USING GEOGEBRA TO TEACH GRADE 11 ANALYTICAL GEOMETRY TO ENHANCE PERFORMANCE IN BOHLABELA DISTRICT OF MPUMALANGA PROVINCE

by

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I declare that the above thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references

I further declare that I submitted the thesis to originality checking software and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at Unisa for another qualification or at any other higher education institution.

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DEDICATION

This thesis is dedicated to my father, Alhaji Mahama Leppode, and my mother, Dari Rahmatu Mahama, who brought me into this world, gave me love, peace, and happiness, and always supported me to the best of their abilities. May Allah be with you for the rest of your lives.

Not forgetting my wife, Hawa Aminu Mohammed, who has always stood by me throughout this academic journey.

To all these distinguished individuals, May Almighty Allah richly bless you

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ABSTRACT

The objectives behind the study were, firstly, to investigate the effectiveness of using GeoGebra in teaching Grade 11 Analytical Geometry to enhance learner performance. The research tested the alternative hypothesis (H_1): The use of GeoGebra significantly enhanced learner performance in Grade 11 Analytical Geometry. However, the control group used examples in the textbook and lessons were delivered in the traditional way while experimental group used GeoGebra. The study was grounded in Van Hiele's theory, learners develop their knowledge of Analytical Geometry following five levels including visualisation, analysis, abstraction, deduction and rigor. of geometric thought and Vygotsky's Zone of Proximal Development (ZPD), which provided a theoretical framework for understanding learning progression and scaffolding in geometry education.

An explanatory sequential mixed-methods design was employed, combining quantitative and qualitative approaches. Purposive sampling was used to select 80 learners, 40 from an experimental group (taught using GeoGebra) and 40 from a control group (taught conventionally). The findings aimed to determine whether GeoGebra-based instruction led to statistically significant improvements in learner performance compared to traditional methods.

The pre-test and post-test were administered to both study groups to assess significant differences following the intervention. Lesson observations were conducted during the intervention, alongside semi-structured interviews with learners before and after implementation. Question coefficients were used to evaluate difficulty levels in the pre-test and post-test, with positive values indicating underperformance and negative values reflecting good performance. The results, analysed at a 95% confidence level ($p < 0.05$), revealed statistically significant differences in performance between the two schools across both assessments. The coefficients determined the significance of pre-test and post-test outcomes between the groups.

The study found a statistically significant difference in performance between the experimental and control groups, with the experimental group (coefficient = -5.891852) outperforming the control group (coefficient = 5.2511648) at $p < 0.05$ ($p = 0.000$). Additionally, the experimental group showed significant improvement in the post-test (coefficient = -5.891852) compared to the pre-test (coefficient = 12.569011) following the GeoGebra intervention ($p < 0.05$, $p = 0.000$). The results from pre- and posts-tests demonstrated that GeoGebra effectively enhanced learner performance in Analytical Geometry. Teacher interviews further supported this conclusion, confirming that the intervention led to improved learner outcomes. Based on these results, the study recommends that mathematics teachers integrate GeoGebra into their instructional strategies for teaching Analytical Geometry.

Keywords: analytical geometry; effective teaching and learning; GeoGebra intervention; performance; pre-test and post-test

ABBREVIATIONS

CAPS:	Curriculum and Assessment Policy Statement
DBE:	Department of Basic Education
FET:	Further Education and Training
HOD:	Head Of Department
HOT:	Higher Order Thinking
ICRSME:	International Consortium for Research in Science & Mathematics Education
MMU:	Multimedia University
MST:	Mathematics, Science and Technology
NSC:	National Senior Certificate
OECD:	Organization of Economic Cooperation and Development
SECMEQ:	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SPSS:	Statistical Package for Social Science
TIMSS:	The Trends in International Mathematics and Science Study
ZPD:	Zone of Proximal Development

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CHAPTER 1: AN OVERVIEW OF THE STUDY

1.1 INTRODUCTION AND BACKGROUND TO THE STUDY

The poor performance of mathematics remains a critical concern for many nations, as it threatens the foundational development of communities and national progress (Mokgosi & Ramapela, 2021). For learners at the FET level, particularly those in Grade 11, mastering mathematical knowledge, skills, and fundamental concepts is crucial for improving performance, especially as they approach their final year of schooling.

South Africa's global mathematics performance remains alarmingly low. According to TIMSS (2023), South African Grade 9 learners ranked among the bottom five out of 44 participating countries. Although the national average improved slightly from 389 in 2019 to 397 in 2023, previously higher-performing provinces, such as the Western Cape and Gauteng, saw declines, dropping from 370 to 362. The International Mathematics Union (IMU, 2020) emphasises that weak foundational mathematics skills at the primary and secondary levels across many African countries hinder learners' potential, resulting in high attrition rates in university-level mathematics courses.

In South Africa, mathematics pass rates have never exceeded 70% (DBE, 2025), underscoring the urgent need for intervention. Addressing this crisis is vital to cultivating the skilled technocrats required to drive national development and bridge existing gaps in critical sectors.

Mathematics performance remains low, ranking last (54.6%–63.5%) among the eleven most popular subjects from 2019 to 2023, compared to English First Additional Language, which achieved 93.0% (DBE, 2023). Table 1 illustrates these trends. (DBE, 2023).

Table 1.1: ***Performance of Mathematics 2019-2023***

Year	2019	2020	2021	2022	2023
Results	54.6%	53.8%	57.6%	55.0%	63.5%

The table above illustrates mathematics performance from 2019 to 2023. The data show that overall performance has not reached 65%, which includes 30% of students who did not meet the university's eligibility criteria for mathematics or science-related courses.

Given the poor performance of learners in mathematics, the government instituted programmes to mitigate this through various school intervention programmes, such as Mathematics, Science, and Technology (MST) from the Dinaledi project, which was initiated in 2001 (DBE, 2016a). Originally, Dinaledi was designed for learners in Grades 10 to 12 who study mathematics and science. MST was also introduced to support learners in Grades 8 to 9 in mathematics and natural sciences. The Mpumalanga Department of Basic Education also introduced a "group studies" programme (2016). The ten best learners in mathematics and science from each school were selected to attend lessons on Saturdays and Sundays and then return to their respective schools to teach fellow learners and improve mathematics performance (DBE, 2014). Another intervention, called "The Count," was introduced to champion the same course, where learners from various schools were selected to attend lessons on Saturdays and Sundays to improve their mathematics and science performance in the province. The programmes were provided at the secondary school level (DBE, 2016). "Winter and summer" studies were also introduced to curb the situation of poor performance in mathematics.

Despite targeted interventions by the Department of Basic Education (DBE) and the Mpumalanga Department of Education, mathematics performance remains critically low in the Bohlabela District, particularly in the Bushbuckridge region. An analysis of School Subject Reports (2019–2023) reveals persistent underperformance, with seven out of eleven schools in the Thulamahashe Circuit consistently achieving poor results in the NSC Grade 12 mathematics examinations over the past five years (Schools A–G). This trend highlights systemic challenges that require urgent, evidence-based solutions.

Table 1.2: ***The performance of schools A-G in the Thulamahashe circuit***

SHOOLS	2019	2020	2021	2022	2023
A	37.6%	27.2%	23.6%	26.9%	28.4%
B	26.0%	85.3%	26.5%	36.7%	20.0%
C	26.1%	75.7%	53.0%	51.9%	46.9%
D	50.5%	49.6%	46.1%	50.3%	45.8%
E	44.6%	79.2%	50.6%	42.2%	35.6%
F	50.0%	55.6%	89.7%	59.0%	60%
G	22.6%	37.5%	54.2%	37.5%	35.6%

Their poor performance in mathematics ultimately hindered them from achieving the required scores to pursue STEM degrees in higher education. Research by Ubah and Bansilal (2019) and Alex and Mammen (2018) confirms that weak mathematics performance often prevents learners from advancing into engineering, science, and technology-related fields.

Geometry remains one of the most challenging topics in mathematics for learners. Mokotjo and Mokhele (2021) highlight that many students struggle with Geometry, a view supported by Luneta (2020), who identifies it as a particularly difficult subject in secondary school. Both Analytical and Euclidean Geometry present challenges, although analytical geometry is often considered less demanding. Despite this, many learners continue to perform poorly in this area, as observed over 19 years of teaching experience.

Analytical Geometry is a branch of mathematics that combines geometric principles with algebraic computation (Sugandi & Benerd, 2020). It encompasses the study of linear functions, circles, ellipses, hyperbolas, and transformations, including equations of straight lines and curves (Adjetey & Tuffour, 2024). Mastery of this topic is essential for learners, as its concepts form the foundation for advanced mathematical disciplines such as calculus, trigonometry, and algebra, as well as applications in physics, astronomy, and engineering (Sugandi & Benerd, 2020). Additionally, Kuzle and Glasnović Gracin (2020) argue that studying Analytical Geometry enhances critical thinking and logical reasoning, thereby improving problem-solving skills and overall mathematical performance.

The integration of technology, particularly dynamic software like GeoGebra, has been identified as an effective tool for enhancing the teaching and learning of Analytical Geometry (Sugandi & Benerd, 2020). GeoGebra's interactive features help learners visualise geometric concepts on the Cartesian plane, facilitating deeper understanding (Bayaga et al., 2019). Mokotjo and Mokhele (2021) emphasise that strategic use of technology can address learning challenges and improve mathematical proficiency, a practice increasingly adopted in countries such as South Africa.

In the South African National Senior Certificate (NSC) examinations, Analytical Geometry is assessed in Paper 2, and its principles are often integrated with Euclidean Geometry and Trigonometry (DBE, 2023). The Department of Basic Education (DBE) emphasises the importance of algebraic manipulation and the ability to connect geometric concepts, noting that weak performance in Analytical Geometry negatively impacts overall mathematics achievement. Diagnostic reports from 2019 to 2023 consistently reveal learner difficulties in this topic, jeopardising their eligibility for critical university programs such as Medicine, Engineering, and Accounting, fields vital for socio-economic development. Addressing these challenges is therefore imperative to ensure academic success and future career opportunities.

1.2 PROBLEM STATEMENT

Analytical Geometry remains a challenging topic for many learners (Adjetey & Tuffour, 2024; Luneta, 2015). While geometry curricula worldwide cover various branches, such as Euclidean, differential, and algebraic geometry, existing literature primarily emphasises the historical significance of geometry rather than addressing learners' performance struggles in Analytical Geometry (Jones, 2002; Pierce, 2014). These challenges may stem from traditional teaching methods, which often fail to cater for diverse learning needs. To improve understanding, educators should adopt more effective instructional strategies, particularly when teaching geometric properties and relationships involving points, lines, and angles on the Cartesian plane.

Integrating modern technology, such as GeoGebra, can significantly enhance learners' understanding of Analytical Geometry. GeoGebra is dynamic, user-friendly software that simplifies complex mathematical concepts, making it suitable for learners at different levels (Roberts, 2023). Studies indicate that GeoGebra aids in visualising and solving problems related to parallel and perpendicular lines, angles of inclination, and other geometric relationships on the Cartesian plane (Sugandi & Benerd, 2020). Additionally, it supports the development of higher-order cognitive skills, such as reasoning, analysis, and evaluation, which are essential for mastering Analytical Geometry (Sarah & Batiibwe, 2024; Kuzle & Glasnović Gracin, 2020).

Despite the Department of Basic Education's efforts to improve mathematics performance, Analytical Geometry remains a persistent challenge (DBE Diagnostic Report, 2019-2023). Limited research has been conducted and hence, the researcher was motivated to diagnose the challenges that learners commit, hindering their performance in the analytical geometry, particularly in Bohlabela District, Mpumalanga. This study explores the potential of GeoGebra concept to address these challenges, aiming to enhance learners' problem-solving skills and prepare them more effectively for matric examinations.

1.3 HYPOTHESIS AND RESEARCH QUESTIONS

The study tested the following hypothesis to find out the significant difference between the two schools and the significant difference of

their post-test and their pre-test.

H0 – There is no significant difference between the experimental and control schools after using GeoGebra as an intervention employed in this study.

H1 – There is a significant difference between the experimental and control schools after using GeoGebra as an intervention employed in this study.

H0 – There is no significant difference between the pre-test and post-test when GeoGebra is used as an intervention in the experimental school

H1 – There is a significant difference between the pre-test and post-test when GeoGebra is used as an intervention in the experimental school.

Using a sample of Grade 11 learners from two schools in the Bohlabela District of Mpumalanga Province, the researcher aimed to investigate whether learners' performance in Analytical Geometry was enhanced with the use of GeoGebra application as an intervention.

1.3.1 Main research question

The researcher has used several research questions to encompass both quantitative and qualitative approaches.

- What is the effectiveness of using GeoGebra to teach Analytical Geometry to enhance learner performance?
- How does the use of GeoGebra as an intervention influence learner performance in Analytical Geometry?

1.3.2 Sub-questions of the research

- How do teachers use Geogebra to teach Grade 11 Analytical Geometry?
- What are the benefits of using Geogebra to teach Analytical Geometry (if any)?
- What are challenges experienced when using Geogebra to teach Analytical Geometry (if any)?
- How can teachers use Geogebra to teach Analytical Geometry to change their practices to enhance learner performance?

1.4 OBJECTIVES OF THIS STUDY

- To investigate the effectiveness of GeoGebra in enhancing Grade 11 learners' performance in Analytical Geometry.
- To examine the impact of GeoGebra on learners' understanding of key Analytical Geometry concepts.
- To explore how Grade 11 mathematics teachers integrate GeoGebra in teaching Analytical Geometry.
- To identify the challenges and benefits of using GeoGebra in teaching Analytical Geometry.
- To propose strategies for using GeoGebra to improve teaching practices and learner performance.

1.5 SIGNIFICANCE OF STUDY

This study holds significant value for learners, teachers, textbook authors, subject advisors, and the field of mathematics education as a whole. It will provides insights into effective teaching strategies, particularly in Analytical Geometry, focusing on lines and angles in the Cartesian plane.

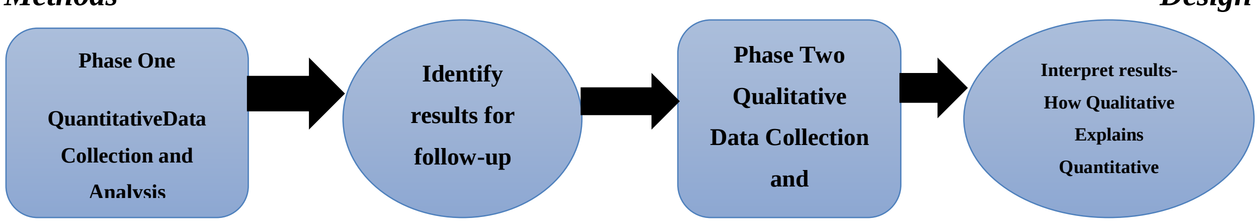
By identifying common learner difficulties and addressing learning gaps through GeoGebra, the study enhances instructional methods and improves student performance.

Stakeholders, including the Department of Basic Education (DBE) and subject advisors, benefit from the findings, which can guide teacher workshops and curriculum improvements. Additionally, the study serves as a foundation for future research and informs textbook development at the Further Education and Training (FET) level, ultimately contributing to better mathematics performance nationwide.

1.6 RESEARCH METHODOLOGY

The researcher employed a mixed-methods approach, combining both quantitative and qualitative methods within a single study. The core assumption of this form of inquiry is that integrating quantitative and qualitative data provides a complete understanding of the study (Creswell & Creswell, 2018). The study adopted the systematic "explanatory sequential design" method to collect data, first quantitatively and then qualitatively (QUANT and qual). Thus, the quantitative data was collated and analysed, and then qualitative data was used to explain the outcome (Creswell & Creswell, 2018). The data collated through systematic random sampling were based on a pre-test and were analysed statistically and then qualitatively. However, prior to the test, the researcher piloted the test instrument with 30 selected learners from two different schools to assess the viability and credibility of the questions.

Fig. 1.1: ***Flowchart of the Basic Procedures in Implementing an explanatory sequential Mixed Methods Design***



In accordance of the quantitative research question, the researcher followed the steps below to collate quantitative data;

- Obtain permission from school authorities
- Identify grade 11 learners administer the pre-test
- Administer the test out of contact hours
- The scripts are marked to analyze the results by using descriptive statistics which led to selection of learners for interview of qualitative analyses

In accordance of the quantitative research question, the researcher followed the steps below to collate quantitative data;

- The significance of the results
- Identify grade 11 learners who will be selected for the interview
- Design qualitative protocols

- state qualitative research questions which are in line with the quantitative results
- obtain permission from school authorities
- purposefully select qualitative samples that can help explain

- Summarize and interpret both quantitative and qualitative results
- Discussion is done to what extent qualitative results explain quantitative results

Adapted from Creswell & Plano Clark (2018)

The researcher followed the above chart of explanatory sequential design to successfully collate and interpret data very well. "Credibility is the extent to which the results approximate reality and are judged to be accurate, trustworthy, and reasonable" (McMillan & Schumacher, 2020). After marking the scripts, the researcher developed a semi-structured interview with nine (9) learners to uncover the challenges and difficulties learners portrayed while answering the questions. The researcher also conducted a classroom observation to understand how learners learn about problems involving lines and angles on the Cartesian plane in Analytical Geometry. According to Creswell and Plano (2018), qualitative data helps explain the mechanisms underlying the quantitative results in more depth. After collecting the quantitative and qualitative data, the researcher merged them and subsequently used inferential statistics to interpret the quantitative results (Creswell & Creswell, 2018). The researcher also conducted classroom observation to find out how learners learn concepts of lines and their relations on the Cartesian plane. The researcher followed and weighed all the values obtained in the quantitative result to establish the validity of the design approach. Hence, the researcher could collate a successful data set to reach a successful conclusion.

1.7 POPULATION AND SAMPLING TECHNIQUES

Approximately 80 learners from two schools in the Thulamahashe circuit, located in the Bohlabela District of Mpumalanga Province, participated in this study. These learners formed two study groups: the experimental group, comprising 40 learners, and the control group, also consisting of 40 learners, which together made up the 80-sample population of the study.

1.8 DATA COLLECTION PROCEDURE

Under this, the researcher administered the pre-test and post-test to both the experimental and control groups. A semi-structured interview was also conducted with the two teachers prior to the intervention, and a follow-up semi-structured interview was conducted with one teacher in the experimental group.

1.9 DATA ANALYSIS AND INTERPRETATION

Data analysis is a process that relies on methods and techniques to take raw data, extract relevant insights, and identify information to transform facts and figures into actionable initiatives for improvement (Durcevic 2020).

1.9.1 Quantitative Data Analysis

Quantitative data from learners' tests were analysed to assess their understanding of lines and angles in the Cartesian plane. The researcher compared pre-test and post-test results using percentage-based interpretations (Creswell & Creswell, 2018) to evaluate group performance, supplemented by coefficient-based analysis. A single-sample z-test was applied to examine confidence intervals. Descriptive statistics summarised and organised the data (McMillan & Schumacher, 2020), with post-test results tabulated for clarity. Inferential statistics, which builds on descriptive analysis, enable population-level predictions and provide deeper quantitative insights (McMillan & Schumacher, 2020). The Statistical Package for the Social Sciences (SPSS) and Stata Release 17 were used for generalisation, leveraging their robustness for larger populations (MMU, 2020).

1.9.2 Qualitative Data Analysis

Content analysis of the participants' views were used in this research work, whereby the researcher phrased the interview questions in a way that was friendly to the participants. In this way, the researcher understood how learners answered the diagnostic test on lines and angles on a Cartesian plane (Creswell & Creswell, 2018). According to Bhatia (2018), qualitative data also involves analysing documented information in the form of texts, media or physical items, and it is also used to analyse responses from the interviewee. Moreover, the researchers assigned codes L1 to L9 to the participants, corresponding to learners 1 to 9. The coding is in place to conceal their identity. Generating a small number of themes or categories for a research study is known as coding (Creswell & Creswell, 2018).

1.10 ETHICAL CONSIDERATIONS

The International Consortium for Research in Science & Mathematics Education (ICRSME, 2020) refers to ethical reasoning as the cognitive activity of determining right and wrong when facing a dilemma that affects other living beings. Therefore, the researcher sought permission from the university by first applying for an Ethical Clearance Certificate, as well as from the Department of Education and the management of the selected schools, before administering any tests to the learners. A letter of information and informed consent from the researcher was handed to the Head of Department (HOD) of the Schools, which was signed by both parties to authenticate the research procedure. The consent and cooperation of the learners' mathematics teachers, the learners participating in the study, and the parents/guardians of the learners were sought before conducting the study. Potential participants must be competent to make an informed decision about participation and be free from any coercion. The researcher also allowed learners to complete the informed consent form, which helped document their willingness to participate in the study and address ethical issues related to their participation.

1.11 LIMITATIONS AND FUTURE STUDIES

The primary limitations of this study were that it was conducted in only three high schools in the Thulamahashe circuit of Bohlabela District, Mpumalanga Province, and that it did not adequately represent the country as a whole. The schools were chosen because they offer mathematics subjects and MST schools. Other schools which are not MST schools were left out. The learners were all from African communities, struggling to pass mathematics, and were close to the researcher. All schools have similar demographics. Therefore, with different demographics, schools may exhibit varying school cultures, which can result in differences in outcomes from this research. The results were, thus, not generalizable across various communities representing the countries' demographic constitutions.

1.12 DEFINITIONS OF KEY CONCEPTS

Analytical Geometry: This is a branch of mathematics where a pair of points is plotted as lines and angles displayed in a Cartesian plane.

Angles: Space created by the meeting of two lines

Cartesian plane: Two perpendicular number lines, the X-axis horizontal and the Y-axis vertical lines, were ordered, and a pair of numbers can be indicated.

Concept: The basic understanding of a phenomenon or knowledge.

Enhancement: Improving on a performance for a positive result.

GeoGebra: Modern scientific software using to teach mathematical concepts for easy understading.

Lines: A continuous dotted line from a particular point to another point

Mathematics: This deals with numbers, measurements and quantities.

Parallel lines: Lines that have a constant distance between them.

Performance: Distinct achievement in a particular phenomenon

Perpendicular lines: Two lines meeting at an angle of 90^0

Syllabus: A guided document meant to be studied within a stipulated

DIVISION OF CHAPTERS

Chapter one presents the introduction, background, purpose statement, research questions and objectives, problem statement, significance of the study, definition of terms, division of chapters and chapter summary.

1.13 CHAPTER SUMMARY

This chapter demonstrated how Vygotsky's Zone of Proximal Development (ZPD) and Van Hiele's levels of learning theory can enhance learners' performance in Analytical Geometry and mathematics at any learning stage. It also highlighted that addressing learning difficulties enables learners to confidently solve problems involving lines and angles in the Cartesian plane, thereby improving their overall performance in the subject.

The chapter provided an introduction and background to the study, along with an overview of the problem statement, research questions, and the study's purpose and objectives. Additionally, it covered the literature review, research methodology, and ethical considerations. The next chapter will explore the theoretical framework underpinning this study.

CHAPTER 2: THEORETICAL AND CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

The previous chapter outlined the foundational elements of the study, including the introduction, background, problem statement, significance, purpose, research questions, objectives, and methodology, detailing the data collection procedures and instruments used. Chapter 2 examines Van Hiele's levels of geometric thought and Vygotsky's ZPD, alongside other key educational theories. By analysing these frameworks, this chapter highlights their relevance in shaping the study's theoretical foundation and their implications for research design and implementation.

2.2 DEFINITIONS OF THEORIES IN EDUCATION

According to Abend (2023), educational theories challenge and expand existing knowledge within defined assumptions to predict, explain, and interpret phenomena, thereby clarifying the existence of research problems. Johnson (2019) adds that theories help organise empirical facts, constructing a framework for understanding phenomena. Further, the ibid emphasises that theories establish meaningful relationships among concepts, providing a structured approach to comprehending phenomena and organising the unknown. The selected theories guided the researcher in analysing why learners struggle with solving lines and angles problems on the Cartesian plane in Analytical Geometry.

2.2.1 The importance of theories in education

The importance of theories in education lies in their capacity to strengthen academic readings and support research through reasoning and discussion (UK essay 2018). Additionally, theories empower researchers to gather a spectrum of ideas, assess their compatibility or divergence, and subsequently reflect on how these notions align with personal thoughts and experiences.

Silva et al (2024) emphasise the role of theory in educational research and teacher education, asserting that educators must recognise theory's influence on educational practice and research. Shi and Blau (2020) stress the importance of teachers being knowledgeable in educational theories, as this develops wisdom in addressing instructional concerns, ensuring effectiveness, relevance, and a focus on educational principles rather than political considerations. Schunk (2020) articulates that learning theories enhance instructional purposes and shape the understanding of the types of learning applied and their implications. Harasim (2017) contends that educators benefit from understanding theories by facilitating reflective teaching, refining their practices, and promoting professional improvement. They align their work with educational theories and integrate them into educational practices. Vinz (2022) further argues that theories establish a vital connection between educational research and practices, offering tools to translate research findings into practical recommendations. Adom and Hussein (2018) agree that a theoretical framework helps situate and formalise theories to provide direction and context. The authors emphasise that theory serves as a lens through which the research is envisioned, offering a comprehensive understanding of its framework and application.

2.2.2 How to apply learning theories in teaching

According to Western Governors University (WGU) (2021), teachers should establish the necessary strategies and techniques to use theories in classroom teaching and management. Teachers should endeavour to comprehend theories to apply them effectively in their instructional lessons. Understanding theories will help teachers connect with learners with different abilities (Shi & Blau 2020). This enables teachers to better support diverse learners by tailoring instruction to meet the varying abilities and needs of students.

2.3 THEORETICAL FRAMEWORK

The theoretical framework serves as the foundational blueprint for research, integrating established theories within a field that align with and inform the hypothesis of the study. As Adom and Hussein (2018) suggest, researchers often "borrow" this framework to structure their inquiry, ensuring a coherent and theoretically grounded approach. George (2023) further emphasises that the theoretical framework anchors the study in relevant theories related to human endeavour, providing direction for interpreting, explaining, and generalising findings. In this study, the theoretical framework consolidates key theories that enhance learners' understanding of lines and angles within the Cartesian plane in Analytical Geometry. It examines the underlying factors that influence learner outcomes and identifies effective instructional strategies. By applying Van Hiele’s theory of geometric reasoning, Vygotsky’s ZPD, and scaffolding techniques, the framework supports deeper comprehension and improved performance in the subject. These theories collectively guide the research, offering a structured lens through which to analyse and address learning challenges in Analytical Geometry.

2.4 CONSTRUCTIVIST THEORY

Constructivist theory posits that learners actively construct their own knowledge rather than passively receiving it from teachers. According to Kurt (2021), learners develop schemas to organise acquired knowledge, while Guy-Evans (2025) emphasises that knowledge is a personal and social construction of meaning from unstructured experiences.

In this study, the researcher explored Van Hiele's (1950) theory of the five levels of geometric learning and Lev Vygotsky's (1978) concepts of the ZPD and scaffolding. These frameworks were used to examine how learners construct knowledge and understanding when studying lines and their relationships on a Cartesian plane in the context of analytical geometry.

2.5 VAN HIELE'S THEORY

The researcher in this study proposes to use Van Hiele's model of theory to ground the challenges that grade 11 learners experience in learning Analytical Geometry of lines. Additionally, to untangle the difficulties learners face when answering questions on Analytical Geometry, we need to consider thinking. The theory hierarchically develops Geometrical thinking levels of learners. The figure below shows level 1 to level 5, which helped each learner in Grade 11 to understand Analytical Geometry.

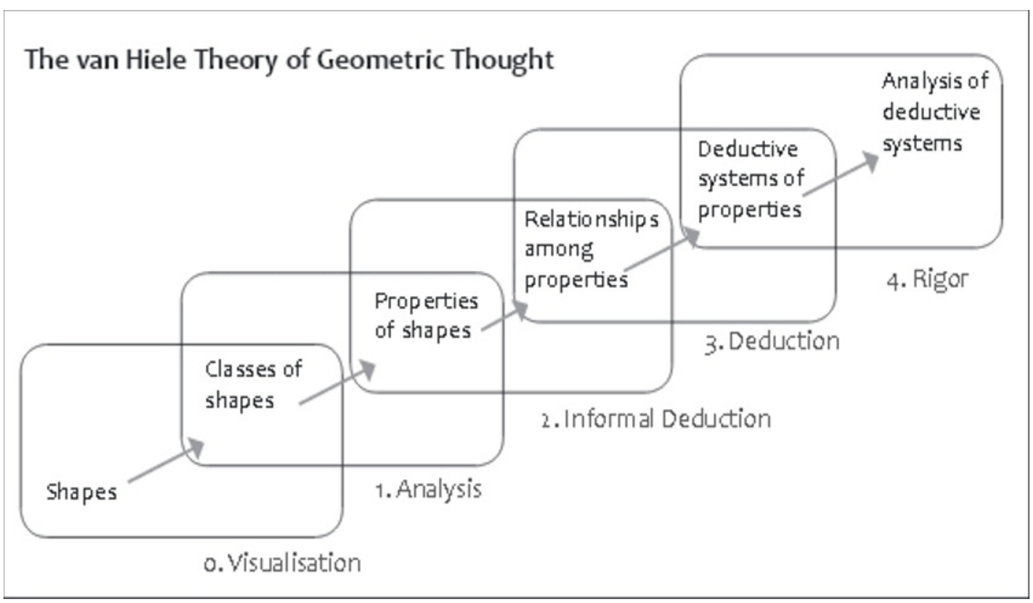


Figure 2.1: ***Van Hiele's theory of geometric thought*** (Van der Walle, 2004, 347)

Pierre van Hiele’s model of geometric thought outlines five progressive levels of geometric reasoning, providing a structured framework for understanding how learners develop geometric thinking. This model is particularly relevant in the study of Analytical Geometry, as it helps explain how learners comprehend fundamental concepts such as points, lines, and angles on the Cartesian plane. Yalley et al. (2021) emphasise that each level in Van Hiele’s hierarchy builds upon the previous one, reinforcing the sequential nature of geometric understanding.

Complementing this theory, constructivism posits that learning is an active process of knowledge construction rather than passive absorption (Kurt, 2021). In mathematics, learners continuously refine their cognitive schemas to integrate new concepts, such as those in Analytical Geometry (Bosman & Schulze, 2018). Van Hiele’s levels of thought align with this constructivist approach, particularly when applied to the study of lines and their relationships on the Cartesian plane. Yudianto et al. (2020) support the application of Van Hiele’s theory in Analytical Geometry, particularly when combined with dynamic tools such as GeoGebra, to enhance learners' conceptual understanding of geometric problems. By integrating Van Hiele’s model with constructivist principles, this study seeks to elucidate how learners develop a deeper understanding of Analytical Geometry through structured reasoning and interactive learning.

Learners' ability to work with geometry is significantly influenced by their prior learning experiences in the subject (Yudianto, 2019). Van Hiele’s theory (1986, 1999) describes five hierarchical levels of geometric understanding. The first level, visualisation, involves recognising geometric shapes based on their overall appearance. The second level of analysis focuses on identifying the properties of these shapes. At the third level, informal deduction, learners begin to understand relationships between geometric objects, such as lines and angles in a Cartesian plane. The fourth level, deduction, requires learners to apply formal axioms, definitions, and theorems to analyse geometric concepts. Finally, the fifth level, rigor, involves constructing and understanding formal deductive proofs (Wang & Kinzel, 2014, as cited in Yudianto, 2018; Yudianto, 2020).

In this study, Van Hiele’s theory was used to support learners in developing a deeper understanding of lines and angles in a Cartesian plane. The integration of GeoGebra software facilitated algebraic and geometric reasoning, enabling learners to overcome conceptual challenges and improve their performance in analytical geometry. By visualising and manipulating geometric relationships dynamically, learners gained greater proficiency in solving related problems.

2.5.1 Level 1 (Visualisation)

According to Van Hiele (1986), visualisation is a form of nonverbal thinking, where shapes are judged and perceived by their appearance. The (ibid) further states that shapes are generally viewed as they are, not necessarily thinking of the construct or property that made the appearance or the shapes. Visualisation is also defined as understanding or seeing initial objects in learners' minds (Yalley et al., 2021). For this study, based on Van Hiele's theory, a straight line on a Cartesian plane at this stage can be considered a straight line drawn without its properties, whether it has a positive or negative gradient. It is worthwhile to note that, from the outset, Level 1 served as the foundation for all subsequent levels, which the researcher followed to lay the groundwork for the geometric thinking cognitive domain and the functions of lines and angles on the Cartesian plane.

Van Hiele's level 1 assisted the researcher in identifying the extent of learners' knowledge about lines and angles on a Cartesian plane using GeoGebra. According to Watan and Sugiman (2018), the level of visualisation is related to the object's appearance as a whole. Learners may know lines and angles on the Cartesian plane in Analytic Geometry language without cognisance of the relations among their variables. According to Machisi and Feza (2021), at this level, learners only recognise geometric figures like triangles, rectangles, squares and rhombuses by their physical look and not by their properties. Van Hiele's theory suggested that learners must be made to understand that there are properties built into each of the objects and lines they see. At this first level, the researcher, by Van Hiele's theory and using GeoGebra, helped learners to know, for example, that the equation of a straight line is of the form e.g. y = 3x +9, y= $\frac{1}{2}x + 12$ and y = -3x -11. Some learners were unfamiliar with the algebraic form of a straight line equation. As such, the researcher provided the necessary approaches and suggestions that helped these learners overcome their difficulties. Learners at this level should be encouraged to progress by paving the way for a conceptual understanding of Analytical Geometry.

2.5.2 Level 2 (Analysis)

At this level, learners begin to identify the properties of figures or objects by their parts or by building, but have not yet established the relationship between objects in geometry and other objects outside of geometry (Van Hiele, 1986). According to González et al. (2021), learners identify the names and traits of objects, such as parallelograms and rectangles, without observing the mutual relationships between their variables, and, as such, cannot define and describe the objects. At level 2, learners knew the appearance and nature of the object, which helped the researcher guide them through the appropriate steps to identify the object's traits, shape, and figure (Yalley et al., 2021; Khalil et al., 2019).

At this stage, with the help of the GeoGebra application, the researcher assisted learners in identifying and discovering these properties in the formulae of lines and the size of angles, which enabled learners to answer questions about identifying these traits. According to Khalil et al. (2019), considering the linear equation of lines and angles, these two lines can form, i.e., $y = 2x + 9$ and $y = -6x + 8$. According to Van Hiele's theory, learners may not be aware of the instructional objectives of these two equations. Therefore, by familiarising learners with these traits of variables, and with the help of the GeoGebra application, they were enabled to answer level 2 types of questions, paving the way for understanding and answering level 3 questions. Otherwise, it will be difficult for them to understand the next level of questions on lines and angles on a Cartesian plane. From the above linear equation of straight lines, the "2" stands for gradient or slope and "9" represents the y-intercept of the equation. The same applies to the second equation. Lastly, in Analytical Geometry, when following Van Hiele's theory, learners must understand both algebraically and geometrically how to solve line equations to overcome any challenging questions on levels 1 and 2.

2.5.3 Level 3 (Abstraction or informal Deduction)

According to Van Hiele (1980), this is where the learner can identify the properties or construct of objects and establish the interrelationships of properties within figures. It is where classes of figures are recognised, definitions of objects are meaningful and informal arguments are followed. Still, at this stage, learners cannot fully understand the importance of deduction as a whole or define axioms (González et al., 2021). In respect, Van Hiele gave an example in a quadrilateral, where parallel opposite sides necessitate that opposite angles are equal. Also, with figures, a square is recognised as a rectangle, for it has all the properties of a rectangle. In this regard, the researcher employed an appropriate theoretical framework to build upon what learners could identify within the construct of lines and angles on the Cartesian plane, helping them recognise the properties and constructs they could not.

Yudianto et al. (2018) also asserted that at this level, learners' deductive reasoning has just begun, and they know geometric forms, understand the properties and can sequence geometric forms with one another. For example, the square is also a rectangle. As such, at this stage, learners should be able to understand the sequencing of geometric forms, even though deductive thinking has not yet developed or is just beginning to emerge. At this stage, learners still cannot explain why the two diagonals of a rectangle have the same length, particularly in relation to the lines and angles formed in the Cartesian plane. Learners can use the transformation logic, “square being a type of rectangle”. Still, they cannot tell whether opposite sides have the same gradients because they are parallel or their diagonals are equal in length. Therefore, the GeoGebra software helped learners understand the properties of a quadrilateral and identify parallel lines that are perpendicular to each other within quadrilaterals. Understanding the properties of quadrilaterals helped learners answer questions on lines and their formation on the Cartesian plane. Van Hiele’s theory suggests that a learner at this level should be able to recognise lines that have distinctive intercepts. For instance, $y = 2x + 1$ and $y = 2x + 4$ are two distinct parallel lines because their slopes are equal, but they have different intercepts. Learners at this stage do not know whether two lines with other intercepts could be parallel and should be made aware of such concepts. With the guidance of GeoGebra, the researcher suggested ways learners can answer questions on the properties of various quadrilaterals.

2.5.4 Level 4 (Formal Deduction)

Van Hiele (1986) defines this level as the significance of deducing geometric terms in the context of axiomatic terms being understood and the possibility of defining a proof more than once being realised. Through the conversion of axioms, theorems and definitions have not yet been apprehended. In relation to this study, learners must be able to tell the properties of the equation of a line. That is, “if the two lines are parallel to each other”, then the lines have the same gradient, and learners must be able to think reversely, that when two lines have the same gradient, it means they are parallel. If a learner could not reason along that construct at this stage, the GeoGebra software paves the way for the teacher to employ strategies that help the learner understand the concept.

According to Fitriyani and Widodo (2018), the objects of thought at level 4 are relationships among properties of geometric objects. Here, learners’ understanding acquired at level 3 can be applied to deduce and build axiomatic systems, definitions, theorems, consequences, and postulates (Bossé et al., 2021). According to Silmi and L (2022), the structure of a complete axiom system, including definitions, theorems, consequences, and postulates, now becomes an explicit object for learners' thinking. They can apply them to solve questions of lines and angles on the Cartesian plane. However, at this level, learners still do not understand the concept and should be taught how to construct proofs and conjectures through the necessary procedural formulas. At this stage, learners can tell that opposite sides of square figures and rectangles have equal gradients because they are parallel and diagonals are equal in length. The learner can apply this in answering questions on any topic with parallel and perpendicular lines. The learner can reason formally and look at different possibilities within the context of lines and their angles on the Cartesian plane in the Analytical Geometry system.

According to (Naufal et al., 2021), learners of this stage must know that for the axiomatic rule for two perpendicular lines. Learners should be taught that if two inclined lines are perpendicular, then the product of their algebraically calculated value of gradients must be equal to -1. Moreover, in the equation of the line, y=m x+c, learners must know the role of "m" and "c" in an abstract way rather than concretely to help them think cognitively. These ground learners enable them to perform well in analytical geometry, particularly in understanding lines and angles on a Cartesian plane.

2.5.5 Level 5 (Rigor)

Van Hiele (1986) defines this level as the ability of a person to have reversible thinking, where non-Euclidean geometry can be studied mostly abstractly. The (ibid) continues that learners would have various axiomatic thinking systems at this stage. A learner of rigor can progress from Level 1 to Level 4. However, the rigour level is not meant to be studied at secondary school but at colleges and universities (Van Hiele, 1986). Including it for learners pursuing Analytical Geometry studies at a higher level is worthwhile, as learners understand the use of "indirect proof and proof" (González et al., 2021). Additionally, learners at this level can transform different systems of mathematics and compare other axiomatic systems, as well as lines and angles in a Cartesian plane (Sari et al., 2020). With Van Hiele's levels enumerated, the above Levels, 1 to 4, with the help of GeoGebra, enabled the researcher to guide each learner in understanding problems concerning lines with ease, leading them to comprehend any problem pertaining to lines in Analytical Geometry.

2.6 VYGOTSKY THEORY OF LEARNING MATHEMATICS

Vygotsky's (1934) proposal for learning has served as the foundation for many researchers and theories in cognitive development for several years, particularly for the theory of sociocultural development (McLeod, 2018). Dogbe et al. (2024) viewed Vygotsky's idea of learning and teaching as essentially social activities that take place between social actors in socially constructed situations. Vygotsky's theory focuses on classroom interactions where teaching and learning activities occur through communication and demonstrations (Rigopouli et al., 2025). Knowledge and its transfer, as well as the classroom setting, are expected to contribute to understanding the subject matter (Falikman, 2021). Vygotsky (1962) emphasises that social development and its cultural context are inseparable; therefore, the most effective way to explore mental processes is through understanding the concept of mediation. Paaskesen (2020) encourages teachers to employ learner-centred pedagogical strategies and create an atmosphere that fosters cordial communication between the teacher and learners. Vygotsky's theory of ZPD is one approach the researcher used to ensure learners have the opportunity to make meaningful contributions to Analytical Geometry thinking.

2.6.1 Vygotsky Zone of Proximal Development

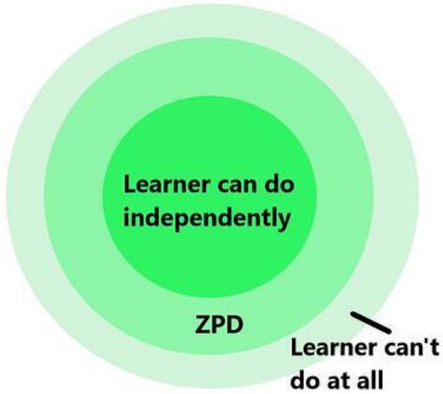
Vygotsky's work is defined by three major periods. Thus, the early period began in the early 1920s. The second period was in the mid-1920s and continued until late 1927 or early 1928, when Vygotsky significantly revisited his earlier ideas. The most productive period began in 1929 and proceeded until mid-1934. According to Vasileva and Balyasnikova (2019), Vygotsky's dedication to theoretical work shows a willingness to actualise ideas and test them experimentally. He developed the ZPD around the late 1920s. He defined it as "*the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers''* (1978).

According to Kempen (2020), the ZPD is the interval between a learner's capability to learn independently and the help they need from more capable peers or the teacher. In the same breath, Sarikas (2020) states that ZPD is the set of skills or knowledge a learner cannot attain on their own but can achieve with the help or guidance of someone else. Establishing the learners' ZPD helped the researcher identify the challenges they could face when trying to identify the construct of an equation of a straight line on a Cartesian plane. If a learner cannot identify the properties of a straight line, for example, parallel lines with the same gradient, they should be given help by a more capable peer to help them understand this concept.

Vygotsky (1896-1934) sought to understand people and how they learn in their social environment and created a theory on social learning development. Vygotsky has come to terms with the fact that educators can control most factors in an educational setting, including tasks, behaviours and responses. He asserted that learning occurs through interactions with others in our communities, such as peers, adults, teachers, and other mentors. As a result, they encouraged more interactive activities to promote cognitive growth, such as productive discussions, constructive feedback and collaboration with others in learning Analytical Geometry. Vygotsky stated that culture is a primary determinant of knowledge acquisition and proposed that learners study from the notions and attitudes modelled by their culture (Kurt 2020). This notion helped the researcher understand how teachers interact with their learners and the types of directions they provide when teaching and learning Analytical Geometry of lines, whether studying individually or in a group.

Vygotsky's (1978) notion of the ZPD is developed to account for children's learning potential and investigates its applications to teachers' professional development. The contributions of ZPD to scaffolding have been explored extensively, and controversial issues have been addressed. There is a consensus that the ZPD and socio-cultural theory of mind are at the heart of how teaching and learning can help learners progress in the learning process, such as in the context of Analytical Geometry. Therefore, explorations and investigations are needed to reflect the implications of ZPD in instructional development (Shabani et al., 2020). ZPD is usually represented as a series of concentric circles, which illustrate that the smaller circle represents the skill a learner can use to perform a task independently without help. The middle circle represents the task the learner can do independently with the help of a more capable peer or teacher; if not, s/he cannot perform the task. The last circle represents a skill beyond the learners' capability, even if they should seek help (Sarikas, 2020).

Figure 2.2: ***The level of ZPD thinking of a learner***



Vygotsky described the ZPD as the gap between the developmental levels achieved through independent problem-solving under guidance or in collaboration with more capable peers (McLeod, 2024). At this point, the theory suggests that educators should attempt to identify learners' ZPD level when instructing them in Analytical Geometry of lines and angles on the Cartesian plane, so that they can perform well in the topic. Small (2017) states that teachers are not using the educational time optimally if they are teaching beyond or are providing tasks that are already outside learners’ ZPD

2.6.2 The Importance of ZPD

It is essential to understand what Vygotsky’s ZPD can offer in terms of developing a child’s instructional learning process. This applies to developing a child’s learning process, especially when learning the Analytical Geometry of lines and angles on the Cartesian plane. According to McLeod (2024), ZPD helps to find the mental functions in the mature mind of a child, which is still in the developmental process (at its embryonic stage and is sure to mature in the future). If teachers can offer help, they should be encouraged to consider what learners will need to know in the future, rather than their current level of knowledge (Wibowo et al., 2024). As a researcher, I was concerned about the academic development of learners. Hence, such a construct of learners’ analytical geometry of lines and angles helped the researcher interpret the data. Thus, the study ***Enhancing the performance of GRADE 11 learners in relation to lines in Analytical Geometry on a Cartesian Plane in Bohlabela district, Mpumalanga***.

According to Sarikas (2020), Vygotsky formulated ZPD to develop appropriate teaching interventions of GeoGebra software that involve principles for possible instructional grouping of learners and identification of specific interventions for individual learners. For this study, the interventions of GeoGebra software included measures that identify learners' present ZPD to help them move from current difficulties to a level where they could understand the Analytical Geometry of lines and angles on a Cartesian plane with the help of more capable peers or the teacher, most especially those struggling in Levels 3 and 4.

2.6.3 Scaffolding or Instructional Scaffolding

According to Vygotsky (1978), scaffolding instruction is the "role of teachers and others in supporting the learners' development and providing support structures to get to that next stage or level". The (ibid) further explains that scaffolding is a teaching strategy in which an educator supports a learner's ability to build on prior instructional academic work. Scaffolding helped the researcher in the quest to show learners the challenges they faced in answering the test items that were administered. Scaffolding provides unique assistance that helps learners prepare for and understand incoming concepts and skills. The "new concept" that learners might be introduced to was a knowledge booster for acquiring more concepts of Analytical Geometry of lines on the Cartesian plane.

Teachers systematically build learners' learning experiences and knowledge as they acquire new skills through instructional scaffolding (Traver et al., 2019). The (ibid) defines instructional scaffolding as the teacher's support process for learners to enhance learning and master a skill or task they are learning. Sarika (2020) also adds that instructional scaffolding, also known as "Vygotsky scaffolding" or just "scaffolding, **"*is*** *a teaching strategy which helps learners to study by working with a teacher or a more capable peer to master a skill of their goals.* With regard to the above, scaffolding allowed the researcher to constructively suggest to stakeholders **in** education and teachers t**hat they** close the gap created in learners' learning of Analytical Geometry.**"**

2.6.4 Importance of scaffolding

Vygotsky (1978) idealised learning as an exercise involving social interaction for communicating from one person to another (Wibowo et al, 2024). McLeod (2025) views Vygotsky's ideas on teaching and learning as important social activities that occur within socially formed conditions. One can see that both views of Vygotsky's theory centre on the interactions within the classroom, where learning and teaching activities, such as the analysis of lines on the Cartesian plane, take place. Knowledge is transferred from one person to another through verbal communication and demonstration. In the classroom, the teacher should act as a facilitator of knowledge, such as when facilitating the use of GeoGebra software in teaching Analytical Geometry. According to Traver et al. (2019), cognition is developed through sociocultural theory interaction as learners learn the concepts and principles of a Cartesian plane. According to the researcher, those who were unable to develop cognitive skills at a particular level of learning lines and angles benefited from the help of more capable peers. According to McLeod (2025), the role of teachers and others is to support learners' development and provide support structures to facilitate progression to the next stage or level of learning. Other, more capable peers with knowledge and understanding of the subject matter can help partners who need an explanation and understanding of the subject (Anwar et al., 2024). Scaffolding occurs when a teacher or adult guides or assists a learner in solving a task that exceeds the learner's understanding or capability (Sarikas, 2020). Regarding the above, learners who found it challenging to understand Levels 3 and 4 of lines and angles in Analytical Geometry on the Cartesian plane relied on more capable peers to master their skills in answering questions. In other words, attaining knowledge and skills of Analytical Geometry can be acquired by a learner from an adult's guidance or a capable peer. A teacher can effectively use scaffolding as a tool in instructional delivery in the classroom (Anwar et al, 2024). This helped the researcher provide teachers with suggestions to bridge the gap learners face in learning Analytical Geometry of lines and angles on a Cartesian plane. Sarikas (2020) proposes four tips a teacher can apply when applying to scaffold, and these are:

- ***Know Each Student's ZPD***

Before applying ZPD and scaffolding strategies successfully**,** it is imperative to ascertain learners' current level of knowledge. Otherwise, it will be challenging to teach them within their ZPD, or facilitating them with scaffolding will be difficult. According to Sarikas (2020), learners' baseline knowledge can be determined by quizzing them, administering a test, or discussing with them to identify what they don't know and what they do know. It should also be noted that learners have different ZPD levels for different stages of Analytical Geometry. Hence, practical lessons such as group work help the teacher provide guidance and monitor each learner's ZPD.

- ***Encourage Group Work***

Grouping learners to work together can be a highly productive way to utilise scaffolding rules in the classroom. This opens ways for them to learn from each other. The more capable peers also learn while helping the weaker ones to improve by explaining the concepts to them. To have effective scaffolding lessons, teachers should try creating groups with learners with different skill abilities and learning levels to maximise how much learners can learn from each other. More capable peers can help other learners who have difficulties solving problems. As a facilitator, the teacher should ensure each learner actively participates in the group setting. The teacher should consistently ask learners in the group for their opinions, especially those who are often overshadowed by their peers. end-ended questions that encourage them to find a solution independently, rather than telling them the next step.

- ***Have Students Think Aloud about decisions they make in solving problems***

According to Sarikas (2020), the best way to discover learners' current skills (ZPD) in solving questions is by encouraging learners to communicate their thoughts and making them active learners. This approach proved helpful to the current researcher during the interviews with learners. The researcher urged learners to discuss their decisions regarding the lines and angles of Analytical Geometry on the Cartesian plane. Learners were also encouraged to think about what should be done next and what they were unsure about in solving the problem.

2.7 CHAPTER SUMMARY

The foundational theories and educational perspectives underpinning this research have been carefully outlined to elucidate the role of theories in practice, particularly in the context of learning mathematics. The study focuses on Analytical Geometry, with an emphasis on lines and their relationships, employing Van Hiele's levels of thought as the primary theoretical framework. This model systematically guides learners through five progressive levels of understanding, ensuring a structured and comprehensive mastery of geometric concepts while reducing potential difficulties in problem-solving.

To further enrich the conceptual explanation of Analytical Geometry, Vygotsky's sociocultural theory (1978), particularly the notions of the ZPD and scaffolding, was integrated. This perspective aligns with constructivist principles, reinforcing Van Hiele's framework by emphasising social interaction and guided learning. The synergy of these theories facilitates a more adaptive and practical instructional approach, fostering deeper conceptual comprehension and engagement with mathematical content.

CHAPTER 3: LITERATURE REVIEW

3.1 INTRODUCTION

The previous chapter explored the theoretical and conceptual framework of Van Hiele's levels of geometric learning, supported by Vygotsky's ZPD and scaffolding theory, which guides learners in understanding the geometric concepts of Analytical Geometry related to lines on the Cartesian plane. This chapter presents a comprehensive review of the literature, offering an in-depth analysis of the study. It examines the foundational principles of Analytical Geometry concerning lines on the Cartesian plane, including the derivation of straight-line equations, the conditions for perpendicular and parallel lines, and the role of slope (gradient) in defining these lines. Additionally, it addresses the challenges learners face in algebraic manipulation, which often hinder their performance in this topic. Furthermore, the chapter explores the role of dynamic geometry software, particularly GeoGebra, in enhancing conceptual understanding. These technological aid is analysed in terms of its' potential to facilitate deeper engagement and visualisation in learning Analytical Geometry.

3.2 BACKGROUND, CONTENT AND DEFINITIONS

This section begins with a review of the literature on document analysis and how learner performance can be enhanced, specifically in Analytical Geometry of lines on the Cartesian plane, as well as in mathematics. According to Taylor (2020), a literature review is an account of what has been published on a topic by accredited scholars and researchers. It presents a review of selecting the correct literature to discuss the conceptual framework used in a research study. Creswell and Creswell (2018) assert that a literature review highlights the importance of the study and serves as a standard for comparing the outcomes of other related studies. The literature reviewed explains the fundamental factors that contribute to poor performance in Analytical Geometry, specifically on lines and angles on a Cartesian plane. Related works of literature outlining the development of learners' cognitive capacity and their knowledge and comprehension of Analytical Geometry on lines in the Cartesian plane are also discussed.

3.3 THE GENERAL PERFORMANCE OF GEOMETRY IN SOUTH AFRICA CONTEXT

According to Yudianto (2018), geometry encompasses various branches, such as analytic geometry, Euclidean geometry, circle geometry, and projective geometry, as well as hyperbolic, parabolic, and spherical geometry—many of which relate to ellipsoids. However, in South Africa (SA), only analytic and Euclidean geometry are studied, with a focus solely on lines and angles in the Cartesian plane.

According to Naidoo and Kapofu (2020), geometry is one of the most difficult topics in mathematics, prompting ongoing research into the challenges learners face. Studies indicate that Euclidean geometry is particularly demanding for secondary school students (Luneta, 2020; Mamiala et al., 2021), while analytical geometry also poses difficulties for beginners (Dinayusadewi & Agustika, 2020). The question of how to effectively teach analytical geometry remains a key concern for educators and researchers (Taylor & Francis, 2020). Additionally, prospective mathematics teachers often struggle with geometric concepts due to their abstract nature (Andika et al., 2020). Tsao (2018) notes that learners often find geometry challenging, particularly in reasoning and problem-solving, which can lead to discouragement and poor performance. This issue is especially prevalent among South African secondary school students (Makhubele, Nkhona, & Luneta, 2020).

As a researcher, one may ask what causes analytical geometry to be challenging for learners, leading to poor performance in the topic. Renowned researchers have identified several causes that contribute to learners' poor performance in Analytical Geometry, specifically in the context of lines on the Cartesian plane. Some of these causes include:

1) Reasoning - this is thinking along the correct concept of the topic
2) Perception - this is the opinion about the topic
3) Attitude - feeling pessimistic or optimistic about the topic
4) Teaching strategy refers to the approach and handling of a topic by teachers.

3.4 CAUSES OF POOR PERFORMANCE IN ANALYTICAL GEOMETRY

Analytical geometry involves the study of geometry using a coordinate system and encompasses algebraic notions and notations (Mamiala et al, 2021). Learning geometry can be challenging, and many students struggle to develop a solid understanding of geometry concepts, reasoning, and problem-solving skills (Uygun, 2020). Dinayusadewi & Agustika (2020) state that the lack of knowledge in learning geometry often causes discouragement among students, which invariably leads to poor performance.

To reason about a particular concept in mathematics is essential. This is because thinking in terms of concepts will enable one to apply the correct formulas and solutions to their problems. Analytical geometry is a topic that has formulas and theorems to be applied before a learner can succeed in answering the questions. It is this view that Mamiala et al. (2021) suggested that three possible reasons exist why learners may not perform well in Analytical Geometry. The (ibid) states that three reasons could result from learners' poor performance and postulates those three reasons as:

Correct reasoning: If a learner cannot do estimations on his/her algebraic manipulations on Analytical Geometry questions correctly, and cannot also find the conceptual relationship in the solution.

Flawed reasoning: A learner's level of Analytical Geometry reasoning that is Medium, meaning the learner is correct in the estimation and correct in terms of the relationship/concept, but less correct in arriving at the correct solution, will lead learners to poor performance.

Poor reasoning: If a learner's level of Analytical Geometry reasoning is less correct, finding conceptual relationships and making wrong estimations will make a learner perform poorly in Analytical Geometry of lines and angles in a Cartesian plane.

As a result, Tsao (2018) asserts that it is important to cultivate and develop geometrical reasoning if one wants to be able to solve Analytical Geometry questions correctly. The writers again emphasised the importance of developing good communication in a topic like Analytical Geometry of lines and angles, as that will enable learners to relate the connection between algebraic calculation and Analytical Geometry and its concepts. Learners in Analytical Geometry need to be able to relate the language of Analytical Geometry to the concepts of the topic to avoid performing poorly on the topic.

Similarly, Makofane and Maile (2019) state that some factors that could cause learners' poor performance in mathematics include fear, lack of time, and insufficient financial and human resources. The (ibid) states that learners' performance will decrease if they pursue their studies under the above-mentioned difficult conditions. According to Mamiala et al (2021), the problems that cause learners' poor performance in mathematics and Analytical Geometry as a topic in mathematics could be as follows:

- ***Lack of soliciting help***: most learners fail to seek help or clarification when needed. Tsao (2018) suggests that educators from both primary and secondary schools should attempt to identify learners who are struggling to learn and provide targeted interventions, such as extra lessons, or involve the necessary stakeholders, including parents, to address the situation.
- ***Learner's interest:*** Most learners develop some level of inferiority and hesitation towards analytical geometry because they find it difficult, boring and complicated. Such attitude does not augur well for the topic and will hinder one's progress. One should be aware that having an interest in a subject plays a crucial role in achieving success in that area, particularly in mathematics and analytical geometry.
- ***Not practising Maths:*** Failure to practice Analytical Geometry is the biggest challenge with learners. According to Pambudi et al. (2022), merely observing an educator working mathematics on the board does not necessarily mean you understand it. Practising maths, particularly Analytical Geometry, will teach learners the necessary mathematics skills and algebraic manipulations to lead them to perform well in the Cartesian plane.
- ***Lack of Relevant Previous Knowledge (RPK):*** Building upon Relevant Previous Knowledge (RPK) in Mathematics concepts is very important. This means it is worthwhile for teachers to build upon learners' previous knowledge for any lesson they are to begin (Ningsih & Retnowati, 2019). Most learners struggle with Analytical Geometry because they lack the fundamental algebraic manipulation skills required for the topic.
- ***Lack of asking questions for answers:*** The lack of full participation by many learners, particularly when they do not understand the material being taught, is a significant problem (Clarke & Roche, 2018). This could result from them feeling shy or lacking confidence in asking questions they do not understand in the presence of their peers during the lesson. Learners' full participation brings enthusiasm, affection towards what they are learning, and their cognitive level contributes to their good performance in mathematics.
- ***Inability to pay attention in class:*** The most essential aspect of mathematics lessons is listening attentively to the teacher. According to Huang & Wang (2023), mathematics

needs much attention. Specifically, the most challenging facet of the lesson and more complex procedures are to be explored by learners during and after the lesson. It is worthwhile to mention that at this point, most learners experience attention disorders such as attention-deficit hyperactivity disorder (ADHD), which makes it difficult for them to adjust to the lesson delivery, and this could make them miss important steps of the lesson.

- ***Unqualified educators**:* Most of the time, dynamic handling of maths practically will develop a learner's interest in the subject. Unfortunately, that is what most of our teachers lack in the delivery of Analytical Geometry (Kusmaryono & Wijayanti, 2023). Most teachers fail to make the lessons on Analytical Geometry of lines on the Cartesian plane practical, making it difficult for learners to grasp the necessary concepts. Teachers with such qualities are either little trained or not trained at all. School management should allocate resources to train teachers in mathematics teaching, enabling learners to grasp the necessary concepts of Analytical Geometry when answering questions (Hua et al., 2018). It is also a problem in many developed countries, such as the USA and other European countries, where learners perform poorly, especially those from disadvantaged backgrounds (Smith, 2004). According to Van Geel et al. (2022), assertion, poor performance in mathematics is mainly observed in low-income communities, which affects their performance in Analytical Geometry. In SA, the black poor communities are those who do not do well in Analytical Geometry. According to Reddy (2006), as cited in Makofane and Maile (2019), blacks experience the poorest performance in mathematics and Analytical Geometry. Similarly, Ysseldyke (2003), as cited in Makofane and Maile (2019), reveals that two-thirds of the learners in low-income communities have mainly demonstrated a lower fundamental level of mathematics achievements in the US. Generally, Model C schools obtain better results in mathematics (Makofane & Maile 2019).

Other factors that could cause poor performance of Analytical Geometry of lines on the Cartesian plane. These could include:

- ***The school environment:*** A poor environment could be one of the reasons why learners still experience poor performance in Analytical Geometry, a topic in mathematics (Liu et al., 2022). An environment that is sufficiently motivated to study will attract interest, playing a crucial role in achieving high goals, especially in mathematics and analytical geometry, such as lines and angles on a Cartesian plane. The learning environment should stimulate learners' interest in Analytical Geometry. It should be a tendency to reduce frustration when solving Analytical Geometry questions involving lines on a Cartesian plane, regardless of the level.

3.5 ATTITUDE OF LEARNERS TOWARDS STUDYING ANALYTICAL GEOMETRY

The positive or negative endurance of a system of emotional feelings and tendencies is known as attitude (Schmitt-Cerna et al, 2024). Giannoulas and Stampoltzis (2021) affirm that attitude is the position of readiness and the tendency to respond to a particular stimulus, whether it is emotional feelings toward a situation, object, or institutional group. Studying Mathematics, especially Analytical Geometry of lines and angles on the Cartesian plane, requires the right mentality and a promising approach for learners. Without this mentality, it will be challenging for every learner to excel in the subject. Some learners have the notion that mathematics is complex, which can lead to fear when learning the subject. Hanfi (2008), as cited in Makondo and Mankondo (2020), attribute the fear of mathematics to "Mathophobia," which will lead learners to poor performance, such as a negative attitude. Dapaah and Agyei (2018) assert that several factors contribute to learners performing poorly in mathematics, including attitude, confidence, interest, and the perception of the usefulness of mathematics. Learners with a negative attitude towards mathematics topics, such as Analytical Geometry, tend to perform poorly (Atteh et al., 2020). A negative attitude can impact mathematics self-efficacy and also generate unnecessary anxiety when answering questions. According to Mamiala et al. (2021), mathematics self-efficacy and anxiety related to mathematics at the learners' level can significantly impact mathematics performance.

3.6 LEARNERS' PERCEPTION OF ANALYTICAL GEOMETRY

According to Reddy et al (2020), perception is the information created by our senses to understand the environment. An opportunity is created for our meaningful environment as a result of this understanding (Naiker et al, 2020). The challenges in completing geometrical activities, such as opinions on studying geometry, teaching, and materials, are activities deemed to be perception (Schmitt-Cerna et al., 2024). Giannoulas and Stampoltzis (2021) reiterated that perceptions are built on how we understand stimuli and are shaped by how we attach importance to our milieu. Therefore, teachers and researchers must explore the mathematical perception of learners to strengthen their teaching of Analytical Geometry (Naidoo & Kapofu, 2020).

Perception plays a vital role in learning, especially in mathematics. A positive perception can help learners of Analytical Geometry of lines on the Cartesian plane perform better in mathematics, whereas a negative perception towards Analytical Geometry will affect their performance. Naiker et al. (2020) indicate that performing or scoring well in Analytical Geometry requires positive perception. Giannoulas and Stampoltzis (2021) suggest that better performance can be achieved by improving a positive perception. It is worthwhile to note that perception has a significant impact on learning, both positively and negatively. According to Atteh et al. (2020), “perceptions affect performance and performance in turn affects perception”. The (ibid) notes that the achievement of goals and objectives could be affected by the perception of performance, especially when it is negative. As such, perception should be kept at a manageable level to avoid affecting learners’ performance in mathematics.

3.7 METHODS OR TEACHING STRATEGIES IN ANALYTICAL GEOMETRY BY TEACHERS

Passing mathematics greatly depends on the teacher's method and strategy in delivering the subject matter. Most teachers still dominate the lesson delivery, when, in fact, learners are supposed to be the dominant force to grasp the necessary concepts of the topic (Anwar et al., 2024). The (ibid) further notes that textbooks dominate most teaching. Teachers depend on textbooks, which makes the lesson theoretically more dependent on its content than on its practicality.

To help learners perform well, expert and pedagogical content knowledge teachers who can effectively handle the topics and understand their learners are essential for teaching mathematics. In this regard, Kusmaryono and Wijayanti (2023) emphasise that teacher expertise and content knowledge are interwoven with their pedagogical knowledge, which enables learners to understand the topics they are teaching. Teachers who are not well-equipped find it challenging to help learners, and their pedagogical strategies often fail to provide learners with concrete examples that enable them to understand the material being taught (Taylor, 2022).

Opus et al. (2025) notice that in teaching Analytical Geometry and equations of lines on the Cartesian plane, teachers should indulge in the practical teaching of the topic. The authors compared textbooks used in China and the United States of America and noticed that American textbooks contain more practical activities, providing a wider range of knowledge about the concepts. According to Hua et al. (2018), books promote the practicality of learning mathematics, featuring colourful visuals and pictures that draw learners' attention to the practical aspects of life and help them remember what they have learned. In the same light, Olkun (2022). Affirms that visual and practical textbooks, with their rich background, make learning maths presentable and interesting. The author also notes that books include engaging exercises and effectively integrate technology.

According to Opus et al. (2025), teachers should be well-versed in Analytical Geometry and able to relate the topic's content to other subjects, thereby stimulating learners' interest. The (ibid) further states that teachers should not neglect learners’ practical understanding in favour of indoctrination and rote learning. Teachers should teach Analytical Geometry practically so that learners can master the application pathways of using the content theorems to solve real life questions and problems. If Analytical Geometry is not taught practically, learners cannot apply their knowledge skilfully and solve Analytical Geometry questions effectively. Teachers of Analytical Geometry should tailor their instructional delivery to the thinking level of learners. In support, Van Hiele (1999) notes that learners’ lack of pattern thinking results in lower conceptual thinking in Analytical Geometry. The instructional delivery path should always follow learners' behavioural thinking and foster their conceptual development from one level to another (Van Hiele,1999).

3.7.1 Ability to answer questions on shapes and numbers in Analytical Geometry

Analytical Geometry is composed of shapes with coordinates, numbers, and points that describe figures, such as triangles, quadrilaterals, and plane figures, on a Cartesian plane. All these figures have their individual properties that one must identify to answer questions. According to Hua et al (2018), combining shapes and numbers under Analytical Geometry questions and solving them is the most important thing to teach learners. Teaching learners to practically transform geometrical problems into mathematical problems is also essential, and this rarely occurs in most lesson delivery.

Hua et al. (2018) assert that when teaching the equation of lines in Analytical Geometry, teachers should not initially consider binary first-order equations when solving for the distance between the intersections of straight lines. This creates a deficiency in that the characteristics of Analytic Geometry are not systematically understood. For example, the intersection of straight lines and the problem of points on the surface of straight lines. Each straight line corresponds to a binary first-order equation, which can be transformed into solving the intersection of two equations. Teachers should incorporate appropriate teaching methods to help learners understand the concept and perform well on the topic. In that case, it will also pave the way to enhance performance and boost learners' attitudes toward learning the topic, which will lead to good performance (Mamiala et al, 2020). The (ibid) argues that teaching that encourages learners to articulate geometric ideas and discuss problem-solving in Analytical Geometry can lead to improved communication skills, which eventually enhance their performance in Analytical Geometry.

3.7.2 Forming equations of lines from abstract geometric questions

The challenging question for learners of Analytical Geometry is how to extract equations from abstract questions to be solved mathematically. According to Scristia et al. (2021, 2022), learners find it challenging to extract geometric problems. A way of teaching learners is to enable them to form equations of lines from mathematical geometric problems, which is a challenge for most teachers. For learners to be successful, teachers should adopt strategies for talking and guide learners to understand geometric equations. Sumarni et al. (2020) state that training learners to distinguish between the relationship of Analytical Geometry and its equation of lines is very important. This will help mitigate learners' difficulties when solving Analytical Geometry questions and enable them to perform well.

Qualified pre-service teachers and their competency are crucial for achieving effective outcomes (Opus et al., 2025). According to these researchers, to teach Analytical Geometry of lines and angles effectively, teachers must master the substantive and syntactic structures of mathematics. They must not only present students with the accepted facts, concepts, and principles of different branches of mathematics, but they must also be able to explain why a particular mathematical principle is deemed warranted.

3.7.3 Strategies to improve performance in Analytical Geometry of lines on the Cartesian plane

Enhancing performance in every subject or topic requires strategies and methods to facilitate this process. In light of this, various writers are devising means to help learners struggling with Analytical Geometry of lines and angles on the Cartesian plane enhance their performance (Widiastuti & Kurniasih, 2021). Cabri 3D can teach line equations in space. They found that the program is potentially very useful for learning equations of lines on a Cartesian plane in Analytic Geometry.

Additionally, Suparman (2022) identified Cabri 3D as an effective tool for teaching Analytic Geometry and demonstrated that it is an effective tool for determining the equations of lines of special planes and their normal vectors. The Cabri 3D is a teaching aid, as stated by the above researchers, that can help any Analytical Geometry teacher solve problems involving lines in the Cartesian plane to enhance their students' performance in mathematics. However, he did not discuss how Cabri 3D could be used to find equations of parallel and perpendicular lines.

Hartono (2020) introduced an innovative strategy leveraging a sketchpad to enhance learner performance in Analytical Geometry, particularly in understanding lines on the Cartesian plane. Research indicates that high school mathematics teachers who adeptly employ geometric sketchpad tools can significantly streamline instructional processes, thereby boosting classroom efficiency (Hwang et al., 2020). Furthermore, the geometric sketchpad proves invaluable in teaching the positional relationships of straight lines within the Cartesian plane, offering dynamic visualisation and interactive learning opportunities. When utilised effectively, this teaching aid has been shown to dramatically improve learners' comprehension and performance in mastering the connections between lines and angles in Analytical Geometry.

According to Badawi and Muhammad (2021), hands-on activities enable learners to discover concepts in Analytical Geometry, leading to improved independent performance. Analytical Geometry engages multiple senses, visual, instinctive, and aesthetic, while capturing learners' interest through real-world shapes and constructions (Siyepu & Mtonjeni, 2018). Jatisunda et al. (2021) emphasise that hands-on activities allow learners to investigate, construct, and participate in creating drawings while observing geometric shapes in their surroundings. These activities foster higher-order thinking and a deeper understanding of equations of lines and angles on the Cartesian plane. However, the study could have further demonstrated how such activities help learners identify properties of plane shapes (e.g., quadrilaterals, triangles) and solids, thereby advancing their comprehension through Levels 3, 4, and 5 of Van Hiele’s theory.

In this study, the researcher focused primarily on GeoGebra as the central instructional tool, aligning its use with Van Hiele’s five-level theory to enhance learners' performance in Analytical Geometry, specifically in understanding lines on the Cartesian plane.

3.8 GEOGEBRA

GeoGebra is a dynamic software tool designed to teach mathematics and enhance performance in mathematics topics, such as geometry, statistics, algebra, and calculus, at the secondary school and college levels (Mudaly & Fletcher, 2019). It is used to foster mathematical visualisation concepts that bring learners a clear understanding and make room for mathematical instructional materials. According to the Encyclopaedia definition, "GeoGebra is an interactive geometry, algebra, statistics and calculus application, intended for learning and teaching mathematics and science from primary school to university level". This makes GeoGebra a highly suitable dynamic software tool that can help learners of Analytical Geometry of lines and angles improve their performance in the topic.

According to Ma and Yang (2022), GeoGebra enables mathematical experiments that foster active, learner-centred, mathematically conceptual problem-solving exploration and teaching discovery, enhancing performance in mathematical topics such as Analytical Geometry of lines on the Cartesian plane (Roberts, 2023).

Teaching Analytical Geometry has undergone a new emergence due to GeoGebra, and this dynamic software provides assistance in integrating the Concepts of lines and how they relate to each other, allowing learners to better understand the concepts. Some of the Geogebra aspects are:

- **Virtual handling:** According to Yorganci (2018), Geogebra can manipulate virtual, abstract concepts of Analytical Geometry, such as lines and angles, to concretise them through diverse representations.
- **The connection of geometric thinking and algebra** (Sudihartinih and Wahyudin 2019). Geogebra is direct software that affords the concurrent exhibition of algebraic and geometric objects along with the process
- **Using sliders and animation** can enable us to see the process of the concept of Analytical Geometry distinctly and uninterruptedly.
- **Idea processing**: Geogebra can act on objects in Analytical Geometry through diverse outcomes, which enables the actualisation of ideas in more accurate and precise ways, providing spontaneous feedback in a precise manner.
- **Analytic and synthetic views:** Through GeoGebra, learners can translate and observe ideas related to different concepts of Analytical Geometry, including lines and angles in

the Cartesian plane, from both synthetic and analytical perspectives across various transformations.

The young age group of technology, otherwise known as the digital age group, is emerging so fast in schools that it is making teaching and learning of mathematics in some specific topics straightforward and easy to learn by the use of gadgets like smart cell phones, tablets and other software where GeoGebra is part of them that plays a smooth process that facilitates teaching and learning in classrooms (Roberts, 2023), especially, Analytical Geometry of lines and angles in Cartesian Plane. For this reason, GeoGebra is used to facilitate and understading of Van Hiele's levels of learning Analytical Geometry.

According to Sugandi and Bernard (2020), the teacher is paramount in ensuring that the mathematics curriculum in schools is implemented and taught effectively for learners to understand the topics being taught. To achieve this effectiveness, which is measured by the progress of learners in their examination results, and to utilise technology like GeoGebra to make things easier, it is rarely practised in many schools. The traditional practice of teaching mathematics remains prevalent in most schools across the country. According to Yorganci (2018), a powerful tool is currently a technology that involves learners and is at the same time recognised by the teachers of the National Council of Teachers of Mathematics (NCTM) (1989, 2000), namely GeoGebra. According to Roberts (2023), GeoGebra is a compelling tool which can be used to explore algebra and Geometry Dahal and Neupane (2022). GeoGebra is easily adaptable by learners and offers a flexible user interface. This demonstrates how GeoGebra software can facilitate learners' understanding of the teacher when teaching Analytical Geometry of lines and angles on the Cartesian plane.

3.9 GEOGEBRA APPLICATION

The study incorporated GeoGebra as an intervention to address the poor performance of Grade 11 learners in Analytical Geometry, specifically lines and angles in a Cartesian plane. GeoGebra is a dynamic software tool that enables teaching geometry from the lower primary to the higher university level (Diaz-Nunja, Rodríguez-Sosa & Lingán, 2018). Badawi and Muhammad (2021) also confirm the definition of GeoGebra as an online, free, dynamic software that is applied in learning algebra, calculus, and geometry. GeoGebra is a dynamic software that combines Geometer's Sketchpad and Cabri, and utilises a computer system.

After administering and marking the pre-test, the researcher implemented GeoGebra (Ljajko, 2018) in the experimental group to teach Analytical Geometry, focusing on lines and they relate to each other on the Cartesian plane, with the aim of improving learners' post-test performance. Díaz-Nuña et al. (2018) suggest that GeoGebra's integration of algebra and geometry enhances mathematical skills. The researcher utilised GeoGebra's interactive features (Pamungkas et al., 2020) to facilitate a deeper understanding of Analytical Geometry. By exploring and manipulating the tool (Nzaramyimana, 2021), the researcher fostered an engaging learning environment that strengthened learners' grasp of lines and angles on the Cartesian plane.

3.10 USING GEOGEBRA TO ENHANCE CONCEPT TEACHING IN ANALYTICAL GEOMETRY

GeoGebra is a software and a teaching aid used to help learners understand the concept of lines and angles in Analytical Geometry. Roberts (2023) affirmed that researchers explored the potential of GeoGebra for teaching lines and their formation of angles on the Cartesian plane, a concept in Analytic Geometry. According to Nzaramyimana (2021), in learning Analytic Geometry, GeoGebra is effective in teaching topics such as the equations of lines and their relationships (parallel lines and perpendicular lines) in the Cartesian plane, as well as the angles that lines may form in our current study, and offers convenience for prospective mathematics teachers.

Additionally, Pamungkas, Rahmawati, and Dinara (2020) reiterate that GeoGebra facilitates an understanding of Analytical Geometry concepts, enhances learning activities, and provides learners with easy access to answers, which motivates their studies. According to Ljajko (2018), the best tool is dynamic software. According to Dahal and Neupane (2022), analytical geometry concepts are easily grasped by learners, enabling them to act flexibly and enhance relational thinking rather than instrumental thinking regarding objects. Sugandi and Bernard (2020) asserted that, in Analytical Geometry, graphs can be drawn dynamically as the images of concepts can be elicited meaningfully through dynamic software. These researchers' findings are sufficient to confirm that GeoGebra is a software that can help learners of analytical geometry, specifically lines and angles on the Cartesian plane, reduce the errors and misconceptions they make.

3.11 USING GEOGEBRA TO MAKE COMPLEX AND PROBLEM-SOLVING QUESTIONS OF ANALYTICAL GEOMETRY OF LINES ON THE CARTESIAN PLANE EASIER

Living in our current global village, technology and mathematical thinking skills are closely correlated with human advancement in technology (Khalil et al, 2019). Such advancements in technological thinking make it easier to solve complex problems that would be impossible without the knowledge of complex mathematical thinking.

According to Khalil et al. (2019), GeoGebra is a dynamic software that offers diverse features for learning Analytical Geometry of lines, which can be effectively used to teach lines and angles in a Cartesian plane. Through the manipulative use of GeoGebra software, analytical geometry concepts related to lines and angles can be concretised, allowing abstract mathematical concepts to be understood by learners.

Establishing how GeoGebra will help learners understand Van Hiele's theory levels is important. According to Khalil et al. (2019), learners can be supported by their teachers using GeoGebra to enhance their understanding of Analytical Geometry of lines and angles on a Cartesian plane by assigning tasks according to Van Hiele's level of learning. This helps learners develop deductive thinking and conjecture on problems as well. To enhance learners' performance in Analytical Geometry, teachers must embrace GeoGebra dynamic software with dynamic processes to help overcome any difficulties in Van Hiele's level of thought. It is in this regard that Sugandi and Bernard (2020) encourage teachers to address the needs of each learner by designing dynamic software activities that cater to their difficulties according to van Hiele's level of learning in Analytical Geometry, specifically lines on the Cartesian plane. According to Dahal and Neupane (2022), utilising GeoGebra in Analytical Geometry enhances learning efficiency and effectiveness. According to Nzaramyimana (2021), the dynamic way of using GeoGebra facilitates the diverse way learners' ideas and thoughts are used to solve problem-solving questions in Analytical Geometry. It is emphasised that GeoGebra will enhance the prospective solving of problem-solving questions in Analytical Geometry, which will eventually improve their performance in the topic. According to Khalil et al. (2019), GeoGebra Software completes learning in Analytical Geometry. The GeoGebra software picture on the Cartesian plane below was used to solve equations of the straight line of Analytical Geometry on the Cartesian plane, and those questions satisfied the reliability requirements and distinguished features of the difficulty index.

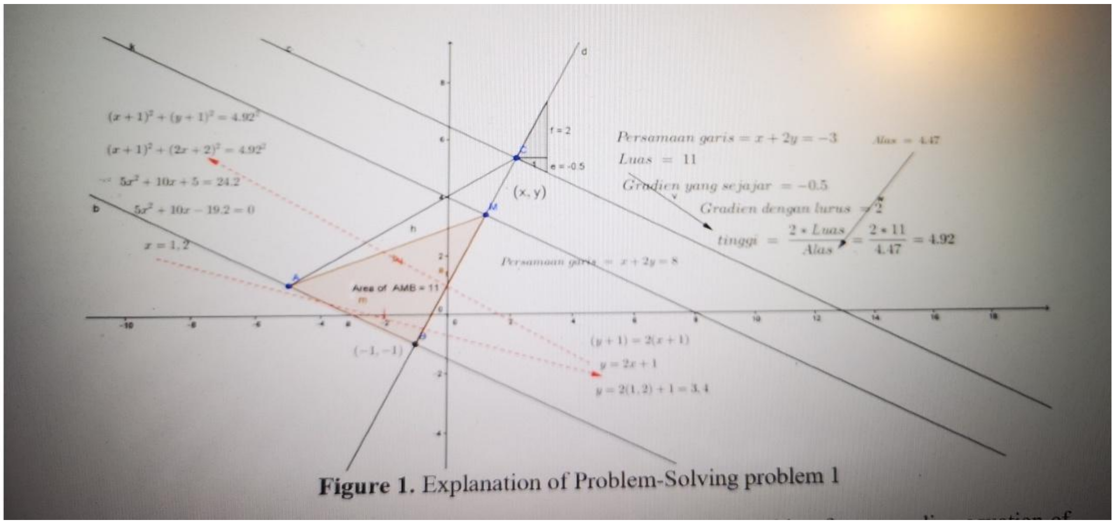


Figure 3.1: ***Explanation of Problem-solving***

Applying GeoGebra to the Cartesian plane encourages learners to develop and explore concepts in Analytical Geometry, deeply mastering mathematical concepts by visualising real-world images of points for solving questions on the Cartesian plane. Learners quickly understood the concept of the equation of straight lines and could comprehend problem-solving problems easily by displaying the pictures on the Cartesian plane. Considering the following points on the Cartesian plane, A (-5,1), B (-1, -1), and C (x, y), is to determine the position of the equation of the points. The figure above illustrates the steps involved in problem-solving and calculating the equations of lines using two different points on the Cartesian plane. Sugandi and Bernard (2020) provided examples of steps used to help learners grasp the concept of Analytical Geometry quickly. The first step provides basic knowledge about straight-line equations at the two points AB, using the formula of the equation of the line from two points, namely points A (-5.1) and B (-1, -1), so that the equation AB x + 2y = -3 is obtained using GeoGebra. The 2nd was to give learners instructions related to the broad line drawing as a clue to the base AB using the two-point distance formula and the proof of the AB segment command. The 3rd step was determining other measurements related to the area besides the base and area. Learners were asked to determine the height of the triangle, given that the base of the triangle was 4.47 units. This was obtained by applying the formula for the triangle's height, which yields an area of 2*11/√5 = 4.92 units. Determining the result of height means that the learner is describing where the point is outside the AB line. After finding a point outside AB, the learners suspect that there must be a perpendicular line from the baseline, which also needs to be found. In the fourth step, to see the perpendicular line, the learner first determines one of the arbitrary points but is in the AB line equation, for example, when choosing point B (-1, -1), a perpendicular line is made by finding the gradient of AB, which is AB = 2 and the AC line equation, $y = 2x + 1$ is obtained. The fifth step makes any point in the AC line equation such that point B and point C are adjusted to the length of height 4.92 and then adjusted to the learner's calculation by using the formula of the distance between two points and substitution y with x, and the result $x = 1.2$ and $y = 3.4$ which is an attribute of point position.

The sixth step was to enter the point into the GeoGebra command by writing (1.2, 3.4). After that, it was illustrated that the right point on the AC line can be seen on the GeoGebra screen. The seventh step involved creating one more line from the GeoGebra command menu, specifically a Parallel line to point (1.2, 3.4) or point M, which resulted in the equation $x + 2y = 8$. This was then adjusted to the final settlement step carried out by learners. The eighth stage was to re-evaluate whether the area of the triangle is 11. To prove this, we used the GeoGebra command menu. According to Sugandi and Bernard (2020), after that, the learners realised that the GeoGebra answer and their manual answer were the same, demonstrating how GeoGebra could simplify complex problems. It was discovered that using GeoGebra makes learning Analytical Geometry easier in terms of problem-solving questions. GeoGebra also makes it easier to answer Analytical Geometry questions and learn Analytical Geometry concepts in real-life situations.

CHAPTER SUMMARY

This chapter demonstrated how general performance in Analytical Geometry needs to be improved and the causes which led to the poor performance in mathematics at any learning stage. It also highlighted learners' attitude and perception towards mathematics that is making them to perform poorly. Teaching strategies are also discussed extensively that addressed the gaps created in learning Analytical Geometry.

The chapter provided an introduction of GeoGebra, GeoGebra application that serves as an interventional tool that was used in order to mitigate the effect of the poor performance in Analytical Geometry of lines, hence, enhancing performance in the topic.

CHAPTER 4: METHODOLOGY

4.1 INTRODUCTION

The previous chapter presented a literature review on learners' poor performance in Analytical Geometry, highlighting the challenges in understanding key concepts and proposing GeoGebra software as a potential solution. This chapter outlines the research methodology employed in the study, conducted across two schools. Adopting a mixed-methods approach, the study integrates both quantitative and qualitative data to provide a comprehensive analysis. The research is grounded in a pragmatist paradigm, which prioritises practical solutions and allows for methodological flexibility to address the research problem effectively.

The chapter outlines the research designs for both quantitative and qualitative approaches, followed by the data collection instruments employed. Sampling techniques are explained, along with the procedures for ensuring reliability, validity, dependability, and trustworthiness of the data. Finally, ethical considerations are discussed to underscore the integrity of the study. By combining numerical data with in-depth insights, this methodological framework ensures a robust examination of GeoGebra's impact on learners' understanding of Analytical Geometry.

4.2 RESEARCH METHODOLOGY

Creswell and Creswell (2018) define research methodology as specific themes that comprise data collection, interpretation, and analysis that researchers proffer for their studies. The researcher specifically delved into conducting a pre-test and post-test on learners on Analytical Geometry of lines and angles to be informed as to why they performed poorly in Analytical Geometry in Grade 11. The researcher employed Van Hiele's theories, along with the appropriate rules based on principles and methods that required specifics, to understand the basis for Grade 11 learners' poor performance in Analytical Geometry. According to Ullah and Ameen (2018), research methodology refers to "the collection of methods or rules" applied to research a particular problem and the aggregation of "principles, theories and values" that govern the entire path to research.

4.3 RESEARCH PARADIGM

The actions guided by a fundamental set of views are referred to as paradigms or sometimes as "world view" (Guba 1990, as cited in Creswell & Creswell 2018). This paradigm defines the guidance of action and worldview of a researcher, making the complex real world sensible and thinkable (Bibi et al., 2022). The paradigm also explores the nature of knowledge and reality, as well as the beliefs and general values in social settings (Kuhn 1970, as cited in Kaushik & Walsh 2019). The researcher adopted the pragmatist worldview, which sought to apply knowledge of this kind in a practical manner rather than based on theory.

This research study explored the various paradigms of ontology, epistemology, methodology, and methods that served as a lens through which the researcher examined the collated data. The figure below illustrates the process of knowledge acquisition for the study.

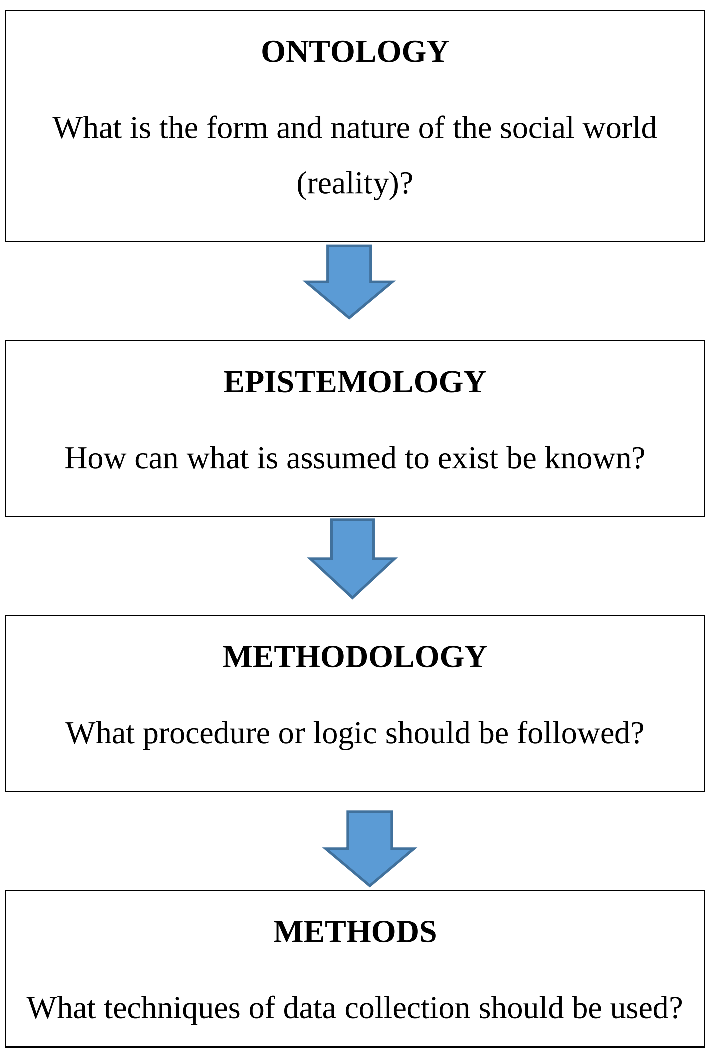


Fig.4.1 ***The relationship between ontology, epistemology, methodology and methods*** (Adapted from Coe et al., 2017:16)

The information above helped the researcher access information directly from participants, using classroom observations and interviews, in their quest to learn about Analytical Geometry of lines on the Cartesian plane.

Ontology

In ontology, the researcher explored the diverse teaching strategies employed by educators in instructing Analytical Geometry of lines, acknowledging that individuals' perceptions of reality and its meaning vary (Leavy, 2020). The ontological perspective enabled the researcher to examine how teachers approach the subject differently in the classroom (Leavy, 2017; Hassan & Mingers, 2018). Through open-ended interviews and observations of the social world, the researcher gained insight into how participants' experiences within their social contexts shaped their instructional methods for teaching Analytical Geometry on the Cartesian plane. This approach facilitated an understanding of individual interpretations of reality and the social construction of knowledge (Burkholder et al., 2020). The study revealed how teachers conveyed the content of Analytical Geometry to learners and the depth of their conceptual understanding.

Epistemology

Epistemology examines the acquisition of knowledge, the interpretation of the world, and the relationship between the knower and the known (Hassan & Mingers, 2018). Classroom observations and open-ended interviews provided the researcher with insights into educators' lived experiences in teaching Analytical Geometry. This epistemological approach also clarified how knowledge is applied, emphasising the dynamic interaction between the researcher and participants (Leavy, 2020). As Burkholder (2020) asserts, knowledge arises through perception, experience, and logical reasoning, whether inductive or deductive. By leveraging epistemology, the researcher identified the essential knowledge required for learners, determined appropriate interventions, and assessed how these interventions could enhance comprehension and performance in Analytical Geometry.

Methodology

Methodology refers to the structured framework guiding the planning, execution, and analysis of a research study (Sreekumar, 2023; Mbuagbaw et al., 2020). In this study, the researcher synthesised quantitative and qualitative data within a constructivist paradigm, shaping the research from conception to data interpretation.

Methods

Methods encompass the techniques for collecting, processing, and analysing research data (Greener & Martelli, 2018). The researcher employed a mixed-methods approach, utilising an explanatory sequential design and tailored data collection techniques to gather pertinent information. These methods established clear boundaries for the study, ensuring focused and meaningful findings.

4.3.4 Pragmatist paradigm

A pragmatic paradigm emphasises practical solutions over theoretical abstractions, focusing on actionable knowledge to address real-world problems (Kaushik & Walsh, 2019). As a pragmatist, the researcher prioritises identifying practical strategies, such as administering pre-tests and post-tests, to assess and improve Grade 11 learners' performance in Analytical Geometry through GeoGebra, rather than relying solely on theoretical frameworks to address the problem (Liu, 2022).

Rooted in the works of Peirce, James, Dewey, and later authors like Creswell & Creswell (2018), pragmatism values research concepts only if they drive tangible action (Turyahikayo, 2021). In this study, the researcher examines how teachers instruct Analytical Geometry and proposes GeoGebra as an intervention to bridge learners' gaps (Shan, 2021). The goal is to mitigate challenges in understanding lines on the Cartesian plane by applying effective solutions.

Pragmatists argue that there are valid ways of interpreting the world, and no single perspective can fully capture the complexity of human ideas (Liu, 2022). They emphasise that research must address real-world problems by prioritising the research question and adopting flexible methodologies to gain a comprehensive understanding (Rossman & Wilson, 1985, as cited in Creswell & Creswell, 2018). Unlike rigid philosophical stances, pragmatism allows researchers to transcend traditional divides, integrating both quantitative and qualitative approaches to enrich their findings (Creswell & Creswell, 2018).

This flexibility is particularly evident in mixed-methods research, where combining different data collection and analysis techniques, such as tests, interviews, and observations, enhances the depth and breadth of insights. In this study, the researcher adopted an explanatory sequential mixed-methods design to thoroughly examine the challenges that Grade 11 learners face in understanding Analytical Geometry of lines on a Cartesian plane. The choice of methodology, including the use of GeoGebra software as an intervention, was driven by its effectiveness in addressing the research question and improving learner performance.

Pragmatists reject the notion of a singular, absolute truth, aligning with mixed methods researchers who advocate for diverse data collection and analysis strategies rather than limiting themselves to a single approach (e.g., purely quantitative or qualitative). By embracing methodological pluralism, this study demonstrates how pragmatism facilitates a more nuanced and practical approach to investigating educational challenges.

The researcher employed a mixed-methods approach to provide a comprehensive analysis of data on the analytical geometry of lines on a Cartesian plane. According to Creswell and Creswell (2018), integrating quantitative and qualitative data allows for a deeper understanding of research problems. Mixed-methods researchers justify their methodological choices by emphasising both purpose and rationale. Pragmatists, as noted by Elgeddawy and Abouraia (2024), prioritise practical outcomes over rigid adherence to singular truths, focusing instead on diverse perspectives in data collection and analysis. In this study, the researcher applied pragmatism to evaluate learners' performance in Analytical Geometry, ensuring the methods aligned with the research questions (Johnson & Christensen, 2020).

While multiple paradigms exist in modern research (Kaushik & Walsh, 2019), this study adopted a pragmatist lens, which integrates ontology (assumptions about reality), epistemology (the nature of knowledge), and methodology (approaches to inquiry). Pragmatism acknowledges multiple realities (Creswell & Creswell, 2018), making it a suitable approach for investigating the analytical geometry of lines and angles on the Cartesian plane to enhance mathematical performance.

4.4 RESEARCH APPROACH

4.4.1 Quantitative Method

The quantitative method approach appears to examine theories deductively, assessing the relationships among variables, and can also protect against biases in a research study (Creswell & Creswell, 2018). Quantitative variables can be measured using instruments administered, where statistical procedures are applied to analyse the collected data (McMillan & Schumacher, 2020).

The research questions to be answered from the collected data could be a driving force in selecting the quantitative method (Lim, 2024). Such questions identified according to (Kraus et al. 2022) that encourage hypothesis testing by quantitative statistical method could be (a) the degree of relationship among two or more variables, and b) the measurement and structure of constructs. At the backdrop of this study, the researcher included a quantitative approach to help provide data for an interventional approach and also to answer some of the research questions like: "What are the benefits of using GeoGebra to teach Analytical Geometry of lines on Cartesian plane? It also helped measure the extent to which Analytical Geometry concepts were understood by grade 11 learners from poor-performing schools in the Bohlabela District. The quantitative data collected also facilitated the collection of qualitative data through interviews with learners about their test results.

4.4.2 Qualitative Method

Qualitative research involves collecting data from the individual natural setting, where the data is analysed inductively, based on general patterns/themes identified by the participants, which are then interpreted by the researcher (Creswell & Creswell, 2018). The researcher went to the natural learning environment of the learners to observe how they are taught/learn Grade 11 Analytical Geometry. This is done to get insight into what happens during the learning process that is causing their poor performance. Qualitative research is a distinguished phenomenon where behaviour is studied in natural settings. It has no external influence on its study outcome because the inhabitants' behaviour occurs in their natural settings, in this case, the school where grade 11 learners learn (McMillan & Schumacher, 2020).

A qualitative method is an approach to exploring a study that aims to understand the meaning of an individual or a group, as well as their problems, in their natural settings (Creswell and Creswell, 2018). Given this, the researcher interviewed 12 learners and the two teachers to ascertain their problems after writing the pre-test and administering the post-test. In an article by McNamara (2022), interviews are beneficial for gathering in-depth information about the topic under study, providing insight into the story behind a participant's experience. It is, therefore, useful to conduct interviews with the learners to understand their difficulties in learning Analytical Geometry. The interview also serves as a primary source of information data collection (McMillan & Schumacher, 2020). The researcher needs to get the natural essence of Grade 11 learners' problems in learning

Analytical Geometry of lines and angles on a Cartesian plane. An excellent way to accumulate information is through an interview, whether on a topic a researcher is working on, as it provides an elaborate method of gathering information that other methods cannot (DeCarlo, 2022). The above reasons motivated the researcher to conduct interviews with Grade 11 learners and the two teachers.

Observation, which involves observing and recording what occurs naturally at the research site location to gather data from participants, is a method used to collect data (McMillan & Schumacher, 2020). I visited the natural environment (schools, classrooms) of Grade 11 learners and two teachers to observe for 45 minutes and understand how they learn Analytical Geometry. The researcher made a critical observation to understand the teaching of Analytical Geometry to Grade 11 learners and how learners perceive their teachers' delivery of the topic. This helped the researcher to establish the fact (first-hand information (Bhandari 2020) on whether the teacher is faltering in the systematic process of delivering the lesson according to Van Hiele's process of learning Analytical Geometry (Gokalp 2016) or is the learners who are not good observers or listeners. Notes were taken during lessons and kept for qualitative data collection. Doing so helped the researcher pinpoint the source of some of the poor performance problems in grade analytical geometry. This research was conducted to gain a deeper understanding of the problem, rather than relying solely on one source of information for my data.

4.4.3 Mixed-method

The reason for combining both quantitative and qualitative methods as a mixed-methods approach is that neither can sufficiently address the trend in data analysis (Creswell & Creswell, 2018; McMillan & Schumacher, 2020). Integrating quantitative and qualitative databases provides insight into research problems and questions (Creswell & Creswell, 2018). Jane (2024) confirms with the above writers that it is prudent for a researcher to collect multiple research data using various research methods. Johnson and Turner (2003) and Johnson and Christensen (2020) affirmed that mixed methods improve a research study compared to individual quantitative and qualitative methods, giving distinctive knowledge, strengths, and weaknesses. Ngulub (2020) argues that mixed methods are the most appropriate and effective approach for answering research questions in a study, regardless of the goal, compared to using individual quantitative and qualitative methods.

For this study, it is worthwhile to acknowledge that the researcher has chosen mixed-method approach for' numerous purposes' more than any other single method, quantitative or qualitative (Creswell & Creswell 2018; Ngulube 2020). Some of the benefits, according to researchers, are:

- Mixed-methods includes the integration of both qualitative (open-ended) and quantitative (closed-ended) data in response to research questions (Johnson, Onwuegbuzie & Turner 2007, cited in Creswell & Creswell 2018)
- Mixed-methods research also offers innovation advantages, multiple perspectives, and insightfulness into research phenomena, facilitating efficient work in the face of current research challenges (Timans et al., 2019).
- The mixed method complements the respective strengths and weaknesses of the single-method approach (Regnault, Willgoss, & Barbic, 2019).
- Mixed-methods can be drawn from quantitative and qualitative data to compare different perspectives (Creswell & Creswell, 2018)
- Mixed-methods can explain quantitative data results by qualitative follow-up data collection and analysis.
- Mixed-methods are the most appropriate methods, and arguably the best, for answering research questions in a research study for any goal-achieving purpose, rather than using individual quantitative and qualitative methods (McMillan & Schumacher, 2020).

4.4.4 Research Design

Research designs provide a specific pathway for the processes of a research study that seeks mixed-methods, quantitative, and qualitative approaches (Denzin & Lincoln, 2011, as cited in Creswell & Creswell, 2018). The researcher sought to find a method for answering the research questions. As stated by Johnson and Christensen (2020), a research design is a pathway or a strategy that shows how a researcher addresses his/her research problem. To address the Grade 11 learners' problem in learning Analytical Geometry of lines on the Cartesian plane, the researcher should draw or find a roadmap that reveals the difficulties learners encounter in answering Analytical Geometry questions.

4.4.5 Explanatory sequential mixed-method design

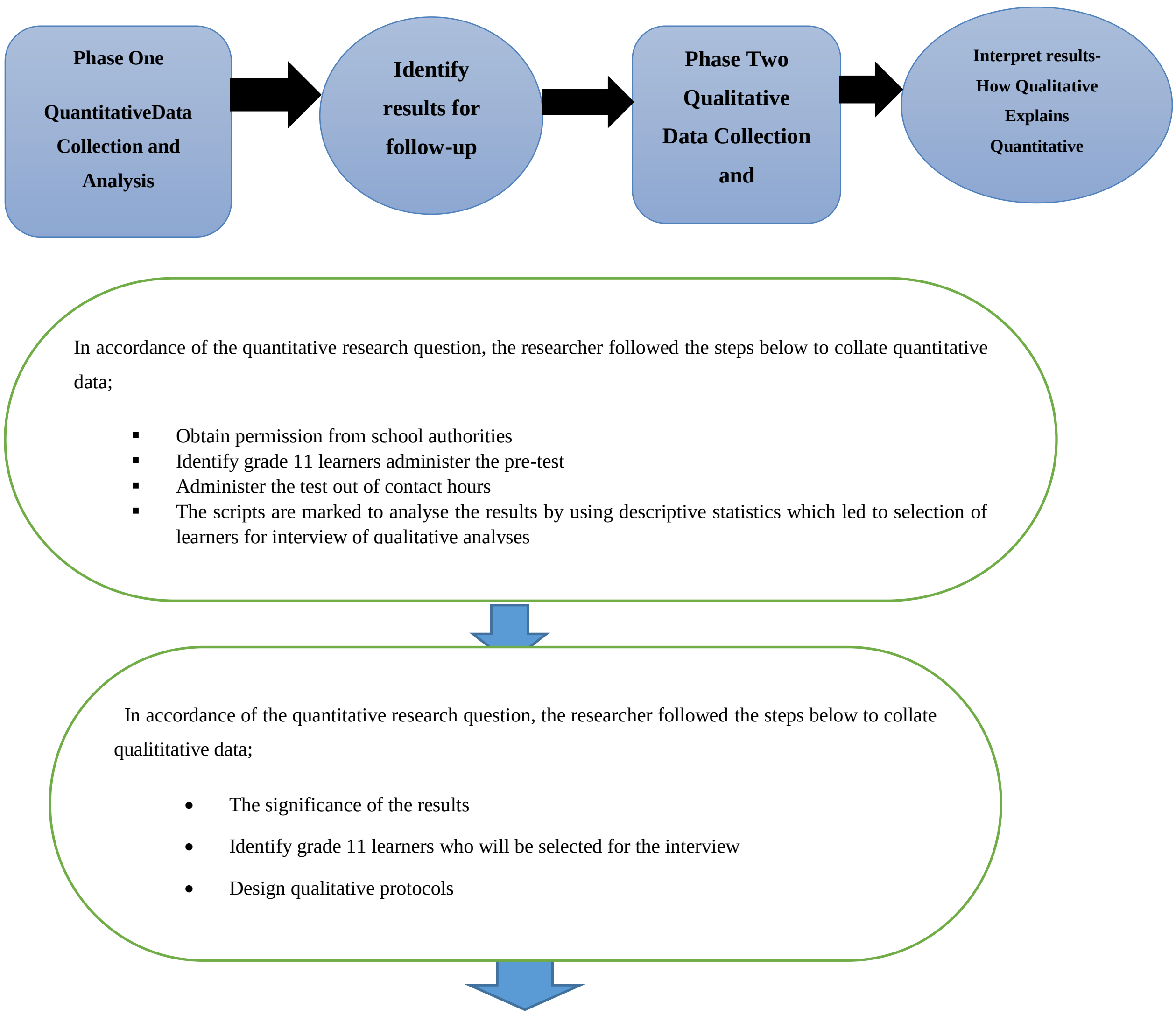
Mixed-methods research seeks to achieve a holistic understanding of the research problem by combining two different data collection methods, quantitative and qualitative, within a single study

(Jane, 2024). According to Johnson, Teddlie, and Tashakkori (2021), the mixed-methods approach reduces the biases inherent in single-data collection methods and yields more accurate inferences from research data. The explanatory sequential mixed-method design comprises two phases of data collection. ,. In this approach, an individual researcher with a strong background in quantitative research collects data and analyses it, and then uses qualitative data to further explain the results of the quantitative data (Creswell & Creswell, 2018).

The researcher employed a mixed-methods approach to understand Grade 11 learners, examining their performance, learner difficulties in the form of challenges in answering questions, and difficulties encountered when learning Analytical Geometry. The core assumption of this form of inquiry is that integrating qualitative and quantitative data provides a more comprehensive understanding of the data than either approach alone (Leavy, 2022).

The researcher's priority was using a systematic explanatory sequential mixed-method design method to collate data (quantitative, then qualitative; QUANT and qual) where pre-test and post-test are administered. Thus, the quantitative data are collated and analysed, after which the qualitative data are used to explain the data outcome (Creswell & Creswell, 2018). The data collected was based on the pre-test and post-test responses written by learners. Later, the data was analysed statistically, and then, based on results, questions were asked along the Cartesian plane of the five levels of Van Hiele's study of Analytical Geometry. The researcher then marked the pre-test and analysed the test to understand the challenges learners' face that hinder their performance in Analytical Geometry. However, before the pre-test, the researcher piloted the test instrument with 30 selected learners from different schools to ascertain the validity of the questions. After the scripts were marked, the researcher developed a semi-structured interview with nine (9) learners to uncover the challenges learners portrayed while answering the questions. The researcher conducted a classroom observation to understand how learners learn problems on lines and angles on the Cartesian plane in Analytical Geometry. According to Creswell & Creswell (2018), qualitative data help explain the mechanisms underlying the quantitative results in greater depth. After the quantitative and qualitative data were collected separately, the researcher merged them and subsequently used inferential statistics to better explain the quantitative results (Creswell & Plano Clark, 2018). The researcher also conducted classroom observations to determine how learners grasp concepts of lines in the Cartesian plane after the pre-test and after the post-test. The researcher weighed all the values obtained in the quantitative result to establish the validity of the design approach. Hence, in this manner, the researcher was able to collate a successful dataset for the research to reach a successful conclusion. Below is the chart that guided the researcher to follow for the data of the study

Fig. 4.2 ***Flowchart of the Basic Procedures in Implementing an explanatory Mixed Methods Design*** **(**Adapted from Creswell & Plano Clark,2018)



- state qualitative research questions which are in line with the quantitative results
- obtain permission from school authorities
- purposefully select qualitative samples that can help explain quantitative results
- qualitative results are analyzed to answer qualitative research questions

↓

- Summarize and interpret both quantitative and qualitative results
- Discussion is done to what extent qualitative results explain quantitative results

The researcher followed the above chart of explanatory sequential mixed-method design to collate and interpret data successfully.

4.5 POPULATION AND SAMPLING TECHNIQUES

The schools were purposefully selected based on their poor performance. According to McMillan and Schumacher (2020), purposeful sampling involves selecting a representative or informative element from a population that represents the topic of interest. For these reasons, the researcher decided to select the schools purposefully based on their poor performance in mathematics, hence, the systematic random sampling technique was used.

Approximately 80 learners from two groups were selected from 105 learners from the two schools for this study to participate in the exercise at the Thulamahashe circuit in Bohlabela District, Mpumalanga Province. The 80 participants represented the population and satisfied the criteria for which the research was intended since it is impossible to work with a huge population, and were within the researcher's control. The researcher employed purposive sampling, whereby a fourth learner potential participant was selected from each row in the class in each school (Lim, 2024), until 40 learners from each of the two schools were reached to administer the pre-test and post-test. These activities of selecting learners and administering of the pre-test and post-test were done after the schools hours. The researcher used purposeful sampling for this study, in the sense that the characteristics of the sample are homogeneous, i.e., the learners have a similar nature, and whereby every fourth learner on the row would be sampled to avoid being bias in the face of participants to participate in the pre-test and post-test that were administered. Thus, in this type of sampling, elements are selected at a regular subject interval, which also requires less time to complete and is less costly (Leavy, 2022). Grade 11 was chosen because the knowledge acquired in Analytical Geometry at this level is what applies in answering Grade 12 questions in NSC exams, as evident in all Analytical Geometry questions in NSC examinations (DBE, 2019-2024). The learners' pre-test papers were analysed to identify the challenges portrayed by learners. The researcher then randomly selected six (6) learners who obtained a mark below 30% to participate in the semi-structured interview. This was aimed at helping to discover the sources of these mistakes and consolidate the data from learners' scripts, and also helped the researcher answer the research question: "How can teachers use Geogebra to teach Analytical Geometry to change their practices to enhance learner performance?

4.6 INSTRUMENTATION

Instrumentation refers to any change that occurs in collecting and analysing data to measure the dependent variable (Dhlakama & Murairwa, 2024; Johnson & Christensen, 2020). The researcher used pre-tests and post-tests to measure the extent of learners' knowledge concerning lines in a Cartesian plane, which was set by the researcher and quality assured by experienced mathematics teachers. The pre-test and post-test instruments were subjected to face and content validity and were reviewed by two experienced mathematics teachers for proper scrutiny. Interviews and tests of a method, as guided by pilot directions, add more dependability and credibility to the research work (Malmqvist et al., 2019). Before administering the instrument, it was piloted at a school that did not participate in the primary research. Piloting the test instrument allowed the researcher to test the test items, take time to complete questions, and identify any weaknesses in the study before administering it to the participants. Therefore, the researcher deemed it necessary to ascertain the current level of learners' learning and identify the reasons why learners consistently perform poorly in Analytical Geometry, specifically in the areas of line formation in a Cartesian plane.

Under the qualitative approach, the researcher employed semi-structured interviews (see Appendix M) with learners who scored below 30% in Analytical Geometry. This method allowed for an in-depth exploration of how these learners organised their thought processes when engaging with the concept of lines on the Cartesian plane. The interviews were designed not only to uncover learners' cognitive strategies but also to examine their opinions, beliefs, and personal experiences that shaped their understanding (Cohen et al., 2018). By using open-ended questions, the researcher encouraged free expression, enabling participants to articulate their struggles in their own words. Follow-up probing questions were used where necessary to clarify responses and deepen understanding (Damyanov, 2023).

This interactive approach provided rich insights into the specific challenges learners faced, particularly in grasping fundamental principles of Analytical Geometry. The open-ended nature of the questions allowed the researcher to explore underlying reasoning patterns, misconceptions, and cognitive barriers that traditional assessments might not reveal. As a result, the study gathered credible and nuanced qualitative data, which McMillan and Schumacher (2020) define as data that accurately reflect participants' realities and are deemed trustworthy and valid.

To triangulate findings and enhance data reliability, the researcher also conducted classroom observations. This method was essential in identifying why and how learners made mistakes when working with lines in Analytical Geometry (Clements, 2023). Observations took place in natural classroom settings (Dean, 2019), allowing the researcher to witness first-hand the instructional dynamics and learner interactions before and after the pre-test. This approach revealed behavioural and instructional factors that interviews alone could not capture, such as learners' engagement levels, problem-solving approaches, and teachers' pedagogical strategies (Cohen et al., 2018).

Additionally, the researcher observed teachers’ instructional methods, particularly their use of GeoGebra as an intervention tool in teaching Analytical Geometry. The observations focused on how teachers integrated GeoGebra into their lessons, how learners responded to the technology-enhanced instruction, and whether the intervention improved conceptual understanding and performance.

This dual approach, combining interviews with classroom teachers provided a comprehensive understanding of the factors contributing to poor performance in Analytical Geometry. The findings not only highlighted learners' conceptual difficulties but also shed light on teaching strategies that may need refinement to enhance comprehension (see Appendix N). By merging these qualitative methods, the study ensured data richness, credibility, and practical relevance in addressing the challenges faced in mathematics education.

4.7 DATA COLLECTION PROCEDURE

Data collection is a gradual process of collecting measurements or observations, whether for academic purposes, business, or government. The data collection enables a researcher to gain first hand information or insight into a study (Bhandari, 2020). The researcher, with the permission and support of the mathematics teachers (see Appendix D), administered the pre-test (see Appendix L) instrument to the learners who constituted the respondents. Data for the pre-test was collected to understand the Grade 11 learners' performance in Analytical Geometry of lines on the Cartesian plane. Following the pre-test, a semi-structured interview was conducted to identify the root cause of the challenges encountered during the pre-test. The researcher also did two classroom observations to familiarise himself with how analytical geometry is taught and learned. The classroom observation was conducted to assess how the teachers of Analytical Geometry influence learners' understanding of the topic, which is contributing to their failure. Thus, the observation was conducted after the pre-test and after the post-test. The post-test was finally administered to learners in the two groups at the same time to determine changes in learners' performance after the integration of GeoGebra intervention to align with the study design. The researcher solicited the help of the control group teacher to conduct the post-test whilst at the same time, the researcher also conducted the test in experimental group concurrently.

4.7.1 Quantitative Data Collection

Quantitative data is where target data are collected and analysed with regard to numbers (McMillan & Schumacher 2020). Before the pre-test was administered to the participating school, the researcher first piloted the questions with a school that is not participating in the research exercise. According to Ahmad (2020), piloting helps you gauge the clarity of questions and their suitability. For example, questions are not offensive and help you estimate the time required for a response, which is necessary to inform participants before initiating your interview or data gathering. The Analytical Geometry pre-test was followed by a content analysis of learners' written tests to extract data from their scripts. The test also revealed the challenges learners faced in exposing how their learning gap in Analytical Geometry needs to be addressed. After implementing the intervention, the researcher also conducted a post-test to collate data for analysis of learners' performance in Analytical Geometry. The intervention has revealed the efficacy of Van Hiele's theories, as well as the effectiveness of GeoGebra's implementation, as evidenced by the lower values of co-efficient in the post-test compared to the higher co-efficient values in the pre-test (Kuehn, 2022). Additionally, how learners respond to Analytical Geometry in relation to lines and angles in a Cartesian plane, and hence, the teaching methods that offer opportunities to learners in Analytical Geometry classrooms.

4.7.2 Qualitative Data Collection

Qualitative data collection is an in-depth study that utilises face-to-face interviews or observational techniques to gather data from individuals in their natural settings (McMillan & Schumacher, 2020). A semi-structured interview was conducted with eight learners (four from each of the two schools), consisting of content-based questions that required respondents to answer questions about specific problems. These problems include the possible factors related to the different types of mistakes learners committed when solving questions on lines and angles on a Cartesian plane of Analytical Geometry. Secondly, the researcher developed open-ended interview questions to determine why learners make mistakes when solving questions on lines and angles on a Cartesian plane in Analytical Geometry. These question is in response to research question.

- How does the use of GeoGebra as an intervention influence learner performance in Analytical Geometry?

The researcher also conducted classroom observations to gather first-hand information and ascertain how learners learn the concepts of lines and angles on a Cartesian plane.

4.7.3 Semi-structured interviews

As indicated earlier, the researcher aimed to determine how learners perceived the pre-test they wrote and how they struggled to perform well in some interview questions. The semi-structured interview is conducted to gather details about learners' challenges (Upadhyay & Lipkovich, 2020). In this case, the Analytical Geometry of lines on a Cartesian plane is used to understand the learner's problem. The interviews conducted with four learners from each school were conducted twice, once after the pre-test and once after the post-test to understand learners' level of geometric concept. The researcher's interview questions were open-ended and detailed, designed to elicit more information from Grade 11 Analytical Geometry learners (Sanders, 2024). The researcher took field notes with the participant's permission after audio recording, as the audio copy could be deleted or lost (Creswell & Creswell, 2018). The researcher sequentially arranged the interview questions, ensuring flexibility and smooth running of the interview process (Tribe, 2020).

4.7.4 Classroom observations

Observation serves as another way of getting primary data. Observing is an intentional way of watching and listening systematically and collecting data (Kumar, 2019). According to Ekka (2021), observation helps to study the personality trait and the function performed by a learner, eliciting information that cannot be obtained by questioning the individual, as the individual may feel uncomfortable answering certain questions during the interview. The researcher observed learners after the pre-test and after the post-test and also took notes in the form of field note was taken in the classroom on how Analytical Geometry is taught to Grade 11 learners, which contributed to learners' poor performance (Creswell & Creswell, 2018). The researcher, before the observation period, had what specific demand he was going to observe based on the research questions. According to McMillan & Schumacher (2020), the reason for specificity must be measurable to define the behaviour of the unit that can be analysed for objective recording. Given this, the researcher's focus was on how teachers can utilise GeoGebra to teach the Analytical Geometry of lines on the Cartesian plane, aiming to improve student performance.

The researcher also pays close attention to the behavioural changes on the part of the participants. In the sense that participants may change their behaviour if they realise that an observer is observing them (Kumar, 2019). The potential change in behaviour of participants prompted the researcher to observe them both outside the classroom (i.e., on school premises) and inside the classroom during lessons.

The researcher also anticipated some problems that could be triggered during the observational process. Some of the problems, according to Ekka (2021), could be:

- There could be possible bias on the observer's part, making it difficult to draw inferences from participants and observational verification.
- The risk of missing recording may occur. This could happen while paying attention to participants and recording or transcribing the recording afterwards.
- There could also be different interpretations from different observers; therefore, it is essential to consider the specific needs of the observation.

In a classroom setting, the researcher is aware of the anxiety that teachers and learners may experience and may not trust the researcher (Johnson & Christensen, 2020). To address this, the researcher assures everyone that the observation is for research purposes only and is not intended to be intimidating. To answer the research question, "How can teachers use GeoGebra to teach Analytical Geometry of lines on Cartesian plane to improve performance?, It is imperative to be critical about how learners receive and react during lesson delivery, as well as how the teacher conveys the concept of Analytical Geometry of lines on a Cartesian plane to learners. A unit must be observed (Kumar, 2019; Johnson & Christensen, 2020) to avoid confusion with the numerous behaviours that a participant may exhibit during the observation period.

4.8 DATA ANALYSIS AND INTERPRETATION

Data analysis is a process that relies on methods and techniques to take raw data, extract relevant insights, and identify information to transform facts and figures into actionable initiatives for improvement (Durcevic, 2020).

4.8.1 Quantitative Data Analysis

From the learner's pre-test and post-test they wrote, quantitative data was analysed to give a general understanding of the extent of knowledge of lines of learners in the Cartesian plane of the research problem (learners 'test scripts). The researcher used the analysis of the variables indicated and the coefficients of the variables to analyse the results of both groups (Creswell & Creswell, 2018). The researcher also used a **single-sample z-test** to explain the outcome of the confidence intervals for data analysis (see Appendix P). Large numbers of observations were reduced, summarised and organised by scientific statistical analysis (McMillan & Schumacher, 2020). Thus, the pre- and post-test results were tabulated to provide a concise understanding of learners' performance. Inferential statistics depend on descriptive statistics and are also used to make predictions about the population from which the sample is drawn, to analyse and provide further explanations of the outcomes that emerge from the quantitative data (Johnson & Christensen, 2020). The researcher used the Statistical Package for the Social Sciences (SPSS) of Stata Release 17 to generalise the population. A broader population is better generalised by SPSS (MMU 2020).

4.8.2 Qualitative Data Analysis

Content analysis of the participants' views was employed in this research work, where the researcher framed the interview questions in a manner that was friendly to the participants. In this way, the researcher understood how learners answered the pre-test on lines and angles on a Cartesian plane (Creswell & Creswell 2018). According to Bhatia (2018), qualitative data also involves analysing documented information in the form of texts, media or physical items, and it is also used to analyse responses from the interviewee. While answering the pre-test questions, the researcher analysed the responses on committing challenges on lines on the Cartesian plane.

The qualitative data analysis was conducted concurrently with data collection, involving interviews, observations, recordings, and transcriptions of recorded information to safeguard the data (Johnson & Christensen, 2020). In qualitative data analysis, the researcher made meaning of participants' issues, took notes of trends, categorised notes, regularised notes and formed themes (Islam, 2024; Creswell & Creswell, 2018). According to Creswell and Creswell (2018), qualitative data views are interactive and not always followed in a sequential manner.

The figure below illustrates how Creswell and Creswell (2018) arranged the data from the lower level to the upper level, which the researcher has adopted. The researcher analysed the qualitative data based on the pre-test and post-test results from 80 learners in two schools, as well as classroom observations and semi-structured interviews.

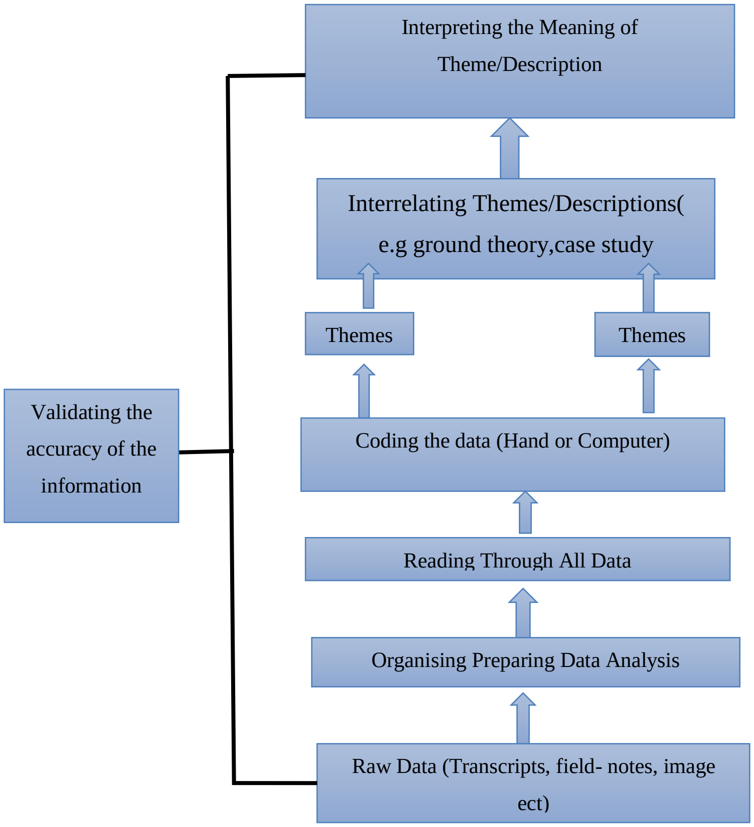


Figure 4.1: ***Data analysis in qualitative research*** (Creswell 2014:317)

Fig.4.3 The researcher, in line with the overview of the processes of qualitative data analysis by Creswell and Creswell (2018), followed these to analyse data:

- Collating and preparing the data for analysis involves recording and writing notes during observation and interview sessions, as well as gathering various information from interviewees.
- The researcher reads all the gathered information to ascertain the meaning of the various segments. In addition, the researcher read the participants' information to discern specific from general meaning and reflected on what is accurate, credible, and reliable to produce sensible data.
- From the above, the researcher repeated the scanning process for information and then drew themes, grouping them into categories of the same kind.
- Coding of data: This involves organising the data, which entails writing texts to describe categories and labelling them for easy analysis.

- Re-arranging the themes to incorporate new information, the researcher created a new arrangement to categorise themes into appropriate coding groups and select the most suitable wording for categorising the themes.
- The researcher abbreviated and numbered the categorised information.
- The researcher grouped categories, analysed data, and recorded appropriate data information.

Creswell and Creswell (2018) also advocate for special codes for development for analysing textual transcripts. These codes could include:

- Codes to be expected. Readers are expected to see these codes on the type of transcript information.
- Unexpected codes: These codes are not expected to be present in the information prior to the study.
- Unusual codes or of conceptual interest: These are codes that readers are interested in and codes of the concepts of Analytical Geometry of line and angle on the Cartesian plane.

The researcher for the study sought advice from the supervisor based on prior codes and emerging ones. The coding was there also to hide `participants' identities. Generating minor themes or categories for a research study is known as coding (Creswell & Creswell, 2018).

4.9 METHODOLOGICAL VALUES

In this section, the researcher discusses the reliability, credibility, and trustworthiness of the research work.

4.9.1 Validity

According to Smith and Noble (2024), validity refers to the integrity and application of the methods undertaken and the precision with which the findings accurately reflect the data. The researcher followed and weighed all the values obtained in the quantitative result to establish the validity of the design approach. The researcher also took enough time to interact with the participants in their natural environment to gain more experience and ascertain valid findings. According to Creswell and Creswell (2018), spending more time in the research field enables the researcher to develop an in-depth understanding. As a researcher, the more time spent in a participant's setting, the more valid and accurate the findings. To ensure validity, the researcher ensured that the interviewees discussed what they gained from the pre-test they had written. Thus, the empirical is in constant agreement with the theoretical. Furthermore, all other investigation aspects were assigned to learners' expression in analysing and interpreting data by Van Hiele's levels. Johnson and Christensen (2020) confirmed that giving appropriate inferences, interpretations and intentions based on test scores is termed validity. This study employed a combination of quantitative and qualitative research techniques to ensure the validity of all information.

4.9.2 Reliability

Reliability refers to the consistency of the analytical procedures employed (Smith & Noble, 2024). The researcher reviewed the transcripts to ensure they did not contain any obvious transcription errors. The pre-test used was the Analytical Geometry Grade 11 Paper 2 provincial paper, which produced consistent results. Before administering the test, the researcher piloted the test instrument to ensure that the test items were consistent with the poor performing schools. Piloting helps researchers gauge the clarity of questions, their suitability, for example, ensuring that questions are not offensive or biased. Also, it helps estimate the time required for response, which is necessary to inform participants before initiating the interview or data collection (Bhalla et al., 2023). The pre-test and interview questionnaires were verified by independent colleagues who are mathematics experts to ascertain the level at which they are intended. Their constructive suggestions and inputs allowed me to add, subtract or reframe some of the test instruments. The researcher ensured that codes were meaningful, memos were written to accompany them, and the data were continuously compared with the codes (Creswell & Creswell, 2018). The tests are instruments for the national examinations from the CAPS syllabus of the DBE for Grade 11 learners and also meet the examination guidelines as stated in the mathematics policy document, which has produced consistent results and covers all Van Hiele's levels of Analytical Geometry Concepts.

4.9.3 Validity and Reliability of the Research Instrument

The validity of a test instrument is the degree to which items in an instrument reflect the content universe to which the instrument was generalised or the extent to which a measure is related to an outcome (Smith and Noble, 2024). The researcher made sure experts in mathematics evaluated the test item to include all the concepts of lines and angles that measure the levels of Van Hiele's theories and remove undesirable items from the instrument. The researcher ensured that the test instrument accurately measured the content and predicted the criterion by adhering to concepts of lines and angles, thereby answering the research questions. The researcher also ensured the use of test items whose results were consistent with the concepts of lines on the Cartesian plane. They confirmed their scores were stable by piloting them before administering them to the participants (Bhalla et al, 2023; Creswell, 2018).

4.9.4 Dependability

The researcher ensured that an external audit was conducted on the test instruments and the collected data. The researcher ensures the stability of the findings over time and involves participants' evaluation of the findings, as well as their interpretation and recommendations for the study, such that all are supported by the data collected from the participants (Korstjens & Moser, 2018). To support the above assertion, Haq et al. (2023) define dependability as the stability of data over time and conditions, as well as an evaluation of the quality of the integrated processes of data gathering, data analysis, and theory generation.

4.9.5 Credibility and Trustworthiness

"Credibility is the extent to which the results approximate reality and are judged to be accurate, trustworthy, and reasonable" (McMillan & Schumacher 2020). The credibility of the results is approximated, reasonable and objective, which was judged to be accurate, trustworthy, and reasonable. The trustworthiness of data refers to the degree of confidence in the data, interpretation, and methods used to ensure the quality of a study (Adler, 2022). The quantitative and qualitative (QUAN → qual) data were credible because the test came from a provincial Grade 11 question paper used for formal examinations from the CAPS syllabus. The researcher's prolonged engagement in the field or research site with learners, combined with the use of peer triangulation, negative case analysis, member checks, and persistent observations, also fostered the credibility of the study, making it trustworthy (Dominic et al., 2023). Examination guidelines, which specify the percentage of knowledge acquisition in mathematics, were also met, as stated in the South Africa policy document of mathematics, which made it more credible. Challenges exhibited by participants were identified using Van Hiele's levels of Analytical Geometry, as outlined in the scripts written by the participants. Classroom experiences and observations by the researcher also helped to analyse learners' test scripts to ascertain the credibility of the research work.

4.10 ETHICAL CONSIDERATIONS

The International Consortium for Research in Science & Mathematics Education (ICRSME 2020) refers to ethical reasoning as the cognitive activity of determining right and wrong when facing a dilemma that affects other living beings. Therefore, the researcher personally sought permission from the university by first applying for an Ethical Clearance Certificate (see Appendix A), the Department of Education (see Appendix B), and the management of the selected groups before administering any tests to the learners. A letter of information and informed consent from the researcher was handed to the school's Head of Department (HOD), which both parties signed to authenticate the research procedure (see Appendix G). The consent and cooperation of the learners' mathematics teachers, the learners who participated in the study (see Appendix H), and the parents/guardians of the learners were obtained before conducting the study. The researcher asked the learners to complete an informed consent form to help document their willingness to participate in the study and address any ethical issues related to their participation. Learners and teachers were assured that the confidentiality of any information the researcher collected would not be used against them in any way whatsoever. Christensen and Johnson (2020) affirmed that protecting the confidentiality of participants and the data is an ethical requirement of the researcher. The researcher also assured the learners and their teachers that participation in the research work is voluntary, and any participant can withdraw from the study at any time if they feel like doing so. This is confirmed by Creswell and Creswell (2018) when they said, “Avoid disclosing information that would harm participants”. Learners also need to be informed of the possibilities of non-confidentiality and the repercussions for the data, not just for themselves. The researcher and the teachers signed the consent form, and the learners and anyone involved sealed the agreement.

CHAPTER SUMMARY

This chapter presented the methodology that suits the study and drives the study till its’ end. It also espoused the right paradigm which gave right worldviews of the research. The right research approach that suits the study was introduced to drive the study to a success. The rightful design, which is explanatory sequential mixed-method design was used to cater for the quantitative and qualitative which were used in the study. Population and sampling techniques were all involve in this chapter. Data collection, which includes semi-structured interviews, classroom observation were all discussed under this chapter.

The data was analysis and its' interpretation were all discussed extensively under this chapter to give true meaning to the study. Methodological values like, validity, reliability, validity and reliability, dependability, credibility and trustworthiness were all discussed to give true value and meaning to the study. Ethical consideration was also discussed in this chapter in order not to violate the ethical rules in a research work.

CHAPTER 5: DATA ANALYSIS

5.1 INTRODUCTION

In the previous chapter, the researcher discussed the methodology of this study, which employed a mixed-methods approach. The qualitative and quantitative results of the pre-test and post-test administered to eighty (80) Grade 11 learners in the Bohlabela District of the Mpumalanga Province were reported. The researcher conducted interviews with six (6) learners and eight (8) lesson observations, four (4) in each school. This chapter discusses the sub-divisional segments, including the pre-test and post-test, interviews with learners (qualitative data), classroom observations, and the statistical analysis of the data (quantitative data). The study, in its inception, noted to answer the following research questions:

- RQ1. How do teachers teach Analytical Geometry of lines on the Cartesian plane using GeoGebra?
- RQ2. What are the benefits of using GeoGebra to teach Analytical Geometry of lines on the Cartesian plane?
- RQ3. What are the challenges experienced when using GeoGebra to teach Analytical Geometry?
- RQ4. How can teachers use GeoGebra to teach Analytical Geometry of lines on the Cartesian plane to improve performance?

This study further tested the above hypotheses. The researcher adopted an intervention mixed-method design. The pre- and post-tests were administered to determine learners' performance. The researcher first administered a pre-test with 80 Grade 11 learners to determine their academic level in geometric properties and relationships between points, lines and angles on the Cartesian plane in Analytical Geometry. The researcher again used the SPSS tool to analyse the results, ascertain learners' levels, utilise the interview data, and make baseline lesson observations to understand learners' challenges with Grade 11 Analytical Geometry on the Cartesian plane. After analysing the quantitative data, qualitative data were collected through semi-structured interviews and classroom observations to better explain the quantitative data outcome using content analysis by interventional mixed-method design.

Prior to embarking on the main study, the researcher piloted the test items with 30 learners to ensure validity and correct any associated errors that could compromise the value of the test items. Piloting helps researchers get an idea of the clarity and suitability of the questions, for example, whether they are offensive or not. It also helps to estimate the time of response, which is necessary to inform participants about before data collection (Durcevic, 2020).

5.2 SUMMARY OF THE INDICATORS/DESCRIPTORS FOR ANALYISING DATA FROM THE THEORETICAL FRAMEWORK

Table 5.1 below presents a comprehensive conceptual and theoretical framework, which was discussed in Chapter 2 of this study. Chapter two discussed the theoretical framework, looking at Van Hiele's (1958) theory and Vygotsky's (1978) ZPD and scaffolding. The constructs were also divided into the discussion of quantitative and qualitative data (pre-test and post-test, lesson observations, and unstructured interviews).

Table 5.1: Summary of constructs from the theoretical framework

Theory	Construct	Definition	Descriptors
Van Hiele's	**Level 1 (Visualisation):** visualisation is related to appearance of the object as a whole.	Visualisation is nonverbal thinking where Shapes are judged and seen by the way they appear and generally viewed as they are and not necessary thinking of the construct or property that made the appearance or the shapes.	- Leading learners to identify the shapes of diagrams without considering their properties.
	Level 2 (Analysis): At level 2, leaners identify the trait of the object, shape and figure which they are made up of.	Learners identify the properties of figures or objects by their parts or builds but no yet established the relationship between	-Learners identify the names and trait of objects like parallelogram, rectangles and related objects

		objects in geometry and other objects outside geometry	without observing the mutual relationships between their variables
	Level 3 (Abstraction): learners are expected to think deductively on lines and angles on the lines form in Cartesian plane.	This is where learners are able to identify the properties or construct of objects and what they are made up of.	-Learners are made to establish the relationships of properties both within figures where classes of figures are recognised -Lead learners to the meaningful definitions of object, arguments of the informal are to follow
	Level 4 (Formal Deduction): At this level, the suitability of deduction as a way of building Analytical Geometry in axiomatic systems, definitions, theorems, consequences and postulates are understood.	This is where the significance of deducing geometric terms in the context of axiomatic terms is being understood and the possibility of defining a proof in more than once is realised though the convers of axioms, theorems. However, definitions	-Learners must be able to tell about the properties of the equation of a line. That “if the two lines are parallel to each other”, then the lines will have the same gradient -Learners must be able to think reversely, that

		have not yet been understood.	when two lines have the same gradient, it means they are parallel to each other
	Level 5 (Rigor): Learner has reversible thinking ability and understand the formal aspects of deduction at this stage.	This is the level of the ability of a person to have a reversible thinking where non-Euclidean and mostly abstract oriented geometry can be studied.	-At rigor level, learners are able to go through from level 1 to level 4. -The rigor level are not meant to be studied at secondary school level but at the colleges and university
Lev Vygotsky's sociocultural theory	**ZPD:** A set of skills or knowledge a learner cannot do by himself/herself but can do with the help or guidance of someone else.	The distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers	Lead learners to determine (RPK): -Ask learners appropriate questions -Answering learners' questions -Giving learners the chance to ask their questions.

	Scaffolding: A teaching strategy which helps learners to study by working with a teacher or a more capable peer in order to master a skill of their goals and perform the task independently.	The role of teachers and others in supporting the learners' development and providing support structures to get to that next stage or level by himself	To lead learners to: -Use visual aid to understand concepts - Use group learning exercises where more capable peers help the less competent to acquire the skills.

5.3 ANALYSIS OF DATA FROM THE PILOT STUDY

In this study, the researcher piloted the pre-test and post-test to assess the efficiency and effectiveness of the instruments (see Section 4.5). In piloting the tests, the researcher used ten previous DBE (2017) questions with 30 learners from a different school, which was not part of the study. Initially, it was estimated that the learners would take 50 minutes to complete the test. However, no single learner was able to complete the questions within the time given. It, therefore, meant that the time allocated for the test was inadequate for Grade 11 learners to finish and obtain a passing mark. According to Bhalla et al. (2023), piloting helps researchers gauge the clarity and suitability of the questions. Additionally, it helps estimate the time required for a response, which is necessary to inform participants before data collection begins. The researcher then adjusted the time allocation to one and a half hours to ensure that any Grade 11 learner had enough time.

The researcher also noticed numerous questions that were repeated and assessed the same concepts. Hence, the researcher trimmed those questions to equal assessable concepts. The researcher also realised that being unable to finish writing the test within the stipulated time could also mean that the questions were too many for the allocated time. Hence, the number of questions was also minimised. Bhalla et al. (2023) assert that piloting helps determine the usefulness and effectiveness of an instrument, providing more reliable feedback. The (ibid) also states that accurate time range and correct wording of the task items are some of the advantages of piloting. Additionally, the pilot provided the researcher with the opportunity to work and properly administer the main research pre-test and post-test. Below is Table 5.1, which shows the piloted results.

Van Hiele's cognitive levels are represented in the first column, the description of the questions is shown in the second, the third column is arranged in accordance with the hierarchy, and the last column represents the percentage of achievements by learners. Constructivists view knowledge formation as an active process that creates cognitive structures through interactions with the environment (Amineh & Davatgari, 2015).

Table 5.2: ***Learners who achieved Cognitive levels of pilot test items***

Van Hiele's Cognitive Level	Description	Test question	Number of learners achieved
1	At this stage, learners are only required to calculate the distance between two points and also find the gradient of two points in Cartesian plane.	1and 2	15 (75%)
2	Learners at basic routine calculate for the equation of the straight line.	3 and 4	3(15%)
3	Learners at this stage are required to find the equation of a straight line in relation (parallel, perpendicular) with another line whether being quadrilaterals or triangles.	5	2 (10%)
Van Hiele's Cognitive Level	**Description**	**Test question**	**Number of learners achieved**
4	Learners are required at this stage to use properties of quadrilateral to answer questions for equations of straight lines. Gradient of lines midpoint rules to answer questions. Thus, parallel, perpendicular and diagonal lines to answer questions.	6	0(0%)

N=30

5.4 QUANTITATIVE ANALYSIS OF THE DATA

The section discusses and explains how the researcher determined learners' performance. The researcher categorised and used pre-tests and post-tests as the primary instruments to assess learners' responses to the test items and code those responses (Creswell & Creswell, 2018). The researcher also used percentages, codes and absolute numbers to view learners' performance. The researchers' main aim in doing this was to address the main research questions of the study:

- What are the benefits of using Geogebra to teach Analytical Geometry (if any)?
- What are the challenges experienced when using Geogebra to teach Analytical Geometry (if any)?

5.4.1 Descriptions of test items

The table below presents a description of the test items assessing skills and knowledge, as well as a detailed description of the process each learner follows while answering questions. The descriptions of the process are as follows:

- Attempt to answer (ATA)
- Never attempted (NA)
- Answered correctly (AC)
- Did not answer correctly (DAC)

5.5 ANALYSIS OF PRE-TEST RESULTS

The quantitative analysis of the pre-test results encompasses the key skills and knowledge areas related to the concept of Analytical Geometry of lines on the Cartesian plane that learners possessed at the Grade 11 level. It also revealed the conceptual performance of each test item. Therefore, the tables display the outcome of the pre-test for each answer on the test item that learners in both groups provided in their attempt to answer questions 1 to 5 (see Appendix H). The researcher followed Levene's (2021) data categorisation of test items to describe learners' responses to pre-tests. The researcher used the pre-test to measure the performance of learners and identify mistakes they committed before an appropriate intervention was implemented to mitigate their effects.

5.5.1 Learners' responses to Q1a and Q1b

Table 5.1 shows analyses of Q1a and b results. Question 1 was used to assess the concept of the gradient of two points on the Cartesian plane. Q1b was also used to assess the application of the gradient of two points by going through the process of finding the gradient of the two points before concluding on the Cartesian plane. For example, for Q1b, learners were to determine whether two lines were parallel or perpendicular to each other. These questions (Q1a, b) are grouped because they are related in a way that learners are tested on the concept of gradients in different situations.

Table 5.3: Distribution of learners' responses to Q1a and Qb

QUESTIONS	**GROUPS**	**PRE-TEST**							
		ATA		**NA**		**AC**		**DAC**	
1a	Experimental group	40	100%	0.0	0%	10	25%	30	75%
	Control group	40	100%	0.0	0%	14	35%	26	65%
Q1b	Experimental group	15	37,5%	25	62.5%	5	33.33%	10	66.67%
	Control group	32	80%	8	20%	15	46.88%	17	53.12%

Table 5.3 above indicates that the experimental group and control group in the ATA category attempted to answer the question at a 100% rate. The gradient of two points was observed in only 10 out of 40 learners in the experimental group and 14 out of 40 in the control group under AC. The results showed a very low positive outcome for both groups, indicating that most learners had difficulty answering questions related to the gradient of a line. A lack of conceptual understanding leads to learners not performing positively in Analytical Geometry (Seloraji 2017). It is important, therefore, to say that learners from both groups struggled to answer questions on the analytical geometry of lines on the Cartesian plane. As indicated by DBE's (2019) diagnostic report, learners are unable to solve questions on gradients successfully. Some learners were unable to quote the gradient formula, and some could not substitute it correctly into the formula, even when the formula was written correctly. Learners demonstrated a lack of skill in performing calculations accurately.

5.5.2 Learners' responses to Q2a, Q2b and Q2c

Question 2a was urging learners to find the length of two points. Question 2b is about how learners could find the value of one of the co-ordinates of the axes whilst the length is given. Question 2c urged learners to find the midpoint of the figure before calculating the length (see Appendix H). Most learners were not able to answer the questions. The three questions are grouped because they share a similar concept, requiring learners to calculate the length between two points on the Cartesian plane. Learners could solve the problem if they could calculate the midpoint of the diagonal lines. Learners lacked the skills to calculate such questions, which led them to commit errors in answering the questions. According to Luneta (2020), "Students have difficulties in understanding the instructional strategies adopted by the teacher", leading learners to commit mistakes when working on some questions.

Table 5.4: Distribution of learners' responses to Q2a, Q2b and Q2c

QUESTIONS	GROUPS	PRE-TEST							
		ATA		NA		AC		DAC	
Q2a	Experimental group	40	100%	0.0%	0.0%	6	15%	34	85%
	control group	40	100%	0.0%	0.0%	7	17.5%	33	82.5%
Q2b	Experimental group	40	100%	0.0%	0.0%	2	5%	38	95%
	Control group	40	100%	0.0%	0.0%	2	5%	38	95%
Q2c	Experimental group	40	100%	0.0%	0.0%	1	2.5%	39	97.5%
	Control group	40	100%	0.0%	0.0%	2	5%	38	95%

Table 5.4 above indicates that the experimental group and control group in the ATA category attempted to answer the question at a 100% rate. The length of two points was observed in only 6 out of 40 learners in the experimental group and 7 out of 40 in the control group under AC for Q2a. The percentage under DAC indicates a higher percentage answering the question, indicating lower performance in the concept.

The results showed a very low positive outcome for both groups, indicating that most learners had difficulty answering concepts related to the length of a line. A lack of conceptual understanding led to learners not performing positively in Analytical Geometry (Seloraji, 2017). It is important, therefore, to say that learners from both groups struggled to answer questions on the Analytical Geometry of lines on the Cartesian plane of lengths. Ozkan and Ozkan (2018) indicated that learners have difficulties with the length between two points when they are not able to provide correct information about the question. Quoting the distance formula correctly was a problem for some learners, whilst others could not correctly substitute the coordinates' values, creating a gap. The percentages obtained by learners from both schools were very low, and they were expected to answer with ease. Challenges often arise when a rule is applied incorrectly or when a learner over-generalises and under-generalises or presents an alternative conception of the situation (Hansen, Drews, Dudgeon, Lawton, & Surtees, 2017).

5.5.3 Learners' responses to Q3a and Q3b

Table 5.5 is an analysis of Q3a and Q3b results. Question 3 was used to assess learners' understanding of the equation of a straight line on the Cartesian plane. Q3b was also used to assess the application of the equation of a straight line by finding the gradient of the line, which is either two lines being parallel or perpendicular, to be calculated before the conclusion of the answer on the Cartesian plane (see Appendix H). For example, for Q3b, learners were to determine whether two lines were parallel or perpendicular to each other. These questions (Q3a and Q3b) are grouped because they are related in a way that learners are tested on the concept of gradients in different situations before the equation of the line is calculated.

QUESTIONS	**GROUPS**	**PRE-TEST**							
		ATA		**NA**		**AC**		**DAC**	
Q3a	Experimental group	30	75%	10	25%	6	20%	24	80%
	Control group	32	80%	8	20%	14	43.75%	18	56.25%

Q3b	Experimental group	22	55%	18	45%	6	27.27%	16	72.73%
	Control group	33	82.5%	8	20%	15	45.45%	18	54.55%

Table 5.5: Distribution of learners' responses to Q3a and Q3b

Table 5.5 above indicates that the experimental group in category ATA saw 75% of which 30 learners attempted to answer the question, whilst the control group had 32 learners, representing 80% who attempted the question.

The results show a very low positive outcome for both groups, indicating that most learners had difficulty answering questions about the equation of a line. It is essential, therefore, to note that learners from both schools struggled to answer questions on the Analytical Geometry of lines on the Cartesian plane, particularly in finding the equation of a straight line. As indicated by DBE's diagnostic reports, learners showed an inability to solve questions on straight-line equations. Some learners were unable to accurately quote the formula, and others struggled to substitute the variables into the formula. Ramdjid et al. (2022) state that the lack of understanding in learning geometry often causes discouragement among students, which invariably leads to their poor performance.

5.5.4 Learners' responses to Q4a and Q4b

Questions 4a and Q4b were about finding the coordinates of points on a figure on the Cartesian plane. Q4a involved finding the coordinates of a point on the intersection of two diagonal lines, which required calculating the midpoint of one of the lines. At the same time, Question 4b required learners to calculate another coordinate, but it was necessary to use the equation of the line before the coordinate could be found. The two questions are grouped because they share a similar concept, involving the calculation of points' coordinates, but require the application of other concepts prior to the calculation. If learners can calculate for other concepts, such as the gradients of two points, midpoints of lines, and the equation of straight lines, they will be able to tackle this problem.

Table 5.6: Distribution of learners' responses to Q4a and Q4b

QUESTIONS	GROUPS	PRE-TEST			
		ATA	**NA**	**AC**	**DAC**

Q4a	Experimental group	7	17,5%	33	82.5%	0	0.00%	7	100%
	Control group	13	32.5%	27	67.5%	1	7.69%	12	92.31%
Q4b	Experimental group	6	15%	34	85%	0	0.0%	6	100%
	Cntrol group	9	22.5%	31	77.5%	1	11.11%	8	88.89%

Table 5.6 above displays the results of Q4a and Q4b. The concepts required applications of other conceptual knowledge before a learner could calculate the values. Results from the data collected in Q4a above indicate that AC answered correctly, with only one learner from the control group answering the question correctly, which represents 7.69%, the lowest for that concept. A larger fraction of the learners from both groups did not attempt (NA). This indicates that a gap needs to be closed to mitigate the effects of poor performance. There was a huge gap in the pre-test for both groups, with the control group performing better than the experimental group. The number of learners who attempted the question from the experimental group was lower compared with the control group. Such responses suggested that learners had inadequate skills to identify what was needed to interpret and analyse the facts and mathematical ideas (Sharma & Verma 2017). These poor performances suggest that the actual level of development of most learners was lacking.

5.5.5 Learners' responses to Q5

Question 5 is designed to assess learners' understanding of determining the equation of a line when it is parallel to another. The question required learners to understand the properties of the given figure (a parallelogram). The pre-test results from the experimental group showed that 7.5% of learners, representing only 3 learners, attempted the question, whereas 20% of School 2, which comprises 8 learners, attempted to answer the question. Application questions were difficult for learners to understand. Based on the results from the table below, learners from both groups were unable to demonstrate and apply their understanding of major mathematical concepts, the applications of geometry concepts, and use them in the context of answering questions (NCTM, 2020b).

Table 5.7: Distribution of learners' responses to Q5

QUESTIONS	GROUPS	PRE-TEST							
		ATA		NA		AC		DAC	
Q5	Experimental group	3	7.5%	37	92,5%	0	0.0%	3	100%
	Control group	8	20%	32	80%	1	12.5%	7	87.5%

Table 5.7 above shows that 3 learners of ATA, 7.5% from the experimental group, and 8 learners of ATA, representing 20%, were able to attempt to answer Q5. Of these, 37 (92.5%) and 32 (80%) learners from the experimental group and control group, respectively, could not attempt the question. The higher percentage of (NA) gives a course of concern. One should ask how such a gap could be closed to pave the way for exemplary performance in the Analytical Geometry of lines in a Cartesian plane. This also suggests a lack of logical conceptual knowledge among learners, preventing them from connecting new ideas with what they already know (Math Assessment Resource Service, 2017). Kilpatrick et al. (2001) propose that once learners completely comprehend concepts and procedures, such as calculating the equation of a line, they can apply these ideas and processes to dissimilar concepts.

5.6 LEARNER RESPONSES TO QUESTION 1-5 ITEMS

Table 5.2 represents the performance of both groups on various cognitive concepts for each test item in the pre-test. One can notice that the highest scores by learners are on questions 1a and 1b, which require the concept of the gradient of two points. The lowest score on the pre-test is 0% for question 5, indicating that no one was able to answer it.

Table 5.8: Distribution of learners' percentages and absolute numbers of Attempts to answer (AT), Never attempted (NA), Answered correctly (AC) and Did not answer correctly (DAC) on pre-test for both groups.

Table 5.8: ***Description of pre-test scores according to knowledge assessed***

ITEM NO.	**PARTICIPANTS**	**PRE-TEST**			
Question	**GROUPS**	**ATA**	**NA**	**AC**	**DAC**
Q1a	Experimental group & control group	80 (100%)	0 (0.0%)	24 (30%)	56 (70%)
Q1b	Experimental group & control group	47(58.75%)	33(41.25)	20 (42.55%)	27 (57.45%)
Q2a	Experimental group & control group	80 (100%)	0 (0.0%)	13(16.25%)	67 (83.75%)
Q2b	Experimental group & control group	80 (100%)	0 (0.0%)	4 (5%)	76 (95%)
Q2c	Experimental group & control group	80 (100%)	0 (0.0%)	3 (3.75%)	77 (96.25%)
Q3a	Experimental group & control group	62 (77.5%)	18(22.5%)	20 (32.26%)	42 (67.74%)
Q3b	Experimental group & control group	55(68.75%)	26(32.5%)	21(38.18%)	34 (61.82%)
Q4a	Experimental group & control group	20 (25%)	60 (75%)	1 (5%)	19 (95%)
Q4b	Experimental group & control group	15(18.75%)	65(81.25%)	1 (6.67%)	14 (93.33)
Q5	Experimental group & control group	11 (13.75%)	69(86.25%)	0 (0.0%)	11 (100%)

Table 5.8 presents the pre-test results for both the experimental group and the control group learners. One could notice that the learners attempted all the questions. However, from the pre-test, most of them were unable to apply the necessary conceptual knowledge to pass the test item. Only 24 learners, which is 30%, were able to correctly answer the concept of the gradient of AC from both groups. Seloraji (2017) stated that a lack of understanding of the concept of Analytical Geometry could cause discouragement in learners, which invariably leads to poor performance.

The majority of learners in both groups did not fully comprehend the concept of distance between two lines. Only 20 learners answered the question correctly about AC, and the number was the same when it came to finding the coordinates of either x or y when the actual length was given. Makhubele, Nkhoma, and Luneta (2015) asserted that challenges could arise since students are in a hurry, lack conceptual knowledge, and have an inadequate command of basic skills, concepts, and facts.

The percentage of learners who answered AC correctly in terms of solving for the equation of lines was poor. This may be since learners need to first find the gradient of the line and identify a point before the question can be answered correctly. Most learners got the calculations wrong because they over- or undergeneralised the rules of both gradient and the equation of the straight line (Hansen, Drews, Dudgeon, Lawton, & Surtees, 2017). When it came to finding the coordinates of a point in a quadrilateral of 4a and 4b, where learners had to apply other concepts before answering the question, it was a challenge. Out of the 35 (43.75%) learners who attempted to answer the questions (ATA), only one learner answered them each correctly (AC), which is 5% and 6.67% for each 4a and 4b, respectively. The results of the pre-test suggested that learners in both groups faced challenges in conceptual aspects of Analytical Geometry when solving for the gradient of a line, calculating the distance of a line, determining the coordinates of a line, and finding the equation of a straight line. This showed that challenges must be corrected in learners' Analytical Geometry. Challenges related to a subject should be identified before it is taught, and it should be determined where and why the learner has encountered these challenges (Ozkan et al., 2018).

After the pre-test, the researcher marked learners' test scripts to ascertain mistakes that surfaced. After scrutinising learners’ scripts, many conceptual violations that led to learners performing poorly were identified. The discussions below explain the sampled picks representing the number of learners who had challenges.

5.6.1 Question 1 (a): Calculate the gradient/slope of A (-2, 5) and B (1, -2) on the Cartesian plane

It is required that Grade 11 learners of Analytical Geometry of lines be able to state the gradient formula, substitute it, and then calculate the answer. The learners were expected to possess the skills and knowledge to answer the questions. The researcher used these schemes to analyse the wrong answers learners obtained. Figures 5.1 and 5.2, respectively, illustrate the challenges presented by some learners, L8S2 and L17S1.

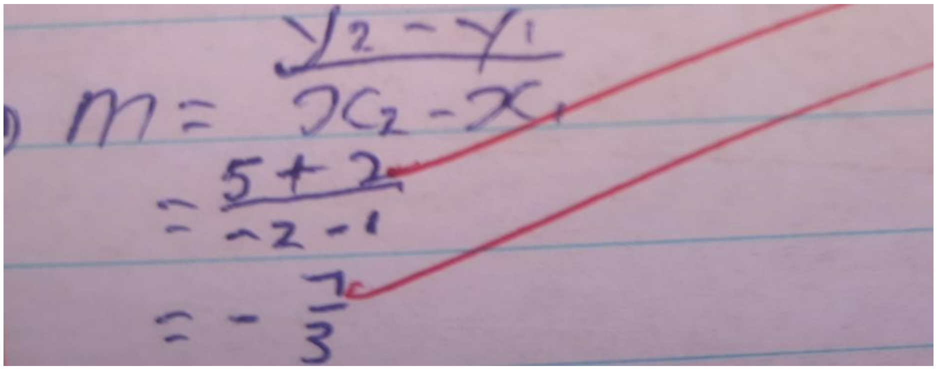


Figure 5.1: ***Solution to by L8S2 to question 1(a)***

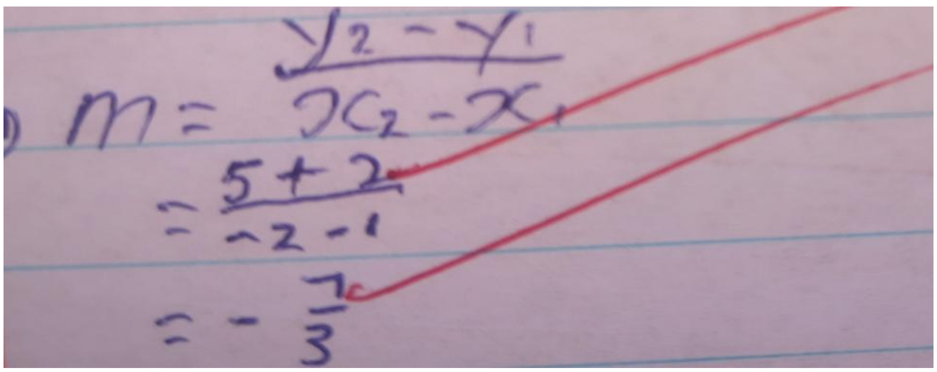


Figure 5.2: ***Solution to by L17S1 to question 1(a)***

The analysis of the L8S2 shows that the learner lacks a concept of coordinating numbers. S/he does not know which of the numbers is *x*-coordinate or *y*-coordinate. A similar thing applies to L12S2, who do not know the conceptual formula of gradient. S/he thinks any of the numbers could be used in the formula, rendering the calculation wrong. L17S1 had a solid grasp of the concept of the gradient of a line and the algebraic manipulation required to obtain an accurate answer. S/he perfectly substituted the co-ordinates into their correct positions and algebraically computerised them well until s/he got the correct answer. L25S2 also had a similar conceptual problem of not knowing the correct places of the values. For him/her, the first number is for the numerator, and the other is for the denominator. While L28S1 substituted the numerator correctly, s/he could not match with the $x_1 \rightarrow y_1$ and $x_2 \rightarrow y_2$. According to him/her from step 2, any *x*-value could be at any place. This led to the incorrect calculation of the answer. L32S1 multiplying the co-ordinates shows a total lack of knowledge of the gradient. The learner thought that any operation sign could be used at any place for any calculation, which led to incorrect computation of the marks.

5.6.2 Question 2: Calculate the value of *x* if the length of the distance between A (12, 9) and B (*x*,7) is 9cm

In this question, learners needed to be able to correctly quote the formula for distance and substitute it into the formula, equating the whole formula to 9 cm. Taking squares of both sides of the equations will lead to the correct computation of the answer.

A solution provided by a grade 11 learner, L3S1

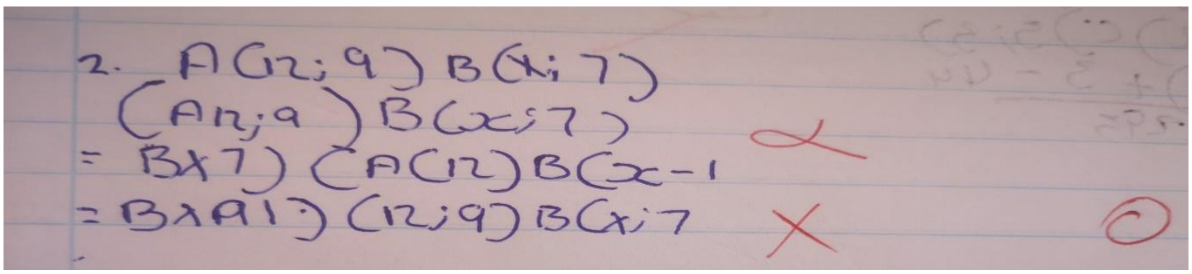


Figure 5.3: ***L3S3's solution to Q2***

A solution provided by learner L18S2

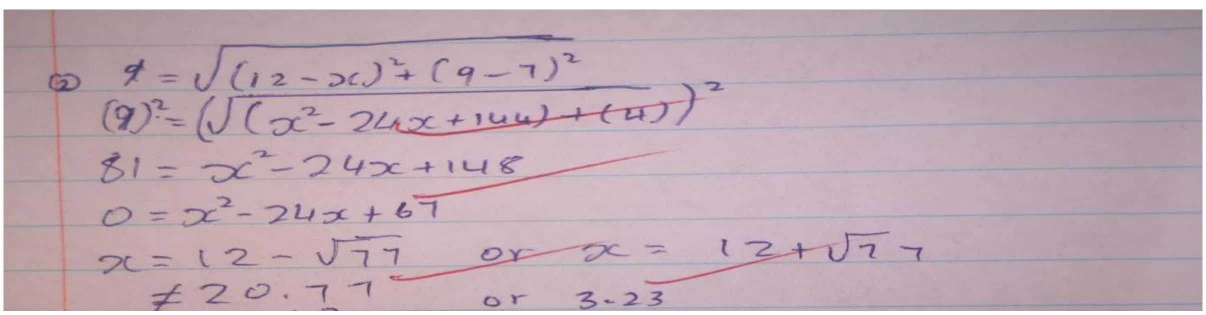
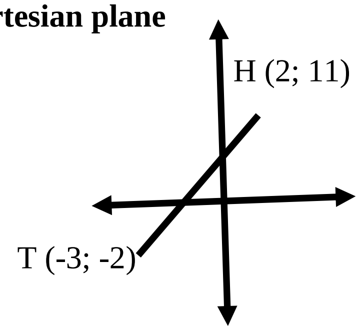


Figure 5.4: ***L18S2's solution to Q2***

The mistakes presented by the six learners on the test items above are similar to the mistakes committed by L24S1 and L33S1. Most learners in solving question 2 had a similar problem of not being able to substitute the coordinates into the correct formula. L3S1, L18S2 and L20S2 could not write the formula for the gradient in the pre-test, let alone do the correct computation. Learners who could write the formula (L24S1 and L26S2) could not substitute the co-ordinates into the formula for correct algebraic manipulation. L20S2 multiplied the numbers in the co-ordinates to get the answer. Hence, his/her calculation was wrong. According to Tanner and Jones (2000), mistakes about a subject should be identified before teaching the subject, and it should be determined where and why the learner has made the mistake (as cited in Educational Review by Ozkan, Ozkan, & Karapıcıak, 2018). The above challenges surfaced during the pre-test. The aforementioned authors confirm that challenges exist for learners in the Analytical Geometry of lines in the Cartesian plane.

5.6.3 Question 3. Calculate the equation of line HT on the Cartesian plane



The question required Grade 11 learners to find the gradient of the two points on the line and identify only one point on line before the equation of a line could be calculated.

A solution provided by a learner in grade 11, L6S2

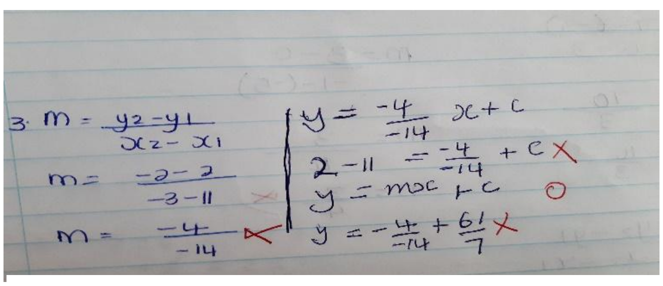


Figure 5.5: ***L6S2's solution to Q3***

A solution provided by a grade 11 learner, L30S1

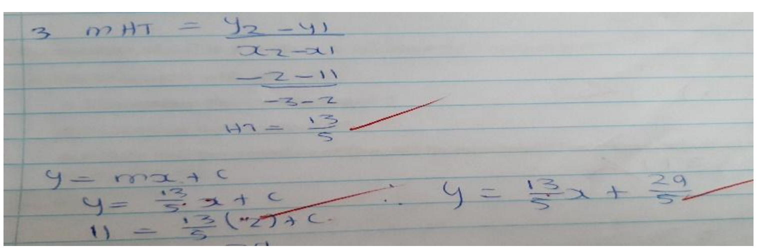


Figure 5.6: ***L30S1's solution to Q3***

L13S1 and L22S2 have the same mistakes; they only differ in their level of difficulty, which most learners found challenging. These learners were able to quote the formula correctly but failed to know that before the equation of a straight line could be seen, the correct calculation of gradient and any point on the same line had to be established. This indicates that they lacked the knowledge necessary to find the equation of the line. L6S2, like many others, could not substitute the two coordinate values into gradient formulas before calculating for the equation of a straight line. According to Van Hiele, a learner can only achieve level 2 if s/he understands level 1. L19S2 was able to write the correct formula for the equation of a straight line, like many other learners, but could not determine that there are essential steps to finding the gradient used for the equation of a straight line. The mistake indicates that s/he lacks an understanding of how the equation of a straight line is calculated. The challenges of L19S2 are common in many Grade 11 learners of Analytical Geometry of lines. L30S1 was able to successfully calculate the equation of a straight line with ease, which is very rare among many Grade 11 learners. The above challenges and their related mistakes are problems that many mathematics learners face, and teachers must address them to ensure they are avoided.

5.6.4 Question 4. In each of the following, determine whether AB∥CD, AB⊥CD or neither

a) A (-1; 5), B (-2; 3), C (9; 10), D (5; 2)

This question assesses learners' understanding of Van Hiele's levels 3 and 4 to determine whether learners of Analytical Geometry of lines can effectively apply their knowledge of gradient to identify lines that are related to each other in some way. Many learners found it difficult to answer the test items in the pre-test.

A solution provided by a learner in grade 11, L10S1

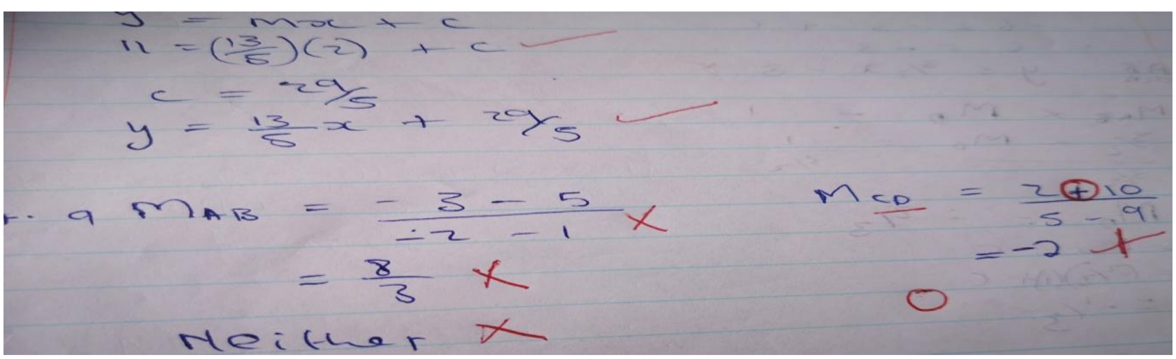


Figure 5.7*: L10S1's solution to Q4*

A solution provided by a learner in grade 11, L16S2

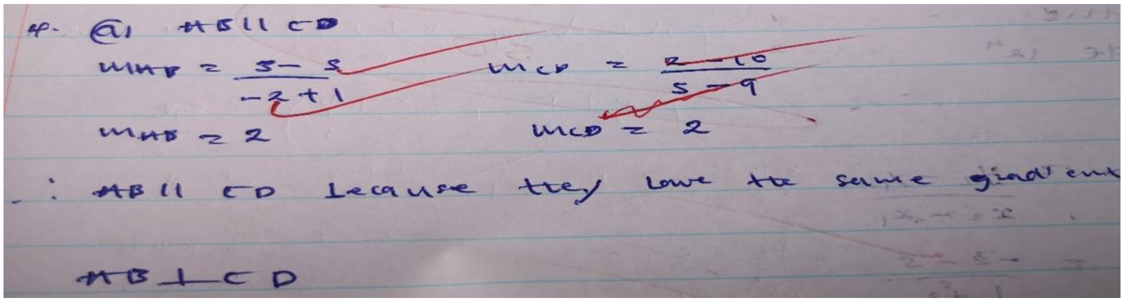
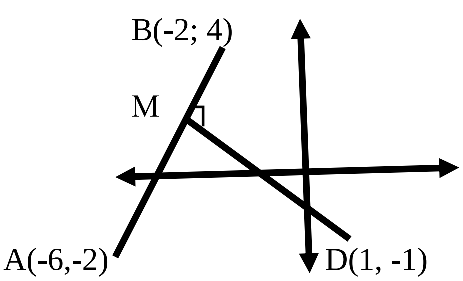


Figure 5.8: ***L16S1's solution to Q4***

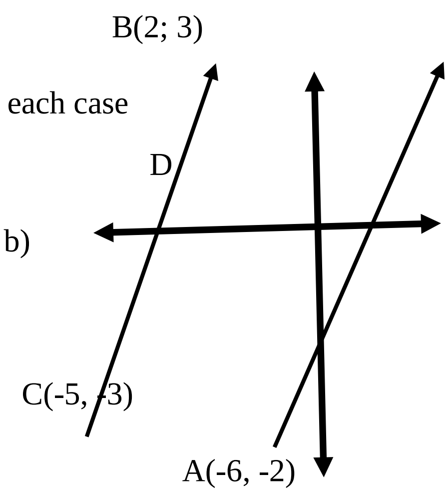
Question 4 (a) comprises two parts that require grade 11 learners to determine whether the two lines are parallel to each other. The two solutions provided above are from L10S2 and L16S1, and several learners provided similar solutions. Learners must compare by first finding the gradients of the two lines and then giving their conclusion. However, most participants were unable to provide the answer during the pre-test.

The mistakes presented by six learners on the test items are similar to the mistakes and difficulties committed by L24S2 and L34S1. Most learners solving question 2 have a similar problem of not substituting the co-ordinates into the correct formula. L10S2, L16S1, L14S2 and L28S2 could not write the formula for the gradient in the pre-test, let alone do the correct computation. Learners who could write the formula (L24S2 and L34S1) could not substitute the co-ordinates correctly into the formula for correct algebraic manipulation. L14S2 added (+) as the numerator of the gradient instead of minus (-). Hence, the calculation above. Learners and Ozkan, Ozkan & Karapıcak (2018), as cited in Educational Review, confirmed that mistakes by learners exist in the Analytical geometry of lines on the Cartesian plane.

5.6.5 Determine the equations of the line CD and MD in each case



b)



In questions 5a and 5b, learners are expected to identify the relation the lines on the Cartesian plane have. The relation will lead learners to identify the appropriate gradients of each pair of lines, which will enable them to find the equations of the lines (line CD and MD) accurately.

A solution provided by a learner in grade 11, L4S1

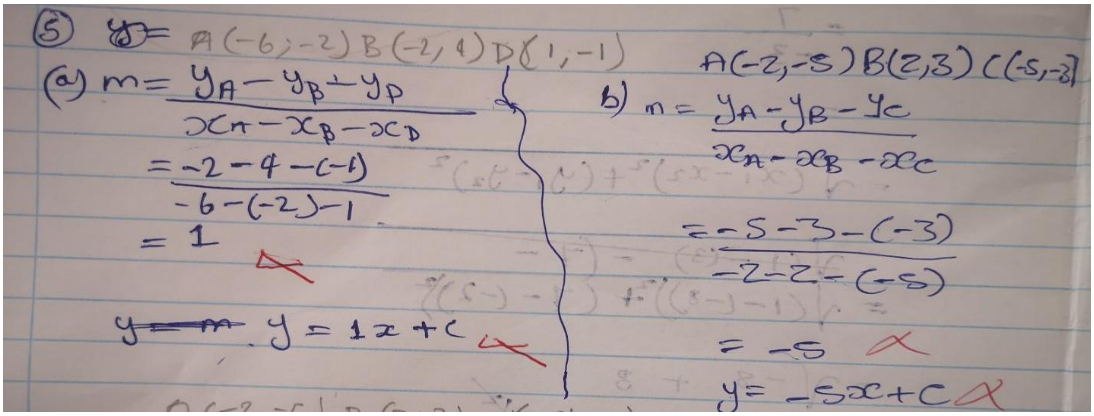


Figure 5.9: ***L4S1's solution to Q***

A solution provided by a grade 11 learner, L15S2

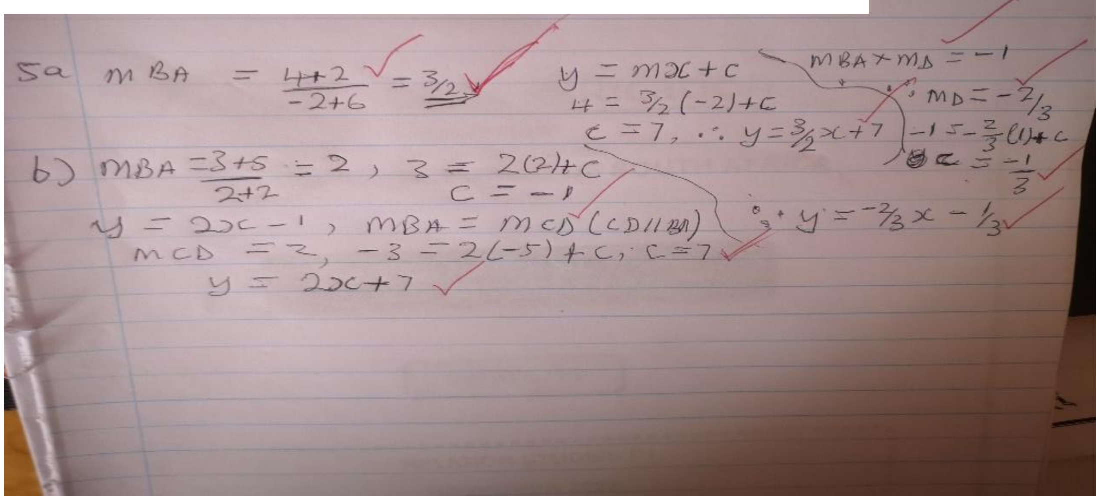


Figure 5.10: ***L15S1's solution to 5***

Based on the analysis of the challenges presented, L4S1 was unsure how to calculate the equation of a line when two lines are parallel or perpendicular to each other. S/he did not even know how the concept of a gradient of a line and how the equation of the line could be calculated. S/he lacked the conceptual knowledge of the algebraic manipulation of the answer. This is similar to L8S2. They both could not find the gradient of the straight line. Only L8S2 could quote the correct formula but failed to substitute the right values into the right positions for the correct gradient. Hence, s/he committed a conceptual error. L15S1 assimilated the processes of finding the equation of another line when it is either parallel or perpendicular. S/he performed the accurate processes of algebraic manipulation to get the right answers. L20S2 lacked the knowledge to find the equation of a straight line. The learner used the raw values in the straight-line equation without first identifying the gradient of the lines. S/he lacked total knowledge of the process of finding the equation of the straight line on the Cartesian plane. L29S1 calculated the midpoint of the lines and used a gradient instead of the gradient formula. L38S2 committed careless mistakes when s/he was able to substitute correctly into the correct formula of the gradient but still got the answer wrong. Challenges that lead to incorrect solutions about a subject should be identified before teaching that subject, and it should be determined where and why the learner has made mistakes (as cited in Educational Review by Ozkan, Ozkan, & Karapıcak, 2018).

5.7 SEMI-STRUCTURED INTERVIEWS ON PRE-TEST OF LEARNERS ON CHALLENGES AND DIFFICULTIES ON VARIOUS QUESTIONS

The researcher conducted interviews with six (6) learners on each question and the difficulties and mistakes they committed on the concept they were tested on. Conducting the interview provided the researcher with insight into the source of the problem (Clements, 2023).

5.7.1 Interviews on question 1 and responses from learners

The analysis of the six (6) learners in Grade 11, L8S2, does not have any concept of coordinates of numbers. S/he does not know which of the numbers is *x*-coordinate or *y*-coordinate. A similar thing applies to L12S2, who do not know the conceptual formula of gradient. They replied that any of the numbers could be used in the formula, which renders their calculation wrong. L17S1 had a solid grasp of the concept of the gradient of a line and demonstrated accurate algebraic manipulation, presenting an accurate answer. S/he perfectly substituted the co-ordinates into their correct positions and algebraically computerised them well until s/he got the correct answer. L25S2 also had a similar conceptual problem of not knowing the correct places of the values. The learner indicated that the first number represents the numerator, and the second number represents the denominator. While L28S1 substituted into the numerator correctly, s/he could not match with the $x_1 \rightarrow y_1$ and $x_2 \rightarrow y_2$. According to him/her, any x-value could be at any place. This led to the incorrect calculation of the answer. L32S1 multiplying the co-ordinates shows a total lack of knowledge of the gradient. The learner thought that any operation sign could be used at any place for any calculation, which led to incorrect computation.

5.7.2 Interviews on question 2 and responses from learners

The challenges and mistakes presented by the six learners on the test items are similar to those committed by L24S1 and L33S1. Most learners in solving question 2 had a similar problem of not being able to substitute the coordinates into the correct formula. L3S1, L18S2, and L20S2 could not write the formula for the distance between two lines in the pre-test, for the correct computation. L24S1 and L26S2 could write the formula but were unable to substitute the coordinates correctly into the formula for correct algebraic manipulation. These learners alluded that multiplying the bracket will get the distance, and if the distance of a line is to be calculated, multiplying a number by the other will give the value of the length. Hence their calculation. L33S1 failed to use the distance formula; s/he quoted the gradient formula instead. According to Matindike and Makonye (2023), mistakes about a subject should be figured out at the early stage of any subject, and it should be identified where and why the learner made the mistakes (as cited in Educational Review by Ozkan, Ozkan & Karapıcak 2018).

5.7.3 Interviews on question 3 and responses from learners

Analysis for L13S1 and L22S2 has the same mistakes, only deferred in their challenges. Those learners were able to quote the formula for the equation of a straight line correctly, but failed to understand that, before the equation of a straight line could be found, they needed to perform the correct calculation of gradient and any point on the same line. This shows that they lack some knowledge that could lead to finding the equation of the line. L6S2, like many others, could not substitute the two coordinate values into gradient formulas before calculating for the equation of a straight line.

S/he alluded that the same number should be together (CE). According to Van Hiele, learners could only achieve level 2 if they understand level 1. A lack of understanding of level 1 made it difficult for learners to calculate the equation of a straight line. L19S2 was able to write the correct formula for the equation of a straight line, like many others, but could not find the gradient to use in the equation of a straight line. The mistakes s/he gave indicated that s/he lacked the concept of how the equation of a straight line is calculated. The mistakes of L19S2 are encountered by many learners in Analytical Geometry of lines in grade 11. According to them, any of the coordinate values could be used to calculate the answer. This made it difficult for them to reach the accurate answers. L30S2 was able to successfully calculate the equation of a straight line with much ease, which is rare among many grade 11 learners of Analytical Geometry of lines on the Cartesian plane. L22S2 alluded that calculating length and gradient is the same as solving the equation of a straight line.

5.7.4 Interviews on question 4 and responses from learners

Question 4 (a) comprises two questions that require grade 11 learners to determine whether the two lines are parallel to each other or not. The two solutions provided in (figure 4) above are from L10S2 and L16S1, and several learners provided solutions similar to that of L10S2. Learners stated that they did not consider finding the gradients before reaching their conclusion. Learners were required to make the comparison by first finding the gradients of the two lines and drawing their conclusion. However, most participants were unable to provide the answer during the pre-test.

L24S2 and L34S1 gave similar explanations. Most learners in solving question 4a had a similar problem of not being able to substitute the co-ordinates into the correct formula. L10S2, L16S1, L14S2, and L28S2 could not write the formula for the gradient in the pre-test, let alone do the correct computation. L24S2 and L34S1 could write the formula but were unable to substitute the coordinates correctly into the formula for correct algebraic manipulation. L14S2 added (+) as the numerator of the gradient instead of minus (-). Hence his/her calculation. When interviewed, their reason was that co-ordinates could be used anywhere in the formula to get the answer. The above mistakes and challenges significantly contributed to the poor performance of learners during the pre-test.

5.7.5 Interviews on question 5 and responses from learners

Based on the analysis of the mistakes and challenges presented in items 5a & 5 b, L4S1 did not know how to calculate the equation of a line when two lines are parallel or perpendicular to each other. S/he did not know the concept of a gradient of a line or how the equation of the line could be calculated. S/he lacks the conceptual knowledge of the algebraic manipulation to get the answer. This is not different from L8S2. Neither learner could successfully find the gradient of the straight line. Learners were expected to interchange the values into the correct formula (CE). When they were questioned, they stated that any value in the question can be used for the answer. Hence, they are committing conceptual mistakes. L15S1 was the only one who assimilated the processes of finding the equation of another line when it is either parallel or perpendicular to another line. S/he performed the accurate processes of algebraic manipulation to get the right answers. L20S2, like most learners, lacked the knowledge of finding the equation of a straight line. The learner used the raw values in the straight-line equation without first finding the gradient of the lines. S/he replied to the interview question that I did not know what to do. S/he lacked total knowledge of the process. L29S1 calculated the midpoint of the lines and used them as a gradient instead of the gradient formula, saying that any method could be used to find the equation since the points are on the line. L38S2 committed a careless mistake when s/he was able to substitute correctly into the correct formula of the gradient but still got the answer wrong. When s/he was asked why commit such a mistake, s/he replied, “I did not know I was wrong.”

5.8 OUTCOMES OF CLASSROOM OBSERVATION ANALYSIS AFTER THE INTERVIEW ON PRE-TEST

The classroom observation was conducted to determine how Grade 11 learners learn about Analytical Geometry of lines on the Cartesian plane. The researcher visited the classroom (s) (the research site) where the learners' activities were taking place (Creswell & Creswell, 2018) to gather first-hand information about how learners were taught Analytical Geometry of lines on the Cartesian plane. The teachers asked the learners general questions that allowed them to freely express themselves (Creswell & Creswell 2018). The researcher interacted directly with learners in their natural settings, allowing for face-to-face conversations (McMillan & Schumacher, 2020). The researcher's focus was to understand the meaning of learning from learners, specifically how they comprehend the concept of Analytical Geometry of lines on the Cartesian plane, rather than relying solely on the literature (Johnson & Christensen, 2020; McMillan & Schumacher, 2020). The researcher focused on the following areas for comprehensive classroom and field observations (Creswell & Creswell, 2018; Johnson & Christensen, 2020).

- Observing classroom practices of the learners (how learners pay attention to the lesson delivery by their teacher)
- Observing learners’ skills (how learners practise examples their teacher gave, their application of Analytical Geometry formulas in solving questions)

- Learners’ attitude towards lessons (are learners playful, or do they pay attention during lesson delivery)
- Effective learning in the classroom (is every learner actively involved in the learning process?)
- Content and presentation (what content of Analytical Geometry is and how it is presented to learners)
- Learner involvement (these include interested learners and participation)
- The resources for learning (are there enough resources and handouts for learners?)
- Classroom management (including classroom rules and seating plans)
- The conducive atmosphere for learning (including learner inclusiveness and dignity)
- The objective of the lesson (involving resources, teaching scheme, aims and objectives and content)
- The evaluation of learner achievement (including clarity and after classroom lesson achievement).

Below, the researcher explains the processes that occurred during the observational period. Also, the activities of teachers at both schools and their learners are explained in detail.

5.8.1 Observing teaching methods and styles in the classroom

Passive forms of teaching mostly dominated the learning in both groups. Learners passively received instructions from teachers in both the experimental group and the control group. A chalkboard and chalk dominated the teaching environment. Vygotsky’s (1978) scaffolding strategy was not followed for the presentation of mathematics lessons. Learners were just information receivers. For example, the teachers started their lessons with examples of the topic which was to be taught during that lesson. One such example in the 3rd lesson was the equation of a straight line; find the equation of the line A (1, 8) and B (-3, -1). S/he continued to explain the steps in detail before arriving at the answer. The teacher from the experimental group occasionally reminded learners that they were far behind the syllabus and had to complete it. This signifies that the teacher was particular in completing the syllabus without taking into cognisance the learners' understanding. S/he did not give himself/herself enough opportunity to observe and listen carefully to the learners’ ideas and alternative conceptions. This indicates that learners' knowledge was not built on Van Hiele’s (1986, 1999) stages of learning. Essentially, the teaching methods employed by both groups were mainly lecture-based. There was not enough time for the classwork, and no establishment of previous knowledge before the commencement of the lesson.

5.8.2 Observing classroom interactions

Questioning and answering were the main styles of interaction between the teachers and their learners. The style of questioning by the teacher from control group was not in line with the development of Analytical Geometry of lines on the Cartesian plane as postulated by Vygotsky (1978). The teacher in the control group's questions were not evenly distributed; instead, s/he did not engage all learners in the subject at hand. S/he posed questions like, "*Who can answer this question"?* Leaving the weak learners behind during the lesson. The strategies used by the teacher in the control group and the teacher from the experimental group were not environmentally conducive to afford learners opportunities to interact and share mathematical ideas that can enhance their conceptual understanding of the Analytical Geometry of lines on the Cartesian plane. The teachers did not understand and/or use the rules of group studies. Group studies are employed to enable weaker learners to learn from more capable ones (Vygotsky, 1978). However, this was not the case; the weak learners were making noise and talking without paying attention to the mathematical concept being discussed. For example, the teacher in the experimental group gave work to the learners in groups to determine whether the lines were perpendicular or parallel. The instructions went like this:

In each of the following pairs, determine whether the lines $AB \parallel CD$, $AB \perp CD$ or neither.

In groups of five, the researcher observed that, in each group, only the capable individuals were striving to solve the question, while the others made noise. The teacher was unable to control the situation. When the researcher asked why s/he did not control them, the teacher said, "Leave them, sir; those are not serious", meaning s/he lacked the professional tactics to control learners in the classroom. This should have indicated to the teacher that those learners lack motivation to learn (Bandura, 1986). Classroom interaction through questioning and answering was not able to motivate the learners' interests in relation to the tasks discussed.

5.8.3 Observing the type of teaching and learning resources used

The teacher from the control group mainly uses the textbook, study guide, and chalkboard to present his/her lessons without the support of any other teaching aid. It is, therefore, important to support teaching and learning with other resources to enhance learners' understanding (Kapur, 2020). It was observed that only the teachers from both schools were using the textbooks, with no learners having some of the textbooks. The learners' lack of textbooks created a significant gap, leading to challenges and mistakes on the topic, as indicated in the interview section. Questions and answers were selected from textbooks and study guides due to the scarcity of other resources for learners, such as computers and tablets. Resources like overhead projectors and computers were non-existent to aid the learning process in both schools. According to Parker and Thomsen (2019), different resources and real objects help learners improve their understanding and enhance their performance. Therefore, using Information and Communication Technology (ICT) is a valuable resource that any modern teacher can utilise to extract various information to enhance teaching and learning, or to help learners of mathematics understand the concept of Analytical Geometry of lines very well. The lack of availability of textbooks compelled the teachers from both groups to copy tasks from the textbooks after their lessons; this constraint made it difficult for learners when it came to references and during their private studies.

5.8.4 Observing how classroom assessment is done

Teachers from both groups did not give the learners a chance to ascertain whether they understood what they were taught before providing solutions to the questions. For this reason, learners find it challenging to understand where they went wrong. As noted by Marinho et al. (2019), assessment procedures should help learners and teachers identify where they are incorrect and where they can improve from there. Both teachers gave learners exercises to try and see where they fell short. This aligns with what Van Den Berg, Bosker and Suhre (2018) postulate, that continuous assessment enhances learners' performance if properly managed. However, the element of differentiation to cater to learners' varying levels of understanding was non-existent. According to Van Den Berg, Bosker and Suhre (2018), assessments given to learners should be differentiated to cater for individual learning abilities. Doing so enables learners to understand the concepts at their own pace, incorporates different perspectives, and provides support to learners who experience learning challenges. Vygotsky (1978) encourages teachers to motivate learners to identify the learning gap, so that they can discover the key relationship to the mathematical concept themselves. A teacher from Experimental group employed scaffolding as a strategy by moving around while learners were writing classwork and guiding those who experienced difficulties. However, s/he was quick to answer questions when learners were still trying to find their solutions. Both teachers also provided feedback to individual learners instead of addressing the whole group, which helped learners develop their solutions.

Teachers from both schools gave most of the exercises and homework from their study guides and textbooks and wrote these activities on the chalkboard due to the scarcity of textbooks for learners. Due to this, learners were required to present their solutions on the chalkboard after completing their work, while some solutions were intended to be completed at home to provide slow learners with the opportunity to understand the mathematical concept at their own pace.

5.8.5 Observation during the intervention before the post-test

The researcher conducted observations during the interventional period before administering the post-test to ascertain whether the learners understood the interventional strategy (Ekka, 2021) aimed at mitigating poor performance in Analytical Geometry of lines on the Cartesian plane.

5.9 THE INTERVENTION STRATEGY

In conjunction with the teachers, the researcher held discussions a day before the pre-intervention to discuss strategies that could be applied to enhance learners' performance in Analytical Geometry of lines on the Cartesian plane. The two teachers were then introduced to the intervention. However, the Experimental group was to be introduced to GeoGebra as an intervention whilst the control group used the conventional way of learning. The reason was to equip them with the skills to use GeoGebra software, which could help mitigate the poor performance of Grade 11 learners in Analytical Geometry of lines on the Cartesian plane.

5.10.1 Classroom observations during the intervention process

The researcher’s observation during the intervention was to ascertain whether the Experimental group teacher successfully implemented the GeoGebra software to enhance learners’ performance. The observation helped to identify the difference in the intervention's teaching to determine whether the intervention enhanced the learners’ performance. The observation at this stage was also done to help answer the following research questions:

- How does the use of GeoGebra as an intervention influence learner performance in Analytical Geometry?

The Experimental group teacher strategically planned how to teach he/her learners to enhance their performance and answer the research questions. The classroom observation data collected were used to achieve the objective of the study.

- To investigate the effectiveness of GeoGebra in enhancing Grade 11 learners' performance in Analytical Geometry.
- To examine the impact of GeoGebra on learners' understanding of key Analytical Geometry concepts.
- To explore how Grade 11 mathematics teachers integrate GeoGebra in teaching Analytical Geometry.
- To identify the challenges and benefits of using GeoGebra in teaching Analytical Geometry.

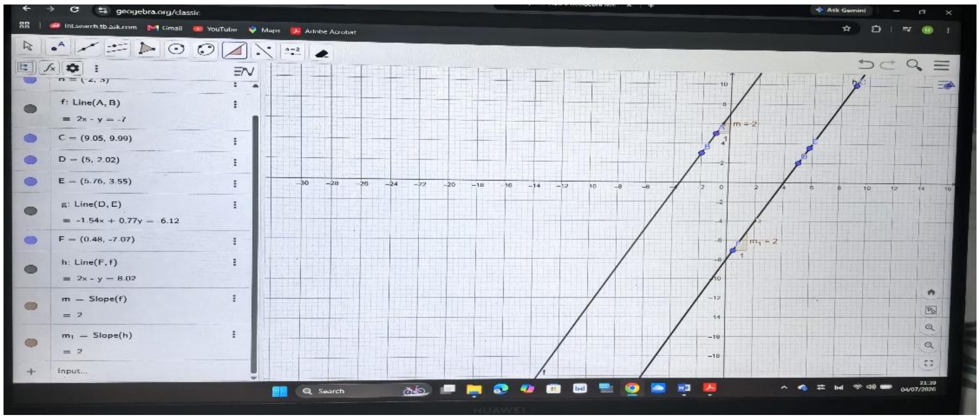
5.9.2 Lesson preparations for intervention

Based on the strategy, the two teachers prepared eight lessons on the Analytical Geometry of lines on the Cartesian plane using GeoGebra software, which is intended to be used during intervention sessions to enhance learners' performance. They prepare the lessons on topics like *"gradients on a line, the length of a line or between two points, equations of a straight line, perpendicular line and parallel lines applications on the topics".* The intervention process took place two times a week with the Experimental group. Each teacher was observed in their respective group. Presenting the intervention would help differentiate the teacher's teaching according to the learning abilities of their learners. This also helped the teacher identify whether the intervention was beneficial to weak learners and whether the average ones were showing improvement. The teacher from the Experimental group created an enabling environment during the intervention by allowing learners to ask questions and directing them to arrive at solutions. This showed that a conducive environment encourages learning positively and improves learners' performance (Gamlem, 2019). Learners were highly motivated, and the confirmation that the intervention improved their understanding of the topic was evident.

5.9.3 Observing teacher-learner interaction

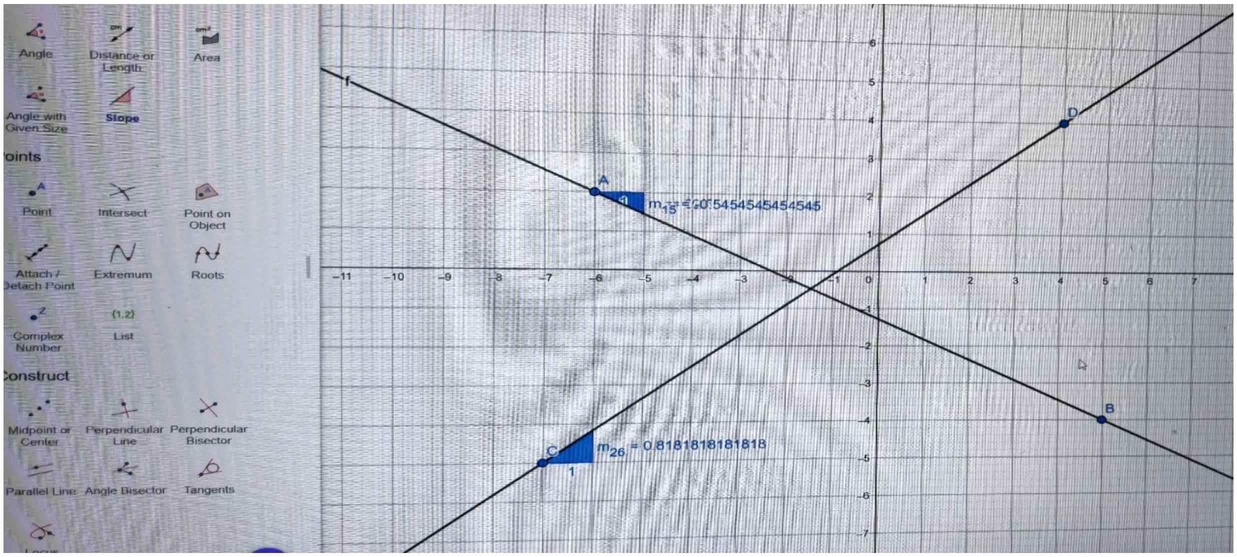
After the teachers prepared their lessons for the interventions, the researcher observed the way they interacted with their learners when teaching Analytical Geometry of lines. The interaction, as usual, began with questioning and answering by the teachers of both the Experimental group and the Control group, along with their learners. But the Experimental group teacher introduced his/her to GeoGebra software.

Below is some of the steps the teacher took his/her learners through to prove that, when two lines are parallel to each other, their gradients are the same, or they have equal gradients.



During the intervention, the researcher observed that the Experimental group teacher began grouping the learners and implemented scaffolding (Vygotsky, 1978) to encourage learners to discover answers to questions themselves by asking probing questions. That way, s/he made learners discover and gain support themselves without giving them answers.

The first week was a bit difficult for learners as they were not used to that new teaching style until the teacher started asking questions based on their previous knowledge to arouse their interest. However, the learners later enjoyed the new style of teaching, which utilised modern GeoGebra software technology, making them understand the concepts very well from their Experimental group teacher. The diagram below shows how GeoGebra was used to prove to learners that, points on the same line produced the same gradient/slope. The m_{15} and m_{26} signifies the number of times the slope was found on the same line with different points on the line which yielded the same value of gradient as indicated in the diagram. The diagrams serves as one of the numerous evidence of GeoGebra usage during the interventional periods.



Providing learners with several teaching methods motivates them and encourages their engagement with various learning approaches in their studies (Molina et al., 2018). The teacher from the Experimental group started with questions on previous knowledge, such as, *What is needed before calculating the equation of a straight line? What are the necessary conditions to calculate the equation of one line which is parallel to another line?* Again, the teacher from Control group posts his/her previous knowledge questions: *What is the formula for the gradient between two points? How does one locate the gradient/slope of an equation of a straight line?* The Experimental group teacher sought to ascertain whether learners still remembered these important parts of the equation of a line and whether they still recalled their previous lessons on the topic.

Both teachers were distributing their questions evenly; this time, they were not concentrating only on able learners. They also asked questions which learners occasionally respond together, showing their interest in the lessons, or they choose learners they feel are weak to respond to the questions. The researcher also observed that most learners, unlike before, could ask their teachers for clarifications on concepts they did not understand. For example, a learner asked the teacher from the control group in lesson 4 about the equation of a line, “If there is another formula that could be used to find the equation of a line?” signifying that they are not only recipients but also motivated to contribute to the lesson. The researcher observed that the teacher introduced the formula $y-y1 = m(x-x1)$ as an alternative to the equation of a straight line, which is derived from the formula $y = mx + c$. The researcher noticed that, at the end of the lesson, the learners from both groups took the study by themselves and only asked when they needed clarity.

5.9.4 Observing assessment during the intervention

Homework and class exercises were used to instantly determine whether learners had made progress. Both teachers provided learners with activities after each lesson's presentation during the intervention to monitor their progress. At this point, the Experimental group teacher was using GeoGebra to demonstrate the presentation as learners sat in a group. They also ascertain the benefits of using GeoGebra to teach Analytical Geometry of lines on the Cartesian plane. The teacher from the Experimental group also assigned homework in lessons 2, 3, 5, and 8, and provided classwork in lessons 1, 4, 6, and 7. In contrast, the teacher from the Control group assigned homework in different classes. Both teachers used textbooks and other handouts to design assessments that suited Van Hiele's (1958) levels of Analytical Geometry of lines on the Cartesian plane. The works were either given during the interventions or after the interventional presentation. And the homework was after the lessons. Both teachers made photocopies for their learners as they did not have enough textbooks for each learner.

5.9.5 Post-intervention lesson observations

The researcher conducted a post-intervention lesson observation to reflect on the intervention and determine whether learners could actively participate in the lessons and make meaningful contributions. It was also to determine whether learners were satisfied with the introduction of GeoGebra software and its impact on their performance in the post-test. Whether learners were motivated during this period to take up learning on their own in their future studies. Whether the learners have acquired the impact of the concrete materials their teachers used in teaching them during the intervention, especially the Experimental teacher who used GeoGebra software in their intervention. Morris (2016) reiterates that it is essential for learners to have opportunities to demonstrate what they have learned using various modalities, such as words, pictures, spoken language, gestures, or tangible materials.

5.9.6 Observing teacher-learner interaction

During the intervention, the data collected by the teachers of two groups showed that they were able to impact the knowledge gained by their learners. As usual, teachers from both schools introduced their lessons through questions and answers, which revealed what learners knew and could be introduced to, as well as the new knowledge of the Analytical Geometry of lines on the Cartesian plane. According to Vygotsky (1978), scaffolding instruction is the "role of teachers and others in supporting the learners' development and providing support structures to get to that next stage or level". The main idea was to transfer learners' knowledge from one point to another that they were unable to reach on their own (Traver et al., 2019). The teachers' questions were evenly distributed among learners, which also prompted learners to share the knowledge they already possessed for their understanding of the topic, unlike before the intervention. The teacher from Experimental group asked learners a question: "W*hat is the gradient/slope of two lines which are parallel to each other?"* S/he wanted learners to say lines would have the same gradient or slope if they were parallel to each other. By using the GeoGebra demonstration, it was clearly shown how those lines on Cartesian planes were formed, and learners understood how to interpret them to answer questions based on Analytical Geometry. Throughout the deliveries of teachers' lessons from the Experimental group, the lessons were solely learner-centred, allowing learners to construct their knowledge, unlike before the intervention. The strategy used by the Experimental group teacher during the post-intervention classroom observation differed from that used during the pre-intervention period. After the interventions, both teachers gained experience in diagnosing learners' challenges and mistakes (Seloraji, 2017).

5.9.7 Observing learner-learner interaction

Learners from the two schools were given the opportunity to interact with each other and share ideas on solving questions related to analytical geometry on the Cartesian plane after the post-intervention observation, to determine how effectively they could answer questions. Whether they constructed their own knowledge on the topic could lead to improvement in their performance. This also provided them with a free environment in which to develop their best solutions (Jameel & Ali, 2016), which was not the case before the pre-test. An environment that is sufficiently motivated to study will attract interest, playing a crucial role in achieving high goals, especially in mathematics and analytical geometry, such as lines and angles on a Cartesian plane (Gamlem, 2019).

Learners from the Experimental groups were now actively engaged in their group discussions while solving the questions. This was because they were exposed to scaffolding (Vygotsky, 1978), a type of teaching approach implemented by their teacher using GeoGebra software during the intervention process. The most capable individuals can now facilitate the learning process to help their smaller groups reach the answers. Each learner in the group was active, and ideas were actively shared to enhance the understanding of all. Whilst the control group teacher continued with his/her approach, he/she was using. The teachers were walking around and only intervened when learners needed help (Traver et al., 2019). The teachers also ensured a clear set of rules for the learning process to make the discussion outcomes valuable (Sarikas, 2020).

5.9.8 Observing teaching and learning resources

The teachers in their respective groups adapted methods that utilised handouts and concrete objects (Kösa & Karakuş, 2016) in teaching their lessons. They both recognised that to improve their learners' performance, they needed to make lessons relevant and practical to the learners' understanding, thereby enhancing their performance. They have realised that using traditional teaching with textbooks and other handouts was not helping learners understand the Analytical Geometry of lines on a Cartesian plane. Therefore, the teachers refrained from using textbooks and handouts solely in their teaching. The Experimental group teacher used GeoGebra to get the advantage of his/her learners understanding the Analytical Geometry concept more. The GeoGebra software was the gadget that captivated learners' understanding and led to their understanding. The experimental group of teachers seemed to have realised the importance of incorporating ICT as an opportunity to improve classroom teaching (Sugandi & Bernard, 2020).

5.9.9 Observing classroom assessment

Both teachers assigned classwork and homework to their learners as a diagnostic and formative assessment to test their understanding of the Analytical Geometry concept (Tookoian, 2018). Both teachers used the classwork to monitor their learners' progress in understanding the immediate concept. According to Hua et al. (2018), solving practical questions requires a practical approach to the question and an understanding of its basic theoretical principles. Therefore, the teachers used the classwork to get the practicality of what they implemented during the intervention. Monitoring learners' work was not done like before. Formative assessment took place during lessons, enabling teachers to determine whether learners understood the material and make adjustments as needed (Staake, 2023).

5.10. THE POST-TEST RESULTS AFTER THE INTERVENTION

In collaboration with the two teachers, the researcher administered a post-test to determine the effectiveness of the intervention. We administered a post-test after the intervention to determine whether the learners' performance had improved.

5.10.1 Outcome of post-test analysis

Table 5.9 below presents the post-test results, which are discussed in relation to the research objectives and aims of administering interventions to improve the performance of Grade 11 learners in Analytical Geometry of lines on the Cartesian plane. The data corresponds with the responses learners gave. The table also presents a description of the test items assessing skills and knowledge, as well as a detailed description of each learner's process in answering the questions.

The description of the process follows:

- Attempt to answer (ATA)
- Never attempted (NA)
- Answered correctly (AC)
- Did not answer correctly (DAC).

Table 5.9: Distribution of learners' responses to Q1a and Q1b post-test

QUESTIONS	**GROUPS**	**POST-TEST**							
		ATA		NA		AC		DAC	
Q1a	Experiement group	40	100.0%	0	0.0%	40	100%	0	0%
	Control group	40	100%	0	0.0%	40	100%	0	0%
Q1b	Experiement group	40	100.0%	0	0.0%	40	100%	0	0%
	Control group	40	100%	0	0.0%	40	100%	0	0%

The results in Table 5.9 indicate that, after the intervention, learners were able to answer the question on the concept of gradient. The table above outlines various descriptions of skills and knowledge that learners acquired. All of them attempted the questions. The concept of calculating for the gradient given points indicated being embedded in learners' understanding.

5.10.2 Learners' responses to Q2a, Q2b and Q2c

Table 5.10 shows analyses of Q2a, Q2b, and Q2c results. Question 2a was used to assess learners' understanding of the concept of the length of a line on the Cartesian plane. Q2b was also used to assess the application of a line to find the coordinate of a point of a line by going through the process of finding the value of one coordinate of a point on the Cartesian plane. The Table indicates the learners had comprehended the concept of length.

QUESTIONS	GROUPS	POST-TEST							
		ATA		NA		AC		DAC	
Q2a	Experiment group	40	100%	0	0.0%	400	100%	0	0.0%
	Control group	40	100%	0	100%	40	100%	0	0.0%
Q2b	Experiment group	38	95%	2	5%	6	15.79%	32	84.21%
	Control group	40	100%	0	0%	15	37.5%	25	62.5%
Q2c	Experiment group	40	100%	0	0.0%	40	100%	0	0.0%
	Control group	40	100%	0	100%	40	100%	0	0.0%

Table 5.10: Distribution of learners' responses to Q2a, Q2b and Q2c

Table 5.10 above indicates that the experimental group in category ATA saw 100% of learners attempt to answer question 2a, and the control group had 40 learners representing 100% who attempted the question. In total, all learners were able to answer the questions correctly, indicating that the intervention was effective, especially for the experimental group, where this was not the case in the pre-test. The results show a positive outcome for both schools, indicating that most learners understood the concept of calculating length. It is important, therefore, to say that learners from both schools absorbed the concept of length in the analytical geometry of lines on the Cartesian plane. This suggests that most learners had adequate skills to interpret and solve questions on the concept of finding the length (Sharma & Verma, 2017).

Table 5.11: Distribution of learners' responses to Q3a and Q3b

QUESTIONS	GROUPS	POST-TEST							
		ATA		NA		AC		DAC	
Q3a	Experiement group	40	100%	0	0.0%	40	100%	0	0.0%

	Control group	40	100%	0	0.0%	40	100%	0	0.0%
Q3b	Experiement group	38	95%	2	5%	38	100%	0	0%
	Control group	38	95%	2	5%	38	100%	0	0%

The results from the data collected in Q3, as shown in Table 5.11 above, indicate that the experimental group achieved 100% and the control group also achieved 100% for AC. More learners in the experimental group as well as the control group were able to answer Q3. The responses of the schools to the questions were parallel. Both schools, 1 and 2, had an equal number of learners who passed all the questions on the concepts they were assessed on. However, 2 learners from each group could not answer the questions correctly (AC), which represents 5%. This suggests a gap needs to be filled.

Table 5.12: Distribution of learners' responses to Q4a and Q4b

QUESTIONS	GROUPS	POST-TEST							
		ATA		NA		AC		DAC	
Q4a	Experiement group	37	92,5%	3	7.5%	34	91.89%	3	8.11%
	Control group	40	100%	0	0.0%	37	92.5%	3	7.5%
Q4b	Experiement group	38	95%	2	5%	35	92.11%	3	7.89%
	Control group	39	97.5%	1	2.5%	38	95%	1	2.5%

Table 5.12 above for Q4a shows that very few learners still lacked conceptual knowledge, signifying that finding the co-ordinates of such questions was difficult to deal with. Although three learners from both groups were unable to answer Q4a, an excellent percentage of learners performed better. The AC for both experimental group and Control group was 92.5%, a remarkable percentage. Both groups were not bad either; they recorded percentages of 87.5% and 95%,

respectively. Q4b shows that most learners developed procedural knowledge and expanded their comprehension of mathematical concepts, interpreting mathematical questions and finding solutions to problems (Math Assessment Resource Service, 2017). The highest percentages for learners in both Q4a and Q4b indicate a clear comprehension of the intervention in AC, which is why the GeoGebra intervention was implemented. However, the other learners who could not attend the AC indicate a gap that needs to be filled. There are other learners who answered the questions partially (DAC), and it is necessary to determine why they were unable to complete the questions. Learners who still lacked the knowledge to answer coordinate questions did not comprehend the intervention and the different ways of solving such questions to achieve the same correct result (Jäder & Johansson, 2024).

Table 5.13: Distribution of learners' responses to Q5

QUESTIONS	GROUPS	POST-TEST							
		ATA		NA		AC		DAC	
Q5	Experiment group	30	75%	10	25%	8	26.67%	22	73.33%
	Control group	36	90%	4	10%	12	33.33%	24	66.67%

Table 5.13 above shows that AC for learners ranges from 20% and 30% for the experimental group and the control group, respectively. The findings for ATA displayed 75% for the experimental group and 90% for the control group. Higher percentages are shown for the A, indicating that the two groups still need to fill the gap that was created for such applicable questions of Q5. Higher percentages of DAC for both groups indicated that learners do not yet comprehend the concepts and procedures (Skolverket, 2022; Kilpatrick et al., 2001), such as when there is no point on the line, one has to find the equation of a line. Furthermore, it indicates that learners still lack the basic application of ideas from one concept to another (Math Assessment Resource Service, 2017).

5.11 ITEM ANALYSES OF PRE- AND POST-TEST RESULTS

The researcher discusses the results of the pre-test and post-test separately for each question, from Q1 to Q5, compares the scores, and also tests the statistical significance between the two study schools (Durcevic, 2020). The researcher presented the outcome (Creswell & Creswell, 2018) analysis in the table, highlighting the coefficient values. A negative co-efficient value means the question was simple or easy for learners to answer. The positive c-efficient value also signifies a difficult question for learners. A case where a question number did not appear with the value in the analysis table signifies that the performance for learners in that question is the same, or within the range of equal numbers in value. The reason Strata 17 does not reveal any value is that learners' performance in that question is equal. Either they performed equally well or not better.

The confidence level is 95%. A p-value of $P < 0.05$ signifies a significant effect in the test. In the table, school 1 is named "experimental group", and school 2 is "control group" in the interpretation of the results of each question from each school. Table 5.14 presents Q1a and Q1b, as well as the post-test results. As previously noted, the coefficient values that are not provided indicate that the two study groups performed similarly.

5.11.1 Item analysis of pre-test and post-test for experimental group and control group

Table 5.14: Q1a – Q1b

	School 1 Experimental Group			School 2 Control group		
Questions	**Pre-test**	**Post-test**	**P-Value**	**Pre-test**	**Post-test**	**P-Value**
Q1a	.4873205	-	0.055	.8163358	-	0.000
Q1b	2.530228	-	0.000	1.791183	-	0.000

In the pre-test, Q1a assessed learners' ability to find the gradient of a line. The results showed a significant difference between the two groups ($p < 0.05$). The experimental group performed better (coefficient = 0.487) than the control group (coefficient = 0.816), indicating that the control group struggled with this concept. Conversely, in Q1b, which tested the application of the gradient, the Control group outperformed the experimental group (coefficient = 1.791 vs. 2.530, $p < 0.05$). This suggests that the experimental group faced greater difficulty in applying the gradient to solve problems.

The post-test results showed no significant difference between the two study groups in Q1a and Q1b, indicating similar performance. Both the experimental group (coefficient = -) and the control group (coefficient = -) showed equal improvement after the intervention. The post-test demonstrated comparable performance in applying the gradient of a line, with a significant p-value of 0.000 (< 0.05) at a 95% confidence level.

Table 5.15: Q2a – Q2c

	School 1 Experimental Group			School 2 Control group		
Questions	**Pre-test**	**Post-test**	**P-Value**	**Pre-test**	**Post-test**	**P-Value**
Q2a	2.111164	-1.119916	0.000	1.207325	1.769897	0.032
Q2b	1.309915	-1.7566	0.000	1.207325	1.207325	0.000
Q2c	1.309915	-1.7566	0.000	1.207325	1.207325	0.000

In Q2a and Q2b, which assessed learners' ability to calculate coordinates given the magnitude of a line, the experimental group performed significantly worse than the control group ($p < 0.05$), indicating initial difficulties with coordinate determination. Similarly, in Q2c, which required calculating the length of a line, both groups struggled, but the experimental group performed significantly worse ($p < 0.05$).

Following the intervention, the experimental group demonstrated significant improvement. In Q2a and Q2b, they outperformed the control group ($p < 0.05$), suggesting enhanced proficiency in solving coordinate-based problems. Additionally, in Q2c, the experimental group performed far better than the control group ($p < 0.05$), which continued to exhibit difficulties.

The intervention effectively improved the experimental group's performance in coordinate geometry and line length calculations, while the control group continued to face persistent challenges. The results highlight the intervention's success in addressing initial learning gaps.

Table 5.16: Q3a – Q3b

	School 1 Experimental Group			School 2 Control group		
Questions	**Pre-test**	**Post-test**	**P-Value**	**Pre-test**	**Post-test**	**P-Value**
Q3a	0.6605836	(0.6605836)	0.013	1.310626	(1.310626)	0.000
Q3b	1.629707	-1.119916	0.000	1.513077	(1.513077)	0.000

In Q3a, the experimental group (coefficient = 0.661) performed significantly better than the Control group (coefficient = 1.311), with a p-value of 0.013, indicating that the control group had greater difficulty in solving the equation of a straight line. However, in Q3b, the experimental group (coefficient = 1.630) scored lower than the control group (coefficient = 1.513), with a p-value < 0.001, suggesting that the experimental group initially struggled more with this specific task.

In the post-test, the experimental group (coefficient = 0.661) again outperformed the control group (coefficient = 1.513) in Q3a, with a p-value < 0.001, reinforcing that the control group continued to face challenges. For Q3b, the experimental group (coefficient = -1.120) showed significantly better performance than the control group (coefficient = 1.513), with a p-value < 0.001. The negative coefficient for the experimental group suggests that the use of GeoGebra software improved their ability to solve the problem effectively, while the Control group still struggled.

Table 5.17: Q4a – Q4b

	School 1 Experimental Group			School 2 Control group		
Questions	**Pre-test**	**Post-test**	**P-Value**	**Pre-test**	**Post-test**	**P-Value**
Q4a	-	-	-	-	2.02115	0.000
Q4b	-	-1.404558	0.000	-	2.02115	0.000

Question In the pre-test, Question 4a assessed learners' ability to apply the equations of perpendicular straight lines to find the coordinates of a point. The analysis showed no significant difference between the experimental and control groups, with both groups demonstrating similar difficulties ($p < 0.05$, $p = 0.000$). Similarly, Question 4b, which tested the midpoint concept, revealed no significant difference between the groups, with both finding the question challenging ($p < 0.05$, $p = 0.000$).

In the post-test, Question 4a again evaluated learners on perpendicular lines. A significant difference emerged, with the experimental group continuing to struggle as much as the control group (coefficient = 2.02115, $p < 0.05$, $p = 0.000$). However, for Question 4b, which revisited the midpoint concept, the experimental group performed significantly better than the control group (experimental coefficient = -1.404558, control coefficient = 2.02115, $p < 0.05$, $p = 0.000$). This indicated that the control group faced greater difficulties in applying the midpoint concept compared to the Experimental group. (Naido & Kapofu, 2020).

Table 5.18: Q5

	School 1 Experimental Group			School 2 Control group		
Questions	**Pre-test**	**Post-test**	**P-Value**	**Pre-test**	**Post-test**	**P-Value**
Q5	2.530228	1.265738	0.000	-	-0.5610322	0.000

Question 5 (Q5) assessed learners on the equation of a straight line in the pre-test when the lines are parallel to each. The analysis results revealed a significant difference between the two study groups. The experimental group had difficulties in solving the question, similar to those of the control group (as the difficulties for the groups appeared to be the same), with confidence limits. This showed that the groups have difficulties in solving the equation of the straight line when the lines are parallel to each other (Naido & Kapofu, 2020) and not stolen results.

Question 5 (Q5) again assessed learners on the equation of a straight line in the pre-test when the lines are parallel to each. The analysis results revealed a significant difference between the two study groups; the experimental group$(coeffiecient\ value = 1.265738)$ performed below the the control group $(coeffiecient\ value = -0.5610322)$, with the $p - value < 0.05, equals\ to\ 0.000$ at 95% confidence limit. This showed that the experimental group have difficulties in solving the equation of the straight line when the lines are parallel to each (Naido & Kapofu, 2020).

5.11.2 Summary of pre- and post-test results based on scores between the experimental (School 1) and the control (School 2) groups

Table 5.19: ***Summary of pre- and post-test results based on scores between the experimental (School 1) and the control (School 2) groups***

	School 1 Experimental Group			School 2 Control group		
Questions	**Pre-test**	**Post-test**	**P-Value**	**Pre-test**	**Post-test**	**P-Value**
Q1a	0.4873205	-	0.055	0.8163358	-	0.000
Q1b	2.530228	-	0.000	1.791183	-	0.000
Q2a	2.111164	-1.119916	0.000	1.207325	1.769897	0.032
Q2b	1.309915	-1.7566	0.000	1.207325	-	0.000
Q2c	1.309915	-1.7566	0.000	-	-	0.000
Q3a	0.6605836	-	0.013	1.310626	-	0.000
Q3b	1.629707	-1.119916	0.000	1.513077	-	0.000
Q4a	-	-	-	-	2.02115	0.000
Q4b	-	-1.404558	0.000	-	2.02115	0.000
Q5	2.530228	1.265738	0.000	-	-0.5610322	0.000
Total	**12.5690611**	**-5.891852**		**7.8458718**	**5.2511648**	

The results showed the analysis of the two study groups, revealing different scores between the pre-test and post-test for the total results. The experimental group's results showed a significantly different score between the pre-test and post-test $p{-}value$ = 0.000 less than 0.05 at a 95% confidence level. The result coefficient value of the $(coeffiecient\ value = 12.5690611)$ s showed that, in total, the questions were difficult for them to answer during the pre-test section. However, the coefficient value of the negative value of $(coeffiecient\ value = -5.891852)$ shows that the questions became much easier to solve (Korstjens & Moser, 2018) after the implementation of the GeoGebra software.

On the other hand, the control group showed a slightly different score between the pre-test and the post-test (p-$value$ = 0.000), suggesting an improvement in the post-test compared to the experimental group ($p{-}value$ =). This indicates a significant improvement in the post-test with a p-value of 0.000 in the control group. The results of the experimental group for Q1 indicated that learners improved moderately well in solving for the gradient of a line or between two points on the Cartesian plane due to the intervention.

The analysis of the results for Q1b in the table revealed an appreciable improvement in the coefficient value ($coeffiecient\ value = 2.530228$) of the pre-test for the experimental group in circumstances where learners had similar performance according to the system in the post-test. Unlike the control group, in the pre-test, learners found Q1b moderately challenging, with a coefficient value of ($coeffiecient\ value = 1.791183$) $2.530228, which$ is better than the experimental group, who found the coefficient value of Q1b to be 2.530228. This indicates more difficulty for learners in the experimental group, where $p{-}value$ = 0.000, which is the same at a 95% confidence level for all groups. Therefore, the control group indicated a statistically significant improvement in the post-test compared to the experimental group, which showed lesser improvement in the post-test.

The analysis of the results in the table above for the experimental group between the pre-test and the post-test for Q2a revealed a statistically significant ($coeffiecient\ value = 1.309915$) difference in the pre-test, which was simpler to deal with in the post-test, with a coefficient value ($coeffiecient\ value = -1.119916$) indicating it was easy to solve, also with $p{-}value$=0.000 at 95% confidence level for the experimental group. On the other hand, the control group's results analysis also indicated a statistically significant difference between the pre-test and post-test $p{-}value$=0.032 below the $p{-}value$ 0.05 at a 95% confidence level. The results showed an improvement in the two study groups in the post-test, suggesting that the learners still had difficulties with calculating the length between two points on the Cartesian plane. Though the two study groups improved significantly, the experimental group showed improved with a coefficient value of ($coeffiecient\ value = -1.119916$) more than the control group with a coefficient value of ($coeffiecient\ value = 1.769897$) showing a challenging problem for learners the control group. This also suggests that the GeoGebra intervention had a positive impact on improving the learners' performance in Q2a in calculating the length/distance of a line. However, a bit of challenge still exists in the experimental group.

The data in Table 5.19 for Q2b of the experimental group revealed a significant difference $p\text{−}value$=0.000 below the $p\text{−}value$ 0.05 at a 95% confidence level between the pre-test of the coefficient value of $(coeffiecient\ value = 1.309915)$ and the post-test coefficient value of $(coeffiecient\ value = -1.7566)$. This indicates that the question was very difficult for learners in the pre-test session and became much easier for them in the post-test session after the intervention. Similarly, the analysis of the control group's results between the pre-test and post-test indicated also significant difference of coefficient value of $(coeffiecient\ value = 1.207325)$ in the pre-test and of $p\text{−}value$=0.000 below the $p\text{−}value$ 0.05. Therefore, the results for Q2b showed a significant improvement in the post-test for the experimental group compared to the control group, suggesting that the intervention positively impacted the learners' ability to calculate the coordinates of a point in the Cartesian plane in the experimental group.

The data for Q3a that were analysed for the two study groups revealed a statistically significant difference between the pre-tests and post-tests. Whilst the experimental group had a p-value (p-value=0.013), the control group also had a p-value $p\text{−}value$=0.000 below the $p\text{−}value$ 0.05 at a 95% confidence level. The results of the two study groups indicated a significant improvement in the post-test. Though the two study groups showed a significant improvement in the post-test, the experimental group, with a coefficient value of $(coeffiecient\ value = 0.6605836)$ improved better than the control group, with a coefficient value of $(coeffiecient\ value = 1.310626)$ suggesting the intervention had an effect in improving the learners' calculating the equation of a straight line.

The analysis of the results for Q3b in terms of p-values also revealed no statistical significant difference between the pre-test and post-test for the experimental group $p\text{−}value$=0.000 below the $p\text{−}value$ 0.05 at a 95% confidence level. However, the coefficient of each group showed significant improvement between the pre-test and post-test. In the question, the experimental group has a significant value of $(coeffiecient\ value = 1.629707)$ in the pre-test to $(coeffiecient\ value = -1.119916)$ showing much easier for learners in the post-test. On the same page, the control group's results of the coefficient value of $(coeffiecient\ value = 1.513077)$ in the pre-test and a value $(coeffiecient\ value = -)$ where all learners scientifically showed the same level of performance in the post-test indicated a statistical significant difference between the pre-test and post-test scores $p\text{−}value$=0.000 below the $p\text{−}value$ 0.05 at 95%

confidence level. This suggested an improvement in the post-test. The analysis of the results revealed a significant improvement in the post-test of the experimental group when compared to the control group. The results suggest that the intervention positively impacted the experimental group compared to the control group (with no intervention) in calculating the equation of a straight line on the Cartesian plane in Analytical Geometry.

The analysis in Q4a showed that the learners showed the same level of performance in both pre-tests in both groups and of the p-value=0.000 below the $p\text{–}value$ 0.05 at 95% confidence level. However, the control group still grappled with the question (Q4a), hence the coefficient value of ($coeffiecient\ value = 2.02115$). This indicated that the question of the application of gradient in solving the equation of a straight line on the Cartesian plane was difficult to solve for the group.

The analysis of the results for Q4b in terms of p-values revealed no statistical significant difference between the pre-test and post-test of the experimental group $p\text{–}value$=0.000 below the $p\text{–}value$ 0.05 at a 95% confidence level. The result of the pre-test showed the same level of performance in both pre-tests in both groups. However, the experimental group showed tremendous performance during the post-test section with a coefficient value of ($coeffiecient\ value = -1.404558$) indicating easier for learners to solve than the control group with a coefficient value of ($coeffiecient\ value = 2.02115$) which indicated more difficulty for learners in the control group to solve. Therefore, the results for Q4b showed a significant improvement in the post-test for the experimental group, suggesting that the GeoGebra intervention positively impacted the learners' understanding of how to apply the properties of quadrilaterals to solve for the coordinates of a point using the midpoint concept.

The analysis of the results in Table 5.19 above for the experimental group between the pre-test and the post-test for Q5 again revealed a statistical significance coefficient value of ($coeffiecient\ value = 2.530228$) in the pre-test, which is also proved to be difficult to deal with in the post-test with a value of ($coeffiecient\ value = 1.265738$). This was also indicated with a $p\text{–}value$=0.000 at 95% confidence level. On the other hand, the analysis of the control group's results also indicated a statistical significant difference between the pre-test and post-test at $p\text{–}value$=0.000 below the $p\text{–}value$ 0.05 at a 95% confidence level. The results showed a significant improvement in the two study groups in the post-test, but simpler for a control group with a coefficient value of ($coeffiecient\ value = -0.5610322$) in the post-test, which suggested that the learners understood how to apply the properties of two parallel lines to calculate the gradient and equation of the various sides of a diagram on the Cartesian plane.

In summary, the analysis of the results from the pre-test and post-test in the two study groups revealed a statistically significant difference in scores for the experimental group (p-value = 0.000), which is below the p-value of 0.05 at a 95% confidence level. Additionally, the control group demonstrated a statistically significant difference between the pre-test and post-test, with $p{-}value$ = 0.000, which is below the $p{-}value$ of 0.05 at a 95% confidence level. The results revealed that both study groups improved significantly in the post-test, suggesting that learners generally improved in solving Grade 11 Analytical Geometry. Although the two study groups showed significant improvement, the experimental group demonstrated a greater improvement in terms of coefficient value analysis in the learners' performance in Grade 11 Analytical Geometry compared to the control group. The results suggested that the intervention had a positive impact on Grade 11 learners in the experimental group's understanding of Analytical Geometry of lines on the Cartesian plane.

5.12 CHAPTER SUMMARY

This chapter presented the outcome of the study, first collecting data from the pre-test and post-test (quantitative) the learners wrote for data analysis. Lesson observations and semi-structured interviews (qualitative) were used to determine the reasons behind the poor performance of Grade 11 learners in Analytical Geometry of lines on a Cartesian plane. The pre-test and post-test, in conjunction with semi-structured interviews, lesson observations, a literature review, and a theoretical framework, helped the researcher conduct the analysis. This was done to determine whether there was a significant improvement in learners' marks after the post-test and following the implementation of the GeoGebra intervention. The researcher also considered the categories Attempt to Answer (ATA), Never Attempted (NA), Answered Correctly (AC), and Did Not Answer Correctly (DAC) to analyse the data from questions Q1a-Q5.

At a significance level of $p \leq 0.001$, the analysis revealed a statistically significant improvement from pre-test to post-test, confirming that the intervention had a meaningful effect within the 95% confidence interval. When comparing the two groups, the experimental group outperformed the Control group in the post-test, as evidenced by a marginally lower coefficient value (experimental: -5.891852 vs. Control: 5.2511648, $p < 0.05$). Since a lower coefficient indicates greater ease in solving questions, the experimental group demonstrated significantly better post-test performance than the Control group.

The researcher also presented the qualitative data analysis to identify the underlying factors that contributed to the study's vivid outcomes. The responses from the learners in the semi-structured interview, aimed at ascertaining the fundamental difficulties and mistakes, were analysed for a deeper understanding and interpretation (see Figures 5.1 to 5.8). Further three classroom observations were carried out to assess how teaching and learning take place. Chapter 6 presents the next chapter, which will present findings from both quantitative and qualitative data from the literature and the recommended theories, as well as the recommendations and conclusions of the study.

CHAPTER 6: DISCUSSION OF FINDINGS

6.1 INTRODUCTION

The previous chapter discussed the data analysis of the quantitative and qualitative results. Statistical tables for the pre-test and post-test were provided to justify the outcome of both groups to ascertain which group performed better after the GeoGebra intervention. The results clearly showed that the GeoGebra intervention had an impact on the performance of learners from the experimental group to which the intervention was administered.

In this chapter, the researcher discusses the outcomes of the quantitative and qualitative data collection from Chapter 5. The semi-structured interviews with learners, as well as the field notes and classroom observations made during and after lesson presentations, were discussed in the previous chapter. Additionally, the pre-test and post-test were analysed and discussed in terms of the study's adopted framework. Furthermore, in this chapter, the researcher interprets the contextual framework of the theories and the literature of the study to answer the research questions.

6.2 QUALITATIVE FINDINGS

In this section, the researcher discusses the outcomes of the classroom observation and the semi-structured interviews, based on the qualitative data generated from the two participating groups. The interviews were conducted to ascertain learners' understanding of the various concepts in Analytical Geometry of lines. The interviews were also conducted to reveal the challenges and mistakes they had and committed during answering the pre-test and after the intervention. The observations made in the classroom were intended to understand how learners study Analytical Geometry of lines, which is hindering their performance in exams.

6.2.1 Observing classroom practices

Prior-intervention observation

The teachers, before their lessons, did not assume the positions of good practice teachers, as good practice teachers usually introduce lessons based on previous knowledge/lesson. The teachers just wrote the topic of the day on the chalkboard, with the comment that, "Today we are going to learn Analytical Geometry", and the children will also respond, "Yes, sir". These attitudes of the teachers show they were still using the traditional method of teaching. The teachers are simply the masters of the learning process (Chen, 2025). They instruct learners on what to do and do not allow learners to independently process the acquisition of knowledge. According to Wang (2022), traditional methods limit learners' participation in the learning process, ultimately hindering the cultivation of individual creativity in knowledge acquisition, which in turn leads to poor learner performance.

Learners differed during the delivery of lessons before the intervention by their teachers in both groups. The skills and practices demonstrated by learners during the process were minimal, and only a few paid attention and participated. Most learners were passive, while a few took notes instead of learning from more capable peers and the teacher (Onyishi & Sefotho, 2020, Vygotsky, 1978). This means that teachers did not give attention to low achievers. Teachers were paying more attention to higher-achieving learners at the expense of those with lower achievement.

Teachers asked questions to get the correct answers from learners and selected learners whom they thought could answer the questions correctly. When an incorrect answer is given, they fail to follow up on why such an answer was provided, instead correcting the learners without asking them to justify their responses. The teacher's questioning style did not allow them to understand the learners' thinking, thereby hindering their ability to support them effectively.

One can say that lessons were purely teacher-centred, with very little participation from learners (Chen, 2025). Very few learners practised what the teachers shared during lessons before the intervention. According to Van Geel et al. (2022), if educational systems could function effectively, there would be a greater emphasis on subject-specific expertise and knowledge of the content. The ineffective teaching of Analytical Geometry of lines before the implementation of the intervention revealed that teachers lacked a strong command of the content (Graham et al., 2021). The teachers could not apply multiple differentiation in the teaching methodologies, which could bring every learner on board (Jager et al., 2022). In terms of interaction, there was minimal and little help from capable learners to support weaker ones (Vygotsky, 1978) before the intervention.

Mid-intervention findings

During pre-intervention observation, most slow learners were marginalised. They were less involved in classroom interactions/activities, which prevented them from achieving well in the subject under discussion. However, learners began to get motivated during the introduction of the GeoGebra software intervention (Sudihartinih & Wahyudin, 2019). The teachers who were not grouping learners during the pre-intervention time now know how to group learners for the GeoGebra intervention. Learners now interact with each other to gain a deeper understanding of the topic. Learners now enthusiastically participate in the lesson being delivered at that moment (Sugandi & Bernard, 2020). Enthusiasm from the learners during the intervention indicated learner participation learning or learner-centeredness (Ma & Yang, 2022). The GeoGebra application has brought creativity to the teaching of Analytical Geometry of lines, as learners can now voluntarily answer questions under discussion (Pamungkas & Nugroho, 2020). At this time, no learner is seen sleeping or bored. This reason is that GeoGebra catches the visual sense of learners, which enthuses them to express their understanding of the concept being taught (Zulnaidi et al., 2020)

Post-intervention observation

The intervention transformed the classroom dynamics, with teachers fostering a mutually respectful and engaging learning environment. By ensuring an equitable distribution of questions, they actively involved all learners, enhancing participation and motivation. The focus on skill and knowledge development during the intervention not only strengthened the teachers' confidence in delivering Analytical Geometry and related mathematical concepts but also amplified their ability to inspire students.

The integration of GeoGebra as an instructional tool immediately captured students' interest, particularly in the experimental group, where heightened engagement led to deeper comprehension (Sugandi & Bernard, 2020). Observations confirmed that the intervention refined teaching practices, equipping educators with innovative strategies such as interactive visualisations and collaborative group work (Sudihartinih & Wahyudin, 2019). This shift not only supported struggling learners through peer-assisted learning but also made the classroom experience more dynamic and enjoyable for both students and teachers.

GeoGebra’s visual and interactive approach proved instrumental in solidifying learners' grasp of Analytical Geometry concepts. As Ma and Yang (2022) emphasise, geometry’s appeal lies in its ability to engage multiple senses, visual, intuitive, and aesthetic, by connecting abstract concepts to real-world contexts. The use of relatable shapes and dynamic constructions sustained student interest, reinforcing understanding and retention. Overall, the intervention revitalised the teaching and learning process, making it more effective, inclusive, and stimulating.

The data, which was collected during classroom observations before, during and after the intervention, are to answer the research objectives as follows:

- To understand how teachers teach Analytical Geometry using GeoGebra
- To establish the benefit of using GeoGebra to teach Analytical Geometry
- To identify the challenges experienced by teachers using GeoGebra to teach Analytical Geometry
- To suggest how teachers can use GeoGebra to teach Analytical Geometry.

The pre-, prior, and post-intervention periods, during which classroom data were collected and their emerging themes discussed, are presented below.

6.2.2 Teaching methods

Before the intervention, teaching practices in both groups were predominantly teacher-centred, hindering learners' ability to grasp Analytical Geometry concepts. This lack of dynamic and engaging pedagogical delivery contributed to poor learner performance (Luneta, 2015). Misconceptions often arose because students struggled to understand the instructional strategies employed by teachers (Luneta, 2020), leading to minimal participation in classroom discussions and activities.

Research in South Africa highlights a critical factor behind this underperformance: many mathematics teachers lack sufficient content knowledge (Zulnaidi et al., 2020). Effective mathematics instruction requires teachers to have a strong command of the subject matter (Manigbas et al., 2024). Without this foundational knowledge, teachers were unable to employ diverse teaching strategies, resulting in disengaged learners and uninspiring classroom interactions during the pre-intervention phase.

Classroom observations further revealed that teachers rarely assessed learners' prior knowledge, preventing them from identifying and addressing misconceptions in Analytical Geometry. This oversight limited their ability to diagnose learners' challenges when working with lines on the Cartesian plane. The observations aimed to evaluate teachers' content mastery and its impact on learners' comprehension and performance.

Following the intervention, teaching methods shifted to include whole-class and small-group discussions, which proved more effective in capturing learners' attention. These strategies increased engagement, with more learners actively participating in discussions. Improved instructional approaches sparked greater interest in Analytical Geometry (Copur-Gencturk & Li, 2023), fostering enthusiasm and classroom involvement. Additionally, the refined teaching strategies enhanced learners' independent problem-solving skills (Manigbas et al., 2024), demonstrating the positive impact of the intervention.

6.2.3 Classroom interaction

The teachers of both groups encouraged learners to actively participate in all classroom discussions. They ensured that each learner in the class understood the concepts of Analytical Geometry. The teachers' main objective was to provide the necessary support to all learners so that they could engage in meaningful learning in their everyday practice of Analytical Geometry of lines on the Cartesian plane (Vygotsky, 1978). The teacher's lesson delivery during the intervention effectively supported the environment, encouraging learners to express their understanding.

The classroom interaction took the questioning style form, which the teacher of the experimental group adopted during the intervention to understand what learners knew or did not know, to ascertain the type of assistance they could provide (Vygotsky, 1978). The teachers asked questions which probed learners' understanding of Analytical Geometry and gave support where necessary (Hamdunah & Imelwaty, 2019). However, the two teachers' teaching did not support a progressive learning environment prior to the intervention. Instead, the teachers paid attention only to more capable learners and neglected the slower learners. Only those learners who raised their hands intending to answer the questions were considered. Whilst the control group teacher continues to teach Analytical Geometry of lines in the best way possible.

During the intervention, the teacher of the experimental group asked questions that involved all learners, providing a supportive and progressive environment for all students in the classroom (Sugandi & Bernard, 2020). During the intervention, the teacher from the experimental group did not just listen to learners' answers, he/she followed up with probing questions to ascertain whether the particular learner's correct answer came from the Analytical Geometry of lines concept or if the wrong answer given by the learner was followed to ensure that they are on the right path. Whilst the control group continue his/her normal teaching strategy. The teacher was testing learners' genuine understanding of concepts to determine whether they had any misconceptions about Analytical Geometry. The effective interaction between the teacher and learners during the intervention appeared to have developed learners' interest in Analytical Geometry of lines. The teacher's and learners' interaction was improved during the intervention.

6.2.4 Teaching and learning resources

Teachers employed similar resources, such as textbooks, but adopted diverse instructional approaches to deliver content. Effective utilization of resources is crucial for improving learner performance (Miremba, 2024). Etienne et al. (2023) emphasise that teaching and learning materials, including textbooks, charts, and three-dimensional aids, play a crucial role in enhancing academic outcomes when utilised effectively. Conversely, Zottor et al. (2022) emphasise that ineffective resource use hinders mathematics performance.

A lack of essential resources, such as textbooks, limits effective mathematics instruction. To reinforce learning, additional exercises from various books on Analytical Geometry should be integrated into classwork and homework. Modern tools, such as GeoGebra and projectors, were instrumental in clarifying concepts like slope, equations of straight lines, and parallel/perpendicular lines. These technologies simplified the teaching of Analytical Geometry during the intervention, ultimately improving learner performance.Technology like GeoGebra has provided teachers with an important tool for use in mathematics (Sugandi & Bernard, 2020). It makes learning fun, interesting and more effective than just using textbooks, worksheets and handouts (Zulnaidi et al., 2020). These teaching materials should complement each other to make learning effective.

Teachers ought to teach the mathematics subject well, understand the learners' ways of thinking, diagnose the learners' takes and their sources, and display knowledge of various alternative ways of presenting the topics. Teachers must interact with all the learners to understand their responses, probing them to understand what learners thought about the concepts. Again, this will allow teachers to diagnose the mistakes learners make in Analytical Geometry and address them accordingly.

6.2.5 Classroom assessment

Classwork and homework informed teachers and learners of the decisions about classroom instruction during teaching. Classwork and homework help to check whether learners have absorbed the knowledge and concepts the teacher is imparting. It also helps to check whether the methodology the teacher uses in teaching makes an impact. Providing learners with activities also helped the researcher collect data for both the pre-test and post-test, as well as offer feedback on what had transpired during the intervention period (Ozkan et al., 2018; National Council of Teachers of Mathematics, 2020). Meanwhile, the two teachers used these exercises to measure the understanding level of learners in Analytical Geometry of lines and ascertain learners' progress in mathematics. The assessments also helped teachers make informed decisions about learners' progress and enhance learners' performance (Bosman & Schulze, 2018; Zulnaidi et al., 2020).

In terms of feedback to learners, the two teachers employed different approaches. The teacher of the experimental group allowed learners to present their solutions on the board for all to discuss and deliberate on, aiming to determine how much they had improved (Wisniewski et al., 2020). However, the teacher in the Control group provided feedback on the chalkboard without considering whether the learners had grasped the concepts or not. The teacher in the experimental group seemed to understand the importance of feedback, as it helped him uncover the learners' challenges and mistakes underlying the concepts taught during the intervention. The feedback of the teacher in school is more learner-centred (Winstone et al. 2021). However, the teacher in the Control group seemed not to have understood his/her learners' challenges, as evidenced by the solutions provided. The teacher did not use the feedback session to understand the challenges faced by learners and their mistakes. This attitude made it difficult for the teacher to be informed about what learners grasped in terms of the concepts of Analytical Geometry of lines.

Additionally, during the intervention, the teacher in the Control group was unable to monitor learners' progress due to their passivity during teaching, learning, and feedback sessions (Chen, 2025). However, the teacher from experimental group was able to monitor learners' progress during the intervention, due to the learners' active participation during lessons and feedback sessions. The learners voluntarily observed, provided solutions, and asked each other questions during the feedback to justify their answers. If teachers can monitor learners' progress during assessments, they can receive ongoing feedback (Burns et al., 2021). Teachers should correct learners' tasks with them to help identify the difficulties that may arise.

6.3 SEMI-STRUCTURED INTERVIEWS WITH TEACHERS

In the methodology section of Chapter Four, it was noted that the researcher conducted semi-structured interviews with two teachers to explore the instructional methods and strategies they had used prior to the intervention, which may have contributed to learners' difficulties in analytical geometry and mathematics overall. These interviews enabled the researcher to refine questions and clarify responses in real-time as the discussion progressed. Additionally, interviews were carried out during and after the intervention. The mid-intervention discussions aimed to examine shifts in teaching approaches and their immediate effects on classroom instruction. The post-intervention interviews, meanwhile, assessed the intervention's long-term influence on learner performance, providing insights into its overall efficacy.

The researcher sought to answer the following objectives:

- To understand how teachers teach Analytical Geometry using GeoGebra
- To establish the benefit of using GeoGebra to teach Analytical Geometry
- To identify the challenges experienced by teachers using GeoGebra to teach Analytical Geometry
- To suggest how teachers can use GeoGebra to teach Analytical Geometry

The researcher deliberated on the outcomes of the semi-structured interviews.

6.3.1 Grade 11 learners' performances in mathematics

The pre-test results show that the performance of both groups was not good. The data also revealed other factors that contributed to the poor performance of mathematics learners of Analytical Geometry. The factors included the mathematical content knowledge possessed by the teachers (Van Geel et al., 2023), the learners' attitude towards mathematics, the background of the children, and thus poverty, as well as teacher-learner material (TLM).

A question comprises two questions that require grade 11 learners to determine whether the two lines are parallel to each other or not. Similar solutions were provided by some Grade 11 learners, L10S2 and L16S1, and several learners provided solutions similar to those of L10S2 to the question. Learners must compare by first finding the gradients of the two lines and giving their conclusion. However, most could not provide the answer during the pre-test.

The challenges and mistakes presented by the researcher to learners on the test items were similar to those committed by other learners (e.g., L24S2 and L34S1) during the pre-test session, and they provided similar explanations. Most learners faced a similar problem when solving questions, namely being unable to substitute the coordinates into the correct formula (Taylor & Francis, 2020). L10S2, L16S1, L14S2, and L28S2 (see Figures 5.5 and 5.6) were unable to write the formula for the gradient (Luneta, 2020). Learners who were able to write the formula (L24S2 and L34S1) could not substitute the co-ordinates correctly into the formula for correct algebraic manipulation. L14S2 added (+) as the numerator of the gradient instead of minus (-). Hence his/her calculation. The above challenges and mistakes surfaced during the pre-test. The learners and the above writers confirmed that challenges and mistakes exist in the Analytical geometry of lines on the Cartesian plane.

Firmanti et al (2025) states that the lack of understanding in learning geometry often causes discouragement among students, which invariably leads to poor performance. The poor performance in mathematics was due to all the factors stated earlier, like the learners' attitudes, teachers' teaching methods, and the children’s backgrounds. Hanfi (2008), as cited in Makondo and Mankondo (2020), attributes the fear of mathematics to “Mathophobia,” which leads to poor performance and negative attitudes. Dapaah and Agyei (2018) asserted that several reasons contribute to learners performing poorly in mathematics, including attitude, confidence, interest, and the perception of the usefulness of mathematics. Learners with a negative attitude towards mathematics topics, such as Analytical Geometry, tend to perform poorly (Dapaah & Agyei, 2018).

Passing mathematics greatly depends on the teacher's method and strategy in delivering the subject matter. Most teachers still dominate the lesson delivery, when, in fact, learners are supposed to be the dominant force in grasping the necessary concepts of the topic (Chen, 2025). For learners to perform well, experts and pedagogical content knowledge teachers who can effectively handle the topics and understand their learners are a must. Jensen et al. (2016) emphasised that the teacher's expertise and content knowledge should be interwoven with their pedagogical knowledge as this will help learners understand the topic. Teachers who are not well-equipped find it challenging to support learners (Manigbas et al., 2024; Jensen et al., 2016). Hua et al. (2018) notice that in teaching Analytical Geometry and equations of lines on the Cartesian plane, teachers should indulge in the practical teaching of the topic.

According to Hua et al. (2018), books promote the practicality of learning mathematics by featuring colourful visuals and pictures that draw learners' attention to the practical aspects of life, making them more likely to remember what they have learned. In the same vein, Ma & Yang (2022) affirm that visual and practical textbooks, with their rich background, make learning mathematics presentable and interesting. The author also notes that books include engaging exercises and effectively integrate technology.

It is also a problem in many developed countries, such as the USA and other European countries, when learners perform poorly (Gupta, 2019). According to Mohammadpour and Yon (2024), poor performance in mathematics is most prevalent in low-income communities. In Malaysia, rural communities often struggle with mathematics, similar to the poor communities in South Africa. The dynamic handling of maths often develops a learner's interest in the subject. Unfortunately, that is what most of our teachers lack in the delivery of Analytical Geometry (Wang, 2022). Most teachers fail to make the lessons on Analytical Geometry of lines on the Cartesian plane practical, which makes the topic difficult for learners to grasp. According to Reddy (2006), as cited in Makofane and Maile (2019), Africans experience the poorest performance in mathematics, particularly in Analytical Geometry. Similarly, Gupta (2019) reveals that most learners in low-income communities in the US demonstrated a lower fundamental level of mathematics achievement. In SA, ex-model C schools generally obtain better mathematics results (Van Geel et al., 2022).

6.3.2 The learners' knowledge of mathematics

The performance of learners in both groups before the intervention showed a lack of mathematical understanding of the concepts of Analytical Geometry of lines. Seloraji and Kwan (2017) identify numerous mistakes learners make in Analytical Geometry, which is why they are highlighted. These displays of learners' conceptual knowledge deficiencies (Luneta, 2020) were evident during the classroom observations before the intervention, where learners were passive recipients of their teacher's instruction. It was noted that the teachers' methods did not directly involve learners (Sun, 2019), and this lack of involvement resulted in their poor performance. Brod (2021) argues that learners taught how to develop conceptual comprehension can formulate knowledge into a complete whole and connect new and old ideas. This was not in line with what learners displayed before the intervention.

Although the learners were deficient in conceptual knowledge of Analytical Geometry of lines prior to the intervention exercise, there was a remarkable improvement during their interventional period when the teacher of the experimental group implemented GeoGebra. The teachers' approaches towards teaching during the intervention aligned with learners' cognitive knowledge

(Bosman & Schulze, 2018) and reflected the constructivist belief that learners should be allowed to construct their understanding of knowledge. The intervention appeared to have assisted learners in developing conceptual knowledge, which often worked in conjunction with the procedural knowledge gained during the intervention.

Procedural fluency builds on a basic conceptual understanding and strategic reasoning and is probably the one in which learners grasp the concept of the new knowledge more (Simonsmeier et al, 2021). The learners' involvement during and after the intervention improved their understanding of the concepts of Analytical Geometry of lines, which signified the effectiveness of the GeoGebra intervention in learning the Analytical Geometry of lines on the Cartesian plane.

6.3.3 The learners' commitment

The learners from the two groups appeared not to be serious or committed to learning mathematics, especially before the intervention. Their non-seriousness led to their poor performance. The non-seriousness could be due to the teachers' attitude, perception, and ineffective methods (Agyei & Dapaah, 2018). Learners' seriousness towards classwork and school activities in general was appalling during the pre-intervention period, hence their poor performance. Learners' background, including their environment, the educational background of their parents, as well as their society and parents' economic background, are all factors that can affect their performance in mathematics (Gupta, 2019).

During the intervention, the learners from the experimental group were motivated. They had an interest in mathematics due to how teachers introduced and handled the pedagogical aspect of Analytical Geometry of lines (Zulnaidi et al., 2020). During the intervention, learners were supported and encouraged to practice mathematics, especially the Analytical Geometry of lines, and they submitted their classwork. All of them showed interest in every activity, culminating in their improvement in mathematics. Shulman & Shulman (2017) reiterate that effective schools provide a supportive environment to foster academic learning, boost learners' confidence, and ensure the well-being of their learners is good. The learners from the experiemental group seemed to have gained self-confidence and were motivated to do mathematics during the interventional process. Studies have indicated that motivation goes hand in hand with learners' performance (Ferlazzo, 2025).

6.4 THE QUANTITATIVE RESULTS

The quantitative scores were achieved through the combination of Excel and Stata, which comprises Stata release 17. The P-value between the pre-test and post-test scores of both groups is discussed below. The improvement of scores by the schools upon the implementation of the intervention is discussed below. The challenges and mistakes of various concepts are also discussed.

6.4.1 The pre- and post-tests

The rationale for administering the pre-test and post-test was to measure the level of improvement the intervention had chalked for learners' performance. It is also to determine whether the correction of the challenges and mistakes identified in the pre-test by learners has been effective during the post-test through the implementation of the intervention. Stata release 17 is a statistical software package which was used to analyse the data. This was used to determine whether there is a significant difference between the coefficients of the two groups that may be due to Analytical Geometry concepts. This is used in hypothesis testing to determine whether the research process has an effect on the population or if there will be a statistically significant difference between the research groups (Bevans, 2020). The researcher, therefore, declared significant if p < 0.001. The analysis of learner performances per item, before and after the pre-test and post-test, is presented below.

6.4.1.1 Question 1 results before and after intervention strategy

The outcome of the pre-test analysis showed that, in the pre-test section, the control group $coeffiecient\ value = 2.60751888)\ participants$ performed significantly differently from the experimental group $(coeffiecient\ value = 3.0175485)\ with\ the\ p - value < 0.05, p$

$0.0321\ at\ 95\%\ confidence\ limit$. The results indicated that the control group performed better than the experimental group before the intervention, which depicted a lower coefficient value. The learners in the groups showed conceptual difficulties in answering questions Q1 with the positive coefficient. These displays of learners' conceptual knowledge deficiency (Luneta, 2020) in the concept of the gradient in Analytical Geometry appeared to have caused their underperformance during the pre-test section.

The post-test results after the implementation of the GeoGebra intervention revealed no significant difference between the experimental and control $(coeffiecient\ value = -)$ for both groups in terms of performance. The results indicate that the two groups performed exceptionally well in the concept of a line's geometry. This means that learners can now organise their knowledge of the gradient of a straight line logically.

6.4.1.2 Question 2 item results before and after intervention strategy

The outcome of the pre-test analysis again showed that they control group $coeffiecient\ value = 3.621975)$ performed significantly differently from each other experimental group $(coeffiecient\ value = 4.730994),\ with\ p - value < 0.05, p = 0.003\ at\ 95\%\ confidence\ limit)$ in terms of their coefficient values. The results indicated that the control group performed better than the experimental group before the intervention, as evidenced by a lower coefficient value. The learners in the groups exhibited conceptual difficulties in answering question Q2 with positive coefficients before the intervention. This difficulty may be due to a lack of conceptual understanding of the Analytical Geometry of lines. Firmanti et al (2024) stated that the lack of understanding in learning geometry often causes discouragement among students, which invariably leads to poor performance.

The post-test results in Q2 show a significant difference between the experimental group and the control group, with the experimental group performing better than the control group at a 95% confidence level. The negative coefficient of the experimental group indicates that the learners in the group have acquired the conceptual knowledge to calculate the coordinate value in the Cartesian plane, given the magnitude of the length, following the implementation of the intervention. NCTM (2020) indicated that conceptual knowledge involves knowledge of concepts in which related concepts are understood in a relationship of coordinate values. Learners from the experimental group demonstrated a greater understanding of the conceptual aspects of Analytical Geometry of lines on the Cartesian plane than learners in the control group.

6.4.1.3 Question 3 item results before and after intervention strategy

The outcome of the pre-test analysis showed that, in the pre-test section, they control group $(coeffiecient\ value = 2.823703)$ performed significantly differently from one experimental group $(coeffiecient\ value = 2.2902906), with\ p - value < 0.05, p =$

$0.04\ at\ 95\%\ confidence\ limit\ an$other in terms of their coefficient values. The learners in the two groups exhibited conceptual difficulties in answering question Q3 with positive coefficients. These displays of learners' conceptual knowledge deficiency (Wang, 2025) in the concept of the gradient in Analytical Geometry appeared to have caused their underperformance during the pre-test section.

The post-test revealed a significant difference in performance between the two schools in Q3. The experimental group showed a clear conceptual understanding of the question ($coeffiecient\ value = -0.4593324$) compared to the control group ($coeffiecient\ value = 2.823703), with\ p - value < 0.05, p = 0.000\ at\ 95\%\ confidence\ limit$). Pamungkas, Rahmawati & Dinara (2020) stated that GeoGebra is an effective tool for teaching and learning space geometry that can improve the achievement of learning space topics in geometry. The experimental group teacher with the GeoGebra intervention seemed to have his/her learners grasp the needed concept of the equations of a straight line better than the control group.

6.4.1.4 Question 4 item results before and after intervention strategy

The outcome of the pre-test analysis showed that, in the pre-test section, the control group and experimental group assessed learners to apply the equations of perpendicular straight lines to find the coordinates of a point in a diagram in the pre-test. The analysis results revealed no significant difference between the two study groups ($coeffiecients\ value = -$), this means that learners' responses were almost the same throughout the question, and their answers were of equal weight (Mudaly & Fletc,her 20,19) which they did not perform well during the pre-test section. the experiemental group and the control group found the question difficulties. This showed that the groups appeared to have difficulties in solving the equation of the straight line. Learners, due to this resulted in committing all kinds of mistakes, which led to their poor performance (Makhubele, Nkhona, & Luneta, 2020).

The post-test revealed a significantly different performance after the intervention in the analysis. The experimental group ($coeffiecient\ value = -1.404558$) of the intervention yet again performed better than the control group ($coeffiecient\ value = 4.0423, with\ p - value < 0.05, p = 0.000\ at\ 95\%\ confidence\ limit$). Learners in experimental group were able to apply competency strategies to algebraically manipulate and use ideas to apply the equations of perpendicular straight lines to find co-ordinate of a point in a diagram in the post-test to solve the question (Garg, 2017).

6.4.1.5 Question 5 item results before and after intervention strategy

The outcome of the pre-test analysis again showed that, the control group ($coeffiecient\ value = -$) performed significantly differently from the experimental group ($coeffiecient\ value = 2.53022, p = 0.03134\ at\ 95\%\ confidence\ limit$ in terms of their coefficient values. The question assessed learners on the equation of a straight line when the lines are parallel to each. The learners in the groups exhibited conceptual difficulties in answering question Q5 in the pre-test, as indicated by the positive coefficients before the intervention. This difficulty might be the absence of a conceptual understanding of the Analytical Geometry of lines. Firmanti et al (202) states that the lack of understanding in learning geometry often causes discouragement among students, which invariably leads to poor performance. This indicates that learners from both schools lacked the necessary concept of parallel lines before the intervention, which prevented them from passing Analytical Geometry.

The post-test revealed a significantly different performance between control group ($coeffiecient\ value = -0.5610322$) performing better than the experiemental group ($coeffiecient\ value = 1.265738), with\ p - value < 0.05, p = 0.000\ at\ 95\%\ confidence\ limit$) difference in Q5. The experimental group still grapple with the equation of a straight line when the lines are parallel to each other after the intervention. This means that there is a need to find a solution to mitigate the gap which is being created. The practice of teachers not being abrasive with the content of knowledge and its dynamism in pedagogical delivery makes it impossible for learners to perform well in Analytical Geometry of lines (Dinayusadewi & Agustika, 2020). Teachers must use a combination of teacher-centred and learner-centred approaches to provide the learners with direction to their understanding by using the GeoGebra software in teaching Analytical Geometry (Sugandi & Bernard, 2020).

6.5 IMPROVEMENT OF THE LEARNERS' PERFORMANCE

This study aimed to investigate the effectiveness of the GeoGebra intervention in teaching Grade 11 Analytical Geometry of lines on the Cartesian plane, to enhance learners' performance. After implementing the GeoGebra intervention with the experimental group, there was a positive impact on the learners' performance in Analytical Geometry of lines in the Cartesian plane of Grade 11 group. In this regard, there was a significant improvement in answering questions about the gradients of a line, the distance of a line, the coordinates of a point on the Cartesian plane and the equation of a straight line. As noted earlier, the pre-test results showed that learners in both groups experienced difficulties in solving Analytical Geometry problems prior to the intervention. As a result, the two groups appeared to lack conceptual and procedural knowledge in solving Analytical Geometry concepts (Lunata 2020). The questions in the pre-test contained concepts from lower grades, such as grade 8, but the Grade 11 Analytical Geometry learners were still unable to obtain correct answers. No learner was able to answer Van Hiele's level 4 (Formal Deduction) question in the pre-test session (Yudianto et al. 2018).

As mentioned earlier, the results of the post-test showed an improvement in the Analytical Geometry score difference between and within the two groups, as indicated by the pre-test and post-test results. The post-test results indicated a statistically significant difference (p $-value=$ 0.0001) in the significant confidence level of the two groups. Although the experimental group did not perform well in the pre-test, they performed strongly in the post-test in terms of performance compared to the control group, due to the effectiveness of GeoGebra.

6.6 ANSWERING THE RESEARCH QUESTIONS

This study aims to address the following main research question, as stated in Chapter.

- What is the effectiveness of using GeoGebra to teach Analytical Geometry to enhance learner performance?
- How does the use of GeoGebra as an intervention influence learner performance in Analytical Geometry?

The sub-research questions are used to address the main research question that arises from the objectives of this study. The sub-research questions, as indicated, are answered in the following sections.

6.7 PEDAGOGICAL APPROACHES

The classroom observation revealed that, before the intervention, the practices of both groups of teachers mainly were teacher-centred, which made it difficult for learners to grasp the Analytical Geometry concept being taught. This practice of teachers not being abrasive with the grip of knowledge content and its dynamism of pedagogical delivery makes it impossible for learners to perform well in Analytical Geometry of lines (Dinayusadewi & Agustika, 2020). Learners appeared not to have been adequately supported by the teacher before the intervention (Vygotsky, 1978). In addition, the teachers in both groups did not deem it essential to administer a diagnostic test to learners to ascertain their prior knowledge, which could have provided learners with the right approach to answering each Analytical Geometry concept. Due to this, most learners were unable to participate in classroom activities. They could not engage in classroom discussions during the lessons whenever they were asked questions.

During the intervention, the teacher in the experimental group integrated both teacher-centred and learner-centred approaches to facilitate understanding of Analytical Geometry using GeoGebra software (Sugandi & Bernard, 2020). In contrast, the control group teacher adhered to conventional teaching methods. The experimental group teacher provided hands-on trial questions to assess comprehension, while the control group teacher limited instruction to a single example of the Analytical Geometry of Lines concept.

When learners struggled, the experimental group teacher employed scaffolding (Vygotsky, 1978), allowing students to arrive at solutions independently. Cooperative learning was also utilised, with small groups collaborating under the guidance of more proficient peers (Traver et al., 2019). The teacher intervened only when necessary, reinforcing the process of knowledge construction (Vygotsky, 1978). This approach aligned with constructivist principles, encouraging learners to build their understanding (Bosman & Schulze, 2018).

In contrast, the control group teacher maintained traditional instruction without the use of GeoGebra. The experimental group's interactive environment fostered greater learner engagement, with students questioning and clarifying problem-solving procedures. As a result, participation in the experimental group significantly improved compared to pre-intervention levels.

6.8 POTENTIAL BARRIERS

In teaching and learning, the use of GeoGebra software poses a familiar problem in the study and its framework in particular. During the initial lessons, most learners were not actively involved in the learning process. This is because most of them were unfamiliar with many of the Analytical Geometry concepts and could not communicate effectively to contribute to the lesson. The finding suggests that the GeoGebra software presents additional responsibilities and work-related problems (Uwurukundo, Maniraho & Tusiime, 2020). Apart from teaching, the teachers had another commitment in their respective groups, participating in extracurricular activities such as sports and other events. Working with the GeoGebra software for the first time during the intervention caused some anxiety for the teacher from the experimental group.

GeoGebra enhances reasoning skills, attitudes, and abstract thinking while improving the functions of all six senses, which are crucial for better performance in Analytical Geometry (Wassie & Zergaw, 2019). Additionally, it serves as an effective tool for teaching space geometry, boosting learners' achievement in the subject (Pamungkas, Rahmawati & Dinara, 2020). However, many learners lacked access to devices like tablets or laptops, limiting their ability to utilise GeoGebra effectively.

Teachers in the experimental group faced challenges in differentiating instruction due to rigid assessment schedules (Yuen, Luo, & Wan, 2023). The pressure to complete the Assessment Teaching Plan (ATP) left little time to address individual learning needs. While effective teaching requires varied assessments tailored to diverse learners (Satyarini, Padmadewi & Santosa, 2022), teachers struggled with curriculum demands, divided attention across subjects, and time constraints. As a result, they were unable to design fair, adaptable activities that accommodated different learning abilities. Despite the importance of differentiated mathematics instruction, all learners were assessed uniformly, disregarding their varying capabilities.

6.8.1 Change in learner performance

Analysis of the pre-test, post-test, classroom observations, and semi-structured interviews demonstrated significant improvement in learner performance within the experimental group following the GeoGebra intervention. The post-test results confirmed that the intervention enhanced learners' understanding and application of Grade 11 Analytical Geometry concepts related to lines on the Cartesian plane. Additionally, learners exhibited stronger conceptual comprehension, enabling them to select and justify appropriate procedures for solving problems, as supported by Schunk (2020). These findings underscore the effectiveness of the GeoGebra intervention in fostering both skill development and deeper mathematical reasoning.

Classroom observations conducted during and after the interventions revealed a marked improvement in learner participation, particularly in Analytical Geometry lessons focused on lines in the Cartesian plane. Post-intervention engagement was notably more dynamic and effective, with both the control and experimental groups actively engaging in tasks structured around Vygotsky's scaffolding principles (Traver et al., 2019). Encouraging a productive struggle allowed learners to grapple with complex concepts, thereby fostering a deeper understanding and more meaningful outcomes. Conversely, excessive direct assistance risks cultivating passivity rather than active engagement, ultimately diminishing the efficacy of learning (Sarika, 2020).

The statistical results indicated that the total coefficient for the experimental group performed poorly in the pre-test $(coeffiecient\ value = 12.5690611)$ compared to the control group's percentage before the interventions in the pre-test $(coeffiecient\ value = 7.8458718)$. The results between the two groups in the post-test revealed that the experimental group obtained a total negative coefficient value of $(coeffiecient\ value = -5.891852)$signifying that, after the intervention, the questions were generally easier for them to answer than the control group. Which got a total positive value of $(coeffiecient\ value = 5.2511648)$. The results revealed that the learners in the experimental group had the procedural knowledge to solve Analytical Geometry of lines questions (Shunk, 2020). Moreover, the learners in the experimental group appeared to have gained conceptual and procedural knowledge of Grade 11 Analytical Geometry (Kurt, 2020).

6.8.2 Change in teaching practice

The qualitative data analysis from classroom observations and semi-structured interviews revealed that teachers in both the experimental and control groups modified their teaching practices after adopting new strategies introduced during the intervention (Lim, 2024). This finding aligns with research by Gokalp (2023), which highlights that collaborative environments, such as teacher clusters, facilitate the exchange of expertise, experiences, and resources, ultimately fostering pedagogical innovation.

Moreover, the teachers gained knowledge of how to use questioning and answering with their learners to understand their learners' thinking by prompting questions that clarified their conceptual understanding of Analytical Geometry of lines (Sugandi & Bernard, 2020). The experimental group teacher during the intervention should assess their learners' prior knowledge before the start of the new lesson and should not just look for answers to the questions (Jameel & Ali, 2016). The teacher also takes cognisance of his/her learners' knowledge when asking questions and their thinking abilities.

The teacher from the experimental group also began to involve learning with GeoGebra tools after the intervention, which revealed what he/she gained from the intervention. (Sugandi & Bernard, 2020) assert that intervention strategies develop and support schools with stimuli used for the improvement of a teacher's teaching practice in the process of collaboration with other teachers. Therefore, the teacher's teaching strategies will positively impact their teaching practices through the intervention, as new practices are obtained during the interventional process.

6.9 CHAPTER SUMMARY

This chapter presents the findings from both quantitative and qualitative data collected in the study. The researcher synthesised pre-test and post-test results, classroom observations, semi-structured interviews, literature review insights, and theoretical perspectives to conclude. Quantitative data indicated poor performance in the pre-test, which was further supported by teacher interviews identifying key challenges. Classroom observations before the intervention revealed that learners in both groups struggled with Analytical Geometry, adopting a passive role in teacher-centred lessons focused on procedural problem-solving.

Post-intervention observations showed notable improvements in learner engagement within the experimental group. Learners became more active, asking questions, participating in group discussions, and completing assigned tasks, indicating increased interest and understanding. The use of GeoGebra software enhanced motivation, as learners collaborated, justified solutions, and engaged in peer learning through small-group discussions,a new strategy introduced during the intervention.

Teachers in both groups adopted more interactive approaches, probing learners' reasoning and incorporating relevant prior knowledge before lessons. However, the experimental group teacher employed more scaffolding techniques, making lessons more dynamic and effective. While both groups showed improvement in post-test results, the experimental group performed better than the control group, as confirmed by statistical analysis. The findings suggest that integrating GeoGebra software and learner-centred strategies positively impact performance and engagement in Analytical Geometry analysis.

CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

7.1 INTRODUCTION

The study aimed to investigate mechanisms for improving the performance of Grade 11 learners in Analytical Geometry on the Cartesian plane, a topic widely regarded as challenging (Luneta, 2020). Addressing barriers such as diverse learning environments, language difficulties, and instructional obstacles (Yuen, Luo, & Wan, 2023), the research introduced GeoGebra as an intervention tool to enhance conceptual understanding (Seloraji & Kwan, 2017). Additionally, collaborative learning strategies were implemented to foster peer-supported knowledge acquisition (Pamungkas, Rahmawati, & Dinara, 2020).

The study employed an experimental design, comparing an experimental group (exposed to GeoGebra and collaborative learning) with a control group. Pre- and post-tests were administered to assess performance differences, with statistical analysis revealing distinct outcomes between the two groups. Instructional scaffolding played a key role, with teachers providing structured support, timely feedback (Molina, 2018), and an inclusive learning environment (Meyer & Rose, 2014; Maghfirah & Mahmudi, 2018) to encourage mathematical reasoning and engagement (Gamlem, 2019).

This final chapter synthesises the study's rationale, evaluates the effectiveness of GeoGebra as a teaching strategy, acknowledges limitations, discusses implications of the framework, and proposes recommendations for future research.

7.2 IMPORTANCE AND DESIGN OF THE STUDY

This study investigated whether integrating the GeoGebra application could improve learner performance in Analytical Geometry of lines in the Cartesian plane, guided by Van Hiele's theory of geometric learning. Using a intervention mixed-method design, the research compared pre-test and post-test results between an experimental group (exposed to GeoGebra) and a control group. The study also explored the benefits and challenges of using GeoGebra in teaching Analytical Geometry.

Initial pre-test findings indicated poor performance in both groups, prompting semi-structured interviews with teachers and classroom observations to identify learning difficulties. Following the GeoGebra intervention, a post-test was administered to assess any improvement in learner performance. The study emphasised guiding teachers on leveraging GeoGebra to enhance understanding at each Van Hiele learning level.

7.3 MAIN FINDINGS

The pre-test results indicated that both groups initially demonstrated low proficiency, with a statistically significant difference in performance between them. This highlighted the need for targeted intervention to address their learning gaps. Quantitative data confirmed that learners in both groups performed poorly on the pre-test, reinforcing the necessity for instructional support.

Following the intervention, post-test results revealed a statistically significant improvement in the experimental group, which received the GeoGebra-based instruction. This suggests that the intervention may have enhanced learners' understanding of Analytical Geometry concepts related to lines on the Cartesian plane. The experimental group's progress can be attributed to their engagement with the intervention, which facilitated a deeper conceptual grasp of the subject and encouraged active practice. These findings suggest that the GeoGebra intervention had a positive impact on learner performance, facilitating knowledge acquisition and skill development in Analytical Geometry.

Quantitative and qualitative data collected prior to the intervention revealed significant gaps in learners' mathematical knowledge (DeCarlo, 2022). The teachers' initial approach to teaching Analytical Geometry, primarily through lecture-based methods, proved ineffective, as they spent most of the time delivering content without actively engaging students. This passive instructional style resulted in limited learner participation, with many students remaining disengaged (Tsang, 2020). Additionally, teachers failed to build on learners' prior knowledge to scaffold new concepts, a critical step in effective lesson planning (Piper et al., 2022). The pre-test results further underscored these deficiencies: most students struggled with fundamental concepts in Analytical Geometry and were unable to answer application-based questions, demonstrating a lack of familiarity with the subject matter.

During the intervention of the experimental group, the use of resources such as Grade 11 textbooks to complement the GeoGebra software was another key factor that yielded positive improvements in the learners' academic performance (Lee & Kung, 2018). In addition, the implementation of scaffolding in the group to progress from one level to the next in Van Hiele's theory greatly impacted the improvement of learners' post-test scores. The strategy of employing scaffolding to identify learners' ZPD in their group settings also helped. It is, therefore, reasonable to suggest that the appropriate use of strategies and teaching and learning resources is essential in promoting effective learning in the classroom.

The qualitative data collected during the intervention indicated a noticeable increase in enthusiasm among learners as they engaged with GeoGebra software in small-group settings. This enthusiasm was evident in their active participation and collaborative interactions within their mathematics groups. Peer discussions not only enhanced learners' confidence in addressing concepts related to Analytical Geometry of lines in the Cartesian plane (Behnsen, 2018) but also created an environment where idea-sharing and mutual support flourished.

The small group learning approach played a pivotal role in fostering engagement, as learners voluntarily assisted one another in understanding Analytical Geometry, leading to deeper individual comprehension. Furthermore, classroom dynamics were significantly influenced by the deliberate use of scaffolding strategies, which stimulated critical thinking and encouraged learners to propose solutions to problems on the chalkboard independently.

These findings underscore the importance of incorporating cooperative learning strategies that promote peer interaction, enabling learners to navigate challenging concepts in Analytical Geometry collaboratively. Such an approach not only helps them recognise and address common misconceptions but also equips them with the skills to solve problems they previously found difficult.

The statistical analysis confirmed that the study successfully achieved its objectives in examining and documenting how teachers instruct Grade 11 Analytical Geometry of lines on the Cartesian plane. The objectives were clearly articulated, and potential barriers to implementation were systematically identified, analysed, and presented in a structured format for scientific interpretation.

The study emphasised the significance of integrating GeoGebra software as a pedagogical strategy, highlighting its role in addressing challenges learners face when mastering Analytical Geometry concepts. This approach established a robust framework for future instructional strategy, enabling the researcher to quantitatively assess its impact on improving learners' academic performance in Grade 11 Analytical Geometry, specifically in relation to lines on the Cartesian plane. The findings provide a data-driven foundation for refining teaching methodologies and enhancing student outcomes in this domain.

7.4 RESEARCHER'S VOICE AND CONCEPTUAL FRAMEWORK

The pre-test administered yielded quantitative results revealing significant learner underperformance. Analysis revealed substantial gaps in learners' understanding of analytical geometry concepts related to lines on the Cartesian plane, which contrasts sharply with curriculum expectations. Qualitative investigations, including semi-structured interviews and classroom observations, identified inadequate implementation of conceptual frameworks for effective teaching as a key factor in the issue. Further examination of test scripts revealed recurring error patterns, highlighting underlying conceptual and procedural challenges.

A critical finding was the absence of Zone of Proximal Development (ZPD) assessment before instruction. Kempen (2020) defines ZPD as the gap between an individual's independent problem-solving ability and their potential for growth with guidance. Winkler (2021) emphasises its role in diagnosing prior knowledge before introducing new content. Classroom observations, however, revealed that teachers routinely neglected this step, likely contributing to pervasive calculation errors and misconceptions in the pre-test. Addressing such gaps before advancing to new topics is essential to prevent cumulative knowledge deficits.

Scaffolding, as theorised by Vygotsky (1978), emerged as a pivotal strategy for addressing identified challenges. By tailoring support within learners' ZPD, scaffolding bridges the transition from current understanding to mastery of analytical geometry concepts. Sarikas (2020) underscores its potential to amplify conceptual clarity when aligned with ZPD principles. This approach not only rectifies immediate errors but also equips learners to assimilate subsequent topics more effectively. The following sections elaborate on these outcomes within their theoretical frameworks. (Sarikas, 2020).

The literature highlights scaffolding as an effective teaching strategy for accommodating diverse learners and fostering their academic growth in accordance with their individual potential. This insight guided the researcher in recommending scaffolding techniques to address learning gaps in

Analytical Geometry, specifically concerning lines on a Cartesian plane. Sarikas (2020) outlines four key strategies for effective scaffolding implementation:

- ***Assess Each Student's Zone of Proximal Development (ZPD)***

Teachers must first identify learners' existing knowledge levels before applying scaffolding strategies. Without this understanding, tailoring instruction to their ZPD becomes ineffective. Sarikas (2020) suggests using quizzes, tests, or discussions to gauge baseline knowledge. Since learners exhibit varying ZPDs across different Analytical Geometry concepts, structured group activities can help teachers monitor and support individual progress.

- ***Promote Collaborative Group Work***

Group work enhances scaffolding by enabling peer learning. Structuring groups with mixed abilities allows advanced learners to reinforce their understanding by guiding peers while struggling students benefit from peer explanations. Teachers should facilitate active participation, ensuring all learners contribute, particularly those who may be overshadowed by more vocal peers.

- ***Provide Minimal Intervention***

A common pitfall in scaffolding is offering premature solutions, which stifles independent problem-solving. Traver et al. (2019) emphasise the importance of allowing productive struggle, letting learners grapple with challenges before intervening. Instead of direct answers, teachers should pose guiding questions, encouraging learners to reflect on their reasoning and explore solutions autonomously.

- ***Encourage Verbalisation of Thought Processes***

Sarikas (2020) advocates for think-aloud strategies to assess learners' ZPD and deepen engagement. During interviews, the researcher prompted learners to articulate their reasoning about lines and their relationships in Analytical Geometry. This approach not only revealed their comprehension but also fostered active learning by having them evaluate their next steps and uncertainties.

Despite these proven strategies, classroom observations revealed minimal implementation of scaffolding (Tables 5.11 and 5.12), indicating missed opportunities to support learners effectively. To improve, teachers should integrate these techniques, building confidence in teaching complex mathematical concepts. Developing teacher efficacy through structured interventions can empower educators to manage challenges in Analytical Geometry instruction, thereby fostering both personal and general teaching efficacy (Ma & Yang, 2022). Teachers with high efficacy are better equipped to guide learners through difficulties, minimising errors and enhancing understanding.

7.5 RECOMMENDATIONS FOR EFFECTIVE TEACHING

The findings of this study, supported by the theoretical framework and existing literature, suggest that tailored instructional strategies can enhance learners' performance in Analytical Geometry, particularly in underperforming schools. Research indicates that effective teaching methods should foster direct teacher-learner interaction, promote conceptual understanding, and encourage active engagement (Lin et al., 2020). Additionally, structured peer collaboration and differentiated instruction can address diverse learning needs while empowering students to take ownership of their learning (Mallipa, 2018). Teachers should act as facilitators, guiding learners through problem-solving rather than providing direct solutions (Sarika, 2020).

To improve outcomes in Analytical Geometry, the following strategies are recommended:

- ***Algebraic Manipulation Mastery***
    - Ensure learners develop a strong foundation in algebraic operations, as these are essential for solving problems in Analytical Geometry.
    - Teach problem-solving techniques explicitly, including step-by-step approaches and common "tricks" for simplifying equations.

- ***Formula Application***
    - Emphasise the correct use of formulas (e.g., distance, midpoint, gradient) by demonstrating their application in different contexts.
    - Train learners to identify when and how to apply specific formulas, properties, and geometric principles.
- ***Keyword Familiarisation***
    - Reinforce key geometric terms (e.g., parallel, perpendicular, angle bisector) to enhance comprehension of problem statements.
    - Incorporate vocabulary exercises and real-world examples to solidify conceptual understanding.

- ***Integration with 2D Geometry***
    - Strengthen learners' knowledge of 2D shapes (e.g., triangles, quadrilaterals) and their properties to facilitate problem-solving on the Cartesian plane.
    - Use visual aids and coordinate geometry applications to reinforce connections between algebra and geometric figures.

By implementing these strategies, educators can address common learning gaps, boost confidence, and improve academic performance in Analytical Geometry among struggling learners. Further research should explore the impact of these methods in diverse classroom settings. The suggestions above are practicable and achievable if Van Helie's five levels are effectively applied in the teaching process of Grade 11 learners on Analytical Geometry of lines (Nzaramyimana, 2021). The researcher recommends that the systematic application of scaffolding through GeoGebra software will enable learners to overcome hurdles in learning Analytical Geometry of lines on the Cartesian plane.

The five levels of Van Hiele's theory of learning Analytical Geometry for learners are suggested below:

- ***Level 0 (Visualisation)***

According to Yundianto et al. (2018), the level of visualisation is related to the object's appearance. Learners know lines and angles on the Cartesian plane in Analytic Geometry language without cognisance of the relations among their variables.

- ***Level 1 (Analysis)***

At level 2, learners identify the traits of the objects, shapes, and figures that make them up. They placed their names and analysed the traits of objects without observing the mutual relationships between their variables and could not define and describe the object completely (Khalil et al., 2019).

- ***Level 2 (Abstraction)***

According to Yudianto et al. (2019), at this level, learners are expected to think deductively about lines and angles that the lines form in the Cartesian plane. Learners can use the transformation logic, that is, "square being a type of rectangle", but cannot tell whether opposite sides have the same gradients because they are parallel or their diagonals are equal in length.

- ***Level 3 (Formal Deduction)***

At this level, the suitability of deduction as a method for building Analytical Geometry in axiomatic systems, including definitions, theorems, consequences, and postulates, has been understood (Yudianto et al., 2018).

- ***Level 4 (Rigor)***

According to Yudianto et al. (2019), learners at this level have reversible thinking ability and understand the formal aspects of deduction. At this stage, the learner is capable of going through level 1 to level 4. With Van Hiele's levels enumerated, the above led each learner to understand problems concerning lines and their relationships in Analytical Geometry much more easily.

Using GeoGebra to enhance performance in Analytical Geometry

Teaching Analytical Geometry has undergone a new emergence due to GeoGebra, and this dynamic software provides assistance in integrating the Concepts of lines and angles, allowing learners to understand these concepts better. Some of the GeoGebra aspects are:

- **Virtual handling:** According to Yorganci (2018), Geogebra can manipulate virtual, abstract concepts of Analytical Geometry, such as lines and angles, to concretise them through diverse representations.
- **The connection of geometric thinking and algebra** (Sudihartinih & Wahyudin, 2019). GeoGebra is a direct software that allows the concurrent display of algebraic and geometric objects along with the process.
- **Using slides and animation** can enable us to see the process of the concept of Analytical Geometry distinctly and uninterruptedly.
- **Idea processing**: GeoGebra can act on objects in Analytical Geometry through diverse outcomes, which enables the actualisation of ideas in more accurate and precise ways, providing spontaneous feedback in a precise manner.
- **Analytic and synthetic views:** Through GeoGebra, learners can translate and observe ideas related to different concepts of Analytical Geometry, including lines and angles in the Cartesian plane, from both synthetic and analytical perspectives across various transformations.

The above are the aspects of learning that GeoGebra can enhance in learners to develop their skills and knowledge, thereby improving their performance in Analytical Geometry of lines. According to Sugandi & Bernard (2020), GeoGebra enables mathematical experiments that foster active, learner-centred, mathematically conceptual problem-solving exploration and teaching discovery, enhancing performance in mathematical topics such as Analytical Geometry of lines on the Cartesian plane (Uwurukundo, Maniraho & Tusiime, 2020; Prodromou, 2015).

7.6 LIMITATIONS OF THE STUDY

Although the study successfully enhanced learners' conceptual understanding of Analytical Geometry through the use of GeoGebra software, its scope was limited to a single school in the Thulamahashe circuit of Mpumalanga Province. Consequently, learners from other schools and provinces were excluded, potentially restricting the generalisability of the findings. The study focused on strategies to improve the performance of FET learners in diverse mathematics classrooms, relying primarily on pre-test and post-test results, lesson observations, and interview responses from teachers and learners.

However, the absence of additional research instruments may have limited data collection, possibly omitting valuable insights. Furthermore, the methodology did not include learners from other schools, which could have influenced the outcomes and broader applicability of the intervention. These limitations suggest the need for further research with a more extensive and varied participant pool to validate the findings.

7.7 SUGGESTIONS FOR FUTURE RESEARCH

Key Findings and Recommendations

The study revealed significant differences in post-test performance between the two schools following the intervention, with the experimental group performing better. The success of the intervention can be attributed to several key factors, including the effective use of GeoGebra software, the identification of learners’ Zone of Proximal Development (ZPD), structured scaffolding, and the application of Van Hiele’s five levels of geometric thought. Mathematics teachers should adopt these strategies to help Grade 11 learners overcome common challenges and errors in Analytical Geometry of lines.

Implications for Future Research

To further enhance mathematics instruction, future studies should:

- Expand the Scope of Intervention – Investigate the benefits of similar interventions across multiple grades rather than focusing solely on Grade 11.
- Examine Teaching Practices – Explore how teachers in different schools approach the teaching of Analytical Geometry on the Cartesian plane to identify best practices and areas.
- Evaluate Large-Scale Interventions – Conduct research involving clusters of schools to assess the broader impact of successful intervention frameworks.

Addressing Learner Challenges

Ineffective teaching methods and learning gaps contribute significantly to learners' struggles in Analytical Geometry. To address these issues, further research should focus on:

- Effective Teaching Strategies – Identifying instructional approaches that enhance Grade 11 learners' performance in solving problems related to lines on the Cartesian plane.
- Root Causes of Mistakes – Investigating the primary challenges that lead to learner errors and underperformance in this topic.
- Scaffolding Implementation – Assessing the extent to which teachers employ scaffolding techniques when teaching Analytical Geometry concepts.

By addressing these areas, educators can refine their teaching methods, ultimately improving learner outcomes in mathematics.

7.8 CONCLUSION

This study investigated effective strategies for teaching Grade 11 Analytical Geometry of lines to enhance learner performance in underperforming schools. Focusing on teacher development, the research aimed to shift instructional practices by introducing proven, research-based teaching methods during an intervention period. The findings provide actionable insights for the Department of Basic Education (DBE) on implementing similar interventions across districts and schools to improve mathematics instruction.

Additionally, the study highlights the broader implications of such interventions for school management, mathematics educators, and teachers across disciplines, demonstrating how structured support can elevate overall teaching quality. The results offer participating schools a validated strategy to drive meaningful improvement in learners' academic achievement in mathematics.

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APPENDICES

APPENDIX A: ETHICAL CLEARANCE CERTIFICATE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2022/03/09

Ref: **2022/03/09/60095016/18/AM**
Name: Mr MMM Seidu
Student No.: 60095016

Dear Mr MMM Seidu

Decision: Ethics Approval from 2022/03/09 to 2025/03/09

Researcher(s): Name: Mr MMM Seidu
E-mail address: 60095016@mylife.unisa.ac.za
Telephone: 0784399525

Supervisor(s): Name: Dr. T.P. Makgakga
E-mail address: makgasw@unisa.ac.za
Telephone: 0123376004

Title of research:

Enhancing the performance of grade 11 learners Analytical Geometry in Bohlabela district, Mpumalanga province

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2022/03/09 to 2025/03/09.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2022/03/09 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX B: LETTER TO THE PROVINCIAL EDUCATION OFFICE

Dear Sir/Madam,

I am Mohammed Musah Mahama Seidu a permanent mathematics teacher of Magigwana Secondary School in Thulamahashe circuit and currently doing Master's Degree in Mathematics Education at University of South Africa (UNISA). I am conducting a study title, enhancing the performance of Grade 11 learners in Analytical Geometry of lines and angles on Cartesian plane, which I identify some schools in your district, which are constantly performing averagely, low in your district. It is a study conducted to fulfil academic requirement only. I have chosen this school for convenience and cost-effective reasons. I am therefore humbly asking for your permission to undertake this project at some of the schools in the district.

The participants of the study are the Grade 11 learners. They will only be required to write a short pre-test and post-test on Grade 11 Analytical Geometry after the completion of that topic. It will be a standardized test, set by me and moderated by a subject advisor and /or HOD. There will also be a short classroom visit to observe leaners' responses to teaching and learning activities in the classroom. Lastly, few learners will be selected for interview to understand their thoughts, feelings and/or reasons underlying their outputs in the test. The pre-test and post-test, classroom observation and interview of learners will all take place on different days which will be scheduled in advance. The maximum time to be used to conduct the test is 45 minutes. The classroom observation will be once and will last for 45 minutes. It will also take approximately 60 minutes to complete the interview with all the selected learners.

I promise to make sure teaching and learning is not disturbed. For these reasons, all activities will be conducted after school hours (thus, between 14:30H00 and 15:30H00). My observation in the classroom will be just sitting and observing how learners respond to Analytical Geometry concept and will not interfere with teaching and learning process. Confidentiality will strictly be maintained. This is assured because names of learners, teacher(s) and the school as a whole will not be mentioned or attached to the information collected. However, the consent forms signed may be reviewed for other research or legal purposes. Also, information collected from schools may be presented in publications or reports in summary form.

It is my hope that, the study will boost the confidence of learners in answering Analytical Geometry problems after their errors and mistakes have been identified and corrected. This will hopefully mitigate the issue of underperforming in mathematics in the district and in the province as a whole.

Attached to this letter are ethical clearance from the University of South Africa (UNISA).

Below are my contact details:

Email: seedoor3@gmail.com ; Cell number: (+27) 784399525

Supervisor's details:

Name: Dr TP Makgakga

Email: makgasw@unisa.ac.za

Tel. number (office): (+27) 12 429 4293

Hope to her from you soon

Yours sincerely,

Seidu MMM

Letter to the District education office

Dear Sir/Madam,

I am Mohammed Musah Mahama Seidu a permanent mathematics teacher of Magigwana Secondary School in Thulamahashe circuit and currently doing Master's Degree in Mathematics Education at University of South Africa (UNISA). I am conducting a study title, enhancing the performance of Grade 11 learners in Analytical Geometry of lines and angles on Cartesian plane, which I identify some schools in your district, which are constantly performing averagely, low in your district. It is a study conducted to fulfil academic requirement only. I have chosen this school for convenience and cost-effective reasons. I am therefore humbly asking for your permission to undertake this project at some of the schools in the district.

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APPENDIX C: LETTER TO THE PRINCIPAL

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The participants of the study are the Grade 11 learners. They will only be required to write a short pre-test and post-test on Grade 11 Analytical Geometry after the completion of that topic. It will be a standardized test, set by me and moderated by a subject advisor and /or HOD. There will also be a short classroom visit to observe leaners' responses to teaching and learning activities in the classroom. Lastly, few learners will be selected for interview to understand their thoughts, feelings and/or reasons underlying their outputs in the test. The pre-test and post-test, classroom observation and interview of learners will all take place on different days which will be scheduled in advance. The maximum time to be used to conduct the test is 45 minutes. The classroom observation will be once and will last for 45 minutes. It will also take approximately 60 minutes to complete the interview with all the selected learners.

I promise to make sure teaching and learning is not disturbed. For these reasons, all activities will be conducted after school hours (thus, between 14:30H00 and 15:30H00). My observation in the classroom will be just sitting and observing how learners respond to Analytical Geometry concept and will not interfere with teaching and learning process. Confidentiality will strictly be maintained. This is assured because names of learners, teacher(s) and the school as a whole will not be mentioned or attached to the information collected. However, the consent forms signed may be reviewed for other research or legal purposes. Also, information collected from schools may be presented in publications or reports in summary form.

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Yours sincerely,

Seidu MMM

APPENDIX D: LETTER TO THE SUBJECT TEACHER

STUDY TITLE: Enhancing the performance of grade 11 learners in relation with lines and angles in analytical geometry on a cartesian plane in Bohlabela district, Mpumalanga province

I am Mohammed Musah Mahama Seidu, a full time teacher in at Magigwana secondary School. I am currently doing Master's in Mathematics Education Degree at University of South Africa (UNISA). I am conducting a study titled "Enhancing the performance of grade 11 learners in relation with lines and angles in analytical geometry on a cartesian plane in Bohlabela district, Mpumalanga province" to explore grade 11 learners' performance and challenges faced in Analytical Geometry of lines and angles on cartesian plane and misconception errors committed during answering question on the concepts. Among some of the schools in this district includes yours. It is a study conducted to fulfil academic requirement only. I have chosen this school for conveniency and cost-effective reasons. I'm therefore humbly asking for your permission to undertake this project at your school.

The participants of the study are the grade 11 learners. They will only be required to write a short pre-test and post-test on Analytical Geometry after the concept is taught. This will be followed by a short interview of some selected learners to understand their thoughts. I hope the study will identify learners' challenges and ultimately contribute to possible ways of minimizing lower performance in the topic. This study may provide a long-term benefit like improvement on our teaching approach for Analytical Geometry to enhance learners' performance in mathematics as a whole.

It is my promise to make sure teaching and learning is not interfered with. Confidentiality will strictly be maintained. This is assured because names of learners, teacher(s) and the school as a whole will not be used or attached to the information collected. However, the consent form signed by you may be reviewed for other research or legal purposes. Also, information collected together with those collected from other schools may be presented in publications or reports in summary form.

Participation is not compulsory. As a participant, you may stop at any time for whatever reason without a penalty. You may also ignore or choose not to answer some questions you don't want.

For any question about your participation in this study in the future, contact me through the details below:

Email: seedoor3@gmail.com ; Cell number: (+27) 784399525

Supervisor's details:

Name: Dr TP Makgakga

Email: makgasw@unisa.ac.za

Tel. number (office): (+27) 12 429 4293

Yours sincerely,

Seidu MMM

APPENDIX E: LETTER TO THE PARENT/GUARDIAN

Dear Parent/Guardian,

I am Mohammed Musah Mahama Seidu, a fulltime teacher in at Magigwana Secondary School. I am currently doing Master's in Mathematics Education Degree at University of South Africa (UNISA). I am conducting a study to explore grade 11 learners' performance and challenges faced in Analytical Geometry and misconception concepts among some of the schools in this district including that of your wards. It is a study conducted to fulfil academic requirement only. I have chosen this school for conveniency and cost-effective reasons. I'm therefore humbly asking for your permission to undertake this project at your school.

The participants of the study are the grade 11 learners. They will only be required to write a short pre-test and post-test on Analytical Geometry after the concept is taught. This will be followed by a short interview of some selected learners to understand their problem. I hope the study will identify learners' challenges and ultimately contribute to possible ways of enhancing their performance.

I promised to make sure teaching and learning is not interfered with. Confidentiality will strictly be maintained. This is assured because names of learners, teacher(s) and the school as a whole will not be used or attached to the information collected. However, the consent form signed by you may be reviewed for other research or legal purposes. Also, information collected together with those collected from other schools may be presented in publications or reports in summary form.

Participation is not compulsory. As a participant, your child may stop at any time for whatever reason without a penalty.

For any further queries or clarifications, don't hesitate to contact me.

My details: Email: seedoor3@gmail.com ; Cell number: (+27) 784399525

Supervisor's details: Name: Dr TP Makgakga; Email: makgasw@unisa.ac.za

Tel. number (office): (+27) 12 429 4293

Yours sincerely,

Seidu MMM

APPENDIX F: ASSENT LETTER TO THE LEARNER

STUDY TITLE: Enhancing the performance of grade 11 learners in relation with lines and angles in analytical geometry on a cartesian plane in Bohlabela district, Mpumalanga province.

Dear Learner,

I am Mohammed Musah Mahama Seidu, a full time teacher at Magigwana Secondary School. I am currently doing Master's in Mathematics Education Degree at University of South Africa (UNISA). I am conducting a study to find out about grade 11 learners' performance and challenges faced in Analytical Geometry of lines and angles on cartesian plane and misconception concepts among some of the schools in this district including yours. It is a study I am doing to fulfil academic requirement only. I have chosen this school because I can easily reach it without wasting a lot of time and money. I'm therefore humbly asking for your permission to be part of this project at your school.

It is grade 11 learners who are needed in the study. If you choose to be part of it, you will only be made to write a short pre-test and post-test on the topic which will take 45 minutes, after the concept is taught. This will be followed by a short interview of some selected learners to understand their thoughts. It will take about 60 minutes to finish the interview. I hope the study will help us as teachers to teach you Analytical Geometry a certain way that will improve your performance in mathematics.

Your private matters like performance in the test and answers to interview questions will remain secret because your name will not be used against what you will say or attached to anything you will write.

Please if you need any further explanations, below are my details:

Email: seedoor3@gmail.com ; Cell number: (+27) 784399525

Supervisor details:

Name: Dr TP Makgakga; Email: makgasw@unisa.ac.za ; Tel. number (office): (+27) 12 429 4293

Note: Do not sign the assent form until all your questions have been answered.

Seidu MMM

APPENDIX G: CONSENT FORM FOR THE PRINCIPAL

CONSENT FORM FOR PRINCIPAL (**Return slip**)

I, ____________________________ the principal of____________________________ School, I would like to permit Mr. Seidu MMM to involve the Grade 11 learners and teacher(s) in his research on the condition that all data collected should be treated with total confidentiality. The name of the participants (that is teachers and learners) as well as the school should not be mentioned in the analysis of the data. The participants (that is teachers and learners) would be permitted to withdraw from the study at any time.

Principal's Signature: __________________________________Date: ____________

Researchers' Signature:_________________________________Date:____________

APPENDIX H: CONSENT FORM FOR THE PARENT/GUARDIAN

CONSENT FORM THE PARENT/GUARDIAN (**Return slip**)

I, ____________________________ the parent/guardian of____________________________ , permit Mr. Seidu MMM to involve my Grade 11 child in his research. My child data can cover the following aspects;

Please put a tick in the appropriate column

Description	Yes	No
I give consent my child to participate in achievement test		
I give consent my child to be observed during lessons		
I give consent my child to answer interview questions		

Data collected on my child should be treated with total confidentiality. The name of my child should not be mentioned in the analysis of the data. My child would be permitted to withdraw from the study at any time.

Parent's/ Guardian's Signature: _______________________________Date: ___________

Name of child: ____________________ Child's Signature: _____________Date:__________

Researcher's Signature:_________________________________ Date:______________

APPENDIX I: CONSENT FORM FOR THE SUBJECT TEACHERS TO PARTICIPATE

CONSENT FORM FOR THE TEACHER (**Return slip**)

I, ______________________________ a grade 11 mathematics teacher at______________________________ school, accept to be a participant of Mr. Seidu MMM research study in Grade 11 mathematics. I consent for the following data to be collected during my lessons;

Please put a tick in the appropriate column

Description	Yes	No
I give consent my learners to participate in achievement test		
I give consent my lessons can be observed during lessons		
I give consent to assist the researcher to conduct the test		

Data collected should be treated with total confidentiality. My name should not be mentioned in the analysis of the data and I would be permitted to withdraw from the study at any time.

Teacher's Signature: ____Date: _____________

Researcher's Signature: ____Date: _____________

APPENDIX J: ASSENT FORM FOR THE LEARNER PARTICIPANTS IN THE STUDY

ASSENT FORM FOR THE LEARNER PARTICIPANT (**Return slip**)

I, ______________________________ of ______________________________ school, have read, understood and accept to be a participant of Mr. Seidu MMM research study in Grade 11 mathematics. I give my consent for the following data to be collected during my participation;

Please put a tick in the appropriate column

Description	Yes	No
I give consent to be given achievement test		
I give consent to be observed during lessons		
I give consent to respond to interview questions		

Data collected on me should be treated with total confidentiality. My name should not be mentioned in the analysis of the data and I would be permitted to withdraw from the study at any time.

Learner's Signature: ____Date: _____________

Parent's/ Guardian's Signature: ________________________________Date: ______________

Researcher's Signature: __________________________________Date: _______________

APPENDIX K: INFORMATION SHEET

PARTICIPANTS INFORMATION SHEET

Title: Enhancing the performance of grade 11 learners in relation with lines and angles in analytical geometry on a cartesian plane in Bohlabela district, mpumalanga province.

Dear prospective participant,

I am Seidu MMM and I am doing research with Dr SW Makgakga, a lecturer in the Department of Mathematics Education towards a Master's Degree in Education at the University of South Africa. We are inviting you to participate in a study entitled 'Enhancing the performance of grade 11 learners in relation with lines and angles in analytical geometry on a cartesian plane in Bohlabela district, Mpumalanga province.

The outcome of the research will be available to curriculum developers and policy makers to inform their curriculums and policies respectively and give insight to teachers on teaching Analytical Geometry of lines and angles in a cartesian plane.

Bohlabela is one of the underperforming districts in the province and also contributes a large number of learners every year to write matric examination. Analytical Geometry is among challenging concepts learners face in matric examinations (DBE, 2019). However, at grade 11 learners are expected to be able to handle the topic very well to enhance their performance (DBE, 2014). Based on this reason, coupled with the distance and school-based performance in the past matric examination, I have randomly selected grade 11 learners in the district for this research, with a permission from the provincial office of Mpumalanga.

From the first school, 10 learners will serve as pilot group. The remaining two schools will contribute 30 learners each for the main research and will be put together to form the sample size (60 learners) required for the research. The nature of participation on the part of learners involves writing of a short-standardized pre-test and post-test on Analytical Geometry of lines and Angles on a cartesian plane after the concepts are taught according to the curriculum, as the first phase of the research. In the second phase, few learners will randomly be selected to respond to a semi-structured interview questions to explain their understandings and responses to the test questions.

Teachers will assist the researcher to conduct the tests in a manner that meets the standard. At least one lesson each will also be observed only for assessment of teaching and learning environment.

Participating in this study is voluntary and you are under no obligation to consent to participation. If you decide to take part, you or your parent or guardian will be given this information sheet to keep and be asked to sign a written consent form and/or assent form. You are free to withdraw at any time and without giving any reason.

A potential benefit of taking part in this study includes exposure to knowledge like some of the challenging questions learners will encounter in the test which can enlighten them for a better performance in the future. Teachers will be aware of learners' errors and misconceptions when solving Analytical Geometry of lines and angles on the cartesian plane problems to inform them of how to approach the teaching of these concepts to improve learners performance. In the long term, the findings will be available to inform curriculum developers, policy makers, authors of textbooks and teachers to improve mathematics education. This means all role players in the study will be helping in contributing to the body of educational knowledge.

There is no foreseeable risk or harm for participating in this study. In other words, there is no negative consequences for participation. The covid 19 protocols will strictly be adhered to for participants protection.

All data collected are going to be treated confidentially. Names of the participating schools, teachers or learners will not be disclosed in any form. Codes will be used instead of participants' names. Your name will not be recorded anywhere and no one will be able to connect you to the answers you give. Hence your answers will be given a code or pseudonym and you will be referred to in this way in the data, any publication, or other research reporting methods such as conference proceedings. Apart from the researcher, it is only identified members of the research team, who will know about your involvement in this research.

The data will be directly accessible to only the researcher and his supervisor. Your answers may also be reviewed by the people responsible for making sure that research is done properly, including the transcriber, external coder, and members of the Research Ethics Review Committee. Otherwise, records that identify you will be available only to people working on the study, unless you give permission for other people to see the records. Data collected will only be used by Unisa as a requirement for the award of Master of Education degree in Mathematics Education. Data will be used for other purposes such as research report, journal articles, conference proceedings and even for publications but all participants' information will remain anonymous. Apart from the researcher, it is only identified members of the research team, who will know about your involvement in this research.

Hard copy of the data including your answers will be stored by the researcher for a period of five years in a locked cupboard cabinet in *UNISA* for future research or academic purposes; electronic information will be stored on a password protected computer. Future use of the stored data will be subject to further Research Ethics Review and approval if applicable. Data collected will be destroyed if necessary. *Hard copies for instance, will be shredded and/or electronic copies will be permanently deleted from the hard drive of the computer through the use of relevant software programmer.*

There will be no cost on the part of the participants and there will be no payment or financial reward offered to them apart from the aforementioned benefits.

This study has received written approval from the Research Ethics Review Committee of the *COLLEGE OF EDUCATION,* UNISA. A copy of the approval can be obtained from the researcher if you so wish.

If you would like to be informed of the final research findings, please contact Mr. Seidu MMM on 0784399525 or email address: seedoor3@gmail.com. The findings are accessible for 3 years.

Should you require any further information or want to contact the researcher about any aspect of this study, please contact Mr. Seidu MMM, Email: seedoor3@gmail.com, Cell number: 0784399525.

Should you have concerns about the way in which the research has been conducted, you may contact my Supervisor: Dr TP Makgakga, Email: makgasw@unisa.ac.za , Tel. number (office): (+27) 12 429 4293. Alternatively, contact the research ethics chairperson of the college of education research ethics review committee: Prof AT Motlhabane, Email: motlhat@unisa.ac.za, Tel. number (office): (+27) 12 429 2840.

Thank you for taking time to read this information sheet and for participating in this study.

Mohammed Musah Mahama Seidu.

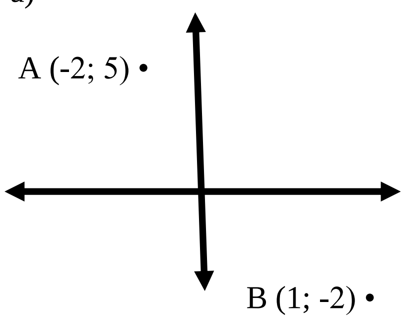
APPENDIX L: PRE-TEST AND POST-TEST

LEARNER CODE------------- AGE ------------ SCHOOL CODE ------------- DATE-------------

TOPIC: Analytical Geometry lines on the cartesian plane.   DURATION: 50 MINUTES

1. Calculate the gradient/slope of A and B on the cartesian plane

a)

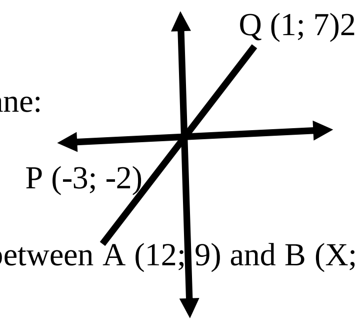


b) In each of the following, determine whether AB∥CD, AB⊥CD or neither.

b) A (-1; 5), B (-2;3), C (9; 10) D (5; 2).

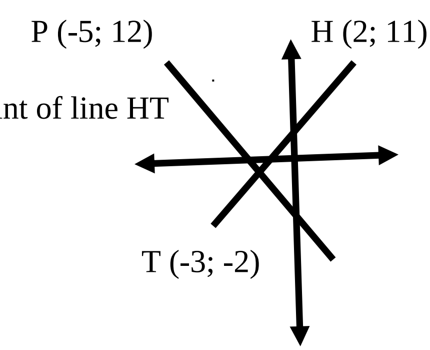
c) A (3; -3), B (6; -7), C (-5; 0), D (-1; 3).

a) Find the distance between Q and P on the cartesian plane:



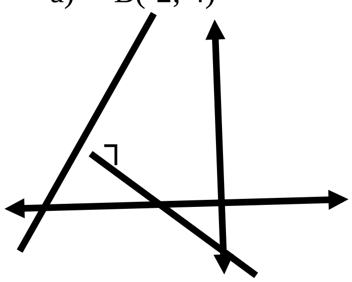
b) Calculate for the value of X if the length of the distance between A (12; 9) and B (X; 7) is 9cm

c) Calculate for the length of line PD if D is midpoint of line HT

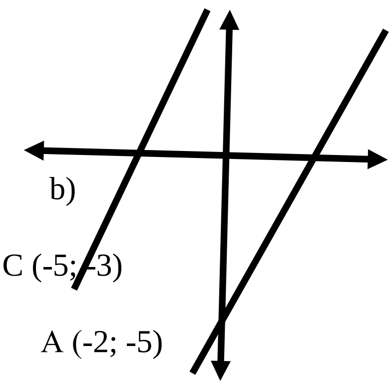


3. Determine the equation of the line in each case

a)

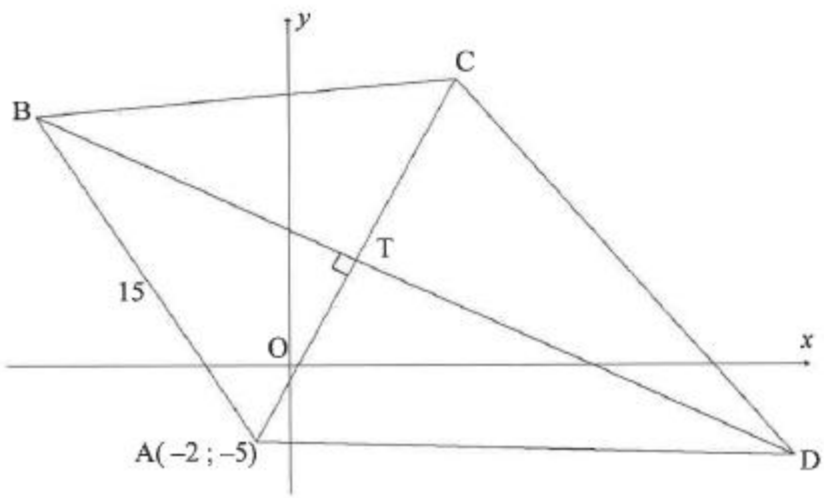


b)



4. A (-2; -5), B, C and D Are the vertices of quadrilateral ABCD such that diagonal AC is perpendicular to diagonal BD at T. AB∥DC and also BC∥AD

The equation of BTD is given by $2\gamma + x = 18$ and AB = 15 units.



If the equation of AC is $y = 2x - 1$, a) calculate the coordinates of T If ABCD is a kite with AB = BC:

b) Determine the coordinates of C

5. From Q4a, how will the equation of line AD be calculated if only the gradient of BC is given and a point is on the line AD.

APPENDIX M:MATHEMATICS: INTERVIEW SCHEDUCE – GRADE 11

LEARNER CODE________________ **SCHOOL CODE___________**

TOPIC: ANALYTICAL GEOMETRY **DURATION: 20 Minutes**

Part 1

Researcher: Please read the question to me.
Learner……………………………..

Researcher: Tell me what the question is asking you to do.
Learner……………………………..

Researcher: Show me or explain to me how you worked out the answer to the question.
Learner……………………………

Researcher: Do you know any other approach or method to answer this question?
Learner……………………………..

Part 2
Researcher: Do you understand your mathematics teacher?
Learner……………………………..

Researcher: Do you like mathematics and do you practice it often?
Learner……………………………..

Researcher: Is there anybody at home who monitors your studies?
Learner……………………………..

Researcher: Is there anybody at home who can, and helps you with mathematics task?
Learner……………………………..

Researcher: Is there anything that disturbs your studies at home?
Learner……………………………..

APPENDIX N: LESSON OBSERVATION SCHEDULE

LESSON OBSERVATION SCHEDULE

Teacher (CODE) observed: ______________________________________

Observer: ______________________________________

Subject: **Mathematics**

Grade: **11**

Topic: ______________________________________

Duration of lesson: ______________________________________

Date: ______________________________________

	Category	**Comment**
1	Is the classroom conducive for effective teaching and learners?	
2	Does lesson begin on time in an orderly, organized fashion	
3	To what extent learners' prior knowledge of the topic is tested and used as a starting point?	
4	To what extent learners are made to participate in the lesson?	
5	Are examples and/ or scenarios used appropriate to facilitate understanding?	
6	Do learners' questioning and answering demonstrate enough understanding of concepts?	
7	Is the time enough for the lesson?	
8	To what extent learner demonstrate a sense of enthusiasm and excitement in the lesson?	

APPENDIX O: COVER LETTER

The Ethics Review Committee (ERC) 14 -03-2022

College of Education

UNISA

To the Committee,

EXPLANATION OF HOW THE COMMITTEE'S COMMENTS ARE ADDRESSED

This letter serves as explanations of how the applicant has clarified and/or revised certain aspects of the Ethics Application Form submitted earlier according to the Committee's comments.

Please refer to the table below for detailed clarifications and explanations:

ITEM	COMMITTEE'S COMMENT	APPLICANT'S CLARIFICATIONS AND/OR RESPONSES
3.1	• 3.1 PROVISIONAL TITLE: The provisional title is too long. Should be reduced to at least a maximum of sixteen words. "Enhancing the performance of grade 11 learners in relation with lines and angles in Analytical Geometry on a cartesian plane in Bohlabela district, Mpumalanga province". We recommend the following title: "Enhancing the performance of grade 11 learners in Analytic Geometry in Bohlabela District, Mpumalanga province"	The title is reduced to 16 words and reads as: **"Enhancing the performance of grade 11 learners in Analytic Geometry in Bohlabela District, Mpumalanga province"** by the committee's recommendation.
3.7.3	• 3.7.3 SAMPLE SIZE AND PARTICIPANT SELECTION: The candidate ticked teachers in section 3.7.2 but in section 3.7.3 teachers do not feature. The candidate is requested to justify the selection of the 80 learners as participants in this study.	The applicant has provided reasons for his reason for the selection of 80 for the sample size (see 3.7.3).

3.7.4	• DATA COLLECTION INSTRUMENTS: d) Interview: The candidate stated that “Twelve Learners (six learners from each of the three participated schools) …” but in 3.7.3 the candidate mentioned that “80 Grade 11 learners from two schools will be participating…”	The “three” suppose be “two” instead and the necessary corrections has been infected.
3.8	• THE PROCESS OF DATA COLLECTION: The candidate indicated that teachers are involved in 3.8, but they are not included in section 3.7.3.	The applicant has responded to the committee’s request and did just that (see 3.7.3)
3.9	• DATA ANALYSIS: The student is requested to clearly explain how he will analyse data.	The applicant has clearly shown how he could analyse the data when collected (see 3.9)
4.2	• 4.2 The student did not describe the steps to be undertaken in case of adverse events or when injury or harm is experienced. Anything may happen during the research. Hence, he should explain possible steps to be taken when an injury or harm happen	The applicant has responded immediately to the committee’s recommendation under this section (see 4.2)
4.6	• 4.6 The student should be clear if there will be any financial costs or not. His statement “… no cost will be incurred apart from driving to the field and printing of the question papers” is confusing.	The student has clarified that, there will be no cost incur by anyone or any entity (see 4.6).
4.9	• 4.9 The student is requested to delete the information and use words such as “I will advise my supervisor who will advise the UNISA CEDU research ethics committee”	The applicant has infected the corrections suggested buy the committee (4.9).

The applicant hopes his revisions, clarifications and/or explanations will be understood by the Committee.

APPENDIX P: DETAIL STATISTICAL ANALYSIS

5.12.1 ITEM ANALYSIS OF PRE-TEST FOR EXPERIMENTAL GROUP

Experimental group & test=="Pre"

One-parameter logistic model **Number of obs = 40**

Log likelihood = -95.415892

Table 5.14

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
Discrim	2.24928	.5533918	4.06	0.000	1.164652	3.333908
Diff						
q1a	.4873205	.2536768	1.92	0.055	-.009877	.9845179
q3a	.6605836	.2648128	2.49	0.013	.14156	1.179607
q2b	1.309915	.3458589	3.79	0.000	.6320444	1.987786
q2b	1.309915	.3458589	3.79	0.000	.6320444	1.987786
q3b	1.629707	.4060267	4.01	0.000	.8339093	2.425505
q2a	2.111164	.5246404	4.02	0.000	1.082888	3.13944
q1b	2.530228	.6686175	3.78	0.000	1.219762	3.840694
q5	2.530228	.6686175	3.78	0.000	1.219762	3.840694

5.12.2 Item analysis of pre-test for the control group

Control group & test=="Pre"

One-parameter logistic model **Number of obs = 40**

Log likelihood = -49.873226

Table 5.15

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
Discrim	7.869888	3.383982	2.33	0.020	1.237405	14.50237
Diff						
q1a	0.8163358	.1175766	6.94	0.000	.5858899	1.046782
q2a	1.207325	.1941311	6.22	0.000	.826835	1.587815
q2b	1.207325	.1941311	6.22	0.000	.826835	1.587815
q3a	1.310626	.2103333	6.23	0.000	.8983801	1.722872
q3b	1.513077	.2564356	5.90	0.000	1.010472	2.015681
q1b	1.791183	.3557694	5.03	0.000	1.093888	2.488478

5.12.3 Item analysis of post-test for experimental group

Experimental group & test=="Post"

One-parameter logistic model Number of obs = 40

Log likelihood = -101.56122

Table 5.16

	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
Discrim	1.508786	.3606669	4.18	0.000	.8018922	2.215681
Diff						
q2b	-1.7566	.4926293	-3.57	0.000	-2.722135	-.7910641
q21(c)	-1.7566	.4926293	-3.57	0.000	-2.722135	-.7910641
q4b	-1.404558	.4277916	-3.28	0.001	-2.243014	-.5661022
q3b	-1.119916	.3835274	-2.92	0.003	-1.871616	-.3682162
q2a	-1.119916	.3835274	-2.92	0.003	-1.871616	-.3682162
q5	1.265738	.3954624	3.20	0.001	.4906458	2.04083

5.12.4 ITEM ANALYSIS OF POST-TEST FOR CONTROL GROUP

Control group & test=="Post"

One-parameter logistic model Number of obs = 40

Log likelihood = -47.780615

Table 5.17

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Discrim	-2.508969	1.111666	-2.26	0.024	-4.687795	-.3301426
Diff						
q5	-5610322	.2621596	-2.14	0.032	-1.074856	-.0472089
q2a	1.769897	.4825717	3.67	0.000	.8240738	2.71572
q4b	2.02115	.560163	3.61	0.000	.9232508	3.119049
q4a	2.02115	.560163	3.61	0.000	.9232508	3.119049

Table 5.20: ***Experiemental group marks differences between pre-and post-tests***

Outcome variable: marks (Marks), n=80

Covariate	Coef.	Std. Err.	P>|z|	95% Conf. Interval
Occasion				
Post-test*	0			
Pre-test	-17.850	1.547	<0.001	(-20.882 to -14.818)
Constant	20.850	1.181	<0.001	(18.534 to 23.166)

Table 5.21: ***Control group's marks differences between pre-and post-tests***

Outcome variable: marks (Marks), n=80

Covariate	Coef.	Std. Err.	P>|z|	95% Conf. Interval
Occasion				
Post-test*	0			
Pre-test	-27.050	1.455	<0.001	(-29.901 to -24.199)
Constant	27.975	1.125	<0.001	(25.769 to 30.181)

Table 5.22: ***The difference between group 1 and 2 after adjusting for test occasion***

Outcome variable: marks (Marks), n=160

Covariate	Coef.	Std. Err.	P>|z|	95% Conf. Interval
Occasion				
Post-test*	0			
Pre-test	-22.450	1.152	<0.001	(-24.708 to -20.192)
School				
1*	0			
2	2.525	1.152	0.028	(0.267 to 4.783)
Constant	23.150	1.061	<0.001	(21.070 to 25.230)

APPENDIX Q: APPROVAL FROM DBE BOHLABELA DISTRICT OFFICE

education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

Ikhamanga Building, Government Boulevard, Riverside Park, Mpumalanga Province
Private Bag X11341, Mbombela, 1200.
Tel: 013 766 5552/5115, Toll Free Line: 0800 203 116

Litiko le Temfundvo, Umnyango we Fundo | Departement van Onderwys | Ndzawulo ya Dyondzo

MR MMM SEIDU

PO BOX 702
HLUVUKANI
1363
078 439 9525
Seedoor3@gmail.com

Dear Sir

APPROVAL OF REQUEST TO CONDUCT ANALYTICAL GEOMETRY (GRADE 11)

Kindly be informed of the approval of your request to conduct analytical geometry in some of our schools under Bohlabela District. The analytical geometry to be based on our grade 11 learners.

Furthermore, please be informed that the Mpumalanga Department of Education will require access to your research findings and recommendations. You are advised to communicate with your chosen schools and ensure that no inconvenience is experienced at any given time. Teaching and learning must not be negatively affected as a result of this Research.

Your professionalism in this regard will be highly appreciated. Good luck on your research; your interest on matters of the District is applauded.

MS L.N GOBA
DISTRICT DIRECTOR: BOHLABELA

06/04/2022
DATE

APPENDIX R: EDITING CERTIFICATE

Unit 3 West Square Business Park
407 West Avenue
Randburg
2194

27 February 2024

TO WHOM IT MAY CONCERN

This serves to confirm that I have edited and made the necessary corrections and emendations to the thesis:

ENHANCING THE PERFORMANCE OF GRADE 11 LEARNERS IN RELATION TO LINES IN ANALYTICAL GEOMETRY ON A CARTESIAN PLANE IN BOHLABELA DISTRICT, MPUMALANGA PROVINCE

by

SEIDU M.M.M.

Sincerely

J Musi
Editor

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PAPER NAME

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AUTHOR

MOHAMMED MUSAH MAHAMA SEIDU

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