Pretorious, 2024). It does not commit to one reality or method, supporting both qualitative and quantitative approaches (Pretorious, 2024). Pragmatism blends subjective and objective perspectives, recognizing the influence of researchers' and participants' values but also valuing objective evidence (Pretorius, 2024). Pretorius (2024) mentions that this approach focuses on practical outcomes and solving real-world problems. Dewey (1998) states that reality is shaped by human perception and is therefore subjective. Dewey (1998) also argues that knowledge is constructed to better manage our presence and participation in the world. The role of knowledge is thus to bring about change.

Pragmatists generally agree that all knowledge in this world is socially constructed, but some versions of those social constructions are more consistent with individual experience than others (Morgan, 2014). Pragmatism was appropriate for this study because it enabled the integration of quantitative and qualitative methods to explore students' pre-existing mathematics knowledge, identify patterns of errors, and understand the reasoning underlying those errors.

I chose pragmatism because it allowed me to employ a diagnostic test to assess students' pre-existing knowledge quantitatively. It enabled the qualitative analysis of alternative conceptions displayed by students in each content area examined. Additionally, unstructured interviews were conducted and analysed qualitatively to understand the underlying reasons behind students' solutions. In terms of ontology, pragmatism assumes that reality is both singular and multiple depending on the context of inquiry. In this study, students' mathematical knowledge was understood through both objective evidence obtained from the diagnostic test and subjective explanations obtained from interviews. Epistemologically, pragmatism recognises that knowledge can be generated through both quantitative and qualitative approaches. Axiologically, the study acknowledged the values and experiences of both the researcher and participants in understanding students' pre-existing mathematical knowledge. Methodologically, pragmatism supported the use of an explanatory sequential mixed-methods design to explore students' mathematical knowledge using both quantitative and qualitative data.

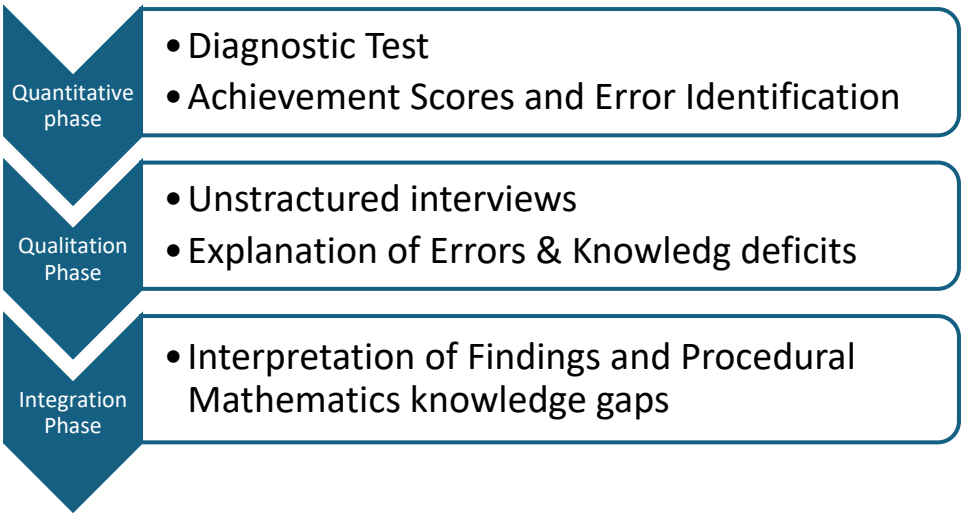
3.3. Research design and approach

This study adopted an explanatory sequential mixed methods design to gather data. The design was appropriate because the quantitative phase identified students' levels of mathematical knowledge and patterns of errors, while the qualitative phase provided deeper insight into the thinking processes and experiences underlying those errors. The study integrated the two datasets through triangulation and convergence of findings. Quantitative findings from the diagnostic test identified patterns in students' mathematical knowledge and achievement, while qualitative interview findings were used to elaborate and explain the conceptual and procedural difficulties reflected in students' responses.

According to Cresswell et al. (2011), this design involves first collecting quantitative data and then collecting qualitative data to elaborate on the quantitative results. This approach uses both quantitative and qualitative data with their analysis enhancing and explaining the statistical results by exploring participants' views in more depth (Cresswell, 2014). The use of different data collection and analysis methods to study the same phenomenon enriches and enhances research results (Molina-Azorin, 2016).

In this study, quantitative data was collected first using a diagnostic test. Participants' test scripts were evaluated and ranked based on their performance, according to the categories

established by DHET (2007): Not achieved (0-29%), Elementary (30-39%), Moderate (40-49%), Adequate (50-59%), Substantial (60-69%), Meritorious (70-79%), and Outstanding Achievement (80-100%), to provide an indication of students' foundational mathematics knowledge based on their performance in the diagnostic test. The same scripts were analysed qualitatively to identify alternative conceptions and errors displayed by students. Then, unstructured interviews were conducted to explore participants' thinking processes used to complete the diagnostic test, to better understand the errors and alternative conceptions displayed in their scripts.



3.4. Sampling

Mbewe (2013) describes a sample as a group chosen from the target population. Du Plooy-Cilliers et al. (2014) define it as accessible participants within a population. Selecting the best sampling method was essential for obtaining useful data.

3.4.1. Population

The target population of this study was the 350 NC(V) Level 2 Engineering students registered at a TVET college in Ekurhuleni District Municipality. This TVET college was selected because it was easily accessible to the researcher, saving both time and effort. This population was chosen because previous studies reported poor performance in NC(V) Level 2 Mathematics (Ngoveni, 2018) and (Dolley 2015). As students enter the college at Level 2, their pre-existing knowledge needs to be assessed so that they can be supported as early as possible (Ngoveni, 2018). The achievement of students in NC(V) Level 2 might influence the throughput rate for Level 4.

3.4.2. Sample

Convenience sampling was used to select 150 NC(V) Level 2 Engineering students from the population of 350 registered engineering students. The sample of 150 students was formed by taking one class of 30 students from each engineering department: civil engineering, electrical infrastructure, information technology, Automotive and Engineering related design. A convenience sample of 150 was chosen because participants from every department were willing to participate in the study

From the sample of 150 students, a purposive sample of 21 participants was selected for unstructured interviews. Participants were selected across different achievement categories to ensure representation of varying levels of mathematical performance and to obtain diverse explanations of students' mathematical reasoning. Purposive sampling involves the selection of certain units or cases based on a specific purpose, rather than random selection (Tashakkori & Teddie, 2003). According to Maxwell (1996), purposive sampling involves deliberate selection to obtain key data that could not be obtained using other methods.

Table 2 shows the original design for selection of participants for the unstructured interviews: 3 participants would be selected from each grading group, once the scripts were graded and ranked. However, following the marking of the scripts, there were insufficient students whose tests fell in the substantial, meritorious, and outstanding achievement categories for purposeful sampling to be feasible. Therefore, adjustments were made to grading groups 1-4 to accommodate the remaining students, as shown in the 'revised' column in Table 2.

Table 2 - Purposive sampling of participants for unstructured interviews

Grading group	Percentage	Description	Purposive sample (no. of participants) Original	Purposive sample (no. of participants) Revised
1	0%-29%	Not achieved	3	3
2	30%-39%	Elementary achievement	3	4
3	40%-49%	Moderate achievement	3	7
4	50%-59%	Adequate achievement	3	5
5	60%-69%	Substantial achievement	3	1
6	70% - 79%	Meritorious achievement	3	1
7	80%- 100%	Outstanding achievement	3	0

3.5. Collection of data

Data was collected using a diagnostic test instrument and an unstructured interview instrument. A diagnostic test (Appendix F) was written by all 150 NC(V) engineering students. The scripts

were coded S1 to S150 and were analysed for errors and alternative conceptions. Campbell et al. (2015) state that a diagnostic test allows examiners to determine a student's individual strengths, weaknesses, knowledge, and skills before the teaching of new knowledge. A diagnostic test also allows researchers to determine learning difficulties or gaps in students' understanding and identify students' strengths and weaknesses while measuring the level of mathematics knowledge of new entrants to a TVET college. In this study, data was collected before students began the Level 2 teaching and learning programme, but after they had been allocated to their engineering streams.

3.5.1 Data collection instruments

As the minimum requirement to enrol for NC(V) Level 2 is a Grade 9 pass, I used the Gauteng Department of Education November 2019 Grade 9 standardised examination paper as a diagnostic test to assess students' knowledge of various mathematical concepts acquired in previous grades. The diagnostic test was also used to explore the alternative conceptions and errors displayed by students when answering mathematics questions and provided insights into areas where a lack of pre-existing knowledge was identified.

This diagnostic test consisted of nine questions.

Question 1 was a multiple-choice question that tested different concepts of the grade nine curriculum. Sub-question (1.1) tested the number system content, (1.2) the concept of ratios, (1.3) their knowledge of algebra, (1.4) content area of financial mathematics, (1.5) knowledge of patterns, (1.6) knowledge in probabilities, (1.7) the knowledge of geometric shapes, (1.8) ratios, (1.9) tested students' knowledge of the concept of straight line geometry, and (1.10) their knowledge of geometric shapes.

Question 2 tested students' knowledge of simplifying the algebraic expressions and numbers: Sub-question (2.1) tested students' ability to simplify ratios, (2.2) evaluation of expression, sub-question (2.3.1) to simplify the square root and cube root, and (2.3.2) to expand the expression and simplification by addition and subtraction of like terms.

Question 3 tested students on the factorization of algebraic expressions. Sub-question (3.1) dealt with taking out the common factor, (3.2) the difference of two squares, (3.3) quadratic trinomial, and (3.4) grouping of terms.

Question 4 tested students' knowledge of solving x using (4.1) linear equations, (4.2) exponential equations, and (4.3) quadratic equations.

Question 5 tested students' knowledge of word problems by translating them into algebraic equations and solving for an unknown. Sub-question (5.1) was a word problem conversion to an equation, (5.2) testing students on financial math, and (5.3) and time and distance content area.

Question 6 and sub-questions (6.1) and (6.2) tested students' knowledge of straight-line graphs and interpretation of graphs.

Question 7 tested students' knowledge and understanding of data handling and probabilities. Sub-questions focused on: (7.1.1) on data handling (stem-and-leaf plot), (7.1.2) on calculating the range, and (7.1.3) on data handling and the interpretation of data. Sub-question (7.2) and sub-question (7.2.1), (7.2.2), and (7.2.3) tested students' knowledge of the concept of probabilities.
Question 8 tested students' knowledge of geometry. They were expected to calculate for (8.1.1) and (8.1.2) the values of angle with reasoning and (8.1.3) the interpretation of a triangle. Sub-question 8.2 tested students' knowledge of congruency and (8.3) similarities of two triangles, while sub-question (8.4.1) tested the knowledge of students on using a mathematical set to construct a triangle and (8.4.2) tested its interpretation of a triangle. Finally, sub-question (8.5) tested the concept of translation.

Question 9 tested students' knowledge of geometric shapes. Sub-question (9.1) tested them on the theorem of Pythagoras. The other sub-questions focused on determining (9.2.1) the area of a rectangle, (9.2.2) the area of a triangle, (9.2.3) the area of a parallelogram, and (9.2.4) the area of a trapezium. Sub-question (9.3) tested the abilities of students to calculate the total surface area of a cylinder.

Unstructured interviews were also used (See Appendix G). Unstructured interview questions are not prearranged (Chauhan, 2022). The purpose an unstructured interview instrument is to support the researcher in developing a real sense of a student's understanding of a situation.

The researcher conducted unstructured interviews with participants selected on the basis of having displayed alternative conceptions and errors in their solutions, to explore their thinking when solving problems. This instrument provided an understanding of the student's experiences and knowledge in mathematics up to Grade 9.

3.5.2 Data collection process

On the first day of the academic year, the researcher visited the TVET college to explain the aim of the study to the 150 selected students and give them the consent form to sign. The data collection process took place during the first week of the first academic term before formal Level 2 mathematics instruction had commenced. The diagnostic test was administered to the 150 participants who signed the consent forms. The students arrived at the college and started in their registered class before all 150 students were assisted by their mathematics lecturers and a researcher at the college hall where the test was written. The assistance by the college and their lecturers was pre-arranged with college staff and management to ensure smooth transition as per the ethics requirements. The total of 150 scripts were marked and analysed for alternative conceptions and errors. Based on this analysis, 21 students were purposively selected for unstructured interviews and the researcher arranged with the college to interview these students. The researcher was provided with an office at the TVET college to conduct the interviews. Student responses were recorded using Microsoft Teams.

3.6. Data analysis

Data analysis involves interpreting data to answer the main research question (Bowers, 2021). The goal is to understand and explain the collected information. This section details the methods used to analyse both quantitative and qualitative data.

3.6.1 Quantitative analysis

The data that was collected using a diagnostic test was analysed using students' achievement (test scores) to calculate the mean, variance, and standard deviation. Descriptive statistics were considered appropriate because the quantitative phase aimed to describe students' levels of pre-existing mathematics knowledge rather than establish causal or predictive relationships. Their performance was then categorized using the achievement levels determined by DHET (2007): Not achieved (0%-29%); Elementary achievement (30%-39%); Moderate achievement (40%-49%); Adequate achievement (50%- 59%); Substantial achievement (60%-69%); Meritorious achievement (70%-79%) and Outstanding achievement (80%-100%). Students who attained

30% and above were classified as ‘achieved’ and students who scored less than 30% were classified as ‘not achieved’.

3.6.2 Qualitative analysis

Movshovitz et al.’s (1987) model of errors and alternative conceptions was used to analyse the alternative conceptions and errors identified in participants’ responses. The model uses five categories: incomplete answers, misused data, technical errors, logical invalid inference and distorted definitions. Table 3 presents the analytical framework used to code the alternative conceptions and errors displayed by participants.

Table 3 - Coding of alternative conceptions & errors using Movshovitz et al.’s (1987) model

Error type	Criterion	Code
Incomplete Answer	Answered portions of the question but did not provide a full solution or conclusion as required by the question. Part of the question was answered correctly; the rest was incorrect.	1
Misused Data	Concluded from the data given in the item inappropriately. Students added or ignored some data and showed no signs of understanding the question.	2
Technical Error	Computational error, error manipulating elementary algebraic symbols, careless error, error using processes and skills usually mastered in a prerequisite course	3
Logically invalid inference	Made a mistake following a procedure or using a skill that is usually mastered at an earlier point in the same course. Error originating from Alternative conception of previously learned material	4
Distorted Definition	Altered a definition that was relevant to the solution of the problem.	5

Participants’ responses to each question and sub-question were examined and coded according to the dominant error pattern evident in the solution process using the categories proposed by Movshovitz-Hadar et al. (1987). This allowed the researcher to see each error and decipher the alternative conception displayed by the participant.

The unstructured interview responses were analysed using thematic analysis, where codes, categories, and themes were formed. The use of error analysis aligned with the constructivist framework underpinning this study. Constructivism provided a lens for understanding how students’ prior cognitive structures influenced their interpretation of mathematical concepts, while error analysis enabled the systematic identification of misconceptions and procedural difficulties reflected in students’ responses.

3.7. Quality measures

This section describes how the research ensured the validity and reliability of the quantitative data collection instruments and analysis methods and reliability, credibility, authenticity, and dependability of the qualitative methods.

Validity refers to how well the equipment used measures what it is intended to measure. A test is considered valid if it satisfactorily performs its intended function (Mbewe, 2013). In this study, I used the 2019 November Grade 9 examination paper which had been validated by the Gauteng Department of Education. Content validity was ensured through the use of a standardised Grade 9 examination paper developed and validated by the Gauteng Department of Education.

Reliability is the degree to which an instrument produces stable and consistent results (Cannon & Newble, 2002). The Grade 9 examination paper was considered to be reliable as it had been administered previously to Grade 9 scholars in the province of Gauteng. The interview process was structured to ensure consistency, clarity, and relevance of the questions to the study objectives. Reliability was strengthened through the consistent administration of the diagnostic test to all participants under similar conditions.

Credibility means that qualitative data is interpreted accurately (Self, 2014). I ensured the credibility of the interview data collected in this study by providing interviewees with copies of their translated recordings to ensure that their views were correctly interpreted.

Authenticity refers to ensuring that all voices of all participants are heard (Cresswell & Poth, 2016). I also ensured that the diagnostic test was authentic in terms of accommodating students' diverse cognitive skills by selecting a paper set by the Department of Education of 2019, which is committed to being diverse and inclusive. The administration of the diagnostic test was supported by mathematics lecturers to ensure standardized invigilation conditions for all participants.

Dependability is the utilization of different instruments to collect and acquire data from participants (Kvale, 2007). Dependability was enhanced through methodological triangulation by combining diagnostic test data with interview responses to obtain a more comprehensive understanding of students' mathematical knowledge and experiences.

3.8. Ethical considerations

I applied for ethical clearance with my supervisor, following the institution's research ethics policy. The approved ethics clearance certificate (Ref: 2023/10/11/60042672/03/AM) (see Appendix A) was presented to the principal of the college when requesting permission/authorization to collect data in the engineering department. All the details of the study, the aim, the period, and the significance were detailed in the permission letter I wrote to the Principal. Students were asked via the participation sheet (Appendix C) about their voluntary participation in the study. The participants were provided with consent forms (Appendices D & E) which they completed and signed before participating in the writing of the diagnostic test. The first consent form (Appendix D) was signed by students who were 18 years old or older, and the second form (Appendix E) was signed by parents of student who were under 18 years of age. Participants were assured that their test scores would only be used for the purpose of this study and would not be shared with anyone other than the researcher and the supervisor. Participants were also assured of confidentiality and anonymity, and each was designated a number for the recording of their results. Participants were advised that they had the right to withdraw from the study at any time without any prejudice.

3.9. Conclusion

This chapter has presented the methodology and research design of the study. The sample and sampling procedures were discussed. The chapter outlined the mixed methods procedures used to investigate students' pre-existing mathematics knowledge, alternative conceptions, and mathematical experiences. The validity and reliability of the instrument and various ethical issues were also highlighted and discussed. In the next chapter, the researcher discusses the analysis of the data and the findings of the study.

CHAPTER 4: DATA ANALYSIS

4.1 Introduction

This chapter presents data obtained from diagnostic tests and unstructured interviews. The discussion is arranged according to the themes identified after the data were processed through coding, categorization, and thematic analysis. The first section outlines students' responses and the alternative conceptions they demonstrated, followed by information from unstructured interviews. Results related to each research objective are analysed in this section.

4.2 Students' achievement levels

To determine the students' achievement levels, their test scores were recorded on a spreadsheet. The researcher calculated percentages for each achievement category (see Table 4), as well as mean, variance, and standard deviation (see Table 5) to assess the mathematics competence of students entering the TVET college.

Table 4 presents students' achievement (test scores) using the grading categories established by DHET (2007).

Table 4 - Student grading categories and achievement

Grading category	Achievement	Description	Number of students	Percentage of students
1	0%-29%	**Not achieved**	123	82%
2	30%-39%	Elementary achievement	13	8%
3	40%-49%	Moderate achievement	7	5%
4	50%-59%	Adequate achievement	5	3%
5	60%-69%	Substantial achievement	1	1%
6	70% - 79%	Meritorious achievement	1	1%
7	80% -100%	Outstanding achievement	0	0
			150	100%

Table 5 indicates that 27 (18%) students achieved 30% or higher while 123 (82%) students did not achieve the pass mark as per the guideline stipulated by the Department of Higher Education and Training (2007). The distribution of test scores suggests a likelihood of poor performance in NC(V) Level 2 overall. Further, the data in the table highlights inadequacies in the base knowledge required for progression to the next level, as stipulated by DHET (2007) guidelines.

Table 5 – Mean and standard deviation of number of students

Measurements of central tendency	No. of students achieved pass mark	No. of student not achieved pass mark	Total
Number of students	27	123	150
Percentage difference	18%	82%	100%
Mean	-		16
Variance	-		244.79093
Standard deviation	-		15.645796

Table 5 presents a calculated mean score of 16, variance of 244.79, and standard deviation of 15.65. The mean score of 16 indicates that on average, the students scored 16 marks on the diagnostic test. Variance is a measure of the average squared deviation from the mean; in this case, a variance of 244.79 indicates the wide spread of the scores. Standard deviation is calculated by taking the square root of the variance which, in this case, is ±15.65. The standard deviation reflects the average deviation of the scores from the mean, meaning the majority of students scored within ±15.65 marks of the mean, or between 0 and 31.65 marks. The results indicate that the majority of the NC(V) Level 2 students (82%) were categorized at Level 1 (not achieved).

4.3 Analysis of students' difficulties, errors and misconceptions

Students' responses were categorized as correct, partially correct, incorrect, or blank. As outlined in Chapter 3, the analysis draws on the error classification framework adapted from Movshovitz-Hadar et al. (1987), with an additional category of incomplete answer errors introduced to reflect patterns emerging in this dataset.

4.3.1 Analysis of students' difficulties, errors, and alternative conceptions (Question 1)

Question 1 was a multiple-choice item that assessed students' pre-existing knowledge across a range of Grade 9 mathematics topics, including scientific notation, percentage increase, algebra, financial mathematics, number patterns, probability, geometry, ratios, and perimeter. It functioned as a broad diagnostic task to gauge students' readiness for subsequent items in the assessment.

Table 6 summarizes student performance across the ten sub-questions of Question 1. The results show that the average percentage of correct responses was only 28%, with 71% of responses being incorrect and 1% of questions left unanswered. These figures reflect

widespread difficulty engaging with the foundational mathematical concepts that students are expected to carry into NC(V) Level 2.

Table 6 - Student responses for Question 1

Question	Topic	Correct answer (%)	Incorrect answer (%)	Partially correct (%)	No answer (%)
1.1	Scientific notation	61%	39%	0%	0%
1.2	Percentage increase	31%	69%	0%	0%
1.3	Algebra	33%	66%	0%	1%
1.4	Financial maths	25%	74%	0%	1%
1.5	Patterns	47%	53%	0%	0%
1.6	probability	9%	90%	0%	1%
1.7	Geometric shapes	18%	81%	0%	1%
1.8	Ratios	12%	87%	0%	1%
1.9	Geometry	29%	71%	0%	0%
1.10	Perimeter	19%	81%	0%	0%
	Average	**28%**	**71%**	**0%**	**1%**

Table 6 shows that across the 10 sub-questions, students' performance indicates that an average of 28% achieved the correct answer and an average of 71% did not achieve the correct answer for the question. For the individual question responses, students were strongest on sub-question 1.1 (scientific notation), with 61% of students answering correctly. However, substantial weaknesses were observed in other areas. For example, only 9% of students answered the probability item (1.6) correctly, while just 12% correctly solved the ratio question (1.8). Similarly, sub-questions involving geometry (1.7 and 1.9) and perimeter (1.10) showed that over 70% of students were unable to apply their prior knowledge to basic spatial reasoning tasks.

Question 1 gives a general indication of students' level of prior knowledge; the findings suggest that these students enter Level 2 with significant conceptual and procedural gaps, especially in topics involving abstract reasoning, measurement, and proportional relationships. The low scores across the majority of the sub-questions highlight the need for further exploration into the types of errors and misconceptions that students carry with them from earlier grades. These initial findings suggest that students may struggle not only with specific concepts but also with applying known procedures under assessment conditions. In subsequent sections of question 1, the analysis delves deeper into the types of difficulties observed through open-ended responses and interviews, providing further insight into both the conceptual misunderstandings and procedural breakdowns that may underline these results.

4.3.2 Analysis of students' difficulties, alternative conception, and errors (Question 2.1)

Question 2.1 required students to simplify decimal ratios by converting the decimals to fractions and reducing the resulting ratio to its simplest form using common factors:

2.1. ***Write*** 0.36 : 0,84 *in the simplest form.*

Sub-question 2.1 [Number Systems]

Table 7 shows the performance of 150 students on sub-question 2.1. Only 8 students (5%) answered correctly, while 7 students (5%) provided partially correct answers. A large number—106 students (71%)—answered incorrectly, and 29 students (19%) did not attempt the question. Among 113 (75%) students who displayed errors, 106 students made misused data errors and 7 students displayed incomplete answer errors.

Table 7 – Ratios: Types and frequencies of errors [Question 2]

Response	**Errors**					**Frequencies and percentages**
	Misused data	**Logically invalid inference**	**Distorted theorem or definition**	**Technical Error**	**Incomplete answer**	
Correct answer	-	-	-	-	-	8 (5%)
Incorrect answer	106	0	0	0	0	106 (71%)
Partially correct					7	7 (5%)
No answer	-	-	-	-	-	29 (19%)
Total						**150 (100%)**

Figure 16 shows an extract of a student's solution demonstrating incomplete answer errors. The student correctly converted 0.36 and 0.84 to fractions but failed to simplify the resulting ratio. This suggests that the student may have thought the conversion step was the final requirement and did not recognise the need to reduce the ratio to its simplest form. The error indicates a lack of procedural knowledge, particularly in applying the final step of simplification through identifying using common factors to reduce ratios to their simplest form. It also points to a conceptual gap, as the student may not have fully understood that a ratio represent a multiplicative relationship that must expressed in its simplest form, similar to fractions,

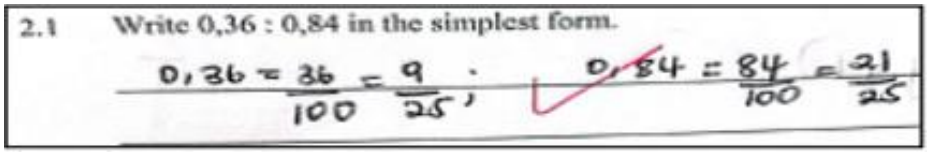


Figure 16 - Example of incomplete answer error for Question 2.1 (Participant S27)

Figure 17 shows an extract of a solution by a student who made a misused data error. Instead of simplifying the ratio, the student added the two decimal numbers and arrived at 1.2, which was then expressed as the ratio 1:2. This indicates a fundamental conceptual misunderstanding of what a ratio represents, specifically confusing a ratio as an additive relationship rather than a comparison between two quantities. Rather than recognizing a ratio as a comparison between two quantities, the student treated the task as a straightforward addition problem, followed by an arbitrary transformation of the sum into a ratio. This error reflects both a misinterpretation of the task and a procedural flaw in executing irrelevant operations.

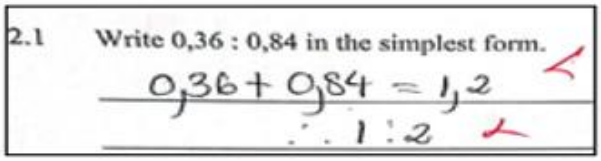


Figure 17 - Example of misused data error (Participant S48)

The researcher conducted unstructured interviews with the students whose responses are shown in Figures 16 and 17, as well as with others who answered the sub-question incorrectly. The purpose of the interviews was to explore their understanding and the reasoning behind the steps they followed when attempting to simplify decimal ratios. Of the 21 selected students, 3 answered this sub-question correctly and 18 answered incorrectly. The researcher interviewed 18 students who answered this question incorrectly. The table below presents the researchers' questions and the students' responses.

Table 8 - Unstructured interview questions: [Question 2.1]

Unstructured question (Sub-question 2.1)	Students' responses	codes	Categories	Themes
Can you explain why you solved this question this way?	S6: I did not know what the question meant by simplifying, Ma'am.	Terminology confusion (S6)	**Task misunderstanding**: ▪ Terminology confusion (S6) ▪ Misinterpreted task (S48) ▪ Topic misidentification (S23) ▪ Guessing due to uncertainty (S24)	Lack of conceptual understanding
	S22: I did simplification in high school but don't remember the steps to do when simplifying now.	Difficulty recalling prior concepts (S22)	**Memory Recall Challenges** ▪ Difficulty recalling prior concepts (S22) ▪ Memory lapse (S37).	
	S48: I saw the question is about ratios, but I thought I needed to add them together and get the answer	Misinterpreted task (S48)		
	S27: I understood that to simplify decimal ratios firstly I must change the decimal fraction to common fractions. Then I was stuck and couldn't remember the next step. But I was aware that my answer was incomplete.	Partial understanding (S27)	**Incomplete Procedural Knowledge** ▪ Partial understanding (S27) ▪ Partial recognition, lack of procedure knowledge (S16) ▪ Forgotten procedure, error in steps (S85) ▪ Unexpected outcome (S26) ▪ Calculator dependence without understanding (S70) ▪ Random calculation without understanding (S142)	Gaps in procedural knowledge
	S37: I did grade 9 six years ago. Hence, I don't remember the topics from grade 9. The	Memory lapse (S37).	**Misapplication of Prior Knowledge** ▪ Use of prior knowledge (fractions) (S11)	

	whole paper was difficult for me.		Misunderstood procedure (S33) ▪ Confusion due to unexpected result (S138)	
	S41: I started by finding a number that goes into 0,36 and 0.84, which was 0,12. Then I divided both decimals by 0,12 and I got 3:7. Then I thought: it is wrong; I need to keep my answers in decimals. Then I removed the 0 from 0.12. Then I divided by 12, which gave me 0.03:0.07 as the final answer.	Confusion in simplification steps (S41)	**Avoidance and Hesitation** ▪ Avoidance (S4) ▪ Confusion in simplification steps (S41) ▪ Avoidance due to low confidence Not understanding the question (S56)	
	S23: I did not understand the question—which topic was it. I thought its time, but I was confuse on how to change it if the question doesn't specify if we should change it to hours, minutes or second.	Topic misidentification(S23)		
	S24: I was not really sure what the question wants. Then I multiplied the numbers together and got $\frac{189}{625}$. Then changed that to 0.30	Guessing due to uncertainty (S24)		
	S16: Ma'am. because of this symbol : I knew this is ratio but I was not sure how to simplify the numbers. Then I divided 0.36 by 0.84 and got $\frac{3}{7}$. Then I did not know how to change that back to ratio form.	Partial recognition, lack of procedure knowledge (S16)		
	S142: I was blank in this question, Ma'am. Didn't know what to do. I just pressed the decimal in my	Random calculation without understanding (S142)		

	calculator and write the answers.			
	S11: I thought we need to convert the decimal numbers to fractions. That's what I did.	Use of prior knowledge (fractions) (S11)		
	S70: I used my calculator to change the decimal numbers to its simplest form.	Calculator dependence without understanding (S70)		
	S26: I changed the decimal numbers to fractions, I'm surprised why my answers are wrong.	**Unexpected outcome** (26)		
	S85: I converted the decimal ratios to fractions and then I divided the first fraction by the second fraction. But I see my mistakes now: when I was multiplying, I didn't use the inverse.	Forgotten procedure, error in steps (85)		
	S4: I did not know what to write Ma'am then I decided to leave the question blank	Avoidance (S4)		
	S33: Ma'am I thought the question is about to simplify the ratio by changing the decimal to fractions and simplify the fractions	Misunderstood procedure (S33)		
	S138: Why my answer is wrong, Ma'am? Because I changed the decimal ratios to fractions. What was needed in this question, Ma'am?	Confusion due to unexpected result (S138)		
	S56: I jumped the question, Ma'am. I didn't know what to do.	Avoidance due to low confidence (S56)		

4.3.2.1. Analysis of students' responses to Question 2.1 (Ratios)

The analysis of students' responses to Question 2.1 revealed significant challenges to students' ability to simplify ratios, which were evident in both the quantitative data and qualitative insights from interviews and detailed examination of students' test scripts. Of the 150 students assessed, only 8 (5%) provided the correct answer, while 7 (5%) solved the question partially correctly. A majority of 106 students (71%) answered incorrectly, and 29 (19%) left the question blank. These results suggest widespread difficulties in understanding and solving ratio problems, particularly in applying proportional reasoning and recognising ratios as multiplicative comparisons.

Tanudjaya and Doorman (2020) found that Grade 9 learners lacked the ability to demonstrate proportional reasoning using ratios. Their findings are in line with the findings of this study.

Qualitative analysis of students' test scripts highlighted two main types of errors: incomplete answer errors and misused data errors. Incomplete answer errors were observed in the responses of students like S27, who successfully converted decimals to fractions but failed to simplify the ratio further, indicating a lack of understanding of the full sequence of ratio simplification steps, particularly the identification of common factors and reduction to lowest terms. On the report released by TIMMS compiled by Reddy (2022), the findings were that the Grade 9 students possibly lacked the procedural knowledge to find the highest common factor, together with division from earlier grades. The TIMMS report confirms the findings of this report regarding the lack of procedural knowledge of these Grade 9 students.

Misused data errors were evident with students who treated ratios as additive relationships rather than multiplicative comparisons, indicating a misunderstanding of proportional reasoning. This aligns with a finding by Fauziah and Cahya (2020) that students made mistakes during addition, subtraction, multiplication, or division operations; they did not comply with the rules of the arithmetical operation order; they misapplied a commutative and associative property in calculating a division operation; or they misapplied an additive or multiplicative inverse or a cross-multiplication procedure.

These findings indicate that students' pre-existing knowledge of ratios is insufficient, particularly in proportional reasoning and simplification, which may hinder their ability to engage successfully with Level 2 mathematics

4.3.2.2. Interview findings: Students' difficulties with simplifying ratios (Question 2.1)

The interview responses provided valuable qualitative insights into the difficulties students encountered with simplifying ratios. The analysis identified two overarching themes: lack of conceptual understanding and gaps in procedural knowledge. Each theme was explored using categories that highlight distinct aspects of students' challenges with this mathematical concept.

4.3.2.2.1. Lack of conceptual understanding

This theme captures the difficulties students experienced with comprehending the foundational principles of ratios and interpreting the question. Several categories emerged within this theme

Task Misunderstanding: Several students misunderstood the nature of the problem, either due to confusing terminology, misidentifying the topic, or experiencing uncertainty that resulted in guessing. For instance, S6 said: *"I did not know what the question meant by simplifying*". S48 misinterpreted the task, explaining: "*I saw the question is about ratios, but I thought I needed to add them together and get the answer*". Similarly, S23 demonstrated topic misidentification, commenting: "*I thought it was about time, but I was confused about the units.*" S4 admitted to guessing due to uncertainty, saying: "*I was not really sure what the question wants. Then I multiplied the numbers together and got* $\frac{189}{625}$ *, then changed that to 0.30.*" Collectively, these responses indicate that significant barriers for students were not only limited conceptual knowledge of ratios as multiplicative relationships, but also difficulty interpreting task requirements.

Recall Challenges: Another notable issue was students' inability to recall prior knowledge from earlier educational experiences. For example, S22 explained*, "I did simplification in high school but don't remember the steps now.*" S37 said: "*I did grade 9 six years ago; hence, I don't remember the topics from grade 9. The whole paper was difficult for me*". These responses suggest that gaps in long-term retention compounded students' conceptual difficulties, limiting their ability to recall and apply previously learned procedures such as simplifying ratios and identifying common factors. Collectively, these responses highlight the notable barriers posed by a lack of conceptual understanding, which hindered students' ability to engage meaningfully with ratio problems.

4.3.2.2.2. Gaps in procedural knowledge

This theme concerns the difficulties students encountered applying the correct steps necessary to simplify ratios, even when some initial understanding was demonstrated. Three categories emerged under this theme.

Incomplete Procedural Knowledge: Some students demonstrated an incomplete understanding of the procedural steps required to simplify ratios. This was apparent in situations where learners started the process correctly but were unable to finish it or carried out steps without fully understanding their intent. For example, S27 remarked, *"I understood that to simplify decimal ratios firstly I must change the decimal fraction to common fractions. Then I was stuck and couldn't remember the next step, but I was aware that my answer was incomplete".* S16 also recognized part of the method, stating, *"I remembered the fractions part but not how to finish it".* Other students relied heavily on calculators to perform calculations without fully understanding the underlying principles. S70 reported: "*I used my calculator to change the decimal numbers to its simplest form".* S142 attempted an unknown calculation: *"I was blank in this question, Ma'am, didn't know what to do. I just pressed the decimal in my calculator and write the answers".* These responses indicate that while some procedural knowledge was partially present, but insufficient to correctly complete ratio simplification, particularly in executing sequential steps and applying division by common factors.

Misapplication of Prior Knowledge: Other students attempted to apply familiar strategies but did so incorrectly. For instance, S33 stated, "*I thought the question was about simplifying the ratio by changing decimals to fractions. That's what I did".* Similarly, S138 remarked, *"Why my answer is wrong, Ma'am, because I changed the decimal ratios to fractions? What was needed in this question, Ma'am?".* These errors highlight students' confusion regarding the appropriate methods needed to solve the problem and suggest that students relied on prior knowledge that was either incomplete or not relevant to the requirements of the task.

Avoidance and Hesitation: Some students exhibited avoidance, either skipping the question entirely or demonstrating a lack of confidence in their abilities. For instance, S4 admitted, *"I did not know what to write, Ma'am. Then I decided to leave the question blank"*, while S56 stated, *"I jumped the question, Ma'am. I didn't know what to do".* In addition, S41 expressed doubt and stated that they repeatedly second-guessed their answers: *"I started by finding a number that goes into 0,36 and 0.84, which was 0,12. Then I divided both decimals by 0,12 and I got 3:7. Then I thought: it is wrong, I need to keep my answers in decimals. Then I*

removed the 0 from 0.12. Then I divided by 12, which gave me 0.03:0.07 as the final answer". These responses suggest that procedural gaps, compounded by low confidence, notably affected students' willingness to engage with the question.

These findings obtained from the interviews reveal that procedural gaps, particularly in executing ratio simplification steps and applying proportional reasoning, combined with low confidence, significantly affected students' performance. Addressing these inter-related issues requires a dual focus on improving both conceptual clarity and procedural fluency to ensure that students are better equipped to engage with foundational mathematical concepts such as ratios.

4.3.3 Analysis of students' difficulties, alternative conceptions, and errors (Question 2.2)

Question 2.2 required students to substitute a given value for *x* into an algebraic expression and simplify the result using exponent rules and the correct order of operations.

$$\mathbf{2.2.}\ \boldsymbol{Calculate}\colon\ -4x^2\ if\ x=-\frac{1}{2}$$

Test item 1 - Sub-question (2.2.) Substitute x

Table 9 shows the performance of 150 students on this sub-question. A total of 55 students (37%) answered correctly, while 11 students (7%) gave partially correct answers, 55 students (37) answered incorrectly, and 29 students (19%) did not attempt the question. Among the 66 who either answered incorrectly or partially correctly, 44 students made misused data errors, and 22 students made technical errors.

Table 9 – Substitution: Types and frequencies of errors [Question 2.2]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	incomplete answer errors	
Correct answer	-	-	-	-	-	55 (37%)
Incorrect answer	44	0	0	11	0	55 (37%)
Partially correct	-	-	-	11	-	11 (7%)
No answer	-	-	-	-	-	29 (19%)
Total						**150 (100%)**

Of the 21 selected scripts, only 4 students answered the question incorrectly, the rest answered correctly. Figure 18 shows an extract of a student's solution showing a misused data error. The

student misinterpreted the expression $[-4x^2]\ as\ [4-x^2]$ during substitution, changing the structure of the expression. This suggests a misunderstanding of how to handle negative coefficients and exponents, particularly the order of operations when substituting values into algebraic expressions. Within the misinterpreted expression, $(4-x^2)$, the student also failed to square correctly before subtracting, reflecting a lack of procedural knowledge in applying the order of operations.

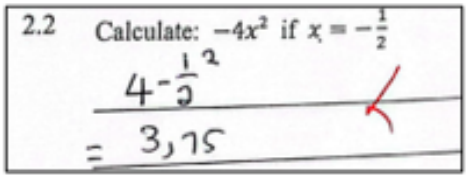


Figure 18 – Example of misused data (Participant S4)

Figure 19 shows an extract of a student's solution demonstrating a misused data error. The student substituted directly without squaring the variable term. This indicates confusion between linear and exponential terms, as the student overlooked the exponent altogether, reducing the quadratic expression to a basic subtraction problem. This error reflects a conceptual misunderstanding of the role of exponents, specifically failing to recognise that the exponent applies to the entire substituted value.

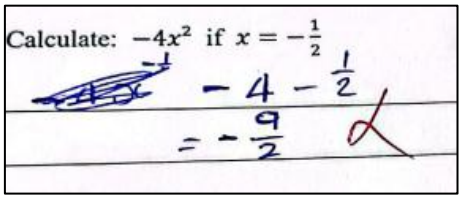


Figure 19 – Example of misused data (Participant S142)

Figure 20 shows an extract of a student's solution demonstrating technical errors. The student dropped the negative sign associated with (-½), changing it to a positive value during the substitution process. As a result, the expression was evaluated incorrectly. This suggests a misunderstanding of how negative rational numbers are handled during substitution, particularly in maintaining correct signs during multiplication and exponentiation.

The error reflects a procedural gap, as the student misapplied the sign during computation, indicating difficulty executing the substitution process accurately when negative fractions are involved.

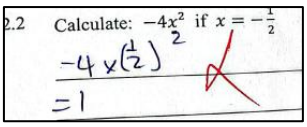


Figure 20 – Example of technical error (Participant S37)

Figure 21 shows an extract from a student's solution demonstrating a misused data error. The student substituted $x = -1/2$ but failed to square the value; instead, they multiplied the numerator by the exponent as 1 x 2 = 2, which led to the incorrect final answer of −4. This error reflects a procedural misunderstanding of how exponents apply during substitution.

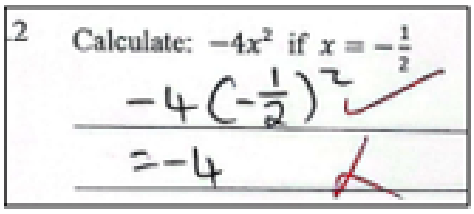


Figure 21 – Example of misused data error (Participant S16)

The researcher interviewed students whose incorrect solutions are presented in Figures 18 to 21, to explore their reasoning and level of understanding. The table 10 below presents the researchers' unstructured interview questions and the students' responses.

Table 10 - Unstructured questions: Question 2.2

Researcher's unstructured question: Can you explain why you solved this question this way?			
Student responses	Codes	Categories	Themes
<u>S4:</u> I thought since the value of x is negative, I was supposed to minus it from the 4 and write the answer.	▪ Misinterpretation of negative values ▪ Confusion about subtraction versus multiplication ▪ Incorrect operation applied to x	Misinterpretation of operations and expressions	Conceptual misunderstandings
<u>S16:</u> I understand substitution, Ma'am, but I thought the square outside the brackets only affects the numerator. Hence, I multiplied the numerator only by 2, and that gave me the answer of 1, and I multiplied it by -4.	▪ Misunderstanding exponent rules ▪ Partial application of substitution rules ▪ Misapplied multiplication	Errors in substitution process (s16)	Procedural challenges
<u>S33:</u> I punched everything on the calculator, and I got the	▪ Reliance on technology ▪ Lack of procedural understanding	Procedural or calculation challenges	

answer. It was easy for me like that; I did not know how to solve it step by step.	▪ Inability to verify results manually		
S37: I did know what to do here, but I made a mistake by omitting the sign of the given value. I just substituted and calculated.	▪ Sign omission error ▪ Incomplete substitution ▪ Misstep during calculation	Over-reliance on technology	

4.3.3.1.Analysis of students' responses to Question 2.2 (Substitution)

The analysis of students' responses to Question 2.2 revealed significant challenges in terms of their ability to simplify the expression, which were evident in both the quantitative data and qualitative insights from interviews and detailed examination of students' test scripts. Of the 150 students assessed, 55 (37%) answered correctly, 11 (7%) partially correctly, 55 (37%) incorrectly, and 29 (19%) left blanks. These results suggest widespread difficulties understanding and solving problems dealing with this concept; in the literature, studies consistently highlight the challenges students face with algebraic expression. The sample of 150 students in this study demonstrated that these Grade 9 students were unable to apply mathematical rules correctly, particularly the order of operations, exponent rules, and substitution procedures This aligns with the findings of a study by Luneta (2016). Similarly, Stemele and Asvat (2024), in a study titled '*Exploring learner errors and misconceptions in algebraic expressions with grade 9 learners through the use of algebra tiles', found* that learners could substitute numbers in algebraic expressions but could not correctly perform the calculations and manipulations to obtain the correct value. This study also found that these Grade 9 learners struggled to substitute negative numbers, as demonstrate in the answers and errors for Question 2.2.

Furthermore, qualitative analysis of students' test scripts highlighted two fundamental errors: namely, misused data error and technical errors. Misused data errors were observed in the answers of students like S142, who understood -4 and x^2 as separate terms and subtracted the given value of x from -4, ignoring the exponents of *x,* indicating a lack of understanding of substitution and evaluation, particularly how variables, coefficients, and exponents interact.
Technical errors were evident in the answers of student S16, who successfully substituted the value of *x* but was unable to square a fraction. Similarly, Moyo and Machaba (2020) found that

Grade 9 students lacked procedural and conceptual knowledge, possibly resulting in students being unable to multiply fractions without considering fractions as numbers by themselves. Students treated fractions as a function of addition or subtraction and proceeded to find the lowest common denominator (LCD) in order to multiply the fractions. Moyo and Machaba (2020) arrived at similar findings and concluded that students were unable to distinguish between fractions and function. These findings indicate that students' pre-existing knowledge of algebraic substitution is limited, particularly in applying exponent rules and the order of operations, which may negatively affect their performance in Level 2 mathematics.

4.3.3.2. Interview Findings: Students' difficulties evaluating a expression (Question 2.2)

The interview responses provided valuable qualitative insights into the difficulties students encountered when evaluating an expression. The analysis identified two key themes: conceptual misunderstandings and procedural challenges. Each theme is supported by categories that reflect distinct patterns in students' difficulties.

4.3.3.2.1. Conceptual misunderstanding of algebraic rules

This theme captures the difficulties students experienced working with the concept of substitution. All difficulties fell into one category: misinterpretation of operations and expression.

Misinterpretation of Operations and Expression: Students displayed a lack of understanding of mathematical operations in substitution, confusing subtraction with multiplication. For instance, S4 said: *"I thought since the value of x is negative, I was supposed to minus it from the 4 and write the answer".* This response indicates not only a lack of knowledge of substitution but also a misunderstanding of how many terms the expression contains and how they interact. The student treated substitution as a subtraction problem, ignoring the multiplication and exponentiation involved.

4.3.3.2.2. Procedural challenges

This theme reflects the difficulties students encountered when applying the correct steps to substitute the given value and simplify the expression. Even when students showed an initial understanding, many struggled to carry out the full procedure. Three distinct categories emerged under this theme.

Errors in substitution: Some students demonstrated a partial understanding of substitution but lacked the necessary knowledge of exponential laws. Student S16 explained: *"I understand*

substitution, Ma'am, but I thought the square outside the brackets only affects the numerator. Hence, I multiplied the numerator only by 2, and that gave me the answer of 1, and I multiplied it by -4". This response shows confusion about how squaring distributes over a fraction. The student understood substitution conceptually but misapplied exponentiation, leading to an incorrect final answer.

Procedural or calculation challenges: Student S33 showed correct understanding of the substitution process but relied heavily on the calculator, showing a lack of confidence with manual computation. They stated: *"I know substitution, but after substitution I punched everything on the calculator, and I got the answer. It was easy for me like that: I did not know how to solve it step by step".*

This response indicates that although the student understood the substitution step, they could not simplify the expression manually. This heavy reliance on the calculator, without understanding how each operation works, may affect the student's ability to solve mathematical problems that require working through procedures without technological assistance.

Negligence: Student S37 made careless errors despite knowing the concept. They reported: *"I did know what to do here, but I made a mistake by omitting the sign of the given value. I just substituted and calculated".* This student showed understanding of substitution but neglected to include the negative sign of the value. The mistake appears to have resulted from a lack of attention to detail or failure to check their work, rather than a conceptual error.

These findings reveal procedural gaps, compounded by a lack of confidence and understanding of the application of mathematical operations, that significantly hindered students' ability to solve the problems correctly. Addressing these inter-related issues requires a dual focus on improving both conceptual clarity and procedural confidence to ensure that students are better equipped to engage with foundational mathematical concepts such as ratios.

4.3.4. Analysis of students' difficulties, alternative conceptions, and errors (Question 2.3.1)

Question 2.3.1 required students to calculate square root and cube root, applying root operations and relevant exponential laws within the content area of number system.

2.3. ***Simplify***:

2.3.1. $\left(\sqrt{16}\right)^2 - \sqrt[3]{4^6}$

Test item 2 - Sub-question (2.3.1) Integers

Table 11 presents the performance of 150 students on sub-question 2.3.1. A total of 52 students (35%) answered correctly, 12 students (8%) gave partially correct answers, 76 students (51%) answered incorrectly, and 10 (6%) did not attempt to answer the question. Of the 88 students who answered incorrectly or partially correctly, 65 students made misused data errors and 23 made technical errors.

Table 11 - Student responses [Question – 2.3.1]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	52 (35%)
Incorrect answer	65	0	0	0	0	76 (51%)
Partially correct	-	-	-	23	-	12 (8%)
No answer	-	-	-	-	-	10 (6%)
Total						**150 (100%)**

Figure 22 shows an extract from a student's solution to this question that demonstrated data errors. The student incorrectly introduced the inverse of a cube root and manipulated the expression to arrive at $4^6=4096$, suggesting a misunderstanding of the properties of a cube root. The introduction of an exponent in this context indicates a fundamental misconception of how root operations function, particularly in distinguishing between square roots and cube roots. This response reflects a lack of procedural knowledge, specifically the incorrect application of exponent rules when attempting to eliminate cube roots. The error also suggests that the student may have overgeneralized strategies used for square roots, incorrectly assuming that square root–square cancellation rules apply in the same way to cube roots.

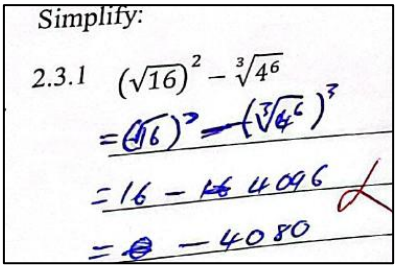


Figure 22 – Example of misused data error (Participant S11)

Figure 23 shows an extract from a student's solution demonstrating a technical error. The response demonstrates that the student understood the general concept of root operations and the inverse relationships between exponents and roots. The student attempted to apply exponential laws but made a computational error when dividing the exponents. Specifically, the student incorrectly wrote 3 as the new exponent for the second term—a mistake likely resulting from a momentary error in calculation rather than a conceptual misunderstanding. This error reflects a procedural slip, specifically an incorrect division of exponents when applying cube root operations.

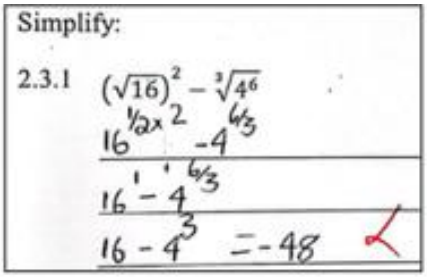


Figure 23 – Example of technical errors (Participant S27)

The researcher conducted interviews with the students whose responses are shown in Figure 22 and Figure 23, as well as with other students who answered this sub-question incorrectly. These students' scripts were among the 21 scripts selected to explore students' reasoning and level of understanding. The table12 below presents the researchers' unstructured interview questions and the students' responses.

Table 12 - Unstructured questions: Question 2.3.1

Researcher's unstructured question: What were you thinking when you were answering this question?			
Students' responses	**Codes**	**Categories**	**Themes**
S27: I understand how to calculate roots, Ma'am, but I made a mistake while I was dividing the exponents of the second term: I wrote 3 instead of 2. So my answer was wrong because of that.	▪ Error dividing exponents (S27) ▪ Mistake in calculation of cube root (S27)	**Errors in Exponent and Root Operations** ▪ Error dividing exponents (S27) ▪ Incorrect assumption about cube roots (S11) ▪ Incorrect multiplication of exponents (S6) ▪ Misstep in finding the cube root (S37) ▪ Misunderstanding of the second term (S22) ▪ Overcomplication of exponent rules (S23)	Conceptual misunderstanding of exponent and root rules
S11: I know that if you have a square root and a square, they cancel each other. Then I applied the same knowledge in the cube root to remove the root and calculate 4^6. On the next step, I found the difference of two terms.	▪ Correct understanding of square root and square cancelling each other (S11) ▪ Incorrect assumption that cube root works the same way (S11)	**Misinterpretation of Cancellation Rules** ▪ Correct understanding of square root and square cancelling each other (S11) ▪ Correct cancellation of square root and square (S22) ▪ Incorrect assumption that cube root works the same way (S11)	
S87: I didn't know what to do, Ma'am. I decided to jump the question and answer the next question.	▪ Avoidance due to lack of understanding (S87)	**<u>Procedural Challenges</u>** ▪ Mistake in calculation of cube root (S27) ▪ Uncertainty about the method (S6)	Procedural gaps in problem-solving

		▪ Ignored squared term in the first term (S37)	
S56: I pressed everything on the calculator, Ma'am, and I got the correct answer without understanding the steps to be shown to get the correct answer.	▪ Reliance on a calculator for the correct answer (S56) ▪ Lack of understanding of the required steps (S56)	**<u>Avoidance or Over-Reliance on calculator</u>** ▪ Avoidance due to lack of understanding (S87) ▪ Reliance on a calculator for the correct answer (S56) ▪ Lack of understanding of the required steps (S56)	
S6: I decided to multiply the exponents outside the brackets by the exponents inside the brackets. I was not sure what to do. **S37: I ignored the squared on the first term and calculated the square root of 16, which is 4. Then calculate the cube root of 4^6, which is 16, and find the difference between the answers. I did not know that I need to remove the square root first on the first term.**	▪ Incorrect multiplication of exponents (S6) ▪ Uncertainty about the method (S6)		
S22: First, I knew that the square root and squared cancels each other and the answer is 16. Second term, I multiplied 4 x 6 = 24; then I found the cube root of 24.	▪ Correct cancellation of square root and square (S22) ▪ Misunderstanding of the second term ($4 \times 6 = 24$ instead of exponentiation)(S22)		
S23: I changed 16 to 4^2, then I multiplied the exponent of 4 by the exponent 2 outside the bracket$(4^2)^2$. And then I changed the next term to $(4^6)^3$ then multiplied the exponent and then I calculated the difference of their answers.	▪ Rewriting base as a power of 2 ▪ Overcomplication of exponent rules (S23)		

4.3.4.1. Analysis of students' responses to Question 2.3.1 (Simplifying roots)

The analysis of students' responses to Question 2.3.1 revealed significant challenges in their ability to simplify roots, evident in both the quantitative data and qualitative insights from the interviews and detailed examination of students' test scripts. Among the 150 students assessed, 52(35%) provided the correct answers, 12(8%) solved the question partially correctly, 76(51%) students answered incorrectly, and 10(6%) left the question blank.

These results suggest widespread difficulties in understanding and solving root expressions, particularly in applying exponent laws and distinguishing between square and cube root operations. The findings of this study are in alignment with those of a study conducted by Stemele and Asvat (2024) on the struggles learners have with adding and subtracting integers. In their study, students also tended to utilize various methods to solve for the square root. Hiebert (2013) found that students misused data by using several methods to solve the square roots.

Qualitative analysis of students' test scripts highlighted two main types of errors: technical errors and misused data errors. Technical errors were observed in the responses of students like S27, who successfully removed the square root and cube root using correct rules of exponents but committed technical errors when dividing the exponent of the second term, $\frac{6}{3} = 3$. Learners misapplied the rules of exponents, particularly in dividing exponents when simplifying expressions involving roots.

These results were in line with the findings of Stemele and Asvat (2024). Ulusoy (2019) found that students misunderstood the rules of exponents and were unable to solve problems with zero or negative exponents.

Furthermore, misused data errors were evident for students who cancelled the square root and introduced the exponent of 3 on the second term, since they wanted to cancel the cube root and 3. There was evidence that they appeared to know the general process, but misapplied exponent rules, particularly when attempting to cancel roots by incorrectly introducing exponents. Similarly, Hiebert (2013) found that students misused data by using several methods to solve the square roots and or just brute force. Putri et al. (2024) found that many students struggled to complete the given task correctly because they did not follow the exponent rules. They reported that when operating the root forms, students tended to directly add or subtract the numbers inside the roots.

4.3.4.2.Interview findings: Students' difficulties simplifying roots (Question 2.3.1)

The interview responses provided valuable qualitative insights into the difficulties students encountered when simplifying roots. The analysis identified two overarching themes: conceptual misunderstanding of exponent and root rules, and procedural gaps in problem-solving. These themes, supported by specific categories, highlight different aspects of students' challenges working with this mathematical concept.

4.3.4.2.1. Conceptual misunderstanding of exponents and roots

This theme captures the difficulties students experienced grasping the foundational principles of roots and interpreting the requirements of the question. Two key categories emerged within this theme.

Errors in exponent and root operations: A recurring issue displayed by students was confusion when calculating the exponents of roots. Some students appeared uncertain about how to correctly simplify expressions involving roots and powers. For example, Student S11 said: *"I know that if you have a square root and a square they cancel each other. Then I applied the same knowledge in the cube root to remove the root and calculate 4^6. On the next step, I found the difference of two terms".* This response suggests a misapplication of the rule for cancelling square roots and squares, where the same logic was incorrectly extended to cube roots. Similarly, Student S37 stated: *"I ignored the squared on the first term and calculated the square root of 16, which is 4, then calculate the cube root of 4^6, which is 16, and find the difference between the answers. I did not know that I need to remove the square root first on the first term".* These responses indicate that these students lacked both procedural fluency and conceptual understanding of how exponents and roots interact, particularly in multi-step expressions involving both square and cube roots.

Misinterpretation of cancellation rules: Some students believed that exponents and roots could be cancelled in all cases, showing over-generalisation of exponential laws. Student S22 explained: *"First, I knew that the square root and squared cancels each other and the answer is 16. Second term, I multiplied 4 x 6 = 24, then I found the cube root of 24"*. This student correctly applied the cancellation rule to the first term but misunderstood the operation needed for the second term. Student S11 also applied this rule incorrectly to a cube root, stating*: "I know that if you have a square root and a square, they cancel each other. Then I applied the same knowledge in the cube root to remove the root and calculate 4^6. On the next step, I found the difference of two terms"*. These responses reflect a lack of conceptual clarity about when

and how exponent rules apply. Students demonstrated a misunderstanding of when cancellation rules apply, particularly overgeneralising square root–square relationships to cube roots without recognising the role of matching indices.

4.3.4.3. Procedural gaps in problem-solving theme

This theme reflects the difficulties students encountered in applying the correct steps to simplify roots using exponential rules. Even when some students showed partial understanding, they struggled to execute the procedure accurately. Two categories emerged within this theme.

Procedural challenges: Some students showed knowledge of the concept but made procedural errors when simplifying the expression. For example, Student S27 explained: *"I understand how to calculate roots, Ma'am, but I made a mistake while I was dividing the exponents of the second term: I wrote 3 instead of 2. So my answer was wrong because of that"*. This shows a minor calculation error, not a conceptual gap, but one that still led to an incorrect answer. Student S6 described feeling uncertain about the method: *"I decided to multiply the exponents outside the brackets by the exponents inside the brackets. I was not sure what to do"*. This highlights confusion about which exponential rules to apply, particularly when simplifying expressions involving powers and roots.

Avoidance or Over-Reliance: Other students either skipped the question or relied heavily on calculators, often without understanding the underlying processes. Student S87 said: *"I didn't know what to do, Ma'am. I decided to jump the question and answer the next question."* This indicates avoidance behaviour, likely due to a lack of confidence or understanding. Student S56 explained: *"I pressed everything on the calculator, Ma'am, and I got the correct answer without understanding the steps to be shown to get the correct answer."* While this student was able to produce an answer, they lacked the ability to explain or verify the steps taken. Such over-reliance on technology without understanding weakens students' procedural fluency and reduces their capacity to solve unfamiliar or more complex mathematical tasks.

These interview responses confirmed that students struggled with both conceptual understanding and procedural accuracy when simplifying roots and using exponent rules. Misinterpretation of cancellation rules, confusion about laws of exponents, over-reliance on calculators, and avoidant behaviours all contributed to their difficulties. These findings indicate that students' pre-existing knowledge of exponent and root relationships is limited, particularly in applying exponent laws and distinguishing between different root operations, which may hinder their success in Level 2 mathematics.

4.3.5. Analysis of students' difficulties, alternative conceptions, and errors (Question 2.3.2)

Sub-question 2.3.2 required students to simplify algebraic expressions involving the expansion of binomial products and the combination of like terms.

> **2.3.2.** $\boldsymbol{Simplify}$: $(2x+5)(x-3)+(x-1)^2$

Simplification using the foil method

Table 13 summarizes the performance of 150 students on Question 2.3.2. A total of 34 students (23%) answered correctly, while 87 students (58%) answered incorrectly. Eighteen students (12%) provided partially correct answers, and 11 students (7%) did not attempt to answer the question. Among the 105 students who answered either incorrectly or partially correctly, 46 committed technical errors, 41 committed misused data errors, and 18 displayed incomplete answer errors.

Table 13 – Algebra: Types and frequency errors [Question – 2.3.2]

Difficulties	Errors					Frequency and percentages
	Misuse of data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	34 (23%)
Incorrect answer	41	0	0	46	0	87 (58%)
Partially correct	-	-	-		18	18(12%)
No answer	-	-	-	-	-	11 (7%)
Total						**150 (100%)**

Figure 24 shows an extract from a student's solution where an error involved misusing data. This student appeared to treat the binomials (2x+5) and (x−3) as separate coefficients to be multiplied directly, generating incorrect terms such as 5x, 2x, and an invalid exponent term, $\mathbf{7x^3}$. This suggests the student misapplied the FOIL method, particularly by multiplying terms incorrectly and introducing invalid exponents during expansion. Additionally, the student omitted the expression $(x-1)^2$, indicating a lack of procedural understanding in managing multi-step algebraic expressions. The response reflects a lack of procedural knowledge, specifically in the application of exponent rules and distributive property during expansion.

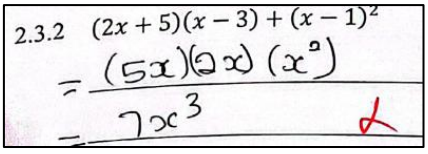


Figure 24 - Example of misused data (Participant S27)

Figure 25 shows an extract from a student's solution which was incomplete. This student correctly applied the FOIL method to both (2x+5)(x−3) and the expanded form of $(x-1)^2$, demonstrating a sound procedural understanding at the start. However, the student did not proceed to combine like terms after expansion. This suggests difficulty identifying and grouping like terms after expansion, indicating incomplete procedural knowledge of the final step in algebraic simplification. The omission reflects a conceptual gap in recognizing the structure of simplified algebraic expressions.

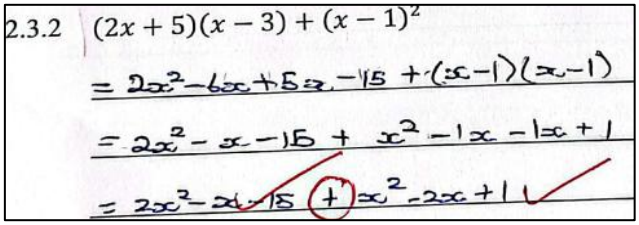


Figure 25 - Example of incomplete answer (Participant S87)

Figure 26 shows an extract from student's solution that demonstrates technical errors. The student accurately removed brackets using the FOIL method and considered the signs appropriately during the expansion process. However, several sign errors occurred when combining like terms. For example, the student grouped terms such as −6*x* and incorrectly changed it to 6x and, similarly, altered constants such as 1 to −1. This suggests confusion between algebraic simplification and equation-solving procedures, particularly the incorrect application of sign-changing rules when combining like terms. The student may have misapplied procedures used for solving linear equations, leading to systematic sign errors. This reflects a lack of procedural knowledge, specifically in applying the distributive property correctly across all terms in binomial expansion.

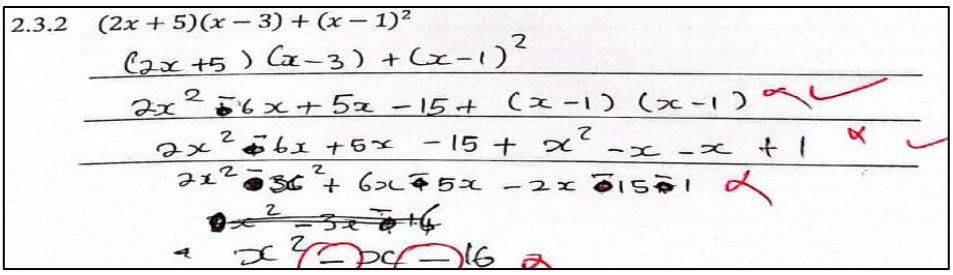


Figure 26 - Example of technical error (Participant S56)

The researcher interviewed the participants whose solutions are shown in Figures 24 to 26, as well as other participants who had answered this sub-question incorrectly, to explore their reasoning and level of understanding. Of the 21 selected scripts,12 students were interviewed with 8 of the students answering the question incorrectly, 2 leaving it blank and 2 providing partially correct answers. The table below presents the researcher's unstructured interview questions and the students' responses.

Table 14 - Unstructured questions: Question 2.3.2

Researcher's unstructured question: Do you understand the FOIL method? Please explain why you solved this question this way.			
Students' responses	Codes	Categories	Themes
S13: Yes, Ma'am, I understand it, but I didn't know how to use the exponent of 2 on the second bracket. Then I decided to square both terms in that bracket and group the like terms afterward.	▪ Misunderstanding how to handle exponents. (S13) ▪ Attempt to square terms individually rather than expanding. (S13)	**Partial or Incorrect Use of FOIL** ▪ Misunderstanding how to handle exponents (S13). ▪ Squaring terms individually instead of expanding (S13). ▪ Error in multiplication using FOIL (S22). ▪ Correct use of FOIL for the first term (S6). ▪ Partial understanding of FOIL (S37). ▪ Correct use of FOIL but failure to group like terms (S87).	Errors applying algebraic procedures
S22: I know the FOIL method, but I made a mistake while I was multiplying the terms; I only realized now after seeing my paper.	▪ Error in multiplication using the FOIL method. (S22) ▪ Realization of mistake after reflection. (S22)	**Conceptual Misunderstanding of Brackets** ▪ Misinterpretation of the second term as a difference of squares (S6). ▪ Incorrect multiplication leading to addition (S37). ▪ Lack of knowledge of FOIL (S27). ▪ Incorrectly added terms instead of expanding (S27). ▪ Grouped like terms before proper expansion (S41). ▪ Removed brackets incorrectly (S41).	
S6: I know FOIL: I used it to simplify the first two brackets. The next	▪ Correct use of FOIL for the first term. (S6) ▪ Misinterpretation of the second term as a	**Errors in Grouping or Combining Like Terms**	

bracket is the difference between two squares. Means one bracket must be positive and the other one must be negative when I factorize it. And then I added the like terms.	difference of squares. (S6)	▪ Incorrect belief that changing signs is necessary for grouping terms (S56). ▪ Failure to group like terms after expansion (S87).	
S37: The FOIL method is about multiplication but I'm not sure how we multiply the terms. I multiplied the terms inside the brackets together then later I added them to get the final answer.	▪ Partial understanding of FOIL (knowing it involves multiplication).(S37) ▪ Incorrect multiplication leading to addition of terms.(S37)	**Avoidance or Lack of Understanding** ▪ Left the question blank (S4). ▪ Avoidance due to lack of understanding (S11). ▪ Lack of understanding of both FOIL and the question (S47).	Lack of conceptual clarity or confidence
S27: I don't know the FOIL method; I removed the brackets by adding the terms inside those brackets, then I got my final answer.	▪ Lack of knowledge of FOIL. (S27) ▪ Incorrectly added terms instead of expanding. (S27)	**Errors from Alternative Approaches** ▪ Knowledge of alternative methods (half-moon method) (S70). ▪ Error in sign when multiplying terms (S70).	
S87: I understand FOIL method I used it to remove the brackets, but I forgot that I need to group the like terms, then add them.	Understand the method but failure to group like terms (S87)		
S56: I know FOIL method. But I thought to add the like terms I needed to change the signs of the other terms.	Incorrect belief that changing signs is necessary for grouping terms. (S56)		
S4: I did not know what needed to be done. I left the question.	Uncertainty about the task, leaving it blank. (S4)		
S11: I jumped the question. Still I don't know what the question is asking.	Avoidance due to lack of understanding of the question. (S11)		
S70: I know halfmoon method. I expanded the second brackets, then used FOIL method to remove brackets. But I	▪ Knowledge of alternative methods (half-moon method). (S70) ▪ Error in sign when multiplying terms. (S70)		

made a mistake when I was multiplying the terms: I wrote positive instead of negative.			
S47: Eish, no Ma'am. And I don't understand the question.	Lack of understanding of both FOIL and the question. (S47)		
S41: Not really, I added the like terms of the first two bracket, then expanded term 2 and added the like terms of their brackets together. That's how I removed brackets.	▪ Attempted grouping of like terms before proper expansion. (S41) ▪ Removed brackets incorrectly. (S41)		

4.3.5.1. Analysis of students' responses to Question 2.3.2 (Simplification of algebra)

The analysis of students' responses to Question 2.3.2 revealed significant challenges in students' ability to simplify algebraic expressions, evident in both the quantitative data and qualitative insights from interviews and detailed examination of students' test scripts. Among the 150 students assessed, only 34 (23%) provided the correct answer, while 18 (12%) solved the question partially correctly. A majority of 87 (58%) students answered incorrectly, and 11 (7%) left the question blank.

These results suggest widespread difficulty in expanding and simplifying algebraic expressions, particularly in applying the distributive property and combining like terms correctly. Mofolo-Mbokane and Ngoveni (2019) revealed that 12 NC(V) Level 2 were given $y(2y+1)=15$. In this question, students could not correctly remove the brackets and could not factorize the quadratic equation after multiplying out. The full sample of 12 students struggled with this equation and most students tended to ignore the *y* outside the brackets as they factorized. These students lack the procedural knowledge for multiplying through the bracket and then finding the value of *y* (Mofolo-Mbokane and Ngoveni, 2019).

Qualitative analysis of students' test scripts highlighted three main types of errors: technical errors, incomplete answer errors, and misused data errors. Students committed technical errors when grouping like terms, particularly in managing negative signs and performing addition and subtraction correctly. Luneta (2016) found that Grade 9 students were unable to operate negative signs when adding or subtracting like terms. In this study, uncomplete answer errors were made by students like S87, who successfully perform the FOIL method and expanded the

second brackets but did not proceed to the final step of combining like terms, indicating incomplete procedural understanding of algebraic simplification. Misused data errors were evident in the responses of students who added unlike terms, indicating a misunderstanding of the concept of like terms and the rules governing algebraic expressions. The outcome of this study reflected the conclusion of Ngoveni (2018) that students showed a lack of procedural knowledge as they added different variables to produce a final answer. Stemele and Asvat (2024) highlight similar challenges in their study, in which Grade 9 learners combined unlike terms—e.g., 5a and 5b.

4.3.5.1. Interview findings: Students' difficulties in simplifying algebraic expressions (Question 2.3.2)

The interview responses provided valuable qualitative insights into the difficulties students encountered when simplifying algebraic expressions using the FOIL method. The analysis identified two main themes: errors in applying algebraic procedures and lack of conceptual clarity and confidence. These themes are supported by specific categories that highlight different aspects of students' challenges with this algebraic concept.

4.3.5.1.1. Errors in applying algebraic procedures

This theme reflects the difficulties students experienced in applying the correct steps for expansion and simplification of binomial expressions. Several categories emerged within this theme.

Partial or incorrect use of FOIL: Many students showed only partial understanding of the FOIL method or used it incorrectly. For instance, Student S13 explained: *"Yes, Ma'am, I understand it but I didn't know how to use the exponent of 2 on the second bracket. Then I decided to square both terms in that bracket and group the like terms afterward".* This shows a misconception about handling exponents, with the student choosing to square individual terms instead of expanding the entire binomial expression. Student S27 also lacked knowledge of the FOIL method, stating: "*I don't know the FOIL method; I removed the brackets by adding the terms inside those brackets, then I got my final answer*". This response indicates a complete misunderstanding of expansion, with the student using addition instead of multiplication. Even students who correctly began the expansion sometimes failed to complete it. For example, S87 said: "*I understand FOIL method. I used it to remove the brackets, but I forgot that I need to group the like terms then add them".*

This suggests procedural gaps in completing simplification after expansion. Collectively, these responses show that students struggled to apply the FOIL method properly, particularly in maintaining the correct multiplication sequence and handling exponents within binomial expressions. Misunderstanding this process can hinder success in algebra, where handling brackets and exponents is foundational.

Conceptual Misunderstanding of Brackets: Some students misinterpreted how to handle brackets, especially to get the second binomial. For instance, Student S6 said: "*I know FOIL: I used it to simplify the first two brackets. The next bracket is the difference between two squares: means one bracket must be positive and the other one must be negative when I factorize it. And then I added the like terms".* This reflects confusion between expansion and factorisation: the student applied a method that was not relevant to the task.

Student S27 explained: "*I don't know the FOIL method; I removed the brackets by adding the terms inside those brackets, then I got my final answer*". Student S41 stated: "*Not really, I added the like terms of the first 2 bracket, then expanded term 2 and added the like terms of their brackets together. That's how I removed brackets*". These responses indicate that students did not clearly understand when and how to expand brackets, particularly confusing expansion with factorisation and applying inappropriate methods. This lack of understanding of brackets and simplification shows the need for clearer instruction and more guided practice for algebraic procedures.

Errors in grouping or combining like terms: Some students made errors combining like terms after expansion. Student S56 stated: *"I know FOIL method, but I thought to add the like terms. I needed to change the signs of the other terms"*. This reflects confusion between algebraic expression simplification and solving equations. The student mistakenly applied the transposition rules used in solving equations to an algebraic expression, demonstrating a procedural misunderstanding, particularly in recognising and correctly combining like terms after expansion.

4.3.5.1.1. Lack of conceptual clarity or confidence

This theme highlights cases where students avoided the task or expressed uncertainty about both the FOIL method and the question itself. Two categories emerged within this theme.

Avoidance or lack of understanding: Some students left the question blank or indicated that they did not understand the task. Student S4 said "*I did not know what needed to be done. I left the question blank".* Similarly, S47 stated: "*I don't understand the question"*. Such responses

suggest a complete lack of familiarity with algebraic simplification, indicating deeper issues with content retention and confidence.

Errors from alternative approaches: Other students tried to use alternative methods but made errors. Student S70 said: "*I know halfmoon method. I expanded the second brackets then used FOIL method to remove brackets. but I made a mistake when I was multiplying the terms: I wrote positive instead of negative*". This error indicates uncertainty about how to manage signs during multiplication — a skill that should be well-established by the time students have reached this level. These responses show that even when the students attempted alternative approaches, lack of understanding of sign rules and operation sequences undermined their solutions.

The interview findings reveal that the students struggled with both procedural steps and conceptual understanding, particularly in applying the distributive property, handling exponents, and combining like terms accurately. Students struggled with both procedural steps and conceptual understanding, particularly in applying the distributive property, handling exponents, and combining like terms accurately. Misapplication of exponent rules, failure to combine like terms, and confusion between expansion and factorization were common. Additionally, avoidance and calculator reliance point to a lack of confidence. These findings indicate that students' pre-existing knowledge of algebraic simplification is insufficient, particularly in applying the FOIL method and combining like terms, which may negatively affect their performance in Level 2 mathematics.

4.3.6. Analysis of students' difficulties, alternative conceptions, and errors (Question 3)

Questions 3.1 to 3.4 tested students' knowledge of the content area of factorizing denoted by the expressions below: taking out a common factor (3.1), the difference of two squares (3.2), quadratic trinomial (3.3), and grouping (3.4).

Factorize fully:	
3.1. $60x^2y - 25xy^2 + 5xy$	(Common Factor)
3.2. $k^2 - 64$	(Difference of two squares)
3.3. $x^2 + 7x - 18$	(Quadratic equation)
3.4. $ax - ay + 4x - 4y$	(Grouping)

Question 3 – Factorization

Table 15 summarizes the performance of 150 students on Question 3.1. A total of 13 students (9%) answered correctly, 88 students (59%) answered incorrectly, 5 students (3%) answered partial correct, and 44 students (29%) did not attempt the question and left blank spaces. Of the 93 students who answered incorrectly and partially correct displayed errors, 85 students committed misused data errors, 5 displayed logically invalid errors, and 3 displayed technical errors.

Table 15 – Factorization: Type and frequency errors [Question – 3.1]

Responses	**Errors**					**Frequencies and percentages**
	Misused data	**Logically invalid inference**	**Distorted theorem or definition**	**Technical Error**	**Incomplete answer**	
Correct answer	-	-	-	-	-	13 (9%)
Incorrect answer	85	1	0	2	0	88 (59%)
Partially correct	-	-	-		5	5(3%)
No answer	-	-	-	-	-	44 (29%)
Total						**150 (100%)**

Figure 27 shows an extract from a student's answer demonstrating a misused of data error. This student did not understand that the question required factorization by removing a common factor. Instead, they used the order of operations and exponential laws to factorize the expression, subtracted the coefficients 60-25, added 5, which equates to 40, and added all the exponents of x and y. The application of this process was also incorrect because they added unlike terms together and, according to the question requirements the student, was unable to take out a common factor. This suggests a lack of conceptual understanding of the highest common factor, particularly the inability to identify common numerical and variable factors across all terms., particularly the inability to identify common numerical and variable factors across all terms. This student confused the concept of exponential law with factorization.

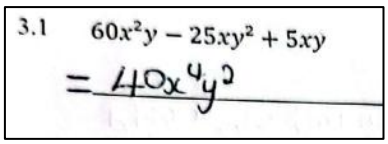


Figure 27 – Example of misused data (Participant S13)

Figure 28 shows an extract from a student's solution which displayed logically invalid errors. The student initiated the solution by grouping all the terms containing the variables *x* and y together and then grouped the constant numbers. The student proceeded to add the terms in the first bracket by applying exponential law and used the order of operators to get the answer for the second bracket. The student also applied the grouping and exponential laws incorrectly but displayed logical invalid errors of alternative errors to understand the question's requirements. This suggests that the student lacked the procedural knowledge required to identify and extract the highest common factor before decomposing the expression. The student confused the concept of exponential laws with factorization, incorrectly applying rules of exponents in a context that required identifying common factors. Laws of exponents are applied when we multiply or divide the same bases, not in addition and subtraction of the terms.

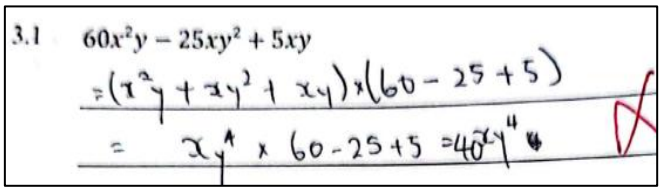


Figure 28 – Example of logically invalid error (ParticipantS27)

Figure 29 shows an extract from a student whose solution was incomplete. The student correctly identified the highest common but failed to correctly classify the terms that would be left in the bracket. The student's incomplete answer error was based on decomposing term (12x-5y) instead of writing (12x-5y+1) as decomposed. As this student identified the common factor as 5xy—which is the same as the 3rd term—the student failed to apply the basic mathematical principle that dividing a term by itself results in one, indicating a gap in understanding factorization structure.

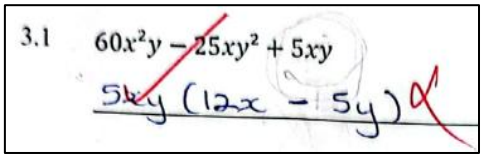


Figure 29 – Example of incomplete answer error (Participant S33)

Figure 30 shows an extract from a student's solution that demonstrates technical errors. The student correctly removed the highest common factor but made a technical error when dividing

the terms inside the brackets. The student divided the first and third terms correctly and made a technical error by writing 5 instead of 5y. Based on the student's response, it is evident that the student had partial procedural knowledge but lacked accuracy in dividing terms correctly within the factorized expression. Focus is required when writing assessments to minimize technical errors as they may affect students' outcomes.

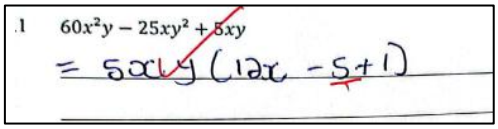


Figure 30 – Example of technical error (Participant S142)

Question 3.2 assessed students' knowledge of factorizing the difference between two squares. Table 16 presents the performance of 150 students in this sub-question. A total of 30 students (20%) answered correctly, while 1 student (1%) answered partially correctly. A large number—77 students (51%)—answered incorrectly, and 42 students (28%) did not attempt to answer the question. Of the 78 students who answered incorrectly, 77 students committed misused data errors and one student displayed technical errors.

Table 16 – Factorization: Type and frequency error [Question – [3. 2]

Difficulties	Errors					Frequencies and percentages
	Misuse of data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	30 (20%)
Incorrect answer	77	0	0	-	0	77 (51%)
Partially correct	-	-	-	-	1	1 (1%)
No answer	-	-	-	-	-	42 (28%)
Total						**150 (100%)**

Figure 31 shows an extract from the solution of a student who misused data. The student incorrectly treated the factorization problem as a 'solving for *k*' problem. Instead of rewriting k^2-64 as a factored expression, the student interpreted the equation as $k^2-64 = 0$ and solved for k, yielding k = 8. The student failed to recognize the structure of a difference of two squares,

specifically the form a2−b2=(a−b)(a+b)=(a−b)(a+b). The response suggests that the student had an inaccurate understanding of factorization, particularly confusing factorization with solving equations.

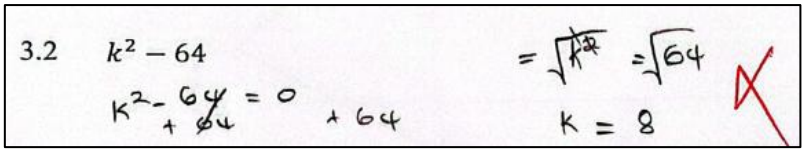


Figure 31 – Example of misused data (Participant S4)

Figure 32 shows an extract from a student's solution that demonstrates technical errors. The correct factorization of $k^2−64$ is (k−8)(k+8). By writing (k−8)(k−8), the student treated $k^2−64$ as a perfect square trinomial, such as $k^2−16k+64$. The student may have overgeneralized factorization patterns, incorrectly treating the expression as a perfect square trinomial instead of a difference of squares. The response suggests that the student lacked the knowledge to recognize and apply the difference of squares formula $a^2 − b^2 = (a−b)(a+b)$.

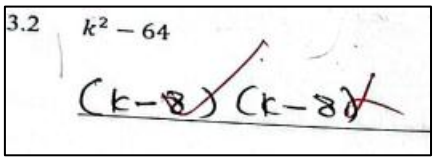


Figure 32 - Example of technical error (Participant S22)

The next question assessed students' knowledge of factorizing a quadratic trinomial. Table 17 summarizes the performance of 150 students for Question 3.3. A total of 48 students (32%) answered correctly, while 69 students (46%) answered incorrectly, 4 students (3%) answered partial correctly, and 29 students (19%) did not attempt to answer the question. Among the 73 students who did not provide a correct answer, 69 made misused data errors, 2 displayed incomplete answers, and 2 displayed technical errors.

Table 17 – Factorization: Type and frequency of error [Question 3.3]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	48 (32%)
Incorrect answer	69	0	0	-	0	69 (46%)
Partially correct	-	-	-	2	2	4 (3%)
No answer	-	-	-	-	-	29 (19%)
Total						**(100%)**

Figure 33 shows an extract from a student's solution who made misused data error. The student misinterpreted the task as solving for x rather than factorizing the expression and incorrectly equated parts of the polynomial to find x, treating the terms as separable equations. The equation $x^2 = -7x+18$ suggests that the student split the expression incorrectly, treating the terms as though they could be calculated separately. The steps $x^2 - x^2 = -7+18$ lack a logical basis and result in $x = 11$, which is mathematically invalid. The response indicates a lack of understanding of the distinction between factorizing and solving quadratic expressions.

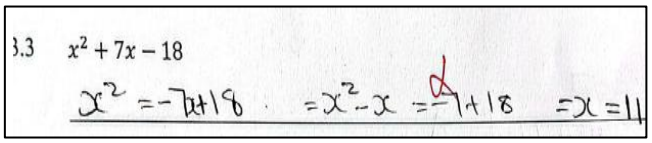


Figure 33 – Example of misused data (Participant S24)

Figure 34 shows an extract from a student's solution that demonstrates an incomplete answer. The student began the factorization process but failed to identify the second factor—the constant in (x−). Leaving a blank space in the bracket after (x−) indicates either that the student did not know how to determine two numbers whose product equals −18 and whose sum equals 7, indicating a gap in procedural knowledge of factorization. Writing =0 implies that the student was also thinking about solving the equation, despite the task only requiring factorization.

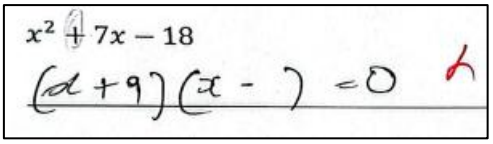


Figure 34 – Example of incomplete answer errors (Participant S85)

Figure 35 shows an extract from a student's solution that demonstrates technical errors. The student correctly determined the factors of the third term but wrote the signs incorrectly as (x+2)(x−9), which is the expected result for factorization. However, the student proceeded to solve the equation (x+2)(x−9)=0 and found the roots x=−2 or x=9, which went beyond the requirement of the task. The student assumed the task involved solving the quadratic equation, indicating a misunderstanding of what factorization entails, particularly treating the expression as an equation to be solved rather than an expression to be rewritten.

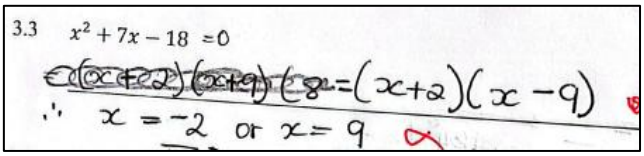


Figure 35 – Example of technical errors (Participant S6)

Finally, Question 3.4 assessed students' knowledge of factorizing by grouping terms. Table 18 presents the performance of 150 students on this question. A total of 19 students (13%) answered correctly, while 78 students (52%) answered incorrectly, 6 students (4%) answered partially correctly, and 47 students (31%) did not attempt to answer the question. Of the 84 students who answered incorrectly, 8 students displayed incomplete answer errors, 70 students displayed misused data errors, and 6 displayed technical errors.

Table 18 – Factorization: Type and frequency error [Question 3.4]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	19 (13%)
Incorrect answer	70	0	0	6	0	78 (52%)
Partially correct	-	-	-	-	8	6 (4%)
No answer	-	-	-	-	-	47(31%)
Total						**150 (100%)**

Figure 36 shows an extract from a student's solution that demonstrates incomplete answer errors. The student demonstrated a clear understanding of grouping and common factors but

did not complete the final step of factorization, specifically extracting the common binomial factor (x−y). The response stops at a(x−y)+4(x−y) without simplifying further; while this is not incorrect, it reflects incomplete factorization. The student may not have understood the need to expand the factored terms to check whether the original expression was recovered.

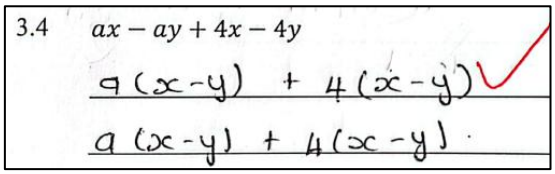


Figure 36 – Example of incomplete answer error (Participant S13)

Figure 37 shows an extract from a student's solution that demonstrates a misused data error. The student started by rearranging terms as ax+4x−ay−4y, but this grouping is invalid because it does not preserve the structure of the original expression. The student incorrectly grouped terms, failing to preserve the structure of the original expression when grouping terms, indicating a misunderstanding of how to group terms correctly. The response shows confusion about how to extract common factors across multiple terms, leading to the erroneous 4ax−4ay. The student appeared to lack a clear grasp of factorization as the process of identifying common factors and grouping terms systematically.

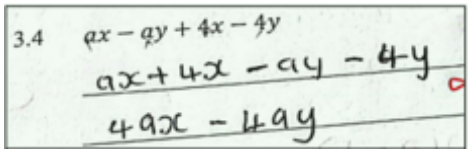


Figure 37 – Example of misused data (Participant S142)

Figure 38 shows an extract from a student's solution displaying technical errors. The student attempted to group terms into two separate binomials, (ax+4x) and (4y−ay), which is mathematically invalid. This grouping does not result in a factored form of the original expression, as it cannot be expanded back to the original terms. The student appeared to lack a clear grasp of factorization as the process of identifying common terms and grouping them correctly.

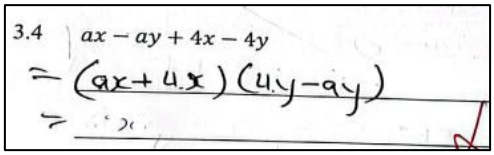


Figure 38 – Example of technical error (Participant S87)

To further explore students' thought processes, the researcher selected 21 scripts and conducted unstructured interviews with these students. Table 20 presents a summary of the researcher's unstructured interview questions and students' responses.

Table 19 - Unstructured questions: Question 3

Researcher unstructured interview: Do you know when to use the methods of factorization? Can you explain why you solved this question the way you did?			
Students' responses	Codes	Categories	Themes
S37: I don't know the methods of factorization; I don't know how to factorize and I didn't know what to do in question 3 as a whole.	Don't know factorization methods (S37)	**<u>Lack of Knowledge</u>** ▪ Don't know factorization methods (S37) ▪ Knows factorization exists but lacks understanding (S33) ▪ Don't know the factorization methods and adds terms incorrectly (S142) ▪ Confusion between solving and factorization (S4) ▪ Don't know the factorization (S48) ▪ Don't know factorization methods (S138) ▪ Don't know factorization methods (S16) **<u>Misinterpretation of the Task</u>** ▪ Confusion between solving and factorization and understanding of the methods (S4) ▪ Confusion between solving and factorization (S70) ▪ Misinterprets terminology like 'factorization' and 'simplification' (S23)	**Conceptual Gaps in Factorization and Related Terminology**

S33: I did factorization in grade 9 but I don't understand the method of factorization. Question 3.1, I knew that I needed to take out a common factor, which is 5xy, but I thought since I took out 5xy as common then I would be left with nothing for the third term. Hence I wrote my answer like this.	Knows about factorization but lacks understanding (S33) Misunderstanding of concept (S33)		
S6: I did factorization but I forgot when to use its different method. It was easy to factorize 3.3 and find the value of x. I got the factors by using my calculator, but the signs are incorrect.	Partial knowledge but does not remember when to use different methods (S6)	**<u>Partial Knowledge and Misapplication</u>** ▪ Partial knowledge but forgets when to use different methods (S6) ▪ Attempts to factorize but forgets steps or methods (S22) ▪ Misunderstands specific methods like grouping (S27) ▪ Partial knowledge of grouping but cannot proceed further (S87) ▪ Knows some methods but misapplies them (S13) ▪ Attempts to factorize but forgets steps or methods (S56) **Challenges with execution** ▪ Relies on guessing or writing random answers (S26) ▪ Relies on guessing or writing random answers(S85)	**Procedural Errors in Applying Factorization**
S142: I don't know the method, and I always get wrong answers when I do factorization. 3.4, I added all the like terms to get the final	Doesn't know factorization methods and adds terms incorrectly (S142)		

answer because I thought that what was needed.			
S22: I don't know the method. In 3.1, I took out a common factor, then I multiplied the remaining terms together. I forgot what to do.	Attempts to factorize but forgets steps or methods (S22)		
S4: Yes, I know them. I did factorization, Ma'am. In 3.2, since I was given a number and a variable, it was easy to find the value of k. I am surprised to see my answer is wrong.	Confusion between solving and factorization (S4)		
S27: I know they are there but I think I needed a revision on the topic. In 3.1, I grouped all the same letters in brackets and the same numbers together– that's what I understand about grouping. And I used their signs to get the answer.	Revision needed (S27) Knows the available methods but unable to use them (S27) Misunderstands specific methods like grouping (S27)		
S24: I don't understand factorization	Does not know factorization methods (S24)		
S70: No, the only thing I remember: when they say factorize, you have to solve for x.	Don't know the method (S70) Confusion between solving and factorization (S70)		
S48: Factorization is difficult. I left blank spaces in Question 3.	Don't know factorization (S48)		
S23: I have a problem knowing exactly what I should do when asked questions like factorize. Lecturers should explain what terms like	Knows factorization exists (S23) Misinterprets terminology like 'factorization' and 'simplification' (S23)		

factorization and simplification mean. The lecturers assume that we know what all these terms mean because this is the knowledge that was done in the previous classes.			
S138: I have no idea of the methods. I left Question 3 blank.	Don't know factorization methods (S138)		
S16: I could not answer Question 3, it was difficult for me.	Don't know factorization methods (S16)		
S26: I did not know what I was writing; I was writing for the sake of filling spaces.	Don't know factorization methods(S26) Relies on guessing or writing random answers (S26)		
S87: I forgot factorization, but I still remember that in grouping I need to put like terms inside the brackets. But I couldn't proceed to the next step.	Partial knowledge of grouping but cannot proceed further S87)	Difficulty proceeding to the next step (S87)	
S13: I know the methods of factorization, and I know when to use them, however I made mistakes in all the questions. In 3.4, I forgot the next, after taking out a common factor in grouping.	Knows some methods but misapplies them (S13)	Careless mistakes (S13)	
S56: I heard about this topic before, but I forgot how to approach it.	Attempts to factorize but forgets steps or methods (S56)	Forgot factorization steps (S56)	
S85: I answered the whole question, but I don't know what I was writing.	No understanding of the concept (S85) Relies on guessing or writing random answers (S85)		

4.3.6.1.Analysis of students' responses to Question 3 (Factorization)

The analysis of students' responses to Question 3 revealed significant challenges in students' ability to factorize algebraic expressions, evident in both the quantitative data, qualitative insights from interviews, detailed examination of students' test scripts. Among the 150 students assessed, the following averages were calculated: 27,5 (18%) correctly answered, 4(3%) partially answered correctly, (52%) answered incorrectly, and 40,5 (27%) left blank.

These results suggest widespread difficulty in factorizing algebraic expressions, particularly in identifying common factors, recognising factorization patterns, and distinguishing between factorizing and solving equations. Mofolo-Mbokane and Ngoveni (2019) concluded that students lacked foundational knowledge of factorization, particularly in recognising factorization structures such as common factors, difference of squares, and quadratic trinomials.

Qualitative analysis of students' test scripts highlighted four main types of errors: misused data, logically invalid, incomplete answer error, and technical errors. Misused data errors were observed in the responses of students like S13, who used the order of operators and exponential laws to factorize the expression of taking out a common factor, subtracted the coefficients 60-25, added 5, which equates to 40, and added all the exponents of x and y. This further indicates a lack of understanding of how to identify and apply the highest common factor in algebraic expressions. Muthma'innah (2024) highlighted that students need help working on questions and understanding how to identify the highest common factor. Logically invalid errors were observed in the work of students like S27, who started by grouping all the terms containing variables x and y together, then grouped the constant numbers and further added the terms in the first bracket by applying exponential law and used the order of operators to get the answer for the second bracket. Burhanzade and Aygor (2015) agreed that students could not correctly factorize expressions due to a lack of understanding of factorization identities and procedures. Incomplete answer errors were evident with students who correctly grouped the terms and took out the correct common factor in each bracket but did not continue to the next step of taking $(x-y)$ as a common factor and have a second bracket $(a+4)$. Technical errors were observed in the work of one student who correctly provided the factors of the expression but exchanged the signs. This reflects a conclusion by Mofolo-Mbokane and Ngoveni (2019) that students lacked procedural knowledge for finding the factors of two squares. Overall, these findings indicate that students' pre-existing knowledge of factorization is limited, particularly in identifying common factors, recognising algebraic structures, and distinguishing between factorization and equation solving. These gaps may significantly hinder their ability to engage successfully with Level 2 mathematics.

4.3.6.2. Interview findings: Students' difficulties with factorization (Question 3)

The interview responses provided valuable qualitative insight into the difficulties students encountered with the concept of factorisation. The analysis identified two overarching themes: conceptual gaps in factorization and related terminology, and procedural errors, particularly in applying correct factorization steps and maintaining the sequence of operations. These themes were supported by specific categories that illuminated distinct aspects of students' challenges with this mathematical concept.

4.3.6.2.1. Conceptual gaps in factorization and related terminology

This theme captures students' difficulties in understanding factorization methods and interpreting the requirements of the question. Two categories emerged within this theme: lack of knowledge and misinterpretation of task.

Lack of knowledge: A recurring issue in students' responses was a limited understanding of factorization methods. For instance, student S37 stated: "*I don't know the methods of factorization. I don't know how to factorize, and I didn't know what to do in Question 3, as a whole."* Similarly, student S4 confused the task with solving for variable *x*, stating: "*Yes, I know it, I did factorization, Ma'am. In 3.2, since I was given a number and a variable, it was easy to find the value of k. I am surprised to see my answer is wrong".* These responses indicate a lack of foundational knowledge of factorization methods and their application. Without foundational understanding, students are unlikely to correctly use algebraic procedures such as solving equations. Strengthening this knowledge base is critical for progressing in algebra.

Misinterpretation of the task: Some students misinterpreted the task and used incorrect procedures. For example, student S23 explained: "*I have a problem knowing exactly what I should do when asked questions like factorize. Lecturers should explain what terms like factorization and simplification mean. The lecturers assume that we know what all these terms mean because this is the knowledge that was done in the previous classes".* Similarly, S70 misunderstood the concept entirely: "*No, the only thing I remember: when they say factorize, you have to solve for x".*

These comments reflect conceptual confusion and highlight the need for clearer instruction and scaffolding around algebraic terminology. These students conflated related concepts such as factorization, simplification, and solving equations, indicating conceptual confusion.

4.3.6.2.2. Procedural errors in applying factorization

This theme reflects students' difficulties executing the factorization method. Although some students demonstrated partial understanding, many struggled to apply correct procedures. Two categories emerged: partial knowledge and misapplication, and challenges with execution.

Partial knowledge and misapplication: Some students demonstrated limited understanding and incorrectly applied known strategies. For example, student S13 said: *"I know the methods of factorization, and I know when to use them, however I made mistakes in all the questions. In 3.4, I forgot the next step after taking out a common factor in grouping".* Student S33 expressed a similar struggle: *"I did factorization in grade 9, but I don't understand the method of factorization. Question 3.1, I knew that I needed to take out a common factor, which is 5xy, but I thought since I took out 5xy as common then I would be left with nothing for the third term. Hence, I wrote my answer like this".* Some relied on calculators to support their working but lacked procedural fluency. For example, S6 remarked: *"I did factorization, but I forgot when to use its different method. It was easy to factorize 3.3 and find the value of x. I got the factors by using my calculator, but the signs are incorrect".*

These examples show that although some of the students began by using the correct process, they had either forgotten the steps or used inappropriate shortcuts. Mechanical methods like calculators helped with computation but do not replace the need for conceptual understanding and process accuracy.

Challenges with execution: Some students indicated that they guessed or skipped the question due to uncertainty. For instance, S48 said: "*Factorization is difficult. I left blank spaces in Question 3."* Similarly, S138 reported: *"I have no idea of the methods. I left Question 3 blank*". These responses reflect low confidence and uncertainty in tackling algebraic expressions. The students who guessed or left questions unanswered may be experiencing prior knowledge difficulties or lack of practice with procedural steps.

The interview responses reveal that students' difficulties with factorization stemmed from both conceptual and procedural gaps. Some misunderstood key terms or confused factorization with other algebraic operations. Others began the process but could not complete it as they had forgotten steps or were reliant on calculators. Several students demonstrated avoidance behaviours, indicating lack of confidence and insufficient prior knowledge to engage with factorization tasks.

4.3.7. Analysis of students' difficulties, alternative conceptions, and errors (Question 4)

Question 4 tested students' ability to solve for *x* in linear, exponential, and trinomial equations. Solving for *x* is a critical component in mathematics, as it requires procedural and conceptual understanding. Mastering this concept enables students to approach a wide range of mathematical problems effectively. Question 4.1 involved solving a linear equation.

4.1. $\mathbf{3x + 5 = 11}$

Question 4.1 – Solve for x [Linear equation]

Table 20 shows the performance of 150 students on Question 4.1. A total of 65 students (43%) answered correctly, 58 (39%) answered incorrectly, 6 (4%) provided partially correct answers, and 21 (14%) did not attempt to answer the question. Table 17 shows that of the 64 students (43%) who displayed errors, 40 made errors involving misused data, 15 made technical errors, and 9 provided incomplete answers.

Table 20 – Solving for *x*: Types and frequency errors [Question 4.1]

Difficulties	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	65 (43%)
Incorrect answer	40	0	0	8	0	48 (39%)
Partially correct	-	-	-	7	9	16 (4%)
No answer	-	-	-	-	-	21 (14%)
Total						**150 (100%)**

The researcher selected three examples of these errors for analysis. Figure 39 shows an extract from a student's solution demonstrating a technical error. The student did not change the sign when transposing 5, although they followed the correct method of solving for *x* in linear equation. This error suggests that the student lacked procedural knowledge for transposing terms, particularly the rule that the sign changes when a term is moved across the equal sign.

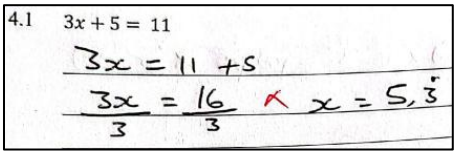


Figure 39 – Example of technical error (Participant S41)

Figure 40 shows an extract from a student's solution where a misused data error occurred. The student incorrectly applied quadratic methods to a linear equation. This error suggests a challenge in differentiating between equation types, particularly distinguishing linear equations from quadratic equations. This may indicate gaps in foundational knowledge of mathematical concepts.

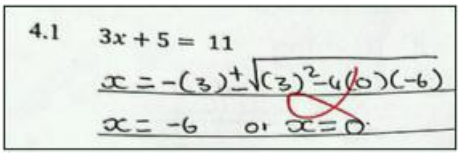


Figure 40 – Example of misused data (Participant S6)

Figure 41 shows an example of an extract from a student's solution that demonstrates an incomplete answer. The student demonstrated procedural knowledge by correctly transposing 11 from the right-hand side to the left-hand side, changing its sign from positive to negative. However, the error occurred in the next step, when dividing both sides by the coefficient *x*. The student incorrectly calculated $\frac{6}{3} = 3$, which indicates a mistake in basic arithmetic or a lapse in attention during division. This error suggests that the student may struggle to apply foundational arithmetic skills accurately, particularly in division when isolating variables.

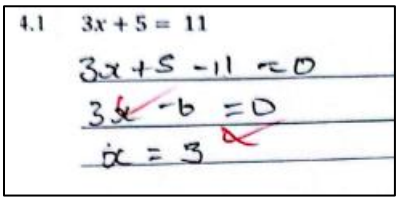


Figure 41 – Example of incomplete answer error (Participant S41)

These findings indicate that students' pre-existing knowledge of solving linear equations is limited, particularly in applying transposition rules and basic arithmetic operations when isolating variables.

Question (4.2) – Exponential equation (Equation 7).

4.2. $x^3 = \frac{1}{8}$

Question 4.2 – Solve for x – Exponential equation

Table 21 shows the results for the 150 students for Question 4.2. A total of 26 students (17%) answered correctly, while 6 (4%) provided partially correct answers. The majority, 80 students (54%), answered incorrectly, and 38 students (25%) did not attempt the question. A total of 118

students (79%) displayed errors, including 62 students who misused data and 24 students who made technical errors.

Table 21 – Solve for x: Types and frequency errors [Question 4.2]

Difficulties	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	incomplete answer errors	
Correct answer	-	-	-	-	-	26 (17%)
Incorrect answer	62	0	0	18	0	80 (54%)
Partially correct	-	-	-	6	-	6 (4%)
No answer	-	-	-	-	-	38 (25%)
Total						**150 (100%)**

Figure 42 shows an example of an extract from a student's solution that demonstrates a misused data error. The student appeared to lack the procedural knowledge for converting a fraction to an exponential form. Instead of expressing $\frac{1}{8}$ as an exponent, the student rewrote the left-hand side as a fraction to isolated x through cross-multiplication. In the next step, the student incorrectly divided both sides by 8 and then inverted $\frac{1}{8}$ to $\frac{8}{1}$, ultimately arriving at $x = 3$, a result that does not logically follow from the equation. This miscalculation suggests confusion between solving linear and exponential equations, particularly in recognising when to apply exponent laws instead of algebraic manipulation such as cross multiplication. Additionally, the student's inversion of the fraction indicates a misunderstanding of how to divide fractions with coefficient. This error highlights a fundamental gap in the student's understanding of exponential laws, particularly the process of rewriting expressions with a common base and equating exponents. The student lacked procedural knowledge of exponential laws, particularly in converting fractions to exponential form and applying negative exponent rules.

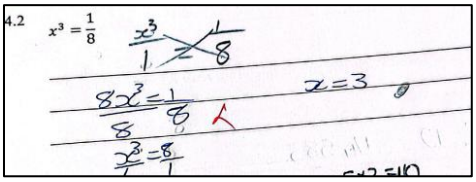


Figure 42 - Example of misused data errors (Participant S37)

Figure 43 show an extract from a student's solution that demonstrates a technical error. The student correctly expressed the denominator of 8 as 2^3 but did not recognize that the exponent

applies to numerator 1. In the next step, the student incorrectly rewrote $\frac{1}{2^3}$ as 2^3, failing to apply the negative exponent rule—the correct transformation should have been 2^{-3}. As a result, the student was unable to correctly equate the exponents. This error suggests a misunderstanding of exponent rules, particularly when converting expressions from fractional form to exponential form using negative exponents.

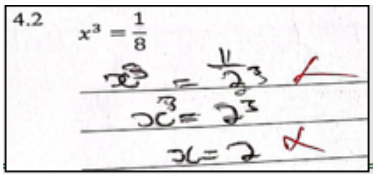


Figure 43 – Example of technical errors (Participant S33)

These findings indicate that students' pre-existing knowledge of exponential equations is insufficient, particularly in applying exponent rules and recognising equivalent exponential forms.

Question 4.3 Quadratic trinomial (Equation 8)

4.3. $x^2 - 7x + 10 = 0$

Question 4.3 – Solve for x – Quadratically

Table 22 shows the results of 150 students assessed on Question 4.3. A total of 45 students (30%) answered correctly, while 71 students (47%) provided incorrect answers. Two students (2%) gave partially correct answers, and 32 students (21%) did not attempt to answer the question. Of the 73 students (49%) who displayed errors, 64 students made misused data errors and 9 made technical errors.

Table 22 – Solving for *x*: Type and frequency of errors [Question 4.3]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	45 (30%)
Incorrect answer	64	0	0	7	0	71 (47%)
Partially correct	-	-	-	2	-	2 (2%)
No answer	-	-	-	-	-	32 (21%)
Total						**150 (100%)**

Figure 44 shows an extract from a student's solution that displays technical errors. The student correctly identified one factor of the equation but made a sign error when determining the second factor. While the overall method for solving quadratic equations was correctly applied, the mistake suggests difficulty handling signs during factorization, particularly when determining the correct signs of binomial factors. The student may have mistakenly assumed that the sign of the third term must be assigned to the larger factor.

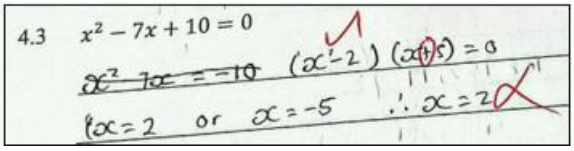


Figure 44 – Example of technical error (Participant S4)

Figure 45 shows an example of an extract from a student's solution that demonstrates a misused data error. The student incorrectly transposed the second and third terms and then divided both sides of the equation by *x*. In the next step, the student cancelled out *x*, leaving *x* on the left-hand side and constant terms on the right-hand side. This suggests a lack of procedural knowledge for solving quadratic equations using factorization, particularly recognising when to factorise and set each factor equal to zero. The student appeared to treat the equation as if it were linear, attempting to isolate *x* rather than recognizing that quadratic equations require factoring and setting each factor equal to zero This misconception indicates confusion between solving linear and quadratic equations, particularly treating quadratic equations as linear by attempting to isolate x directly.

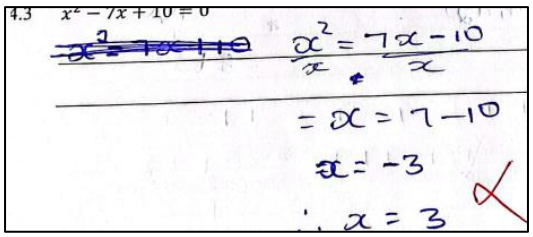


Figure 45 – Example of misused data (Participant S37)

This misconception indicates confusion between solving linear and quadratic equations, particularly treating quadratic equations as linear by attempting to isolate x directly. The researcher interviewed the students whose solutions are shown in Figures 44 and 45, along with others who answered this question incorrectly, to gain insight into their reasoning and their level of understanding of this concept. These interviews were based on a sample of 21 selected scripts. Of these, 9 students answered the entire question correctly. The table below presents the researcher's questions alongside the students' responses.

Table 23 - Unstructured questions: Question 4

Researcher's unstructured question: "Please take me through all your solutions for Question 4"			
Students' responses	Codes	Categories	Themes
S27: I jumped the question; I forgot all the steps of solving for x.	• Did not know how to solve for x (S27) • Forgot all steps (S27)	**Lack of Knowledge/Uncertainty** • Did not know how to solve for x.(S37) Forgot all steps.(S27) • Forgot how to solve for x; did not attempt.(S142) • Did not know how to approach(S23)	*Conceptual understanding issues* (focusing on students' grasp of the method or underlying ideas)
S27: I got 4.1 correct; in 4.2 I didn't understand the question so I left it; in 4.3 I found the factors and applied the sign correctly.	• Left blank/skipped question due to uncertainty (S27) • Confusion in method selection (S27)	**Uncertainty in Approach** • Left 4.2 blank due to uncertainty.(S41) • Skipped question(S4) • Confusion in method selection.(S4) • Left blank(S37) • Jumped the question(S27) • 4.2: Left blank due to non-understanding (S27) • 4.2: Left blank due to lack of understanding (S22)	
S23: It is hard to manipulate the equation to make x the subject—that's why my answer in 4.1 is incorrect. I did not know 4.2. In 4.3, I took -7x to the right-hand side, divided terms containing x by x, then subtracted 7 from -10 to get -3.	• Uncertainty in manipulation (e.g., exponent/fraction changes) (S23) • Technical errors (transposition, sign error, or forgetting formulas)" (S23) • Difficulty manipulating the equation" (S23)	• **Technical/Procedural Errors** • 4.1: Technical transposition/sign error.(S48) • 4.2: Uncertainty with cube root manipulation.(S48) • 4.2: Uncertainty in equating bases due to exponent confusion. (S11) • 4.3: Attempted manipulation. (S23) • 4.1: Difficulty manipulating the equati4.2: (S23) • 4.2:Uncertainty in exponent manipulation on (S33) • 4.1: Grouped terms correctly but had a writing/sign error. (S22)	**Procedural Execution Issues (focusing on the actual application and execution of the procedure).**

		• 4.2: Confusion in fraction/exponent manipulation and uncertainty(S6) • 4.1: Used quadratic but forgot the full formula (S6)	
S11: My 4.1 and 4.3 are correct; in 4.2 I changed the fraction into $1/2^3$, then changed the denominator to numerator and the exponent to negative. But I wasn't sure how to equate the bases, so I left it.	• Correct procedure/approach (S11)	**Successful Procedures** • 4.3: Correct (S48) • 4.1 & 4.3: Correct (S11)(S27)(S33) • 4.3: Correct procedure(22) • Correct procedure in 4.1 & 4.3(S41) • 4.3: Correct (S6)	
S48: In 4.1, I made a transposition error—forgot to change the sign, so my answer was wrong. In 4.2, I introduced cube roots on both sides and found 1/3, but I wasn't sure why my answer was wrong. I got 4.3 correct.	• 4.1: Technical transposition/sign error (S48) • 4.2: Uncertainty with cube root manipulation (S48) • 4.3: Correct (S48)		
S142: I did not answer Question 4; I forgot how to solve for x	• Forgot how to solve for x; did not attempt (S142)		
S33: I got 4.1 correct. In 4.2, I changed the denominator to exponents but wasn't sure how to solve for x from there. 4.3 is correct.	• 4.1 & 4.3: Correct (S33) • 4.2: Uncertainty in exponent manipulation (S33)		
S6: In 4.1, I used the quadratic equation, though I don't really remember the formula. In 4.2, I	• 4.1: Used quadratic but forgot the full formula(S6) • 4.2: Confusion in fraction/exponent		

changed x^3 to a fraction then cross-multiplied, but was unsure of the next step: dividing by 8 took me back to the original equation. In 4.3 I got it correct.	manipulation and uncertainty(S6). • 4.3: Correct (S6)		
S37: I did not know how to solve for x. Then I left blank spaces	• Did not know how to solve for x(S37) • Left blank		
S4: It is not easy to understand which method to use when solving for x. I jumped the question.	• Skipped question(S4) • Confusion in method selection(S4)		
S41: In 4.1, I took 5 to the right-hand side—the constant— and divided by 3 to isolate x. In 4.2, I left a blank space because I didn't know how to approach the question. In 4.3, I got the question correct.	• Left 4.2 blank due to uncertainty(S41) • Correct procedure in 4.1 & 4.3(S41)		

4.3.7.1.Analysis of students' responses to Question 4 (Solve for x)

The analysis of students' responses to Question 4 revealed significant challenges in students' ability to solve for *x*, which were evident in both quantitative data and qualitative insights from interviews and detailed examination of students' test scripts. Among the 150 students assessed, on average of 45 (30%) provided correct answers, while on average 5 (3.33%) solved the question partially correctly. Furthermore, a high average of 70 (47%) answered incorrectly, and 30 (20%) left the question blank. These results suggest difficulties in understanding and manipulating equations, particularly in selecting appropriate methods and correctly applying algebraic procedures to isolate the variable. This aligns with the findings of other studies highlighting the challenges students face with solving algebraic equations. Mgidi (2023) concluded that Grade 9 students lacked a clear understanding of variables and their role in equations, particularly when manipulating expressions to isolate the unknown.

Qualitative analysis of students' test scripts highlighted three types of errors: misused data errors, distorted definition errors, and technical errors. Misused data errors were made, particularly when students applied inappropriate methods such as cross multiplication to exponential equations. Distorted definition errors were observed where students applied inappropriate formulas, such as using the quadratic formula for linear equations. This suggests that students lacked understanding of when to use quadratic equations. Ngoveni (2018) found that students did not know how to approach particular types of equations, especially in selecting appropriate methods for linear, exponential, and quadratic equations. One student displayed technical errors were observed, particularly in transposition and sign changes when isolating variables. Mgidi (2023) reported that students did not change the sign when they transposed the term.

4.3.7.2.Interview findings: Students' difficulties solving for x (Question 4)

The interview responses provided valuable qualitative insights into the difficulties students encountered in solving for *x* for different types of equations, including liner, exponential, and quadratic forms. The analysis identified two overarching themes: issues with conceptual understanding (focusing on students' grasp of the method or underlying ideas) and issues with procedural execution (focusing on the actual application and execution of the procedure). Each theme is supported by specific categories that highlight distinct challenges.

4.3.7.2.1. Issues with conceptual understanding

This theme captures students' struggles to understand the general concept of solving for *x* and choosing appropriate methods. Two categories emerged within this theme.

Lack of knowledge: Several students displayed an inability to initiate solving the problem and gave no answer on the test paper. Student S37 said: *"I did not know how to solve for x, then I left blank spaces."* Similarly, S27 said: "*I jumped the question; I forgot all the steps of solving for x."* These responses suggest a lack of retained knowledge of equation-solving procedures, particularly the steps required to isolate variables.

Uncertainty: Other students expressed uncertainty about which method to use or how to begin solving the problem. For instance, student S4 explained: "*It is not easy to understand which method to use when solving for x. I jumped the question."* Student S41 also reported experiencing confusion: *"I left a blank space because I didn't know how to approach the question".* These responses highlight conceptual difficulty in distinguishing equation types and selecting appropriate solution methods. A foundational understanding of how to manipulate algebraic expressions is crucial for solving equations effectively; without this, students are likely to struggle with broader mathematical contexts.

4.3.7.2.2. Issues with procedural execution

This theme reflects students' difficulties applying the correct steps to solve for *x*. Even when students demonstrated an understanding of the concept, many struggled with procedural execution. Two categories emerged within this theme.

Technical/procedural errors: Some students initiated correct strategies but committed procedural mistakes along the way. For example, S11 remarked: *"I changed the fraction into* $1/2^3$*, then changed the denominator to numerator and the exponent to negative. But I wasn't sure how to equate the exponents, so I left it".* Student S33 said: "*I used the quadratic equation, even though I don't really remember the formula*". S48 reported a common algebraic error: *"I made a transposition error—forgot to change the sign. So my answer was wrong".* These examples show that while these students understood the initial direction of answering the question, they struggled with later steps due to incomplete procedural knowledge, especially when handling exponents and signs.

Successful procedure: Some students displayed an adequate understanding of algebraic equations by solving the problem correctly. Student S33 said: *"I understand the procedure to*

answer question 4.1 and 4.3". S27 said: "*I understand how to answer 4.1*". S11 said: *"I used the right procedure to answer questions 4.1 and 4.3."* S48 reported: *"I understand the procedure to answer 4.3. I found the factors and applied the sign correctly".* This suggests that the principles required to solve the question were known and were used by the student to answer the question and solve for *x*.

The interview data reveals that students' difficulties solving for *x* arose from both conceptual and procedural challenges. While some students struggled to recall or identify the correct method, others began the process but were unable to arrive at a correct solution due to sign errors, incomplete steps, or errors in their manipulation of exponent. A small number, however, demonstrated strong procedural fluency and completed the tasks correctly.

Overall, these findings indicate that students' pre-existing knowledge of solving equations is insufficient, particularly in selecting appropriate methods, applying algebraic procedures, and distinguishing between different types of equations. These gaps may significantly hinder their success in Level 2 mathematics.

4.3.8. Analysis of students' difficulties, alternative conceptions, and errors (Question 5)

Question 5 assessed students' abilities to interpret word problems and translate them into mathematical equations. Word problems are often used in mathematics assessments as they require students to apply mathematical concepts to real-life scenarios.

Question 5.1 Word problems and financial mathematics

5.1. A turkey costs 5 times more than a chicken. A farmer bought 20 turkeys and 50 chickens for R1200. Calculate the price of one turkey. **(Budgeting)**

5.2. Calculate how much I must invest at 7% p.a. simple interest to have R 15 000 in my account at the end of 6 years. **(Financial Mathematics)**

5.3. A Taxi covers a distance in 3 hours at an average speed of 80km/h. How long will it take to cover the same distance if the average speed is 60 km/h?
(Distance)

Question 5 – Word Problems

Table 24 summarizes the performance of 150 students on Question 5.1. Only 5 students (3%) answered correctly, while 75 students (50%) answered incorrectly. No students provided partially correct answers, and 70 students (47%) did not attempt the question. All 75 of the students who answered incorrectly displayed misused data errors.

Table 24 – Word Problem: Type and Frequency Errors [Question 5.1]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	incomplete answer errors	
Correct answer	-	-	-	-	-	5 (3%)
Incorrect answer	75	0	0	0	0	75 (50%)
Partially correct	-	-	-	-	0	0(0%)
No answer	-	-	-	-	-	70 (47%)
Total						**150 (100%)**

The researcher selected one example of this error for analysis. Figure 46 shows an extract from a student's solution where misused data errors were made. The student added the number of chickens and turkeys, multiplied the sum by 5, and then divided by R1200. This error indicates a lack of procedural knowledge for translating word problems into mathematical equations, particularly identifying relevant variables and forming appropriate relationships between quantities. Additionally, the student struggled with computational aspects, particularly applying basic arithmetic operations correctly within the context of a word problem.

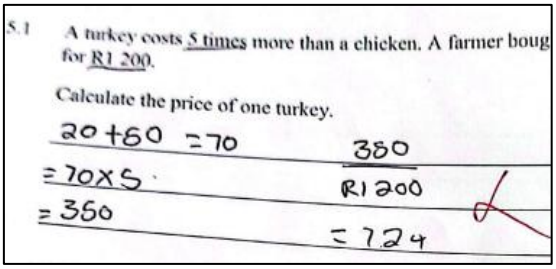


Figure 46 – Example of misused data error (Participant S37)

These findings indicate that students' pre-existing knowledge of translating word problems into mathematical representations is limited, particularly in identifying relationships between quantities and forming equations.

Question 5.2 Simple and compound interest

Table 25 shows the results of the 150 students for Question 5.2. Eight students (5%) answered correctly, 100 students (67%) answered incorrectly, 2 students (1%) provide partially correct answers, and 40 students (27%) did not attempt to answer the question. Of the 102 students

(68%) who displayed errors, 77 misused data, 23 made technical errors, and 2 made incomplete data errors.

Table 25 – Financial Mathematics: Type and frequency errors [Question 5.2]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	8 (5%)
Incorrect answer	77	-	-	23	0	100 (67%)
Partially correct	-	-	-	-	2	2 (1%)
No answer	-	-	-	-	-	40 (27%)
Total						**150 (100%)**

Figure 47 shows an extract from a student's solution demonstrating a technical error. Although the student correctly identified the formula for simple interest, the student swapped the values of A and P, demonstrating confusion between the accumulated value and the initial investment. This error indicates a lack of conceptual understanding of financial terminology, particularly the distinction between principal (P) and accumulated amount (A) in interest calculations.

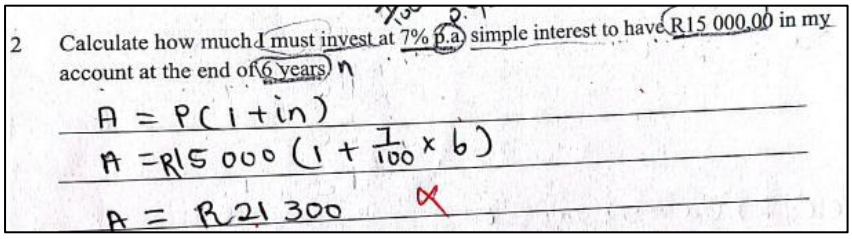


Figure 47 – Example of technical error (Participant S23)

Figure 48 shows an extract from a student's solution where the answer was incomplete. The student correctly substituted values into the formula but made errors using a calculator, leading to an incorrect final answer. This suggests difficulty manipulating equations, particularly in substituting values correctly and isolating the unknown variable.

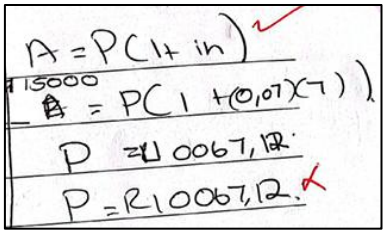


Figure 48 – Example of technical error (Participant S85)

Figure 49 shows an extract from a student's solution that demonstrates a misused data error. The student's response was mathematically incorrect, demonstrating a fundamental lack of understanding of the formula, substitution process, and the use of a calculator. This student likely lacked familiarity with financial mathematics concepts, particularly the correct application of interest formulas and substitution procedures.

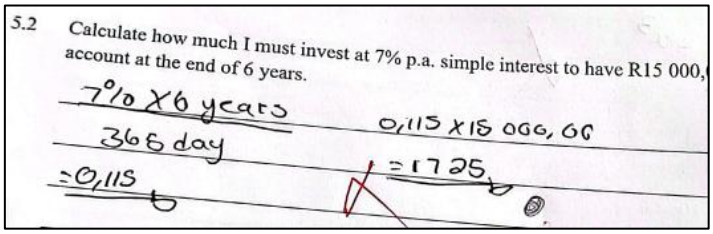


Figure 49 – Example of misused data error (Participant XX)

These findings indicate that students' pre-existing knowledge of financial mathematics is insufficient, particularly in understanding formulas, interpreting variables, and applying substitution correctly.

Question 5.3: Speed, distance, and time. This question assessed students' ability to calculate distance and time using the formula $S = \frac{D}{T}$ with an understanding the meaning for these letters on the formula. Table 26 shows that out of 150 students, 9 students (6%) answered correctly, 76 students (51%) answered incorrectly, 10 students (6%) provided partially correct answers, and 55 students (37%) did not attempt to answer the question. Among the 86 students (57%) who displayed errors, 65 students committed misused data errors, 11 students made technical errors, and 10 students had incomplete data errors.

Table 26 – Distance, speed & time: Types and frequency errors [Question 5.3]

Difficulties	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	9 (6%)
Incorrect answer	65	-	-	11	-	76 (51%)
Partially correct	-	-	-	-	10	10 (6%)
No answer	-	-	-	-	-	55 (37%)
Total						**150 (100%)**

Figure 50 shows an extract from a student's solution demonstrating a misused data error. Instead of using the correct speed formula, the student applied a ratio method, leading to an incorrect solution. This indicates a misunderstanding of the question requirements, particularly selecting the appropriate formula (S = D/T) instead of applying unrelated methods such as ratios.

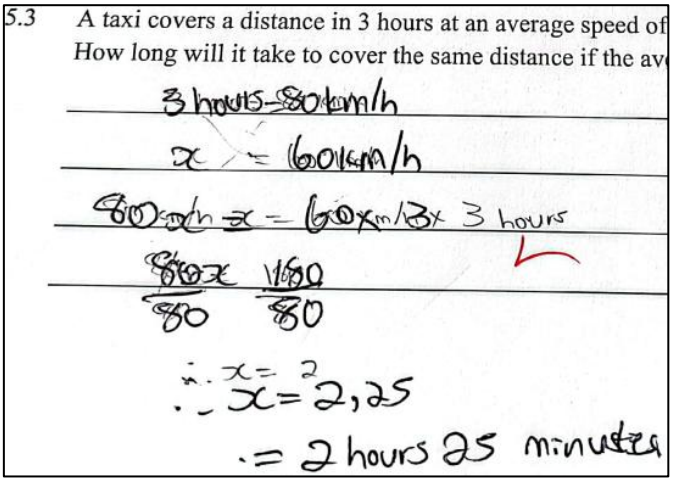


Figure 50 – Example of misused data (S70)

Figure 51 shows an extract from a student's solution that demonstrates a technical error. While this student demonstrated an understanding of speed (S), distance (D), and time (T), they structured the formula incorrectly, exchanging the values for distance and speed. This suggests the student experienced difficulty applying algebraic manipulations, particularly in rearranging formulas to isolate the required variable.

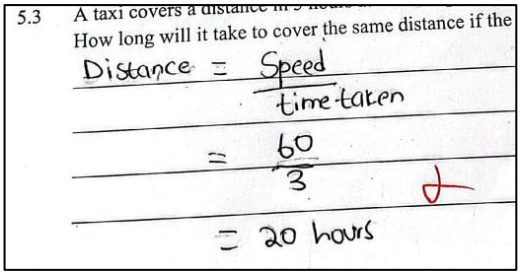


Figure 51 – Example of technical error (Participant S56)

Figure 52 shows an extract from a student's solution that demonstrates an incomplete answer error. The student correctly applied the formula and substituted values but failed to complete all required steps. Instead of using the calculated distance to determine time, the student assumed that finding the distance was the final step. This suggests difficulty understanding multi-step problem-solving, particularly recognising that intermediate results must be used to complete subsequent steps.

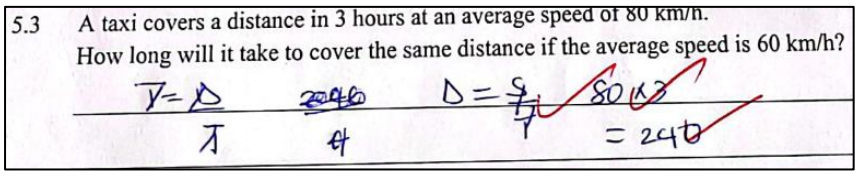


Figure 52 – Example of incomplete answer (Participant S11)

These findings indicate that students' pre-existing knowledge of applying formulas in context is limited, particularly in selecting appropriate formulas and completing multi-step calculations.

To better understand students' difficulties, the researcher conducted unstructured interviews with 21 students to explore why they had approached in the way they had. Table 28 presents the researcher's questions with the students' responses.

Table 27 - Unstructured questions: Question 5

Researcher unstructured interview: What were you thinking when you were answering Question 5?			
Students' responses	Codes	Categories	Themes
S37: 5.1 I did not know what I was doing, I just added all the given numbers. All the questions were difficult for me.	• Did not understand, added numbers (S37)	**Lack of conceptual understanding** • Did not understand, added numbers (S37) • Did not understand question, lacked prior knowledge, left blank responses **(multiple students)**	**Conceptual understanding**
S22, S4, S142, S24, S16, S41, S48, S33, S13, S26, S87, S47 who left blank spaces in 5.1: The question was difficult for me ma'am, I left it, **I did not know what to write, I jumped the question,** **I did not understand what was needed in the question,** **I don't remember doing this topic in grade 9, I was clueless.**	• Did not understand question, lacked prior knowledge, left blank responses (Multiple Students)	**Guessing or Filling Answers without Understanding** • Did not understand; guessing or filling of answers (S138)	
S138: To be honest, Ma'am, I didn't understand question 5.1. But I didn't want to leave blank spaces, so I wrote whatever that was in my mind—and I can't explain it now.	• Did not understand; guessing or filling of answers (S138)	**Misinterpretation of Concepts** • Misinterpretation of concepts; confusion between principal (P) and amount (A) (S23) • Misinterpretation of question; incorrectly used ratios (S70) • Incomplete calculation due to misunderstanding of instructions (S11)	

S23: In 5.2, I knew the formula, but I confused A and P; I didn't know which amount represented principal (P) or accumulated amount (A)"	• Misinterpretation of concepts; confusion between principal (P) and amount (A) (S23)	**Incomplete or Incorrect Substitution of Value** • Incorrect substitution of values (years) (S85) • Incorrect recall or application of speed formula (S56)	**Procedural Challenges in Applying Mathematical Models.**
S85: I understand 5.2, but I made a mistake of substituting the wrong number of years— which is 7— which I don't know where I got it.	• Incorrect substitution of values (years) (S85)	**Forgotten or Inaccessible Procedures** • Forgotten formula for simple interest (S6) • Forgotten formula or substitution steps; left blank **(Multiple students)**	
S27: I did not have any idea of how to answer the question. I just wrote what I was thinking is right. I multiplied all the given values.	• Did not understand, randomly added numbers (S37)		
S6: I understand simple interest, but I forgot the equation to calculate the simple interest. I wrote it incorrectly.	• Forgotten formula for simple interest (S6)		
S22,S4, S142, S24,S16, S41, S48, S33, S13, S26, S87, S47 who left blank spaces in 5.2; **I forgot the formula to calculate simple interest and I decided to leave the question.** **I wrote the formula, but now I could not substitute the values; then, I jumped the question.** **I didn't know where to start, that is why I left it blank. I was clueless' I left the question.**	• Forgotten formula or substitution steps; left blank **(Multiple students)**		

S70: Sub-question 5.3, I used ratio approach: I thought the question is about ratio.	• Misinterpretation of question; incorrectly used ratios (S70)		
S56: I know the letters we use to build the formula of speed, but I wrote it incorrectly.	• Incorrect recall or application of speed formula (S56)		
S11: 5.3 I understand the concept, but I did not know that I needed to continue and calculate time.	• Incomplete calculation due to misunderstanding of instructions (S11)		
S47, S87, S26, S13, S33, S48, S41, S16, S24,S142, S4, S22s who left blank spaces in 5.3: **Is this not physics, Ma'am? I don't remember doing this topic in high school.** **I did not understand the question.** **I was blank and left the question.**			

4.3.8.1.Analysis of students' responses to Question 5 (Word problems)

The analysis of students' responses to Question 5 revealed significant challenges in students' ability to translate word problems to mathematical equations, which were evident in both quantitative data and qualitative insights from interviews and detailed examination of students' test scripts. Among the 150 students assessed, only 5 (3%) provided the correct answer to Question 5.1, while 75 students (50%) answered incorrectly, no students provided partially correct answers, and 70 students (47%) did not attempt to answer the question. For Question 5.2, 8 students (5%) answered correctly, while 100 students (67%) answered incorrectly. Additionally, 2 students (1%) provide partially correct answers, and 40 students (27%) did not attempt the question. For Question 5.3, 9 students (6%) answered correctly, 76 students (51%) answered incorrectly, 10 students (6%) provided partially correct answers, and 55 students (37%) did not attempt the question. These results suggest widespread difficulties in understanding and solving word problems, particularly in translating verbal information into mathematical equations and selecting appropriate solution strategies. (Emanuel, Kirana, and Chamida, 2020).

Qualitative analysis of students' test scripts highlighted three main types of errors: misused data errors, incomplete answer errors, and technical errors. Misused data errors were observed, particularly where students combined given values incorrectly without establishing meaningful mathematical relationships. Emanuel, Kirana, and Chamida (2020) mentioned that students struggle to generate appropriate formulas from word statements, particularly in identifying known and unknown variables. Incomplete answer errors were observed where students initiated correct procedures but failed to complete all required steps of the calculation. Technical errors were observed, particularly in algebraic manipulation and incorrect use of formulas. This aligns with the finding of Mathaba (2019) that students faced difficulty applying operational signs and lacked the necessary knowledge of algebraic manipulation to solve the equations correctly, thus making the unknown variable the subject of the formula.

4.3.8.2. Interview findings: Students' difficulties with word problems (Question 5)

The interview responses provided valuable qualitative insights into the difficulties students encountered with solving word and financial mathematics problems. The analysis identified two main themes: conceptual misunderstanding of word problems and procedural errors during calculation. These themes, supported by specific categories, highlight the core challenges students faced when approaching contextual mathematical tasks.

4.3.8.2.1. Conceptual misunderstanding of word problems

This theme captures the difficulties students experienced in understanding word problems, interpreting problem statements, and connecting them to appropriate mathematical strategies.

Lack of conceptual understanding: Several students mentioned that they did not understand how to translate word problems into equations or to how to begin solving the problem. For instance, the majority of students said: *"The question was difficult, I did not understand, I don't remember doing this topic in grade 9, I was clueless". Student S37 said "I did not know what I was doing; I just added all the numbers.* Multiple students (S22,S4, S142, S24, S16, S41, S49, S33, S13, S26, S87, and S47) expressed confusion about the topic, stating: *"All questions were difficult", "The question was difficult for me ma'am. I left it", "I didn't know what to write, I jumped the question, I did not understand what was needed", "I don't remember doing this topic in grade 9. I was clueless."* These responses suggest that the students lacked a foundational understanding of how to interpret word problems and translate them into mathematical expressions and had poor recall of previously taught concepts. related to interpreting word problems, indicating content gaps.

Guessing or answering without understanding: Some students guessed so that they could write down a response. Student S138 stated: " *To be honest, Ma'am, I didn't understand question 5.1 but I didn't want to leave blank spaces. So I wrote whatever that was in my mind—and I can't explain it now."* This response reflects a strategy of guessing without understanding, indicating an inability to interpret the problem context and identify relevant mathematical operations.

Misinterpretation of concepts: Particularly misunderstanding variables such as principal (P), amount (A), and the relationship between speed, distance, and time. For example, Student S70 said: *"In 5.3, I misunderstood the question; I used ratios incorrectly instead of solving speed".* Similarly, S23 said: "*In 5.2, I knew the formula, but I confused A and P; I didn't know which amount represented principal (P) or accumulated amount (A)".* Student S11 said: "*In 5.3, I understood the concept but didn't realize I needed to continue and calculate time".* These responses indicate that these students found it challenging to apply appropriate mathematical models and correctly interpret key variables, particularly misunderstanding variables such as principal (P), accumulated amount (A), and the relationship between speed, distance, and time.

Procedural errors in applying mathematical models

This reflects procedural errors in applying mathematical models, particularly difficulties in substituting values correctly and executing multi-step calculations when translating word problems into equations. . Although students demonstrated an initial understanding of the task required, many struggled to execute the correct procedural steps accurately. From the responses of students, two categories emerged within themes; Incomplete or incorrect substitution of values and forgotten or inaccessible procedures.

Incomplete or incorrect substitution of values: Some students substituted incorrect values into correct formulas, suggesting that they became confused when they attempted to identify which numbers aligned with which variables. For example, S85 said: *"I understood 5.2, but I mistakenly substituted 7 for the number of years, and I don't know where it came from".* Student S56 said: *"I know the formula letters for speed, but I wrote the formula incorrectly".* These examples reflect partial understanding and poor procedural accuracy, particularly in identifying correct values for substitution and applying formulas appropriately. These students appeared to recall formulae but could not remember their proper application or structure.

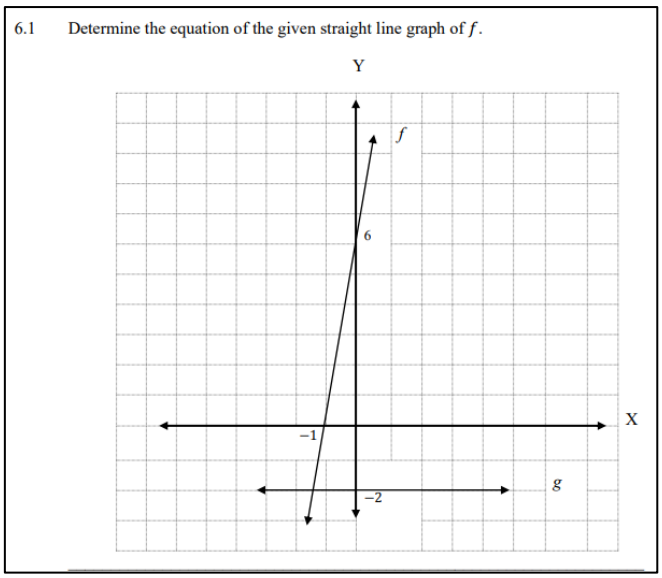
Forgotten or inaccessible procedures: Some students could not remember the formulae or steps required to solve the problems and provided an incorrect answer to the question. For example, S6 said: *"I understand simple interest, but I forgot the equation; I wrote it incorrectly".* Eight other students (S4, S16, S22, S26, S41, S47, S48, S87, S142) indicated that they had forgotten the formula or had not been able to proceed with substitution and left the answer paper blank. For example, S22 said: "*I forgot the formula or substitution steps and therefore left blank spaces".* S16 said: *"I wrote the formula, but I couldn't substitute the values, so I jumped the question." Similarly, S87 said: "I didn't know where to start, so I left it blank."* These responses suggest forgotten or inaccessible procedures, particularly difficulty recalling formulas and applying them correctly within problem contexts. The findings show that these students struggled with both understanding and executing mathematical procedures, particularly in translating real-world situations into mathematical models, selecting appropriate formulas, and carrying out multi-step calculations. Common issues included misinterpreting variables, substituting incorrect values, confusion—resulting in guessing, and forgetting steps. As a result, many did not provide an answer to the question. While a few students showed partial understanding, the majority displayed conceptual and procedural weaknesses. These

gaps, especially in translating real-world scenarios into equations, hindered students' ability to problem-solve.

Overall, these findings indicate that students' pre-existing knowledge of word problems and financial mathematics is significantly limited, particularly in translating real-world situations into mathematical models, selecting appropriate formulas, and executing multi-step procedures. These gaps may severely hinder their ability to solve applied mathematical problems at Level 2.

4.3.9. Analysis of students' difficulties, alternative conceptions, and errors (Question 6)

Question 6 assessed students' abilities to determine the equation of a straight-line graph.



6.2. *Write down the equation of the graph of g.*

Question 6.2 - Straight-line Graph

Table 28 shows the performance of 150 students on Question 6.1. Only 3 students (2%) answered correctly, while 49 students (33%) answered incorrectly, and 4 students (3%) provided partially correct answers. A large number of students, 94 (62%), did not attempt to answer the question. Among the 53 students (35%) students who displayed errors, 40 committed misused data errors, 9 made technical errors, and 4 provided incomplete answer errors.

Table 28 – Straight-line equation: Type and frequency of errors [Question 6.1]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	3 (2%)
Incorrect answer	40	0	0	9	0	49 (33%)
Partially correct	-	-	-	-	4	4 (3%)
No answer	-	-	-	-	-	94 (62%)
Total						**150 (100%)**

Figure 53 shows an extract from a student's solution that demonstrates technical errors and incomplete answers. The student correctly determined that the gradient must be calculated first, but mistakenly used addition instead of subtraction when substituting values into the gradient formula. Additionally, while the student understood the requirement to substitute a given point to determine the y-intercept, they failed to substitute the calculated gradient value into the final equation. This suggests that they may have incorrectly assumed that the gradient equation always contains positive signs.

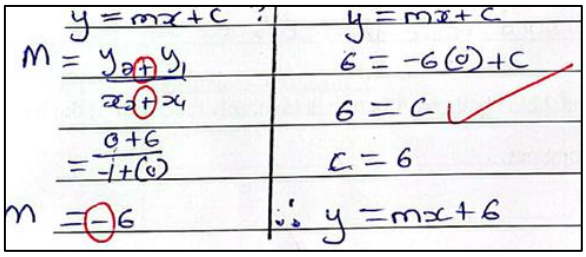


Figure 53 – Example of technical errors (Participant S13)

Figure 54 shows an extract from another student's solution, demonstrating a technical error. Although the student knew that the gradient must be calculated first and correctly identified the formula, they misplaced the *x*-values in the denominator and interchanged the values. This error appears to be a careless mistake, as the numerator was substituted correctly.

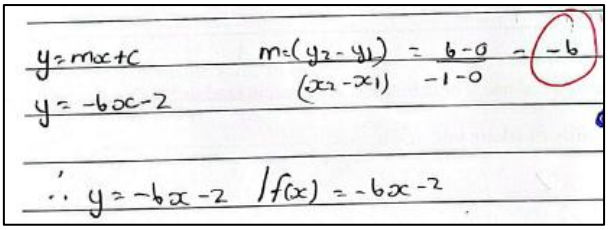


Figure 54 – Example of technical errors (Participant S138)

Figure 55 shows an extract from a student's solution demonstrating a misused data error. The student lacked the procedural knowledge required to calculate the gradient and substituted incorrect values into the equation. This suggests that the student may have mistakenly believed that they could determine the equation by substituting the *x*- and *y*-values into the standard formula without first calculating the gradient.

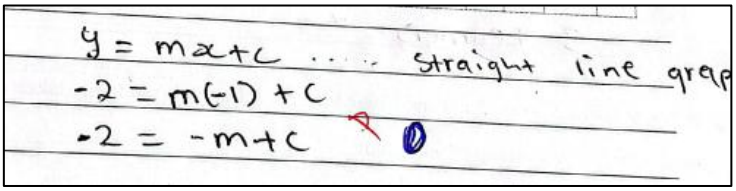


Figure 55 - Example of misused data error (Participant S142)

Table 29 shows the performance of 150 students on Question 6.2. A total of 19 students answered correctly, while 53 students (35%) answered incorrectly. A large portion, 78 students (52%), did not attempt to answer the question. All 53 who answered incorrectly made misused data errors.

Table 29 - Straight-line equation: Type and frequency of errors [Question 6.2]

Difficulties	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	19 (13%)
Incorrect answer	53	-	0	-	-	53 (35%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	78 (52%)
Total						**150 (100%)**

Figure 56 shows an extract from a student's solution demonstrating a misused data error. The student incorrectly identified the gradient as 2 and the *y*-intercept as -1, demonstrating a lack of procedural knowledge for straight-line graphs. This response suggests that the students lacked prior foundational knowledge, making it difficult for them to develop a conceptual understanding of the topic. Additionally, the student did not recognize that a horizontal line always has a gradient of zero.

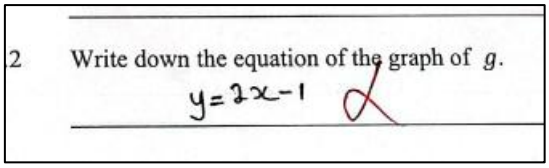


Figure 56 – Example of misused data errors (Participant S142)

To explore students' approaches to solving the problem more deeply, the researcher conducted unstructured interviews with some of the students. Table 30 presents a summary of the researcher's unstructured interview questions and students' responses regarding their thought processes and errors.

Table 30 - Unstructured questions: Question 6

Researcher's unstructured interview: Please define a gradient for me. Do you understand the effect of the gradient on the graph? When m>0, m<0,and m=0? Can you explain why you solve this question this way?			
Students responses	Codes	Categories	Themes
S13: I made a mistake when I was writing the formula of gradient and that cause me to get wrong answer: and I got the y-intercept correct.	• Mistake writing the gradient formula (S13)	**Lack of Conceptual Understanding (No knowledge, forgot the topic)** • Lack of knowledge, forgetting topic, uncertainty about method (**multiple students)** • Lack of knowledge in interpreting and solving graph-related questions (S37)	**Conceptual misunderstanding**
S138: I confused the value of both of the x, then I read the y-intercept from the second graph. I didn't understand why the second line is there.	• Confusion of graph elements (x-values/y-intercept), misunderstanding second graph line (S138)	**Misinterpretation of Concept (Confusion between elements of the graph)** • Confusion of graph elements (x-values/y-intercept), misunderstanding second graph line (S138)	
15 students (SS27, S4, S6, S70, S23, S24, S16, S11, S41, S48, S33, S56,S26,	• Lack of knowledge, forgetting topic, uncertainty about	**Partial Knowledge; Uncertainty (Unsure of steps or method)**	

87, S47) who left blank spaces in 6.1: • **I have no idea how to find the equation of the straight-line.** • **What I know is that I need to change y=mx+c to numbers. I was not sure how and I decided to leave the question.** • **I forgot this topic, Ma'am, that is why I left blank space.** • **I don't know the question.**	method **(multiple students)**	• Uncertainty in point selection, forgotten steps (S142)	
S142: What I know, Ma'am ,is that we must take the points from the graph to substitute to y=mx+c, but I am not sure which points to take. I forgot this topic	• Uncertainty in point selection, forgotten steps (S142)	**Mistakes in writing Formula** • Mistake writing the gradient formula (S13) **Misreading or Misinterpreting graph information** Incorrect identification of gradient values from graph (S22)	**Procedural Errors**
S22: I read the value of the gradient from the graph as 2. Since the given value in this line is -2, I don't know where I got this 1.	• Incorrect identification of gradient values from graph (S22)		
S37: I still don't know how to answer the question regarding to graphs.	• Lack of knowledge in interpreting and solving graph-related questions (S37)		

4.3.9.1. Analysis of students' responses to Question 6 (Straight line graph)

The analysis of students' responses to Questions 6.1 and 6.2 revealed significant challenges in their ability to calculate the equation of a straight line, evidenced by quantitative data, qualitative insights from interviews, and detailed examination of students' test scripts. Of the 150 students, for Question 6.1 only 3 students (2%) answered correctly, while 49 students (33%) answered incorrectly, 4 students (3%) provided partially correct answers. A large number of students, 94 (62%) did not attempt to answer the question. For question 6.2, a total of 19 students answered correctly, while 53 students (35%) answered incorrectly. A large portion, 78 students (52%), did not attempt to answer the question. These results suggest widespread difficulty in determining the equation of a straight-line graph, particularly in calculating the gradient, identifying the y-intercept, and correctly applying the formula y=mx+c. Ngwabe (2018) concluded that Grade 9 students were unaware of the procedure for finding the equation of the graph. Ngwabe (2018) reported that these students lacked procedural knowledge of a straight-line graph.

Qualitative analysis of students' test scripts highlighted two main types of errors: technical errors and misused data. Technical errors were evident, particularly in substituting coordinate values into the gradient formula and maintaining correct order of operations. This error appears to be a careless mistake, as the numerator was substituted correctly. Misused data errors were evident, particularly where students incorrectly identified gradients and failed to recognise that a horizontal line has a gradient of zero. The outcome of this study is reflected the conclusion of Ngwabe (2018) that students lack procedural knowledge of straight-line graphs, particularly in calculating gradients from coordinates and substituting values correctly into the equation y=mx+c. Interview findings: Students' difficulties with straight line graphs (Question 6)

The interview responses provided valuable qualitative insights into the difficulties students encountered finding the equation of a straight line. The analysis identified two overarching themes: conceptual misunderstanding and procedural errors. These themes, supported by specific categories, illuminate distinct aspects of students' challenges with this mathematical concept.

4.3.9.1.1. Conceptual misunderstanding

This theme captures the difficulties students experienced in comprehending the foundational principles of straight-line graphs and interpreting what the question required. Three categories emerged within this theme.

Lack of conceptual understanding: A recurring issue among the students was an inability to determine the equation of a straight-line graph. This was evident from the responses of 15 students (S4, S6, S11, S16, S23, S24, S26, S27, S33, S41, S47, S48, S56, S70, S87) who said things like: *"I don't have an idea on how to find the straight-line equation"; "all I know is that I need numbers to substitute into y=mx+c but unsure how to find those numbers"; "I forgot the topic; I don't understand the question".* Student *S37* also revealed a lack of knowledge, stating: *"I still don't know how to answer graph-related questions"*. These responses indicate a significant lack of conceptual understanding of straight-line graphs, particularly the relationship between gradient, intercept, and the equation y=mx+c. **Misinterpretation of concepts:** Some students misinterpreted the task and confused key elements of the graph. For example, student S138 explained: "*I confused x-values and mistakenly read the y-intercept from a second graph; I didn't understand why a second line was present*". This suggests a misunderstanding of how to interpret graph features, particularly identifying the correct x- and y-values and distinguishing between multiple lines on a graph.

Partial knowledge and uncertainty: Some students were unsure how to proceed, particularly for selecting the correct points or recalling the steps to find the equation. For example, S142 shared: *"What I know, Ma'am, is that we must take the points from the graph to substitute to y=mx+c, but I am not sure which points to take. I forgot this topic".* While students were taught this topic in Grade 9, many showed no retained understanding. Such responses underscore the need to revisit this content, particularly the process of selecting correct points from a graph and applying them in the equation y=mx+c.

4.3.9.1.2. Procedural errors

This theme reflects the difficulties students encountered applying the correct method to find the equation of the line. Although some students demonstrated an initial understanding of the task, many struggled to carry out the correct procedural steps. Two categories emerged within this theme: mistakes in writing formulae, and misreading or misinterpreting graph information.

Mistakes in writing formulae: Some students demonstrated awareness of the necessary steps but made technical mistakes when writing the gradient formula. Student S13 explained: *"I made a mistake writing the formula for gradient, and that caused the wrong answer; but the y-intercept was correct*". Even though the student showed partial understanding, their inability to apply the formula correctly indicates a gap in their procedural knowledge, particularly in writing and applying the gradient formula and substituting values accurately. This suggests that the topic should be retaught in more detail.

Misreading or misinterpreting graph information: Other students misread the information presented in the graph. For example, S22 said: "*I read the value of the gradient from the graph as 2. Since the given value in this line is -2, I don't know where I got this 1".* This indicates confusion about how to extract relevant values from graphs, particularly identifying correct coordinates and using them appropriately in calculations.

The interview findings indicate that these students struggled with both understanding the concept of straight-line graphs and executing the procedural steps required to determine their equations, particularly calculating gradients, identifying intercepts, and correctly applying the formula y=mx+c. Key issues included a lack of foundational knowledge, confusion over graph elements, incorrect representation of formulae, and misreading of information.

Overall, these findings indicate that students' pre-existing knowledge of straight-line graphs is limited, particularly in understanding the relationship between gradient and intercept, interpreting graphical information, and applying algebraic procedures to determine equations. These gaps may significantly hinder their ability to engage successfully with Level 2 mathematics.

4.3.10. Analysis of students' difficulties, alternative conceptions, and errors (Question 7)

Question 7 assessed students' understanding and prior knowledge of key topics related to interpreting data and computing various statistical measures, including stem-and-leaf plots, range and mean calculations, percentages, and probability concepts.

Question 7.1.1: Stem-and-leaf graphs

7.1 The following are the marks obtained by a group of learners in a test out of 60.

12	21	23	35	42	54	57	31	43	35	46	37	55

7.1.1 Draw a stem-and-leaf plot to organize the data.

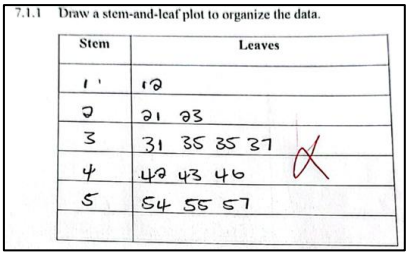
Test item 3 – Stem-and-leaf graphs

Table 31 summarizes the performance of 150 students on this question. A total of 19 students (13%) answered correctly, 41 students (27%) answered incorrectly, and only 3 students (2%) provided partially correct answers; 87students (58%) did not attempt to answer the question. Among the 44 students (29%) who displayed errors, 32 students committed misused data errors, 9 students displayed distorted theorem or definitions errors, and 3 students made incomplete answer errors.

Table 31 – Stem and Leaf: Types and Frequency Errors [Question 7.1.1]

Difficulties	**Errors**					**Frequencies and percentages**
	Misused data	**Logically invalid inference**	**Distorted theorem or definition**	**Technical Error**	**Incomplete answer**	
Correct answer	-	-	-	-	-	19 (13%)
Incorrect answer	32	0	9	-	0	41 (27%)
Partially correct	-	-	-	-	3	3 (2%)
No answer	-	-	-	-	-	87 (58%)
Total						**150 (100%)**

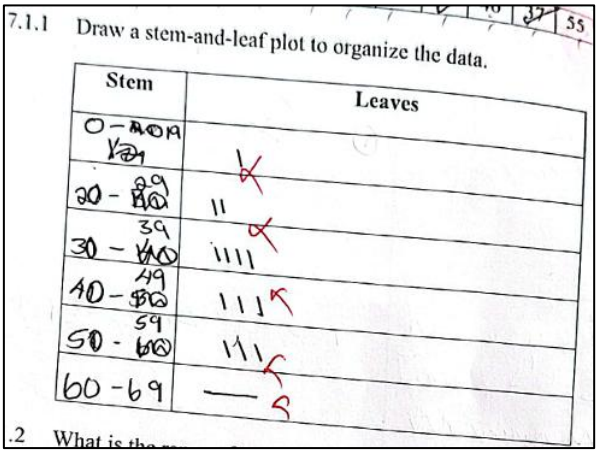
Figure 57 shows an extract from a student's solution demonstrating a misused data error. The student correctly placed the stem numbers in the appropriate column but incorrectly wrote both the tens and units in the leaves. This suggests a misunderstanding of the stem-and-leaf plotting process, as only units should appear in the leaf section. The student appears to confuse the stem-and-leaf diagram by mistakenly placing both digit under the leaves columns instead of separating them.


7.1.1 Draw a stem-and-leaf plot to organize the data.

Stem	Leaves
1	12
2	21 23
3	31 35 35 37
4	42 43 46
5	54 55 57

Figure 57 – Example of misused data (Participant S22)

Figure 58 shows an extract from a student's solution demonstrating a distorted definition error. Instead of plotting a stem-and-leaf graph, the student constructed a tally chart. The student also wrote grouped data in the stem column instead of listing tens from 1 to 9. This error suggests a fundamental misconception about the difference between a stem-and-leaf diagram and a frequency tally chart.


7.1.1 Draw a stem-and-leaf plot to organize the data.

Stem	Leaves
0 – 9	I
20 – 29	II
30 – 39	IIII
40 – 49	III
50 – 59	III
60 – 69	—

.2 What is the

Figure 58 – Example of distorted definition (Participant S37)

Question 7.1.2: Calculating the range of the data

7.1.2 What is the range of the marks?

Question 7.1.2 – Range

Table 32 shows the performance of 150 students on this question. A total of 19 students (12%) answered correctly, while 79 students (53%) answered incorrectly. No students provided partially correct answers, and 52 students (35%) did not attempt to answer the question. Among the 79 students who answered incorrectly, 74 students made misused data errors and 5 students made technical errors.

Table 32 – Range: Type and frequency of errors [Question 7.1.2]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	19 (12%)
Incorrect answer	74	0	0	5	0	79 (53%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	52 (35%)
Total						**150 (100%)**

Figure 59 shows an extract from a student's solution demonstrating a misused data error. The student incorrectly calculated the range by adding all data points together, then dividing by the total number of values. This suggests confusion between calculating range and computing mean, indicating a lack of procedural knowledge for distinguishing between these statistical concepts.

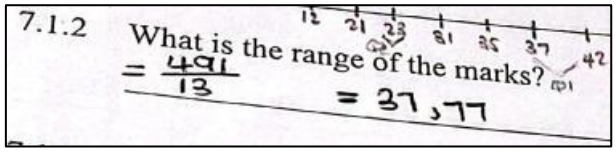


Figure 59 – Example of distorted definition (Participant S27)

Figure 60 shows an extract from another student's solution demonstrating a misused data error. This student incorrectly identified the highest number (57) as the range instead of computing the difference between the highest and lowest values. This error suggests that the student misinterpreted the definition of range, confusing it with identifying the maximum value.

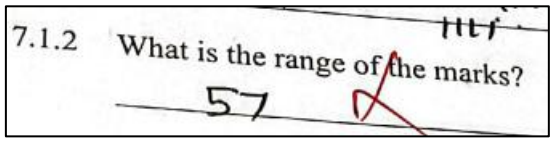


Figure 60 – Example of misused data (Participant S37)

Figure 61 shows an extract from a student's solution demonstrating technical errors. Instead of using the correct highest value (57), the student mistakenly identified 55 as the largest number, leading to an incorrect range calculation. This error suggests a possible careless mistake or misreading of the dataset, preventing the correct application of the range formula.

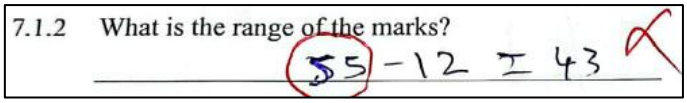
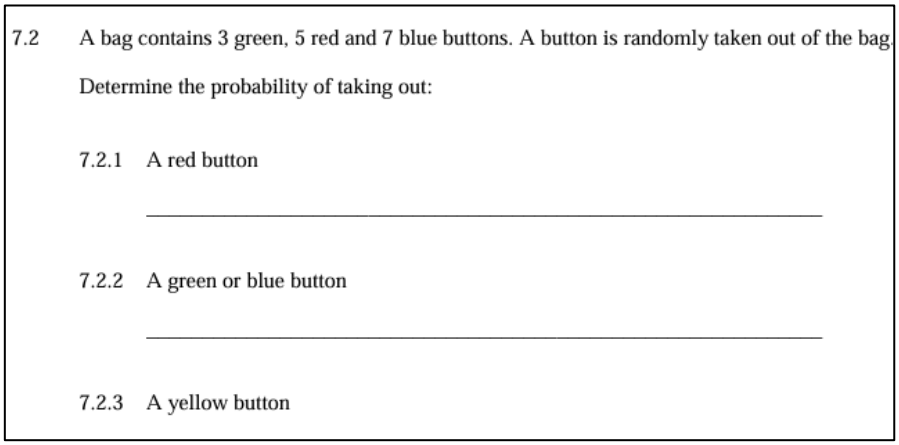


Figure 61 – Example of technical error

Question 7.2.: Probability



Question 7.2 - Probability

Table 33 presents the performance of 150 students on Question 7.2.1, which required students to determine the probability of selecting a red button from a bag. A total of 37 students (25%) answered correctly, 72 students (48%) students answered incorrectly, and 41 students (30%) did not attempt to answer the question. All 72 of the students who answered incorrectly made misused data errors.

Table 33 – Probability: Type and frequency of errors [Question 7.2.1]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	incomplete answer errors	
Correct answer	-	-	-	-	-	37 (25%%)
Incorrect answer	72	0	0	-	0	72 (48%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	41 (27%)
Total						**150 (100%)**

Table 34 presents the performance of 150 students on Question 7.2.2, which required students to determine the probability of selecting a blue or green button. A total of 28 students (19%) answered correctly, while 76 students (51%) answered incorrectly, and 46 students (30%) did

not attempt to answer the question. All of the 76 students who answered incorrectly made misused data errors.

Table 34 – Probability: Type and frequency of errors [Question 7.2.2]

Difficulties	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	28 (19%)
Incorrect answer	76	0	0	0	0	76 (51%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	46 (30%)
Total						**150 (100%)**

Table 35 presents the performance of 150 students on Question 7.2.3, which required students to determine the probability of selecting a yellow button. A total of 53 students (35%) answered correctly, while 49 students (33%) answered incorrectly, and 48 students (32%) did not attempt the question. All 49 students who answered incorrectly made misused data errors.

Table 35 – Probability: Type and frequency of errors [Question 7.2.3]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	53 (35%)
Incorrect answer	49	0	0	0	0	49 (33%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	48 (32%)
Total						**150 (100%)**

Figure 62 shows an extract from a student's solution demonstrating a misused data error. The student's response indicates a lack of procedural knowledge for calculating and a poor understanding of the question. This error suggests that the student lacked fundamental conceptual knowledge of probability and how it is computed. For example, when computing the probability of selecting either a green or blue button, the student should have added the individual probabilities of green and blue. When computing the probability of selecting a

yellow button, the student should have recognized that yellow buttons were not included in the sample space, leading to a probability of zero.

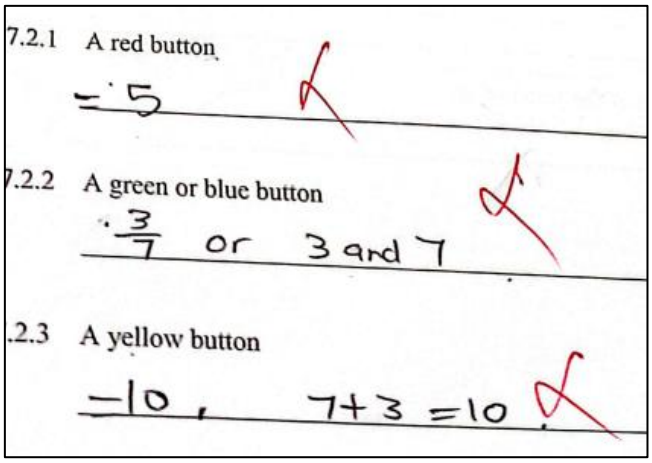


Figure 62 – Example of misused data error (Participant S16)

Analysis of the 21 selected scripts revealed that 19 students answered Questions 7.1.1 and 7.1.2 correctly, only 2 students answered incorrectly and 9 students answered Questions 7.2.1 to 7.2.3 correctly with the same 9 students answered correctly, while 3 students attempted the question but provided incorrect answers. The remaining students did not attempt to answer the question. The researcher invited these students to participate in unstructured interviews to understand the reasoning behind their approaches to solving these questions. Table 37 presents a summary of the researcher's unstructured interview questions and students' responses regarding their errors and misconceptions.

Table 36 - Unstructured questions: Question 7

Researcher's unstructured interview question: Why did you answer this question the way you did?			
Students responses	Codes	Categories	Themes
S37: In question 7.1.1, I forgot which one is the stem and leaf diagram—hence, I plotted a tally. I need revision.	• Forgot specific statistical representation (stem-and-leaf), used incorrect method (tally) (S37)	**Partial Knowledge, Misinterpretation of Concepts** • Forgot specific statistical representation (stem-and-leaf), used incorrect method (tally) (S37) • Forgotten details of statistical method (stem-and-leaf) (22) • Used inappropriate method (mean calculation instead of correct representation) (S27) • Misunderstanding definition of statistical concept (range) (S37)	**Conceptual misunderstanding**
S22: I forgot stem and leaf, Ma'am. I was not sure which numbers to write under stem column and leaf column in question (7.1.1)	• Forgot details of statistical method (stem-and-leaf) (S22)	**Lack of Conceptual Understanding** • Misunderstanding of probability concept, counting instead of calculating probabilities (S16) • Lack of understanding; confusion about probability concepts; uncertainty about correct method **(Multiple students)**	
S27: I forgot which one is which. Then this method of adding all the values, then divide by the total number of data, came first to my mind.	• Used inappropriate method (mean calculation instead of correct representation) (S27)	**Uncertainty in Method Selection** • Lack of understanding; confusion about probability concepts; uncertainty about correct method **(Multiple students)**	

		• Forgot details of statistical method (stem-and-leaf) (S22)	
S47: I thought range is the highest number of the data.	• Misunderstood definition of statistical concept (range) (S37)	**Incorrect selection of data values** • Procedural error (incorrect selection of highest number in range calculation) (S22)	**Procedural Errors**
S26: I know the formula to calculate range, but I thought I should use the last highest number in the table of data values. I saw 57, it is not a mistake to choose 55 in question	• Procedural error (incorrect selection of highest number in range calculation) (S22)	**Misapplication or Incorrect use of Formula** • Used inappropriate method (mean calculation instead of correct representation) (S27)	
S16: I don't understand probability. I thought they want me to write how many red, blue, green balls in the bag; hence, I wrote their number.	• Misunderstanding of probability concept, counting instead of calculating probabilities (S16)		
15 students who left blank spaces: Probability is difficult. **I did not understand that topic from high school.** **I didn't know what they were asking.** **I did not know what to write.** **At first I thought they want ratios, but I didn't know how to answer it.**	• Lack of understanding; confusion about probability concepts; uncertainty about correct method **(multiple students: S4, S142, S6, S70, S23, S24, S11, S41, S48, S33, S56, S13, S138, S85, S87)**		

4.3.10.1.Analysis of students' responses in Question 7 (Data handling and probability)

The analysis of students' responses of Question 7 revealed significant challenges in students' ability to calculate statistics, which were evident in both quantitative data and qualitative insights from interviews and detailed examination of students test scripts. Among the 150 students, a total of 19 students (13%) answered the stem-and-leaf question correctly, while 41 students (27%) answered incorrectly. Only 3 students (2%) provided partially correct answers, and 87 students (58%) did not attempt to answer the question. For the question on range, a total of 19 students (12%) answered correctly, while 79 students (53%) answered incorrectly. No students provided partially correct answers, and 52 students (35%) did not attempt the question. For Questions 7.2.1-7.2.3, which required students to determine the probability of the ball, an average of 39 students (26%) provided the correct answers, while no student provided partial answers. An average of 66 students (44%) answered incorrectly, and an average of 45 (30%) left the question blank. These results suggest widespread difficulties in understanding and calculating statistical concepts, particularly constructing stem-and-leaf diagrams, distinguishing between range and mean, and applying probability rules correctly. A TIMSS (2019) report concluded that Grade 9 students have challenges with data handling and probability, particularly interpreting data representations, calculating statistical measures, and applying probability formulas. Reddy's (2019) findings are in line with the findings of this study.

Qualitative analysis of students' test scripts highlighted three main types of errors: misused data errors, distorted definition errors, and technical errors. Misused data errors were observed, particularly where students incorrectly structured stem-and-leaf diagrams by placing both tens and units in the leaf column instead of separating them appropriately.

Distorted definition errors were evident where students confused statistical concepts, such as representing data using tally charts instead of stem-and-leaf diagrams or applying the mean formula when calculating range. This suggests confusion between calculating the range (difference between highest and lowest values) and computing the mean (average of data values), indicating a lack of conceptual differentiation between statistical measures. This reflect the findings from TIMMS (2019) report that Grade 9 students are unable to find mean because they lack understanding of the word 'mean'.

Technical errors were observed, particularly where students misread data values and incorrectly identified the highest or lowest values when calculating the range.

4.3.10.2. Interview findings: Students' difficulties with data handling (Question 7)

The interview responses provided valuable qualitative insights into the difficulties students encountered with statistics concepts. The analysis identified two main themes: conceptual misunderstanding and procedural errors. These themes are supported by specific categories, highlight different of students' challenges with this mathematical concept.

4.3.10.2.1. Conceptual misunderstanding

This theme captures the difficulties students experienced comprehending the foundational principles of statistical representations and interpreting what the question required. Three categories emerged within this theme: partial knowledge or misinterpretation of concepts, lack of conceptual understanding, and uncertainty in method selection.

Partial knowledge or misinterpretation of concepts: Some students misinterpreted the task entirely. For example, student S37 stated: *"Ma'am, I forgot which one is the stem and leaf diagram. Hence, I plotted a tally. I need revision".* This response reflects confusion between two distinct statistical representations. Similarly, student S22 said: *"I forgot stem and leaf, Ma'am. I was not sure which numbers to write under stem column and leaf column".* These statements show limited conceptual knowledge of statistical representations, particularly how stem-and-leaf diagrams are constructed by separating tens (stems) and units (leaves). Student S27 confused the concept of range with mean: "*I forgot which one is which. Then this method of adding all the values, then divide by the total number of data, came first to my mind.*" Student S37 expressed confusion: *"I thought range is the highest number of the data".* These responses reveal a misunderstanding of statistical definitions, particularly failing to distinguish between range as a difference and mean as an average.

Lack of conceptual understanding: Another recurring issue was students' struggle with the probability questions. For example, fifteen students (S4, S142, S6, S70, S23, S24, S11, S41, S48, S33, S56, S13, S138, S85, S87) left the sub-questions blank, providing reasons such as: *"Probability is difficult, Ma'am"(S70), "I did not understand that topic from high school"(S41), "I didn't know what they were asking"(S48), "I did not know what to write"(S6), "At first I thought they want ratios, but I didn't know how to answer it"(S24).* These responses suggest a significant lack of prior knowledge of basic probability concepts, particularly understanding probability as a ratio of favourable outcomes to total possible outcomes. Student S16 demonstrated a misconception about how to calculate probability: *"I don't understand probability. I thought*

they want me to write how many red, blue, green balls in the bag; hence, I wrote their number". This response indicates a tendency to count items instead of computing probabilities as fractions or ratios, reflecting a misunderstanding of probability as a measure of likelihood.

Uncertainty in method selection: Some students showed confusion about how to choose the most appropriate method. Student S22 stated: *"I forgot stem and leaf, Ma'am. I was not sure which numbers to write under stem column and leaf column*". This reflects not only poor recall but also conceptual uncertainty in selecting appropriate statistical methods for representing and analysing data. These students' difficulty selecting the correct approach also suggests a lack of fluency with statistical representations.

4.3.10.2.2. Procedural errors

This theme addresses the difficulties students encountered when attempting to apply correct steps to solve statistical problems. Even when students showed some understanding of the concept, they struggled with execution of the steps. Two categories emerged within this theme.

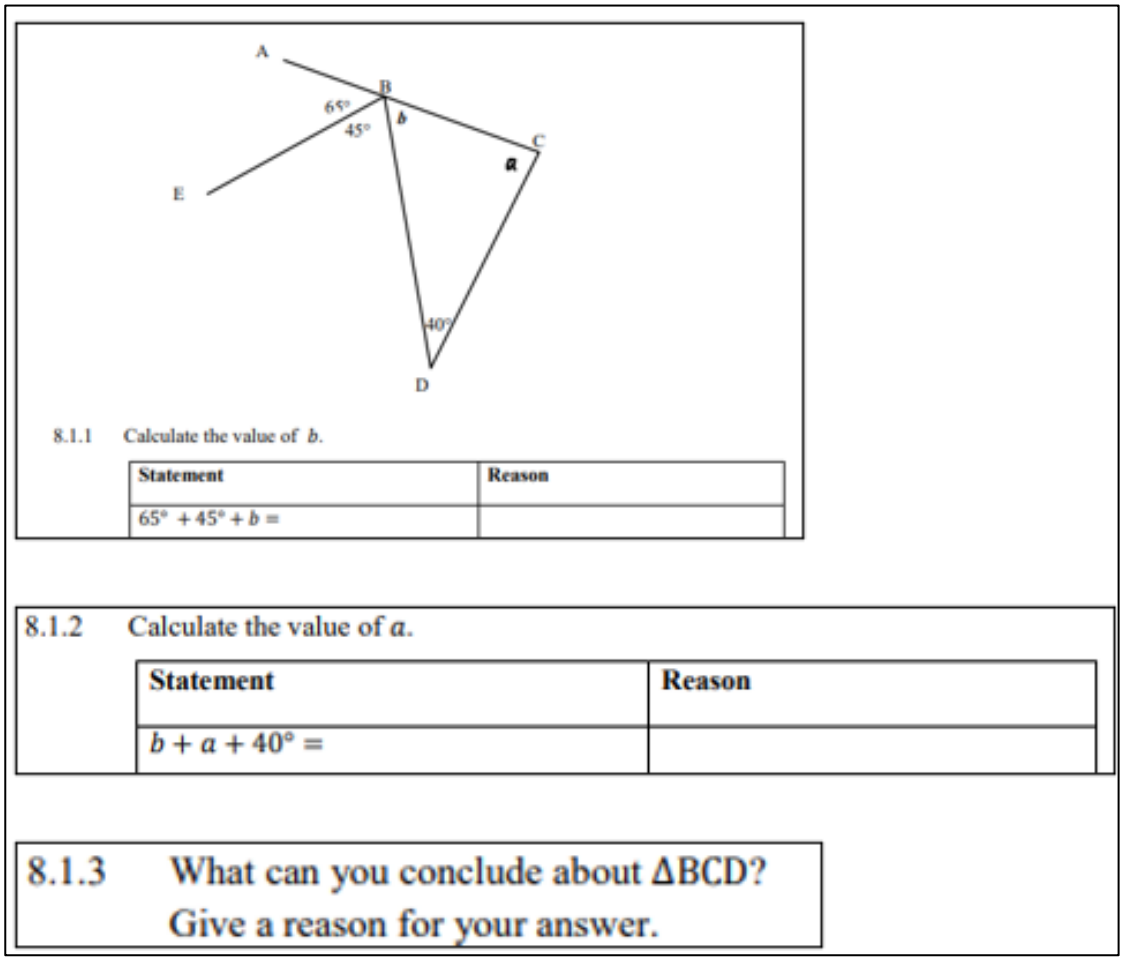
Incorrect selection of data values: Student S22 said: *"I know the formula to calculate range, but I thought I should use the last highest number in the table of data values. I saw 57. It is not a mistake to choose 55".* This response reveals a procedural misunderstanding—the student correctly recalled the need to find the range but incorrectly assumed the last number in a table is always the largest. This suggests gaps in their ability to read data and apply a concept correctly.

Misapplication or incorrect use of formula: Some students recognised the need for a formula but applied the wrong one. For example, Student S27 said: *"I forgot which one is which. Then this method of adding all the values then divide by the total number of data came first to my mind.".* This shows that the student used the method for calculating the mean when they were meant to calculate the range. The substitution of one statistical process for another suggests incomplete conceptual differentiation and insufficient procedural reinforcement.

The findings reveal that these students experienced a range of difficulties interpreting data and calculating statistical measures. Conceptually, students misinterpreted tasks, confused statistical terms (e.g., range vs. mean), and showed limited understanding of representations like stem-and-leaf diagrams. Procedurally, they struggled to recall or apply correct formulas, misread data, or selected inappropriate methods. Addressing these inter-related issues requires a dual focus on improving both conceptual clarity and procedural fluency to ensure that students are better equipped to engage with foundational mathematical concepts such as data handling.

4.3.11. Analysis of students' difficulties, alternative conceptions, and errors (Question 8)

Question 8 assessed students' knowledge of straight-line geometry to determine the unknown angles of triangles.



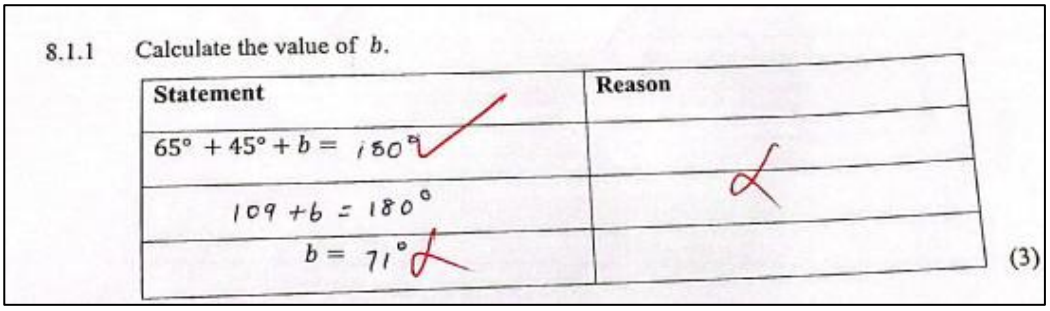
Question 8.11 – 8.1.3 – Straight line geometry

Table 37 summarizes the performances of the 150 students on Question 8.1.1. A total of 37 students (25%) answered correctly, 54 students (36%) answered incorrectly, and 15 students (10%) provided partially correct answers. Many students, 44(29%), did not attempt to answer the question. Among the 69 students (46%) who made errors, 40 made misused data errors, 15 made technical errors, and 14 provided distorted definitions.

Table 37 – Geometry of 2D shapes: Types and frequency of errors [Question 8.1.1]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	37 (25%)
Incorrect answer	40	0	14	-	0	54 (36%)
Partially correct	-	-	-	-	15	15 (10%)
No answer	-	-	-	-	-	44 (29%)
Total						**150 (100%)**

Figure 63 shows an extract from a student's solution demonstrating an incomplete answer error. The student understood that the sum of the adjacent angles on a straight-line equals 180º but made an arithmetic mistake when adding 65º + 45º, incorrectly obtaining 109º instead of 110º. This miscalculation led to an incorrect answer for angle *b*. The student failed to provide the reasons for equating the sum of the angles to 180º, which is imperative in geometrical answers.

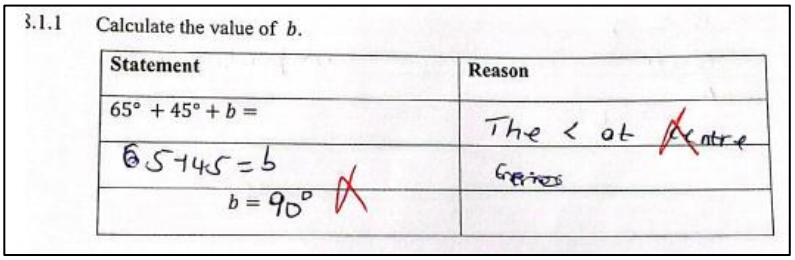

8.1.1 Calculate the value of b.

Statement	Reason
$65° + 45° + b = 180°$	
$109 + b = 180°$	
$b = 71°$	

(3)

Figure 63 – Example of incomplete answer (Participant S48)

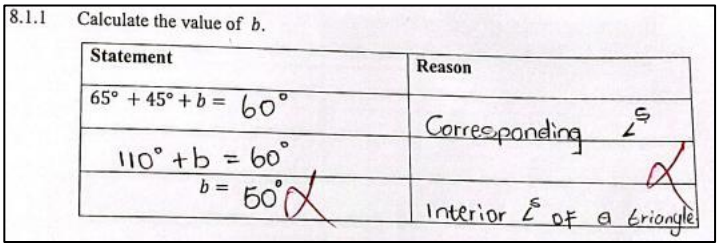
Figure 64 shows an extract from a student's solution demonstrating a misused data error. The student transposed *b* to the right-hand side of the equation and incorrectly added the given angles together. This response indicates a lack of procedural knowledge for geometry, as the student did not recognize that the sum of adjacent angles on a straight line must equal 180º. Additionally, the reasoning provided did not align with the correct mathematical approach.


8.1.1 Calculate the value of b.

Statement	Reason
$65° + 45° + b =$	The < at Centre
$65 + 45 = b$	Angles
$b = 90°$	

Figure 64 - Example of misused data (Participant S4)

Figure 65 shows an extract from a student's solution demonstrating a distorted definition error. This student incorrectly equated the sum of the given angles to 60º, a value that is not present in the data. The student's justification referenced corresponding angles, which indicates a misconception about angle relationships. Corresponding angles occur in an F-shaped pattern, suggesting that the student likely misunderstood the geometric properties being tested.


8.1.1 Calculate the value of b.

Statement	Reason
$65° + 45° + b = 60°$	Corresponding $\angle$s
$110° + b = 60°$	
$b = 50°$	Interior $\angle$s of a triangle

Figure 65 – Example of distorted definition (Participant S142)

These two examples indicate that these NC(V) Level 2 students lacked the necessary procedural knowledge to identifying the relevant concepts and remembering the information to answer the questions.

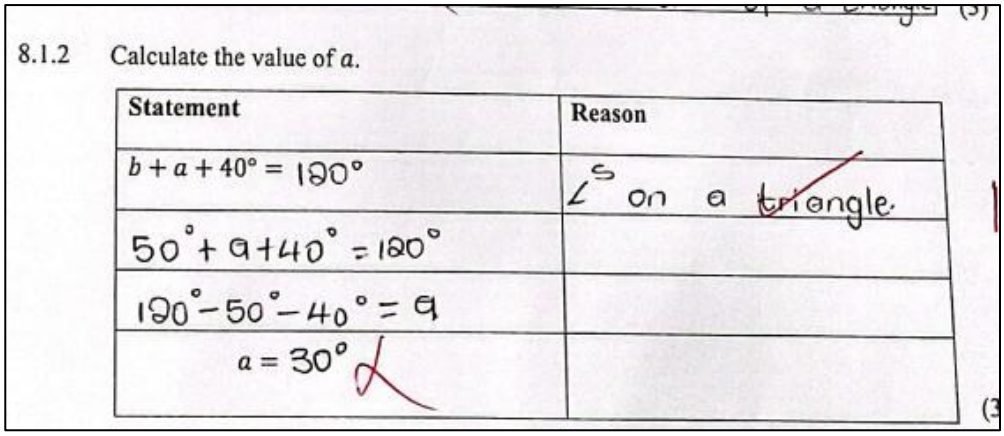
Question 8.1.2 used a value calculated in Question 8.1.1 to determine another unknown angle a. Thus, an error in Question 8.1.1 would like result in an error in Question 8.1.2, too. Table 38 summarizes the performance of 150 students on Question 8.1.2. A total of 29 students (19%) answered correctly, while 45 students (30%) answered incorrectly, 17 students (12%) provided partially correct answers, and many students, 59(39%), did not attempt to answer the question. Of the 62 students (41%) who displayed errors, 23 made misused data errors, 21 made incomplete answer errors, and 18 provided distorted definitions.

Table 38 – Geometry of 2D shapes: Types and frequency of errors [Question 8.1.2]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	29(19%)
Incorrect answer	23	0	18	-	4	45 (30%)
Partially correct	-	-	-	-	17	17 (12%)
No answer	-	-	-	-	-	59 (39%)
Total						**150 (100%)**

Figure 66 shows an extract from a student's solution demonstrating incomplete answer and misused data errors. The student correctly stated that the sum of the interior angles of a triangle

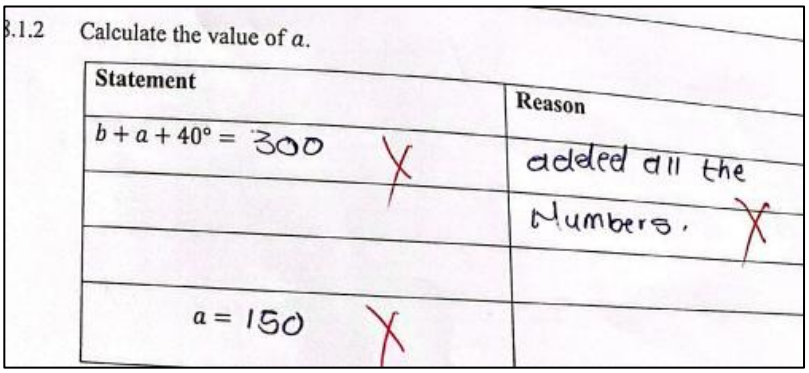
equals 180º but miscalculated the sum, obtaining 120º instead. Despite this miscalculation, the student provided a correct justification for their answer. This suggests that the student understood the reasoning behind the rule but lacked the procedural knowledge to apply it correctly.


8.1.2 Calculate the value of a.

Statement	Reason
$b + a + 40° = 180°$	$\angle^s$ on a triangle.
$50° + 9 + 40° = 180°$	
$180° - 50° - 40° = 9$	
$a = 30°$	

Figure 66 -Example of incomplete answer error (Participant S6)

Figure 67 shows an extract from a student's solution that demonstrated a misused data error. The student failed to recognize that the sum of the interior angles of a triangle must equal 180º and instead calculated a total of 300º by adding together all the given numbers in the diagram. This response suggests a lack of both procedural and conceptual knowledge, as the student demonstrated difficulty distinguishing between different angle properties.


8.1.2 Calculate the value of a.

Statement	Reason
$b + a + 40° = 300$	added all the
	Numbers.
$a = 150$	

Figure 67 – Example of misused data (Participant S22)

Figure 68 shows an extract from a student's solution demonstrating a distorted definition error. The student incorrectly stated that the sum of the interior angles of a triangle is 360º, which is the rule for a quadrilateral, not a triangle. This confusion suggests that the student misinterpreted the properties of different shapes, leading to incorrect reasoning.

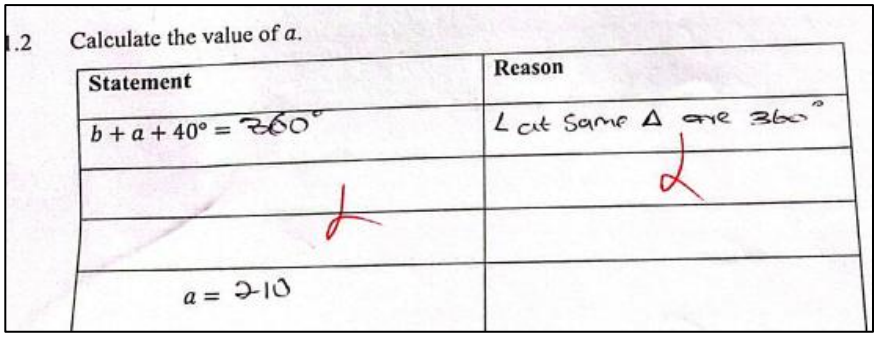


Figure 68 – Example of distorted definition (Participant S27)

Table 39 summarizes the performance of 150 students on Question 8.1.3. A total of 19 students (13%) answered correctly, 48 students (32%) answered incorrectly, and 7 students (5%) provided partially correct answers. The majority of the students, 76 (51%), did not attempt to answer the question. Among the 55 students (37%) who displayed errors, 32 made misused data errors, 16 made distorted definitions errors, and 7 provided incomplete answer errors.

Table 39 –Geometry of 2D shapes: Types and frequency of errors [Question 8.1.3]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical error	Incomplete answer	
Correct answer	-	-	-	-	-	19 (13%)
Incorrect answer	32	-	16	-	0	48 (32%)
Partially correct	-	-	-	-	7	7 (5%)
No answer	-	-	-	-	-	76 (51%)
Total						**150 (100%)**

Figure 69 shows an extract from a student's solution demonstrating a misused data error. The student failed to recognize that isosceles triangles have equal angles opposite equal sides. Instead, the student incorrectly stated that line ABC is equal to another given angle because they both add up to 180º, demonstrating an alternative conception of triangle properties.

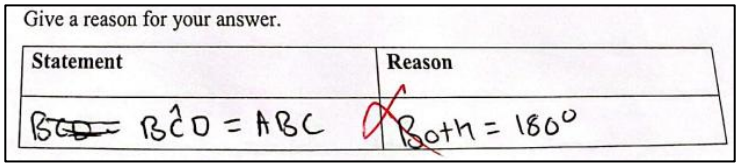


Figure 69 – Example of misused data error (Participant S37)

Figure 70 shows an extract from a student's solution demonstrating an incomplete data error. While the student correctly identified the triangle as isosceles, they failed to provide an appropriate

reason to support their answer. Instead, the student claimed that "all sides are not equal," which is a property of a scalene triangle, not an isosceles triangle. This suggests that the student confused the properties of different types of triangles.

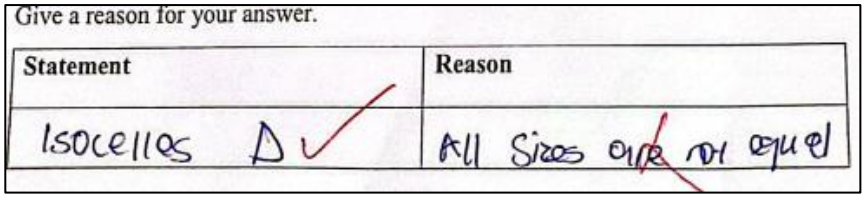


Figure 70 – Example of incomplete answer error (Participant S70)

Figure 71 shows an extract from a student's solution demonstrating a distorted definition error. The student failed to recognize that the triangle was isosceles because two angles were equal. Instead, the student incorrectly wrote BC = BD and justified the answer with the statement that the triangle was equilateral. Since there were no given side lengths, this response indicates a misconception about the relationship between angles and sides in different triangle classifications.

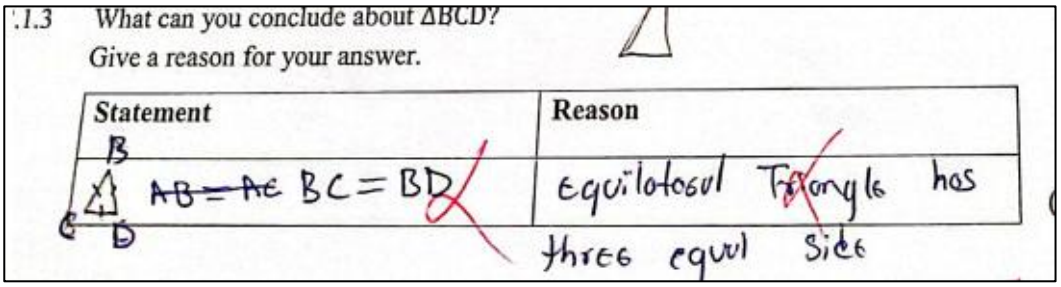
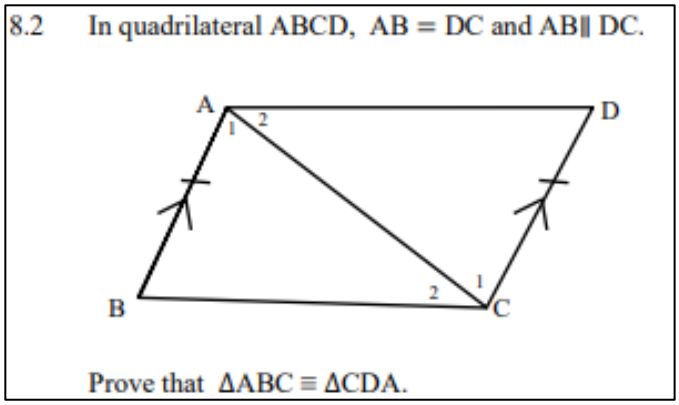


Figure 71 – Example of distorted definition (Participant S23)

Question 8.2 required students to apply the principles of congruency to prove that two triangles were identical in shape and size. The results indicate that many of the students struggled to identify and apply the correct conditions for proving congruence, leading to errors in their justifications.



Question 8.2 – Congruency between triangles

Table 40 summarizes the performance of 150 students on this question. Only 4 students (3%) answered correctly, while 40 students (27%) answered incorrectly, and 30 students (20%)

provided partially correct answers. The majority of the students, 76(51%), did not attempt the question. Among the 70 students (47%) who displayed errors, 31 students committed misused data errors and 39 students made incomplete answer errors.

Table 40 – Geometry of 2D shapes: Types and frequency of errors [Question 8.2]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	4 (3%)
Incorrect answer	31	0	0	0	33	40 (27%)
Partially correct	-	-	-	-	6	30 (20%)
No answer	-	-	-	-	-	76 (51%)
Total						**150 (100%)**

Figure 73 shows an extract from a student's solution demonstrating an incomplete answer error. The student provided only one condition to prove congruence, while three were required. This suggests a lack of conceptual understanding, as the student did not recognize that multiple conditions are necessary to establish congruence.

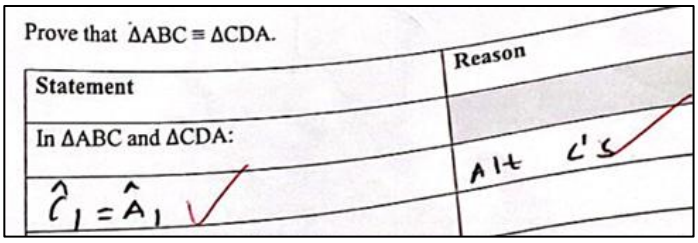
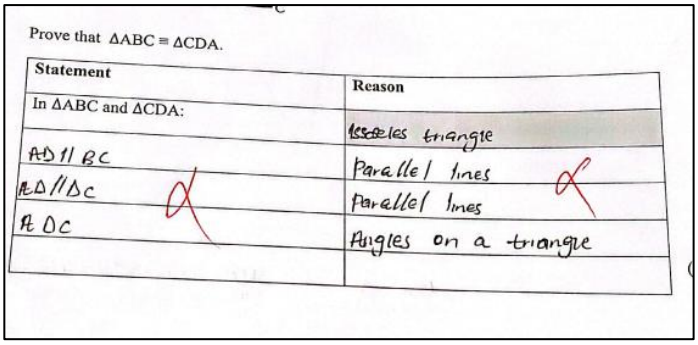


Figure 72 – Example of incomplete answer error (Participant S11)

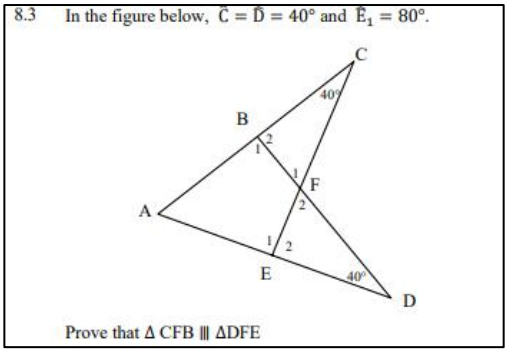
Figure 73 shows an extract from a student's solution demonstrating a misused data error. The student attempted to prove that the triangles were parallel by stating that AD ∥ BC and AD ∥ DC, although this information was no provided in the diagram. This response suggests a lack of procedural knowledge, as the student did not understand the concept of parallel lines and their relevance to triangle congruence.


Prove that ΔABC ≡ ΔCDA.

Statement	Reason
In ΔABC and ΔCDA:	Isosceles triangle
AD // BC	Parallel lines
AB // DC	Parallel lines
A D C	Angles on a triangle

Figure 73 – Example of misused data (Participant S41)

Question 8.3: similarity between two triangles. This question assessed students' knowledge of identifying similarities between two triangles and giving the necessary reasons for each statement provided.



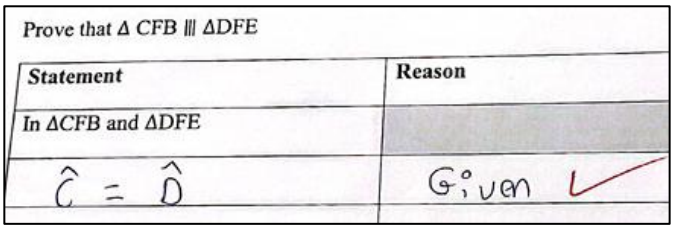
Question 8.3 – Similarity of triangles

Table 41 summarizes the performance of 150 students on this question (8.3). Only 5 students (3%) answered correctly, while 39 students (26%) provided incorrect answers, and 22 students (15%) answered partially correctly. The majority of the students, 84 (56%), did not attempt to answer the question. Of the 61 students who displayed errors, 35 students made incomplete answer errors and 26 students made misused data errors.

Table 41 – Geometry of 2D shapes: Types and frequency of errors [Question 8.3]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer errors	
Correct answer	-	-	-	-	-	5 (3%)
Incorrect answer	26	0	-	-	13	39 (26%)
Partially correct	-	-	-	-	22	22 (15%)
No answer	-	-	-	-	-	84 (56%)
Total						**150 (100%)**

Figure 75 shows an extract from a student's solution demonstrating an incomplete answer error. The student provided only one condition to establish similarity instead of the required minimum of three. This indicates a lack of conceptual understanding, as proving similarity between triangles requires the use of at least three conditions to justify the relationship between the two triangles.


Prove that Δ CFB ||| ΔDFE

Statement	Reason
In ΔCFB and ΔDFE	
$\hat{C} = \hat{D}$	Given ✓

Figure 74 – Example of incomplete answer error (Participant S16)

Figure 76 shows an extract from a student's solution demonstrating a misused data error. Instead of proving similarity, the student attempted to prove congruency by incorrectly equating angles and sides that were not in proportion. The student added angle values that were not relevant to the problem and provided justifications that did not align with the geometric requirements for similarity. This suggests a lack of procedural knowledge, as the student failed to distinguish between similarity and congruency in geometric proofs.

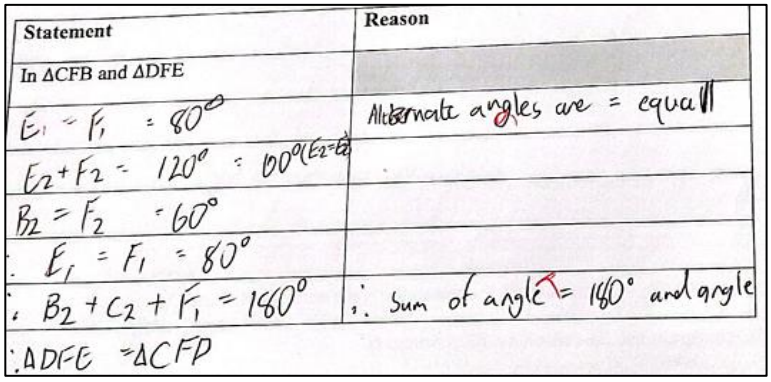

Statement	Reason
In ΔCFB and ΔDFE	
$E_1 = F_1 = 80°$	Alternate angles are = equal
$E_2 + F_2 = 120° = 60°$ (E_2=B)	
$B_2 = F_2 = 60°$	
$\therefore E_1 = F_1 = 80°$	
$\therefore B_2 + C_2 + F_1 = 180°$	$\therefore$ Sum of angle = 180° and angle
$\therefore$ ΔDFE = ΔCFD	

Figure 75 – Example of misused data (Participant S22)

Question 8.4.1: Constructing triangles using mathematical instruments. This question evaluated students' ability to construct two triangles according to the provided instructions using mathematical instruments such as a ruler, compass, and protractor. These tools are used for drawing precise angles and sides of any given question related to construction of triangles.

8.4.1 Use a pair of compasses, ruler and a sharp pencil to construct triangle ABC in the frame below, so that AB = BC = 7 cm and $\hat{A} = 65°$.

Question 8.4.1 – Isosceles Triangle

Table 42 summarizes the performance of 150 students on this question. A total of 27 students (18%) answered correctly, while 17 students (11%) answered incorrectly. No students provided partially correct responses, and 106 students (71%) left the question blank. All 17 of the students who answered incorrectly made misused data errors.

Table 42 – Construction of triangle: Types and frequency of errors [Question 8.4]

Difficulties	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	27 (18%)
Incorrect answer	17	0	0	-	-	17 (11%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	106 (14%)
Total						**150 (100%)**

Figure 77 shows an extract from a student's solution demonstrating a misused data error. The student did not follow the instructions given and failed to use the correct mathematical tools. Instead of using a ruler to measure 7 cm and a protractor to construct an angle of 65º, the student attempted to draw the triangle freehand. This suggests a lack of procedural and conceptual knowledge regarding the proper use of construction tools in geometry.

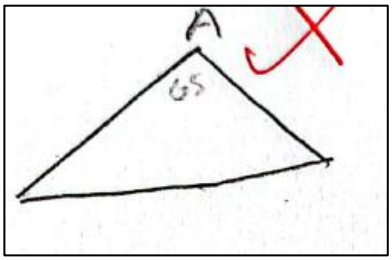
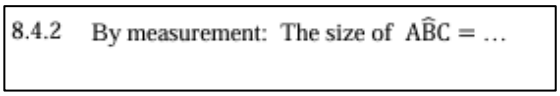


Figure 76 – Example of misused data error (Participant S13)

Question 8.4.2: Measuring angle B from a triangle student had constructed.



Question 8.4.2 – Angle measurement

Table 43 summarizes the performance of 150 students on this question. Only 5 students (3%) students answered questions correctly, while 42 students (28%) answered incorrectly. No student

provided partially correct answers, and 103 students (69%) did not attempt to answer the question. All 42 of the students who answered incorrectly made misused data errors.

Table 43 – Construction of triangles: Types and frequency of errors [Question 8.4.2]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	5 (3%)
Incorrect answer	42	0	0	-	-	42 (28%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	103 (69%)
Total						**150 (100%)**

Figure 78 shows an extract from a student's solution demonstrating a misused data error. The student failed to measure the correct angle because the constructed triangle was inaccurate. This error suggests that the student may not have brought the required mathematical tools for the test or lacked the skills to use them properly. As a result, the student's construction errors led to incorrect angle measurements.

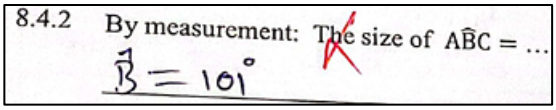
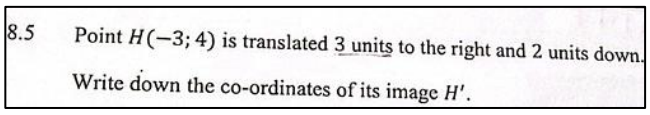


Figure 77 – Example of misused data (Participant S23)

Question 8.5: Translation: This question evaluated students' understanding of translating and transforming triangles according to given instructions. Translation involves shifting a shape based on its coordinates, while transformation refers specifically to reflecting the shape.



Question 8.5 - Translation

Table 44 summarizes the performance of 150 students on this question. A total of 12 students (8%) answered correctly, while 33 students (22%) provided incorrect answers. No students answered partially correctly, and 105 students (70%) left the question blank. All 33 students who answered incorrectly made misused data errors.

Table 44 – Translation: Types and frequency of errors [Question 8.5]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	12 (8%)
Incorrect answer	33	0	0	-	-	33 (22%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	105 (70%)
Total						**150 (100%)**

Figure 79 shows an extract from a student's solution demonstrating a misused data error. The student failed to correctly apply the translation instructions, misinterpreting the direction of movement. Instead of translating the *x*-value to the right by three units and shifting the *y*-value down by two units, the student did the opposite—moving the *x*-value to the left by two units and shifting the *y*-value up by three units. This response indicates both procedural and conceptual deficiencies, particularly in understanding ordered pairs, where the first value represents the *x*-axis movement, and the second value represents the *y*-axis movement.

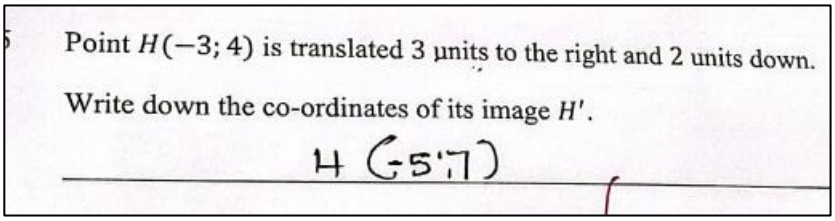


Figure 78 – Examples of misused data (Participant S48)

To further explore students' thought processes, the researcher selected 21 students based on their scripts for unstructured interviews. Table 46 presents a summary of the researcher's unstructured interview questions and students' responses.

Table 45 - Unstructured questions: Question 8

Researcher unstructured interview: What were you thinking when you were answering Question 8?			
Students responses	Codes	Categories	Themes
S48: I made a mistake when I was adding the angles, and I forgot to write a reason.	• Procedural error; forgot justification (S48)	**Lack of Conceptual Understanding** • Lack of knowledge; random response (S4) • Forgotten concepts; random answer (S142) • Lack of understanding; random answers (S22) • Lack of knowledge; conceptual difficulty (S27) • Guessing without understanding (S70) • Lack of understanding; copying question as response (S11) • Lack of understanding; no attempt (S37,S24, S16, S41, S33, S56, S13, S26, S138, S85, S87, S47)	**Conceptual misunderstanding**
S4: I don't know what I was writing.	• Lack of knowledge; random response (S4)	**Guessing due to lack of knowledge** • Lack of knowledge; random response (S4) • Forgotten concepts; random answer (S142) • Lack of understanding; random answers (S22) • Guessing without understanding (S70) • Lack of understanding; copying question as response (S11)	

S142: I forgot this topic, I wrote what came first to my mind.	• Forgot concepts; random answer (S142)	**Partial Knowledge (Forgetfulness)** • Partial recall of concepts; forgotten geometry concepts (S6)	
S6: The only part I remember in geometry is the reason that says 'sum of the angles of the straight-line'—but I forgot the 'sum of angles of the triangle'.	• Partial recall of concepts; forgotten geometry concepts (S6)	**Mistake in angle calculation** • Procedural error; forgotten justification (S48)	**Procedural Errors**
S22: I was just answering for the sake of answering; I don't understand geometry.	• Lack of understanding; random answers (S22)	**Forgotten justifications** • Procedural error; forgotten justification (S48)	
S27: Geometry is difficult.	• Lack of knowledge; conceptual difficulty (S27)	**Practical constraints (equipment)** • Practical constraint preventing solution (S23)	
S70: I was guessing all the answers throughout.	• Guessing without understanding (S70)		
S11: I didn't understand question 8, I was coping what is on the question and writing it as my answer.	• Lack of understanding; copying question as response (S11)		
S24: This is the difficult topic ever—that's what I can say.	• Difficulty in understanding Geometry (S22)		
S23: I didn't have the equipment to	• Practical constraint		

draw the triangle.	preventing solution (S23)		
Responses from students who did not answer questions: They all said they don't understand Geometry. (S37,S24, S16, S41, S33, S56, S13, S26, S138, S85, S87, S47)	• Lack of understanding; no attempt (S37,S24, S16, S41, S33, S56, S13, S26, S138, S85, S87, S47)		

4.3.11.1. Analysis of students' responses to Question 8 (Geometry of lines and triangles)

The analysis of students' responses to Question 8 revealed significant challenges in students' ability to calculate geometry, which were evident in both quantitative data and qualitative insights from interviews and detailed examination of students' test scripts. Among the 150 students assessed, an average of 17 students (11%) answered correctly, while an average of 11 students (7%) partially answered the question, and an average of 40 students (27%) answered incorrectly. A total of 82 students (55%) did not attempt to answer the question. These results suggest widespread difficulty understanding and solving problems that involve geometry concepts, which is consistent with the findings of previous studies highlighting the challenges of students in calculating unknown angles. Reddy (2019) concluded that Grade 9 students lack basic knowledge of general geometry principles and the procedural knowledge to apply the principles of angles.

Qualitative analyses of students' test scripts highlighted three main types of errors: misused data, incomplete answer error, and distorted definition. Misused data errors were observed in the responses of students like S22, who added angle values that were not relevant to the problem and provided justifications that did not align with the geometric requirements for similarity. This suggests a lack of procedural knowledge of the concept. Bowie, et.al (2022) concluded that Grade 9 students lack the knowledge of polygons. Incomplete answer errors were evident in the responses of students like S16, who provided only one condition to establish similarity instead of the required minimum of three. Distorted definition errors were evident in the response of S23, who failed to recognize that the triangle was isosceles because two angles were equal. Instead, the student incorrectly wrote BC = BD and justified the answer by stating that the triangle was equilateral. This aligns with the finding of Dagli et al. (2016) that students failed to recognize an isosceles triangle when it was compared with a right-angled triangle.

4.3.11.2. Interview findings: Students' difficulties with geometry (Question 8)

The interview responses provided valuable qualitative insights into the difficulties students encountered solving geometry questions, including straight-line geometry, triangle properties, congruence, similarity, construction, and translation. The analysis identified two overarching themes: conceptual misunderstanding and procedural errors. These themes, supported by several categories, illuminate key challenges students faced with this mathematical topic.

4.3.11.2.1. Conceptual misunderstanding

This theme captures the difficulties students experienced in comprehending the foundational principles of geometry and understanding what the question required. Three categories emerged within this theme.

Lack of conceptual understanding: Many students expressed not knowing how to approach the questions or simply wrote random answers. For instance, student S22 stated: *"I was just answering for the sake of answering, I don't understand geometry".* Similarly, S4 stated: *"I don't know what I was writing".* Multiple students echoed similar sentiments, such as "we don't understand geometry." These responses reflect a fundamental lack of understanding of geometry, indicating unfamiliarity with both mathematical content and the reasoning processes required to engage with the tasks.

Guessing due to lack of knowledge: Some students guessed answers as they did not remember the topic adequately. Student S70 said: *"I was guessing all the answers throughout".* Similarly, S22 said *"I was just answering for the sake of answering; I don't understand geometry".* Student S142 also said: "*I forgot this topic; I wrote what came first to my mind."* These responses indicate disengagement and a lack of confidence, which may be the result of inadequate foundational exposure to geometry.

Partial knowledge: Some of the students could recall specific geometric rules but struggled to apply them correctly. For example, Student S6 stated: *"The only part I remember in geometry is the reason that says 'sum of the angles of the straight line', but I forgot the 'sum of angles of the triangle'*". Student S27 stated, broadly: *"Geometry is difficult".* These responses suggest that although some students retained isolated facts from their previous exposure to geometry, they lacked the broader conceptual understanding needed to interpret and solve geometric problems holistically.

4.3.11.2.2. Procedural errors

This theme reflects students' difficulties applying the correct steps to solve geometry questions, even when they understood the task conceptually. Two categories emerged under this theme.

Mistakes in angle calculation: Student S48 highlighted a procedural misstep: *"I made a mistake when I was adding the angles, and I forgot to write a reason".* This reflects a lapse in both calculation accuracy and the procedural requirement to provide mathematical justification — a crucial element in geometry.

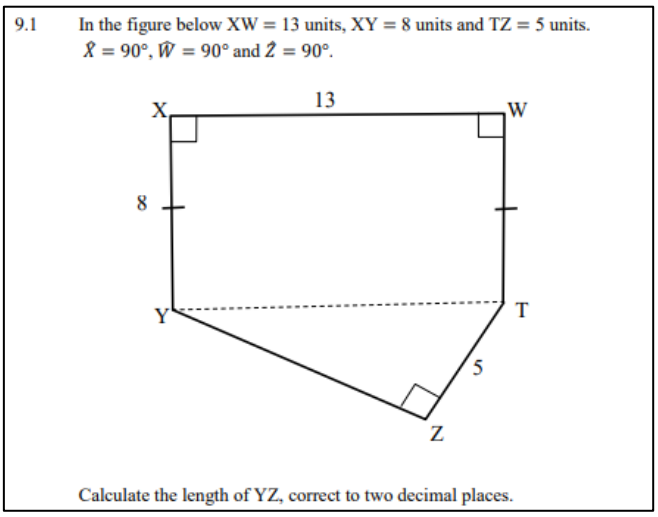
Practical constraints: At least one student reported being unable to complete the question due to missing equipment. For instance, student S23 noted: *"I didn't have the equipment to draw the triangle".* This suggests a practical issue that contributed to the student's inability to engage with the task, especially those involving construction or measurement.

The interviews revealed that most of the students faced significant challenges with both conceptual understanding and procedural execution in geometry. Many lacked a foundational knowledge of geometric properties, misunderstood what was required, or guessed answers without meaningful engagement. Others could recall isolated rules but were unable to apply them properly. Additionally, some students faced practical barriers, such as lacking drawing tools, which prevented them from completing construction tasks.

4.3.12. Analysis of students' difficulties, alternative conceptions, and errors (Question 9)

Question 9 tested students' knowledge of the Theorem of Pythagoras, area, and surface area.

Question 9.1: Application of the Theorem of Pythagoras. This question assessed students' knowledge for finding an unknown side of a right-angled triangle by applying the theorem of Pythagoras.



Question 9.1 - Theorem of Pythagoras

Table 46 shows the performance of 150 students on Question 9.1. Only 11 students (8%) answered correctly; 30 students (20%) answered incorrectly, 8 students (5%) answered partially correctly, and 101 students (67%) did not attempt to answer the question. Of the 38 students (25%) who answered partially or incorrectly, 23 displayed misused data errors, 7 displayed distorted definiti6ons, and 8 provided incomplete answers.

Table 46 – Student Responses: Type and frequency of errors [Question 9.1]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	11 (8%)
Incorrect answer	23	-	7	-	0	30 (20%)
Partially correct	-	-	-	-	8	8 (5%)
No answer	-	-	-	-	-	101 (67%)
Total						**150 (100%)**

Figure 80 shows an extract from a student's solution demonstrating a misused data error. The student lacked a procedural understanding of the Theorem of Pythagoras and attempted to solve for the unknown side by subtracting all the given sides instead of applying the correct formula. This response indicates a lack of a conceptual understanding of the theorem's structure and application.

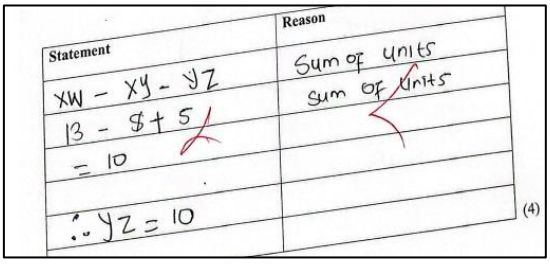


Figure 79 – Examples of misused data errors (Participant S13)

Figure 81 shows an extract from a student's solution demonstrating a distorted definition error. The student incorrectly added all the given side lengths and equated the result to 180°, confusing the sum of side lengths with the sum of interior angles. This error demonstrates a lack of both procedural and conceptual knowledge. Procedurally, the student failed to apply the correct formula for the Theorem of Pythagoras and did not identify the hypotenuse or understand the relationship between the triangle's sides. Conceptually, the student confused the geometric principles of triangles: specifically, the sum of angle rule with the rules for the measurement of right-angled triangles, indicating a misinterpretation of the question's requirements.

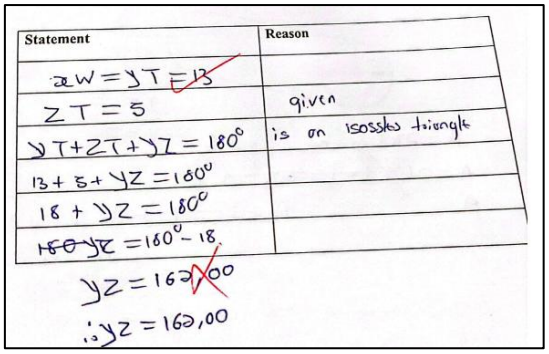


Figure 80 – Example of distorted definitions (Participant S56)

Figure 82 shows an extract from a student's solution demonstrating an incomplete answer error. Although the student applied the correct steps to calculate the unknown side, they did not provide mathematical reasoning or justification for their solution. This omission indicates a possible lack of conceptual clarity about the importance of providing both the solution and a valid explanation.

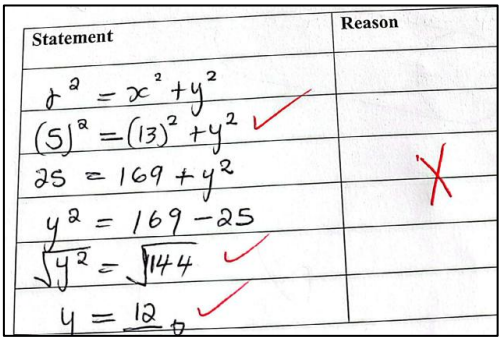
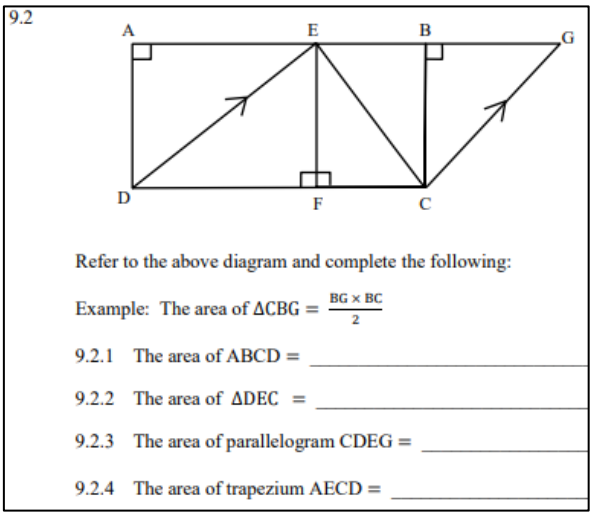


Figure 81 – Example of incomplete answer errors (Participant S47)

Questions 9.2.1 to 9.2.4: Calculations of the areas of a rectangle, triangle, parallelogram, and trapezium: These questions assessed students' knowledge of using formulae to calculate the areas of a rectangle, triangle, parallelogram, and trapezium using the given shapes.



Question 9.2 - Area of Triangle, Rectangle, Parallelogram & Trapezium

Table 47 shows the average performance of the 150 participants across the four questions and on each question. Out of 150 students, on average only 4 students (3%) answered correctly, 73 students (49%) answered incorrectly, and 73 students (49%) did not attempt the questions. There were no partially correct responses recorded. Among those who answered incorrectly, an average of 28 students made misused data errors, 35 made technical errors, and 10 made logically invalid inferences.

Table 47 – 2D Shapes: Type and Frequency of Errors [Questions 9.2.1 – 9.2.4]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	Av 4 (3%)
Incorrect answer	28	10	0	35	0	Av 73 (49%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	Av 73 (49%)
Total						**150 (100%)**

Figure 83 shows an extract from a student's solution demonstrating misused data errors. This students lacked both the procedural knowledge and the conceptual framework to calculate areas correctly for all the required shapes, including rectangles, triangles, parallelograms, and trapeziums. This highlights the critical role of conceptual understanding in solving problems involving area calculations.

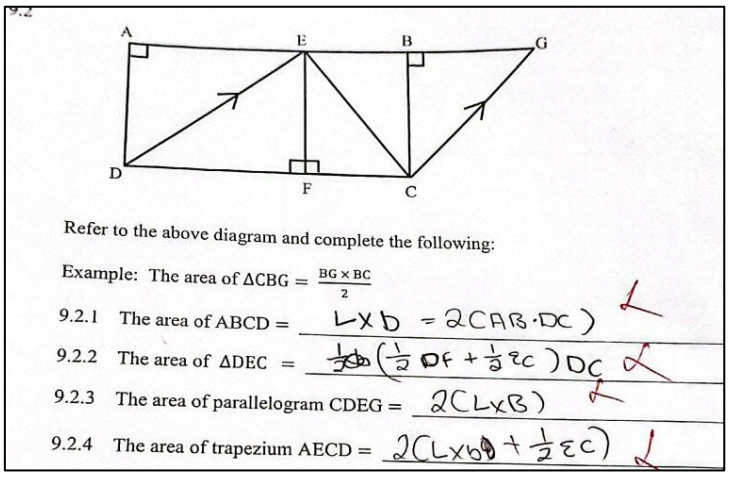


Figure 82 – Examples of technical errors (Participant S85)

Figure 84 shows an extract from a student's solution demonstrating misused data errors. Although the student appeared to know the formulae for rectangles and triangles, they applied

data incorrectly when solving for the area of a parallelogram and lacked the procedural knowledge needed to calculate the area of a trapezium. This suggests inconsistency in the application of known formulae across different shapes.

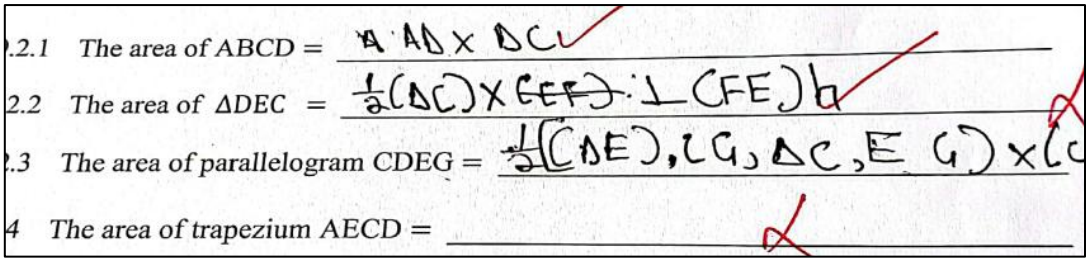
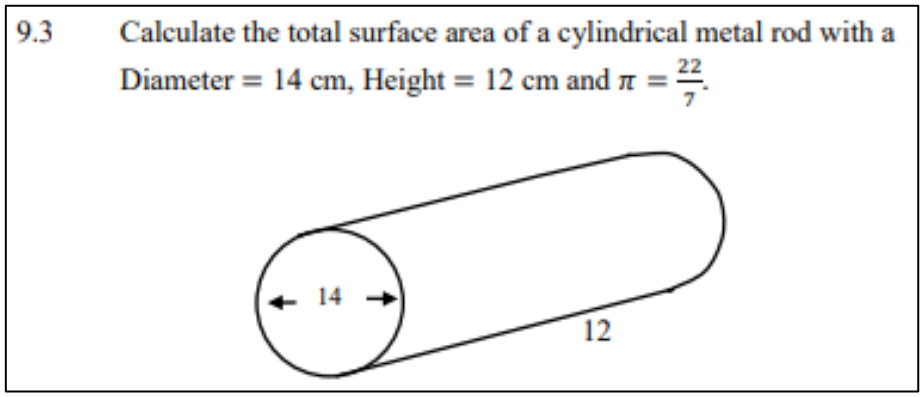


Figure 83 – Example of misused data error (Participant S87)

Question 9.3: Calculation of a cube's surface area: On this question, students were assessed to calculate the surface area of the 3D shape (cylinder).



Question 9.3 – Surface Area (Tube)

Table 48 summarizes the performance of 150 students for Question 9.3. Only 4 students (3%) answered correctly, 60 students (40%) answered incorrectly, and 86 students (57%) did not attempt to answer the question. No students gave partially correct answers. Of the incorrect answers, 43 students made misused data errors, 14 students made technical errors, and 3 students displayed distorted definitions.

Table 48 – Surface Area of a Cylinder: Types and frequency of error [Question 9.3]

Responses	Errors					Frequencies and percentages
	Misused data	Logically invalid inference	Distorted theorem or definition	Technical Error	Incomplete answer	
Correct answer	-	-	-	-	-	Av 4 (3%)
Incorrect answer	43	-	3	14	-	Av 73 (49%)
Partially correct	-	-	-	-	-	0 (0%)
No answer	-	-	-	-	-	Av 73 (49%)
Total						**150 (100%)**

Figure 85 shows an extract from a student's solution demonstrating a misused data error. The student failed to recall the formula to calculate the surface area of a cylinder and lacked the necessary procedural knowledge to identify the value of the radius. This student failed to recall or recognize the correct formula and substituted the length of the cylinder as the radius, confusing the structure of 2D and 3D shapes. The student treated the cylinder as a flat circle, suggesting a significant gap in their conceptual knowledge of spatial forms.

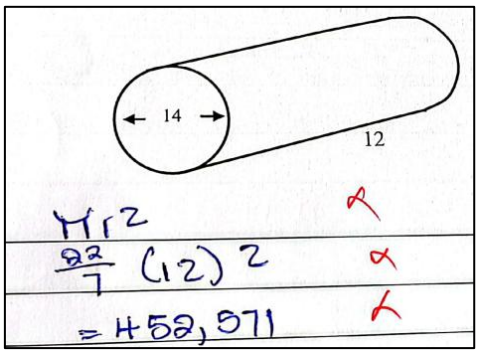


Figure 84 – Examples of misused data error (Participant S48)

Figure 86 shows an extract from a student's solution demonstrating a technical error. The student knew the correct formula for the surface area of a cylinder but omitted the factor of 2 in the circular component and was unsure about squaring the radius. The student substituted the radius and height correctly but erased the exponent, indicating uncertainty about the formula structure.

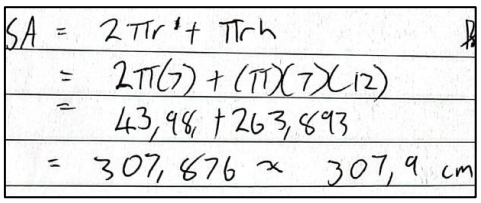


Figure 85 – Examples of technical error (Participant S33)

Figure 87 shows an extract from a student's solution demonstrating a distorted definition error. This student calculated the surface area as if for a sphere, rather than a cylinder. Their confusion between the two 3D shapes suggests that the student lacked procedural understanding and could not distinguish between the surface area properties of a sphere and a cylinder.

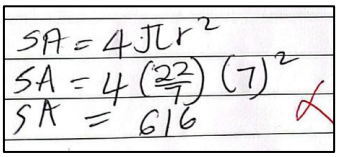


Figure 86 - Examples of distorted definitions (Participant S138)

The researcher conducted unstructured interviews with students whose scripts were analysed to understand their thinking and knowledge about the topic of measurement. Table 49 shows a summary of the unstructured interview questions and student responses.

Table 49 – Unstructured questions: Question 9

Researchers unstructured interview: Do you know the formulas to calculate the areas of Pythagoras, 2D and 3D measurement shaped area			
Students' responses	Codes	Categories	Themes
S37: I don't know the formulas—I was out of the school for 6 years.	• Lack of knowledge due to extended absence (S37)	**Lack of Conceptual Understanding (No knowledge or forgotten concepts)** • Lack of knowledge due to extended absenteeism (S37) • Forgot shape identification (S27)	**Conceptual Misunderstanding**
S22: I don't remember the formulas to calculate area and volume.	• Forgot formulas for area and volume (S22)	**Partial knowledge (Difficulty recalling or forgetting formulae and shapes)** • Forgot formulae for area and volume (S22) • Difficulty recalling multiple formulas (S4)	
S27: Ma'am I don't even remember the types of shapes.	• Forgotten shape identification (S27)		
S4: Shapes are difficult because they have many	• Difficulty recalling multiple formulas (S4)		

formulas to remember.			

4.3.12.1. Analysis of students' responses to Question 9 (Shapes)

The analysis of the students' responses to Question 9 revealed significant challenges in students' ability to calculate measurements, which were evident in both quantitative data and qualitative insights from interviews and detailed examination of students' test scripts. Among the 150 students assessed, only an average of 6 students (4%) provided correct answers, while an average of 3 (2%) solved the question partially correctly. A majority of 82 students (55%) left the question blank, and 59 (39%) answered incorrectly. These results suggest widespread difficulties understanding and solving problems that used this concept, which is consistent with the findings of other studies highlighting the challenges students face with shapes. A study by Kgopane (2021) concluded that Grade 9 students struggle to recognize basic geometric shapes and are unable to define basic geometrical shapes correctly and calculate their volumes.

Qualitative analysis of students' test scripts highlighted four main types of errors: misused data error, technical errors, distorted definition, and incomplete answer errors. Technical errors were observed in the responses of students like S33, who knew the correct formula for the surface area of a cylinder but omitted the factor of 2 in the circular component and was unsure about squaring the radius, indicating a lack of knowledge in formulas. Kgopane (2021) found that students performed poorly in the content area of shapes and showed a lack of pre-existing knowledge of 3D shapes; were unable to calculate the area of 3D shapes, and were confused about the formula of the area of 3D shapes. In this study, misused data errors were evident in the responses of students like S48, who failed to replace the formula to calculate the surface area of a cylinder and lacked procedural knowledge to identify the value of the radius. This student failed to recall or recognise the correct formula and substituted the length of the cylinder as the radius, confusing the structures of 2D and 3D shapes. Distorted definition errors were observed in the responses of students like S138, who calculated the surface area as if it were for a sphere rather than a cylinder. This confusion between the two 3D shapes suggests that the student lacked procedural understanding and could not distinguish between the surface area properties of a sphere and a cylinder. Incomplete answer errors were evident in the responses of students like S47, who applied the correct steps to calculate the unknown side but did not provide mathematical reasoning or justification for their solution.

4.3.12.2.Interview findings: Students' difficulties with measurement (Question 9)

The interview responses provided valuable qualitative insights into the difficulties students encountered with the concept of measurement, which included the application of the Pythagorean Theorem, calculating areas of 2D shapes, and computing surface areas of 3D shapes. The analysis identified one overarching theme: conceptual misunderstanding, supported by two categories that highlight the specific challenges experienced by the students.

4.3.12.2.1. **Conceptual misunderstanding**

This theme captures the difficulties students experienced with understanding the foundational principles of measurement and interpreting the requirements of the question. Two categories emerged within this theme.

Lack of conceptual understanding: Multiple students demonstrated a lack of conceptual knowledge related to measurement. They were unable to recall or use appropriate formulae for calculating area, volume, or surface area. For example, S4 remarked: *"Shapes are difficult because they have many formulas to remember".* Student S22 stated: *"I don't remember the formulas to calculate area and volume".* Similarly, S27 noted:*" Ma'am, I don't even remember the types of shapes".* A particularly notable issue was the impact of prolonged absence from school on content retention. Student S37 shared *"I don't know the formulas; I was out of school for 6 years".* These responses suggest not only a lack of exposure to, or poor retention of, key measurement concepts but also a forgetfulness of mathematical confidence. The inability to recall shape types and associated formulae suggests that foundational knowledge was either never firmly established or has not been maintained through practice.

Partial knowledge: Some students exhibited partial knowledge but struggled to apply it effectively. They indicated that they were aware that multiple formulae exist but were unable to remember or distinguish between them. For instance, S4 remarked: *"Shapes are difficult because they have many formulas to remember".* Student S22 stated: "*I don't remember the formulas to calculate area and volume".* These responses indicate that the students may have been exposed to these concepts in the past but could not recall or retrieve them during the assessment. The high number of required formulas — especially across different shapes and dimensions — appears to have overwhelmed their procedural fluency.

The interview findings reveal that the challenges the students experienced with measurement resulted primarily from a lack of conceptual understanding and poor recall. Many students were

unable to identify geometric shapes or recall the corresponding formulas for area and surface area. Prolonged absence from formal education further compounded the issue for some, while others, although partially familiar with the content, could not apply it due to confusion over the variety of formulas involved**.**

4.3.12.2.2. Strategies to improve NC(V) Level 2 Mathematics students' performance

Table 51 below displayed students' responses when the researcher asked what they thought could be done to improve their performance in mathematics and whether the students would be willing to participate in interventions to improve their performance.

Table 50 – Ways to improve NC(V) Level 2 Mathematics performance

Researcher unstructured interview: What do you think can be done to improve mathematics L2 achievement? AND Are you willing to take part in an initiative aimed at improving your performance in Mathematics L2?			
Students' responses	Codes	Categories	Themes
S37: I think we need to practice mathematics on a daily basis; that is what I am willing to do.	• Regular daily practice (S37)	**Consistence Practice and Reinforcement** • Consistent daily practice of mathematics (S41, S142) • Regular daily practice (S37)	**Student-based Initiatives**
S22: Lecturers must be clear when they teach mathematics. Sometimes we don't understand them—they are too fast. I will participate in the process of improving mathematics.	• Lecturer training and better lesson presentations (S22)	**Student Attendance and Commitment** • Regular attendance and avoiding absenteeism (S13)	
S27: The number of students per each class need to be reduced to 20 so that the lecturers will be able to attend to students individual. Sometimes I feel like my mathematics lecturer can take his time to re-explain the topic to me only. I don't have a problem with being part of the process of improving mathematics.	• Need for individual attention or tutoring (S27)	**Student Attitudes and Beliefs** • Encouraging positive attitudes towards mathematics (S6)	
S4: Lecturers do not care how they present their lesson to us, they rush to finish the syllabus. I so wish they understand that we don't learn at the same time.	• Lecturer training and better lesson presentations (S4)	**Lecturer Competence and Teaching Quality** • Lecturer training and better lesson presentations (S4, S11, S22)	**Lecturer Based Initiatives**
S142: Student don't practice mathematics, and we need to practice it every day. I am willing to participate in the process of improving mathematics.	• Consistent daily practice of mathematics (S142)	**Individualized Support and Attention** • Clear explanations by lecturers; individual attention (S56, S85)	

		• Extra time for mathematics (S24) • Need for individual attention or tutoring (S27)	
S6: We need to change our attitude towards mathematics. Because we believe that mathematics is difficult, so we fail it before even we write.	• Encouraging positive attitudes towards mathematics (S6)	**Structured Revision and Preparation** • Revision sessions before assessments (S70)	
S70: Lecturers must do revision classes before we write our tests. Otherwise, for now, we are on our own.	• Revision sessions before assessments (S70) • Revision sessions, extra support for late registrants (S70)	**Innovative and Multimedia Teaching Approaches** • Change teaching methods to multimedia approaches (videos) (S47)	
S23: I registered a bit late and some of the topics were already taught so I had to learn those topics on my own. If the college can have a catch-up plan that will be better. The achievement of mathematics will improve. You are more than welcome to invite me in the process of improving mathematics.	• Late registration support, extra catching-up required (S23) • Revision sessions, extra support for late registrants (S23)	**Supplementary Support for Late Registration** • Revision sessions, extra support for late registrants (S70, S23) • Late registration support, extra catching-up required (S23)	**Institution based Initiatives**
S24: Extra lessons in the morning and afternoon will help us to perform in mathematics and I am willing to attend them.	• Extra time for mathematics (S24)	**Additional Academic Support (Tutoring)** • Hiring extra mathematics tutors for exam preparation (S16, S27)	
S16: The college need to hire tutors to help us with mathematics before we write our exams. I am willing to take part in improving mathematics.	• Hiring extra mathematics tutors for exam preparation (S16)	**Continuous Assessment and Monitoring** • Weekly assessments or tests (S138) • Daily work (S26) • Organize resources such as previous papers before exams (S48)	**Assessment driven Initiatives**

		• Open book examination (S33)	
S11: Some mathematics lecturers must be sent for training because they cannot explain properly when they teach. Yes, I am willing to participate in the improvement of mathematics.	• Lecturer training and better lesson presentations (S11)		
S41: Mathematics needs practice, that's the only way we can pass it. I am willing to participate.	• Consistent daily practice of mathematics (S41)		
S48: Lecturers must organize previous papers to assist student when they prepare for their exams. I am willing to take part.	• Organize resources such as previous papers before exams (S48)		
S33: Change mathematics examinations to open book because this subject is difficult.	• Open book examination (S33)		
S56: If lecturers can accommodate all students in their lesson, we can do better in mathematics. Yes, I am willing.	• Clear explanations by lecturers; individual attention (S56)		
S13: We need to be in school every day, because absenteeism affect our performance: once you miss school you miss a topic and no one is going to explain that topic to you.	• Regular attendance and avoiding absenteeism (S13)		
S138: We must write weekly tests to test our understanding. I am willing to take part.	• Weekly assessments or tests (S138)		
S26: Lecturers must give us classwork and homework not only assess us during formal tests.	• Daily work (S26)		
S85: Students must start early to prepare for mathematics examination.	• Clear explanations by lecturers; individual attention (S85)		
S47: Lecturer must be creative. We are in 2024 they must create videos for each lesson so that we can always go back to watch where we are experiencing challenges. I will definitely participate.	• Change teaching methods to multimedia approaches (videos) (S47)		

4.3.13. Interviews findings on ways to improve NC(V) Level 2 Mathematics students' performance

The interview responses provided valuable insights into how students believe mathematics performance at NC(V) Level 2 could be improved. The findings are organized into four overarching themes: student-based initiatives, lecturer-based initiatives, institution-based initiatives, and assessment-driven initiatives. Each theme is supported by categories that reflect specific recommendations drawn from students' experiences and suggestions.

4.3.13.1. Student-based initiatives

This theme captures students' own reflections about what they could do to improve their performance in mathematics. Three key categories emerged.

Consistence practice and reinforcement: Students emphasized the importance of daily practice. For example, S37 noted *"I think we need to practice mathematics on a daily basis. That is what I am willing to do".* Similar responses were given by S41 and S142. These responses highlight that regular, consistent practice reinforces conceptual understanding and may reduce poor recall of foundational knowledge.

Student attendance and commitment: Attendance emerged as a critical factor in learning continuity of students. Student S13 stated: *"We need to be in school every day because absenteeism affect our performance. Once you miss school, you miss a topic, and no one is going to explain that topic to you".* This underscores the importance of consistent attendance and how missing instructional time leads to persistent knowledge gaps.

Student attitudes and beliefs: Students acknowledged that mindset plays a role in performance. For example, Student S6 remarked *"We need to change our attitude towards mathematics because we believe that mathematics is difficult, so we fail it before even we write".* Such reflections indicate that fostering positive mathematical identities and confidence is essential to supporting learners' engagement and achievement.

4.3.13.2. Lecturer-based initiatives

This theme presents students' views on how lecturers' teaching practices influence their achievement. Four categories were identified.

Lecturer competence and teaching quality: Students raised concerns about the clarity and pace of lessons. Student S4 noted: *"Lecturers do not care how they present their lesson to us:*

they rush to finish the syllabus. I so wish they understand that we don't learn at the same time". S11 said: *"Some mathematics lecturers must be sent for training because they cannot explain properly when they teach. Yes, I am willing to participate in the improvement of mathematics".* Student S22 explained: *"Lecturers must be clear when they teach mathematics. Sometimes we don't understand them—they are too fast"*. These responses suggest that teacher training in instructional clarity and differentiated pacing is essential.

Individualized support and attention: Students highlighted the need for personalised support. For example, S56 noted: *"If lecturers can accommodate all students in their lesson, we can do better in mathematics".* Student S27 stated: *"The number of students per each class need to be reduced to 20 so that the lecturers will be able to attend to student's individual. Sometimes I feel like my mathematics lecturer can take his time to re-explain the topic to me only".* These responses point to the importance of tailored support and smaller class sizes in enhancing understanding.

Structured revision and preparation: The need for formal revision sessions was raised by students. Student S70 explained: *"Lecturers must do revision classes before we write our tests. Otherwise, for now, we are on our own".* This suggests that institutionalizing structured review sessions can boost preparedness and reduce test anxiety.

Innovative and multimedia teaching approaches: Students suggested leveraging technology to enhance learning. Student S47 recommended: *"Lecturer must be creative. We are in 2024: they must create videos for each lesson so that we can always go back to watch where we are experiencing challenges".* This call for multimedia teaching reflects a desire for more flexible and accessible learning methods, aligned with current digital learning trends.

4.3.13.3. Institution-based initiatives

This theme reflects students' views on what colleges could do to improve learning outcomes. Two categories emerged.

Supplementary support in the case of late registration: Students who registered late requested catch-up programs. For instance, S23 remarked: *"I registered a bit late and some of the topics were already taught, so I had to learn those topics on my own. If the college can have a catch-up plan that will be better".* This suggests the need for institutional strategies to support late registrants through bridging or remedial interventions.

Additional academic support (tutoring): Students proposed the introduction of extra lessons and tutors. Student S16 said: *"The college need to hire tutors to help us with mathematics before we write our exams".* This highlights a demand for sustained academic support to supplement formal instruction, especially around examination periods.

4.3.13.4.Assessment-driven initiatives

This theme includes students' ideas about how assessments could be structured to improve understanding and performance.

Continuous assessment and monitoring: Some students suggested weekly assessments. For example S138 remarked: *"We must write weekly tests to test our understanding. I am willing to take part."* S26 stated: *"Lecturers must give us classwork and homework—not only assess us during formal tests".* These suggestions point to the benefits of ongoing assessment for identifying learning gaps and reinforcing understanding. However, the Department of Higher Education (DBE, 2011) cautions that the design of such systems must balance regular feedback with the development of learners' independence and critical thinking.

Students' suggestions during interviews focused on increased practice, better instructional delivery, more support for late registrants, access to tutors, and more regular assessment. These views indicate a desire on the part of students for a more responsive, supportive, and student-centred learning environment. Addressing these concerns through a combination of student-driven strategies, improved teaching practices, institutional adjustments, and assessment reforms could significantly contribute to enhanced mathematics achievement.

4.4. Analysis and answers to research questions

The findings of the data analysis presented above were used to address the four research sub-questions posed in this study.

4.4.1. Answer to research sub-question one

The first research sub-question was: What is the level of pre-existing mathematics knowledge of new entrants to a NC(V) programme in the TVET college?

The analysis presented in Section 4.2 demonstrates that the participants' level of pre-existing mathematical knowledge at the time they started NC(V) Level 2 at the college was low. Only 18% of 150 participating students who wrote the diagnostic test achieved a score of 30% or higher, which is the minimum requirement set by the Department of basic Education (DBE,

2011) guidelines for acceptable performance. The remaining 82% of the participants fell below this threshold, indicating a widespread failure to meet basic competency levels.

As shown in Table 1, the majority of students (123 of 150) fell into the category ‘not achieved’ (0-29%), while a small minority achieved at an elementary or moderate level. Only two students achieved scores classified as substantial or meritorious. These figures suggest that most of the students entered the NC(V) Level 2 programme with an inadequate foundational knowledge of mathematics.

The statistical overview provided in Table 2 supports this conclusion. The mean score of 16, coupled with a standard deviation of ±15.65, reveals that the student performance was generally low and tightly clustered around a poor average. The variance of 244.79 further indicates a wide spread of scores, suggesting inconsistent mastery of knowledge even among the small number of students who performed better.

Together, these findings confirm that these students generally possessed a weak mathematical foundation at the beginning of their NC(V) Level 2 studies, which was likely to hinder their success. The data highlight an urgent need for foundational support and bridge programmes to address students’ knowledge gaps before they engage with the demands of the NC(V) curriculum.

4.4.2. Answer to research sub-question two

The second research question was: What alternative conceptions and errors do students display when solving previous grade mathematics questions?

To investigate alternative conceptions and errors, responses to nine questions were analysed using the Movshovitz–Zaslavsky–Inbar model. The analysis revealed incomplete responses in 10% of cases—students often started correctly but failed to arrive at solutions for questions on some topics, including ratios, algebraic simplification, and geometry (e.g., Figures 1, 4, 8).

The majority (65%) of responses contained misused data errors, where students either omitted necessary information or introduced irrelevant data, altering the problem's requirements. This error type was prevalent across all content areas (e.g., Figures 1, 4, 10, 13, 22).

Technical errors appeared in 17% of responses, manifested as incorrect algebraic manipulations, sign mistakes, or wrong operations across substitution, factorization, and measurement tasks (e.g., Figures 2, 4, 9, 10).

Logically invalid inferences were observed in approximately 2% of responses, such as improper factorization steps (Figure 5), while 6% reflected distorted definitions and task misunderstandings, including reallocating stem-and-leaf data to tally formats or confusing angles with lengths in geometry (Figures 16, 17, 23, 38).

Collectively, these patterns illustrate that these students not only made technical and procedural mistakes but also misinterpreted problems and applied incorrect conceptual frameworks. This suggests persistent gaps in both their procedural fluency and conceptual understanding, reinforcing the need for targeted instructional strategies that address problem interpretation alongside computational accuracy.

4.4.3. Answer to research sub-question three

The third research question was: What is the nature of students' pre-existing mathematical knowledge acquired up to Grade 9?

To address this question, interview data from Sections 4.3.2 to 4.3.12 were analysed. Two main themes emerged from students' responses: lack of conceptual understanding and gaps in procedural knowledge. These themes were consistent across the various mathematical domains evaluated in the diagnostic assessment.

The first theme, lack of conceptual understanding, reflected students' struggles to grasp foundational principles and interpret mathematical tasks accurately. Many learners expressed uncertainty about terms such as 'simplify', 'substitute', or 'factorise', as shown in Sections 4.3.2, 4.3.3, and 4.3.5. In Section 4.3.6, students showed confusion around identifying number patterns, indicating that they had not internalized basic algebraic structures. Others, particularly in Sections 4.3.8 and 4.3.10, struggled to differentiate between related concepts such as congruency and similarity, further highlighting conceptual confusion from earlier grades. Some learners also linked these difficulties to significant gaps in their education, suggesting poor long-term retention, as seen in Sections 4.3.2 and 4.3.9.

The second theme, gaps in procedural knowledge, revealed that even when students could recall concepts, they often failed to apply the correct steps for problem-solving. In Section 4.3.3, for example, students substituted values but mishandled signs or exponents. In Sections 4.3.4 and 4.3.7, common issues included combining unlike terms and errors during factorization. A number of learners reported that they had stopped trying to solve a problem midway as they were uncertain about the next step (see Sections 4.3.5 and 4.3.6). Procedural issues were further complicated by over-reliance on calculators, as shown in Sections 4.3.3 and 4.3.10, where learners bypassed manual steps that could have reinforced their understanding.

Together, these findings indicate that the students' experiences in mathematics through Grade 9 were marked by interrupted conceptual development and insufficient procedural fluency. Many lacked confidence interpreting and solving basic problems, often relying on guessing or avoiding questions altogether. The reflections in Section 4.3.12, where students suggested more revision and clearer explanations in earlier years, confirm the presence of both instructional and experiential gaps. These insights suggest a need for strengthened foundational teaching and more sustained engagement with core mathematical practices in earlier grades.

4.4.4. Answer to research sub-question four

The fourth research question was: What strategies may improve achievement in NC(V) Level 2 Mathematics?

Based on the analysis of test responses and interview data presented in Sections 4.3.1 to 4.3.12, several strategies can be proposed to address persistent underperformance in NC(V) Level 2 Mathematics. First, the students displayed widespread procedural errors and conceptual misunderstandings, as seen in questions involving algebraic simplification, solving equations, and ratio calculations (Sections 4.3.2, 4.3.4, and 4.3.7). This highlights the need to strengthen both conceptual and procedural knowledge through the targeted revision of foundational topics that they should have been taught in previous grades. Second, many students relied heavily on calculators, even for basic operations, indicating a lack of fluency in manual computation. As noted in Sections 4.3.3 and 4.3.6, this dependence could be addressed by emphasising mental arithmetic and step-by-step working, thus promoting a deeper engagement with mathematical procedures. Third, in the interview’s students expressed confusion about terminology and question requirements (Sections 4.3.2, 4.3.8), highlighting the importance of explicit teaching of mathematical language and symbols to improve the interpretation of questions.

Furthermore, the students showed signs of low confidence and anxiety, often skipping questions or second guessing their answers (Sections 4.3.5, 4.3.11). To address this, teachers could incorporate formative feedback and supportive interventions that foster resilience and self-efficacy in solving mathematical problems. Finally, Section 4.3.12 highlights the students' own suggestions for improvement, including the need for more practice, clearer explanations, and a review of Grade 9 content. Taken together, these findings underscore the importance of a holistic and student-informed approach to remediation, one that addresses both cognitive gaps and affective barriers to learning.

4.5. Conclusion

This chapter has presented the analysis of data collected using a diagnostic test and unstructured interviews. The data revealed that the NC(V) Level 2 students experienced numerous difficulties on the diagnostic test, which assessed their mastery of the previous level (Grade 9), demonstrating a lack of conceptual and procedural knowledge related to nearly all the concepts covered in the assessment. As these concepts form the foundation on which the NC(V) Level 2 syllabus builds, this chapter's objective was to determine the level of understanding which can impact achievement either way. Furthermore, this chapter has identified critical fundamentals that required students, lecturers and institutions to collaborate to achieve a common goal for all. The next chapter discusses the findings derived from the data presented in this chapter.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1. Introduction

This chapter summarises how the research objectives and questions outlined in Chapter 1 have been addressed and fulfilled. It discusses key findings and the implications of these findings. The chapter makes recommendations for lecturers, the institution, and students involved in the teaching and learning process of NC(V) Level 2 Mathematics and reflects on the limitations of the study.

5.2. Summary of the study

The purpose of this study was to explore the influence of the pre-existing mathematics knowledge of students entering NC(V) Level 2 Mathematics on their performance and achievement during Level 2 and to recommend approaches to improving their mastery. To achieve the aim of this study, the following research question was formulated: *What gaps in pre-existing mathematics knowledge do NC(V) Level 2 students have when they enter a TVET college?*

The following sub-questions were used to answer this question:

(1) What is the level of pre-existing mathematics knowledge of new entrants at a TVET college?

(2) What alternative conceptions and errors do students display when solving mathematics questions at the level of their previous year of study?

(3) What is the nature of students' pre-existing mathematical knowledge acquired up to Grade 9?

(4) What strategies may improve achievement in NC(V) Level 2 Mathematics?

Piaget's (1896) theory of constructivism and Movshovitz-Hadar et al.'s (1987) error analysis framework provided the theoretical framework for this study. An explanatory sequential mixed method design was used. The target population consisted of 350 NC(V) Level 2 Engineering students at a TVET college in Ekurhuleni district; 150 students were selected using convenience sampling to take part in a diagnostic test and 21 were purposively sampled for interviews based on their test results. The diagnostic test covered all mathematics topics included in the Grade 9 curriculum: ratios, integers, substitution, algebraic expression, algebraic equations, financial math, word problem, algebraic function, geometry, statistics and measurements. Data analysis combined quantitative methods (descriptive statistics including

mean, variance, and standard deviation) and qualitative methods, where Movshovitz-Hadar et al.'s (1987) model was used to analyse test scripts and interview transcriptions.

The study found that students lacked both conceptual and procedural knowledge across several Grade 9 mathematics topics resulting in weak foundational knowledge that was likely to hinder their success in NC(V) Level 2 Mathematics. Participants exhibited all aspects of the Movshovitz–Zaslavsky–Inbar model of alternative conceptions and errors in their responses to all diagnostic test questions; these included misinterpreting questions and applying incorrect conceptual frameworks.

The findings also revealed that the students' experiences with mathematics up to Grade 9 had been marked by interrupted conceptual development and insufficient procedural fluency. Many lacked confidence interpreting and solving basic problems, often relying on guessing or avoiding questions altogether. When asked during interviews how their mathematics competence could be improved, participants identified daily practice, extra lessons, and catch-up plans for late registration.

5.3. Discussion of the findings

The findings presented and analysed in the preceding chapter are examined with reference to the theoretical framework outlined in Chapter 3 and the research questions introduced in Chapter 1.

5.3.1 The level of existing mathematics knowledge of new entrants in the TVET college

The findings from the diagnostic test indicate that the majority of participants entered the NC(V) Level 2 programme with limited mathematical proficiency. Only 18% met the 30% benchmark set by the Gauteng Department of Education (GDE) Government notices No. 1115 and 1116 in Government Gazette No. 36042 of 28 December 2012, indicating that they had significant gaps in their foundational knowledge. These results mirror the findings of the TIMSS assessments (Reddy, 2022), which consistently report that South African Grade 9 learners struggle with basic mathematical operations.

In Türkiye, a study by Yayla and Bangir-Alpan (2019) found that Grade 9 students experienced difficulty with mathematics due to insufficient foundational knowledge and difficulty understanding the teaching style of educators. This suggests that this problem is not unique to South Africa but is a more pervasive issue. This highlights the urgent need for foundational

support and structured bridge programmes before students engage with the demands of the NC(V) curriculum. From a constructivist perspective, these findings suggest that many students entered NC(V) Level 2 without sufficiently developed prior cognitive structures required to support new mathematical learning.

5.3.2 Alternative conceptions and errors in mathematical problem solving

The findings revealed that students exhibited both conceptual and procedural errors across all topics assessed in the diagnostic test. These included algebraic expressions, exponents, ratios, equations, word problems, functions, geometry, and measurement. These errors occurred in recognizable patterns rather than as isolated mistakes; the patterns included misused data, technical errors, incomplete responses, logically invalid errors, and distorted definitions. Students frequently substituted values incorrectly, misapplied exponent rules, or attempted operations (such as squaring fractions or roots) without understanding the underlying principles

In this study, some students demonstrated errors in substitution and the handling of exponents. For example, some students incorrectly separated algebraic terms and misapplied substitution procedures. Similarly, students like S16, who were able to substitute but failed to square a fraction, demonstrated procedural fluency without conceptual depth. These patterns correspond with the findings of Moyo and Machaba (2020), where South African Grade 9 students misapplied rules for fractions and confused operations of multiplication by finding common denominators where this was not required.

The study also revealed misuse of calculators and over-reliance on memorized procedures without comprehension. Students like S33 relied on calculators but lacked the ability to validate or interpret their results. This aligns with the findings of Beros et al. (2024) and Castaldi et al. (2020), who caution that excessive calculator use can suppress students' conceptual engagement, impairing their ability to build flexible problem-solving strategies.

When working with ratios, students in this study were able to convert decimals to fractions but often failed to use the highest common factor (HCF) method to simplify. This aligns with the findings of Tanudjaya and Doorman (2020), who reported similar proportional reasoning gaps among Grade 9 students in Indonesia. Reddy (2022) also found that many South African Grade 9 learners struggled with finding common factors when simplifying ratios.

Students also struggled with exponent and root operations, often misapplying exponent rules. Some used exponent rules incorrectly—for example, cancelling roots by arbitrarily adjusting exponents (e.g. raising a term to power 3 incorrectly). Stemele and Asvat (2024), Ulusoy (2019), and Putri et al. (2024) similarly documented persistent exponent-rule misunderstandings among secondary students.

Students also struggled with exponent and root operations, often misapplying exponent rules. Some students were able to expand expressions using the FOIL technique but then combined unlike terms, indicating confusion in their procedural approach. Studies by Luneta (2016), Ngoveni (2018), Mofolo-Mbokane and Ngoveni (2019), and Stemele and Asvat (2024) reported similar algebraic groupings of adding like terms and difficulty removing brackets—further highlighting conceptual misalignment. In this study, students frequently misused transposition rules when engaging with equations and unknown variables—failing to change signs or applying procedures at inappropriate times. Ngoveni (2018) found the same challenges with algebraic fluency that NC(V) Level 2 students demonstrated in this study when solving equations; Mgidi (2023) also found the same challenge about t the Grade 9 students that they failed to change signs when transposing terms. Mehlo (2026) highlighted that NC(V) level 2 students are unable to simplify expressions, manipulate equations, and solve problems due to conceptual and procedural errors.

Students in this study also struggled to interpret word problems correctly. They had difficulty converting real-world contexts into symbols—often conflating ratios with speed/formulaic concepts. Financial word problems were particularly challenging to them. This resonates with the findings of Emanuel et al. (2020) and Putri et al. (2024), who emphasize how context and translation errors hinder problem-solving. When completing graphical and functional tasks, many students lacked procedural fluency—for example, failing to recognize that a horizontal line has a gradient of zero. This reflects the broader challenge highlighted by Ngwabe (2018) that students experience difficulty moving from graphic visuals to symbolic expressions.

For data handling and geometry, some students misread stem-and-leaf plots, confused angle measurements, and struggled with shape-based measurement—evidenced by skipping questions or misidentifying formulae for cylinders and polygons. These errors correspond with the findings of Reddy (2019), Bowie et al. (2022), Dagli et al. (2016), and Kgopane (2021) on the misconceptions of geometry of Grade 9 students.

Overall, the combined evidence points to deep-rooted misconceptions and fragmented knowledge and procedural skills. To address these, the study reinforces the need for instructional approaches that integrate conceptual explanations with step-by-step procedural reinforcement, such as the use of visual representations, error analysis, and formative assessment techniques to explicitly address and correct misconceptions.

5.3.3 Students' experiences with mathematics learning up to Grade 9

The findings drawn from Sections 4.3.2 to 4.3.12 reveal that students' experiences with mathematics learning up to Grade 9 were characterized by a lack of conceptual understanding and gaps in procedural knowledge. This was reflected across topics such as ratios, algebra, exponents, geometry, statistics, measurement and functions, pointing to significant foundational weaknesses that continued to affect learners' mathematical performance at the NC(V) Level 2.

Many students struggled to grasp foundational mathematical concepts. For instance, students misunderstood basic terms such as 'simplify', 'factorise', and 'substitute', as demonstrated in their responses to algebraic and numerical tasks. This aligns with findings by Luneta (2016), who reported that South African Grade 9 learners often confused conceptual rules, particularly in algebraic simplification. Similarly, Tanudjaya and Doorman (2020) highlighted students' difficulties with proportional reasoning, which resonates with this study's findings on ratio simplification.

In addition to conceptual gaps, procedural difficulties were widespread. Students frequently misapplied the FOIL method, mishandled signs, and failed to complete problem-solving sequences. As Ngoveni (2018) observed, students at NC(V) Level 2 often struggle with identifying and combining like terms, which was echoed in this study's findings on algebraic expression tasks. Mofolo-Mbokane and Ngoveni (2019) also found that procedural errors in factorization were common due to insufficient prior knowledge—a challenge similarly reflected in this cohort. Moreover, the incorrect application of rules governing exponents and algebraic equations points to both conceptual and procedural confusion. Ulusoy (2019) and Stemele and Asvat (2024) identified similar issues, reporting that students misused exponential rules and failed to solve problems involving negative or zero exponents. In this study, such gaps were compounded by students' over-reliance on calculators, which limited their ability to internalize core mathematical operations.

Further complications arose in problem contexts involving graphs, word problems, and measurements. Students had difficulty translating real-world scenarios into mathematical equations and selecting appropriate formulae—findings that were also reported by Putri et el. (2024) and Kgopane (2021), who documented persistent confusion about geometry and applied contexts among Grade 9 learners.

Collectively, the findings suggest that students' earlier experiences with mathematics did not adequately support the development of stable conceptual understanding and procedural fluency required for NC(V) Level 2 Mathematics. This supports constructivist views that new learning depends heavily on the quality and stability of previously acquired knowledge structures. The persistence of foundational errors and misconceptions supports the need for early and sustained interventions that emphasize not only rote procedures, but deep conceptual engagement and confidence-building. The call for clearer explanations and more revision support, as voiced by students themselves in Section 4.3.12, further underscores the critical role of instructional quality in shaping learners' long-term mathematical trajectories.

5.3.4 Strategies for improving mathematics achievement at NC(V) Level 2

The analysis of interview and test data presented in Sections 4.3.1 to 4.3.12 reveals several interrelated factors that students believed could enhance mathematics performance. These insights, grouped into student-based, lecturer-based, and institutional initiatives, emphasize both cognitive and affective needs that must be addressed to improve achievement at NC(V) Level 2.

A major finding across the diagnostic test responses and interviews was the prevalence of conceptual misunderstandings and procedural errors, particularly in algebraic simplification, equations, and ratio problems (Sections 4.3.2, 4.3.4, and 4.3.7). Many students struggled to recall prior knowledge, execute multi-step processes, and interpret terms such as 'simplify' or 'factorise'. These gaps suggest the need for foundational support programmes that revisit Grade 9 content and build fluency through scaffolded revision. This aligns with Ngoveni's (2018) argument that continuous practice is essential for skill retention and academic improvement.

Another pervasive issue was over-reliance on calculators even for basic computations (Sections 4.3.3 and 4.3.6). Students often bypassed manual calculation steps, undermining their conceptual engagement with mathematical operations. As Beros et al. (2024) argue, excessive

use of calculators can weaken memory and hinder mathematical reasoning. This highlights the importance of balancing calculator use with opportunities for manual computation and reasoning.

Students also reported anxiety, lack of confidence, and uncertainty when interpreting tasks (Sections 4.3.5, 4.3.11), often leading to skipped questions or incomplete answers. These affective barriers point to the need for formative assessments, motivational feedback, and a classroom culture that promotes resilience. The incorporation of weekly assessments, as suggested by students in Section 4.3.12, could help normalise feedback and reinforce progress incrementally.

From an instructional perspective, students highlighted a lack of clarity in teaching methods and ineffective communication of mathematical concepts. Several respondents attributed their struggles to unclear explanations and underprepared lecturers. Dolley (2015) and Ngoveni (2018) stress the importance of capacitating lecturers through workshops and peer-sharing platforms to improve their pedagogical practices. Dolley (2015) further recommends using concrete teaching aids—such as physical 3D models—to enhance understanding, particularly in spatial and visual domains.

Lastly, students perceived stronger institutional-level support as essential. Suggestions included structured revision classes, enhanced tutoring, and consistent monitoring of student progress. A supportive institutional environment that prioritizes both academic and emotional support may mitigate the effects of foundational gaps and improve student motivation.

Overall, the findings indicate that improving mathematics achievement at the NC(V) Level 2 requires a coordinated, multilevel approach. Addressing instructional clarity, providing targeted foundational support, and cultivating learner confidence are critical to fostering meaningful engagement with mathematics.

5.4. Implications of the findings

This section presents a detailed discussion of the implications of the study's findings, outlining possible applications, future research directions, and the conclusions that may be reached.

5.4.1 Implications for foundational mathematics readiness

This study found that 82% of participants—who were new entrants to the NC(V) Level 2—did not achieve the minimum 30% benchmark on the diagnostic test. This raises serious concerns about the mathematical readiness of students enrolling in the NC(V) Level 2 programme. This suggests that current admissions policies may allow students who lack the foundational competencies necessary to engage meaningfully with the Level 2 Mathematics curriculum to be admitted to the programme. In light of this, institutions may need to implement entry-level screening tools and establish bridging programmes specifically targeting core Grade 9 mathematics content. These interventions would provide academically underprepared students with structured support before they encounter more advanced topics.

The results of this study also underscore the need for colleges—and potentially the Department of Higher Education and Training (DHET)—to revisit admissions and support policies. This might include formalising diagnostic testing as part of the entry process and using those results to guide early interventions. Such measures could help identify academically underprepared students early and provide targeted support before progression into more advanced mathematics.

Finally, the findings point to a need for further investigation into the schooling contexts in which these students were educated. Understanding whether feeder schools are adequately preparing learners in Grade 9 mathematics would provide additional insight into the systemic roots of these challenges and could inform collaborative efforts between TVET colleges and secondary schools.

5.4.2 Implications of students' alternative conceptions and errors

The NC(V) Level 2 students who participated in this study exhibited a range of alternative conceptions and procedural errors across all major mathematical domains assessed from the Grade 9 curriculum. This widespread difficulty underscores that many students enter the NC(V) programme with significant gaps in their foundational understanding, which, if unaddressed, may continue to hinder their success in Mathematics at NC(V) Level 2.

It is therefore critical for lecturers to integrate a remedial focus into their lesson planning. This requires explicitly revisiting key Grade 9 concepts and anticipating common misconceptions.

Lecturers should routinely implement formative assessments —such as short class exercises or quizzes—and systematically review them to detect persistent errors. Immediate, targeted feedback and corrective instruction should follow to help students rebuild accurate mathematical understandings and avoid compounding misconceptions as they engage with Level 2 content.

5.4.3 Implications of students' prior experiences with mathematics learning

This study found that participants' previous experiences with mathematics were marked by interrupted conceptual development and insufficient procedural fluency. This implies that TVET colleges need to develop or implement instruments to assess the readiness of students to enrol in the NC(V) Mathematics programme. Such diagnostic tools would enable lecturers to identify students who are underprepared and to implement appropriate support mechanisms early in the programme.

Additionally, lecturers should routinely design classroom tasks that integrate both conceptual and procedural aspects of mathematics, allowing students to develop a deeper understanding of mathematical operations and how to apply them. Emphasizing varied and scaffolded activities could strengthen students' foundational comprehension and ability to perform multi-step calculations with confidence.

5.4.4 Implications for student-informed improvement strategies in mathematics

In this study, participants offered valuable insights into the practical realities of the classroom, which provides a foundation for more responsive teaching and institutional practices. Their suggestions point to the need for collaborative efforts by students, lecturers, and the institution to improve mathematics achievement at NC(V) Level 2.

Students emphasized the value of active learning approaches—such as interactive games, quizzes, and real-life applications—that enhance engagement, motivation, and conceptual understanding. Lecturers should consider integrating these strategies into lesson planning, alongside appropriate technological tools, to create a more stimulating and supportive learning environment.

Participants also reported concerns about the clarity and pace of instruction. Some reported that topics were introduced rapidly and not always explained clearly. This highlights the need for lecturers to use precise mathematical language, regularly check for understanding, and adjust

their pacing to accommodate different rates of learning. Encouraging student participation and creating space for questions could help students to deepen their understanding of complex concepts and reduce the development of alternative conceptions.

At the institutional level, the absence of catch-up mechanisms for late registrants emerged as a significant gap. The college should consider implementing structured support strategies, such as additional after-hours sessions, peer tutoring, or bridge programmes, to assist students who fall behind. In addition, lecturers are advised to embed formative assessment practices into their teaching to monitor progress continuously and provide timely, constructive feedback. Lastly, promoting a positive attitude toward mathematics is essential. Lecturers should actively foster student confidence by recognizing progress, offering encouragement, and framing challenges as opportunities for growth. These student-informed insights should be formally integrated into institutional planning and teaching practices to create a more equitable and effective learning environment.

5.5. Recommendations

Based on the findings of this study, several recommendations are made for institutions and practitioners and for further research.

5.5.1 Recommendations for institutions and practitioners

Implement foundational bridging programmes. TVET colleges should develop or introduce bridging courses tailored for NC(V) Level 2 students. These courses should target learners who achieved the minimum pass mark of 30% in Grade 9 mathematics, helping them build proficiency up to at least 50%. A 30% pass often reflects limited understanding of key mathematical concepts, suggesting a need for foundational remediation.

Strengthen conceptual and procedural instruction. Lecturers should begin lessons by assessing students' pre-existing knowledge using diagnostic questioning. Opportunities should be provided for students to articulate their understanding, which promotes engagement and deeper learning. Instruction should include real-life applications and the integration of technology. Additionally, lecturers should revise essential foundational concepts across all topics to strengthen students' conceptual and procedural understanding.

Enhance teacher training and support. TVET colleges should organize regular workshops and professional development sessions to enhance lecturers' content knowledge and

pedagogical skills. Institutions should also facilitate consistent subject meetings to address content gaps, encourage peer learning among staff, and share effective teaching practices.

Establish regular diagnostic and formative assessment. Institutions should implement recognised diagnostic instruments to assess student readiness for NC(V) mathematics. This would enable lecturers to identify and support students who may struggle with course content. Regular formative assessments, including classwork and homework, should be closely monitored to ensure students are progressing and misconceptions are promptly addressed.

Foster a supportive learning environment. Lecturers should cultivate an inclusive and supportive classroom atmosphere where students feel safe to participate and ask questions. Availability during consultation hours and encouragement of group work can enhance collaborative learning. The use of varied representations and alternative solution strategies should be embedded into teaching practices.

Offer academic tutoring and mentoring. TVET colleges should introduce structured tutoring and mentoring programmes. These could include morning or afternoon support classes focused on reinforcing previously taught content. Such initiatives would allow students to engage with material at their own pace and receive individualized attention.

Involve students in learning feedback. Lecturers should provide timely and constructive feedback. This should begin with acknowledging correct responses before guiding students to understand and correct their errors. Students should be encouraged to revise incorrect responses and resubmit them by a set deadline, fostering accountability and improvement.

Institutionalise revision programmes. Colleges should establish compulsory revision sessions before major assessments or exams. These sessions should be carefully scheduled and aligned with the curriculum to maximize student benefit without disrupting learning in other subjects.

5.5.2 Recommendations for further research

To build on the findings of this study, future research could pursue two key directions.

First, there is a need to design and evaluate targeted intervention programmes, such as structured bridging or remediation courses aimed at addressing foundational misconceptions among NC(V) Level 2 students. Such studies could adopt a longitudinal, mixed-methods approach to assess the effectiveness of these interventions in improving conceptual understanding, procedural fluency, and long-term achievement in mathematics.

Second, while this study focused on students' conceptual and procedural understanding, future research could expand this investigation to include the role of lecturers in influencing mathematics learning outcomes at NC(V) Level 2. Suggested areas for further exploration include:

- **Pedagogical content knowledge (PCK):** Future studies could investigate how lecturers teach mathematical concepts, their ability to diagnose student misconceptions, and how their instruction aligns with curriculum goals.
- **Comparative institutional studies:** A broader comparative study across multiple TVET colleges—especially between higher and lower performing campuses—could uncover systemic patterns in teaching quality and student outcomes.
- **Correlation with student outcomes:** Studies could explore the relationship between lecturer expertise (both content and pedagogical) and student achievement in NC(V) Mathematics, using performance data to assess instructional impact.

5.6. Limitations of the study

This study was conducted at a single TVET college campus with a sample of 150 students, which limits the generalizability of the findings to other TVET institutions.

The diagnostic assessment was administered at the beginning of the NC(V) Level 2 programme, using a Grade 9 question paper to evaluate students' existing mathematical knowledge. While this approach effectively identified foundational gaps, it did not measure progression over time or the impact of instruction during the academic year.

The study did not examine the content knowledge or teaching practices of lecturers, which may also play a significant role in shaping student learning outcomes. The qualitative component, based on student interviews, was limited in scope and may reflect only a subset of learner experiences.

5.7. Conclusion

The aim of this study was to explore the influence of students' pre-existing knowledge of mathematics on their achievement in NC(V) Level 2 Mathematics at a TVET college in the Ekurhuleni District. The study revealed multiple challenges encountered by students when assessed on topics from the South African Grade 9 Mathematics curriculum.

Analysis of participants' diagnostic test results and unstructured interviews with participants revealed both conceptual and procedural gaps in participants' knowledge of Grade 9 Mathematics. Students frequently misused data and demonstrated weak understanding of foundational mathematical concepts, including basic arithmetic, algebraic manipulation, ratio simplification, geometry, and word problems, indicating poor internalisation or retention of mathematics taught in Grade 9.

These results imply that lecturers should identify and prioritise areas that need reinforcement when planning lessons. Strengthening these areas could help improve students' confidence, support knowledge retention, and enhance performance. Applying constructivist principles that build on students' prior knowledge, encouraging active participation, and addressing errors immediately, could also address alternative conceptions.

The study contributes to mathematics education research within the TVET sector by providing empirical evidence of the specific conceptual and procedural gaps that NC(V) Level 2 students bring from prior schooling experiences. It further contributes by identifying common patterns of alternative conceptions and errors across multiple mathematical domains, offering practical insight for lecturers, curriculum planners, and institutions seeking to strengthen foundational mathematics support within TVET colleges. This study provides valuable insight into the level of pre-existing mathematical knowledge of students entering NC(V) Level 2 Engineering. This information can guide TVET colleges lecturers in planning lessons and TVET colleges in refining admission processes, reviewing the Mathematics curriculum, and developing support strategies to improve student readiness and achievement.

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APPENDICES

Appendix A: Approval from Ethics Review Committee

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2023/10/11

Ref: **2023/10/11/60042672/03/AM**
Name: Ms T CAPA
Student No.:60042672

Dear Ms T CAPA

Decision: Ethics Approval from 2023/10/11 to 2026/10/11

Researcher(s): Name: Ms T CAPA
E-mail address: 60042672@mylife.unisa.ac.za
Telephone: 0748229505

Supervisor(s): Name: Dr EG Makwakwa
E-mail address: makwaeg@unisa.ac.za
Telephone: 012 429 4575

Title of research:

INFLUENCE OF PRE-EXISTING MATHEMATICS KNOWLEDGE ON STUDENTS' ACHIEVEMENT AT A LEVEL 2 TVET COLLEGE IN EKURHULENI DISTRICT.

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2023/10/11 to 2026/10/11.

*The **low risk** application was reviewed by the Ethics Review Committee on 2023/10/11 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2026/10/11**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2023/10/11/60042672/03/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,



Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za



Prof Mpine Makoe
EXECUTIVE DEAN
qakisme@unisa.ac.za

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

Appendix B: Request to TVET college for permission to conduct study

Request for permission to conduct research at Ekurhuleni West TVET college (Tembisa campus)

Title of the research: Influence of pre-existing mathematics knowledge on students' achievement at a level 2 TVET college in Ekurhuleni district.

Date: 24 AUGUST 2023

Connie Maleka

Campus Manager

082 529 5747

conniema@ewc.edu.za

Dear Ms Maleka

My name is Thabisa Capa, and I am a student at the University of South Africa (UNISA) pursuing a Master's degree in Mathematics Education. I am supervised by Dr. EG Makwakwa, a Senior Lecturer in the Department of Mathematics at UNISA. We have funding from the Mining Qualifications Authority for effective research outcome of the study. We are inviting you to participate in a study entitled "Influence of pre-existing mathematics knowledge on students' achievement at a level 2 TVET college in Ekurhuleni district".

The aim of the study is to explore the influence of pre-existing knowledge on Level 2 TVET students' mathematics achievement.

Your college has been selected because the researcher saw it fit and has the potential of generating data to answer the research questions and fulfill the aims and objectives of the study. The sample size of 150 NC(V) level 2 registered engineering students will be expected to write a diagnostic test to measure the level of mathematics knowledge of students entering level 2. Secondly, 25 marked scripts will be purposively sampled from the convenience sample to analyze alternative conceptions and errors displayed by students when answering mathematics questions. Lastly, the researcher will conduct unstructured interviews with the 25 students who had their marked scripts purposively selected for analysis of alternative conceptions and errors. The unstructured interviews will help the researcher to understand why students did not solve the questions the way they should have, especially with those questions where they may have erroneously answered or omitted errors, and to understand students' mathematics experiences up to Grade 9.

The benefits of this study are that the findings of this study are anticipated to assist mathematics lecturers in understanding the level and diversity of mathematics knowledge attained by Level 2 students before continuing or deepening their studies in mathematics at Level 2. The results of this study are anticipated to assist level 2 mathematics lecturers in knowing which area content to focus on the

most when they design lesson plans and also to design lesson plans that will revive the prior knowledge of the students which were supposed to have been acquired in earlier grades and existing in the students' mind. The need for potential future research will also be established.

There will be no reimbursement or any incentives for participation in the research.

The feedback procedure will entail sharing the final draft of the research findings with the participants for verification before writing up the final report and publication.

Yours sincerely



Thabisa Capa

M.Ed. Student UNISA

Appendix C: Participation Information Sheet



Date: 23 August 2023

Title: **Influence of pre-existing mathematics knowledge on students' achievement at a level 2 TVET college in Ekurhuleni district.**

My name is Thabisa Capa, and I am a student at the University of South Africa (UNISA) pursuing a Master's degree in Mathematics Education. I am supervised by Dr. EG Makwakwa, a Senior Lecturer in the Department of Mathematics at UNISA. We have funding from Mining Qualifications Authority for effective research outcome of the study. We are inviting you to participate in a study entitled "Influence of pre-existing mathematics knowledge on Students' achievement at a Level 2 TVET College in Ekurhuleni district".

WHAT IS THE PURPOSE OF THE STUDY?

This study is expected to collect important information that could impart knowledge and for potential future research. The purpose of the study is to explore the influence of pre-existing knowledge on Level 2 TVET students' mathematics achievement. I hope that the findings of this study will provide guidance to lecturers on how to prepare their daily lessons to ensure positive outcomes in mathematics level 2.

WHY AM I BEING INVITED TO PARTICIPATE?

You are invited because your contribution and input can assist in reaching the outcomes of the study. I believe that sharing your experience will help me obtain rich data for the study.

I obtained your contact details from the campus manager of the college. There will be 150 participants for the study.

WHAT IS THE NATURE OF MY PARTICIPATION IN THIS STUDY?

Describe the participant's actual role in the study.

150 NC(V) level 2 registered engineering students will be expected to write a diagnostic test (Grade 9 paper) to measure the level of mathematics knowledge of students entering level 2. The duration of the test will be 3 hours. Secondly, 25 marked scripts will be selected from the 150 scripts to analyze alternative conceptions and errors displayed by students when answering mathematics questions. Lastly, the researcher will conduct unstructured interviews with the 25 students who had their marked scripts selected for analysis of alternative conceptions and errors. The unstructured interviews will help the researcher to understand why students did not solve the questions the way they should have, especially with those questions where they may have erroneously answered or omitted errors, and to understand students' mathematics experiences up to Grade 9.

It is possible that the researcher may contact the participant later to clarify certain points, or to request further (shorter) interview to explore some details in greater depth and to verify answers. A concise draft summary of the main findings will be sent to each school.

CAN I WITHDRAW FROM THIS STUDY EVEN AFTER HAVING AGREED TO PARTICIPATE?

Participating in this study is voluntary and you are under no obligation to consent to participation. If you do decide to take part, you will be given this information sheet to keep and be asked to sign a written consent form. You are free to withdraw at any time and without giving a reason.

WHAT ARE THE POTENTIAL BENEFITS OF TAKING PART IN THIS STUDY?

The benefits of participants being involved in this study is that they will positively contribute to show pre-existing- knowledge of earlier grades, and their scripts will assist the researcher in concluding the study. Participation of participants in this study will also assist the researcher in reaching the study's purpose and objectives.

ARE THERE ANY NEGATIVE CONSEQUENCES FOR ME IF I PARTICIPATE IN THE RESEARCH PROJECT?

There are no foreseeable negative inconveniences except the time to be taken participating in this study. The study does not require sensitive information. The results obtained from the participants' scripts regarding showing their pre-existing knowledge in mathematics is believed to pose a low risk.

WILL THE INFORMATION THAT I CONVEY TO THE RESEARCHER AND MY IDENTITY BE KEPT CONFIDENTIAL?

You have the right to insist that your name will not be recorded anywhere and that no one, apart from the researcher and identified members of the research team, will know about your involvement in this research. Your name will not be recorded anywhere and no one will be able to connect you to the answers you give. Your answers will be given a code number and you will be referred to in this way in the data, any publications, or other research reporting methods such as conference proceedings.

Only the researcher and supervisor will have access to the data and they will also sign a confidentiality agreement to maintain confidentiality.

Participants anonymous data may be used for other purposes, such as a research report, journal articles and/or conference proceedings. *A report of the study may be submitted for publication, but individual participants will not be identifiable in such a report.*

HOW WILL THE RESEARCHER(S) PROTECT THE SECURITY OF DATA?

Hard copies of your answers will be stored by the researcher for a period of five years in a locked cupboard/filing cabinet at my house for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. In case of the need to destroy information *(e.g. hard copies will be shredded and/or electronic copies will be permanently deleted from the hard drive of the computer through the use of a relevant software programme).*

WILL I RECEIVE PAYMENT OR ANY INCENTIVES FOR PARTICIPATING IN THIS STUDY?

There will be no payment or reward offered for participating in this study.

HAS THE STUDY RECEIVED ETHICS APPROVAL

This study has received written approval from the Research Ethics Review Committee of the college of education, Unisa. A copy of the approval letter can be obtained from the researcher if you so wish.

HOW WILL I BE INFORMED OF THE FINDINGS/RESULTS OF THE RESEARCH?

If you would like to be informed of the final research findings, please contact Thabisa Capa on 074 822 9505 or email qhoshet@gmail.com. The findings are accessible for 5 years.

Should you require any further information or want to contact the researcher about any aspect of this study, please contact on 074 822 9505, email qhoshet@gmail.com

Should you have concerns about the way in which the research has been conducted, you may contact: Dr. E.G. Makwakwa,: email address: makwaeg@unisa.ac.za

Thank you for taking the time to read this information sheet and for participating in this study.

Thank you.



Capa Thabisa

Appendix D: Parental consent



EXAMPLE OF A LETTER REQUESTING PARENTAL CONSENT FOR MINORS TO PARTICIPATE IN A RESEARCH PROJECT

Dear Parent

Your _____<son/daughter/child> is invited to participate in a study entitled **Influence of pre-existing mathematics knowledge on students' achievement at a level 2 TVET college in Ekurhuleni district.**

I am undertaking this study as part of my Master's research at the University of South Africa. The purpose of the study is to explore the influence of pre-existing knowledge on Level 2 TVET students' mathematics achievement.

and the possible benefits of the study are the improvement of mathematics teaching and learning. I am asking permission to include your child in this study because (he/she) is currently registered for NC(V) level 2 mathematics. I expect to have 150 other children participating in the study.

If you allow your child to participate, I shall request him/her to complete a test with other students who are enrolled for mathematics level 2. The test will be administered at Ekurhuleni TVET College during school hours. The duration of the paper is 2 hours. The test will be invigilated by all mathematics lecturers.

Furthermore, after the test is marked and assessed, students will be randomly selected for further deliberation to understand the misconceptions and errors the student has made. This will be conducted via a one-on-one interview session with the student and will be recorded for analysis and conclusions.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are lecturers will know the level of mathematics pre-existing knowledge they have when they enter NC(V) level 2, and this will assist mathematics lecturers in preparing their lesson. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available by replacing your child with other students.

In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

The benefits of this study are to assist lecturers to know the level of mathematics pre-existing knowledge of the NC(V) level 2 students and that will assist them in preparing their daily lesson plan to improve the achievement of mathematics level 2.

The are no risks expected in this study.

There will be no reimbursement or any incentives for participation in the research.

If you have questions about this study please ask me or my study supervisor, Prof/Dr EG Makwakwa Department of Mathematics, College of Education, University of South Africa. My contact number is 074 822 9505 and my e-mail is qhoshet@gmail.com. The e-mail of my supervisor is makwaeg@unisa.ac.za. Permission for the study has already been given by College administrators and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child: …….

Sincerely

_____	_____	
Parent/guardian's name (print)	Parent/guardian's signature:	Date:
Thabisa Capa		24 August 2023
Researcher's name (print)	Researcher's signature	Date:

Appendix E: Consent form to participate in the study



I, _____ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the writing of the test and unstructured interviews if my script has be selected.

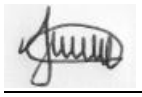
I have received a signed copy of the informed consent agreement.

Participant Name & Surname (please print)_____

_____ _____

Participant Signature Date

Researcher's Name & Surname (please print)_____Thabisa Capa_____



_____ 24 August 2023_____

Researcher's signature

Appendix F: Grade 9 question paper

testpapers.co.za

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION

PROVINCIAL EXAMINATION

NOVEMBER 2019

GRADE 9

MATHEMATICS

NAME OF LEARNER: _____

GRADE / CLASS: _____

TIME: 2 hours

MARKS: 100

19 pages

P.T.O.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions and 19 pages, including the attached FORMULA SHEET.
2. Answer ALL the questions on the question paper.
3. A non-programmable calculator may be used unless otherwise stated.
4. Clearly show all calculations, diagrams and graphs that you have used in determining your answers. Answers only will not necessarily be awarded full marks.
5. If necessary, round-off answers to 2 decimal places, unless otherwise stated.
6. Diagrams are not necessarily drawn to scale. Reasons MUST always be given for statements made when answering geometry questions.
7. Answer questions 2 to 9 in Section B in the spaces provided.
8. Write neatly and legibly.

SECTION A

QUESTION 1

Circle the letter of **the correct answer**.

1.1 Complete: 0,000163 written in scientific notation = …

A $1{,}63 \times 10^{-3}$

B $1{,}63 \times 10^{-4}$

C 163×10^{-3}

D $1{,}63 \times 10^{4}$ (1)

1.2 What will a 20 kg bag weigh if its mass is increased by 40% ?

A 60 kg

B 32 kg

C 28 kg

D 24 kg (1)

1.3 Complete: $-9 - (2x - 3) = \ldots$

A $-18x + 3$

B $-2x - 6$

C $-2x - 12$

D $18x - 3$ (1)

1.4 R600 is invested at 6% per annum compound interest for 6 years.

Complete: A = …

A $\text{R}600(1{,}06)^6$

B $\text{R}600(0{,}06)^6$

C $\text{R}600(1 + 0{,}36)$

D $\text{R}600(0{,}36)$ (1)

P.T.O.

1.5 Study the diagram pattern below. Each diagram was created using matchsticks. The table below gives the number of matchsticks in each figure.

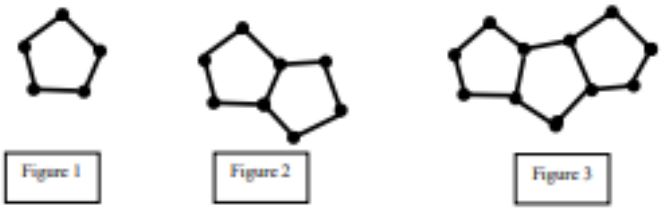


Figure 1 Figure 2 Figure 3

Figure number	1	2	3
No. of matchsticks	5	9	13

How many matchsticks will there be in Figure 6 if the pattern continues?

A $T_n = n + 4$

B $T_n = 4n + 1$

C $T_n = 4n - 1$

D $Tn = -4n + 1$ (1)

1.6 What is the probability of drawing a card marked 5 from a pack of 52 playing cards?

A $\frac{1}{5}$

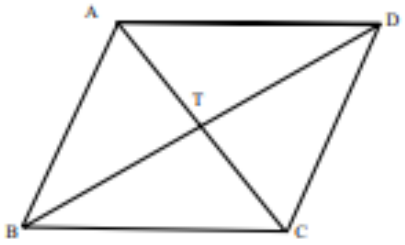
B $\frac{1}{4}$

C $\frac{1}{52}$

D $\frac{1}{13}$ (1)

P.T.O.

1.7 In the figure below AT = TC, BT = DT and AC ⊥ BD.



What kind of quadrilateral is ABCD?

A A kite

B A rhombus

C A parallelogram

D A rectangle (1)

1.8 In ΔPQT, $\hat{P} : \hat{Q} : \hat{T} = 1 : 3 : 5$

Calculate the size of $\hat{T}$.

A 20°

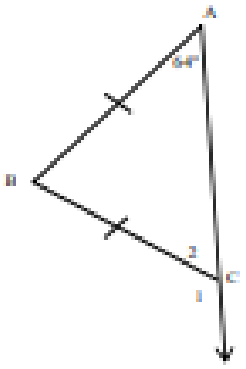
B 60°

C 100°

D 120° (1)

P.T.O.

1.9 In the figure below AB = AC and $\hat{A} = 64°$.



Calculate the size of $\hat{C}_1$.

A 58°

B 64°

C 116°

D 122° (1)

1.10 The length of a rectangle = x cm and the width = y cm.

Both the length and the width of the rectangle are **doubled**.

What is the perimeter of the bigger rectangle?

A $2x+2y$ cm

B $2(x + y)$ cm

C $4(x + y)$ cm

D $x^2 + y^2$ cm (1)

[10]

P.T.O.

SECTION B

QUESTION 2

2.1 Write 0,36 : 0,84 in the simplest form.

(2)

2.2 Calculate: $-4x^2$ if $x = -\frac{1}{2}$

(2)

2.3 Simplify:

2.3.1 $\left(\sqrt{16}\right)^2 - \sqrt[3]{4^6}$

(2)

2.3.2 $(2x + 5)(x - 3) + (x - 1)^2$

(4)

[10]

P.T.O.

QUESTION 3

Factorise fully:

3.1 $60x^2y - 25xy^2 + 5xy$

_____

_____ **(2)**

3.2 $k^2 - 64$

_____ **(2)**

3.3 $x^2 + 7x - 18$

_____ **(2)**

3.4 $ax - ay + 4x - 4y$

_____

_____

_____ **(3)**

[9]

P.T.O.

QUESTION 4

Solve for x.

4.1 $3x + 5 = 11$

(2)

4.2 $x^3 = \frac{1}{8}$

(2)

4.3 $x^2 - 7x + 10 = 0$

(2)

[6]

QUESTION 5

5.1 A turkey costs 5 times more than a chicken. A farmer bought 20 turkeys and 50 chickens for R1 200.

Calculate the price of one turkey.

(5)

5.2 Calculate how much I must invest at 7% p.a. simple interest to have R15 000,00 in my account at the end of 6 years.

(3)

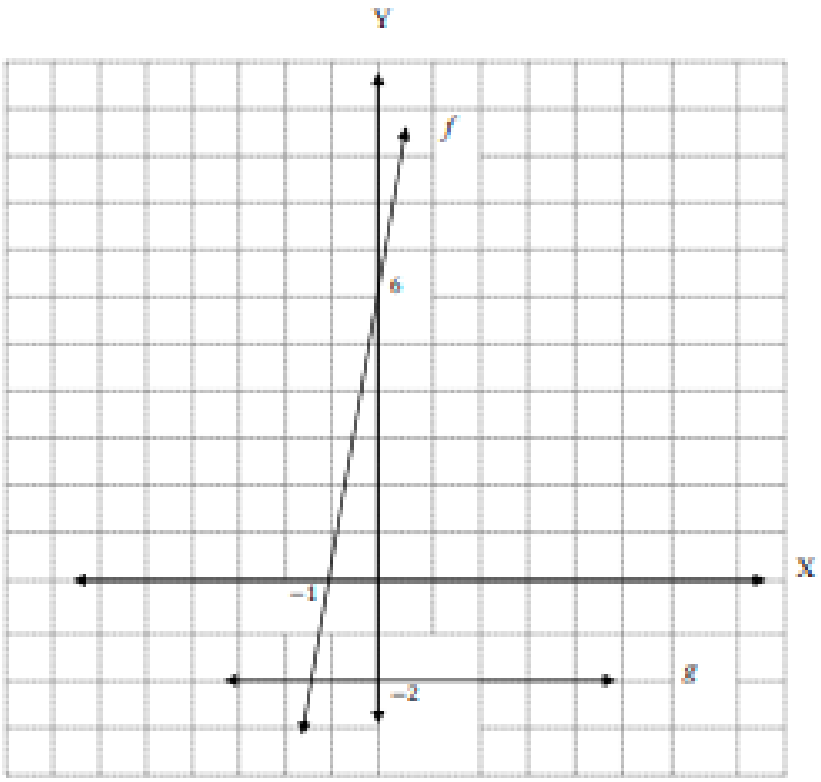
5.3 A taxi covers a distance in 3 hours at an average speed of 80 km/h.
How long will it take to cover the same distance if the average speed is 60 km/h?

(6)

[14]

QUESTION 6

6.1 Determine the equation of the given straight line graph of f. (4)



6.2 Write down the equation of the graph of g.

(1)

[5]

P.T.O.

QUESTION 7

7.1 The following are the marks obtained by a group of learners in a test out of 60.

12	21	23	35	42	54	57	31	43	35	46	37	55

7.1.1 Draw a stem-and-leaf plot to organize the data.

Stem	Leaves

(5)

7.1.2 What is the range of the marks?

_____ (1)

7.1.3 How many learners obtained more than 50% for the test?

_____ (1)

7.2 A bag contains 3 green, 5 red and 7 blue buttons. A button is randomly taken out of the bag.

Determine the probability of taking out:

7.2.1 A red button

_____ (1)

7.2.2 A green or blue button

_____ (2)

7.2.3 A yellow button

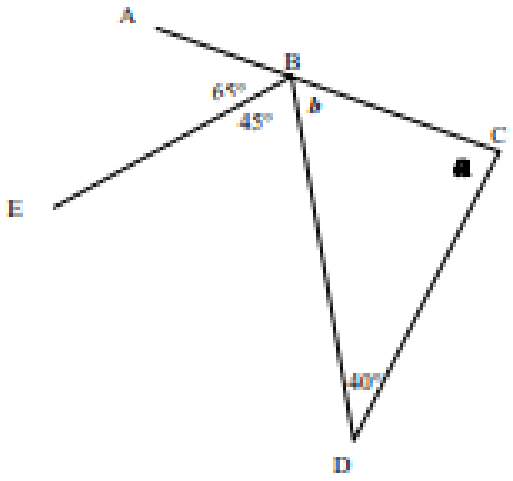
_____ (1)

[11]

P.T.O.

QUESTION 8

8.1 In the diagram below, ABC is a straight line. $A\hat{B}E = 65°$, $D\hat{B}E = 45°$ and $B\hat{D}C = 40°$.



8.1.1 Calculate the value of b.

Statement	Reason
$65° + 45° + b =$	
$b =$	

(3)

8.1.2 Calculate the value of a.

Statement	Reason
$b + a + 40° =$	
$a =$	

(3)

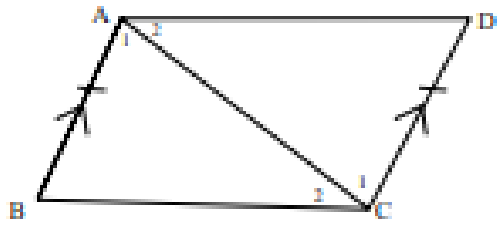
8.1.3 What can you conclude about ΔBCD?
Give a reason for your answer.

Statement	Reason

(2)

P.T.O.

8.2 In quadrilateral ABCD, AB = DC and AB∥ DC.

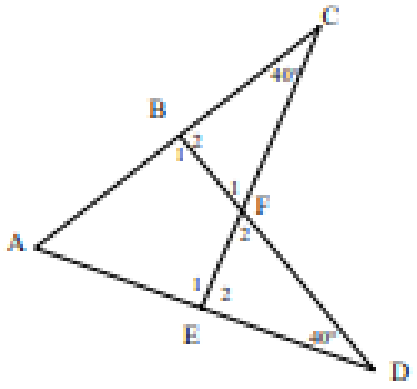


Prove that ΔABC ≡ ΔCDA.

Statement	Reason
In ΔABC and ΔCDA:	

(5)

8.3 In the figure below, $\hat{C} = \hat{D} = 40°$ and $\hat{E}_1 = 80°$.



Prove that Δ CFB ||| ΔDFE

Statement	Reason
In ΔCFB and ΔDFE	

(4)

P.T.O.

8.4 8.4.1 Use a pair of compasses, ruler and a sharp pencil to construct triangle ABC in the frame below, so that $AB = BC = 7$ cm and $\hat{A} = 65°$.

(2)

8.4.2 By measurement: The size of $A\hat{B}C = \ldots$

_____ (1)

8.5 Point $H(-3; 4)$ is translated 3 units to the right and 2 units down.

Write down the co-ordinates of its image H'.

_____

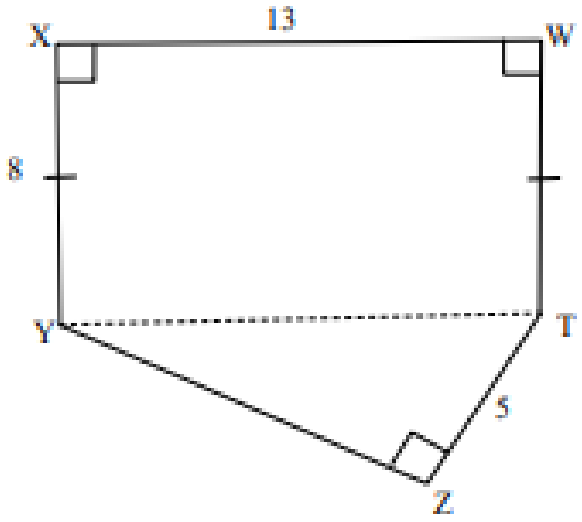
_____ (2)

[22]

P.T.O.

QUESTION 9

9.1 In the figure below XW = 13 units, XY = 8 units and TZ = 5 units.
$\hat{X} = 90°$, $\hat{W} = 90°$ and $\hat{Z} = 90°$.

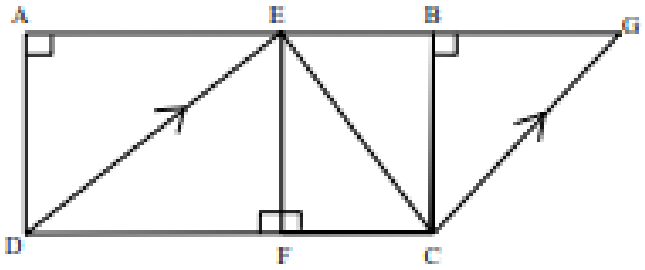


Calculate the length of YZ, correct to two decimal places.

Statement	Reason

(4)

9.2



Refer to the above diagram and complete the following:

Example: The area of $\Delta CBG = \frac{BG \times BC}{2}$

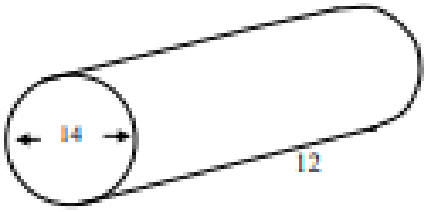
9.2.1 The area of ABCD = _____

9.2.2 The area of ΔDEC = _____

9.2.3 The area of parallelogram CDEG = _____

9.2.4 The area of trapezium AECD = _____ (4)

9.3 Calculate the total surface area of a cylindrical metal rod with a Diameter = 14 cm, Height = 12 cm and $\pi = \frac{22}{7}$.



_____

_____

_____

_____

_____ (5)

[13]

TOTAL: 100

P.T.O.

FORMULA SHEET

Simple Interest:	Compound Interest:
$I = \frac{Prn}{100}$ $A = P(1 + in)$ $A = P(1 + \frac{rn}{100})$	$A = P(1 + i)^n$ $A = P(1 + \frac{r}{100})^n$

	Perimeter	Area
Rectangle	$2(l + b)$	$l \times b$
Circle	$2\pi r$	πr^2
Trapezium	$S_1 + S_2 + S_3 + S_4$	$\frac{1}{2}(sum\ of\ parallel\ sides) \times h$
Triangle	$S_1 + S_2 + S_3$	$\frac{1}{2}b \times \perp hA$

Appendix G: Unstructured interview questions



Unstructured Interview Questions

Section 1

Research Question 2: What Alternative conceptions and errors do students display when solving Mathematics L2 Questions?

Research Question 3: What are students' experiences in mathematics up to Grade 9?

Unstructured Questions

1. Can you explain why you solved this question this way?
2. Can you explain why you used the wrong formula to answer this question?
3. You couldn't write this expression in its simplest form can you explain why?
4. What were you thinking when you were answering this question?
5. Can you explain why you couldn't determine the equation of this graph correctly?
6. You didn't answer this question at all can you explain why?

Research Question 4: How can the low achievement in mathematics L2 be addressed for higher achievement?

7. What do you think can be done to improve mathematics L2 achievement?
8. What are your thoughts on taking part in an initiative aimed at improving your performance in Mathematics L2?

Appendix H: Comparison of knowledge covered in grade 9 and NC(V) level 2

GRADE 9 TOPICS AND NC(V) LEVEL 2 TOPIC SIMILARITIES			
Topic	**Units**	**Grade 9 Topics**	**NCV Level 2 Topics**
Whole numbers	Properties of Numbers	Describe the real number system by recognizing, defining and distinguishing properties of: natural numbers, whole numbers, integers, rational numbers, irrational numbers	Describe the real number system by recognizing, defining and distinguish properties of Natural numbers, whole numbers, integers, rational numbers and irrational numbers • How to use the basic functions of a scientific calculator. • How to apply the order of operations when doing calculations. • How to approximate and round off in the decimal system, to give answers correct to a given number of significant figures, and a given number of decimal places. • How to use estimation to check calculations.
	Calculating using whole numbers	Calculations using all four operations on whole numbers, estimating, and using calculators where appropriate	**How to add and subtract surds**. • How to multiply and divide surds. • How to rationalize denominators to simplify surds.
	Multiples and factors	Use prime factorization of numbers to find LCM and HCF	Find the lowest common denominator of two or more fractions
	Solving problems	**Solve problems in contexts involving**: • Ratio and rate • Direct and indirect proportion	Not in Level 2 NCV

Common Fractions	Solving problems	**Calculations with fractions** • All four operations with common fractions and mixed numbers • All four operations, with numbers that involve the squares, cubes, square roots and cube roots of common fractions. **Calculation techniques** • Revise: - convert mixed numbers to common fractions in order to perform calculations with them - use knowledge of multiples and factors to write fractions in the simplest form before or after calculations - use knowledge of equivalent fractions to add and subtract common fractions - use knowledge of reciprocal relationships to divide common fractions Solving problems • Solve problems in contexts involving common fractions mixed numbers and percentages	**Algebraic fractions:**Simplification of algebraic fractions by taking out a common factor.Multiplication and division of algebraic fractions Addition and Subtraction of algebraic fractions

Decimal fractions	Solving problems	**Calculations with decimal fractions** • Multiple operations with decimal fractions, using a calculator where appropriate • Multiple operations with or without brackets, with numbers that involve the squares, cubes, square roots, and cube roots of decimal fractions. **Calculation techniques** Use knowledge of place value to estimate the number of decimal places in the result before performing calculations. Use a rounding-off and a calculator to check results where appropriate. Solve problems in a context involving decimal fractions	• Identify rational and irrational numbers. • Round off rational and irrational numbers to an appropriate degree of accuracy • Convert rational numbers between terminating and recurring decimals to the form; a,b ∈Z ; b ≠ 0 b a
INTERGERS	Calculation with integers	**Revise:** • Addition and subtraction with integers • Multiplication and division with integers • Perform calculations involving all four operations with integers • Perform calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers.	• Use a scientific calculator correctly to solve expressions involving addition, subtraction, multiplication, division, squares, cubes, square roots and cube roots. • Estimate and approximate physical quantities to solve problems in practical situations. Quantities include length, time, mass and temperature
	Properties of integers	**Revise:** • Commutative, associative and distributive properties of addition and multiplication for integers • Additive and multiplicative inverses for integer	• Identify rational and irrational numbers. • Round off rational and irrational numbers to an appropriate degree of accuracy • Convert rational numbers between terminating and recurring decimals to the form ; a,b ∈Z ; b ≠ 0 b a

EXPONENTS	Calculations using numbers in exponential form	• Revise compare and represent integers in exponential form - compare and represent numbers in scientific notation • Extend scientific notation to include negative exponents. Calculations using numbers in exponential form • Revise the following general laws of exponents:$a^m \times a^n = a^{m+n},\ a^m \div a^n = a^{m-n}, (ab)^n = a^n b^n,\ a^0 = 1$ • Extend the general Extend the general laws of exponents to include: - integer exponents - $a^{-m} = \frac{1}{a^m}$ • Perform calculations involving all four operations using numbers in exponential form, using the laws of exponent	**In this unit students will learn the following:** • How to use the exponential form to write a number as there is any natural number and any real number. • The law for multiplying exponents with the same base. • The law for dividing exponents with the same base. • The identity. • The identity for negative exponents: • Raise a power to a power. • Simplify exponential expressions by applying the exponent laws. • How to raise a power to a negative exponent. • How to use prime factorization to simplify powers • The definition of a fractional exponent. • How to convert from a fractional exponent to root form, also called radical form. • How to convert from a radical form to a fractional exponent

NUMERIC AND GEOMETRIC PATTERNS- NUMERIC PATTERNS	Investigate and extend patterns	Investigate and extend numeric and geometric patterns looking for relationships between numbers including patterns: • represented in physical or diagram form, not limited to sequences involving a constant difference or ratio, of learner's own creation, represented in tables, represented algebraically Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language.	In this unit students learn the following: • How to determine if a sequence is arithmetic. • How to find the common difference. • The formula for n terms of an arithmetic sequence. • How to find the sum of an arithmetic series by using the summing formula. • The definition of sigma notation and how to interpret its meaning
ALGEBRAIC EXPRESSIONS	Algebraic language	**Revise the following**: • Recognize and identify conventions for writing algebraic expressions • Identify and classify like and unlike terms in algebraic expressions • Recognize and identify coefficients and exponents in algebraic expressions • Recognize and differentiate between monomials, binomials and trinomials	Manipulate a formula to change the subject of a formula

	Expand and simplify algebraic expressions NB. Ensure that common fractions and decimal fractions are part of calculations with expressions.	Revise the following: using the commutative, associative and distributive laws for rational numbers and laws of exponents to: • add and subtract like terms in algebraic expressions • multiply integers and monomials by monomials, binomials, trinomials • divide by integers or monomials: monomials, binomials, trinomials • simplify algebraic expressions involving the above operations • determine the squares, cubes, square roots, and cube roots of single algebraic terms or like algebraic terms Extend the above algebraic manipulations to include: • multiply integers and monomials by polynomials, • divide polynomials by integers or monomials, • the product of two binomials, the square of a binomial	Revise the following: using the commutative, associative and distributive laws for rational numbers and laws of exponents to: • add and subtract like terms in algebraic expressions • multiply integers and monomials by monomials, binomials, trinomials • divide the following by integers or monomials: monomials, binomials, trinomials • simplify algebraic expressions involving the above operations • determine the squares, cubes, square roots, and cube roots of single algebraic terms or like algebraic terms Extend the above algebraic manipulations to include: • multiply integers and monomials by polynomials, • divide polynomials by integers or monomials, • the product of two binomials, the square of a binomial

	Factorize algebraic expressions	Factorise algebraic expressions that involve: • common factors • difference of two squares • trinomials of the form: x2 + bx + c ax2 + bx + c, where a is a common factor	• Find products of two binomials. • Find products of binomials with trinomials. • Factorise by identifying and taking out the common factor. • Factorise by grouping in pairs. • Factorise the difference between two squares. • Factorise trinomials. • Simplify algebraic fractions with monomial denominators.
		Simplify algebraic expressions that involve the above factorization processes Simplify algebraic fractions using factorization	Find products of two binomials. • Find products of binomials with trinomials
ALGEBRAIC EQUATIONS	Equations	**Revise the following:** set up equations to describe problem situations • analyze and interpret equations that describe a given situation • Solve equations by inspection • using additive and multiplicative inverses • using laws of exponents • Solve equations by substitution • Use substitution in equations to generate tables of ordered pairs. Extend solving equations to include: • using factorization • equations of the form: a product of factors = 0	**Solve linear equations.** • Solve quadratic equations by factorization. • Solve exponential equations in the form ka m x = (where x is an integer) by using the laws of exponents. • Solve inequalities in one variable and represent the solution in set builder notation, interval notation, and on the number line. • Solve simultaneous equations with two unknowns algebraically and graphically, where both equations are linear

FUNCTIONS AND RELATIONSHIPS	Input and output values	Determine input values, output values or rules for patterns and relationships using: • flow diagrams • tables • formulae • equation	Understand what a Function is: a relationship between two variables X and Y, meaning that for each x-value (independent variable), there is only one y-value (dependent variable).
	Equivalent forms	Determine, interpret, and justify the equivalence of different descriptions of the same relationship or rule presented: • verbally • in flow diagrams • in tables • by formulae • by equations • by graphs on a Cartesian plan	Use the table method and dual method to plot algebraic graphs such as linear, Parabola, Exponential, and Hyperbola. Learn how to determine the critical points of plotting the Parabola graph. Determine the equations of the given graphs: Linear, parabola, hyperbola, and Exponential.
GRAPHS	Interpreting graphs	Extend the focus on features of graphs with special focus on the following features of linear graphs: • x-intercept and y-intercept • gradient.	• That a function is a mathematical relationship between inputs and outputs that maps each input to one, and only one, output. • The difference between a function and a relation. • The standard form of a linear function is (or). • The effect of and (or and) on the graph of a linear function. • What we mean by the domain and range of a linear function and how to represent these in set and interval notation. • How to plot the graph of a linear function using any two points, the dual-intercept method and the gradient-intercept method. • How to find the equation of a linear function
	Drawing graphs	Use tables of ordered pairs to plot points and draw graphs on the Cartesian plane Extend drawing of graphs with special focus on: • drawing linear graphs from given equations • determining equations from given linear graph	

TRANFORMATION GEOMETRY	Transformation	Recognize, describe and perform transformations with points, line segments and simple geometric figures on a co-ordinate plane, focusing on: • reflection in the x-axis or y-axis • translation within and across quadrants	• Use the Cartesian co-ordinate system to plot points, lines and polygons. • Use the Cartesian co-ordinate system to calculate the distance between two points. • Use the Cartesian co-ordinate system to find the gradient of the line joining two points. • Use the Cartesian co-ordinate system to find the co-ordinates of the midpoint of a line segment joining two points. • How to calculate the distance between two points on the Cartesian plane using the distance formula. • How to calculate the gradient between any two points on a straight line. • How to work with the gradients of parallel and perpendicular lines. • That the gradients of parallel lines are the same. • That the product of the gradients of perpendicular lines is equal to . • How to calculate the midpoint between two points on the Cartesian plane • Use transformation geometry to translate units horizontally and units vertically. • Use transformation geometry to reflect graphs about the x-axis, the y-axis and about the lines and . • Find the coordinates of the point after it is translated units horizontally and units vertically. • Find the coordinates of the point after it is reflected about the x-axis, the y-axis, and the lines y = x and y= -x.

GEOMETRY OF STRAIGHT LINES	Angle relationship	Revise and write clear descriptions of the relationship between angles formed by: • perpendicular lines • intersecting lines • parallel lines cut by a transversal	Not in Level 2 NCV
	Solving problems	Solve geometric problems using the relationships between pairs of angles described above	Not in Level 2 NCV
GEOMETRY OF 2D SHAPES AND CONSTRUCTION OF GEOMETRIC FIGURES	Classifying 2D shapes	Revise properties and definitions of triangles in terms of their sides and angles, distinguishing between: • equilateral triangles • isosceles triangles • right-angled triangle	Not in Level 2 NCV
	Construction	Investigate the angles in a triangle, focusing on the relationship between the exterior angle of a triangle and its interior angle	Not in Level 2 NCV
	Classifying 2D shapes	Revise and write clear definitions of quadrilaterals in terms of their sides, angles and diagonals, distinguishing between: • Parallelogram • Rectangle • Square • Rhombus • Trapezium • Kite	Not in Level 2 NCV

	Constructions	Investigate sides and angles. and diagonals in quadrilaterals, focusing on: • exploring the sum of the interior angles of polygons • the diagonals of rectangles, squares, parallelograms, rhombi and kites	In this unit students learn the following: • How to use a calculator and the trigonometric ratios to calculate the length of a side in a right-angle triangle if you know one angle and the length of one other side. • How to use a calculator and the trigonometric ratios to calculate the size of an angle in a right-angled triangle if you know the length of two of the sides. • Solve problems in two dimensions (2D) using the trigonometric ratios.
GEOMETRY OF 2D SHAPES AND CONSTRUCTIONS	Similar and congruent triangles	Through investigation, establish the minimum conditions for congruent triangles. Through investigation, establish the minimum conditions for similar triangles	Not in Level 2 NCV
	Problem solving	Solve geometric problems involving unknown sides and angles in triangles and quadrilaterals, using known properties of triangles and quadrilaterals, as well as properties of congruent and similar triangles	Not in Level 2 NCV
THEOREM OF PYTHAGORAS		Use the Theorem of Pythagoras to solve problems involving unknown lengths in geometric figures that contain right-angled triangles	In this unit students will learn to: • Identify different types of triangles. • Investigate the relationship between the sides of a right-angled triangle to develop the Theorem of Pythagoras. • Use the Theorem of Pythagoras to find missing lengths in right angled triangles
AREA AND PERIMETER OF 2D SHAPES		Use appropriate formulae and conversions between SI units, to solve problems and calculate perimeter and area of: • polygons • circles	**By the end of this unit you will be able to:** • Use a ruler and protractor accurately. • Understand the SI system of measurement. • Convert between different units. **Name the different types of polygons** • Calculate the perimeter and surface area of the following laminas: ◦ Square ◦ Rectangle ◦ Circle ◦ Triangle ◦ Parallelogram ◦ Trapezium ◦ Hexagon

SURFACE AREA AND VOLUME OF 3D SHAPES		Use appropriate formulae and conversions between SI units to solve problems and calculate the surface area, volume, and capacity of: • rectangular prisms • triangular prisms • cylinders	• Calculate the volume of the following geometric objects: ◦ Cubes ◦ Rectangular prisms ◦ Cylinders ◦ Triangular prisms ◦ Hexagonal prisms • Investigate the effect on area of laminas where one or more dimensions are multiplied by a constant factor k. • Investigate the effect on the volume and surface area of right prisms and cylinders, where one or more dimensions are multiplied by a constant factor k

DATA HANDLING	Collect, organize and summarize data	**Collect data** • Pose questions relating to social, economic, and environmental issues • Select and justify appropriate sources for the collection of data • Distinguish between samples and populations, and suggest appropriate samples for investigation • Select and justify appropriate methods for collecting data **Organize and summarize data** • Organize numerical data in different ways in order to summarize by determining: - measures of central tendency - measures of dispersion, including extremes and outliers • Organize data according to more than one criteria	• Define the types of data. • Differentiate between grouped and ungrouped data. • Create an ungrouped data frequency distribution and understand when to use it. • Collect, organize, and interpret univariate numerical data. • Differentiate between the mean, median, and mode. • Calculate the mean, median, and mode for ungrouped data • Measures of dispersion tell us how spread out or dispersed the values in a dataset are. • The range is the difference between the biggest value and the smallest value. • A percentile is the value below which a given percentage of scores fall. The is that value in a data set below which of the values lie and above which of the values lie. • The rank of the value at the percentile for a dataset of values can be calculated using. • Quartiles divide a dataset into four equal groups. • The inter-quartile range is the difference between and, and represents the range in which the middle of the data lies. • The semi-interquartile range is the middle of the inter-quartile range.

	Represent data	**Represent data** • Draw a variety of graphs by hand/technology to display and interpret data including: - bar graphs and double bar graphs - histograms with given and own intervals - pie charts - broken-line graphs - scatter plots	• Represent data effectively, choosing appropriately from: ◦ Construction of frequency distribution/tally chart ◦ Bar and compound bar graphs ◦ Construction of the stem and leaf plot ◦ Histograms (grouped data) ◦ Frequency polygons ◦ Pie charts ◦ Line and broken line graphs

	Interpret, analyse, and report data	**Interpret data** • Critically read and interpret data represented in a variety of ways • Critically compare two sets of data related to the same issue **Analyse data** • Critically analyse data by answering questions related to: - data collection methods - summary of data - sources of error and bias in the data **Report data** • Summarize data in short paragraphs that include - drawing conclusions about the data - making predictions based on the data - making comparisons between two sets of data - identifying sources of error and bias in the data - choosing appropriate summary statistics for the data (mean, median, mode, range) - the role of extremes and outliers in the data	Not in Level 2 NCV

Probability		• Consider situations with equally probable outcomes, and: - determine probabilities for compound events using two-way tables and tree diagrams - determine the probabilities for outcomes of events and predict their relative frequency in simple experiments - compare relative frequency with probability and explains possible differences	Not in NCV level 2
FINANCE		Not in grade 9	• Describe financial concepts related to personal finances, methods of financing, and financial control. • Draw up a projected personal and household monthly budget. • Record actual income and expenditure over a period (one month, six months, or twelve months) and compare to the projected budget. • Identify and explain variances between actual and projected figures. • Provide possible corrective methods of financial control

Appendix I: Certificate of professional editing

CERTIFICATE
OF PROFESSIONAL EDITING

I, Barbara L. Louton, declare that I am a professional editor with a Bachelor of Arts in Professional Writing, a Master of Arts in Development Administration, and 20 years of experience as an editor, researcher, and writer.

I declare that I was contracted by Thabisa Capa, a master's student at the University of South Africa, to complete a professional edit of her dissertation:

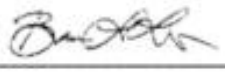
'Influence of pre-existing mathematics knowledge on students' achievement at a Level 2 TVET college in Ekurhuleni district'.

I declare that I have completed a two-phase substantive edit of the document. The edit addressed issues with the logic, structure, language, style, and formatting of the manuscript. Changes were tracked and comments were made in text with recommendations for the client to address.

Disclaimer:
Responsibility for the originality and accuracy of the material presented in the manuscript lies with the author. I have not verified the originality or accuracy of statements, quotations or citations and references presented in the manuscript. Where I have detected inaccuracies, I have rectified them or reported them to the author. I did not edit or proofread the document again after the author made revisions in response to my recommendations.

I can be contacted at:

Cell: 073 766 1139
Email: bellway@gmail.com

Barbara L Louton		10 October 2025
Name	Signed	Date

Appendix J: Turnitin report