THE EFFECT OF WORKING MEMORY-CONTEXT-BASED PROBLEM-SOLVING INSTRUCTION ON CALCULUS PROBLEM-SOLVING SKILLS OF FIRST YEAR STUDENTS AT AN ETHIOPIAN UNIVERSITY

by

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I declare that the above thesis is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

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KEY TERMS DESCRIBING THE TOPIC OF THE THESIS

Problem; Problem-solving; Problem-solving strategy; Problem-solving skills; Conventional problem-solving instruction; Human cognitive architecture; Cognitive load theory; Working memory-context-based problem-solving instruction; Chunking effect; Collective working memory effect; Split-attention effect; Worked example effect; Self-explanation effect

DEDICATION

This work is dedicated to my daughter Tsinat Kumlachew Tadesse, and to my family for their unwavering support.

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ABSTRACT

Mathematics problem-solving, particularly in calculus, continues to pose significant challenges for university students, especially when tasks require conceptual understanding and multi-step reasoning. This study, conducted within a pragmatic research paradigm, examined the effect of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI), grounded in Cognitive Load Theory (CLT), on the calculus problem-solving performance of first-year students at an Ethiopian university. The instructional approach was designed to reduce extraneous cognitive load by integrating worked examples, structured self-explanation prompts, and strategies aimed at minimizing split-attention effects. A two-month intervention involving 79 students was implemented using a static group, post-test-only quasi-experimental design, with 41 students in the experimental group and 38 students in the comparison group. The experimental group received calculus instruction through WMCBPSI while the comparison group followed conventional problem-solving instruction. Data were collected using a standardized functional mathematics achievement test, classroom observations, and semi-structured interviews. The results indicated that students exposed to WMCBPSI significantly outperformed their peers in solving calculus problems, particularly those requiring conceptual reasoning and multi-step solution processes. Qualitative findings further revealed that students perceived the structured nature of WMCBPSI as supportive of deeper engagement with complex problem-solving tasks. The study concludes that WMCBPSI is an effective instructional approach for enhancing calculus problem-solving skills, and that aligning instructional practices with principles of human cognitive architecture offers a practical and theoretically grounded model for improving mathematics instruction in higher education, with implications for adaptation across other mathematical domains and diverse learning contexts.

Key terms

Calculus problem-solving; working memory–context-based instruction; cognitive load theory; worked example effect; self-explanation effect; split-attention effect.

ABBREVIATIONS

Abbreviation	Description
WMCBPSI	Working Memory-Context-Based Problem Solving Instruction
CL	Cognitive Load
CLT	Cognitive Load Theory
CPSI	Conventional Problem-solving Instruction
DoE	Department of Education
ECL	Extraneous Cognitive Load
GCL	Germane Cognitive Load
ICL	Intrinsic Cognitive Load
LO	Learning Outcome
LTSM	Learning and Teaching Support Materials
MPSAT	Mathematical Problem-solving Achievement Test
PS	Problem-Solving
SFMAT	Standardized Functional Mathematics Achievement Test

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CHAPTER ONE

THEORETICAL BACKGROUND AND ORIENTATION OF THE PROBLEM

1.1. OVERVIEW OF THE STUDY

Mathematical problem-solving remains one of the most persistent challenges in higher education, particularly within mathematics instruction. Beyond representing an essential learning outcome, problem-solving cultivates critical reasoning, analytical skills, and conceptual understanding, which are indispensable for mathematical competence (National Council of Teachers of Mathematics, 2018; Schoenfeld, 2016). However, despite its recognized value, students frequently encounter difficulties in applying problem-solving skills effectively, pointing to a pressing need for innovative instructional strategies that address these challenges and foster deeper mathematical proficiency (García et al., 2019; Fuchs et al., 2009).

In this study, problem-solving skills refer to students' ability to understand mathematical problems, apply appropriate strategies, use cognitive and procedural knowledge, execute solution processes, and evaluate solutions effectively (National Research Council, 2001). These skills are operationalized through students' performance on a functional mathematics achievement test assessing conceptual understanding, strategic reasoning, procedural accuracy, and the ability to apply learned strategies to calculus problems (Amalina & Vidákovich, 2023).

Existing research has examined various instructional designs aimed at enhancing problem-solving performance (Dhlamini & Mogari, 2013; Geary & Hoard, 2019), highlighting the significant cognitive demands inherent in mathematical problem-solving (Chen & Kalyuga, 2020; Kala & Ayas, 2023). A growing body of evidence emphasizes that working memory plays a pivotal role in managing these demands. Cognitive Load Theory (CLT) explains that the mental effort—or cognitive load—required to process information is limited by the capacity of working memory, which constrains how learners approach complex problems (Sweller, Ayres, & Kalyuga, 2011; Cowan, 2001). Instruction that reduces unnecessary cognitive load enables students to concentrate on essential processes, thereby enhancing problem-solving effectiveness.

According to Cognitive Load Theory, effective instructional design should deliberately minimize extraneous cognitive load, appropriately manage intrinsic cognitive load according to learners' prior knowledge and task complexity, and promote germane cognitive load to facilitate schema construction and meaningful learning (Sweller, 1988; Sweller et al., 1998, 2011). Rather than simplifying mathematical content, CLT advocates organizing instructional materials in ways that allow learners to devote their limited working memory resources to understanding essential concepts and developing efficient problem-solving strategies (Sweller et al., 2011; van Merriënboer & Sweller, 2005). These principles provide the theoretical foundation for designing instructional interventions that enhance learning without exceeding learners' cognitive capacity.

This research explores the potential of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) as an approach to improve calculus problem-solving skills among first-year university students in Ethiopia. Anchored in CLT, WMCBPSI incorporates strategies such as worked examples, self-explanation, and split-attention reduction, embedding problems in meaningful, real-world contexts. For example, presenting a calculus optimization problem within an engineering scenario can help students link abstract mathematical principles with practical applications, enhancing engagement and lowering cognitive barriers (Karabay & Meşe, 2024; Sweller et al., 2011).

The selection of these instructional strategies is grounded in established Cognitive Load Theory principles. Worked examples reduce unnecessary search processes during early learning, self-explanation prompts encourage learners to actively integrate new information with prior knowledge, and split-attention reduction minimizes unnecessary cognitive processing caused by fragmented instructional materials. Collectively, these strategies support schema acquisition, improve knowledge transfer, and strengthen students' capacity to solve increasingly complex calculus problems.

Furthermore, this study distinguishes between global instructional approaches, which often present mathematical concepts in generalized and decontextualized forms, and contextualized instructional approaches that situate learning within authentic, meaningful environments. Within the Ethiopian higher education context, contextualized instruction is particularly important because it enables students to connect abstract calculus concepts with familiar real-life situations, thereby increasing engagement, reducing unnecessary cognitive burden, and promoting deeper conceptual understanding.

The study‘s central aim is to develop and evaluate an instructional model that minimizes extraneous cognitive load while supporting germane load, thereby improving problem-solving performance in calculus. This will be examined through a two-month instructional intervention, whose duration corresponds to the allocated teaching period for the selected Functions of One Variable unit in the university Calculus course syllabus, comparing WMCBPSI with conventional problem-solving instruction (CPSI) among first-year Ethiopian university students. Evaluation will combine quantitative performance measures with qualitative observations of problem-solving processes, offering both empirical evidence and practical insights into the effectiveness of WMCBPSI for mathematics instruction.

First-year university students were intentionally selected because they are transitioning from secondary school mathematics to university-level calculus, where abstract reasoning, symbolic manipulation, and multi-step problem-solving become substantially more demanding. Establishing effective problem-solving strategies during this foundational stage is expected to improve subsequent learning in calculus and other STEM disciplines.

1.2. MATHEMATICAL KNOWLEDGE AND PROBLEM-SOLVING IN HIGHER EDUCATION

In higher education, mathematical knowledge and problem-solving are intrinsically connected, with problem-solving serving as both a method and a context for deepening university students‘ mathematical understanding. Contemporary research underscores that engaging in problem-solving enhances higher-order cognitive skills, fostering the ability to apply mathematical concepts across diverse contexts (Sweller et al., 2011; Fuchs et al., 2009). Voskoglou (2008) emphasizes that problem-solving constitutes a central dimension of mathematics learning, promoting the transfer of skills and cultivating profound conceptual comprehension.

Although closely related, mathematical knowledge, mathematical problem-solving, and mathematical proficiency represent distinct but complementary constructs. Mathematical knowledge refers to learners' understanding of mathematical concepts, principles, procedures, and relationships. Problem-solving is the process of applying this knowledge to resolve unfamiliar or non-routine mathematical situations, whereas mathematical proficiency encompasses a broader combination of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick et al., 2001). Accordingly, effective problem-solving contributes to mathematical proficiency but should not be regarded as synonymous with it.

The interdependence of problem-solving and mathematical proficiency is well-established. Kilpatrick et al., 2001) contend that problem-solving provides a foundational basis for engaging with core mathematical concepts, including numbers, operations, and functions. Further, Ketterlin-Geller and Chard (2011) argue that effective problem-solving nurtures critical thinking and analytical reasoning, which are essential for advanced mathematical competence. Recent studies (Gilmore, 2023; Karabay & Meşe, 2024) further confirm that problem-solving promotes meaningful engagement with mathematics and reinforces conceptual mastery.

Despite this evidence, many students continue to encounter difficulties in applying mathematical knowledge effectively, which often reflects gaps in their problem-solving capabilities (Pimta et al., 2009). This suggests a need for instructional interventions specifically designed to integrate mathematical knowledge and problem-solving processes. This study proposes that Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) can bridge this gap by offering targeted strategies that improve calculus problem-solving.

The instructional framework of WMCBPSI is informed by Cognitive Load Theory, which posits that learning is optimized when instructional activities are designed to align with the limited capacity of working memory. By minimizing unnecessary cognitive demands, organizing information systematically, and promoting schema construction through meaningful learning activities, WMCBPSI enables students to devote greater cognitive resources to mathematical reasoning and problem solving.

Moreover, the intervention incorporates contextualized mathematical tasks that connect abstract calculus concepts with authentic situations familiar to students. Such contextualization is particularly relevant in Ethiopian higher education, where connecting mathematical ideas to students' educational and real-life experiences can enhance engagement, improve conceptual understanding, and facilitate the transfer of learning to new situations.

By embedding problems in authentic, real-world contexts and addressing cognitive constraints, WMCBPSI aims to strengthen both mathematical knowledge and the development of higher-order problem-solving skills.

Recent contributions to the field suggest that such context-based interventions significantly enhance mathematical understanding and problem-solving performance (Kala & Ayas, 2023; Sweller et al., 2011). These findings also support the view that instructional approaches informed by Cognitive Load Theory not only improve immediate problem-solving performance but also facilitate long-term schema acquisition, thereby enabling learners to solve increasingly complex mathematical problems more efficiently. This research, therefore, examines WMCBPSI as an approach that aligns problem-solving instruction with the cognitive architecture of learners, thereby promoting deeper learning and improving students' calculus problem-solving skills and overall mathematical proficiency.

1.3. HUMAN COGNITIVE ARCHITECTURE AS A LENS FOR UNDERSTANDING PROBLEM-SOLVING IN CALCULUS

This study examines how human cognitive architecture, particularly the interplay of working memory (WM) and long-term memory (LTM), underpins problem-solving in calculus, providing a lens for designing effective instructional interventions. Cognitive Load Theory (CLT) proposes that effective learning—especially for complex domains such as mathematics—depends on instructional design that aligns with the structure of human cognition (Sweller et al., 2011; Kala & Ayas, 2023). Rather than focusing solely on learners' intellectual ability, CLT emphasizes that learning outcomes are strongly influenced by how instructional materials are designed and presented. Instruction becomes more effective when it minimizes unnecessary mental processing, manages the inherent complexity of learning tasks, and promotes cognitive processes that support schema construction and knowledge transfer. In the context of calculus, understanding cognitive architecture is essential for creating instructional strategies that reduce unnecessary mental effort while promoting meaningful problem-solving.

Human cognitive architecture comprises two primary systems: working memory and long-term memory. WM acts as a transient processing system for holding and manipulating information during problem-solving tasks (Baddeley, 2003), whereas LTM stores organized knowledge structures, or schemas, that facilitate efficient retrieval and application in novel situations (Sweller et al., 2019).

Schemas are cognitive structures that organize related concepts and procedures into meaningful units, thereby reducing the burden on working memory during problem solving. As learners acquire experience, repeated practice and meaningful learning enable individual pieces of information to become integrated into increasingly sophisticated schemas stored in long-term memory. Learning occurs when WM integrates new information into existing LTM schemas, enabling transfer and sustained understanding. In calculus problem-solving, WM limitations—typically holding 4 ± 1 elements for about 20 seconds without rehearsal—highlight the importance of instructional designs that minimize unnecessary cognitive load (Cowan, 2001; Baddeley, 2003).

Although several theories explain mathematical learning, including information-processing theory and conceptual change theory, this study adopts Cognitive Load Theory because it provides explicit instructional principles for managing the interaction between working memory and long-term memory during the learning of complex mathematical concepts. This theoretical perspective is particularly appropriate for designing instructional interventions intended to improve calculus problem-solving.

Cognitive load refers to the mental resources required to process information, and when not managed effectively, it can obstruct learning in mathematically demanding contexts such as calculus (van Merriënboer & Sweller, 2010). CLT distinguishes three forms of cognitive load: intrinsic cognitive load, which arises from the inherent complexity of the learning task; extraneous cognitive load, which results from ineffective instructional design; and germane cognitive load, which represents the mental effort devoted to constructing and refining schemas. Effective instruction seeks to manage intrinsic load, minimize extraneous load, and encourage germane load so that learners can devote their cognitive resources to meaningful learning.

Recent empirical work shows that structured instructional approaches significantly reduce extraneous load while enhancing learning efficiency (Kala & Ayas, 2023; Karabay & Meşe, 2024). This study introduces the concept of ―working memory contexts‖ as a way to optimize learning conditions. These contexts consist of instructional strategies designed to minimize unnecessary cognitive demand, foster schema acquisition, and promote automaticity in problem-solving.

The Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) developed in this study operationalizes these principles by integrating worked examples, self-explanation prompts, contextualized mathematical tasks, and instructional materials designed to reduce split attention. Together, these instructional components decrease unnecessary cognitive processing while encouraging learners to organize mathematical concepts into coherent schemas that can be efficiently retrieved and applied to unfamiliar calculus problems.

Mathematics learning benefits from such approaches because WM actively organizes relevant information into schemas, which are stored in LTM for later use. Experts, for example, can identify mathematical patterns rapidly, reducing cognitive effort, whereas novices expend significant WM capacity attempting to process isolated information. However, automaticity should not be interpreted as rote memorization. Rather, it reflects the development of well-organized schemas that free working memory resources, enabling learners to engage in flexible reasoning, adapt previously learned knowledge to unfamiliar situations, and solve increasingly complex mathematical problems. By integrating working memory contexts into calculus instruction, this research aims to create conditions that reduce cognitive strain and enhance schema development, ultimately improving students' calculus problem-solving skills and overall mathematical proficiency.

1.4. WORKING MEMORY AS A THEORETICAL BASIS FOR MATHEMATICAL PROBLEM-SOLVING

Working memory (WM) plays a central role in calculus problem-solving, where learners must hold intermediate results and integrate new information across multiple steps. Understanding WM provides a theoretical basis for designing instruction that supports effective problem-solving in mathematics. Recent cognitive science research underscores the strong link between WM and mathematical problem-solving, although debates continue regarding the direct role of WM in mathematical development and related learning challenges (Allen et al., 2022). Evidence consistently shows that WM capacity plays a decisive role in performing complex, multi-step tasks, such as those required in calculus, where learners must retain partial results while integrating new information to derive solutions (Peng et al., 2016; Allen et al., 2022). Consequently, instructional approaches that are explicitly designed to operate within the limits of working memory are more likely to promote meaningful learning and successful mathematical problem-solving than approaches that overlook learners' cognitive constraints.

WM is often conceptualized as a dynamic workspace for the temporary storage and manipulation of information necessary for problem-solving (Baddeley, 2003). Mathematical proficiency—spanning arithmetic, algebra, geometry, and calculus—relies heavily on the ability to maintain and process relevant information in WM. Theoretical perspectives on WM‘s role in mathematics have evolved considerably. Earlier models, such as McCloskey‘s abstract code model (1992) and Dehaene‘s triple-code model (1992), did not explicitly incorporate WM as a construct. However, contemporary frameworks increasingly integrate WM as essential for explaining individual differences in problem-solving proficiency across age groups and learning contexts (Geary & Hoard, 2019; LeFevre et al., 2021). This evolution reflects growing recognition that successful mathematical learning depends not only on the amount of information learners can temporarily retain, but also on how efficiently working memory interacts with long-term memory to construct, organize, and retrieve mathematical schemas during problem solving.

From a practical standpoint, recognizing WM‘s role in mathematics is vital for designing effective instruction. Instructional approaches that manage WM load—such as scaffolding, worked examples, and context-based problems—can mitigate cognitive overload and enhance learners' problem-solving capacity (Case & Okamoto, 1996; Sweller et al., 2011). Similarly, self-explanation prompts encourage learners to reflect on solution procedures, while split-attention reduction strategies integrate related sources of information into coherent instructional materials. These approaches reduce unnecessary cognitive processing and allow learners to devote greater working memory resources to understanding mathematical relationships and developing effective solution strategies. This insight applies across educational levels, but it is especially relevant in calculus, where cognitive demands are high.

Cognitive science research further clarifies that WM and long-term memory (LTM) work collaboratively to support problem-solving. LTM stores schemas—structured knowledge frameworks that allow learners to categorize problems efficiently and retrieve relevant procedures quickly (Rockwell et al., 2011). Developing robust, domain-specific schemas reduces the load on WM and enables more efficient problem-solving (Sweller et al., 2019). Repeated engagement with meaningful problem-solving tasks strengthens these schemas, enabling learners to recognize underlying mathematical structures, transfer previously learned knowledge to unfamiliar situations, and solve increasingly complex calculus problems with greater efficiency.

Problem-solving models grounded in schema theory (Jonassen, 2011; Schnotz & Kürschner, 2007) emphasize that schema acquisition is as important as solution finding. For example, WM can be taxed not only by calculations but also by auxiliary tasks, such as interpreting diagrams or symbolic representations, which are essential in calculus problem-solving (Raghubar et al., 2010). Neuroimaging studies corroborate this, showing that task presentation, pacing, and format affect how mathematical information is processed and highlight the importance of instructional design in optimizing WM efficiency (Allen et al., 2022). These findings support the central assumption of Cognitive Load Theory that carefully designed instructional materials can improve learning outcomes by reducing extraneous cognitive load while preserving learners' cognitive resources for schema construction and meaningful mathematical reasoning.

In this study, CLT informs the design of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI). This approach aims to cultivate problem-solving schemas by managing WM load and aligning instruction with cognitive capacity (Sweller et al., 2011). Well-structured WMCBPSI activities reduce extraneous WM demands, enabling students to focus on problem-solving strategies and foster transferable skills. Specifically, worked examples support the acquisition of new procedures during initial learning, self-explanation prompts encourage learners to integrate new information with existing knowledge, contextualized mathematical problems enhance conceptual understanding by connecting abstract ideas to authentic situations, and split-attention reduction strategies prevent unnecessary cognitive overload by presenting related information in an integrated format. Together, these instructional components operationalize the principles of Cognitive Load Theory within the proposed instructional intervention.

Moreover, the design aligns with Bloom‘s taxonomy by integrating cognitive, affective, and psychomotor dimensions of learning, all of which are influential in problem-solving (Pimta et al., 2009). Within the cognitive domain, students progress from understanding and applying mathematical concepts to analyzing, evaluating, and creating solution strategies. The affective domain is addressed by fostering confidence, persistence, and positive attitudes toward solving complex calculus problems, while the psychomotor domain is reflected in the accurate execution of mathematical procedures, symbolic manipulation, graphical interpretation, and technology-assisted problem-solving where appropriate. Thus, WMCBPSI supports holistic learning by integrating cognitive processing with students' attitudes and procedural competencies.

It is important to emphasize that the development of automaticity through repeated schema acquisition does not imply rote memorization. Rather, automaticity reduces unnecessary working memory demands, thereby enabling learners to allocate greater cognitive resources to flexible reasoning, conceptual understanding, and the solution of novel mathematical problems.

By applying WMCBPSI, this study seeks to enhance calculus problem-solving skills through structured instructional strategies that manage cognitive load and strengthen schema acquisition.

1.5. STATEMENT OF THE PROBLEM

Problem-solving in mathematics, particularly in calculus, is a cornerstone of higher education and a prerequisite for success in STEM disciplines. Despite its importance, many first-year university students continue to struggle with solving calculus problems effectively. Studies in Ethiopia and comparable contexts have documented persistent challenges, including limited conceptual understanding, overreliance on rote memorization, and difficulty applying mathematical knowledge to multi-step or context-based problems (Mussa & Fekede, 2023; Taye & Mengistu, 2024). These difficulties often result in low academic performance, reduced persistence in STEM programs, and diminished problem-solving confidence.

Within the cognitive domain, students progress from understanding and applying mathematical concepts to analyzing, evaluating, and creating solution strategies. The affective domain is addressed by fostering confidence, persistence, and positive attitudes toward solving complex calculus problems, while the psychomotor domain is reflected in the accurate execution of mathematical procedures, symbolic manipulation, graphical interpretation, and technology-assisted problem-solving where appropriate. Thus, WMCBPSI supports holistic learning by integrating cognitive processing with students' attitudes and procedural competencies.

It is important to emphasize that the development of automaticity through repeated schema acquisition does not imply rote memorization. Rather, automaticity reduces unnecessary working memory demands, thereby enabling learners to allocate greater cognitive resources to flexible reasoning, conceptual understanding, and the solution of novel mathematical problems.

Conventional teaching practices, which emphasize procedural demonstration and routine exercises, frequently fail to address these issues. Such approaches can overload students‘ working memory by presenting information in fragmented ways, preventing meaningful integration of concepts (Sweller et al., 2019; Ibrahim & Adusei, 2022). Cognitive Load Theory (CLT) argues that instructional strategies must be deliberately designed to manage working memory limitations, particularly when learners face complex, multi-step problem-solving tasks (Ouwehand et al., 2025).

Based on Cognitive Load Theory, the instructional components incorporated into WMCBPSI were deliberately selected because each addresses a specific source of cognitive demand. Worked examples reduce unnecessary search processes during initial learning, self-explanation prompts encourage learners to actively integrate new information with their existing knowledge structures, and split-attention reduction strategies minimize unnecessary cognitive processing by presenting related information in an integrated manner. Collectively, these instructional principles enable learners to devote greater cognitive resources to schema acquisition, conceptual understanding, and mathematical reasoning.

Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) offers a promising approach. By integrating worked examples, self-explanation prompts, and strategies that reduce split attention, WMCBPSI supports learners in focusing on essential information while developing transferable problem-solving skills (Zhou & Leppink, 2021; Sweller et al., 2011). However, there is limited evidence on its effectiveness in enhancing calculus problem-solving in Ethiopian universities, where contextual factors such as large class sizes, resource constraints, and varied student preparedness may influence outcomes.

Furthermore, although Cognitive Load Theory has been extensively investigated in international educational settings, limited empirical evidence exists regarding how its instructional principles can be effectively adapted to Ethiopian higher education mathematics classrooms. Differences in curriculum implementation, classroom organization, instructional resources, and students' prior learning experiences necessitate context-specific investigations before international findings can be generalized to Ethiopian universities.

This study, therefore, addresses a critical gap by examining the effect of WMCBPSI on first-year university students‘ calculus problem-solving skills in Ethiopia. The findings are expected to provide evidence-based insights into how CLT-informed instructional design can be adapted to local contexts, contributing to both theory and practice in mathematics education.

1.6. RESEARCH QUESTIONS

This study seeks to address the following research questions:

1. How can Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) be effectively integrated into calculus instruction to support students’ calculus problem-solving skills?
2. What challenges and facilitators arise in the implementation of WMCBPSI in calculus instruction, and how can they be addressed?
3. What effect does WMCBPSI have on students’ calculus problem-solving skills and calculus problem-solving performance?

1.7. AIMS, OBJECTIVES AND HYPOTHESIS OF THE STUDY

1.7.1 Aim of the Study

The aim of this study is to investigate the effect of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI), grounded in Cognitive Load Theory (CLT), on first-year university students’ calculus problem-solving skills and calculus problem-solving performance in Ethiopia. The study addresses persistent challenges related to applying calculus concepts to multi-step and context-based problems, as highlighted in Ethiopian and international mathematics education research (Taye & Mengistu, 2024; Zhou & Leppink, 2021).

1.7.2 Objectives of the Study

To achieve the aim of this study, the following specific objectives are proposed:

1. To design and implement WMCBPSI for teaching calculus problem-solving skills.

2. To identify the challenges and facilitators associated with the implementation of WMCBPSI in calculus instruction.
3. To evaluate the effect of WMCBPSI on students' calculus problem-solving skills and calculus problem-solving performance.

1.7.3 Research Hypotheses

The study is guided by the following research hypotheses:

Null Hypothesis (H_0)

There is no statistically significant difference in students' calculus problem-solving skills and calculus problem-solving performance between those taught through WMCBPSI and those taught through conventional instruction.

Alternative Hypothesis (H_1)

There is a statistically significant difference in students' calculus problem-solving skills and calculus problem-solving performance between those taught through WMCBPSI and those taught through conventional instruction.

1.8. SIGNIFICANCE OF THE STUDY

This study contributes to improving calculus problem-solving skills among university students in Ethiopia, where the application of Cognitive Load Theory (CLT)-based instructional strategies remains underexplored. While international research has consistently demonstrated the effectiveness of CLT-informed approaches in enhancing problem-solving performance (Kalyuga, 2019; Sweller et al., 2019; Zhou & Leppink, 2021), these frameworks have not been systematically adapted or tested in Ethiopian higher education mathematics instruction. Furthermore, studies indicate that findings from international problem-solving research may encounter challenges related to transferability across diverse educational systems (van Loon-Hillen et al., 2012), underscoring the importance of context-specific investigations that account for cultural, curricular, and institutional differences. By introducing and evaluating the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) framework, this study addresses this gap and provides evidence-based strategies for calculus instruction in Ethiopia.

Beyond examining the effectiveness of an existing instructional approach, this study contributes to the refinement of Cognitive Load Theory by demonstrating how its instructional principles can be adapted and operationalized within the Ethiopian higher education context. Accordingly, the study provides contextual evidence regarding the applicability of CLT-informed instructional design under educational conditions that differ from those commonly reported in international research.

The significance of this study is further strengthened by its alignment with Ethiopia's national commitment to learner-centered pedagogies. Recent studies on learner-centered instruction in African higher education emphasize the need for approaches that connect learning to students' lived experiences and interests, build on prior knowledge and competencies, and embed problems within authentic and relevant contexts (Kasanda et al., 2005; Taye & Mengistu, 2024). Although earlier cognitive psychology frameworks emphasized meaningful, interest-driven problem-solving (e.g., Wertheimer, 1959; Bruner, 1960; Hiebert et al., 1996), their application in calculus classrooms remains limited. This study examines whether WMCBPSI can bridge this gap by fostering deeper engagement and supporting robust problem-solving skills.

The findings are also expected to establish a foundation for future research on Cognitive Load Theory-based instructional interventions in Ethiopian mathematics education. Specifically, the study may encourage subsequent investigations involving different mathematical topics, educational levels, instructional technologies, and experimental designs, thereby contributing to the continued advancement of mathematics education research in Ethiopia and comparable educational contexts.

The findings are expected to provide actionable insights for curriculum designers, teacher educators, and policymakers seeking to improve higher education mathematics instruction in Ethiopia and similar contexts.

1.9. DEFINITIONS OF KEY TERMS

Problem: A mathematical task or situation for which the solution method is not immediately apparent, requiring reasoning and the application of relevant mathematical knowledge (Polya, 1957; Jonassen, 2011).

Problem-Solving: The process of applying prior knowledge, reasoning, and appropriate strategies to find a solution to a mathematical problem (Polya, 1957; Ormrod, 2019;

Schoenfeld, 2023).

Problem-Solving Skill: The ability to solve calculus problems accurately and efficiently through the application of conceptual knowledge, procedural knowledge, reasoning, and appropriate problem-solving strategies. In this study, problem-solving skill refers to students' performance on the Calculus Achievement Test (CAT) following instruction (Renkl & Atkinson, 2010; LeFevre et al., 2021).

Learning: The process of acquiring, organizing, and applying knowledge or skills through instruction or experience, resulting in the development and refinement of schemas in long-term memory (Schnotz & Kürschner, 2007; Kalyuga, 2019).

Problem-Solving Instruction: An instructional approach in which mathematical problems serve as the primary means of teaching concepts, promoting active engagement, reasoning, and the application of prior knowledge (Kim & Park, 2021; Widada et al., 2022).

Conventional Problem-Solving Instruction (CPSI): In this study, CPSI refers to the teacher-centred instructional approach used with the comparison group, characterized by teacher explanation, demonstration of solution procedures, and individual student practice with limited collaborative learning and cognitive load support (Putri & Rahayu, 2021; Huang & Wang, 2023).

Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI): In this study, WMCBPSI refers to the instructional approach developed by the researcher based on Cognitive Load Theory to improve students' calculus problem-solving skills. The approach incorporates worked examples, chunking, split-attention reduction, self-explanation, and collaborative problem-solving activities to manage working memory demands, facilitate schema construction, and improve problem-solving performance (Sweller et al., 2019; Retnawati et al., 2022; Zheng et al., 2023).

1.10. ASSUMPTIONS OF THE STUDY

This study was conducted based on the following assumptions:

1. The participants responded honestly and independently to the research instruments administered during the study.
2. The participants understood and interpreted the test items and instructional activities as intended by the researcher.
3. The instructional learning environment represented typical classroom conditions, with students engaging in the learning activities in a natural and typical manner.
4. The theoretical principles of Cognitive Load Theory and Working Memory Theory provided an appropriate theoretical framework for explaining and interpreting students' calculus problem-solving processes in the Ethiopian university context.

1.11. DELIMITATIONS OF THE STUDY

This study was delimited in several ways in order to maintain focus and ensure manageability.

First, the study was confined to first-year, non-repeating students enrolled in calculus courses at one Ethiopian university. This population was deliberately selected to provide a consistent instructional and institutional context for evaluating the effectiveness of WMCBPSI. Therefore, the findings may not be directly generalizable to other universities or educational settings.

Second, the study focused only on calculus topics involving functions of one variable. Other areas of mathematics, such as multivariable calculus and advanced mathematical content, were not included because they were beyond the scope of the study.

Third, the study specifically examined the effect of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI), grounded in Cognitive Load Theory (CLT), on students' calculus problem-solving skills and calculus problem-solving performance. The study did not investigate the effects of other instructional approaches or learner variables, such as motivation, attitudes, or prior mathematical ability, except where they emerged from the qualitative findings.

Finally, the intervention was implemented over a period of two months, consistent with the university academic calendar and the planned instructional programme**.** The findings therefore primarily reflect the short-term effects of the intervention.

1.12. THEORETICAL ORIENTATION AND THESIS STRUCTURE

Problem-solving is a central cognitive activity that enables individuals to identify, evaluate, and implement solutions to achieve specific goals (Wang & Chiew, 2010). In this study, problem-solving skills refer specifically to learners' abilities to apply mathematical knowledge, strategies, and reasoning processes to solve calculus problems effectively.

However, learners may encounter difficulties when faced with complex or poorly organized information, resulting in cognitive overload and reduced problem-solving efficiency. Cognitive Load Theory (CLT) provides a framework for designing instruction that optimizes learning. Initially articulated by Sweller (1988) and refined in subsequent research (Sweller et al., 2019; Zheng et al, 2023), CLT emphasizes reducing extraneous cognitive load caused by inefficient instructional design, thereby freeing working memory resources for schema construction and automation. By reducing unnecessary cognitive demands and organizing learning materials in accordance with human cognitive architecture, CLT supports more efficient use of working memory for learning and problem solving.

Working memory (WM) plays a central role in this process, serving as a transient workspace for processing and integrating information before storage in long-term memory (LTM) (Baddeley, 2012; Sweller et al., 2019). Given WM‘s limited capacity, instructional approaches must align with learners‘ cognitive architecture to maximize learning outcomes.

The primary objective of this study is to apply CLT principles in designing the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) model, aiming to enhance students‘ calculus problem-solving skills by facilitating schema development. The WMCBPSI model applies CLT principles by structuring learning experiences in ways that manage cognitive load and support the development of organized knowledge structures. Schemas—structured cognitive frameworks—enable learners to categorize problems and apply appropriate strategies effectively (Renkl & Atkinson, 2021; Sweller et al., 2019). Constructing and automating schemas is essential for achieving expertise in problem-solving. Learning is defined as the process of encoding knowledge and skills into LTM for later retrieval and application, involving active engagement of WM, where information is organized, rehearsed, and integrated with prior knowledge before consolidation into schemas (Sweller et al., 2019). Schemas serve as mental frameworks that guide problem-solving by enabling rapid recognition of problem types and the selection of relevant strategies (Kalyuga, 2021).

To guide the reader through the thesis, the chapter structure is organized as follows. Chapter 1 introduces the study, outlining the background, problem statement, significance, objectives, and key concept definitions, and establishes the conceptual link between working memory and problem-solving skills. Chapter 2 explores human cognitive architecture, examining the roles of WM, LTM, and schemas in learning and problem-solving. It further discusses the theoretical foundations of CLT and related instructional principles underlying the WMCBPSI model. Chapter 3 presents the detailed theoretical framework grounded in CLT, explaining how cognitive load affects learning and informs instructional design to enhance problem-solving skills. Chapter 4 reviews relevant literature on problem-solving and cognitive load, identifies research gaps, and situates the study within broader research contexts. Chapter 5 describes the research methodology, including design, participants, data collection, and analysis procedures for both quantitative and qualitative data. Chapter 6 presents the quantitative findings, focusing on students' calculus problem-solving performance, while Chapter 7 offers qualitative insights from interviews and classroom observations to complement the quantitative results. Chapter 8 synthesizes and discusses the findings in relation to the study's objectives, theoretical orientation, and implications for mathematics instruction. Finally, Chapter 9 concludes the thesis by summarizing key findings, discussing limitations, and providing recommendations for future research and practice.

CHAPTER TWO

COMPONENTS OF HUMAN COGNITIVE ARCHITECTURE

2.1 INTRODUCTION

This chapter examines the structure and functioning of human cognitive architecture and explains its relevance to developing mathematical problem-solving skills. Insights from cognitive science are essential because they reveal how mental systems operate and how these processes influence learning. Cognitive Load Theory (CLT) emphasizes that instruction is most effective when it aligns with the strengths and limitations of human cognitive architecture (Sweller et al., 2019; Zheng et al., 2023).

This chapter introduces the core elements of cognitive architecture—sensory memory, working memory, long-term memory, and schemas—and highlights their implications for the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) designed and applied in this study. Specifically, the chapter establishes the theoretical rationale for examining how limitations of working memory, the organization of knowledge in long-term memory, and schema development influence students' ability to solve complex calculus problems.

2.2. HUMAN COGNITIVE ARCHITECTURE

Cognition refers to the mental processes that enable individuals to perceive, interpret, store, and utilize information (Rockwell et al., 2011). In cognitive science, cognitive architecture describes the enduring mental structures and mechanisms that govern these processes. Langley et al. (2009) define it as the system of rules and representations that guide how knowledge is acquired, organized, and retrieved.

The core components of human cognitive architecture include:

1. Sensory Memory – Briefly stores raw sensory input, allowing initial perception before information is transferred for further processing.
2. Working Memory (WM) – Temporarily holds and manipulates information during active thinking and reasoning.
3. Long-Term Memory (LTM) – Serves as a permanent store of organized knowledge, typically structured as schemas for efficient retrieval and application.

Working memory plays a central role in problem-solving because it actively processes and integrates information. However, its capacity is inherently limited to only a few elements at a time (Cowan, 2001). When cognitive demands exceed this capacity, learners may experience cognitive overload, which constrains comprehension and learning efficiency (Sweller et al., 2019).

By contrast, long-term memory possesses virtually unlimited capacity and stores knowledge in the form of schemas—organized mental frameworks that enable rapid and efficient problem-solving (Ericsson & Kintsch, 1995). Cognitive Load Theory (CLT) emphasizes the need for instructional designs that reduce extraneous cognitive load while fostering the formation and automation of schemas (Paas & Sweller, 2014; Kalyuga, 2021).

Together, these components explain why learners may experience difficulties when solving unfamiliar mathematical problems. When relevant schemas are insufficiently developed, learners must rely heavily on limited working memory resources, increasing cognitive demands and reducing problem-solving efficiency.

The Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) framework developed in this study was grounded in these cognitive principles. It was specifically designed to minimize unnecessary cognitive load, encourage collaborative reasoning, and facilitate the construction of durable, transferable problem-solving schemas in calculus.

2.3. MODELS OF MEMORY

The scientific study of memory has evolved over time, with successive models offering distinct perspectives on how knowledge is acquired and retained.

Ebbinghaus (1890) pioneered the systematic study of memory, demonstrating that repeated linking of new material to existing knowledge strengthens recall, whereas lack of rehearsal leads to rapid forgetting.

Bartlett (1932) shifted focus toward a constructive view of memory, arguing that remembering involves actively reorganizing information in light of existing knowledge and experiences. This perspective emphasizes the role of meaning and context in memory formation and retrieval.

In the mid-20th century, the information-processing model emerged as a central framework in cognitive psychology. It conceptualizes memory as a system in which information passes through sequential stages—sensory memory, short-term memory, and long-term memory—while undergoing active processing (Baddeley, 2007; Sweller et al., 2019). This model highlights the importance of WM in manipulating information before its integration into long-term memory, which is critical for reasoning and problem-solving.

Although early models primarily emphasized memory as a sequence of storage systems, later models highlighted the active and interactive nature of cognitive processing. This theoretical development provides the foundation for contemporary perspectives, including Cognitive Load Theory, which focuses on how instructional conditions influence the use of limited working memory resources and the construction of knowledge structures in long-term memory.

These theoretical models form the foundation for understanding how students acquire and retain mathematical problem-solving strategies. CLT builds on these insights, stressing the importance of aligning instruction with working memory capacity to promote the transfer of durable schemas into long-term memory (Sweller et al., 2011; Zheng et al., 2023). This alignment underpins the WMCBPSI framework developed in this study, which is designed to support efficient learning and problem-solving in calculus.

2.3.1. Waugh and Norman‘s Model of Memory

In 1965, Waugh and Norman introduced a two-stage model of memory that has since played a foundational role in understanding human information processing. Their framework distinguishes between:

1. Primary Memory – The immediate, short-lived holding of information, analogous to the contemporary concept of working memory.
2. Secondary Memory – The more durable storage of information, corresponding to long-term memory.

Figure 2.1 illustrates the relationship between these two components.

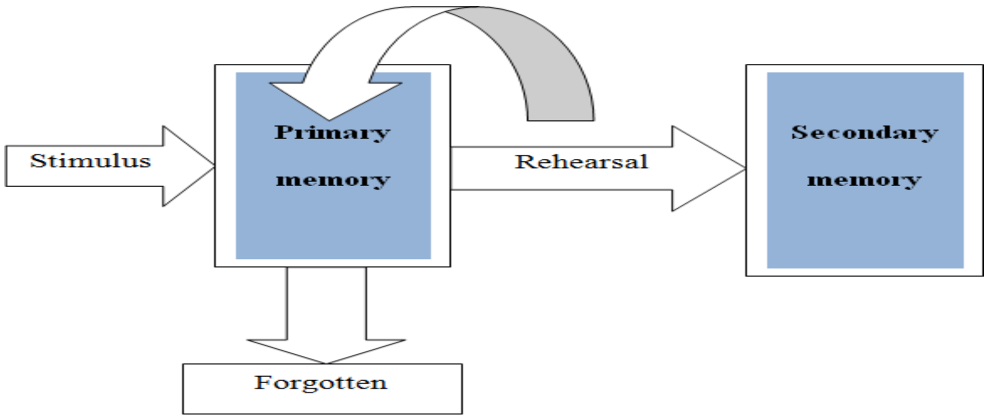

Figure 2.1: Waugh and Norman's Memory Model. Source: Khateeb (2008, p. 5)

According to this model, information entering primary memory requires active rehearsal to be retained. Without timely rehearsal, such input rapidly fades. Moreover, because primary memory has limited capacity, new information tends to displace existing content unless it is actively processed (Waugh & Norman, 1965; Khateeb, 2008).

For longer-term retention, information must be transferred into secondary memory. Rehearsal is the principal mechanism for this transfer. Once information is consolidated in secondary memory, continued rehearsal is no longer necessary for recall.

This framework provides important insights for educational problem-solving. Primary memory operates similarly to working memory in Cognitive Load Theory (Sweller et al., 2019), temporarily handling new information but being highly susceptible to overload during complex tasks. Instructional approaches that manage cognitive load—by reducing extraneous processing demands and encouraging effective rehearsal—facilitate the movement of knowledge into secondary memory. This process underpins the durable storage of problem-solving strategies and mathematical concepts, making them more readily available for future use.

However, contemporary cognitive research views memory as a more interactive system rather than a simple transfer process between separate storage locations. Therefore, this model is considered primarily as a historical foundation for understanding memory limitations, while later models provide a more detailed explanation of active cognitive processing.

A related perspective is presented in Cooper's (1998) model (see Figure 3.3), which aligns working memory with primary memory and long-term memory with secondary memory. Both frameworks highlight the crucial role of rehearsal and the constraints of memory capacity in successful storage and retrieval. Within this study, these insights underscore the importance of instructional strategies—such as those embodied in the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) model—that actively support rehearsal and scaffold the consolidation of problem-solving skills into long-term memory.

2.3.2. Memory Model Proposed by Atkinson and Shiffrin

Atkinson and Shiffrin (1968, as cited in Khateeb, 2008) introduced the multi-store model of memory, which has been highly influential in shaping subsequent perspectives on human cognitive architecture. While more recent research in cognitive science has refined and expanded upon its principles, this model remains a valuable foundation for understanding how information is processed and retained. As illustrated in Figure 2.2, the model differentiates three principal memory systems: the sensory register, the short-term store, and the long-term store.

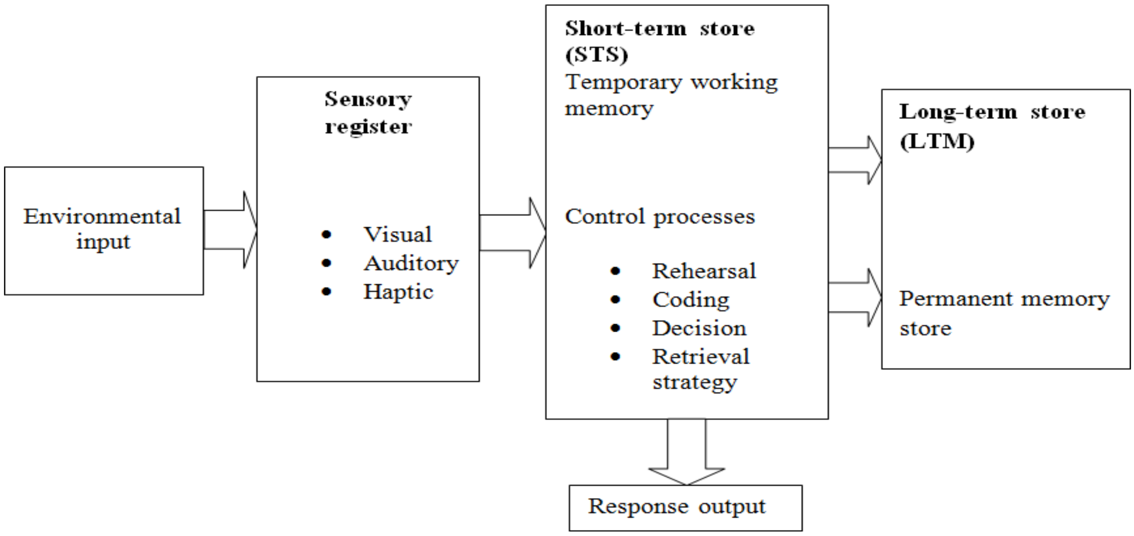

Figure 2.2: Representation of Atkinson and Shiffrin's Multi-Store Memory Model.

Atkinons & Shiffrin (1968, p. 92)

The Atkinson–Shiffrin model has been extensively referenced in memory research and continues to shape our understanding of the progression of information through the cognitive system. Unlike earlier frameworks such as Waugh and Norman‘s (1965) emphasis on primary and secondary memory, Atkinson and Shiffrin introduced the sensory register as the initial gateway for incoming information. Their most significant contribution was the clear differentiation between short-term storage and long-term storage, a distinction that has guided much of the later theoretical development in cognitive psychology.

This framework carries direct relevance for problem-solving and the management of cognitive load. The short-term store—conceptually comparable to working memory within Cognitive Load Theory (Sweller et al., 2019)—serves as the principal mechanism for holding and manipulating new information during learning tasks. Within the context of this study, examining how information is first filtered through sensory input and then transferred to short-term memory helps explain the cognitive strain learners experience when solving problems. As discussed in Chapter 3, cognitive load can be effectively managed through instructional designs that minimize unnecessary mental effort and promote the consolidation of knowledge into long-term memory.

Although the model has been criticized for presenting memory as a linear sequence of stages, its distinction between temporary processing and durable knowledge storage remains useful for explaining why learners require appropriate instructional support when dealing with complex mathematical tasks.

Contemporary scholarship has refined this understanding, particularly with regard to the interaction between sensory and short-term memory systems. Baddeley (2000) advanced the Atkinson–Shiffrin framework by introducing the concept of a central executive along with specialized subsystems, including the phonological loop and visuospatial sketchpad**.** This enriched perspective offers a more detailed account of how learners temporarily process information, aligning closely with this study's aim of supporting working memory efficiency during problem-solving through cognitive load management.

Neuroimaging research has further strengthened these refinements. Rees et al. (2002) identified a functional ―working memory network‖ in the brain, emphasizing its role in higher-order thinking, including problem-solving. These findings highlight the importance of reducing extraneous cognitive load so that working memory resources are available for constructing durable problem-solving schemas—an objective central to the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) proposed in this research.

2.3.2.1. The Sensory Register

In the Atkinson and Shiffrin (1968, as cited in Khateeb, 2008) framework, sensory information from the environment—such as visual, auditory, and tactile stimuli—first enters the sensory register. Here, information is held for only a very brief period before being relayed to short-term memory.

Recent research (Van der Meer, et al., 2010) expands this concept by emphasizing that the sensory register functions not merely as temporary storage but as an initial filter that prioritizes information relevant to the task at hand. This filtering mechanism is essential for efficient problem-solving, as it prevents working memory from being overloaded with irrelevant details. Instructional strategies that reduce extraneous sensory input—by structuring learning environments and materials carefully—can therefore enhance learners' ability to concentrate on core problem-solving processes (Sweller, 2010; Sweller et al., 2019).

In mathematical problem solving, this filtering function is particularly important because learners must identify relevant mathematical information from problem statements while ignoring unnecessary details. Effective instructional design can therefore support attention allocation before information reaches working memory.

From a Cognitive Load Theory perspective, the sensory register plays a critical role in determining whether incoming information will be efficiently processed. Misaligned or excessive sensory input can increase extraneous cognitive load, thereby hindering schema construction in long-term memory (Kalyuga, 2021).

2.3.2.2. The Short-Term Store

The short-term store functions as a temporary workspace where information is consciously rehearsed and manipulated (Atkinson & Shiffrin, 1971). It plays a central role in decision-making and problem-solving, particularly in mathematics, where learners must hold and transform information to apply strategies effectively.

By design, short-term memory has limited capacity, constraining the volume of information that can be actively processed. Cowan (2001) refined this understanding, suggesting that short-term memory capacity approximates four ―chunks‖ of information—a revision of Miller's (1956) earlier estimate of seven items. This insight underscores how cognitive load directly affects working memory efficiency during problem-solving tasks.

Although the terms short-term memory and working memory are sometimes used interchangeably in educational literature, contemporary cognitive science distinguishes working memory as an active processing and manipulation system rather than merely a passive, temporary storage mechanism. This distinction is important for understanding students' difficulties when solving complex calculus problems, as calculus requires not just holding formulas in mind, but actively transforming and coordinating them under heavy cognitive demands.

Given these limitations, instructional designs that reduce extraneous cognitive load—by integrating worked examples, chunking content, and aligning problem complexity with learner expertise—can significantly improve problem-solving performance (Renkl & Atkinson, 2021; Sweller et al., 2019). Such designs allow learners to focus their cognitive resources on processing essential information rather than managing extraneous demands.

2.3.2.3. The Long-Term Store

The long-term store is capable of retaining information over extended periods and is generally considered to have virtually unlimited capacity (Khateeb, 2008). Contemporary research (Squire, 2004) highlights that the hippocampus plays a central role in the consolidation of long-term memories.

Transferring information from short-term to long-term memory requires effective rehearsal and the activation of neural processes that support memory stabilization (Bahrick, 2010). For problem-solving, successful memory consolidation is vital, as it ensures that relevant strategies and domain knowledge are available for retrieval in novel contexts.

Furthermore, studies (Dudai, 2004) have shown that the durability of memory depends not only on repetition but also on the meaningfulness and emotional significance of the information. This insight challenges purely mechanistic views of memory and suggests that instructional approaches embedding problem-solving tasks within meaningful and authentic contexts enhance retention and transferability.

For mathematical learning, this implies that students are more likely to solve unfamiliar problems successfully when conceptual understanding and problem-solving procedures are organized into accessible schemas rather than memorized as isolated procedures.

Such strategies align with Cognitive Load Theory, which emphasizes designing instruction to promote deep schema construction. In the context of this study, embedding calculus problems within relevant contexts is expected to support schema development and thus improve problem-solving efficiency (Sweller et al., 2019; Kalyuga, 2021).

2.4. MEMORY TYPES AS CONCEPTUALIZED TODAY

In this study, Cooper‘s (1998) modal memory model serves as a central framework for examining the cognitive demands placed on students during problem-solving, particularly in mathematics. This model distinguishes three primary types of memory—sensory memory, working memory (short-term memory), and long-term memory—which together form an integrated information-processing structure of human cognition.

The adoption of Cooper’s model in this study is based on its compatibility with Cognitive Load Theory, particularly its explanation of how limited working memory interacts with knowledge structures stored in long-term memory during learning and problem solving.

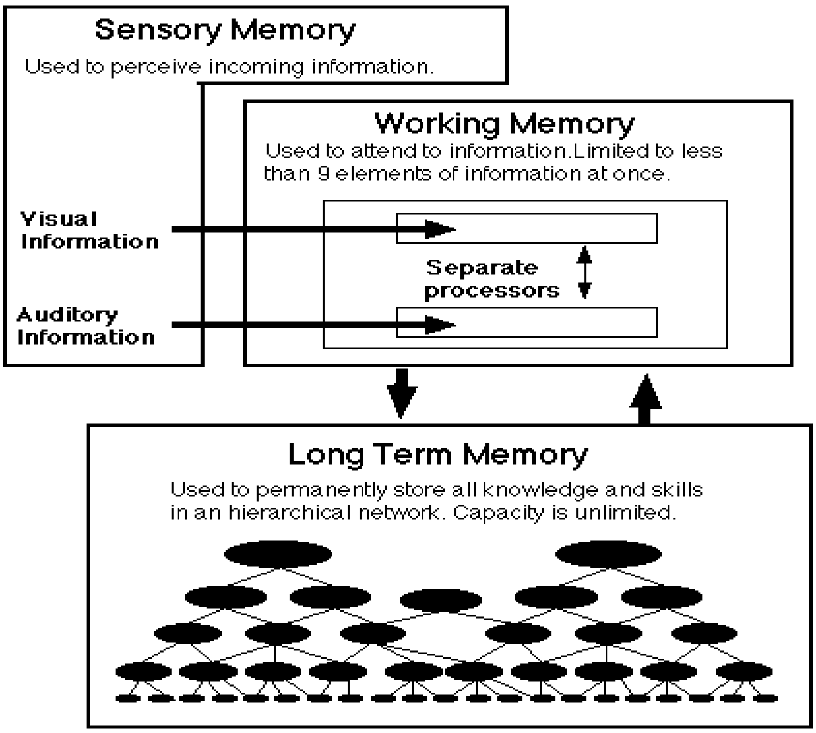

Figure 2.3: Three interconnected memory modes forming an integrated information processing model of human cognitive architecture. Adapted from Cooper (1998), Cognitive Load Theory and Instructional Design, retrieved from

*http://education.arts.unsw.edu.au/CLT_NET_*

Cooper‘s conceptualization remains influential in cognitive science because it clarifies how these memory systems interact during learning and problem-solving tasks. According to this perspective, our perceptions and memories are shaped more by pre-existing knowledge stored in long-term memory than by raw sensory input alone (Cooper, 1998). Long-term memory serves as a repository of schemas—structured knowledge frameworks—that working memory can access and manipulate.

The bidirectional flow illustrated in Figure 2.3 reflects this dynamic exchange between working memory and long-term memory. Such interaction allows learners to reorganize and adapt stored knowledge to address novel problems, generate new ideas, and effectively engage with their learning environment (Logie, 1999; Baddeley, 2012). From a Cognitive Load Theory standpoint, this model reinforces the importance of instructional designs that support efficient retrieval and integration of schemas to reduce cognitive load and enhance problem-solving performance (Sweller et al., 2019).

By adopting Cooper‘s model, this study situates working memory as a critical bottleneck in problem-solving, particularly for complex calculus tasks. Instructional approaches that respect the capacity limits of working memory—while promoting effective schema construction and retrieval from long-term memory—are therefore central to improving students‘ problem-solving skills in higher education mathematics.

2.4.1. Sensory Memory

Sensory memory represents the initial stage of memory processing, where incoming stimuli from the environment are briefly held before further analysis in working memory (Cooper, 1998). This form of memory deals with sensory input—such as visual, auditory, tactile, olfactory, and gustatory signals—lasting only a very short time, typically from milliseconds to a few seconds. For instance, visual impressions last approximately 0.5 seconds, while auditory ones may persist for about three seconds (Cooper, 1998). Without focused attention and meaningful categorization, sensory information quickly fades or is replaced by new input.

Recent findings in cognitive neuro science have refined this concept. Scolari et al. (2009) demonstrate that attention and prior knowledge influence the processing of sensory data. The brain filters sensory input, prioritizing relevant information and suppressing extraneous data. This filtering process is crucial in problem-solving, as concentrating on pertinent information reduces cognitive burden and allows working memory to operate more efficiently (Jolicoeur et al., 2015).

In the context of calculus problem solving, this filtering process is important because students must identify relevant mathematical relationships, variables, and conditions from complex problem situations before engaging in higher-level reasoning.

Sensory information remains unprocessed until it enters working memory, where it is interpreted and integrated with existing knowledge. More precisely, sensory information undergoes initial processing before attention and working memory resources are allocated to task-relevant information. This aligns with the modal memory model, which emphasizes the role of a cognitive processor in organizing and manipulating data. Short-term storage acts as an intermediary, supporting cognitive work before transferring information to long-term memory for permanent retention (Logie, 1999).

2.4.2. Working Memory

The concept of working memory evolved from earlier notions of short-term memory, which regarded it primarily as a temporary storage before information transfer to long-term memory (Sorden, 2005). By the late 1960s and early 1970s, this view shifted toward a more active model, defining working memory as a system that not only stores but also processes and integrates information simultaneously. This allows for the active manipulation of verbal and visual data (Sorden, 2005).

The term "working" underscores this active nature, marking it as a central space where much cognitive activity occurs. Working memory is closely tied to conscious awareness, unlike long-term memory, which largely operates outside conscious perception (Grossberg, 2010). Paas et al. (2003) emphasize that consciousness is limited to the information currently held in working memory, while vast stores in long-term memory remain outside awareness.

A defining limitation of working memory is its restricted capacity and brief duration. Unrehearsed information is typically lost within about 30 seconds (Paas et al., 2003), and capacity is limited to approximately seven (±2) items at once (Miller, 1956). To manage this constraint, information is often grouped into ―chunks‖—meaningful units ranging from simple digits to complex patterns or ideas. For example, phone numbers are chunked to facilitate recall.

However, contemporary research suggests that working memory capacity depends on task demands, prior knowledge, and the nature of information being processed rather than representing a fixed number of independent units.

These capacity constraints are most evident when working memory processes new, unfamiliar information, requiring trial-and-error strategies that slow problem-solving. In contrast, familiar material stored in long-term memory can be recalled efficiently without imposing such limitations (Paas et al., 2003).

Working memory functions through dual processing. Baddeley‘s (2000) model and Paivio‘s dual-coding theory (Artino, 2008) describe it as having separate subsystems for visual and auditory information. This dual-channel structure allows simultaneous processing across modalities, enhancing cognitive capacity (De Jong, 2010; Van Merriënboer & Sweller, 2010). Instructional approaches engaging both channels can therefore improve learning by optimizing cognitive load (Chong, 2005).

Given these constraints, instructional designs must reduce extraneous cognitive load to maximize working memory efficiency, thereby strengthening learners‘ problem-solving skills in mathematics.

2.4.3. Long-Term Memory

Unlike working memory, which has limited capacity and duration, long-term memory (LTM) provides a virtually unlimited repository for information. LTM contains a person‘s accumulated knowledge, skills, and experiences, which can be accessed over extended periods (Paas et al., 2003). All learned material is stored here, and working memory draws from LTM when processing new information. This interaction is fundamental to both learning and higher-order thinking.

Retrieval from LTM can be rapid for well-practiced information. For instance, recalling one‘s own name or the alphabet occurs almost instantly, whereas less frequently used knowledge may require more time to retrieve (Cooper, 1998). The retrieval process can involve complex operations, such as searching for memory traces and activating relevant knowledge, often aided by cues. Recognition often precedes recall—a phenomenon described as the ―tip of the tongue‖ effect (Sutcliffe, 1995).

From a Cognitive Load Theory perspective, LTM‘s capacity to store extensive networks of associations—including categories, procedures, and strategies—has significant instructional implications. Effective teaching should facilitate the acquisition of domain-specific knowledge, skills, and abilities (KSAs) that learners can store in LTM and retrieve when solving problems (Khateeb, 2008). With repeated practice, problem-solving skills can become automated, enabling learners to apply them efficiently to novel tasks.

Therefore, developing students' long-term knowledge structures is particularly important in mathematics education because successful problem-solving depends not only on temporary processing ability but also on the availability of relevant conceptual and procedural schemas.

2.5. SCHEMA THEORY

Long-term memory stores knowledge in the form of schemata (plural of schema), which are cognitive frameworks organizing information based on its application in specific contexts (Ayres & Van Gog, 2009). A schema can be understood as a learned structure that groups related concepts, enabling efficient storage and retrieval. Song (2011) defines a schema as ―a knowledge structure that represents a class of things, events, and situations‖ (p. 16).

Schema theory explains how knowledge is structured and how these structures influence the use of information. Problem-solving schemata, in particular, are essential for organizing and applying knowledge relevant to solving problems (Song, 2011). The quantity and quality of schemata stored in long-term memory significantly influence problem-solving success.

For mathematical problem solving, schemas are particularly important because they allow learners to recognize problem structures, select appropriate solution strategies, and apply previously acquired knowledge to unfamiliar situations.

Learning does not occur in isolation; new information is interpreted in relation to existing schemata. When encountering novel material, the mind integrates it into pre-existing frameworks, determining how it will be processed and stored. For example, Sweller (1994, p.

296) illustrates that when recalling a —tree, individuals tend to rely on a general—tree schema‖ rather than remembering every specific detail such as leaf shape, branch arrangement, or color.

Problem-solving schemata operate similarly, classifying problems according to solution strategies. This allows individuals to process a wide variety of problems efficiently, as schemata store organized knowledge that can be transferred from long-term memory to working memory. While working memory is capacity-limited, schemata allow multiple pieces of information to be processed as single units, thereby reducing cognitive load (Van Merriënboer & Sweller, 2005).

The capacity to group elements reduces the mental effort required for task completion, particularly when schemata become automated (see Section 2.6). Repeated practice with similar problem types leads students to develop increasingly refined problem-solving schemata. Fuchs et al. (2004) assert that —the broader the schema, the greater the probability that individuals will recognize connections between familiar and novel problems and will know when to apply the solution methods they have learned (p. 419). This underscores schema automation as a vital component of effective learning and problem-solving.

2.5.1. Schema Theory and Learning

Learning involves encoding knowledge and skills into long-term memory, where they become available for future retrieval and application (Cooper, 1998). This process begins in working memory, where new information is processed, rehearsed, and linked to existing knowledge. Once transferred to long-term memory, this knowledge is stored within schemas—organized cognitive structures representing interconnected concepts about the world.

Schemas function as mental frameworks that classify and structure knowledge. They develop in complexity through repeated practice, accumulated experience, and formal instruction, resulting in deeper, more specialized expertise. Experts, for example, possess more extensive and refined schemas within their domains than novices, who often have only foundational knowledge.

However, schema development is not achieved through repetition alone; it requires meaningful engagement with knowledge, appropriate instructional support, and opportunities to apply concepts in varied contexts.

Learning is most effective when new information integrates with existing schemas, as this facilitates meaningful application and retention. Conversely, when new information conflicts with existing knowledge or is presented without logical coherence, meaningful learning becomes more difficult, and rote memorization is more likely. For instance, requiring learners to perform a sequence of tasks without rationale or context can hinder deep understanding and retention.

2.5.2. Schema Automation and Problem-Solving Skill

Automation refers to the ability to execute tasks without continuous conscious control from working memory (Sweller et al., 2011). This ability is critical for developing expertise and enhancing problem-solving efficiency. Khateeb (2008, p. 26) notes that ―automation of skill execution is developmental in nature, reflecting that schema development is an evolving process shaped by repeated exposure and practice.

In mathematics problem-solving, developing expertise requires creating problem-solving schemas that classify problems according to their characteristics and solutions (Fuchs et al., 2004). With repeated exposure to similar problem types, learners progressively develop these schemas, which eventually become automated (Van Merriënboer & Ayres, 2005). Khateeb (2008) emphasizes that once automation occurs, solutions are retrieved and applied rapidly, reflecting expert problem-solving capability.

In calculus learning, schema automation is especially relevant because students must coordinate conceptual understanding, procedural knowledge, and strategic reasoning. Without sufficient automation of foundational knowledge, working memory resources may be consumed by basic operations rather than higher-order problem-solving processes.

The contrast between novices and experts is striking. Experts operate with automated processes, allowing them to recognize problem types and deploy solutions effortlessly. Novices, however, often work through problems via trial-and-error and general strategies such as means–end analysis, which can be inefficient (Jonassen, 1997). According to Jonassen (1997), problem-solving schemata help learners internalize the features of different problem types so they can approach problems confidently. Well-developed schemas enable learners to treat new challenges as variations of familiar problems rather than entirely novel tasks.

Automation transforms problem-solving from an effortful process into a fluent and nearly unconscious one, reducing cognitive load and freeing working memory to handle more complex or higher-order tasks.

2.5.3. The Difference between Novices and Experts

The primary differences between novices and experts in any cognitive domain lie in the scope and sophistication of their schemas and the degree of automation in their problem-solving strategies. Experts possess a vast and refined network of schemas acquired through experience, enabling them to address new problems efficiently without extensive conscious deliberation (Chi et al., 1988).

Evidence shows that higher domain expertise enhances the effective use of working memory. For example, experienced chess players can simultaneously hold and manipulate numerous board positions mentally, even under blindfolded conditions, due to their rich knowledge of game patterns (Saariluoma, 1995). Similarly, devoted soccer fans demonstrate remarkable recall of match details compared to casual viewers, a skill that stems from their well-developed sport-related schemas that optimize working memory function (Morris et al., 1995).

Novices, by contrast, have fewer and less developed schemas, limiting their ability to recognize problems based on prior knowledge. As a result, they treat each problem as unique, relying on slower, less efficient trial-and-error approaches. Even when novices identify a solution, execution tends to be laborious and error-prone (Jonassen, 1997).

This difference between novice and expert performance provides a theoretical justification for instructional approaches that guide learners in constructing and automating problem-solving schemas rather than expecting them to discover complex solution strategies independently.

The manner in which instructional content is presented can significantly affect their learning outcomes. Instructional approaches that scaffold learning and minimize extraneous cognitive load can effectively help novices build the necessary schemas and gradually approximate expert performance (Cooper, 1998).

2.6. SUMMARY OF THE CHAPTER

This chapter examined human cognitive architecture, emphasizing the roles of sensory memory, working memory, and long-term memory in learning and problem-solving. It highlighted the limited capacity of working memory and how complex tasks can overload this system, thereby constraining problem-solving efficiency.

The chapter further explained how long-term memory mitigates these limitations through schema formation and automation, enabling information to be processed more efficiently and reducing cognitive load during problem-solving.

The review of memory models, Cognitive Load Theory, and schema theory provides the theoretical foundation for understanding why students may experience difficulties in solving complex calculus problems. It also explains the need for instructional approaches that manage working memory demands while supporting the development of meaningful and transferable problem-solving schemas.

From an instructional perspective, the chapter emphasized the importance of designing learning environments that promote schema development and automation. These theoretical principles provide the basis for examining instructional models and strategies that optimize cognitive load and enhance mathematical problem-solving performance. The next chapter focuses on instructional models and strategies aimed at optimizing cognitive load in problem-solving contexts.

CHAPTER THREE

COGNITIVE LOAD THEORY AND ITS IMPLICATIONS FOR INSTRUCTIONAL DESIGN

3.1. CONCEPTUAL FOUNDATIONS OF COGNITIVE LOAD THEORY

Cognitive Load Theory (CLT) provides a comprehensive framework for understanding how human cognitive architecture influences learning, particularly in mathematics problem-solving (Sweller et al., 2011). Grounded in cognitive psychology, CLT emphasizes that effective instruction must align with the inherent constraints of working memory. Learning outcomes, therefore, depend not only on the content taught but also on how instructional information is structured and presented.

A central concern of CLT is the regulation of cognitive load to prevent learners from being overwhelmed during learning activities. When instructional demands exceed working memory capacity, learners struggle to construct and retain meaningful schemas—organized knowledge structures that allow efficient information processing (Sweller, et al., 2011). CLT highlights the interaction between working memory, long-term memory, and schema acquisition, underscoring the need for instruction that supports the organization and integration of knowledge (Sweller et al., 2011).

Within this framework, CLT distinguishes three types of cognitive load, each influencing learning in distinct ways. Intrinsic cognitive load arises from the inherent complexity of the material and the degree of element interactivity within a task. Germane cognitive load refers to the mental effort devoted to organizing information, constructing schemas, and integrating new knowledge with existing cognitive structures. Extraneous cognitive load stems from poorly designed instructional materials or ineffective presentation formats that divert attention away from learning objectives (Chandler & Sweller, 1991). Effective instructional planning seeks to manage intrinsic load, minimize extraneous load, and foster germane load to optimize learning, particularly in cognitively demanding domains such as mathematics (Sweller et al., 2019).

In calculus problem solving, these principles are particularly important because learners must coordinate conceptual knowledge, procedural steps, and strategic reasoning while operating within the limitations of working memory. Therefore, CLT provides a theoretical explanation for why students may experience difficulty when solving complex problems and why instructional support is necessary.

To address these cognitive constraints, CLT has informed empirically supported instructional strategies. Presenting information in smaller, meaningful units—commonly referred to as chunking—enhances comprehension and reduces unnecessary cognitive demands (Gobet & Lane, 2023). Worked examples, where learners are guided through step-by-step solutions, facilitate schema formation and reduce working memory load during problem-solving (Sweller & Cooper, 1985). Instructional designs that reduce split-attention, integrating related sources of information, further lower extraneous cognitive load and improve learning efficiency (Chandler & Sweller, 1991). Together, these strategies demonstrate that CLT-based instruction can promote deeper conceptual understanding and more effective problem-solving performance than conventional approaches.

3.2. THEORETICAL DEVELOPMENT AND CORE ASSUMPTIONS OF COGNITIVE LOAD THEORY

Cognitive Load Theory (CLT) emerged in the late 1980s as a seminal contribution to instructional design, integrating foundational principles from cognitive psychology into educational practice. Developed primarily by John Sweller, CLT builds on earlier theoretical and empirical research on human memory, including Miller‘s (1956) findings on the limited capacity of short-term memory and Baddeley and Hitch‘s (1974) multicomponent model of working memory, which emphasized the simultaneous processing of multiple information streams. Sweller‘s pioneering work, particularly his 1988 publication, formalized the argument that instructional design must explicitly consider working memory limitations to support effective learning.

At its core, CLT conceptualizes learning as a process constrained by the capacity of working memory. Van Gerven et al. (2003) define CLT as an instructional theory that acknowledges the restricted volume of information and the number of cognitive operations that working memory can manage simultaneously. Similarly, Paas et al. (2003) emphasize that CLT is particularly relevant to complex tasks involving multiple interacting elements, which can easily overwhelm learners if instructional demands are not carefully managed. Cognitive load, therefore, refers to the mental effort required to process instructional information, and learning efficiency declines when this effort exceeds working memory capacity (Sweller et al., 2011).

The applicability of CLT is particularly evident in problem-solving contexts, where learners must process task requirements, conceptual knowledge, and procedural steps concurrently. Chang et al. (2001) describe cognitive load as the mental effort imposed by a task, shaped by both its intrinsic complexity and instructional presentation. Poorly structured or unnecessarily complicated materials can overload working memory, resulting in reduced learning outcomes and ineffective problem-solving performance. Sweller (2005) further notes that task complexity and instructional design interact to determine cognitive load, suggesting that carefully organized and simplified instruction enables learners to allocate cognitive resources efficiently, focusing on essential processes rather than managing avoidable complexity.

Thus, the central contribution of CLT is not simply reducing cognitive effort but optimizing the allocation of limited cognitive resources. Effective instruction should reduce unnecessary processing while maintaining sufficient cognitive engagement to promote schema construction and transfer of knowledge to new problem-solving situations.

3.3. INSTRUCTIONAL APPROACHES FOR MANAGING COGNITIVE LOAD

Effectively managing cognitive load is a central objective of CLT-informed instructional design. Instructional approaches that reduce unnecessary cognitive demands typically focus on organizing content in a coherent sequence, directing learners‘ attention to essential information, and providing timely feedback to reinforce correct reasoning. By structuring learning materials in ways that align with cognitive constraints, instructors enhance learners‘ capacity to process and retain new information.

Chunking instructional content into smaller, meaningful units improves processing efficiency by reducing the number of elements learners must attend to simultaneously (Gobet & Lane, 2023). Similarly, worked examples demonstrate complete solution procedures, reducing extraneous cognitive load and supporting schema development (Sweller & Cooper, 1985). These strategies are particularly valuable in mathematics, where problem-solving tasks often impose high intrinsic cognitive load.

A central mechanism in managing cognitive load is schema acquisition. Schemas are organized mental structures that allow learners to group multiple elements into single cognitive units, reducing the burden on working memory. Instructional designs that support schema acquisition and automation enable learners to solve increasingly complex problems with greater efficiency. Consequently, CLT-based instructional approaches prioritize not only the transmission of information but also the development of cognitive structures that facilitate long-term learning and problem-solving expertise (Sweller, 1988).

However, effective cognitive load management requires more than reducing task difficulty. Instruction should maintain meaningful engagement with complex ideas while providing appropriate support that enables learners to gradually develop independent problem-solving abilities.

By systematically integrating these approaches, instructors can balance cognitive loads—minimizing extraneous load, managing intrinsic complexity, and fostering germane processing—to optimize learning outcomes, particularly in cognitively demanding domains such as calculus problem-solving. Through this deliberate alignment of instructional design with human cognitive architecture, learners are better equipped to develop robust problem-solving skills and apply their knowledge flexibly in novel contexts.

3.4. THE ROLE OF COGNITIVE LOAD THEORY IN PROBLEM-SOLVING INSTRUCTION

Cognitive Load Theory (CLT), originally proposed by Sweller (1988), is a central framework in instructional design grounded in cognitive psychology. It explains how learning is constrained by the limited capacity of working memory and how instructional design can either hinder or facilitate knowledge acquisition. CLT is particularly relevant to problem-solving instruction because such tasks involve complex, multi-step reasoning, requiring learners to coordinate multiple interacting elements simultaneously.

Within CLT, effective learning is achieved by supporting schema construction—the organization of knowledge into structured mental representations that enable efficient information processing. Instruction designed to reduce unnecessary cognitive demands while promoting meaningful processing allows learners to integrate conceptual understanding with procedural fluency. This balance is critical in mathematics and calculus, where poorly designed instruction can easily overload working memory, hindering problem-solving performance (Paas et al., 2003; Sweller et al., 2019).

Therefore, CLT provides not only an explanation of cognitive difficulties experienced by learners but also a framework for designing instructional interventions that support gradual development of mathematical expertise. This perspective directly informs the development of the WMCBPSI framework examined in this study.

3.4.1. Theoretical Foundations of Cognitive Load Theory

CLT builds on foundational research on human memory and information processing. Miller (1956) demonstrated that working memory can handle only a limited number of information elements simultaneously, while Baddeley and Hitch‘s (1974) multicomponent model clarified that cognitive processing capacity is inherently constrained and divided across subsystems.

Sweller (1988) extended these insights to instructional contexts, emphasizing that learning effectiveness depends on how instructional materials interact with working memory limitations. A central mechanism is schema construction and automation, whereby learners ―chunk‖ multiple elements into integrated units, reducing working memory load during problem-solving. As schemas become automated, cognitive resources can be allocated to higher-order reasoning rather than basic processing.

CLT aligns with complementary cognitive perspectives, including Mayer‘s (2008) Cognitive Theory of Multimedia Learning, which emphasizes coordinated verbal and visual processing, and Anderson‘s (2010) Schema Theory, highlighting how structured knowledge underpins expert performance. Together, these perspectives provide strong theoretical justification for applying CLT to problem-solving instruction in cognitively demanding domains such as university-level mathematics.

The integration of these perspectives suggests that successful mathematical learning depends on both managing immediate cognitive demands and developing durable knowledge structures that support future problem solving.

In this study, CLT serves both as a theoretical lens and a practical foundation for the Working

Memory–Context-Based Problem-Solving Instruction (WMCBPSI) framework, which operationalizes CLT principles to reduce unnecessary cognitive demands while fostering deep conceptual understanding and transferable problem-solving skills.

3.4.2. Types of Cognitive Load and Their Instructional Implications

CLT distinguishes three interrelated types of cognitive load—intrinsic, extraneous, and germane—which are essential for designing effective problem-solving instruction (Sweller et al., 2011).

Intrinsic Cognitive Load: Intrinsic load arises from the inherent complexity of the learning material and the degree of element interactivity. Tasks with low element interactivity, such as memorizing isolated facts, impose minimal intrinsic load. In contrast, complex calculus problems, involving multiple variables, interdependent concepts, and sequential reasoning, impose high intrinsic load because multiple elements must be processed simultaneously (Sweller, 2005).

Learners' prior knowledge critically moderates intrinsic load. Experts rely on well-developed schemas, which reduce load by treating complex problems as integrated wholes, whereas novices process each element separately. Instructional strategies such as scaffolding, sequencing tasks from simple to complex, isolated-elements instruction, and worked examples have been shown to manage intrinsic load effectively, particularly for novice learners (Pollock et al., 2002; Chen & Star, 2016).

This distinction is particularly relevant to calculus because the same problem may impose different cognitive demands depending on learners' prior mathematical knowledge and existing schemas.

Extraneous Cognitive Load: Extraneous load arises from how information is presented rather than the task itself. Poor instructional design—such as split attention between multiple sources, redundant explanations, or poorly structured materials—consumes working memory resources without contributing to learning (Chandler & Sweller, 1991). Common sources include split-attention effects, redundancy, ineffective modality use, and tasks exceeding learners' prior knowledge (Van Gog et al., 2009; De Jong, 2010).

Reducing extraneous load requires careful instructional design, including integrating textual and visual information, eliminating unnecessary repetition, using multimodal explanations appropriately, and aligning task difficulty with learners' cognitive readiness.

Germane Cognitive Load: Germane load refers to cognitive effort devoted to schema construction and automation. Unlike extraneous load, germane load directly contributes to learning. Instructional strategies that promote self-explanation, worked-example processing, varied practice, and collaborative problem-solving encourage learners to actively construct and refine schemas (Chi et al., 1994; Paas et al., 2003).

Learners' ability to engage in germane processing depends on effective management of intrinsic and extraneous load. For novices, excessive task complexity or poor instructional design can hinder productive schema construction. Scaffolded and progressively challenging tasks are therefore essential for maintaining cognitive efficiency while promoting deep learning.

3.4.3. Sources of Cognitive Load and Schema Acquisition

Effective problem-solving instruction requires systematic identification of cognitive load sources and deliberate support for schema acquisition. Intrinsic load arises from task complexity and element interactivity, extraneous load from instructional design features, and germane load from learners' active engagement in schema construction. Understanding how these sources interact provides a practical basis for instructional decision-making in mathematics education.

In calculus problem solving, this distinction is particularly important because learners must simultaneously coordinate conceptual knowledge, procedural operations, and problem interpretation. Therefore, effective instruction should not simply reduce cognitive demands but should regulate them in ways that promote meaningful schema construction.

Table 3.1 summarizes the major sources of cognitive load and their corresponding instructional strategies, providing a concise link between theory and instructional practice.

Table 3.1: Sources of Cognitive Load and Instructional Strategies

Load Type	Source	Instructional Implications
ICL	High element interactivity	Scaffolding, sequencing, isolated-elements method
ECL	Split attention, redundancy, poor design	Integrated presentations, multimodal instruction, remove redundancy
GCL	Effort in schema construction	Self-explanation, worked examples, varied practice, peer collaboration

3.4.4. Managing Cognitive Load in Problem-Solving Instruction

Applying CLT to problem-solving instruction involves a coordinated effort to reduce extraneous load, manage intrinsic load, and foster germane load. Instruction should minimize unnecessary processing demands, align task complexity with learners‘ existing schemas, and promote activities that support schema construction and automation.

These principles are central to the WMCBPSI framework, which integrates context-based tasks, worked examples, guided practice, and self-explanation to optimize working memory resources during calculus problem-solving.

The integration of these strategies reflects the central assumption of this study that students' problem-solving difficulties can be addressed by designing instruction that manages working memory limitations while gradually developing more efficient mathematical schemas.

3.4.5. Challenges and Criticisms of Applying Cognitive Load Theory

Despite its strong empirical foundation, CLT has faced several criticisms. Measuring germane cognitive load remains methodologically challenging, and some argue that excessive emphasis on load reduction may oversimplify tasks, limiting opportunities for deeper conceptual engagement (Leppink et al., 2014). Additionally, cultural and contextual factors may influence how learners experience cognitive load, raising concerns about the generalizability of CLT-based instructional designs across diverse educational settings (Kirschner et al., 2018).

These critiques suggest that CLT should be applied flexibly and contextually, with careful attention to learner characteristics, instructional goals, and disciplinary demands.

Accordingly, the present study applies CLT within a context-based calculus problem-solving environment, recognizing that effective instruction requires balancing cognitive support with meaningful learner engagement rather than merely reducing task difficulty.

3.5. LEARNING AND UNDERSTANDING IN TERMS OF COGNITIVE LOAD THEORY

3.5.1. Learning as Schema Construction and Automation

From a Cognitive Load Theory perspective, learning is fundamentally associated with the construction and automation of schemas (Sweller et al., 2011). Schemas enable learners to organize and integrate related elements of information into coherent mental structures that can be processed efficiently within working memory. In calculus, schema formation allows learners to combine formulas, procedures, and conceptual relationships into unified cognitive units, reducing cognitive demands during problem-solving (Paas et al., 2003).

For example, students who have developed a schema for differentiation rules can apply these rules as integrated processes rather than as isolated steps. This integration reduces cognitive load and allows learners to allocate attention to higher-order reasoning and problem interpretation (Kirschner et al, 2006).

Schema automation further enhances learning efficiency. Through repeated practice, schemas become automatic and can be applied with minimal conscious effort (Kalyuga, 2011). As a result, working memory resources are freed for more complex tasks, such as multivariable calculus or applied problem-solving. The interaction between schema acquisition and automation supports adaptive expertise, enabling learners to transfer known strategies to novel and increasingly complex situations (Paas et al, 2019).

Thus, schema construction and automation provide the cognitive mechanism through which instructional interventions can transform learners from dependent problem solvers into more independent and flexible mathematical thinkers.

3.5.2. Understanding as Cognitive Integration

Within CLT, understanding is achieved when learners can simultaneously process and integrate all relevant information elements without exceeding working memory capacity (Schnotz & Kürschner, 2007). In calculus, understanding involves grasping the relationships among concepts such as limits, derivatives, integrals, and their applications. Achieving this level of comprehension requires instructional approaches that manage cognitive load effectively, allowing learners to focus on essential conceptual connections (Sweller, 2020).

Understanding extends beyond isolated schema acquisition; it involves integrating multiple schemas into broader conceptual frameworks. For instance, meaningful understanding of differentiation includes not only procedural fluency but also insight into how derivatives model real-world phenomena, such as rates of change and optimization (Skulmowski & Xu, 2021). Such integration supports transfer of learning, enabling students to apply calculus concepts flexibly across contexts (Sweller et al., 2019).

This perspective supports the use of contextualized mathematical problems because authentic contexts can encourage learners to connect abstract concepts with meaningful applications and strengthen knowledge transfer.

Key Considerations for Instructional Design

To support learning and understanding effectively within a CLT framework, instructional design should carefully balance cognitive loads to prevent working memory overload, with particular attention to minimizing extraneous load while promoting germane processing (Sweller et al., 2011; Paas et al., 2019). Complex calculus content should be delivered through scaffolded instruction that progressively increases task demands and fosters schema development, allowing learners to build their knowledge structures step by step (Chen & Star, 2016).

Moreover, instructional approaches should integrate active engagement strategies, including self-explanation and collaborative discussion, to deepen conceptual understanding and enhance the transfer of knowledge to novel problems (Mayer, 2008; Fiorella & Mayer, 2015). By systematically applying these principles, instructional practices can strengthen both learning and understanding, enabling students to solve calculus problems efficiently and apply their knowledge flexibly in a variety of contexts.

These instructional principles provide the theoretical justification for the design features of the WMCBPSI framework, where context-based problems, guided support, and collaborative reasoning are combined to manage cognitive load and promote deeper mathematical understanding.

Cognitive Load Theory has generated a well-established set of instructional effects that explain how learning and problem-solving performance can be optimized through effective management of working memory. These effects clarify how different instructional strategies influence intrinsic, extraneous, and germane cognitive loads, thereby shaping learners‘ capacity to construct, automate, and apply schemas in complex domains such as calculus.

In problem-solving instruction, these effects provide practical guidance for instructional design decisions. Strategies such as worked examples, completion problems, integrated representations, modality-based presentations, and guidance fading primarily function by reducing extraneous cognitive load, allowing working memory resources to be redirected toward learning-relevant processes. Other strategies, such as managing element interactivity and isolated-elements instruction, address intrinsic cognitive load by structuring task complexity in line with learners‘ prior knowledge. Germane cognitive load is fostered through strategies that promote schema construction and flexible application, including varied examples and imagination-based activities. Table 3.2 presents a synthesized overview of key cognitive load effects and their instructional applications in calculus problem-solving instruction. The table identifies the primary cognitive load source associated with each effect and situates these effects within the context of the present study, which focuses on working memory–context-based problem-solving in calculus. By summarizing how different instructional strategies influence cognitive load, the table highlights the importance of balancing intrinsic, extraneous, and germane loads to support efficient learning and problem-solving performance.

Table 3.2: Cognitive Load Effects and Their Applications in Calculus Instruction

Cognitive Load Effect	Description	Primary Cognitive Load Source
Worked Examples	Studying fully worked examples helps reduce initial cognitive demands, particularly for novices tackling complex problems. This strategy supports schema construction by illustrating step-by-step processes. For advanced learners, however, the benefits diminish, as they require more independent problem-solving (Sweller et al., 2019; Renkl, 2022).	Extraneous
Completion Problems	Learners complete partially solved problems, striking a balance between guided instruction and autonomous learning. This approach promotes active engagement and supports schema development, particularly in STEM education (Atkinson et al., 2000; Fraser et al., 2022).	Extraneous
Split Attention Effect	When related information is presented separately (e.g., text and diagrams), learners expend extra effort to integrate it. Integrating such information into a cohesive format reduces unnecessary working memory demands, enhancing comprehension in mathematics (Sweller et al., 2011).	Extraneous
Modality Effect	Using multiple sensory modalities (e.g., combining visual and auditory information) distributes cognitive processing demands. In calculus, pairing verbal explanations with diagrams or symbolic representations can significantly improve understanding (Dawson et al., 2021; Ayres & Cierniak, 2022).	Extraneous
Redundancy Effect	Eliminating unnecessary or duplicate information reduces cognitive load. Avoiding repetitive explanations in mathematics helps learners focus on essential content and preserves working memory capacity (Chandler & Sweller, 1991; Schnotz & Kürschner, 2007).	Extraneous

Expertise Reversal Effect	Instructional strategies beneficial for novices, such as worked examples, may become less effective as learners gain expertise. Advanced learners benefit more from exploratory and open-ended tasks that promote analytical thinking (Kalyuga, 2007; Nájera et al., 2021).	Extraneous
Guidance Fading	Gradually removing instructional support—progressing from worked examples to independent problem-solving—builds learner autonomy. This technique fosters analytical competence for tackling complex calculus problems (Atkinson et al., 2003).	Extraneous
Goal-Free Problems	Removing explicit goals encourages learners to explore problem structures independently. This fosters conceptual understanding but may require scaffolding for novices to prevent overload (Ayres, 2013; Sweller et al., 2019).	Extraneous
Element Interactivity	High element interactivity increases cognitive demand, as multiple concepts must be processed simultaneously. Structuring tasks to simplify these interactions helps manage intrinsic load, especially in calculus (Sweller, 2010; Fraser et al., 2022).	Intrinsic
Isolated/Interacting Elements	Presenting complex material initially as isolated components before integration helps learners manage intrinsic load. In calculus, this approach is effective when teaching foundational skills before applying them to comprehensive problems (Pollock et al., 2002; Sweller et al., 2019).	Intrinsic
Variable Examples	Exposure to a range of problem types promotes flexible schema formation and facilitates knowledge transfer to novel situations. In STEM disciplines, varied examples strengthen adaptive problem-solving skills (Paas & van Gog, 2006; Fraser et al., 2022).	Germane
Imagination Effect	Encouraging learners to visualize and mentally rehearse problem-solving steps fosters schema construction and deepens conceptual understanding. In calculus, this helps students conceptualize abstract relationships, such as those between functions and their derivatives (Cooper et al., 2001).	Germane

Although individual cognitive load effects address different instructional challenges, their effectiveness depends on coordinated application. In complex domains such as calculus, learners require appropriate support to manage task complexity, avoid unnecessary processing, and construct transferable problem-solving schemas.

A central insight emerging from this synthesis is that effective instructional design requires a dynamic and context-sensitive balancing of cognitive loads. Managing intrinsic and extraneous loads while deliberately fostering germane load ensures that learners‘ limited working memory capacity is used efficiently. This balance supports schema development, automation, and the transfer of problem-solving skills to novel calculus contexts, which are key goals of the present study.

3.6. SUMMARY AND CONCLUSION OF THE CHAPTER

This chapter has presented a comprehensive examination of the theoretical framework underpinning the study, with a central focus on Cognitive Load Theory (CLT) and its relevance to problem-solving instruction in higher education mathematics (Sweller, et al., 2011; Paas & van Gog, 2006). The chapter examined the three primary types of cognitive load—intrinsic, extraneous, and germane—and analyzed how they interact with working memory during complex learning activities, particularly in calculus problem-solving (Sweller et al., 2011; Van Merriënboer et al., 2003).

The discussion highlighted the critical role of instructional design in managing cognitive load to enhance learning efficiency and problem-solving performance (Renkl, 2022). Key instructional strategies reviewed included the use of worked examples to scaffold learning, the reduction of split-attention and redundancy effects to streamline cognitive processing, and the use of varied problem examples to strengthen schema formation and promote transfer of learning (Atkinson et al., 2000; Sweller et al.,2011; Paas & van Gog, 2006). Collectively, these strategies support schema construction, automation, and adaptive expertise**,** which are essential outcomes of effective calculus instruction.

Table 3.2 synthesized major cognitive load effects and their instructional implications, demonstrating how CLT can be operationalized to support mathematics teaching and learning (Sweller et al., 2011; Fraser et al., 2022). A key conclusion is that effective calculus instruction requires a deliberate and dynamic balance of intrinsic, extraneous, and germane cognitive loads. Instructional approaches must be designed to optimize working memory resources while scaffolding learning in ways that foster deep conceptual understanding and flexible problem-solving skills**.**

Overall, this chapter establishes a strong theoretical foundation for the instructional framework of the present study by integrating CLT with empirically grounded instructional strategies.

The reviewed literature demonstrates that effective calculus problem-solving instruction requires alignment between human cognitive architecture, cognitive load management, and schema development. These principles directly informed the design of the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) framework.

The next chapter extends this theoretical foundation by reviewing empirical studies related to cognitive load, instructional strategies, and mathematics problem-solving to identify existing limitations and establish the research gap addressed by the present study.

CHAPTER FOUR

MATHEMATICAL PROBLEM-SOLVING: CONCEPTS, MODELS, AND COGNITIVE FOUNDATIONS

4.1. INTRODUCTION

Problem-solving is a fundamental competency in mathematics education, extending its relevance across academic, professional, social, and societal domains (Schoenfeld, 1985; Schoenfeld, 2016). In contemporary education, mathematical literacy and the capacity to solve complex problems are recognized as central to student development, underpinning critical thinking, reasoning, and higher-order learning (OECD, 2019; Tan et al., 2020).

Despite its significance, traditional mathematics instruction often emphasizes rote memorization and repetitive drills, limiting students' ability to transfer learned skills to novel contexts (Jitendra et al., 2002; Verschaffe et al., 2020; Zhang & Liu, 2023). Such limitations indicate the need for instructional approaches that not only develop procedural competence but also support conceptual understanding, strategic reasoning, and flexible application of knowledge. Cognitive Load Theory (CLT) highlights that effective problem-solving instruction must manage cognitive load, promote schema construction, and provide scaffolds for complex reasoning tasks (Sweller, et al., 2011).

In this study, a problem is defined as a mathematical challenge that demands deliberate cognitive effort, while problem-solving refers to the process by which learners apply prior knowledge and reasoning to reach a solution (Polya, 1957; Jonassen, 2011). Problem-solving therefore involves more than obtaining correct answers; it requires understanding problem structures, selecting effective strategies, monitoring solution processes, and evaluating outcomes. From a cognitive perspective, successful problem-solving depends largely on the availability of organized knowledge structures (schemas) in long-term memory and the efficient use of working memory during reasoning.

Grounded in CLT, effective problem-solving depends on the construction and automation of schemas that guide decision-making and streamline solution processes. Instructional frameworks such as Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) provide scaffolds—through chunking, worked examples, and guided practice.

These approaches are intended to bridge the gap between theoretical principles of cognitive architecture and practical classroom strategies for improving calculus problem-solving performance.

4.2. CONCEPTUAL FOUNDATIONS OF MATHEMATICAL PROBLEM-SOLVING

Mathematical problem-solving is essential not only for academic achievement but also for equipping learners with reasoning skills to address real-life challenges creatively and analytically (Schoenfeld, 2016; Li & Zhang, 2021). However, many higher education programs inadequately prepare students for the complex problem-solving demands of professional and societal contexts (Nguyen et al., 2023). Therefore, effective mathematics instruction should move beyond procedural performance toward approaches that develop conceptual understanding, strategic competence, and adaptive problem-solving abilities.

From a cognitive perspective, mathematical problem-solving relies heavily on working memory, which coordinates information processing, strategy selection, and reasoning during complex tasks (Sweller, 2010; Zhang et al., 2023). Because working memory capacity is limited, learners may experience difficulties when problems contain multiple interacting elements or require simultaneous processing of several concepts. Consequently, problem-solving instruction should consider both the cognitive demands imposed by mathematical tasks and the knowledge structures learners use to manage these demands.

Problem-solving can be conceptualized as a sequence of cognitive processes involving problem representation, strategy formulation, execution, and evaluation (Dhlamini & Mogari, 2011). Effective learners are able to interpret problem conditions, identify relevant concepts, select appropriate strategies, monitor their progress, and evaluate the reasonableness of solutions. These processes are closely associated with schema development, as organized knowledge structures allow learners to recognize problem types and retrieve appropriate solution procedures more efficiently.

In this study, problem-solving skill is operationalized as learners' ability to apply conceptual and procedural knowledge effectively to mathematical challenges. Assessment is aligned with Polya's framework, examining students' ability to understand problems, formulate solution strategies, execute procedures, and evaluate solutions (Polya, 1957). This operationalization is consistent with the cognitive perspective adopted in this research, where successful problem-solving is viewed as the interaction between working memory processes and knowledge structures stored in long-term memory.

Polya‘s classical four-step framework—understanding the problem, devising a plan, executing the plan, and reviewing the solution—remains central in mathematics education (Polya, 1957; Jonassen, 2011). Extensions of this framework, including strategy selection and metacognitive monitoring approaches proposed by later researchers (Jitendra et al., 2002), emphasize the importance of deliberate reasoning throughout the problem-solving process. When combined with cognitive load principles, these stages provide a basis for designing instruction that supports learners in managing cognitive demands, constructing schemas, and gradually developing independent problem-solving expertise (Sweller et al., 2011; Kalyuga, 2022).

4.2.1. Historical Models of Problem-solving

Foundational models of problem-solving developed during the twentieth century provided important insights into how individuals represent problems, search for solutions, and evaluate outcomes. Newell and Simon’s General Problem Solver (GPS) model conceptualized problem-solving as a systematic cognitive process involving problem representation and solution search. The model emphasized the importance of identifying goals, generating possible actions, and reducing differences between the current state and the desired solution state (Newell & Simon, 1972).

Polya’s four-step framework remains one of the most influential models in mathematics education, emphasizing understanding the problem, devising a plan, executing the plan, and reviewing the solution (Polya, 1957; Alio & Harbor-Peter, 2000; Williams, 2003). This framework highlights not only procedural execution but also the importance of reasoning, strategy selection, and reflection during mathematical problem solving. Similarly, Bransford and Stein’s IDEAL model expanded problem-solving instruction by incorporating problem identification, goal definition, exploration of strategies, anticipation of consequences, and evaluation of outcomes (Bransford & Stein, 1993).

Although these models differ in emphasis, they share the view that successful problem-solving involves structured cognitive processes rather than random trial-and-error activity. However, early models provided limited explanation of how learners’ existing knowledge structures and cognitive limitations influence problem-solving performance. Later cognitive theories, particularly Cognitive Load Theory, extended these perspectives by explaining how working memory constraints and schema availability affect learners' ability to process complex problems.

Mathematical problem-solving is widely recognized as a cornerstone of mathematical proficiency, requiring learners to coordinate cognitive resources, heuristic strategies, and self-regulatory mechanisms. A foundational perspective in this area is established by Schoenfeld (1985, 1992), whose framework posits that successful mathematical problem-solving depends on four interacting dimensions: the individual's knowledge base, problem-solving strategies (heuristics), control/self-regulation (metacognition), and belief systems regarding mathematics. Schoenfeld argued that students often fail not due to a lack of mathematical facts, but because they lack executive control and monitoring strategies to manage their cognitive resources effectively during non-routine tasks.

Complementing this cognitive perspective, Lesh and Zawojewski (2007) emphasize that modern problem-solving involves complex, multi-step interpretation and schema generation where students must construct, adapt, and refine mathematical models to make sense of problematic situations. According to Lesh and Zawojewski, authentic problem-solving tasks require going beyond routine procedural execution, urging learners to manage cognitive demands while organizing and internalizing concepts for flexible transfer. Integrating these perspectives into instructional designs—such as Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI)—provides the necessary theoretical scaffolding to support learners' cognitive load management, strategic flexibility, and metacognitive control.

For the present study, these historical models provide the procedural foundation for organizing mathematical problem-solving activities, while CLT provides the cognitive explanation for how learners can be supported during each stage of the problem-solving process.

4.2.2. Contemporary Cognitive Models of Problem-solving

Contemporary models of problem- solving emphasize the interaction of cognitive processes, prior knowledge, metacognitive regulation, and contextual factors (Kirkley, 2003; Zhang & Liu, 2023). A central concept within these models is the role of schemas—organized knowledge structures stored in long-term memory that enable learners to recognize problem characteristics, select appropriate strategies, and execute solutions efficiently (Chi et al., 2021; Kirkley, 2003).

Mayer (1983) conceptualized problem-solving as a goal-directed cognitive process involving the representation, manipulation, and application of knowledge. Similarly, Plato (2003) proposed that problem-solving involves three major processes: problem representation, solution search, and implementation/evaluation. These perspectives emphasize that effective problem solving requires learners to construct meaningful representations of problems before selecting and applying strategies.

Complex mathematical problems often require learners to decompose tasks into manageable components and coordinate intermediate steps toward an overall goal (Newell & Simon, 1972; Plato, 2003). However, successful decomposition and strategy selection depend strongly on learners' prior knowledge and cognitive resources. Learners with well-developed schemas can recognize underlying problem structures more efficiently, whereas novices may rely on inefficient trial-and-error approaches because they must process each element separately (Sweller, 1988; Sweller et al., 2011).

Recent research integrating Cognitive Load Theory and working memory principles demonstrates that instructional support should be aligned with learners' cognitive capacity to enhance schema construction, knowledge transfer, and problem-solving efficiency (Sweller et al., 2019). Instructional frameworks such as WMCBPSI operationalize these principles by combining contextualized problems, worked examples, chunking, and guided practice to support learners during complex mathematical tasks.

Thus, contemporary cognitive models provide the theoretical bridge between problem-solving processes and instructional design. They explain not only what successful problem solving involves but also how instructional environments can be structured to reduce unnecessary cognitive demands and promote the development of adaptive expertise.

4.3. RESEARCH ON PROBLEM-SOLVING

Problem solving is widely recognized as a fundamental cognitive process that supports success across educational, professional, and social contexts (Jonassen, 2000; OECD, 2021). In mathematics education, effective problem solving contributes not only to achievement but also to conceptual understanding, reasoning ability, and the development of transferable cognitive skills (Renkl & Atkinson, 2010). Research consistently indicates that students improve their problem-solving performance when instruction provides structured opportunities to engage with challenging mathematical tasks and develop appropriate strategies (Jurdak, 2006; Maccini & Ruhl, 2000; Scherer & Beckmann, 2014).

A growing body of research emphasizes the value of contextualized and authentic problem-solving tasks. When mathematical problems are connected to meaningful real-world situations, learners are better able to relate abstract concepts to prior experiences, increasing engagement and supporting knowledge transfer (Jurdak, 2006; Schoenfeld, 2016). However, contextual relevance alone does not guarantee effective learning; the cognitive demands of contextualized tasks must also be carefully managed. Complex real-world problems may increase cognitive demands if learners lack sufficient prior knowledge or instructional support. Cognitive Load Theory therefore suggests that context-based instruction should be accompanied by appropriate scaffolding, worked examples, and guided practice to reduce unnecessary cognitive demands and support schema construction (Sweller et al., 2011).

Research on structured problem-solving strategies has demonstrated the effectiveness of instructional frameworks that guide learners through analysis, strategy selection, execution, and reflection. Strategies such as STAR (Search, Translate, Answer, Review) and other multi-step approaches provide learners with systematic procedures for approaching complex tasks while encouraging metacognitive monitoring and strategic reasoning (Maccini & Ruhl, 2000; Gaigher et al., 2006; Gibbons et al., 2021; Kramarski et al., 2020). These findings suggest that effective problem-solving instruction requires a balance between providing sufficient guidance for schema development and gradually promoting learner independence.

The integration of structured strategies with authentic problem contexts has been shown to enhance transfer of learning, adaptive expertise, and critical thinking (Li & Zhang, 2021; Brown et al., 2022). When learners repeatedly engage with meaningful problems supported by appropriate cognitive scaffolds, they develop organized knowledge structures that enable more efficient recognition and solution of future problems. This finding is consistent with CLT, which emphasizes that expertise develops through schema acquisition and automation, allowing learners to allocate working memory resources toward higher-order reasoning rather than basic information processing.

Cognitive factors, particularly working memory capacity, prior knowledge, and self-regulation, play a central role in determining problem-solving success (Perels et al., 2021; Zhang et al., 2023). Learners who can effectively manage cognitive demands and connect new information with existing schemas are better positioned to solve complex problems. Despite these insights, limited research has examined how working memory principles, contextualized mathematical tasks, and cognitive load management can be systematically integrated into calculus problem-solving instruction, particularly in higher education contexts.

Therefore, the existing literature provides strong support for instructional approaches that combine cognitive principles with authentic problem-solving experiences. However, the need remains for empirically examining integrated frameworks such as WMCBPSI, which explicitly connects working memory management, contextual learning, and problem-solving strategy development.

4.4. FACTORS INFLUENCING MATHEMATICAL PROBLEM-SOLVING SKILLS

Mathematical problem-solving skills are influenced by an interconnected combination of cognitive, affective, and instructional factors. Among these, prior knowledge represents a fundamental cognitive resource that enables learners to recognize problem structures, select appropriate strategies, and construct effective schemas. Empirical research indicates that students with stronger mathematical backgrounds are more capable of transferring strategies to unfamiliar problems because their knowledge structures allow more efficient information processing (Bransford et al., 2021; Zhang & Liu, 2023). From a Cognitive Load Theory perspective, prior knowledge reduces the burden placed on working memory by allowing learners to process familiar information as organized cognitive units rather than isolated elements.

Learners’ beliefs, attitudes, motivation, and self-efficacy also influence their engagement with mathematical problems. Positive mathematical beliefs and confidence encourage persistence, strategic exploration, and willingness to engage with challenging tasks, whereas and low self-efficacy may restrict cognitive engagement and negatively affect performance (Malmivuori, 2001; Cifarelli et al., 2010). Although affective factors do not directly determine cognitive capacity, they influence learners’ willingness to invest mental effort, regulate their learning, and persist during complex problem-solving processes.

Instructional design represents another critical factor shaping mathematical problem-solving development. Effective instruction should provide learners with opportunities to acquire conceptual knowledge, develop strategic competence, and apply mathematical ideas in meaningful situations. Bloom’s learning domains highlight that successful learning involves cognitive, affective, and psychomotor dimensions, requiring learners not only to acquire knowledge but also to engage actively and develop confidence in applying that knowledge (Bloom, 1956; Pimta et al., 2009). However, instructional approaches must also consider cognitive constraints; excessive task complexity or insufficient guidance may overwhelm working memory and limit schema development.

Active learning and problem-centered instructional approaches provide mechanisms for addressing these factors by engaging learners in authentic mathematical challenges that require analysis, reasoning, collaboration, and reflection (Jonassen, 2004; Jung et al., 2020; Russo & Minas, 2021; Kim & Park, 2024). When combined with cognitive supports such as worked examples, chunking, and scaffolding, these approaches help learners manage cognitive demands while gradually developing independent problem-solving abilities (Sweller et al., 2019; Zhang et al., 2023).

Collectively, research suggests that mathematical problem-solving competence emerges from the interaction of learner characteristics, cognitive resources, and instructional conditions. Therefore, improving students’ problem-solving skills requires instructional frameworks that simultaneously support knowledge development, motivation, and efficient cognitive processing. This perspective provides the rationale for WMCBPSI, which integrates contextual learning with cognitive load management to support calculus problem-solving development.

4.5. ERRORS IN PROBLEM-SOLVING

Errors in mathematical problem solving provide valuable information about learners’ cognitive processes, conceptual understanding, and strategy use. Rather than being viewed only as failures, errors can reveal difficulties in problem representation, knowledge retrieval, strategy selection, and procedural execution. Research indicates that frequent errors are associated with conceptual gaps, inadequate procedural knowledge, and ineffective application of mathematical principles (Prakitipong & Nakamura, 2006). Therefore, systematic analysis of errors is essential for identifying the specific cognitive obstacles that prevent learners from developing effective problem-solving schemas.

Although the terms errors and misconceptions are often used interchangeably in mathematics education, they represent distinct constructs with different cognitive and instructional implications. Errors are incorrect responses or procedures that may result from inattention, computational mistakes, or inappropriate strategy selection. In contrast, misconceptions are persistent misunderstandings of mathematical concepts or principles that repeatedly give rise to systematic errors. Table 4.1 summarizes the key distinctions between errors and misconceptions.

Table 4.1: Key distinctions between errors and misconceptions. Source: Adapted from Schlöglmann (2007); Prakitipong and Nakamura (2006); Erbaş, Çetinkaya, and Ersoy (2009).

Aspect	Errors	Misconceptions
Definition	Incorrect responses or procedures made while solving a mathematical problem.	Persistent incorrect understandings or faulty conceptions of mathematical concepts or principles.
Nature	Often procedural and may be temporary or situational.	Conceptual and typically stable unless explicitly addressed through instruction.
Causes	Carelessness, computational mistakes, inappropriate strategy selection, time pressure, or lack of attention.	Incomplete or incorrect conceptual understanding, faulty prior knowledge, or incorrect mental models.
Relationship	Errors may occur with or without misconceptions.	Misconceptions frequently lead to repeated or systematic errors.
Instructional Implication	Can often be corrected through feedback, practice, and improved procedural skills.	Require conceptual change through targeted instruction, explanation, and restructuring of existing knowledge.
Examples	Arithmetic mistakes, incorrect substitution into a formula, or computational slips.	Believing that the derivative of a product equals the product of the derivatives, or misunderstanding the concept of a limit.

As shown in Table 4.1, errors and misconceptions are related but distinct constructs. While misconceptions often lead to repeated errors, not all errors arise from misconceptions.

Recognising this distinction is important for selecting appropriate instructional interventions.

Schlöglmann (2007) categorized errors into two primary dimensions: formula application errors and conceptual misconceptions. Formula application errors occur when students incorrectly apply known procedures or fail to recognize the conditions under which a formula is valid, often due to haste, inattention, or superficial understanding. Conceptual misconceptions, in contrast, reflect deeper misunderstandings of mathematical ideas, gaps in strategic knowledge, or difficulties translating real-world problems into formal representations. These misconceptions are often persistent, emphasizing the need for instruction that addresses foundational knowledge while scaffolding higher-order thinking. Addressing such misconceptions requires instructional experiences that create cognitive conflict by challenging learners' incorrect conceptions and guiding them toward scientifically accepted mathematical understanding through conceptual change (Schlöglmann, 2007; Erbaş et al., 2009).

From a cognitive perspective, errors may emerge at different stages of the problem-solving process, including comprehension, strategy formulation, execution, and evaluation. Lannin et al. (2007) emphasized that mistakes can occur throughout these stages and that affective factors such as anxiety, low self-efficacy, or lack of confidence increase error likelihood, especially under time constraints (Zhang et al., 2023). Within the framework of Cognitive Load Theory (CLT), such difficulties may arise when task demands exceed learners' working memory capacity or when learners lack sufficiently developed schemas to organize and process relevant information efficiently. Schoenfeld (1985) similarly noted that limited mathematical knowledge, restricted strategy repertoire, and weaknesses in self-regulation contribute to error-prone problem solving. Learners' perception of problem difficulty also matters: misalignment with prior knowledge increases errors, whereas alignment with existing schemas reduces cognitive load and enhances accuracy (McGinn & Boote, 2003).

Overall, both errors and misconceptions reflect the dynamic interplay of cognitive, affective, and perceptual factors. Whereas errors provide evidence of learners' immediate performance difficulties, misconceptions reveal deeper conceptual problems that require targeted instructional intervention. Accordingly, effective instruction should not only identify and correct procedural errors but also promote conceptual change by helping learners resolve misconceptions through guided feedback, structured problem-solving activities, and reflection. Recognizing this distinction provides a foundation for instructional strategies that enhance understanding, manage cognitive load, strengthen schema development, and cultivate accurate, adaptive problem-solving skills, which are core principles of WMCBPSI and CLT-informed instruction.

4.6. STRATEGIES FOR MINIMIZING ERRORS

Minimizing errors in problem solving requires instructional approaches that target both cognitive and metacognitive dimensions. Learners benefit when instruction emphasizes strategy knowledge, problem categorization, and the relevance of resources, supporting deliberate practice and schema construction (Lannin et al., 2007; McGinn & Boote, 2003; Erbaş et al., 2009; Li & Zhang, 2021).

Developing a repertoire of strategies enables learners to approach problems flexibly and adaptively. Awareness of multiple solution pathways allows students to select the most appropriate approach, reducing procedural and conceptual errors. Instruction that fosters strategic awareness encourages monitoring of reasoning, reflection on intermediate steps, and early detection of mistakes, enhancing both accuracy and cognitive efficiency. Problem categorization further helps learners identify underlying structures rather than relying on superficial features. Recognizing problem types and linking them to appropriate formulas, concepts, and procedures reduces extraneous cognitive load and strengthens schema construction (McGinn & Boote, 2003; Zhang et al., 2023).

From a Cognitive Load Theory perspective, mathematical errors may occur when learners' working memory resources are exceeded by excessive element interactivity, ineffective problem representation, or difficulties retrieving relevant schemas from long-term memory. When learners lack well-developed problem-solving schemas, they often depend on inefficient trial-and-error strategies, increasing cognitive demands and reducing solution accuracy. Therefore, minimizing errors requires instructional approaches that not only correct mistakes but also support the development and automation of appropriate schemas.

Another critical strategy involves guiding learners to evaluate the relevance of knowledge resources. Effective instruction connects strategies, principles, and formulas to specific problem types, fostering metacognitive control and enabling learners to justify method selection. This promotes deeper conceptual understanding and supports transfer of skills to novel contexts.

These approaches align closely with WMCBPSI and CLT. Structuring tasks to optimize working memory, embedding worked examples, using chunking techniques, and promoting collaborative engagement reduces extraneous cognitive load and directs attention to germane processes critical for schema development (Sweller et al., 2019). Through these mechanisms, WMCBPSI addresses common sources of mathematical errors by improving problem representation, supporting strategic knowledge retrieval, and gradually transferring responsibility from guided instruction to independent problem solving. Instruction integrating these principles not only mitigates errors but also fosters adaptive problem-solving, preparing students to navigate complex calculus problems and other higher-order mathematical challenges with confidence and efficiency.

4.7. VIEWS ON PROBLEM-SOLVING

Problem-solving is widely acknowledged as a cornerstone of mathematics education, yet its effectiveness is significantly influenced by the perceptions and beliefs of both teachers and students. Understanding these perspectives is crucial for designing instruction that fosters problem-solving competence and encourages active engagement with mathematical challenges.

4.7.1. Teachers‘ Perspectives

Teachers play a pivotal role in shaping how problem-solving is implemented in classrooms. Their beliefs, attitudes, and prior experiences influence instructional decisions, strategy selection, and the overall learning environment (Saadati et al., 2018). Evidence indicates that teachers often replicate instructional methods they encountered as students, which may reinforce traditional, lecture-centered approaches, posing challenges for adopting innovative problem-solving pedagogy (Thompson, 1992). Resistance can also arise when teachers feel inadequately prepared or lack professional development in problem-solving strategies (Borko, 2004).

Within a Cognitive Load Theory framework, teachers' understanding of learners' cognitive limitations is particularly important because instructional decisions directly influence the amount and type of cognitive load experienced by students. Teachers implementing WMCBPSI must be able to design appropriately sequenced tasks, provide effective scaffolding, and select contextual problems that support schema construction without overwhelming working memory.

Despite these challenges, many teachers recognize the benefits of problem-solving instruction, including enhanced reasoning skills, increased motivation, and improved conceptual understanding for students (Thompson, 1992). Effective implementation requires not only well-designed curricula but also ongoing professional development that equips teachers to foster inquiry, collaboration, and creative engagement (Saadati et al., 2018). In this sense, teachers act as architects of the problem-solving environment, shaping students‘ opportunities to internalize and apply mathematical concepts.

4.7.2. Students‘ Perspectives

Students’ perceptions of problem solving similarly influence engagement and achievement. Some view problem solving as a fixed ability or a rigid set of procedures, whereas others perceive it as a dynamic, exploratory process requiring reasoning and reflection. De Hoyos et al. (2004) framed these contrasting perspectives as the difference between locating a pre-existing answer and actively building a novel pathway to a solution, reflecting whether learners view outcomes as fixed or as constructed through active cognitive engagement.

These conceptions influence learners’ motivation, strategy use, and willingness to tackle complex problems (Zhang et al., 2023). Classrooms that emphasize active engagement, experimentation, and reflection are more likely to foster the latter perspective, encouraging learners to view problem solving as a creative, iterative, and self-regulated process. Teacher guidance plays a crucial role in shaping these perceptions, as supportive learning environments enable students to test hypotheses, justify solutions, and develop transferable problem-solving skills (Kim & Park, 2024; Russo & Minas, 2021).

This perspective is particularly relevant to WMCBPSI because the framework requires learners to actively engage with contextualized problems, interpret information, apply appropriate strategies, and reflect on the effectiveness of their solutions rather than merely reproduce procedures. Consequently, learners who perceive problem solving as a learnable and reflective process are more likely to engage successfully in cognitively demanding tasks, develop effective schemas, and acquire adaptive problem-solving expertise.

By acknowledging the interplay between teacher beliefs and student perceptions, educators can design instruction that aligns with cognitive principles, motivates learners, and fosters adaptive expertise, thereby providing a strong foundation for the implementation of WMCBPSI.

4.8. DEVELOPING EFFECTIVE SKILLS FOR PROBLEM-SOLVING

Developing mathematical problem-solving skills requires more than exposure to tasks; it demands instruction integrating cognitive, contextual, and motivational dimensions. Effective skill development involves guiding learners to analyze, interpret, and strategically approach problems, while connecting abstract concepts to meaningful, authentic contexts (Schoenfeld, 1985; Mousavi & Baghaei, 2022).

4.8.1. Problem-Solving Instruction

Instruction is most effective when it links academic knowledge with real-life applications, positioning problems as both the entry point and the goal of learning (Yin, 2011; NCTM, 2006). Rather than presenting tasks in isolation, instruction should scaffold reasoning, provide reflection opportunities, and encourage multiple solution strategies. This approach strengthens competence, diversifies strategies, and fosters critical thinking (Schoenfeld, 1992; Kramarski et al., 2022).

4.8.2. Working Memory and Context-Based Instruction

WMCBPSI operationalizes these principles by situating problems within learners' prior knowledge and real-life experiences. Cognitive Load Theory emphasizes that learners construct schemas most effectively when instruction reduces extraneous cognitive load and facilitates germane processing (Sweller, 2011). Worked examples and scaffolded guidance enable learners to focus on understanding underlying principles rather than memorizing procedures (Renkl, 2005; Van Gog & Sweller, 2022).

However, contextualization alone does not guarantee reduced cognitive load. Authentic contexts may increase cognitive demands when learners lack sufficient prior knowledge or when contextual information contains unnecessary or distracting details. Consequently, effective context-based instruction requires careful selection of meaningful contexts, appropriate sequencing of learning tasks, and instructional scaffolding to ensure that contextual relevance supports schema construction rather than increasing extraneous cognitive load.

Teachers' awareness of learners' cognitive demands and working memory capacity is essential for effective implementation (Wu & Adams, 2006; Zhang et al., 2023). Integrating contextual relevance and real-world mathematical modelling into instruction enhances learners' engagement and promotes the transfer of problem-solving skills to authentic situations (Kohen & Nitzan-Tamar, 2022; Khumalo, 2009).

4.8.3. Real-World Applications and Teacher Training

Despite its benefits, context-based instruction remains underutilized in mathematics classrooms. Research suggests that science teachers incorporate real-world contexts more frequently than mathematics teachers, revealing pedagogical gaps (Gaoseb et al., 2002). WMCBPSI addresses this by linking abstract concepts to students' experiences, supporting self-regulated problem-solving and independent solution generation while managing cognitive load (Li & Zhang, 2021). Consequently, teacher training is a crucial enabler, equipping educators to create authentic problem-solving environments and guide learners effectively.

4.8.4. The Ethiopian Context

In Ethiopia, the integration of WMCBPSI into mathematics instruction remains limited, and empirical research examining its effectiveness is scarce. In particular, there is limited evidence on the application of Cognitive Load Theory-informed, context-based problem-solving instruction in Ethiopian higher education mathematics classrooms. This gap highlights the need for research that examines instructional approaches capable of integrating cognitive principles with authentic learning contexts to improve students' mathematical problem-solving performance.

Implementing instructional strategies that align with learners' cognitive capacities and authentic contexts can enhance problem-solving performance and support the sustainable development of reasoning skills (Abebe & Yeshanew, 2022; Wu & Adams, 2006). By emphasizing cognitive load management, contextual relevance, and learner engagement, WMCBPSI provides a coherent framework for developing effective, transferable problem-solving skills in higher education and offers a promising approach for strengthening mathematics instruction within the Ethiopian context.

4.9. SUMMARY AND CONCLUSION OF THE CHAPTER

This chapter examined the conceptual and pedagogical foundations of mathematical problem solving and its central role in mathematics education. Classical and contemporary models were reviewed, emphasizing structured problem-solving strategies, schema construction, and the interplay of cognitive, affective, and contextual factors that shape effective problem solving.

Key influences on mathematical problem-solving performance—including error types, teacher beliefs, students' perceptions, and instructional approaches—highlight the complexity of developing problem-solving competence. Synthesizing these perspectives, WMCBPSI was presented as a coherent instructional framework that connects abstract mathematical concepts to authentic contexts, reduces extraneous cognitive load, and promotes self-regulated learning.

The reviewed literature indicates that, although various problem-solving models and instructional strategies have been developed, a gap remains in integrating working memory principles, cognitive load management, and authentic mathematical contexts within a unified framework for calculus problem solving. This gap provides the rationale for examining WMCBPSI as an instructional approach designed to optimize cognitive resources, strengthen schema development, and improve students' calculus problem-solving performance.

Although its implementation in Ethiopia remains limited, WMCBPSI shows strong potential to enhance calculus problem-solving skills, support adaptive reasoning, and facilitate knowledge transfer. Overall, this chapter establishes the theoretical and pedagogical foundation for the present study and provides the basis for the research methodology presented in the following chapter.

CHAPTER FIVE

RESEARCH METHODOLOGY

5.1. INTRODUCTION

The design of a research study is fundamentally shaped by its purpose, as research objectives determine the selection of appropriate methodological approaches, data collection procedures, analytical strategies, and instruments (Creswell & Creswell, 2018). In alignment with this principle, the present study adopted a carefully structured methodological framework to examine the effect of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on the calculus problem-solving skills of first-year university students in Ethiopia.

This chapter presents the methodological decisions undertaken to address the study‘s objectives and to ensure rigor and coherence throughout the research process. It describes the target population and sampling techniques, the instruments used for data collection together with the procedures employed to establish their validity and reliability, and the data collection procedures, including the implementation of the instructional intervention. The chapter also outlines the phased structure of the study, specifying the sequence of the comparison group, pilot study, and experimental group across the respective academic years. Finally, ethical considerations are discussed to demonstrate adherence to established research standards and to safeguard participants‘ rights and the integrity of the study.

The methodological choices presented in this chapter were guided by the research questions and the theoretical assumptions underlying WMCBPSI, particularly the need to examine not only whether the intervention improved students’ calculus problem-solving performance but also how the instructional approach influenced students’ engagement, strategy use, and learning experiences. Therefore, a mixed-methods framework was adopted to provide both measurable evidence of instructional effectiveness and contextual understanding of the learning process.

5.2. RESEARCH METHODOLOGICAL FRAMEWORK

This section presents the methodological framework adopted to investigate the effect of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on first-year university students' calculus problem-solving skills. The framework was developed to align directly with the study's research problem, objectives, and research questions by integrating the philosophical orientation, research design, mixed-methods approach, instructional intervention, data collection procedures, and analytical strategies into a coherent research process through which each methodological decision was selected to address the specific evidence required by the research objectives and questions.

Because the study sought both to determine the effectiveness of WMCBPSI in improving students' calculus problem-solving performance and to explain how students experienced and engaged with the instructional approach, a methodology capable of examining measurable learning outcomes together with contextual learning processes was required. Accordingly, the study adopted a pragmatic philosophical orientation, a quasi-experimental research design, and a mixed-methods approach. This methodological combination enabled the researcher to evaluate instructional effectiveness quantitatively while explaining the learning processes, classroom interactions, and students' experiences qualitatively, thereby providing a comprehensive understanding of the intervention.

5.2.1. Research Philosophy

The study was guided by a pragmatic research philosophy, which emphasizes selecting research methods that best address the research problem rather than adhering exclusively to a single philosophical paradigm (Creswell & Plano Clark, 2018; Morgan, 2022). Pragmatism acknowledges that educational phenomena are multifaceted and that understanding instructional effectiveness often requires the integration of quantitative and qualitative evidence.

The choice of pragmatism was driven by the nature of the research objectives. The study sought not only to determine whether WMCBPSI improved students' calculus problem-solving performance but also to understand the classroom processes and learning experiences through which such improvement occurred. Consequently, neither quantitative nor qualitative methods alone could adequately address the research problem.

The quantitative component generated objective evidence regarding differences in calculus problem-solving performance between students receiving WMCBPSI and those receiving Conventional Problem-Solving Instruction (CPSI). The qualitative component complemented these findings by exploring students' engagement, perceptions, and problem-solving behaviors during the instructional intervention.

Accordingly, pragmatism provided the philosophical foundation for integrating quantitative and qualitative evidence within a coherent methodological framework, ensuring that the study addressed both the effectiveness of the intervention and the instructional processes through which learning outcomes were achieved and providing the philosophical justification for adopting a quasi-experimental mixed-methods design consistent with the study's objectives.

5.2.2. Research Design and Approach

Consistent with the pragmatic philosophical orientation, the study employed a quasi-experimental research design using a static-group comparison with a post-test-only structure. In this design, one group received Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI), while a comparison group received Conventional Problem-Solving Instruction (CPSI). The static-group post-test-only design was selected because random assignment of students to the experimental and comparison groups was not feasible within the existing university classroom organization. Instead, intact classes were used, allowing the instructional intervention to be implemented under authentic educational conditions while maintaining systematic comparison between instructional approaches (Fraenkel et al., 2019; Johnson & Christensen, 2020). This design was therefore appropriate for addressing the principal research objective of determining the effect of WMCBPSI on students' calculus problem-solving skills under naturally occurring classroom conditions while maintaining the authenticity of the instructional setting. Such quasi-experimental designs are widely recognized as appropriate for evaluating instructional interventions in authentic higher education settings because they balance methodological rigor with practical feasibility (Capili, 2024).

The study was implemented in three sequential phases to enhance methodological rigor and minimize contamination between instructional groups. First, the comparison group receiving Conventional Problem-Solving Instruction (CPSI) was studied during the 2022 academic year to establish a baseline representing existing instructional practice. Second, a pilot study was conducted in 2023 using a separate cohort to evaluate the feasibility of the WMCBPSI intervention, refine the instructional procedures, and improve the research instruments. Third, the main intervention study was conducted in 2024, during which students received WMCBPSI. Conducting the comparison and intervention groups in different academic years reduced the possibility of treatment contamination between groups. At the same time, recognizing that data collection across different cohorts may introduce time-related influences, the study minimized these threats by using comparable first-year student cohorts, the same calculus content, equivalent instructional duration, consistent assessment procedures, and standardized data collection protocols throughout all phases of the study.

To achieve the study objectives comprehensively, the research adopted a mixed-methods approach that integrated quantitative and qualitative components. The quantitative component addressed the objective of determining the effect of WMCBPSI on students' calculus problem-solving skills through post-test achievement assessments, providing objective evidence for comparing the performance of the experimental and comparison groups. This quantitative strand directly addressed the research questions concerning the effectiveness of WMCBPSI, whereas the qualitative strand addressed the research questions relating to students' learning experiences, engagement, and instructional processes. The qualitative component complemented the quantitative findings by examining students' engagement, learning experiences, and instructional processes through classroom observations, semi-structured interviews, and video recordings. These qualitative data provided contextual explanations of how students experienced the intervention and how the instructional approach influenced their problem-solving development.

The integration of quantitative and qualitative evidence enabled methodological triangulation, thereby strengthening the credibility, validity, and interpretation of the findings (Tashakkori & Teddlie, 2003; Schoonenboom & Johnson, 2017; Creswell & Creswell, 2023). Quantitative analyses established the extent of the instructional effectiveness of WMCBPSI, whereas qualitative findings explained the cognitive and instructional processes through which the intervention influenced students' calculus problem-solving skills. Consequently, the methodological framework aligned the research problem, objectives, research questions, philosophical orientation, research design, and mixed-methods approach into a coherent methodological framework for evaluating both the effectiveness and implementation of WMCBPSI in an authentic university classroom context, thereby ensuring that every methodological component contributed directly to answering the research questions and fulfilling the study objectives.

5.3. RESEARCH HYPOTHESES

Within the methodological framework described above, the study formulated hypotheses to statistically examine whether Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) leads to significant improvements in first-year students' calculus problem-solving skills compared to Conventional Problem-Solving Instruction (CPSI). The hypotheses were derived directly from the principal research objective and the corresponding research question concerning the effectiveness of WMCBPSI. The hypotheses were tested to evaluate the effectiveness of WMCBPSI relative to traditional instructional approaches.
The null and alternative hypotheses were stated as follows:

Null Hypothesis (H_0): The use of WMCBPSI will not produce a significant improvement in students' calculus problem-solving skills compared to conventional problem-solving instruction.

Alternative Hypothesis (H_1): The use of WMCBPSI will lead to a significant improvement in students' calculus problem-solving skills compared to conventional problem-solving instruction.

These hypotheses guided the quantitative analysis and provided a clear basis for evaluating the instructional effectiveness of WMCBPSI in fostering calculus problem-solving competence among first-year university students, thereby enabling an objective statistical test of the study's principal research objective.

5.4. INSTRUCTIONAL INTERVENTIONS

To implement the research design described above, two instructional interventions were developed and deployed: Conventional Problem-Solving Instruction (CPSI) and Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI). Both interventions covered equivalent calculus content from the first-year university mathematics curriculum, focusing on problem-solving involving functions of one variable. The interventions were implemented over a two-month period, ensuring comparable content coverage, instructional time, and assessment conditions. The primary distinction between the two groups was the pedagogical approach employed. To ensure comparability between the groups, the same curriculum objectives, mathematical topics, instructional duration, and assessment conditions were maintained, while only the instructional approach differed.

5.4.1. Conventional Problem-Solving Instruction (CPSI)

The Conventional Problem-Solving Instruction (CPSI) served as the comparison group and represents the teacher-centered instructional method commonly practiced in Ethiopian university mathematics classrooms. This approach relied primarily on direct instruction, in which the teacher presented concepts, explained problem-solving steps, and demonstrated examples. Students‘ roles were largely passive, involving listening, note-taking, and replicating procedures rather than actively constructing knowledge.

The key features of CPSI included:

Direct Presentation of Concepts: New calculus topics were introduced primarily through lecture-based explanations, with limited opportunities for active engagement or inquiry.

Minimal Link to Prior Knowledge: Lessons rarely connected explicitly to students‘ prior experiences or real-life contexts.

Passive Learning Role: Students mainly observed and recorded problem-solving procedures demonstrated by the instructor.

Limited Problem-Solving Opportunities: Few worked examples were provided, restricting exposure to diverse strategies and representations.

Restricted Interaction: Peer collaboration and group discussion were minimal due to the traditional classroom structure.

Note-Taking Emphasis: Students‘ activities focused on note-taking and memorization rather than reasoning or conceptual understanding.

Assessment-Oriented Approach: Instruction was mainly driven by exam preparation goals rather than promoting conceptual depth and analytical skills.

Although CPSI provided students with procedural guidance and opportunities to practice mathematical procedures, it offered limited support for managing working memory demands, constructing schemas, and applying knowledge flexibly to unfamiliar problems.

Overall, CPSI emphasized procedural fluency and accuracy but offered limited opportunities for contextualized understanding or cognitive engagement. This contrasts with the WMCBPSI model, which was designed to foster collaboration, contextualization, and schema-building strategies that actively engage learners (Putri & Rahayu, 2021; Huang & Wang, 2023).

5.4.2. Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI)

The WMCBPSI model was developed specifically for this study to address the limitations of working memory during mathematical problem-solving. Grounded in Cognitive Load Theory (CLT) (Sweller et al., 2019) and context-based learning principles (Gilbert, 2006), the instructional model aimed to optimize students‘ cognitive processing and promote deeper understanding of calculus by strategically managing intrinsic, extraneous, and germane cognitive load.

The WMCBPSI framework was designed to align learning tasks with students‘ cognitive capacities while maintaining active engagement through contextualized activities. The design of the intervention followed the assumption that effective problem-solving instruction should not only present mathematical procedures but also support learners in organizing, storing, and retrieving knowledge through meaningful schemas.

Its core features included:

Chunking Effect: Complex problems were divided into smaller, logically related segments to facilitate information processing.

Collective Working Memory Effect: Collaborative learning was used to distribute cognitive load across group members.

Worked-Example Effect: Both full and partial worked examples were provided to scaffold learners‘ progression toward independent problem-solving.

Split-Attention Effect: Related information was presented together (text, symbols, and diagrams) to minimize extraneous cognitive load.

Variable Examples Effect: A range of examples was provided to promote flexible schema formation.

Self-Explanation Effect: Students were encouraged to verbalize their reasoning to strengthen schema construction and conceptual understanding.

These features were integrated systematically during instruction rather than applied as isolated techniques. Each element was intended to support a specific cognitive function: reducing unnecessary processing, directing attention toward essential information, and facilitating the development and automation of problem-solving schemas.

These design features of WMCBPSI were intended to cultivate transferable problem-solving schemas, thereby enhancing students‘ ability to apply calculus concepts to novel situations (Sweller et al., 2019; Zheng et al., 2023). By actively managing cognitive load and integrating contextualized learning, WMCBPSI promoted both procedural fluency and conceptual understanding, providing a richer and more engaging learning experience compared to conventional instruction.

5.4.2.1. Implementation of the WMCBPSI Intervention

The WMCBPSI intervention comprised 24 lessons over 8 weeks at 3 lessons per week and each lesson lasting 100 minutes. The lessons were delivered according to a structured sequence of activities designed to progressively develop students’ calculus problem-solving skills while managing working-memory demands. The intervention covered the selected calculus topics and provided repeated opportunities for students to engage with contextualised problems, worked

examples, guided practice, collaborative problem solving, self-explanation, individual problem solving, and feedback.
At the beginning of each lesson, the researcher activated students' prior knowledge by asking questions related to prerequisite concepts and procedures. The lesson topic was then introduced through a contextualised problem or situation to help students connect the mathematical concept with meaningful contexts. Key terms, concepts, and relevant information were identified before students proceeded to the problem-solving activities.

During the teacher-modelling stage, the researcher presented worked examples and demonstrated the problem-solving process step by step. The problems were broken into manageable components, and the researcher explicitly explained the reasoning behind the selection and application of each solution strategy. Relevant information, mathematical representations, and relationships were highlighted to direct students' attention to essential elements of the problem and reduce unnecessary cognitive processing. Full and partial worked examples were used progressively to support students' movement from observing solution procedures to completing parts of the solution independently.

Following teacher modelling, students engaged in collaborative problem-solving activities in small groups. Students worked on contextualised calculus problems that required them to interpret the problem context, identify relevant information, select appropriate mathematical strategies, and justify their solutions. During group activities, the researcher monitored students' progress, asked guiding questions, provided prompts when difficulties arose, and encouraged students to explain and compare their reasoning. Rather than providing immediate solutions, the researcher used scaffolding questions to guide students towards identifying errors and constructing appropriate solution strategies.

Self-explanation was incorporated throughout the problem-solving activities to encourage students to make their reasoning explicit. Students were required to explain why a particular problem-solving strategy was selected, how the mathematical steps were connected, and how the contextual information was translated into mathematical representations. Students explained their reasoning during peer discussions and classroom interactions, while the researcher used probing questions to encourage justification, reflection, and clarification of their thinking.

After the guided and collaborative activities, students were provided with opportunities to solve problems individually. Initially, students worked on problems that shared characteristics with the worked examples and guided activities. As students demonstrated increased competence, the level of teacher support was progressively reduced, and students were presented with less familiar and novel problems requiring them to independently identify relevant information, select appropriate strategies, and complete the solution process. This gradual reduction of scaffolding was intended to support the transition from teacher-supported learning to independent problem solving.

At the conclusion of each lesson, students' understanding and problem-solving performance were assessed through lesson-related problem-solving activities. The researcher reviewed students' responses, identified common errors and difficulties, and provided appropriate feedback and clarification. Where necessary, additional exercises were provided to reinforce concepts and problem-solving strategies that students had not yet mastered. This cycle of modelling, guided practice, collaborative problem solving, self-explanation, independent practice, assessment, and feedback was applied systematically throughout the WMCBPSI intervention.

A detailed example of one WMCBPSI lesson illustrating the implementation of these instructional procedures is provided in Appendix J. Although Appendix J presents one complete lesson, the instructional procedures illustrated in the lesson represent the approach applied throughout the WMCBPSI intervention.

5.4.3. The WMCBPSI Lesson

The WMCBPSI lesson was organized into multiple phases to systematically manage cognitive load and promote effective learning. Instructional methods that explicitly address working memory demands have been shown to significantly enhance mathematical problem-solving performance (Orosco et al., 2024; Russo & Minas, 2021). The lesson phases were structured to ensure gradual movement from teacher-supported learning toward independent problem solving, allowing students to construct, practice, and automate relevant problem-solving schemas.

Design Phase: Instructional tasks were carefully prepared to minimize cognitive overload and facilitate efficient information processing. Complex problems were segmented into manageable units corresponding to learners‘ cognitive capacities, thereby managing intrinsic cognitive load (Cooper, 1998; Kalyuga, 2015). Worked examples and integrated visuals were incorporated to reduce extraneous cognitive load and prevent split-attention effects (Ayres & Sweller, 2005; Paas et al., 2003). Students also engaged in self-explanation and reflection activities to increase germane cognitive load and consolidate new knowledge (Paas & van Merriënboer, 1994; Renkl, 2002**).** These design decisions ensured that mathematical complexity was preserved while unnecessary cognitive demands were reduced.

Instruction Phase: The teacher modeled problem-solving procedures and guided students through worked examples, prompting them to verbalize reasoning processes and highlight key elements. Guided modeling combined with verbalized reasoning has been shown to improve working memory efficiency and promote schema construction (Orosco et al., 2024). The teacher’s role shifted from information transmitter to cognitive guide, supporting students in identifying problem structures and selecting appropriate solution strategies.

Learning Phase: Students applied the strategies through both individual and collaborative tasks. The instructor facilitated discussions that encouraged learners to connect contextual information with mathematical representations. Contextual and collaborative problem-solving approaches improve conceptual understanding in mathematics (Fererde et al., 2024). Collaboration was used not only for interaction but also as a mechanism for distributing cognitive demands and encouraging mathematical communication.

Performance Phase: Students‘ independent problem-solving skills were assessed using novel and complex problems. Evidence suggests that context-based instructional approaches are associated with improved independent reasoning and enhanced problem-solving performance (Eshetu & Assefa, 2019). This phase provided evidence of students’ ability to transfer learned strategies beyond guided examples.

A detailed example of one WMCBPSI lesson is provided in Appendix J to demonstrate how the principles of working-memory management, contextualised problem solving, worked examples, guided practice, self-explanation, collaborative learning, and gradual transition to independent problem solving were applied during classroom instruction. The lesson provided in Appendix J serves as an illustrative example of the procedures followed across the WMCBPSI intervention. To provide a clear overview of the WMCBPSI lesson structure, Table 5.1 summarizes the planned stages and associated activities, illustrating how the phases described above were implemented in practice

Table 5.1: Summary of the Planned Working Memory-Context-Based Problem-Solving Lesson

Lesson Stages	Planned Activities
Introduction (35 min)	Introduction to the topic; explanation of key terms and concepts. Questions to assess students' prior knowledge. Connect the topic to students' real-life experiences.
Body (40 min)	Group work on various problem-solving examples, followed by completion exercises. Discussions on solution strategies for worked examples. Self-explanation activity to enhance understanding.
Conclusion (25 min)	Reflection on the lesson content and key takeaways. Individual or group problem-solving activity to apply newly learned strategies. Evaluation of student success rates and provision of additional exercises for reinforcement.

Note. The activities presented in Table 5.1 represent the planned structure of the WMCBPSI lessons. A detailed example of one WMCBPSI lesson showing how these activities were implemented in practice is provided in Appendix J.

5.4.4. Comparison of WMCBPSI and CPSI

The Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) differs fundamentally from Conventional Problem-Solving Instruction (CPSI) in both design and delivery. While CPSI emphasizes teacher-centered explanations, passive learning, and exam-oriented practice, WMCBPSI focuses on managing cognitive load and promoting active, context-based learning. By integrating collaboration, chunked instruction, and self-explanation, WMCBPSI enables students to construct schemas and transfer problem-solving skills to novel contexts. In contrast, CPSI limits student interaction and often overloads working memory through lengthy, unstructured presentations.

The comparison between the two instructional approaches was based on pedagogical characteristics rather than differences in mathematical content. Both groups studied the same calculus topics for an equivalent duration; however, the organization of learning activities, student participation, cognitive support mechanisms, and problem-solving opportunities differed.

Consequently, WMCBPSI provides a more cognitively efficient and engaging learning environment, supporting deeper understanding, long-term retention, and enhanced problem-solving performance in calculus.

5.4.5. Summary and Final Remarks

The WMCBPSI framework integrates Cognitive Load Theory with context-based learning principles to improve calculus problem-solving by strategically managing intrinsic, extraneous, and germane cognitive load throughout instruction. This structured approach enhances conceptual understanding, schema formation, and independent problem-solving skills.

The instructional model developed in this study therefore represents an integration of cognitive principles and classroom practices rather than a single teaching technique. Its effectiveness depends on the coordinated use of contextual problems, cognitive support strategies, student interaction, and gradual transfer of responsibility from teacher to learner.

Recent research highlights the benefits of combining contextual learning with cognitive regulation strategies for improving mathematical reasoning and problem-solving outcomes (Eshetu & Assefa, 2019; Kalyuga, 2015; Fererde et al., 2024). In the present study, these principles were operationalized through structured lesson phases, cognitive scaffolds, collaborative activities, and contextualized calculus tasks.

Future applications of WMCBPSI should continue to align with emerging cognitive research, ensuring that instructional practices remain adaptive, evidence-based, and responsive to evolving educational contexts.

5.4.6. Static-Group Comparison with Post-Test-Only Design

To evaluate the effectiveness of WMCBPSI relative to CPSI, the study adopted a static-group comparison with a post-test-only design. The design was selected to address the principal research objective and corresponding research question by enabling a systematic comparison of students' calculus problem-solving performance following the two instructional approaches. This quasi-experimental approach was chosen for its practicality within institutional constraints and its capacity to provide a rigorous assessment while minimizing potential biases (Cook & Campbell, 1979; Fraenkel et al., 2019).

Although random assignment was not possible because university classes were already established, several procedures were implemented to strengthen internal validity. These included using comparable first-year calculus students, maintaining equivalent curriculum content and instructional duration, separating intervention periods, and applying identical assessment procedures. These measures ensured that any observed differences in students' post-test performance could be more confidently attributed to the instructional intervention rather than to extraneous factors.

The study involved first-year calculus students at a public university in Ethiopia. Intact classes were used to form the experimental and comparison groups, ensuring consistency in academic background and preventing repeated participation, in line with recommended practices in educational research (Johnson & Christensen, 2020). The comparison group received CPSI in the 2022 academic year, and the experimental group received WMCBPSI in the 2024 academic year. Staggering the interventions across academic years minimized cross-group contamination and strengthened internal validity (Gaigher et al., 2006). To minimize potential time-related influences associated with implementing the study across different academic years, comparable first-year student cohorts, identical calculus content, equivalent instructional duration, standardized assessment procedures, and consistent data collection protocols were maintained throughout the study.

Several considerations justified this design. Limiting the study to a single institution controlled for variations in policies, resources, and curricula (Fraenkel et al., 2019; Gierus et al., 2025). Delivering interventions in separate years reduced the risk of students sharing materials or strategies, preserving internal validity (Shadish et al., 2002). Additionally, using intact classes provided a pragmatic solution where random assignment was infeasible, balancing methodological rigor with practical constraints (Johnson & Christensen, 2020; Ahmed et al., 2024).

While non-randomized, this quasi-experimental design is widely recognized as suitable for educational research. Measures such as matching groups by academic background and separating intervention years mitigated validity threats and enhanced the credibility of comparisons (Cook & Campbell, 1979; Ahmed et al., 2024). Consequently, the selected research design provided a methodologically coherent means of addressing the study's research problem, objectives, and research questions by generating valid evidence regarding the effectiveness of WMCBPSI under authentic university classroom conditions. In summary, the static-group post-test-only design offered a practical and effective framework for evaluating WMCBPSI while maintaining methodological rigor and contextual feasibility (Morgan, 2022; Gierus et al., 2025).

5.4.7. Exploratory Qualitative Design

In addition to the quantitative component, an exploratory qualitative design was incorporated to gain deeper insights into students' attitudes, behaviours, and experiences with CPSI and WMCBPSI. This qualitative component complemented the post-test data by providing a richer understanding of how students perceived, experienced, and engaged with the instructional methods. The inclusion of the qualitative component was consistent with the mixed-methods framework of the study, as learning outcomes alone cannot fully explain the classroom processes, student engagement, and learning experiences through which an instructional intervention influences mathematical problem-solving development.

The qualitative component was included to explore students' experiences, classroom participation, and strategy development, thereby providing evidence regarding the mechanisms through which WMCBPSI may influence students' problem-solving performance. While the quantitative component determined differences in calculus problem-solving achievement between the instructional groups, the qualitative component provided contextual explanations of how and why such differences occurred.

Classroom observations, guided by a structured observation schedule (Appendix B), documented the implementation of both instructional methods and captured student engagement with problem-solving tasks in real time. Observations focused on variations in teaching strategies, cognitive load management, and student participation (Bansilal et al., 2010; Onwu & Mogari, 2004). These observations enabled the researcher to examine the extent to which the intended instructional procedures were implemented and how students interacted with the problem-solving activities during classroom practice.

Semi-structured interviews with purposively selected students further explored learning experiences, perceived challenges, and opinions about the instructional methods, ensuring that participants had direct experience with the interventions (Vithal & Gopal, 2005). Purposive selection was employed because students with different levels of achievement and participation could provide diverse perspectives on the instructional experience. This variation enhanced the richness of qualitative interpretation by allowing exploration of both common and contrasting student experiences.

The combination of classroom observations and interviews enhanced the validity and depth of findings by reducing reliance on a single data source. Triangulation provided a comprehensive understanding of instructional practices, student engagement, and learning experiences, aligning with methodological recommendations for integrating multiple sources of evidence in mixed-methods research (Harries & Brown, 2010; Flick, 2023; Johnson & Onwuegbuzie, 2022).

Exploratory qualitative approaches have been effectively applied in educational research to investigate teaching and learning processes, including teacher development programmes (Onwu & Mogari, 2004), student perceptions of feedback (Bansilal et al., 2010), and students' experiences with instructional strategies (Vithal & Gopal, 2005; Borko, 2004; Kramarski &

Revach, 2009). These studies demonstrate the value of qualitative inquiry in complementing quantitative findings and developing deeper understanding of instructional effectiveness.

The qualitative findings were not intended to replace quantitative measurement but rather to explain, complement, and contextualize the statistical differences observed between the CPSI and WMCBPSI groups. Thus, the exploratory qualitative design strengthened the explanatory capacity of the mixed-methods approach by connecting measurable learning outcomes with students' cognitive, behavioural, and experiential responses to the instructional intervention.

By integrating classroom observations and semi-structured interviews, the exploratory design captured both measurable academic outcomes and students' subjective experiences. This dual perspective enriched the overall evaluation of CPSI and WMCBPSI, providing a robust and context-sensitive assessment that aligns with contemporary educational research advocating the integration of qualitative insights within mixed-methods studies (Paas et al., 2010; Creswell & Poth, 2018).

5.4.8. Study Participants

The study participants comprised first-year undergraduate mathematics students enrolled in calculus courses at an Ethiopian university. Three independent student cohorts participated in different phases of the study across consecutive academic years. The comparison group received Conventional Problem-Solving Instruction (CPSI) during the 2022 academic year, the pilot group participated in the pilot implementation of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) in 2023, and the experimental group received the full WMCBPSI intervention during the 2024 academic year. Using separate cohorts prevented repeated participation and reduced the possibility of treatment contamination between instructional groups, thereby strengthening the internal validity of the quasi-experimental design (Fraenkel et al., 2019; Johnson & Christensen, 2020).

A total of 118 students participated across the three phases of the research. The comparison group consisted of 38 students, the pilot group included 39 students, and the experimental group comprised 41 students. All participants were enrolled in the Department of Mathematics and were registered as first-year undergraduate students studying introductory calculus.

The gender distribution across the three cohorts was relatively balanced. The comparison group consisted of 20 (52.6%) male and 18 (47.4%) female students. The pilot group comprised 21 (53.8%) male and 18 (46.2%) female students. The experimental group included 22 (53.7%) male and 19 (46.3%) female students. The similarity in participant characteristics across the three cohorts enhanced the comparability of the groups and helped reduce the likelihood that demographic differences contributed substantially to observed differences in instructional outcomes (Fraenkel et al., 2019; Johnson & Christensen, 2020).

Table 5.2: Demographic Characteristics of Study Participants

Variable Total (N =118)	Category	Comparison Group (n = 38)	Pilot Group (n = 39)	Experimental Group (n = 41)
Gender 63 (53.4%)	Male	20 (52.6%)	21 (53.8%)	22 (53.7%)
55 (46.6%)	Female	18 (47.4%)	18 (46.2%)	19 (46.3%)
Age (years) 118 (100%)	19–20	38 (100%)	39 (100%)	41(100%)
Department 118 (100%)	Mathematics	38 (100%)	39 (100%)	41 (100%)
Academic Level 118 (100%)	First-year Undergraduate	38 (100%)	39 (100%)	41 (100%)

The demographic characteristics presented in Table 5.2 indicate that the three cohorts were comparable with respect to gender, age, academic level, and field of study. All participants were first-year mathematics students enrolled in calculus courses, and the proportion of male and female students was similar across the comparison, pilot, and experimental groups. Such similarity in participant characteristics contributes to the credibility of comparisons in quasi-experimental research where random assignment is not possible (Fraenkel et al., 2019; Johnson & Christensen, 2020). Therefore, differences observed in calculus problem-solving performance are more plausibly associated with the instructional approaches rather than substantial differences in participant characteristics.

5.4.9. Pilot Study and Refinements

A pilot implementation of the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) was conducted with a separate cohort of 39 students during the 2023 academic year. While a pilot study summary was utilized to test procedures, feedback, observations, and initial performance data gathered during this phase led to several practical adjustments to optimize the final intervention and data collection instruments used in the main study:

- Refinement of Instructional Pacing: Based on initial student feedback indicating that certain multi-step calculus problems caused a temporary cognitive bottleneck, the pacing of the WMCBPSI intervention was adjusted. Specifically, complex calculus tasks were broken down into smaller, more manageable sub-components ("chunking") to better manage working memory loads before progressing to independent problem-solving.
- Clarification of Interview Prompts: Observations during pilot interviews revealed that a few semi-structured interview questions were occasionally misinterpreted by students. These questions were rephrased to use clearer, more accessible language, ensuring students could precisely articulate their experiences regarding problem-solving strategies and group interactions.
- Adjustment of Classroom Observation Checklists: The pilot classroom observations highlighted that certain subtle peer-interaction dynamics and non-verbal problem-solving behaviours were difficult to capture in real time. Consequently, the observation checklist was revised to include specific markers for collaborative discourse and peer-support indicators, and video-recording angles were optimized to ensure comprehensive coverage.
- Time Allocation for Group Tasks: Pilot implementation showed that initial group discussions took longer than anticipated. As a result, the time allocated for collaborative problem-solving activities during the main intervention sessions was appropriately extended to allow students sufficient time to discuss alternative strategies without feeling rushed.

5.5. INSTRUMENTATION

This study used multiple instruments to collect quantitative and qualitative data to evaluate the effectiveness of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) in enhancing students' calculus problem-solving skills. The primary instruments were an achievement test, classroom observations, semi-structured interviews, and video recordings (Appendices A, B, and C). Supporting tools included WMCBPSI lesson plans, the Problem-Solving Skill Level Evaluation Form (Appendix D), and the Solution Strategy Evaluation Form (Appendix E).

Instrument selection was guided by research objectives and the need for comprehensive data triangulation to strengthen validity in a mixed-methods framework (Creswell & Poth, 2018; Paas et al., 2010). Each instrument was selected to address a specific dimension of the research problem. The achievement test measured the outcome of the intervention, whereas observations, interviews, and video recordings examined the instructional process and students' cognitive engagement. The evaluation forms provided additional evidence regarding the quality and nature of students' problem-solving approaches.

5.5.1. Achievement Test

The achievement test assessed students' calculus problem-solving skills and measured the impact of WMCBPSI compared with Conventional Problem-Solving Instruction (CPSI). Administered after the intervention, it included nine questions covering limits, continuity, derivatives, and integration, reflecting core calculus concepts (Appendix A; Steen, 2022).

The test was designed to assess higher-order problem-solving abilities rather than simple procedural recall. Therefore, items required students to interpret problems, select appropriate strategies, justify procedures, and apply calculus concepts in unfamiliar situations

The test was developed based on prior course assessments and expert feedback to ensure content alignment, relevance, and differentiation of students' problem-solving abilities(Rubin & Babbie, 2017; Van Merriënboer & Kirschner, 2018), directly addressing Research Question 3.
Validation and Refinement

Validity was ensured through face, content, and criterion-related procedures. Nine mathematics experts reviewed items for integration across topics and critical thinking, while practitioner feedback emphasized cross-topic applications and flexible strategy use (Boaler & Lamb, 2022; National Academies of Sciences, 2021).

Experts evaluated the items based on alignment with the research objectives, suitability for first-year university calculus students, clarity of mathematical language, cognitive demand, and capacity to distinguish different levels of problem-solving competence.

Adjustments included revising or removing misaligned items and introducing more challenging problems, ensuring suitability for higher-order thinking (Steen, 2022).

Reliability

Pilot testing with 39 students yielded a split-half reliability coefficient of $r = 0.889$ (Spearman–Brown), confirming the test's consistency in measuring calculus problem-solving skills (Cohen et al., 2018).

This reliability coefficient indicated strong internal consistency, suggesting that the test items consistently measured the intended construct of calculus problem-solving ability.

5.5.2. Classroom Observations and Semi-Structured Interviews

These qualitative instruments complemented the achievement test by capturing contextual insights into classroom dynamics, student engagement, and learning experiences with WMCBPSI.

Observations documented teaching strategies, lesson structure, student participation, and peer interactions (Onwu & Mogari, 2004; Bansilal et al., 2010). Semi-structured interviews explored students' perceptions, problem-solving strategies, challenges, and feedback on the instructional methods (Kvale & Brinkmann, 2015).

The observation and interview protocols were aligned with the theoretical framework of WMCBPSI by focusing on indicators such as cognitive load management, collaborative learning, strategy development, self-explanation, and students' perceptions of problem-solving tasks**.** Together, they provided a holistic view of instructional effectiveness.

5.5.2.1. Development of Observation Schedules and Interview Protocols

The observation schedule (Appendix B) and interview protocol (Appendix C) were designed to align with the research objectives and ensure consistency. The observation schedule captured how WMCBPSI was implemented in calculus lessons and identified challenges (Research Questions 1 and 2), following best practices for classroom observation in mathematics instruction (Vithal & Gopal, 2005; Borko, 2004). The interview protocol explored students‘ experiences with WMCBPSI, allowing follow-up on emerging themes while maintaining relevance and depth (Creswell & Poth, 2018).

The observation indicators were developed around three major areas: (1) teacher implementation of WMCBPSI principles, (2) student engagement and interaction patterns, and (3) evidence of cognitive and metacognitive activities during problem solving.

Both tools were informed by literature on classroom observation and interviews to ensure methodological rigor (Gay et al., 2011; Paas et al., 2010).

5.5.2.2. Piloting and Refinement

A pilot study refined the instruments, ensuring validity, reliability, and appropriateness for capturing intended constructs. Observation schedules were tested for clarity and applicability, with adjustments to reduce ambiguity and align with research questions (Paas et al., 2010; Vithal & Gopal, 2005). Interview questions were revised for clarity, enriched with probing prompts, and aligned with quantitative findings (Johnson & Christensen, 2020; Creswell & Poth, 2018).

The pilot phase was particularly important because the qualitative instruments were intended to capture classroom processes associated with a newly developed instructional model. Testing these instruments before the main study ensured that observation categories, interview questions, and recording procedures were appropriate for documenting WMCBPSI implementation. Pilot feedback also guided session duration, interview length, and strategies to minimize observer bias, enhancing data quality and consistency.

Revisions resulting from the pilot included improving the sequencing of interview questions, clarifying observation indicators, and adjusting the level of detail required during classroom documentation. These refinements strengthened the feasibility and consistency of data collection during the main study.

5.5.2.3. Constructs Measured by the Instruments

The instruments captured constructs aligned with the research questions:

Integration of WMCBPSI: Teacher strategies, lesson structure, and implementation fidelity (RQ1).

Classroom dynamics: Student engagement, interaction patterns, participation, and effectiveness of instructional materials (RQ2).

Problem-solving improvement: Measured quantitatively via the achievement test (RQ3).

Additionally, qualitative instruments examined cognitive and metacognitive indicators underlying problem-solving development, including students' strategy selection, explanation of reasoning, collaboration, confidence, and ability to transfer learned approaches to unfamiliar problems.

Convergent validity was ensured through triangulation of observations and interviews, confirming that both qualitative sources consistently captured related constructs such as engagement, attitudes, and perceptions of learning (Creswell & Creswell, 2018; Onwu & Mogari, 2004).

The instruments captured constructs aligned with the research questions:

- Integration of WMCBPSI**:** Teacher strategies, lesson structure, and implementation fidelity (RQ1).
- Classroom dynamics**:** Student engagement, interaction patterns, participation, and effectiveness of instructional materials (RQ2).
- Problem-solving improvement**:** Measured quantitatively via the achievement test (RQ3).

Convergent validity was ensured through triangulation of observations and interviews, confirming that both qualitative sources effectively captured overlapping constructs such as engagement, attitudes, and perceptions of learning (Brasil & Bordin, 2010; Onwu & Mogari, 2004).

5.5.2.4. Reliability

Classroom Observations: Reliability was maintained by consistent use of observation schedules and cross-checking observational data against interviews to ensure coherence (Bansilal et al., 2010; Borko, 2004).

To improve observational consistency, the researcher followed predetermined observation criteria throughout all sessions. Repeated review of classroom records and comparison with interview responses helped reduce subjective interpretation and strengthened trustworthiness of the qualitative findings.

Semi-Structured Interviews: Reliability and trustworthiness were ensured by using the same interviewer, standardized questions, and uniform interview conditions (Creswell & Poth, 2018; Patton, 2022; Lincoln & Guba, 1985). These measures ensured dependable, consistent, and credible qualitative data.

Although the interviews allowed flexibility through probing questions, the same core questions were maintained across participants to ensure comparability of responses.

These measures ensured dependable, consistent, and trustworthy qualitative data.

5.5.3. Video Recording

Video recording complemented observations and interviews by capturing both verbal and non-verbal classroom interactions, including gestures, posture, and tone, providing dense, context-rich data (DuFon, 2002; van Es & Sherin, 2022). Videos allowed real-time observation of teacher–student dynamics (Sherin, 2004; Sherin & van Es, 2009), captured linguistic and non-linguistic information (Heath, Hindmarsh, & Luff, 2010; Roschelle et al., 2021), and created a permanent record for repeated review, reducing the risk of overlooking subtle interactions (Sherin, 2004; Seidel et al., 2019).

In this study, video recordings served as supporting evidence rather than an independent source of measurement. They enabled repeated examination of instructional practices, student participation, and problem-solving interactions, thereby strengthening interpretation of observational findings. This tool enriched the qualitative dataset and enhanced interpretation of students' engagement and problem-solving strategies under WMCBPSI.

Ethical safeguards were maintained during video recording by informing participants about the purpose of recording, obtaining consent, restricting access to research purposes, and ensuring that recordings were stored securely.

5.5.4. Students‘ Problem-Solving Evaluation Forms

To assess students' calculus problem-solving in depth, two complementary evaluation forms were employed, addressing both procedural competence and strategic reasoning.

The inclusion of these forms was necessary because mathematical problem-solving involves not only obtaining correct answers but also understanding the reasoning processes, strategies, and cognitive steps used to reach solutions.

5.5.4.1. Problem-Solving Skill Level Evaluation Form

Based on the TIMSS framework and Polya's problem-solving model (Appendix D), this form evaluated students' ability to apply problem-solving skills in calculus.

The TIMSS scale assessed the quality of problem solving by integrating strategy use, procedural steps, and reasoning depth (Mogari & Lupahla, 2013; Mullis et al., 2021). Polya's model provided a theoretical basis for evaluating progression from understanding the problem to reviewing the solution (Polya, 1945; Schoenfeld, 2021).

The combination of these frameworks allowed classification of students according to the complexity and completeness of their problem-solving processes rather than only the correctness of final answers. Combining TIMSS and Polya's model ensured a holistic measure of both procedural and cognitive aspects of problem-solving.

5.5.4.2. Students‘ Solution Strategy Evaluation Form

Adapted from Jiang and Chua (2010) and Thomson Learning (2008) (Appendix E), this form analyzed the strategies students employed, focusing on reasoning and approaches.
The solution strategy model evaluated both correctness and the complexity of strategies used in problem solving (Mogari & Lupahla, 2013; Cai, 2021).

The analysis considered whether students relied on routine procedures, recognized mathematical structures, constructed appropriate representations, connected concepts, and justified their chosen methods.

While the Skill Level Form evaluated outcomes and accuracy, the Strategy Form examined cognitive reasoning and the steps taken, providing a complete perspective on problem-solving performance. Together, these forms captured both what students achieved and how they approached problem solving.

5.5.4.3. Integration with Other Instruments

Used alongside the achievement test, classroom observations, interviews, and video recordings, these evaluation forms contributed to a triangulated, multidimensional data framework.

This integration enabled examination of both the effectiveness of WMCBPSI and the mechanisms through which the intervention influenced students' mathematical thinking (Onwu & Mogari, 2004; Bansilal et al., 2010; Paas et al., 2010)**.** For example, improvements observed in achievement scores could be interpreted alongside evidence of increased strategic flexibility, explanation, and collaborative reasoning.

This approach allowed assessment of both the effectiveness of WMCBPSI and the cognitive processes and strategies underlying students' problem-solving, enhancing the validity and depth of the findings.

5.6. DATA COLLECTION

Data for this study were collected in two phases using a mixed-methods approach to ensure a comprehensive assessment of the effects of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on students' calculus problem-solving skills.

The data collection process was organized to correspond with the sequential implementation of the study: pilot testing, refinement of procedures, and main intervention. This structure ensured that instruments and instructional procedures were sufficiently developed before collecting final research data.

The first phase involved a pilot study to examine feasibility and refine the methodology, while the second phase, the main study, implemented the intervention under controlled conditions to evaluate its effectiveness.

5.6.1. Pilot Study

The pilot study, conducted during the 2023 academic year, involved 39 first-year calculus students from a different cohort to prevent contamination between the pilot participants and those involved in the main experimental study. The pilot was conducted as a feasibility and refinement stage rather than as part of the main experimental comparison. Its primary purpose was to test all major components of the intervention and research procedures, providing critical feedback for refining instructional design, lesson pacing, instructional materials, assessment instruments, and data collection procedures (Van der Sluis et al., 2022).

During the pilot, instruction followed departmental guidelines, consisting of six 50-minute sessions per week over a period of two months. The duration was determined jointly by the official calculus course schedule and practical research logistics. It allowed completion of the planned instructional content while providing sufficient time to implement and evaluate the WMCBPSI intervention under normal classroom conditions. WMCBPSI was implemented according to the lesson structure and cognitive load management strategies described in Section 5.4.3. The instructional process emphasized scaffolding, worked examples, self-explanation, and the gradual automation of problem-solving strategies.

Quantitative results from the pilot indicated that the experimental group outperformed the comparison group, with mean post-test scores of 58.42 and 43.45, respectively, suggesting that WMCBPSI had a positive influence on students' calculus problem-solving performance. However, these findings were interpreted only as preliminary evidence regarding the feasibility and potential effectiveness of the intervention. The pilot data were not included in the statistical analyses of the main study, and the conclusions of this research are based exclusively on data collected during the 2024 experimental implementation.

The pilot study generated important evidence for improving the intervention and research procedures before the main experimental implementation. Based on classroom observations, student responses, assessment results, and researcher reflections, several modifications were introduced. First, the instructional design was refined by reorganizing lesson activities to improve the balance between teacher modelling, collaborative problem solving, individual practice, and reflection. The sequencing of activities was adjusted to provide students with sufficient time to process complex calculus problems and apply problem-solving strategies effectively.

Second, the WMCBPSI instructional materials were modified to strengthen cognitive load management. Some problem-solving tasks were redesigned by incorporating additional chunking and scaffolding strategies, enabling students to process complex information in smaller and more manageable units. Worked examples were revised, and additional guidance was incorporated into self-explanation activities to help students identify important solution steps and articulate their reasoning more effectively.

Third, the achievement test and other research instruments were reviewed and refined based on the pilot findings. Ambiguous or unclear assessment items were revised to improve clarity and alignment with the study objectives. Interview questions were modified to obtain more meaningful information about students' learning experiences, while classroom observation procedures were refined to ensure systematic documentation of instructional implementation and student engagement.

Fourth, lesson pacing and classroom management procedures were adjusted based on challenges identified during the pilot implementation. Additional time was allocated for discussion, collaborative activities, and feedback to ensure that students could fully engage with the problem-solving processes emphasized in WMCBPSI.

These modifications improved the clarity, feasibility, and consistency of the intervention, strengthened the quality of the research instruments, and enhanced the readiness of the study procedures for the full-scale implementation of the main study conducted during the 2024 academic year.

Thus, the pilot study fulfilled its intended methodological purpose by refining the intervention and research procedures before the commencement of the main experiment.

5.6.2. Main Study

The main study was conducted during the 2024 academic year, incorporating refinements derived from the pilot phase. The study involved an experimental group receiving WMCBPSI and a comparison group receiving Conventional Problem-Solving Instruction (CPSI). To avoid contamination, the interventions were implemented in separate academic years. The instructional period also spanned two months, following the same schedule established during the pilot study, with six 50-minute sessions per week conducted in the morning to maximize student engagement.

The researcher personally conducted all instructional sessions to ensure fidelity and consistency of delivery. Throughout the intervention, classroom observations were conducted to monitor implementation integrity, evaluate student engagement, and examine the adoption of problem-solving strategies. These observations were structured according to the observation protocol detailed in Appendix B, enabling systematic documentation of classroom dynamics and instructional effects.

The WMCBPSI lessons were implemented according to the principles of Cognitive Load Theory, integrating scaffolding, worked examples, self-explanation, and gradual automation to enhance students‘ problem-solving skills (Paas et al., 2010; Sweller, 2023). Observations captured how students responded to instruction, the methods they used to solve problems, their application of prior knowledge to novel tasks, and the integration of concepts across different problem types. This qualitative data complemented quantitative measures, providing a detailed understanding of the instructional impact and classroom interactions (Hattie, 2022; Onwu & Mogari, 2004).

At the conclusion of the intervention, students in both groups completed a two-hour achievement test designed to measure calculus problem-solving skills, including conceptual understanding, procedural fluency, and strategy application, consistent with the TIMSS framework (Mogari & Lupahla, 2013). Participants were assigned unique identifiers to maintain anonymity, and the test was administered under identical conditions to ensure reliability and comparability. This test served as the primary quantitative measure of instructional effectiveness.

To provide further insight into students‘ learning experiences, semi-structured interviews were conducted with a purposively selected sample from both groups. Participants were chosen based on post-test performance, class participation, and willingness to engage, ensuring diverse perspectives. The interviews explored students‘ perceptions of teaching methodology, the difficulty and significance of problem-solving tasks, challenges encountered during instruction, the strategies they employed, and their recommendations for improving problem-solving teaching. These qualitative data complemented the achievement test results, offering a holistic view of both instructional approaches (Kvale & Brinkmann, 2015; Bansilal et al., 2010).

5.7. DATA ANALYSIS

This study employed a mixed-methods approach, integrating quantitative and qualitative analyses to evaluate the effectiveness of the instructional interventions. The analysis was conducted in accordance with the research questions. Quantitative analysis determined whether WMCBPSI produced measurable improvements in calculus problem-solving performance, while qualitative analysis explained students' experiences, engagement patterns, and problem-solving behaviors associated with the intervention. The analysis compared the performance of the experimental group receiving Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) and the comparison group receiving Conventional Problem-Solving Instruction (CPSI), addressing the research objectives comprehensively (Creswell & Plano Clark, 2018; Johnson & Christensen, 2020).

5.7.1. Quantitative Analysis

Quantitative analysis focused on determining whether differences in calculus problem-solving outcomes between the groups were statistically significant. Descriptive statistics, including means, standard deviations, and percentage scores, were calculated for post-test results of both groups, providing an overview of performance differences and highlighting the impact of the WMCBPSI intervention. Descriptive analysis was conducted first to examine overall performance patterns before inferential testing. This allowed identification of differences in achievement levels and provided a basis for interpreting the statistical results (Field, 2022).

An independent sample *t*-test was conducted at a 0.05 significance level. This test was chosen because the experimental and comparison groups were mutually exclusive, assumptions of equal variances were verified, and the *t*-test is robust for small to moderate sample sizes (Field, 2022).

The null hypothesis (H_0) stated that there is no significant difference in calculus problem-solving skills between students taught using WMCBPSI and those taught using CPSI:

H_0: μ Working Memory–Context-Based Problem-Solving Instruction = μ Conventional Problem-Solving Instruction

The alternative hypothesis (H_1) proposed that students taught using WMCBPSI demonstrate significantly better problem-solving skills:

H_1**:** μ Working Memory–Context-Based Problem-Solving Instruction ≠ μ Conventional Problem-Solving Instruction

Decision-making was based on the *p*-value: if $p < 0.05$, H_0 was rejected in favor of H_1, indicating a statistically significant improvement; if $p \geq 0.05$, H_0 was not rejected, implying no statistically significant difference. All analyses were performed using SPSS version 21.0 to ensure accuracy, consistency, and replicability (Field, 2022). Before conducting the independent sample *t*-test, the assumptions of independence of observations and homogeneity of variance were examined to ensure that the statistical procedure was appropriate for the collected data (Field, 2022).

5.7.1.1. Analysis of the Effect Size of the Intervention

Beyond statistical significance, the practical significance of the intervention was assessed using

Cohen's d, which quantifies the magnitude of the effect and provides insight into its educational meaningfulness (Lakens, 2022; Morris, 2023). Cohen's d was calculated as follows:

$$\text{ES(Cohen`s d)} = \frac{\mu_E - \mu_C}{SD_{pooled}}$$

where where μ_E and μ_C are the mean percentage scores of the experimental and comparison groups, respectively, and SD_{pooled} is the pooled standard deviation. The pooled standard deviation was calculated using:

$$SD_{pooled} = \sqrt{\frac{(n_E - 1)\sigma_E^2 + (n_C - 1)\sigma_C^2}{n_E + n_C - 2}}$$

where σ_E *and* σ_C are standard deviations, and n_E and n_C are the sample sizes of the experimental and comparison groups, respectively. Effect size values range from −3.0 to +3.0, with interpretations as follows: values near 0 indicate negligible practical significance; 0.2 represents a small effect; 0.5 denotes a moderate effect; and 0.8 or higher reflects a large effect, indicating substantial educational impact. This analysis ensures that observed differences between WMCBPSI and CPSI are both statistically valid and pedagogically meaningful (Adams & Newton, 2022). Detailed effect size results are presented in Section 6.2.

Although statistical significance indicates whether observed differences are unlikely to have occurred by chance, effect size provides additional information regarding the practical importance of WMCBPSI. This distinction is particularly important in educational intervention research because a statistically significant difference may not always represent a meaningful improvement in classroom practice. Therefore, Cohen's *d* was included to determine the magnitude of the instructional effect and to support interpretation of the educational relevance of the findings.

This analysis ensures that observed differences between WMCBPSI and CPSI are both statistically valid and pedagogically meaningful (Adams & Newton, 2022). Detailed effect size results are presented in Section 6.2.

5.7.1.2. Analysis of Students' Overall Calculus Problem-Solving Skill Levels

Students' calculus problem-solving skills were assessed using the post-intervention achievement test, following the TIMSS framework, which aligns with Polya's (1957) four-step problem-solving model. Students were classified into skill levels based on the frequency and type of strategies employed (Appendix D), consistent with recent approaches for evaluating cognitive and metacognitive development (Boesen et al., 2021); Sengupta & Bhattacharya, 2024).

The analysis of problem-solving skill levels extended beyond final answers by examining the quality of students' reasoning processes. This approach recognizes that mathematical competence involves not only obtaining correct solutions but also understanding problem conditions, selecting appropriate strategies, executing procedures logically, and evaluating solutions. Therefore, skill-level analysis provided deeper evidence of how WMCBPSI influenced students' cognitive development and strategic competence.

5.7.1.3. Analysis of Students' Solution Strategies for Solving Calculus Problems

Beyond overall skill levels, students' specific solution strategies were analyzed using a coding model adapted from established frameworks (Appendix E). Strategies were categorized and quantified across all post-test questions. Each student's solution was coded according to the predefined categories in the coding framework (Appendix E), and the frequency of each strategy was recorded for both the experimental (WMCBPSI) and comparison (CPSI) groups. Comparative analysis was then conducted to identify similarities and differences in the types and frequencies of solution strategies employed by students across the two instructional groups. This analysis enabled systematic examination of students' problem-solving approaches and provided a basis for interpreting variations in strategy use, with the corresponding findings presented in Chapter 6.

The strategy analysis was conducted to identify qualitative differences in students' approaches to solving calculus problems. Particular attention was given to whether students relied mainly on routine procedural applications or demonstrated flexible strategy selection, conceptual connections, and justification of solution methods. Such analysis enabled evaluation of the extent to which WMCBPSI promoted adaptive problem-solving rather than only procedural accuracy.

The coding of solution strategies was conducted systematically across all test items to ensure consistency in interpretation. Strategies were compared between groups based on frequency, diversity, and cognitive complexity, allowing identification of differences in students' problem-solving behaviours associated with each instructional approach.

5.7.2 Qualitative Analysis

Qualitative analysis complemented the quantitative analysis by exploring students' perceptions, experiences, and behaviors during the WMCBPSI and CPSI interventions. Data were collected through classroom observations, semi-structured interviews, and video recordings, providing a comprehensive understanding of student engagement and strategy use under each instructional approach (Braun & Clarke, 2022; Creswell & Poth, 2018). Data preparation involved verbatim transcription of interviews and observation notes, alongside repeated review of video recordings to ensure completeness and accuracy. This process enhanced interpretive sensitivity and reliability of thematic coding (Patton, 2015; Williams & Moser, 2019; Smith et al., 2023; O'Connor & Joffe, 2021).

To ensure a rigorous, transparent, and systematic examination of the qualitative data, the analysis followed the 10-stage thematic content analysis framework adapted from Burnard (1991). Burnard (2004) emphasizes that exclusive reliance on statistical analysis may overlook critical aspects of program implementation and instructional processes; therefore, this structured framework was adopted to provide a richer, more contextualized understanding of students' experiences and to enhance the reliability and validity of the interpretive process.

The analysis followed an integrated inductive-deductive approach. Codes were derived from recurring patterns, theoretical constructs, and emergent student experiences. The systematic stages of Burnard's (1991) framework were applied as follows:

Stage 1: Initial review of classroom observations, semi-structured interviews, and recorded interview sessions to extract relevant notes and remark tags linked to study objectives.

Stage 2: Transcription of interviews, followed by initial reading and identification of general themes using keywords directly aligned with the study’s aims and research questions.

Stage 3: Further review of transcripts and categorization of data under identified themes corresponding to specific objectives.

Stage 4: Refinement of categories by examining similarities and merging overlapping groups to reduce redundancy.

Stage 5: Finalization of categories and subcategories by removing repetitions and inconsistencies to ensure alignment with the study's framework.

Stage 6: Verification of categories against the raw interview data to ensure comprehensive coverage.

Stage 7: Systematic coding of all transcripts based on the finalized categories and subcategories, with results cross-verified in collaboration with colleagues.

Stage 8: Extraction of coded sections from transcripts and consolidation under relevant categories.

Stage 9: Organization of data by sorting and pasting coded excerpts under relevant headings.

Stage 10: Validation of categorized data through respondent feedback to confirm that participant interpretations accurately reflected their intended responses.

Key themes examined through this framework included student engagement and attitudes, instructional adaptations, perceived effectiveness, and problem-solving strategies aligned with Polya's framework (Leahy & Sweller, 2023; Yohannes & Kriek, 2023; Sengupta & Bhattacharya, 2024).

The qualitative analysis followed a systematic coding process to maintain transparency and reduce subjective interpretation. Initial codes were generated from repeated examination of transcripts, observation records, and video data. These codes were then organized into broader categories aligned with the research questions and theoretical framework of WMCBPSI, particularly cognitive load management, engagement, strategy development, and learning experiences.

Classroom observations focused on participation, problem-solving approaches, and instructional reactions. Observation data were systematically coded using the predetermined thematic framework, with additional codes generated inductively where new patterns emerged. The coded observation data were then compared across the WMCBPSI and CPSI groups to identify similarities and differences in classroom interactions, engagement, and problem-solving behaviours.

Semi-structured interviews involved purposively selected students from both groups (anonymized as ES1, ES2; CS1, CS2) and explored perceptions of teaching effectiveness, problem-solving experiences, instructional challenges, and solution strategies. Interview transcripts were analysed thematically following the same coding procedure to ensure consistency across qualitative data sources. Emergent themes were refined through iterative review and comparison with the observation and video data before being organised into overarching categories relevant to the study objectives.

Triangulation of interview, observation, and video data strengthened the credibility, validity, and transferability of the qualitative findings (Tashakkori & Teddlie, 2003; Flick, 2023). Evidence from the three data sources was compared to confirm the consistency of emerging themes and to enhance the trustworthiness of the analysis. The integrated qualitative findings are presented and discussed in Chapter 6 alongside the quantitative results.

The integration of multiple qualitative sources strengthened the credibility of interpretations by allowing comparison of students' reported experiences with observed classroom behaviours. For example, students' statements regarding improved confidence or strategy use were examined alongside classroom evidence of participation, collaboration, and problem-solving behaviours.
Overall, the integration of qualitative and quantitative analyses provided a comprehensive understanding of WMCBPSI's influence on calculus problem-solving processes, student engagement, and instructional dynamics (Greene, 2023; Johnson & Onwuegbuzie, 2022; Chen et al., 2023; Tafesse & Kriek, 2025).

5.8. ETHICAL CONSIDERATIONS

Ethical integrity was central to this study, guiding all phases of research design, implementation, and reporting. The study adhered to established ethical principles to safeguard participants, ensure transparency, and maintain methodological credibility (Creswell & Creswell, 2023; Ederio, 2023; Tetteh, 2023). Participants were fully informed about the study's objectives, procedures, and voluntary nature, and both verbal and written consent were obtained.

Before the commencement of the study, participants in the comparison, pilot, and experimental groups were provided with clear information about the instructional approaches they would receive, including the purpose of the intervention, the nature of classroom activities, the duration of participation, and the data collection procedures. Participants assigned to the comparison group were informed that they would receive Conventional Problem-Solving Instruction (CPSI), while those in the pilot and experimental groups were informed that they would receive Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI). The rationale for the different instructional approaches was explained, and participants were assured that their academic assessment, grades, and standing within the university would not be adversely affected by their participation or by the instructional approach they received. They were also informed of the anticipated educational benefits, including opportunities to strengthen their calculus problem-solving skills, improve conceptual understanding, and enhance problem-solving strategies, as well as potential disadvantages or inconveniences, such as the additional time required for classroom activities, observations, interviews, and completion of assessment tasks. Participants were informed that the instructional interventions were designed solely for educational and research purposes and that no foreseeable physical, psychological, academic, or social harm was expected.

They were explicitly made aware of their right to withdraw at any stage without repercussions, ensuring autonomy and respect for individual choice (Cohen et al., 2018; Bryman, 2016).

Given that the researcher was directly involved in implementing the instructional intervention, particular attention was given to maintaining professional neutrality and minimizing potential researcher influence throughout the study. Classroom interactions, observations, and interviews were conducted objectively to ensure that participants' responses and performance were not influenced by the researcher's dual role as instructor and researcher (Creswell & Creswell, 2023).

To minimize potential risks, a thorough risk–benefit assessment was conducted. The benefits, including enhancing knowledge of effective calculus instruction, outweighed minimal potential discomfort.

Participants were given sufficient opportunity to ask questions about the interventions before providing consent, and all questions were answered to ensure that participation was based on a clear understanding of both the expected educational benefits and any foreseeable inconveniences.

Measures such as creating a supportive, non-judgmental environment during classroom observations and interviews ensured participants' dignity, privacy, and self-esteem were upheld (Adley, 2024). All data—including achievement tests, observation notes, and interview transcripts—were anonymized, with participants identified through codes (e.g., E1 for experimental group, C1 for comparison group), and securely stored to maintain confidentiality (Ederio, 2023).

Participant confidentiality was maintained throughout data management, analysis, and reporting. Identification codes rather than personal identifiers were used in achievement test records, classroom observation notes, interview transcripts, and video-based analyses. Access to research data was restricted solely to the researcher and was used exclusively for the purposes of this study, thereby ensuring secure handling and protection of participants' information (Ederio, 2023; Bryman, 2016).

Formal approval to conduct the study was obtained from relevant Ethiopian university authorities as well as the University of South Africa (UNISA) College of Science, Engineering and Technology – School of Science – Ethics Review Committee (ERC), following the submission of detailed documentation outlining objectives, methodology, and ethical safeguards (Cohen et al., 2018; Bryman, 2016). The Unisa ethics reference number is 6147, (see Unisa ethical clearance certificate Appendix K). Researcher integrity was maintained throughout, with classroom observations conducted unobtrusively and interviews guided by neutral, open-ended questioning to minimize bias (Creswell & Creswell, 2023).

To further reduce potential bias during data analysis, quantitative analyses followed predetermined statistical procedures, while qualitative analyses were conducted through systematic coding and comparison across multiple sources of evidence. The use of methodological triangulation strengthened the credibility and trustworthiness of the findings by ensuring that interpretations were supported by classroom observations, interview data, and video recordings rather than relying on a single source of evidence (Creswell & Creswell, 2023; Tetteh, 2023).

The study was sensitive to participant diversity, respecting variations in age, gender, culture, religion, and socio-economic background. Instructional materials and assessment items were reviewed for cultural appropriateness and to avoid bias, ensuring inclusivity in all interactions (Adley, 2024). Transparency and accountability were further promoted through ongoing communication with participants and university authorities, including opportunities for feedback and sharing preliminary findings (Tetteh, 2023).

Potential ethical dilemmas were addressed proactively. For instance, maintaining neutrality during classroom observations was ensured by systematic documentation without intervention, and initial reluctance from some students to participate in interviews was managed by reassurance of confidentiality, flexible scheduling, and respect for their decision to decline (Creswell & Creswell, 2023).

The ethical procedures adopted were particularly important because this study involved a classroom-based instructional intervention in which educational responsibilities were integrated with research activities. Maintaining professional neutrality, confidentiality, transparency, fairness, and respect for participants' rights ensured that the study complied with accepted ethical standards for educational research while safeguarding participants throughout the research process (Cohen et al., 2018; Adley, 2024).

In sum, ethical considerations in this study encompassed comprehensive informed consent, including explanation of the instructional interventions, their expected educational benefits, possible disadvantages or inconveniences, voluntary participation, the right to withdraw without penalty, confidentiality, sensitivity to diversity, researcher integrity, transparency, and proactive management of potential ethical dilemmas.

These measures collectively ensured that the research was conducted responsibly, protecting participants while upholding the credibility and rigor of the study (Creswell & Creswell, 2023; Ederio, 2023; Adley, 2024; Tetteh, 2023; Cohen et al., 2018; Bryman, 2016).

5.9. SUMMARY AND CONCLUSION OF THE CHAPTER

Chapter 5 provided a comprehensive account of the methodological framework employed to investigate the effect of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on first-year students' calculus problem-solving skills. The study was guided by principles of rigorous research design and ethical integrity, ensuring both validity and reliability of findings (Creswell & Creswell, 2023; Adley, 2024).

The methodological framework was developed to establish alignment among the research problem, objectives, instructional intervention, data collection procedures, and analytical methods. The combination of quantitative and qualitative approaches enabled examination of both the measurable effects of WMCBPSI and the processes through which the intervention influenced students' problem-solving experiences.

The chapter began by detailing the static-group comparison design, selected to facilitate a valid comparison between the experimental group receiving WMCBPSI and the comparison group receiving Conventional Problem-Solving Instruction (CPSI), while minimizing threats to internal validity. This design enabled the researcher to evaluate differences in learning outcomes associated with the instructional interventions. The timeline of the study was clarified, with a comparison group studied in the 2022 academic year, a pilot study conducted in the 2023 academic year to assess feasibility and refine the methodology, and the main experimental study implemented in 2024 under controlled conditions (Ederio, 2023; Mensah & Boateng, 2023).

The chapter also demonstrated how WMCBPSI was operationalized through specific instructional mechanisms, including cognitive load management, contextualized learning, collaborative problem solving, worked examples, and self-explanation activities. These features provided a clear connection between the theoretical foundation presented in earlier chapters and the empirical investigation conducted in this study.

The chapter also described the study participants, including their demographic characteristics, sample composition, and distribution across the comparison, pilot, and experimental groups. A total of 118 first-year mathematics students participated in the study, consisting of 38 students in the comparison group, 39 students in the pilot group, and 41 students in the experimental group. The similarity of participants in terms of age, academic level, department, and gender distribution provided a basis for comparing the instructional groups and interpreting the study findings.

The sampling strategy was described, including the selection of intact classes and the procedures used to establish the comparison, pilot, and experimental groups. These procedures minimized potential selection bias and ensured that the groups provided an appropriate basis for evaluating the effectiveness of WMCBPSI relative to CPSI.

The data collection methods were presented in detail, highlighting achievement tests for quantitative assessment, systematic classroom observations for monitoring instructional dynamics, and semi-structured interviews to capture students' experiences and perceptions (Cohen et al., 2018; Bryman, 2016). The pilot study was critical in refining the intervention and data collection instruments, ensuring methodological readiness and fidelity for the main study (Creswell & Creswell, 2023; Chen et al., 2023).

Instructional procedures were described comprehensively. The WMCBPSI intervention integrated principles of Cognitive Load Theory with structured problem-solving strategies, whereas the comparison group received conventional problem-solving instruction. The careful design and systematic implementation ensured consistency in delivery across sessions and strong alignment with the study's objectives, supporting accurate assessment of the intervention's effectiveness (Leahy & Sweller, 2023; Zewdu et al., 2024).

Ethical considerations were rigorously addressed through comprehensive informed consent procedures, confidentiality protection, sensitivity to participant diversity, researcher integrity, transparency, and proactive management of potential ethical dilemmas. Before participation, students in the comparison, pilot, and experimental groups were informed about the instructional approaches they would receive, the purpose and procedures of the study, expected educational benefits, possible disadvantages or inconveniences, and their right to withdraw without penalty. Participants were also assured that their involvement would not negatively affect their academic assessment or university standing. Formal approval was obtained from relevant university authorities, reinforcing the credibility and ethical rigor of the research process (Tetteh, 2023; Greene, 2023).

Because the researcher was directly involved in implementing the instructional intervention, particular attention was given to maintaining ethical neutrality and minimizing potential researcher influence. Students were informed that participation in research activities was voluntary and that their academic evaluation would not be affected by their decision to participate or withdraw. Furthermore, confidentiality was maintained through the use of participant identification codes, secure handling of research data, and restricted access to information collected during the study.

To reduce possible bias during data interpretation, quantitative analyses followed predetermined statistical procedures, while qualitative findings were developed through systematic coding and comparison of multiple data sources. The use of triangulation further supported objective interpretation by ensuring that conclusions were not based on a single source of evidence.

Although the study employed rigorous procedures to enhance validity and reliability, limitations related to the quasi-experimental design, single institutional setting, and use of intact groups should be considered when interpreting and generalizing the findings. Nevertheless, the methodological safeguards implemented provide a strong basis for evaluating the effectiveness of WMCBPSI within the Ethiopian higher education context.

In conclusion, this chapter establishes a solid methodological foundation for the presentation of findings in Chapter 6. By integrating a carefully designed research framework with comprehensive ethical safeguards, Chapter 5 ensures that the subsequent analysis is grounded in valid, reliable, and ethically sound data.

The following chapter presents the findings of the study, integrating statistical analyses, problem-solving skill evaluations, strategy analyses, and qualitative evidence. These findings provide empirical evidence regarding the effectiveness of WMCBPSI and contribute to understanding how cognitively informed, context-based instruction can enhance calculus problem-solving skills among university students.

CHAPTER SIX

DATA ANALYSIS AND RESULTS: QUANTITATIVE DATA

6.1. INTRODUCTION

This chapter presents the quantitative results obtained from the post-intervention achievement test administered to both the experimental and comparison groups. The primary objective of this analysis is to examine the effectiveness of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) in enhancing students' calculus problem-solving performance.

The data were analyzed using SPSS (Version 21.0), employing both descriptive and inferential statistical techniques to ensure analytical accuracy and reliability. The analyses include comparisons of post-test mean scores between the two groups, followed by an independent samples *t*-test to determine the statistical significance of the observed differences. In addition, effect size (Cohen's ***d***) analysis was conducted to determine the practical significance of the intervention beyond statistical significance, thereby providing evidence of the magnitude of the instructional effect. These findings provide empirical evidence regarding the impact of WMCBPSI on learners' performance in calculus problem-solving tasks.

Recent studies (e.g., Sweller, 2023; Ashcraft & Krause, 2007; Cowan et al., 2024) highlight that instructional approaches aligned with cognitive architecture—particularly those that minimize extraneous cognitive load and strengthen working memory engagement—can significantly enhance mathematical problem-solving outcomes. Such approaches facilitate more efficient schema construction and automation, enabling learners to allocate limited working memory resources more effectively during complex calculus problem-solving tasks (Sweller, 2023; Cowan et al., 2024). Therefore, the analyses presented in this chapter not only test the study's hypotheses but also situate the findings within the broader context of contemporary cognitive and instructional theories.

The quantitative analysis in this chapter specifically addresses Research Objective 3 and Research Question 3 by determining whether students exposed to WMCBPSI demonstrate improved calculus problem-solving performance compared with students taught through Conventional Problem-Solving Instruction (CPSI). The findings are interpreted by considering both statistical evidence and the theoretical assumptions underlying WMCBPSI, particularly the role of working memory management, schema development, and contextualized learning in mathematical problem solving. Accordingly, the results presented in this chapter provide the empirical basis for evaluating the effectiveness of WMCBPSI in improving students' calculus problem-solving performance and for informing the subsequent discussion of the study's findings in Chapter 7.

6.2. RESULTS FROM STUDENTS' POST-TEST SCORES

Table 6.1 provides a summary of the percentage mean scores for both the experimental and comparison groups.

Table 6.1: Summary of Participant Students' Post-Test % Mean Scores

Group	N	Mean	Std. Deviation	Std. Error Mean
Experimental	41	59.76	16.544	2.584
Comparison	38	43.45	21.227	3.443

As shown in Table 6.1, the experimental group (M = 59.76, SD = 16.54, N = 41) outperformed the comparison group (M = 43.45, SD = 21.23, N = 38). This notable difference indicates that students who received Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) demonstrated stronger calculus problem-solving competence than those who received Conventional Problem-Solving Instruction (CPSI). An independent-samples *t*-test indicated that this difference was statistically significant, $t(77) = 3.824$, $p < .001$. These results indicate that students who received Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) demonstrated significantly stronger calculus problem-solving performance than those in the comparison group.

This finding is consistent with recent research suggesting that cognitively structured and context-based instructional interventions enhance mathematical reasoning and performance (Adeniyi & Akinsola, 2022; Ng & Mousoulides, 2023). Instructional approaches that actively engage working memory through meaningful contextualization enable learners to integrate conceptual understanding with procedural fluency, thereby improving their overall achievement levels. The difference in mean scores further suggests that the instructional approach influenced not only students' overall achievement but also their ability to organize, manage, and apply mathematical knowledge during complex calculus problem-solving tasks. Furthermore, the higher variability observed in the comparison group (SD = 21.23), compared with the experimental group (SD = 16.54), indicates greater differences in students' performance under conventional instruction, whereas the lower standard deviation in the experimental group suggests relatively more consistent performance following exposure to WMCBPSI.

Although the descriptive statistics demonstrate a clear performance advantage for the experimental group, inferential statistical analysis was necessary to determine whether the observed difference reflected a true effect of the intervention rather than random sampling variation. Therefore, an independent-samples *t*-test was conducted to evaluate the statistical significance of the difference between the two instructional groups.

6.2.1. Independent Samples t-Test Analysis of Post-Test Mean Score Differences

To determine whether the difference in post-test mean scores between the experimental and comparison groups was statistically significant, an independent samples *t*-test was conducted. The hypotheses were formulated as follows:

Null Hypothesis (H_0): WMCBPSI does not significantly improve students' calculus problem-solving performance; there is no significant difference in post-test scores between the experimental and comparison groups.

Alternative Hypothesis (H_1): WMCBPSI significantly improves students' calculus problem-solving performance, resulting in higher post-test scores in the experimental group.

The results of the independent samples *t*-test are presented in Table 6.2. The analysis revealed a statistically significant difference between the post-test mean scores of the two groups, indicating that students who received WMCBPSI achieved higher performance in calculus problem-solving compared to those who received conventional instruction.

Table 6.2: Summary of t-Test Results

		Levene's Test for Equality of Variances		t-test for Equality of Means						
t		F	Sig.		Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
Score	Equal variances assumed	4.962	.029	3.824	77	.000	16.309	4.265	7.817	24.801
	Equal variances not assumed			3.788	69.896	.000	16.309	4.305	7.723	24.895

The analysis indicates a statistically significant difference between the two groups (p = .000 < .05). Therefore, the null hypothesis is rejected, confirming that WMCBPSI had a statistically significant positive effect on students' calculus problem-solving performance.

The observed *t*-value (3.824) exceeds the critical *t*-value (1.699) at the 5% significance level, confirming that the experimental group's higher mean score is unlikely to have occurred by chance. This result demonstrates the instructional advantage of integrating working memory principles with contextualized problem-solving in calculus learning—a finding consistent with Cognitive Load Theory and research on cognitive instructional design (Paas et al., 2010; Sweller et al., 2019).

Before interpreting the *t*-test results, equality of variances was examined using Levene's test. The test indicated a significant difference in variances (p = .029). Therefore, the results were also interpreted using the unequal variances assumption. The Welch-adjusted test remained statistically significant (t = 3.788, df = 69.896, p < .001), confirming the robustness of the observed difference between the two instructional groups.

This statistical evidence indicates that the difference in post-test performance cannot be attributed solely to random sampling variation. Rather, the significantly higher achievement of the experimental group provides empirical support for the effectiveness of WMCBPSI in improving students' calculus problem-solving performance.

The quantitative findings indicate that the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) had a significant positive effect on students' calculus problem-solving performance. Students in the experimental group achieved substantially higher post-test scores than those taught through traditional instructional methods.

This result supports the study's central proposition that WMCBPSI provides a more effective pedagogical framework for developing problem-solving skills in higher mathematics. The findings further suggest that instructional approaches designed to manage cognitive load and optimize working memory resources enable students to develop stronger conceptual understanding and more effective problem-solving strategies than conventional instructional approaches. The findings align with contemporary research emphasizing the importance of cognitively efficient and working-memory-oriented instructional designs in fostering mathematical understanding and long-term retention **(**Kirschner et al., 2022**;** Cowan et al., 2024**).**

6.2.2. Analysis of Effect Size (ES) of the Intervention Instruction on Post-Test Mean Scores

To assess the practical significance of the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on students' calculus problem-solving performance, the effect size (ES) was computed. Effect size provides a standardized measure of the magnitude of difference between two groups, complementing statistical significance by indicating the educational importance of the observed effect.

Based on the data presented in Table 6.2, Cohen's *d* was calculated, yielding an effect size of 0.862. According to conventional benchmarks proposed by Cohen (1988):

- 0.2 = Small effect
- 0.5 = Moderate effect
- 0.8 or higher = Large effect

The obtained effect size value of 0.862 indicates a moderate-to-large effect, demonstrating that WMCBPSI produced a meaningful and practically significant improvement in students' calculus problem-solving skills compared to the Conventional Problem-Solving Instruction (CPSI).

Recent empirical studies corroborate this result, showing that instructional designs grounded in working memory activation and cognitive load reduction often produce medium-to-large effects on mathematics learning outcomes (Paas et al., 2010; Kirschner et al., 2022; Cowan et al., 2024). Moreover, research in STEM education suggests that effect sizes exceeding 0.80 typically represent interventions that substantially enhance both conceptual understanding and procedural fluency (Ng & Mousoulides, 2023).

The effect size finding is particularly important because statistical significance alone does not indicate whether an instructional intervention produces meaningful educational improvement. The obtained Cohen's *d* value suggests that the advantage associated with WMCBPSI was not only statistically detectable but also sufficiently large to have practical relevance in mathematics classrooms.

Therefore, the computed effect size provides evidence of the practical significance of the differences observed between WMCBPSI and CPSI, indicating that the intervention was associated with meaningful educational benefits beyond statistical significance. The effect size therefore reinforces the practical superiority of WMCBPSI by demonstrating that the observed improvement is educationally meaningful as well as statistically significant. These findings suggest that WMCBPSI has the potential to enhance calculus problem-solving performance and may serve as a promising instructional approach for supporting students' learning in similar contexts. Furthermore, the magnitude of the observed effect indicates that WMCBPSI is capable of addressing persistent learning difficulties in calculus while promoting higher-order problem-solving competence. This finding suggests that WMCBPSI can contribute to addressing challenges in calculus learning by supporting higher-order problem-solving competence, although further research across diverse educational settings is needed to establish the broader applicability of these effects.

6.3. COMPARISON OF RESULTS BY PARTICIPANTS' OVERALL PROBLEM-SOLVING SKILL LEVELS

This section presents a comparison of the overall calculus problem-solving skill levels of participants in the experimental (N = 41) and comparison (N = 38) groups. Students' performances were evaluated using the Trends in International Mathematics and Science Study (TIMSS) framework, which was adapted from Polya's problem-solving model. The assessment scale ranged from 1 (very low) to 5 (advanced), providing a systematic measure of students' cognitive and procedural proficiency in solving calculus problems.

Each participant solved nine calculus problems, resulting in 369 solution cases for the experimental group and 342 cases for the comparison group. Table 6.3 summarizes the distribution of problem-solving skill levels across both groups. Table 6.3 summarizes the distribution of problem-solving skill levels across both groups.

Table 6.3: Summary of Calculus Problem-solving Skill Levels

Group	Statistics	TIMSS scale levels					
		5	4	3	2	1	Total
Experimental = 369	Frequency	44	138	90	59	38	369
	Percent	11.9	37.4	24.4	16.0	10.3	100.0
	Cumulative Percent	11.9	49.3	73.7	89.7	100.0	
Comparison = 342	Frequency	17	80	75	88	82	342
	Percent	5.0	23.4	21.9	25.7	24.0	100.0
	Cumulative Percent	5.0	28.4	50.3	76.0	100.0	

=Number of solution cases for students in experimental group; *=Number of solution cases for students in experimental group*

Key Findings

1. Higher Skill Levels in the Experimental Group

The experimental group demonstrated a greater proportion of students performing at the higher TIMSS levels (3, 4, and 5) than the comparison group. This distribution indicates that WMCBPSI effectively enhanced students' calculus problem-solving skills and conceptual understanding. Specifically, students in the experimental group demonstrated greater ability to interpret mathematical problems, select appropriate solution strategies, apply procedures accurately, and evaluate the correctness of their solutions, reflecting the characteristics of proficient problem solvers described in Polya's problem-solving framework.

2. Proportion of Higher-Level Solutions

- Approximately 73.7% (272/369) of the experimental group's solutions were rated as intermediate (Level 3) or above, compared with 50.3% (172/342) in the comparison group.
- Nearly 90% (331/369) of the experimental group's responses reached at least Level 2, whereas 76% (260/342) of the comparison group's responses achieved this level.

3. Lower Skill Levels in the Comparison Group

The comparison group showed a larger proportion of students performing at the lower levels (Levels 1 and 2) than the experimental group. This pattern suggests that conventional instruction was less effective in promoting advanced mathematical reasoning and higher-order problem-solving processes.

The distribution of responses further indicates that the influence of WMCBPSI extended beyond improving students' final achievement scores. Rather, the intervention appeared to enhance the quality of students' mathematical reasoning by supporting more organized and systematic solution processes. Through instructional features such as contextualized learning activities, worked examples, chunking of complex information, and self-explanation, students were better able to organize mathematical knowledge and apply appropriate solution strategies to calculus problems.

The findings therefore indicate that WMCBPSI improved not only students' overall performance but also the quality of their problem-solving processes, including analytical reasoning, strategy selection, procedural accuracy, and solution verification. These characteristics are consistent with the progression of problem-solving competence described in Polya's framework and support previous findings that cognitively structured and context-based instructional approaches enhance mathematical reasoning and performance (Adeniyi & Akinsola, 2022).

Summary of the Findings

The analysis demonstrated that students in the experimental group attained significantly higher calculus problem-solving skill levels than those in the comparison group, as evidenced by the greater proportion of responses classified at the higher TIMSS performance levels. These findings suggest that WMCBPSI facilitated greater mastery of problem-solving strategies and deeper conceptual understanding than Conventional Problem-Solving Instruction (CPSI). Consequently, the results provide further empirical support for the effectiveness of WMCBPSI as an instructional approach for enhancing university students' calculus problem-solving competence and addressing persistent challenges in mathematics learning.

6.4. COMPARISON OF GROUPS BY PARTICIPANTS' PROBLEM-SOLVING STRATEGIES

This section compares the types and quality of problem-solving strategies employed by participants in the experimental group (N = 41) and the comparison group (N = 38) when solving nine calculus problems in the post-intervention achievement test. The strategies were categorized into 15 types, based on a framework adapted from established problem-solving models (see Appendix E). These categories reflect the cognitive, metacognitive, and procedural dimensions of mathematical problem solving.

The analysis of solution strategies was undertaken to provide deeper evidence regarding how students approached calculus problems rather than relying solely on final achievement scores. While the post-test achievement results indicate whether students performed better, the analysis of solution strategies provides insight into the cognitive processes, reasoning patterns, and strategic decisions that underpinned students' performance during problem solving.

6.4.1. Frequency of Strategies Used

Table 6.4 presents the frequency and percentage of each strategy used by participants in both groups.

Table 6.4: Summary of Solution Strategies Used by Participants in Both Groups

Group	Statistics	Solution strategy categories															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Experimental	Frequency	16	64	-	17	-	72	13	22	45	12	22	24	19	43	-	369
= 369	Percent	4.34	17.34	-	4.61	-	19.51	3.52	5.96	12.20	3.25	5.96	6.5	5.15	11.65	-	100
Comparison	Frequency	38	97	-	17	-	61	2	19	22	9	14	27	16	20	-	342

= 342	Percent	11.11	28.36	-	4.97	-	17.84	0.59	5.56	6.43	2.63	4.09	7.90	4.68	5.85	-	100
Total	Frequency	54	161	-	34	-	133	15	41	67	21	36	51	35	63	-	711
	Percent	7.60	22.64	-	47.82	-	18.71	2.11	5.77	9.42	2.95	5.06	7.17	4.92	8.86	-	100

=Number of solution cases for students in experimental group; *=Number of solution cases for students in experimental group*

Key Observations

- Frequent Strategy Use: Across both groups, the most commonly employed strategies were Logical Reasoning (19.51%), Algebraic Method (17.34%), and Draw a Picture (12.20%).
- Non-use of Certain Strategies: Strategies such as Model Drawing, Guess-and-Check, and Mathematical Induction were not utilized by any participant.
- Group Differences: The comparison group relied more heavily on algorithmic approaches (e.g., Algebraic Method = 28.36%), whereas the experimental group demonstrated a more balanced use of reasoning- and visualization-based strategies.

The observed differences in strategy use suggest that WMCBPSI encouraged students to move beyond routine procedural methods toward more flexible and conceptually oriented approaches to calculus problem solving. Rather than depending predominantly on algorithmic procedures, students in the experimental group employed a wider range of reasoning and visualization strategies, reflecting greater strategic flexibility and deeper conceptual engagement with mathematical tasks.

These findings indicate that WMCBPSI encouraged participants to employ conceptual and integrative strategies, supporting previous research showing that cognitively scaffolded instruction fosters flexible thinking and adaptive expertise in mathematics problem-solving (Cowan et al., 2024; Ng & Mousoulides, 2023). The findings further suggest that the instructional emphasis on contextualized learning and working memory support enabled students to select strategies that were more appropriate for the demands of individual calculus problems, thereby enhancing the quality of their problem-solving processes.

6.4.2. Appropriateness of Selected Strategies

Table 6.5 presents the number and percentage of strategies that were appropriately selected in solving the assigned problems. A strategy was considered appropriate when it effectively connected the known information to the problem's target goal.

Table 6.5: Summary of Solution Strategies Selected Appropriately by Participants in Both Groups

Group	Statistics	Problem-solving strategy categories														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Experimental	Frequency	-	17	-	12	-	68	12	13	45	12	11	24	17	41	-	272
= 369	Percent	-	6.25	-	4.41	-	25	4.41	4.78	16.54	4.41	4.04	8.82	6.25	15.07	-	100
Comparison	Frequency	-	13	-	7	-	53	1	5	22	9	5	27	13	17	-	172
= 342	Percent	-	7.56	-	4.07	-	30.81	0.58	2.91	12.78	5.23	2.91	15.69	7.56	9.88	-	100
Total	Frequency	-	30	-	19	-	121	13	18	67	21	16	51	30	58	-	444
	Percent	-	6.75	-	4.28	-	27.23	2.93	4.05	15.08	4.73	3.6	11.48	6.75	13.05	-	100

=Number of solution cases for students in experimental group; =Number of solution cases for students in experimental group

Key Observations

- Appropriate Strategy Selection Across Instructional Groups: Approximately 73.71% of solution cases from the experimental group involved appropriate strategy selection, whereas 50.29% of solution cases from the comparison group demonstrated this characteristic. This difference suggests that students exposed to WMCBPSI used appropriate problem-solving strategies more frequently within the context of this study
- Top Effective Strategies: Logical Reasoning, Draw a Picture, and Establishing Sub-goals were the strategies most frequently and appropriately employed.

These results indicate that WMCBPSI enabled students to identify and select the most effective strategies to connect given problem conditions to desired outcomes—an ability closely associated with metacognitive monitoring and schema automation (Schoenfeld, 1985; Sweller et al., 2019).

The difference between the two instructional groups demonstrates that WMCBPSI positively influenced one of the most important stages of mathematical problem solving—strategic planning. Students exposed to the intervention were not only more successful in solving calculus problems but also demonstrated greater ability to determine which solution strategies were most appropriate for the problem at hand.

This finding is consistent with the principles of Cognitive Load Theory, which propose that reducing unnecessary cognitive demands enables learners to allocate more working memory resources to meaningful reasoning, strategy selection, and schema construction (Sweller et al., 2019). Consequently, students receiving WMCBPSI were better able to select strategies that effectively linked the given information to the desired solution, thereby improving both the efficiency and quality of their problem-solving processes.

6.4.3. Successful Execution of Problem-Solving Strategies

A strategy was considered successfully executed when the participant demonstrated correct logical reasoning and completed a step-by-step solution that led to an accurate answer. Table 6.6 summarizes the frequency of successfully executed strategies for both groups.

Table 6.6: Summary of Solution Strategies Executed Successfully by Participants in Both Groups

Group	Statistics	Problem-solving strategy categories															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Experimental	Frequency	-	9	-	3	-	49	8	7	31	8	5	21	8	33	-	182
= 369	Percent	-	4.94	-	1.65		26.90	4.39	3.84	17.02	4.39	2.75	11.53	4.39	18.12	-	100
Comparison	Frequency	-	5	-	2	-	33	-	1	10	3	2	22	5	14	-	97
= 342	Percent	-	5.16	-	2.06	-	34.02	-	1.03	10.31	3.09	2.06	22.68	5.16	14.43	-	100
Total	Frequency	-	14	-	5	-	82	8	8	41	11	7	43	13	47	-	279
	Percent	-	5.01	-	1.79	-	29.36	2.86	2.86	14.68	3.94	2.51	15.39	4.65	16.83	-	100

=Number of solution cases for students in experimental group; *=Number of solution cases for students in experimental group*

Key Observations

- Higher Success Rate in the Experimental Group: Approximately 49.32% (182/369) of the experimental group's solutions were successfully executed, compared with 28.36% (97/342) in the comparison group.
- Lower Failure Rate: The comparison group exhibited a higher failure rate (71.64%) compared with the experimental group (50.68%).
- Ineffective Strategies: All participants in the comparison group who employed indirect strategies failed to complete them successfully.

These results suggest that WMCBPSI enhanced not only students' strategy selection but also their execution accuracy. This improvement can be attributed to the instructional model's alignment with working memory theory, which facilitates chunking and efficient information retrieval during multistep reasoning (Sweller, 2023; Cowan et al., 2024).

The difference between the two instructional groups indicates that WMCBPSI supported students throughout the entire problem-solving process, from understanding the problem and selecting an appropriate strategy to accurately executing mathematical procedures. The higher rate of successful strategy execution suggests that students developed more stable and well-organized problem schemas, enabling them to coordinate multiple solution steps without exceeding the limitations of working memory.

In contrast, the comparison group's greater reliance on routine procedural approaches may have limited students' ability to adapt their strategies when confronted with unfamiliar or complex calculus problems. This pattern reinforces the view that successful mathematical problem solving requires not only procedural knowledge but also the ability to organize, monitor, and flexibly apply mathematical knowledge during the solution process.

Therefore, the analysis of strategy execution provides additional evidence that the effectiveness of WMCBPSI extended beyond improving students' achievement scores. The intervention also enhanced the cognitive processes underlying successful mathematical problem solving by strengthening students' ability to execute appropriate solution strategies accurately and consistently.

Overall, the comparative analysis revealed that students in the experimental group employed a wider range of cognitive and heuristic strategies than those in the comparison group, including greater use of visualization, logical reasoning, and schema-based approaches. These findings are consistent with previous studies demonstrating that working memory-supported instruction improves students' adaptability in selecting appropriate solution paths and enhances the efficiency of mathematical reasoning (Kang et al., 2022; Mensah & Boateng, 2023). The observed diversity of strategies indicates that WMCBPSI supports flexible and effective problem-solving behaviours while balancing cognitive load and contextual engagement during calculus learning (Chen et al., 2023; Sweller et al., 2023; Tadesse & Kriek, 2025).

6.5. DISCUSSION AND INTERPRETATION

The comparative data indicate that participants in the experimental group, who received WMCBPSI, demonstrated superior ability to select, apply, and execute appropriate problem-solving strategies. This suggests that the intervention fostered both strategic flexibility and procedural fluency, which are hallmarks of expert problem solvers.

These findings are consistent with contemporary research showing that context-based and cognitively aligned instruction supports the effective transfer of problem-solving strategies across different learning contexts (Kirschner et al., 2022; Hmelo-Silver et al., 2007). By scaffolding cognitive load and supporting the effective use of working memory resources, WMCBPSI enabled students to process complex calculus problems more efficiently and to develop more organized and durable problem-solving schemas.

Students in the experimental group, who received Working Memory–Context-Based Problem-Solving Instruction, demonstrated higher frequencies of appropriate strategy selection and successful strategy execution compared with students in the comparison group. These findings complement the statistical results and suggest that WMCBPSI contributed not only to improved achievement outcomes but also to enhanced strategic competence in calculus problem-solving. The findings therefore reinforce the quantitative results by demonstrating that the intervention improved both students' overall performance and the quality of the cognitive processes underlying successful problem solving.

The quantitative findings also provide support for the theoretical foundation of WMCBPSI. Consistent with Cognitive Load Theory, the improved performance of the experimental group suggests that instructional features such as chunking, worked examples, contextualization, collaborative learning, and self-explanation reduced unnecessary cognitive demands while facilitating schema construction and the efficient use of working memory resources during multistep calculus problem solving.

Furthermore, the results indicate that WMCBPSI promoted deeper mathematical understanding rather than merely improving procedural accuracy. The greater use of appropriate and successfully executed strategies suggests that students developed stronger problem representation, greater strategic awareness, and increased flexibility in applying mathematical knowledge to a variety of calculus problem situations.

These findings are particularly important in university calculus education, where students frequently encounter difficulties because of the abstract nature of mathematical concepts and the substantial cognitive demands associated with multistep problem solving. The evidence from this study suggests that instructional approaches grounded in cognitive principles provide a more effective alternative to predominantly procedural approaches to calculus instruction.

Overall, the evidence indicates that WMCBPSI provides a theoretically supported instructional framework for addressing some of the cognitive challenges associated with learning advanced mathematics. The approach appears to promote more effective problem-solving processes by integrating working memory principles, contextual learning, and strategic reasoning. The evidence therefore supports WMCBPSI as an empirically grounded instructional framework for enhancing calculus problem-solving competence in higher education.

However, the findings should be interpreted within the context of the study design. Because the study employed a quasi-experimental static-group comparison design within a single institutional setting, causal inferences should be made with appropriate caution. Nevertheless, the use of comparable participant groups, separate implementation periods, multiple measures of problem-solving performance, and triangulation with qualitative evidence strengthens confidence in the observed instructional effects.

6.6. SUMMARY AND CONCLUSION OF THE CHAPTER

This chapter presented and analyzed the quantitative data from the post-test results following the instructional intervention. Statistical analyses evaluated the effectiveness of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) in enhancing students' calculus problem-solving skills, addressing Research Objective 3 and Research Question 3 (Sections 1.2 and 1.8).

The findings revealed that students in the experimental group, who received WMCBPSI, demonstrated significantly greater improvement compared with the comparison group. Results from the independent samples t-test (Section 6.2.1) indicated these differences were statistically significant ($p < .05$), confirming the positive impact of the intervention. The effect size analysis (Section 6.2.2) further established its practical significance, showing a moderate-to-large positive effect on students' outcomes.

The comparative analysis using the TIMSS-based framework (Section 6.3) confirmed that WMCBPSI substantially enhanced students' abilities across all skill categories, promoting both conceptual understanding and procedural fluency. Investigation of students' problem-solving strategies (Section 6.4) showed that experimental group participants not only selected strategies more appropriately but also executed them more successfully, indicating development of strategic competence and improved metacognitive monitoring.

Taken together, the quantitative evidence demonstrates that WMCBPSI influenced both the outcomes and the processes of calculus problem solving. Students exposed to the intervention achieved higher post-test scores, demonstrated stronger problem-solving skill levels, and employed more effective solution strategies than students receiving Conventional Problem-Solving Instruction (CPSI).

The findings therefore provide empirical support for the central assumption of this study that integrating working memory principles with contextualized problem-solving instruction enhances mathematical learning by improving cognitive processing, facilitating schema development, and strengthening the strategic application of mathematical knowledge during calculus problem solving.

In summary, the quantitative results demonstrate that WMCBPSI is both statistically significant and educationally meaningful, enhancing performance, strategic competence, and metacognitive skills. The next chapter builds on these findings by presenting qualitative data from interviews and classroom observations, providing deeper insight into how WMCBPSI influences students' learning experiences and cognitive engagement.

CHAPTER SEVEN

RESULTS AND DATA ANALYSIS: QUALITATIVE DATA

7.1. INTRODUCTION

Building upon the methodological framework and data collection procedures detailed in Chapter 5, and supplementing the quantitative outcomes established in Chapter 6, this chapter presents the qualitative findings obtained from classroom observations and semi-structured interviews, conducted to evaluate the effects of the Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) intervention. As described in earlier chapters, the study employed a static group comparison design with a post-test-only approach to assess the effectiveness of the intervention. In addition to quantitative analyses, qualitative data were collected to gain a deeper understanding of how WMCBPSI influenced students‘ problem-solving behaviors, engagement, and skills.

Burnard (2004) emphasizes that exclusive reliance on statistical analysis may overlook critical aspects of program implementation and instructional processes. Therefore, qualitative evidence provides a richer, more contextualized understanding of students‘ experiences, enabling in-depth examination of both instructional and learner-related factors. This chapter specifically addresses Research Questions 2 and 4 and Research Objectives 2 and 4 (see Sections 1.8 and 1.2).

The qualitative analysis followed a structured framework adapted from Burnard (1991), comprising systematic steps designed to ensure a thorough, transparent, and organized examination of the data. This framework enhanced the reliability and validity of the interpretations.

7.2. RESULTS FROM CLASSROOM OBSERVATIONS AND INTERVIEWS

Classroom observations were conducted for both the experimental group, receiving WMCBPSI, and the comparison group, receiving conventional problem-solving instruction (CPSI). Observations were structured and scheduled (see Appendix B) and took place during lessons in which both groups engaged with calculus problems. The primary purpose was to examine students' approaches to problem-solving tasks, their interactions with peers and instructors, and the overall learning environment.

The observations also focused on students' participation, use of problem-solving strategies, collaboration, persistence during problem-solving activities, and responses to instructional support. These observations provided direct evidence of how the instructional approaches were implemented and how students engaged with the learning process.

Observation notes were recorded immediately after each session, focusing on behaviors related to problem-solving strategies, peer collaboration, and instructional dynamics. Video recordings were reviewed where necessary to verify classroom events and minimize the possibility of overlooking important verbal and non-verbal interactions. These notes were subsequently cross-referenced with interview data to enhance the validity of the findings.

Semi-structured interviews were conducted with selected students from both groups to capture personal perspectives on the instructional methods and their perceived impact on problem-solving abilities. Participants were purposively selected to represent a range of post-test achievement levels and classroom participation, allowing diverse learning experiences and perspectives to be explored.

The interview questions were aligned with the research objectives and explored themes such as the perceived effectiveness of WMCBPSI, challenges encountered, and changes in problem-solving skills. The interview questions also examined students' problem-solving strategies, classroom engagement, perceptions of collaborative learning, and suggestions for improving calculus instruction.

Interview transcripts were systematically coded and categorized to identify recurring themes, ensuring alignment with the study's aims. Interview recordings were transcribed verbatim before analysis, and codes were developed from recurring patterns across interview transcripts and classroom observations. These codes were then organised into broader themes corresponding to the study's research objectives and research questions.

The integration of classroom observations, interview responses, and video evidence enabled convergence of findings across multiple qualitative data sources, thereby enhancing the credibility, confirmability, and depth of the interpretations presented in this chapter.

Table 7.1 summarizes the steps followed in the qualitative data analysis, illustrating the systematic process from initial data collection to final thematic interpretation.

Table 7.1: Steps followed to analyze qualitative data

Step	Method of Analysis	Explanation/Example of Application in the Current Study
1	Initial review of classroom observations, semi-structured interviews, and recorded interview sessions to extract relevant notes.	For example, a student‘s statement such as, ―The way of instruction you used is better than the methods I knew before because we also communicate as students,‖ was linked to Objective 4. Similarly, the remark ―This is the best method to teach math‖ was associated with Objective 5.
2	Transcription of interviews, followed by initial reading and identification of themes.	General themes were identified using keywords directly aligned with the study‘s aim, objectives, and research questions.
3	Further review of transcripts and categorization of data under identified themes.	During analysis, themes were grouped into categories corresponding to specific objectives and research questions.
4	Refinement of categories by examining similarities and merging overlapping categories.	Similar categories were merged into broader headings, reducing redundancy while maintaining the integrity of the data.
5	Finalization of categories and subcategories by removing repetitions and inconsistencies.	A definitive list of categories and subcategories was produced, ensuring clarity and alignment with the study's framework.
6	Verification of categories against the interview data to ensure comprehensive coverage.	The finalized list was reviewed alongside the interview transcripts to confirm alignment with all aspects of the collected data.

7	Coding of transcripts based on the finalized categories and subcategories.	Each transcript was analyzed and coded systematically using the agreed-upon list of categories. Results were verified in collaboration with colleagues.
8	Extraction of coded sections from transcripts and consolidation under relevant categories.	Initially identified themes were reassigned to the refined categories where appropriate, ensuring coherence.
9	Organization of data by pasting coded excerpts under relevant categories and subcategories.	Newly established categories were cross-verified and used as a basis for organizing data.
10	Validation of categorized data through respondent feedback.	Selected participants were asked to validate the categorization of their responses, using questions such as, ―Does this quotation from your interview accurately reflect this category?

Adapted from Burnard (1991)

The following sections provide a detailed account of the findings from classroom observations and interviews, highlighting key themes related to the impact of WMCBPSI on students' problem-solving strategies, engagement, and overall learning experience.

7.2.1 Analysis of Student Observations

Classroom observations offered valuable insights into how students in both the experimental and comparison groups approached calculus problem-solving and interacted with their respective instructional methods. The observations provided direct evidence of students' participation, interactions, use of problem-solving strategies, and responses to the instructional approaches throughout the intervention period. Together with the interview data, these observations helped explain the quantitative differences in problem-solving performance reported in Chapter 6.

7.2.1.1 Preparation and Flexibility in Problem Solving

Experimental Group

Students in the experimental group demonstrated greater preparation and flexibility when addressing both familiar and novel problems. They adapted more readily by applying a variety of problem-solving strategies. For example, ES1 noted:

"You taught us using varied examples and different strategies… giving us many exercises to work on independently… now I know how to approach calculus problems."

This response reflects the students' ability to generalize and transfer learned strategies beyond the specific examples provided. Classroom observations confirmed this perception. Students increasingly attempted unfamiliar problems independently, discussed alternative solution methods with peers, and justified their reasoning before arriving at final solutions. These behaviours indicate the development of flexible problem-solving schemas, which are consistent with the objectives of WMCBPSI and the principles of Cognitive Load Theory emphasizing schema construction and transfer of learning.

Comparison Group

Students in the comparison group primarily imitated the researcher's solutions rather than developing independent approaches. CS3 stated:

"I found it difficult when I try to solve other problems."

This suggests a lack of adaptability due to limited exposure to varied problem-solving strategies. Observation records similarly showed that many students hesitated when confronted with unfamiliar calculus problems and frequently waited for teacher guidance before attempting solutions. Their dependence on previously demonstrated examples suggests limited development of flexible problem-solving strategies compared with students in the experimental group.

7.2.1.2 Mode of Instruction and Student Engagement

Experimental Group

Instruction in the experimental group emphasized varied examples and guided practice, fostering active participation. Students frequently collaborated, engaged in group discussions, and sought peer support. Some students initially found group work challenging. ES2 explained:

"It was difficult for us to engage in group discussions… but your probing questions guided and encouraged interaction during problem-solving activities."

This indicates that the instructional approach supported gradual development of collaborative skills and active problem-solving. The WMCBPSI approach also incorporated structured scaffolding, collaborative learning, and guided questioning, which encouraged students to explain their reasoning and compare alternative solution methods. Throughout the intervention, classroom observations showed increasing confidence, higher participation levels, and more frequent mathematical discussions among students. Students became progressively more willing to explain their ideas, evaluate peers' solutions, and provide support during problem-solving activities, indicating deeper engagement with the learning process.

Comparison Group

Interaction and engagement were less evident in the comparison group. Instruction was largely teacher-centered, with students working individually. CS1 commented:

"I have worked on my own all the time and get my answers. If I have problems, I go to you."

This reflects a predominantly passive learning environment with minimal peer interaction. Classroom observations further revealed that opportunities for collaborative learning were limited, resulting in fewer student discussions and reduced active participation. Most classroom communication occurred between the teacher and individual students rather than among students themselves, limiting opportunities to develop collaborative problem-solving skills.

Analysis revealed that WMCBPSI students demonstrated higher active engagement, collaborative learning, and persistence, consistent with studies showing that context-based instruction enhances cognitive engagement and metacognitive regulation (Kang et al., 2022; Zewdu et al., 2024). These findings suggest that the instructional approach not only improved students' participation but also supported the development of self-regulation, strategic awareness, and more effective problem-solving behaviours.

7.2.1.3 Cognitive Load and Instructional Adjustments

7.2.1.3.1 Experimental Group

Instruction in the experimental group was designed to manage cognitive load in line with Cognitive Load Theory (Sweller et al., 2011). Strategies included reducing split-attention effects and encouraging self-explanation, thereby facilitating the development of problem-solving schemas. Additional instructional strategies included the use of worked examples, guided practice, and organising information into manageable chunks to support efficient processing and reduce unnecessary cognitive demands during complex calculus problem solving.

ES1 noted:

"The split-attention effect helps me manage information better,"

while ES3 highlighted the utility of the:

"chunking effect"

for organizing complex information.

These observations indicate that students were not only applying mathematical procedures but were also becoming aware of instructional strategies that supported their learning. As the intervention progressed, students demonstrated increased independence when solving complex calculus problems and required less teacher support, suggesting improved management of working memory resources and stronger development of problem-solving schemas.

7.2.1.3.2 Comparison Group

Instruction in the comparison group did not explicitly address cognitive load. Problems were presented without strategies to manage intrinsic or extraneous load, limiting students' capacity to process and transfer problem-solving skills. CS3 remarked:

"I found it difficult when I try to solve other problems."

Observation records further showed that students frequently experienced difficulties when solving multistep calculus problems independently and often relied on teacher guidance when encountering unfamiliar procedures. This pattern suggests that conventional instruction provided fewer opportunities for developing efficient problem-solving schemas compared with WMCBPSI.

These observations suggest that the experimental group benefited from varied instruction and active engagement, supported by cognitive load considerations, which enhanced their adaptability and conceptual understanding in calculus problem solving. The combination of collaborative learning, structured guidance, and explicit cognitive load management strategies appeared to strengthen students' engagement, flexibility, and strategic reasoning. In contrast, the comparison group's passive and less cognitively attuned approach resulted in reduced adaptability and lower engagement.

Overall, the classroom observations provide qualitative evidence supporting the quantitative findings presented in Chapter 6, demonstrating that WMCBPSI promoted more active engagement, greater strategic flexibility, and stronger problem-solving behaviours than Conventional Problem-Solving Instruction.

7.2.2 Analysis of Student Interviews

Semi-structured interviews with students from both groups provided rich insights into their experiences with problem-solving instruction. The interview questions were designed to align with the research objectives and to explore students' perspectives on how the instructional methods influenced their calculus problem-solving abilities.

The interviews complemented classroom observations by allowing students to explain their experiences, perceptions, confidence levels, engagement, and use of problem-solving strategies. Interview data were analysed thematically using the procedures described in Chapter 5, and recurring patterns were compared with observation findings to strengthen the credibility of the qualitative interpretations.

7.2.2.3 Need for Problem-Solving in Calculus

Students in both groups strongly agreed that problem-solving should be an integral part of calculus instruction. They emphasized that problem-solving skills are essential for fostering deeper conceptual understanding and developing critical thinking abilities.

Although students from both groups recognised the importance of problem solving, students in the experimental group more frequently described problem solving as a process involving understanding concepts, selecting appropriate strategies, and explaining reasoning. In contrast, students in the comparison group tended to emphasise obtaining correct answers and following demonstrated procedures. This difference suggests that WMCBPSI encouraged a broader understanding of mathematical problem solving beyond procedural competence.

7.2.2.4 Perceived Benefits of Problem-Solving Instruction

7.2.2.4.1 Experimental Group

Students recognised clear benefits of WMCBPSI. ES3 explained that learning through texts and graphs simplified problem-solving and facilitated strategy sharing. ES1 noted:

"The course was delivered in a more organized manner, which strengthened connections between concepts and improved my skills."

These responses indicate that students perceived WMCBPSI as supporting conceptual understanding, effective organisation of mathematical knowledge, and development of appropriate problem-solving strategies. Students also reported that contextualised examples, structured guidance, and collaborative activities helped them apply learned strategies to unfamiliar calculus problems.

7.2.2.4.2 Comparison Group

Students in the comparison group appreciated the importance of problem-solving but expressed less confidence in applying strategies. Their responses reflected the limitations of passive, lecture-based instruction, which restricted active engagement and opportunities for practice.

Several students indicated dependence on teacher demonstrations and difficulty transferring learned procedures to new problems. These findings reinforce classroom observation evidence showing limited flexibility in strategy use and greater reliance on teacher guidance.

7.2.2.5 Impact on Future Classes

Students in the experimental group believed that the WMCBPSI method could be effectively applied to other subjects. They appreciated its flexibility and emphasis on diverse problem types. ES1 noted:

“The method encouraged approaching problems from multiple perspectives, deepening conceptual understanding.”

In contrast, students in the comparison group were less optimistic, perceiving the rigid, example-based instruction as less transferable across contexts.

These responses indicate that students viewed WMCBPSI not only as a method for improving calculus performance but also as an approach that promotes transfer of problem-solving skills across different mathematical topics and learning contexts.

7.2.2.6 Challenges in Implementing the Instruction

7.2.2.6.1 Experimental Group

Students acknowledged initial difficulties with group work, which was a new learning format. ES2 explained:

“It was challenging at first, but the instructor’s guiding questions encouraged collaboration and improved strategies.”

However, students reported that these difficulties decreased over time as they became more comfortable discussing mathematical ideas, explaining their reasoning, and learning from peers. This suggests that collaborative learning skills developed progressively throughout the intervention.

7.2.2.6.2 Comparison Group

The primary challenge was a lack of interaction with peers and the instructor. CS3 expressed frustration:

“Solving problems independently was difficult when only a few examples were given.”

These responses suggest that students required more opportunities for guided practice, peer interaction, and exposure to diverse problem-solving approaches than were provided under the conventional instructional method.

7.2.2.7 Student Opinions on Instructional Effectiveness

Students in the experimental group generally expressed positive views, highlighting that WMCBPSI helped them form meaningful connections between concepts and increased their confidence in problem-solving. ES1 observed:

“Identifying connections made problem-solving easier.”

By contrast, students in the comparison group reported that while they could complete exercises, they lacked deeper, flexible understanding.

The findings suggest that WMCBPSI supported both conceptual understanding and strategic flexibility, whereas CPSI primarily supported procedural completion of familiar exercises. These results complement the quantitative evidence showing higher problem-solving performance among students in the experimental group. WMCBPSI students reported greater confidence, clearer conceptual understanding, and enhanced strategic flexibility, supporting evidence that working memory-informed instruction improves both cognitive and affective learning outcomes (Mensah & Boateng, 2023; Paek & Kim, 2024).

7.2.2.8 Advice for Improving Problem-Solving Instruction

Experimental group students recommended incorporating more interactive strategies and varied examples. ES3 suggested including multiple problem types while explaining the reasoning behind strategies.

Some comparison group students preferred more independent work, reflecting their comfort with traditional methods.

Students from both groups also recommended increased opportunities for practice, discussion, and exposure to different problem types. These recommendations closely reflect the major features of WMCBPSI, including active learning, contextual examples, collaborative problem solving, and structured instructional support.

Overall, the interviews indicate that students in the experimental group benefitted substantially from the WMCBPSI approach. Its interactive and flexible design promoted engagement, deeper conceptual understanding, and the development of problem-solving skills. In contrast, the comparison group, constrained by a passive, example-driven approach, demonstrated lower adaptability and reduced confidence in problem-solving.

The responses from the experimental group revealed a highly favorable reception to WMCBPSI, with several consistent themes indicating that students regarded the approach as beneficial and effective in enhancing their calculus problem-solving abilities. Together with classroom observations, these interview findings provide qualitative confirmation of the quantitative results presented in Chapter 6 and demonstrate that WMCBPSI influenced not only students' achievement outcomes but also their learning processes, engagement, and strategic competence.

7.2.3 Additional Insights from Student Interviews

The interview data provided additional insights into students' experiences with WMCBPSI beyond the themes presented in the preceding sections. Several recurring patterns emerged regarding students' engagement, motivation, confidence, collaborative learning, and overall satisfaction with the instructional approach. These themes consistently supported the classroom observation findings and further explained the quantitative improvements reported in Chapter 6.

7.2.3.1 Greater Engagement and Motivation

Many students reported that WMCBPSI significantly enhanced their engagement with calculus and increased their motivation. ES3 stated:

"This method has motivated me to love mathematics,"

while ES7 observed noticeable improvement:

"I always fail math, but this time I did better."

These reflections suggest that WMCBPSI not only increased students' interest in calculus but also had a positive influence on academic performance.

Students attributed this increased motivation to the use of contextualised examples, collaborative activities, and opportunities to solve problems independently. Increased motivation was also reflected in classroom observations, where students demonstrated higher participation, persistence, and willingness to attempt challenging problems.

7.2.3.2 Value of Multiple Examples and Strategies

Students emphasized the importance of being exposed to a variety of examples and solution strategies. ES1 noted that working through diverse examples with step-by-step explanations promoted greater independence in problem solving. ES4 remarked:

"The example problems and their solution strategies were interesting, and I have improved my problem-solving skills."

This feedback indicates that varied examples encouraged deeper understanding and skill acquisition.

These responses indicate that presenting multiple solution methods enabled students to compare strategies, recognise relationships among concepts, and develop greater flexibility when approaching unfamiliar calculus problems. Such experiences are consistent with the principles of

WMCBPSI, which encourage schema development through varied and meaningful learning experiences.

7.2.3.3 Preference for Collaborative and Interactive Learning

Although group work initially presented challenges, it was ultimately valued as an effective learning strategy. ES2 explained:

“I have passed my test because of this way... I really favour it.”

ES6 further highlighted that the interactive approach encouraged consistent attendance and participation, enhancing the overall learning experience.

The interview data suggest that collaborative problem-solving activities promoted not only academic learning but also increased students’ confidence in discussing mathematical ideas, explaining their reasoning, and learning from peers. Classroom observations similarly showed that students gradually became more comfortable participating in group discussions and sharing alternative solution methods.

7.2.3.4 Overall Satisfaction with the Instructional Approach

Most students expressed strong approval of WMCBPSI. ES5 referred to it as:

“The best method to teach maths,”

while ES9 appreciated the structured course presentation, which made calculus concepts more comprehensible.

Participants associated their improved understanding with the structured sequence of instruction, guided practice, and opportunities to apply different problem-solving strategies during classroom activities. These features appeared to support stronger connections between mathematical concepts and solution procedures.

7.2.3.5 Peer Endorsement and Collective Approval

Several students reported that their peers also valued the approach. ES3 noted:

"My friends liked the way you teach calculus. They say it will help us pass mathematics,"

and ES6 added:

"I think all of the students have liked this method of teaching."

Although these comments represent students' perceptions rather than direct evidence from all participants, they suggest that the instructional approach was generally well received within the experimental group and contributed to a positive classroom learning environment.

7.2.3.6 Challenges and Individual Variation

Although most feedback was positive, ES8 indicated that the method did not translate into improved results for them personally, attributing this to insufficient effort. However, they acknowledged that peers benefited, suggesting that the method was effective for most students.

This variation illustrates that instructional effectiveness is also influenced by learner characteristics such as effort, participation, and engagement. Therefore, while WMCBPSI provided supportive learning conditions, individual differences continued to influence students' outcomes.

The feedback from the experimental group suggests that WMCBPSI significantly enhanced engagement, strengthened problem-solving skills, and improved overall calculus performance. Its interactive and flexible design, incorporation of varied examples, and structured guidance contributed to positive student experiences. Nearly all participants recommended wider use of the method, highlighting its potential as an effective instructional approach (Sweller et al., 2011).

However, these qualitative findings should be interpreted together with the quantitative results rather than independently. The interview evidence complements the statistical findings by explaining how WMCBPSI may have contributed to improved problem-solving performance through increased engagement, strategic flexibility, and more effective management of cognitive demands.

7.2.4 Comparison Group Responses

The responses from the comparison group highlighted a notable contrast with the experimental group, revealing dissatisfaction with the traditional, teacher-centred approach to problem-solving instruction. Several recurring themes emerged, reflecting unmet needs and instructional limitations.

The interview responses from the comparison group contrasted markedly with those of the experimental group. Students generally identified limitations associated with the predominantly teacher-centred instructional approach, particularly regarding opportunities for active participation, independent problem solving, peer interaction, and exposure to varied problem-solving strategies. These findings complemented the classroom observation results, which showed lower levels of interaction and greater dependence on teacher guidance among comparison group students.

7.2.4.1 Lack of Engagement and Autonomy

Many students expressed frustration with the limited opportunities for active participation. CS9 stated:

“You did not give us a chance to do on our own... you had dominated the class,”

suggesting a preference for a more student-driven approach. CS2 echoed this sentiment:

“I prefer if you were in a position to give us a chance to do on our own.”

These comments suggest that students desired greater responsibility for constructing their own understanding rather than relying primarily on teacher explanations. The limited opportunities for independent exploration may have restricted the development of self-regulated problem-solving behaviours.

7.2.4.2 Insufficient Problem-Solving Practice

Several students emphasized the limited number of examples provided. CS1 explained:

"Most of the time you gave us one example. I found it difficult when I try to solve other problems,"

highlighting the need for greater variety in practice opportunities. CS8 added:

"I did not see much,"

underscoring the insufficiency of exercises for developing problem-solving skills.

These responses indicate that students required greater exposure to diverse problem situations to develop transferable problem-solving skills. Limited practice with varied examples may have contributed to difficulties when students encountered unfamiliar calculus problems, as observed during classroom activities.

7.2.4.3 Need for Peer Collaboration

Some students stressed the importance of interaction during problem-solving. CS9 suggested:

"I think it was better if you were in a position to allow us to talk to our friends for answers during problem-solving tasks,"

indicating a preference for more collaborative approaches.

This finding reinforces the observation data showing limited opportunities for collaborative learning in the comparison group. The absence of peer discussion reduced opportunities for students to compare strategies, explain reasoning, and learn alternative approaches to solving calculus problems.

7.2.4.4 Frustration with Teacher-Centered Methods

A recurring theme was dissatisfaction with teacher-led instruction, which limited autonomy. CS5 remarked:

"Sometimes I prefer if you listen to us as students because these are our problems,"

emphasizing the need for a more balanced, dialogic classroom environment.

The responses suggest that students preferred a more interactive learning environment in which they could actively contribute to the learning process, receive feedback on their own solution strategies, and discuss difficulties encountered during problem solving.

7.2.4.5 Call for a More Flexible Approach

Students suggested incorporating a variety of examples and allowing space for independent problem-solving. CS1 explicitly recommended more diverse examples, while CS2 stressed the importance of greater flexibility in lesson structure.

Collectively, these recommendations indicate that many features students desired were already incorporated within the WMCBPSI intervention, including varied examples, guided practice, peer interaction, and opportunities for independent strategy development. This further strengthens the contrast between the two instructional approaches.

The comparison group's feedback highlights a clear demand for more interactive, student-centred learning approaches. Their responses underscore the limitations of traditional problem-solving instruction and point to the potential value of active learning, peer collaboration, and varied examples (Prince, 2004).

Overall, the comparison group responses provide qualitative evidence explaining the lower quantitative performance observed in Chapter 6. Students' difficulties were associated not only with mathematical content but also with limited opportunities to develop flexible problem-solving processes, strategic awareness, and independent reasoning.

7.2.4 Key Findings from Experimental and Comparison Groups

The qualitative data revealed clear contrasts between the two groups, emphasizing both the strengths of WMCBPSI and the limitations of the conventional approach. Evidence from classroom observations, semi-structured interviews, and video-recorded classroom interactions consistently demonstrated that the two instructional approaches resulted in different learning experiences and problem-solving behaviours.

1. Active Learning and Engagement: Students in the experimental group reported higher levels of engagement, exploration, and collaboration compared to the comparison group.

Classroom observations confirmed that experimental group students participated more actively in problem-solving activities, discussed alternative solution methods, and demonstrated greater willingness to explain their reasoning. In contrast, students in the comparison group showed more passive participation and relied primarily on teacher explanations.

2. Problem-Solving and Use of Examples: Experimental students valued diverse examples and strategies, which enhanced confidence and adaptability. They reported that contextualised examples, guided practice, and exposure to multiple solution approaches helped them understand calculus concepts more deeply and apply strategies to unfamiliar problems. The comparison group criticized the limited examples and practice opportunities, indicating that insufficient exposure to varied problem situations reduced their confidence and flexibility when solving new problems.
3. Collaboration and Peer Learning: Group work was valued in the experimental group, enhancing problem-solving and reducing anxiety, whereas it was absent in the comparison group. Although students initially experienced challenges adapting to collaborative learning, observations and interviews showed that group discussions gradually improved strategy sharing, mathematical communication, and confidence. In contrast, limited peer interaction in the comparison group reduced opportunities for collaborative knowledge construction.
4. Teacher Role and Instructional Approach: Experimental students appreciated the teacher as a facilitator, while comparison students expressed dissatisfaction with the teacher-centered style. Students receiving WMCBPSI perceived the instructor as a guide who encouraged questioning, discussion, and independent thinking. Conversely, students receiving CPSI described the instructional approach as mainly teacher-directed, with fewer opportunities for active participation and independent reasoning.
5. Motivation and Confidence: The experimental group reported higher motivation and confidence, whereas the comparison group expressed discouragement and lack of independence. Students exposed to WMCBPSI reported increased willingness to attempt challenging problems, improved confidence in selecting appropriate strategies, and stronger conceptual understanding. These improvements were less evident among students receiving conventional instruction.

Overall, the qualitative evidence strongly supports the effectiveness of WMCBPSI in fostering active learning, collaboration, strategic flexibility, autonomy, and confidence in calculus problem-solving. The convergence of observation and interview findings provides qualitative confirmation of the quantitative results presented in Chapter 6, demonstrating that WMCBPSI influenced not only students' achievement outcomes but also the cognitive and behavioural processes underlying successful problem solving.

These results reinforce the value of working memory-based, context-driven instructional strategies in mathematics education (Sweller et al., 2011; Prince, 2004).

7.3. CHAPTER SUMMARY

This chapter systematically analyzed qualitative data from classroom observations and semi-structured interviews to examine the effects of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on students‘ calculus problem-solving skills. The analysis was guided by the study‘s research questions (Section 1.8) and aimed to evaluate the efficacy of WMCBPSI compared to Conventional Problem-Solving Instruction (CPSI).

The qualitative findings complemented the quantitative results presented in Chapter 6 by providing deeper explanations of how WMCBPSI influenced students' engagement, strategy use, classroom interactions, and learning experiences. The integration of multiple qualitative data sources, including classroom observations, semi-structured interviews, and video-recorded classroom interactions, strengthened the credibility and comprehensiveness of the findings through triangulation.

Key Findings

- Classroom Observations: Findings from classroom observations indicated that students in the experimental group, who received WMCBPSI, demonstrated superior problem-solving behaviours compared to the comparison group exposed to conventional teaching methods.

Experimental group students engaged more actively in problem-solving activities and exhibited greater flexibility and adaptability in their approaches. They were observed discussing alternative solution methods, explaining their reasoning, collaborating with peers, and attempting unfamiliar calculus problems with increasing independence. These behaviours indicate that WMCBPSI supported the development of flexible problem-solving schemas and improved students' ability to apply strategies effectively.

In contrast, students in the comparison group demonstrated greater dependence on teacher explanations, fewer peer interactions, and limited flexibility when encountering unfamiliar problems. Their reliance on demonstrated examples suggests fewer opportunities to develop independent problem-solving approaches.

- Interviews: Semi-structured interviews with nine students from each group revealed that students in the experimental group expressed strong support for WMCBPSI, recognizing its effectiveness in enhancing problem-solving skills and conceptual understanding.

Students highlighted the benefits of contextualised examples, structured guidance, collaborative learning, and exposure to multiple solution strategies. They reported increased confidence, motivation, and ability to approach calculus problems from different perspectives.

In contrast, students in the comparison group expressed dissatisfaction with CPSI, citing a lack of opportunities for active participation, peer collaboration, and independent problem solving. They indicated that teacher-centred instruction limited their opportunities to practise diverse strategies, develop confidence, and transfer learned procedures to unfamiliar problems.

- General Support for WMCBPSI: Across data sources, there was a clear consensus that WMCBPSI was more effective in developing problem-solving skills than CPSI. Experimental group students appreciated the diversity of problem-solving strategies, the active engagement opportunities, and the structured guidance that fostered deeper understanding. Many also reported increased motivation to continue learning and practicing mathematics, highlighting the instructional method's motivational impact.

The consistency between classroom observations and interview responses provides strong qualitative evidence that WMCBPSI influenced both the cognitive and affective dimensions of calculus learning. The intervention supported students' strategic competence, metacognitive awareness, collaborative learning, and confidence in solving complex mathematical problems.

The findings of this chapter strongly support the adoption of WMCBPSI as an effective instructional approach for enhancing calculus problem-solving skills. Evidence from both classroom observations and interviews indicates that WMCBPSI fosters active engagement, deeper conceptual understanding, and increased problem-solving proficiency. However, successful implementation may require addressing challenges such as class size and resource constraints, which can influence the approach's effectiveness.

The qualitative findings also provide explanatory support for the quantitative results reported in Chapter 6. While the statistical analysis demonstrated that students exposed to WMCBPSI achieved significantly higher problem-solving performance, the qualitative evidence explains that these improvements were associated with increased engagement, improved strategy selection, collaborative learning opportunities, and better management of cognitive demands during problem solving.

These findings align with prior research advocating for student-centered, active learning approaches in mathematics education (Hattie, 2009; Prince, 2004; Sweller et al., 2011).

The integration of qualitative and quantitative findings provided a more comprehensive understanding of WMCBPSI's influence on students' calculus problem-solving processes, engagement, and instructional experiences. The convergence of achievement outcomes, observed classroom behaviours, and student perceptions strengthened the interpretation of the intervention's effectiveness and provided complementary evidence regarding its educational impact.

Taken together, the findings from Chapters 6 and 7 demonstrate that WMCBPSI enhanced not only students' mathematical achievement but also the underlying cognitive processes that support successful problem solving. By integrating working memory principles, contextualised learning, structured guidance, and collaborative activities, WMCBPSI created a learning environment that supported deeper understanding and more effective application of mathematical knowledge.

The next chapter will discuss these findings in relation to the study's theoretical framework, existing literature, and the practical implications for mathematics teaching and learning in higher education.

CHAPTER EIGHT

DISCUSSION OF RESULTS

8.1. INTRODUCTION

This chapter presents a comprehensive discussion of the study‘s findings in relation to the research objectives, research questions, and the theoretical framework of Cognitive Load Theory (CLT). The study investigated the effects of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on the calculus problem-solving skills of first-year students at an Ethiopian university. The discussion is informed primarily by Cognitive Load Theory (CLT) while also drawing on the principles of working memory, context-based learning, and Polya's problem-solving framework that guided the design and implementation of the instructional intervention.

The discussion synthesizes both quantitative and qualitative results, highlighting their implications for instructional practice, particularly in the context of CLT. Emphasis is placed on understanding how WMCBPSI influenced students‘ engagement, problem-solving performance, and strategy use, and how these outcomes align with cognitive load principles. The chapter integrates the quantitative findings presented in Chapter 6 with the qualitative findings reported in Chapter 7 to provide a comprehensive explanation of how WMCBPSI influenced students' calculus problem-solving skills. Rather than considering achievement outcomes in isolation, the discussion examines how changes in students' engagement, problem-solving strategies, conceptual understanding, and learning experiences help explain the observed improvements in performance.

The findings are interpreted in relation to existing literature to determine the extent to which they support, extend, or differ from previous research on cognitively informed mathematics instruction. The chapter further discusses the implications of the findings for mathematics teaching and learning in higher education, evaluates the study against its research objectives and research questions, and considers the broader educational significance of implementing WMCBPSI in university calculus instruction.

8.2. SUMMARY OF RESULTS

The study involved two groups of first-year calculus students from different academic years. The comparison group ($n = 38$) from 2022 received Conventional Problem-Solving Instruction (CPSI), while the experimental group ($n = 41$) from 2024 was taught using WMCBPSI. A mixed-methods design was employed, integrating quantitative and qualitative data collection tools, including a standardized Calculus Achievement Test (CAT), semi-structured interviews, and classroom observations.

A static group comparison, post-test-only design was implemented. The CAT assessed students' problem-solving performance following a two-month instructional intervention, while qualitative data provided rich insights into students' experiences with each instructional approach.

Cognitive Load Theory (CLT) served as the guiding framework for the design of WMCBPSI. The intervention incorporated strategies to reduce cognitive overload, including the use of varied examples, worked-out solutions, chunking techniques, and self-explanation. These methods aimed to support schema acquisition and automation, thereby improving problem-solving performance and efficiency in calculus tasks.

The quantitative findings demonstrated that students who learned through WMCBPSI consistently outperformed those who received Conventional Problem-Solving Instruction across multiple indicators of problem-solving performance. Similarly, the qualitative findings revealed that students exposed to WMCBPSI experienced greater engagement, confidence, collaboration, and conceptual understanding, providing complementary evidence for the effectiveness of the instructional intervention.

The findings revealed that the experimental group achieved significantly higher problem-solving skills compared to the CPSI group. Specifically, they attained superior post-test scores, demonstrated greater proficiency according to the TIMSS framework, and made fewer errors in solving calculus problems. These differences were statistically significant ($p < .05$), providing strong evidence for the effectiveness of WMCBPSI in enhancing students' calculus problem-solving abilities.

Taken together, the quantitative and qualitative findings indicate that WMCBPSI not only improved students' academic performance but also enhanced the quality of their problem-solving processes and learning experiences. These findings provide the basis for the detailed discussion presented in the following sections.

8.3. DISCUSSION OF FINDINGS

8.3.1. Alignment with Cognitive Load Theory

The study's findings are consistent with the principles of Cognitive Load Theory (CLT), which emphasize the need to manage cognitive load to optimize learning and problem-solving performance (Sweller, 1988; Sweller et al., 2011). WMCBPSI effectively managed cognitive load by providing structured worked examples, multiple problem-solving strategies, and guided opportunities for self-explanation and reflection. This approach enabled students to develop robust problem-solving schemas, reducing cognitive load during subsequent problem-solving tasks and enhancing efficiency in complex calculus procedures.

The quantitative findings presented in Chapter 6 support this interpretation. Students in the experimental group achieved significantly higher post-test scores than those in the comparison group, while the qualitative findings in Chapter 7 indicated that students perceived the instructional approach as helping them organise information, connect concepts, and solve unfamiliar problems more confidently. These complementary findings demonstrate that reducing unnecessary cognitive load while promoting schema construction contributed to improved calculus problem-solving performance.

The data indicate that WMCBPSI significantly improved not only students' overall performance scores but also the quality of their problem-solving processes. Learners exposed to this instructional model demonstrated stronger analytical reasoning, more effective strategy selection, and greater accuracy in verifying solutions—key attributes of advanced problem solvers as outlined in Polya's framework.

The findings therefore provide empirical support for the theoretical proposition that instructional approaches designed around the limitations of working memory enable learners to devote greater cognitive resources to germane processing and long-term schema development. Rather than merely improving examination performance, WMCBPSI appears to have strengthened students' capacity to analyse, plan, and execute multistep calculus solutions efficiently.

These findings align with recent international research showing that contextualized, cognitively structured instruction combined with metacognitive support fosters higher-order mathematical thinking (Hmelo-Silver et al., 2007; Sweller et al., 2019). Specifically, WMCBPSI appears to optimize the interaction among working memory, schema development, and cognitive load management, enabling students to transfer learned strategies more effectively to novel calculus problems.

8.3.2. Active Learning and Schema Development

The success of WMCBPSI aligns with prior research demonstrating that problem-solving skills are best developed through active, scaffolded, and collaborative learning (Hattie, 2009; Van Merriënboer & Sweller, 2005). Exposure to varied worked examples allowed students to integrate new knowledge with prior understanding, supporting the automation of problem-solving strategies. The intervention facilitated greater mastery of problem-solving strategies and deeper conceptual understanding. Additionally, self-explanation encouraged deeper conceptual understanding, consistent with findings on generative learning by Chi et al. (1994).

Evidence from the classroom observations further showed that students in the experimental group became increasingly active participants during problem-solving sessions. They discussed alternative solution methods, justified their reasoning to peers, and demonstrated greater willingness to attempt unfamiliar problems. Interview data similarly revealed that students believed the instructional approach helped them understand relationships among calculus concepts rather than memorising isolated procedures.

These qualitative findings complement the quantitative improvements observed in post-test performance and suggest that WMCBPSI promoted meaningful learning rather than short-term procedural success. The instructional emphasis on collaborative discussion, multiple solution paths, and reflective thinking appears to have facilitated schema refinement, enabling students to transfer previously acquired knowledge to new calculus tasks.

These findings suggest that integrating structured scaffolding with active engagement supports both schema development and long-term retention of problem-solving strategies.

8.3.3. Limitations of Conventional Problem-Solving Instruction

In contrast, students in the CPSI group, taught through traditional, teacher-centered methods, demonstrated lower performance than students who received WMCBPSI. As reported in Chapter 6, a larger proportion of CPSI students performed at the lower TIMSS proficiency levels (Levels 1 and 2), indicating less-developed problem-solving skills.

They had limited opportunities to independently apply problem-solving strategies. One possible explanation is that conventional teacher-centered instruction provides fewer opportunities for students to actively construct knowledge, explain their reasoning, evaluate alternative solution methods, and engage collaboratively in solving complex problems. Consequently, students may rely more heavily on memorized procedures rather than developing flexible problem-solving schemas that can be transferred to unfamiliar calculus tasks.

From the perspective of Cognitive Load Theory, conventional instruction may place greater demands on students' working memory because learners are often expected to process new concepts without sufficient instructional scaffolding, worked examples, or opportunities for guided practice. In contrast, WMCBPSI incorporated chunking, varied worked examples, collaborative learning, and self-explanation activities that supported schema acquisition and reduced unnecessary cognitive load.

The qualitative findings provide additional insight into these quantitative differences. Students frequently described lessons as relying heavily on teacher demonstrations, with relatively few opportunities for collaborative discussion or independent exploration. Consequently, many students reported difficulty applying learned procedures to unfamiliar problems, indicating that knowledge acquired through conventional instruction was less transferable.

The lower proportion of higher-level problem-solving performance observed in Chapter 6 further supports this interpretation. Students in the comparison group relied primarily on routine algorithmic approaches and demonstrated less flexibility in selecting and executing appropriate strategies. This suggests that conventional instruction may have imposed greater cognitive demands during independent problem solving because students had not developed sufficiently integrated schemas for complex calculus tasks.

These results corroborate prior research indicating that lecture-based instruction may not sufficiently foster adaptive problem-solving skills (Bransford et al., 2021; Jonassen, 2000). The findings therefore suggest that the lower performance observed among students in the CPSI group may be associated with the limited cognitive and collaborative support provided by conventional instructional practices, thereby restricting opportunities for effective schema construction and transfer.

8.3.4 Student Experiences and Motivation

Qualitative data reinforced the quantitative results. Experimental group students reported higher motivation, confidence, and positive attitudes toward calculus learning, appreciating the collaborative and interactive nature of WMCBPSI. Conversely, CPSI students expressed dissatisfaction with limited interaction, lack of autonomy, and insufficient practice opportunities.

Students also consistently reported that solving problems collaboratively increased their confidence to attempt more challenging calculus questions. Rather than depending solely on instructor explanations, they described learning from peer discussions, comparing different solution methods, and receiving immediate feedback during classroom activities. These experiences appear to have contributed to greater persistence when solving complex problems.

The observed increase in motivation is particularly important because sustained engagement is essential for developing higher-order mathematical reasoning. Students who perceive instruction as meaningful and supportive are more likely to invest cognitive effort in constructing durable problem-solving schemas, thereby reinforcing the mechanisms proposed by Cognitive Load Theory.

These findings align with studies emphasizing the motivational benefits of active, collaborative learning (Prince, 2004), suggesting that instructional approaches that actively engage learners may enhance both motivation and learning outcomes.

8.3.5 Individual Variation in Outcomes

Notably, a few experimental group students reported limited benefit from WMCBPSI, attributing their outcomes to personal effort and engagement. This highlights the importance of learner motivation and suggests the need for ongoing scaffolding to maximize instructional impact. It also indicates that instructional effectiveness may vary across learners depending on individual differences in engagement and readiness to learn.

Although the overall findings strongly favoured WMCBPSI, these individual differences indicate that instructional innovation alone cannot guarantee equivalent learning outcomes for all students. Differences in prior mathematical preparation, learning habits, persistence, and willingness to participate in collaborative activities may influence the extent to which students benefit from cognitively structured instruction.

This finding is consistent with contemporary educational research suggesting that instructional effectiveness is shaped by interactions between teaching strategies and learner characteristics. Consequently, successful implementation of WMCBPSI should include appropriate scaffolding, continuous formative feedback, and opportunities for differentiated support to accommodate diverse learner needs while maintaining the cognitive principles underpinning the intervention.

8.4. IMPLICATIONS AND RECOMMENDATIONS

The findings of this study have significant implications for mathematics instruction in higher education.

Adoption of WMCBPSI

The evidence strongly supports integrating Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) as a standard instructional approach for enhancing problem-solving skills in calculus and related subjects. The quantitative findings demonstrated statistically significant improvements in students' calculus problem-solving performance, while the qualitative findings showed increased engagement, confidence, and strategic thinking among students exposed to WMCBPSI. Together, these findings indicate that WMCBPSI offers a practical instructional alternative to conventional teacher-centred approaches, particularly in courses where students are required to solve complex, multi-step mathematical problems. Institutions of higher learning may therefore consider incorporating WMCBPSI into undergraduate mathematics curricula as part of broader efforts to improve students' conceptual understanding, problem-solving competence, and academic achievement.

Teacher Preparation

Instructors should be trained to design instruction aligned with the principles of Cognitive Load Theory (CLT), incorporating worked examples, varied problem types, chunking, and self-explanation to support schema development. The successful implementation of WMCBPSI depended not only on the instructional materials but also on the instructor's ability to organise learning activities that effectively managed cognitive load. Consequently, professional development programmes should equip mathematics lecturers with practical skills in designing cognitively efficient lessons, facilitating collaborative learning, and selecting appropriate worked examples that progressively develop students' problem-solving schemas. Such training would enable instructors to translate Cognitive Load Theory into effective classroom practice rather than treating it solely as a theoretical framework.

Active Learning Integration

Lesson plans should embed opportunities for collaboration, peer discussion, and guided self-explanation to foster deep understanding and the transfer of problem-solving skills. The classroom observations demonstrated that students who actively discussed solution strategies and justified their reasoning developed greater flexibility in solving unfamiliar calculus problems. Similarly, interview responses indicated that collaborative learning enhanced students' confidence and encouraged them to explore multiple solution methods. These findings suggest that active learning strategies should become an integral component of calculus instruction rather than supplementary classroom activities. Embedding structured collaborative problem-solving tasks within regular lessons can promote deeper conceptual understanding while simultaneously strengthening students' metacognitive and communication skills.

Addressing Contextual Challenges

Institutional factors, such as class size, resource availability, and student preparedness, should be considered and addressed to ensure effective implementation of WMCBPSI. Although the intervention proved effective, several implementation challenges emerged during the study. Some students initially struggled to adapt to collaborative learning, while large class sizes occasionally limited opportunities for close instructor guidance. Resource constraints may also influence the extent to which instructors can prepare varied instructional materials and worked examples. Universities intending to adopt WMCBPSI should therefore provide appropriate institutional support through manageable class sizes where possible, adequate instructional resources, and professional development opportunities for mathematics instructors. Such support would increase the likelihood of successful and sustainable implementation.

Future Research Directions

Future studies should investigate the long-term effects of WMCBPSI, including the transferability of problem-solving skills to other mathematical domains. Research should also explore adaptations of WMCBPSI for diverse educational contexts and examine strategies for implementing cognitive load management in large classes. In addition, future research should examine whether the positive effects observed in this study are maintained over longer periods and whether students retain and transfer their problem-solving skills to advanced mathematics courses such as differential equations, numerical analysis, and linear algebra. Researchers may also investigate the effectiveness of individual components of WMCBPSI, such as worked examples, collaborative learning, chunking, and self-explanation, to determine the relative contribution of each instructional feature to students' learning outcomes. Comparative studies involving multiple universities or different educational contexts would further strengthen the external validity and generalisability of the findings.

This study provides compelling evidence that WMCBPSI is an effective instructional approach for enhancing calculus problem-solving skills. By aligning instruction with Cognitive Load

Theory, WMCBPSI fosters active engagement, collaboration, and schema development, resulting in measurable improvements in problem-solving performance. These findings contribute to the growing body of literature advocating student-centered, cognitively informed instructional approaches in mathematics education (Hattie, 2009; Sweller et al., 2011; Van Merriënboer & Sweller, 2005). Overall, the implications of this study extend beyond the immediate context in which the research was conducted. They demonstrate that instructional designs grounded in Cognitive Load Theory can substantially improve university students' mathematical thinking and problem-solving competence. Consequently, WMCBPSI represents a practical and theoretically sound instructional model that can contribute to improving the quality of mathematics teaching and learning in higher education institutions, particularly in contexts where students experience persistent difficulties in calculus problem solving.

8.5. DISCUSSION OF THE FINDINGS IN TERMS OF STUDY OBJECTIVES

8.5.1. Study Objective 1: Design and Implementation of WMCBPSI Using CLT

Objective 1 aimed to use Cognitive Load Theory (CLT) as a framework for designing Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) to enhance students‘ calculus problem-solving skills and facilitate understanding of their problem-solving approaches. CLT provided the theoretical foundation for WMCBPSI, guiding instructional design to reduce extraneous cognitive load, manage intrinsic load, and optimize germane load. Both quantitative and qualitative evidence indicate substantial benefits for the experimental group.

The successful implementation of WMCBPSI demonstrates how the principles of Cognitive Load Theory can be translated into practical classroom strategies that support students' mathematical learning. As detailed in Section 5.4.2.1, the 24-lesson intervention was implemented through a structured sequence of activities that included activation of prior knowledge, contextualised problem presentation, teacher modelling, worked examples, guided and collaborative problem solving, self-explanation, individual practice, feedback, and a gradual reduction of instructional support. The intervention was deliberately designed to align instructional activities with the limitations of working memory while supporting schema construction in long-term memory. Evidence from Chapters 6 and 7 indicates that the systematic implementation of these instructional strategies was associated with higher achievement, improved problem-solving strategies, and more positive learning experiences among students in the experimental group.

Mechanisms Underpinning the Effectiveness of WMCBPSI

Chunking Effect:

Complex calculus content was segmented into meaningful units, allowing students to process, store, and retrieve information efficiently. This systematic organization facilitated schema acquisition and improved performance and comprehension (Sweller, 1994; Cooper, 1998). Student feedback highlighted the effectiveness of structured lessons in connecting concepts (ES9).

The classroom observations further showed that students increasingly organised information into manageable components before attempting complete solutions. Specifically, as reported in Section 7.2.1, students were observed identifying the known and unknown quantities, separating relevant from irrelevant information, breaking complex calculus problems into smaller steps, and selecting appropriate solution strategies before proceeding to the complete solution. These observed behaviours indicate that students were increasingly applying the problem-structuring and information-management strategies emphasised during the WMCBPSI intervention. This gradual organisation of information reduced unnecessary cognitive demands and enabled learners to solve increasingly complex calculus problems with greater confidence and accuracy.

Variable Worked Examples:

Multiple solved problems differing in surface features but sharing underlying structures enabled students to construct diverse problem-solving schemas, enhancing flexibility in novel problem-solving situations. Students reported improved understanding and test performance after working through varied examples (Paas & van Merriënboer, 1994; Renkl & Atkinson, 2010).

This instructional feature was reflected in the quantitative findings, as presented in Section 7.2.3.2, where students in the experimental group demonstrated greater success in selecting appropriate solution strategies and applying them accurately across different problem types. The qualitative findings presented in Section 7.2.3.2 similarly showed that students valued exposure to multiple examples because it helped them recognise underlying mathematical structures and move beyond memorising isolated procedures.

Split-Attention Effect:

Integration of textual, algebraic, and graphical information into cohesive presentations reduced cognitive switching, improving efficiency and comprehension. Students were able to link multiple representations, which enhanced conceptual understanding (Ayres & Sweller, 2005; Van Gog et al., 2009). These findings are further supported by the evidence presented in Section 7.2.4, where the integration of multiple representations was reflected in students' observed and reported learning experiences.

Students frequently commented during interviews that presenting information in an integrated manner made calculus concepts easier to understand. As reported in Section 7.2.4, students' interview responses indicated that the integrated presentation of information facilitated their understanding of calculus concepts. These observations suggest that reducing split attention allowed learners to devote more cognitive resources to conceptual understanding instead of coordinating information from multiple disconnected sources.

Group Approach / Collective Working Memory Effect:

Collaborative learning distributed cognitive load across group members, creating a larger collective working memory that facilitated problem-solving. Students actively engaged in group tasks, shared strategies, and refined solutions collaboratively, which reinforced schema acquisition and conceptual understanding (Kirschner et al., 2009a, 2009b; Hinsz et al., 1997).

Both classroom observations and interview findings demonstrated that collaborative problem-solving promoted active discussion, peer explanation, and shared reasoning. As discussed in Section 7.2.3.3, these collaborative interactions appeared to strengthen students' conceptual understanding by enabling them to articulate, compare, and refine their mathematical reasoning.

At the same time, collaboration reduced the cognitive demands associated with solving complex calculus problems individually, as students were able to distribute aspects of the problem-solving process through peer support and shared reasoning. This convergence between the observational and interview findings reinforces the role of collaborative problem-solving in supporting both conceptual understanding and students' ability to manage the cognitive demands of complex calculus tasks.

8.5.2. Study Objective 2: Challenges, Facilitators, and Students' Perceptions of WMCBPSI

Objective 2 focused on identifying the challenges and facilitators of implementing WMCBPSI in calculus instruction. Insights from students' views (previously presented under Objective 4) strengthen the qualitative discussion.

Facilitators of WMCBPSI:

Active and Collaborative Learning: Students in the experimental group reported higher engagement, exploration, and peer collaboration than those in the CPSI group. Group-based problem-solving allowed sharing of strategies, discussion, and reflection.

Diverse Problem-Solving Strategies: Exposure to multiple examples promoted flexibility and strategic competence in applying calculus methods.

Structured and Integrative Instruction: Cohesive lesson presentation, linking textual, algebraic, and visual content, supported understanding and independent application.

Positive Learning Environment: Students reported increased motivation, confidence, and satisfaction with WMCBPSI, which enhanced their learning experiences and willingness to engage with complex problems.

The qualitative findings consistently demonstrated that students regarded these instructional features as major strengths of WMCBPSI. Classroom observations further confirmed that these facilitators promoted active participation, sustained engagement, and collaborative reasoning throughout the intervention period (see Section 7.2.3.3). Together, these findings indicate that effective instructional design and classroom interaction were central contributors to the intervention's success.

Challenges and Considerations:

Large class sizes, limited resources, and varying student preparedness were noted as potential constraints affecting implementation. Addressing these factors is important for scaling WMCBPSI effectively.

Some students also required time to adjust to collaborative learning because they had previously experienced predominantly teacher-centred instruction. During the early stages of implementation, a few students were hesitant to participate actively in discussions or explain their reasoning. However, these challenges gradually diminished as students became familiar with the instructional approach and developed greater confidence through guided practice.

The qualitative evidence demonstrates that WMCBPSI provides a richer and more supportive learning environment than CPSI, aligning with research advocating student-centered, active learning approaches that foster deeper understanding, reflection, and strategic competence (Hattie, 2009; Van Merriënboer & Sweller, 2005).

Overall, the evidence indicates that the facilitators substantially outweighed the implementation challenges. Consequently, Objective 2 was achieved by identifying both the strengths and practical considerations associated with implementing WMCBPSI in university calculus instruction.

8.5.3. Study Objective 3: Quantitative Evaluation of WMCBPSI Impact

Objective 3 evaluated the impact of WMCBPSI on students' calculus problem-solving performance**.** Post-test results indicated that the experimental group achieved significantly higher performance scores than the comparison group ($p < 0.05$), providing evidence of a positive association between WMCBPSI implementation and students' calculus problem-solving performance. The statistical analyses presented in Chapter 6 demonstrated that the observed differences between the experimental and comparison groups were not only statistically significant but also educationally meaningful. The independent-samples t-test confirmed that students exposed to WMCBPSI achieved significantly higher post-test scores than students receiving Conventional Problem-Solving Instruction (CPSI).

Effect size analysis demonstrated a meaningful practical difference between the groups, including greater use of diverse problem-solving strategies and higher levels of solution accuracy among students who received WMCBPSI. Furthermore, the calculated effect size (0.862) indicated that the intervention produced a practically important improvement in students' calculus problem-solving performance rather than a trivial statistical difference.

The quantitative evidence also showed that students in the experimental group demonstrated higher levels of problem-solving proficiency according to the adapted TIMSS framework (see appendix D), selected more appropriate solution strategies, and executed those strategies more successfully than students in the comparison group (see section 6.4.3). These findings suggest that WMCBPSI enhanced both the outcomes and the quality of students' mathematical reasoning, indicating improvements in conceptual understanding as well as procedural competence.

These findings suggest that WMCBPSI has potential as an effective instructional approach for supporting calculus problem-solving development in similar educational contexts.

These findings are consistent with research showing that CLT-informed instructional approaches—incorporating chunking, worked examples, and collaborative learning—enhance mathematical learning outcomes (Paas & van Merriënboer, 1994; Sweller et al., 2011).

The qualitative findings reported in Chapter 7 (see section 7.2) further strengthened these conclusions by explaining how the observed improvements occurred. Students consistently attributed their enhanced performance to collaborative learning, varied worked examples, structured instructional support, and opportunities for guided reflection. The convergence of quantitative and qualitative findings therefore provides strong evidence that WMCBPSI effectively improved calculus problem-solving skills.

Accordingly, Objective 3 was fully achieved because the study demonstrated, through rigorous quantitative analysis supported by qualitative evidence, that WMCBPSI significantly improved students' calculus problem-solving performance compared with Conventional Problem-Solving Instruction.

8.5.4. Conclusion Based on Study Objectives

The findings demonstrate that WMCBPSI, grounded in Cognitive Load Theory, significantly improves calculus problem-solving skills by:

- Enhancing instructional design through CLT principles, which optimize cognitive load and promote schema acquisition (Objective 1).
- Fostering active engagement, collaboration, and strategic competence, while addressing implementation challenges, as revealed in students' feedback (Objective 2).
- Achieving measurable improvements in problem-solving performance, including strategy repertoire and accuracy (Objective 3).

Experimental group students consistently outperformed their peers in both quantitative and qualitative measures. These results extend prior research on schema-based instruction, highlighting the transformative potential of CLT-informed, student-centered approaches for enhancing mathematical thinking, conceptual understanding, and problem-solving skills (Sweller et al., 2011; Van Merriënboer & Sweller, 2005; Jitendra et al., 2002).

Taken together, the evidence presented throughout this study demonstrates that all three study objectives were successfully achieved. The instructional model was systematically designed using Cognitive Load Theory, implemented successfully in university calculus classrooms, and evaluated using complementary quantitative and qualitative methods. The consistency of findings across multiple data sources strengthens confidence in the validity of the conclusions and provides convincing evidence that WMCBPSI represents an effective instructional approach for improving university students' calculus problem-solving skills.

8.6. EVALUATING STUDY RESULTS AGAINST THE RESEARCH QUESTIONS

8.6.1. Research Question 1

How can Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) be incorporated into the teaching and learning of calculus concepts?

The study successfully addressed this question by demonstrating how WMCBPSI can be systematically integrated into calculus instruction. The design of WMCBPSI (Section 5.3.1.1) was grounded in Cognitive Load Theory (CLT) principles, particularly the chunking effect, worked example effect, split-attention effect, and collective working memory effect.

As illustrated in Table 5.1, these principles guided the structuring of instructional materials and activities to minimise extraneous cognitive load while maximising conceptual understanding. WMCBPSI incorporated collaborative group work, varied worked examples, and scaffolded problem-solving tasks, all of which fostered active learning and deeper engagement with calculus concepts.

Evidence from both classroom observations and student interviews confirmed that these instructional features were successfully implemented throughout the intervention. Students reported that the combination of structured explanations, collaborative activities, and multiple worked examples enabled them to understand concepts more effectively and apply problem-solving strategies with greater confidence. Therefore, the study provides a practical model for integrating Cognitive Load Theory into undergraduate calculus instruction.

8.6.2. Research Question 2

What challenges, if any, does the incorporation of WMCBPSI pose in the teaching and learning of calculus concepts?

Several challenges emerged during WMCBPSI implementation in the experimental group:

8.6.2.1. Familiarity with New Teaching Methods:

Students required time to adjust to the interactive, collaborative style, contrasting with traditional, teacher-centred approaches.

8.6.2.2. Social Skill Deficiencies:

Some students lacked the interpersonal and communication skills necessary for effective group problem solving, limiting collaborative engagement.

8.6.2.3. Group Dynamics:

In certain groups, dominant students overshadowed quieter peers, affecting the quality of discussion and collective problem-solving.

8.6.2.4. Large Class Sizes:

Managing group work and providing individual support in larger classes posed logistical difficulties, potentially constraining instructional effectiveness.

These challenges highlight the practical considerations of adopting innovative instructional methods and underscore the need for strategies such as teacher training, scaffolding for group work, and class size management to support successful implementation.

Despite these challenges, qualitative evidence indicated that they diminished over time as students became familiar with collaborative learning and increasingly confident in applying the instructional strategies. Importantly, none of these challenges prevented successful implementation of the intervention or reduced its overall effectiveness. Instead, they represent practical considerations that institutions should address when adopting WMCBPSI on a larger scale.

8.6.3. Research Question 3

Will the incorporation of WMCBPSI have any effect on students' calculus problem-solving skills?

The implementation of WMCBPSI had a substantial positive impact on students' calculus problem-solving abilities. Quantitative data from Chapter 6 revealed statistically significant differences in post-test scores between the experimental and comparison groups ($p < 0.05$). Students exposed to WMCBPSI demonstrated:

1. Higher overall achievement in calculus problem-solving performance.
2. A broader repertoire and greater flexibility in selecting problem-solving strategies.
3. Greater accuracy in solving calculus problems and executing solution strategies successfully.

Qualitative evidence from classroom observations and interviews corroborated these findings. Experimental group students reported increased confidence, higher motivation, and deeper conceptual understanding, highlighting the practical and pedagogical benefits of WMCBPSI.

The convergence of quantitative and qualitative evidence provides particularly strong support for answering this research question. Improvements were observed not only in students' achievement scores but also in the quality of their reasoning, strategy selection, collaborative engagement, and confidence when solving calculus problems. These findings indicate that WMCBPSI influenced both the cognitive processes and observable outcomes of mathematical problem solving.

The study provides robust evidence that WMCBPSI was associated with improved calculus problem-solving outcomes within the context of this study. Both qualitative and quantitative findings demonstrate the greater effectiveness of WMCBPSI compared to conventional problem-solving instruction (CPSI), suggesting that WMCBPSI offers substantial benefits for enhancing students' calculus problem-solving skills.

However, successful adoption requires careful consideration of challenges related to student adaptation, group dynamics, social skill development, and classroom management. These results reinforce the importance of designing instructional strategies informed by Cognitive Load Theory, integrating active, scaffolded, and collaborative learning to maximise problem-solving competence in mathematics education.

Overall, the findings provide comprehensive answers to the study's research questions and demonstrate that WMCBPSI constitutes an effective, theoretically grounded, and practically applicable instructional approach for improving university students' calculus problem-solving skills. The consistency of evidence across different methods of data collection further strengthens the credibility of these conclusions.

8.7. IMPLICATIONS OF THE STUDY RESULTS TO PRACTICE

1. Epistemological Implication

This study underscores the significance of Cognitive Load Theory (CLT) in designing mathematics instruction. CLT asserts that effective teaching should aim to reduce extraneous cognitive load while enhancing learning and problem-solving abilities. A primary contribution of this study lies in its practical application of CLT principles—such as the chunking effect, worked example effect, and split-attention effect—to facilitate the acquisition of problem-solving schemas.

These schemas, stored in long-term memory, enable students to process information more efficiently during problem-solving tasks. Previous research has demonstrated the benefits of worked examples (Gerjets et al., 2004; Renkl et al., 2009; Rourke & Sweller, 2009), and the present study extends these findings by integrating collaborative learning and worked examples with varying solution strategies. By embedding these strategies within instructional design, this study advances the understanding of how cognitive science can inform mathematics education and improve students‘ problem-solving competencies.

Beyond confirming existing Cognitive Load Theory principles, this study demonstrates how these principles can be operationalised within an integrated instructional model specifically designed for university calculus. In doing so, the study contributes empirical evidence showing that cognitive theories can inform practical instructional decisions that improve both students' conceptual understanding and problem-solving performance. This represents an important contribution to mathematics education research, particularly within higher education contexts where studies integrating Cognitive Load Theory with problem-solving instruction remain relatively limited.

2. Methodological Implication

A distinctive feature of this study was the researcher-led implementation of both Conventional Problem-Solving Instruction (CPSI) and Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI). This approach ensured consistency and fidelity in the delivery of instruction, allowing for a more precise comparison between the two methods.

By excluding external instructors, the study minimised potential biases associated with instructor variability. Previous studies (e.g., Gaigher, 2006) often employed external instructors, introducing additional variables that could influence outcomes. Controlling for instructor-related differences strengthened the reliability and internal validity of the findings. Moreover, the study adhered to principles of equating groups in experimental research (Gay et al., 2011), ensuring that observed differences in outcomes could be confidently attributed to the instructional methods rather than extraneous factors.

The integration of quantitative and qualitative methods further strengthened the methodological rigour of the study. Statistical analyses established the effectiveness of the intervention, while classroom observations and student interviews explained how and why the observed improvements occurred. The convergence of findings from multiple data sources enhanced the credibility, trustworthiness, and overall validity of the study's conclusions.

3. Pedagogical Implication

The findings provide strong evidence that WMCBPSI significantly enhances students' problem-solving skills in calculus, outperforming conventional instruction. By incorporating worked examples with varied strategies and fostering active engagement, WMCBPSI enabled students to develop more efficient, flexible, and transferable problem-solving schemas. This instructional design guided students systematically through each stage of problem solving, deepening their conceptual understanding and promoting confidence in applying strategies to novel problems.

Furthermore, the study highlights the value of group-based collaborative learning in mathematics education. Collaborative problem solving allows students to share perspectives, challenge reasoning, and collectively consolidate understanding. Such interaction not only reduces cognitive load but also enriches learning experiences, reinforcing the integration of CLT-informed strategies with active, collaborative pedagogy.

In summary, this study contributes to the growing body of research linking Cognitive Load Theory, collaborative learning, and problem-solving instruction. It strongly advocates for the adoption of WMCBPSI as a transformative pedagogical approach to enhance problem-solving skills, deepen conceptual understanding, and improve academic performance in calculus and related mathematical domains.

For mathematics educators, these findings suggest that effective instruction should extend beyond demonstrating solution procedures. Lessons should intentionally engage students in analysing problems, discussing alternative solution strategies, reflecting on their reasoning, and connecting mathematical ideas across multiple representations. Such instructional practices can promote deeper learning while simultaneously reducing unnecessary cognitive load during complex mathematical tasks.

The study therefore provides practical guidance for curriculum developers, teacher educators, and university lecturers seeking to improve calculus instruction through evidence-based, cognitively informed teaching approaches.

8.8. SUMMARY OF THE CHAPTER

This chapter has discussed and interpreted the results of the study in relation to the research objectives and the theoretical framework of Cognitive Load Theory (CLT). The findings highlight the effectiveness of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) in enhancing the calculus problem-solving skills of first-year students.

By integrating CLT principles into instructional design, the study demonstrated improvements in students‘ information processing, reductions in cognitive load, and deeper engagement in problem-solving activities. These outcomes align with existing research on the benefits of cognitive load–informed instruction for improving learning outcomes (Sweller et al., 2011; Van Gog & Paas, 2012).

The study addressed the limitations of traditional teacher-centred approaches by implementing lessons that incorporated cognitive load–reducing strategies, including chunking, variable worked examples, split-attention reduction, and collaborative learning. This approach enhanced instructional efficiency by aligning teaching with the constraints of working memory, facilitating schema acquisition, and supporting the development of more advanced problem-solving skills.

In conclusion, the findings affirm that WMCBPSI is a promising instructional strategy capable of transforming calculus teaching by reducing cognitive load and improving problem-solving competence. The study provides valuable empirical evidence for applying CLT in mathematics education and suggests that similar approaches could be effectively adapted to other disciplines to promote deeper learning and transferable skill development.

Importantly, the discussion presented in this chapter demonstrates that the study objectives were successfully achieved and that the research questions were comprehensively addressed through the integration of quantitative and qualitative evidence. The consistency of findings across multiple sources strengthens confidence in the conclusion that WMCBPSI represents a theoretically sound and educationally effective instructional approach for improving university students' calculus problem-solving skills.

These findings also lay the groundwork for the final chapter, which synthesises the study‘s conclusions, outlines pedagogical and policy implications, identifies limitations, and presents recommendations for future research and practice in mathematics instruction. Chapter 9 will integrate these insights to provide a comprehensive conclusion to the study.

The discussion therefore serves as the bridge between the empirical findings presented in the preceding chapters and the broader conclusions and recommendations that follow. By interpreting the findings within the framework of Cognitive Load Theory and the wider mathematics education literature, this chapter establishes the scholarly significance of the study and its contribution to improving calculus teaching and learning in higher education.

CHAPTER NINE

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

9.1. SUMMARY OF THE STUDY

9.1.1. Problem of the Study

Students in Ethiopian universities have long faced persistent challenges in solving calculus problems effectively. These difficulties undermine performance in higher-level mathematics courses and hinder academic success in related fields. Despite the importance of calculus problem-solving skills for students' mathematical development, conventional instructional approaches often provide limited opportunities for students to develop flexible strategies, conceptual understanding, and independent problem-solving abilities. Therefore, there is a need for theoretically informed instructional approaches that address the cognitive demands involved in calculus problem solving.

This study addressed this challenge by examining the effect of Working Memory–Context-Based Problem-Solving Instruction (WMCBPSI) on the calculus problem-solving skills of first-year university students. The study was grounded in Cognitive Load Theory (CLT), which provides a framework for designing instruction that manages working memory limitations and supports the development of problem-solving schemas.

The research objectives were:

1. To design a WMCBPSI model grounded in Cognitive Load Theory (CLT) to enhance students‘ calculus problem-solving skills and to explain these skills.
2. To qualitatively compare WMCBPSI with conventional problem-solving instruction (CPSI).
3. To quantitatively assess differences in problem-solving skills between students taught using WMCBPSI and those taught using CPSI.

The study addressed the following research questions:

1. How can WMCBPSI be incorporated into calculus teaching and learning?
2. What challenges arise in incorporating WMCBPSI into calculus teaching and learning?
3. What effect does WMCBPSI have on students‘ calculus problem-solving skills?

This chapter summarises the study‘s results, aligns them with the research aim, objectives, and questions, and presents the study‘s conclusions and recommendations. It synthesises the major findings from the quantitative and qualitative phases of the study, highlights the contribution of WMCBPSI to calculus problem-solving instruction, and identifies implications for practice and future research.

9.1.2. Aggregated Study Results

The study achieved its aim and objectives and effectively addressed the research questions:

- Research Question 1: Section 5.3.1.1 detailed the integration of WMCBPSI into calculus instruction, confirming its feasibility in mathematics classrooms (see Section 8.4.1). The qualitative findings from classroom observations and interviews further supported this integration by showing that students experienced increased engagement, collaboration, and opportunities to apply varied problem-solving strategies.
- Research Question 2: Section 8.4.2 discussed the challenges of implementing WMCBPSI, including students' adaptation to a new instructional style, the need for interpersonal skills in collaborative learning, and difficulties associated with large class sizes. These challenges were examined using classroom observation data and researchers' field notes, which captured students' participation, interaction, adaptation to collaborative activities, and difficulties encountered during the intervention, alongside student interview findings that provided students' own perspectives on these experiences. The combined qualitative evidence indicated that, although WMCBPSI presented implementation challenges, these challenges were manageable through appropriate scaffolding, teacher support, and effective classroom organization.
- Research Question 3: Section 8.4.3 presented quantitative evidence showing that WMCBPSI significantly improved students‘ problem-solving skills compared to CPSI. These quantitative findings were reinforced by qualitative evidence from students' experiences, where experimental group students reported improved confidence, deeper conceptual understanding, and greater flexibility in approaching calculus problems.

Overall, the integration of quantitative and qualitative findings demonstrated convergence of evidence, indicating that WMCBPSI positively influenced students' calculus problem-solving skills through improved instructional design, cognitive load management, active engagement, and collaborative learning opportunities.

9.1.3. Achievement of Study Objectives

- Objective 1: Section 8.3.1 demonstrated that WMCBPSI, grounded in CLT, significantly enhanced students‘ calculus problem-solving skills.
- Objective 2: Section 8.3.2 confirmed through qualitative analysis that WMCBPSI outperformed CPSI in developing problem-solving abilities.
- Objective 3: Section 8.3.3 presented statistical results showing significant improvements in the experimental group‘s problem-solving performance ($p < 0.05$).

Overall, all the study‘s objectives were achieved, offering robust evidence of the effectiveness of WMCBPSI in enhancing calculus problem-solving skills.

9.1.4. Achievement of the Study Aim

The aim of this study—to investigate the effect of Working Memory–Context-Based Problem-Solving Instruction on the calculus problem-solving skills of first-year students—was successfully accomplished. The research design, grounded in CLT and implemented systematically, provided a robust framework for addressing the study‘s aim.

The results demonstrate a significant positive effect of WMCBPSI on students‘ problem-solving abilities. Empirical evidence confirmed that WMCBPSI improves problem-solving skills with statistical significance ($p < 0.05$). Thus, the aim of the study can be concluded as successfully attained.

9.2. CONCLUSIONS

The findings of this study provide compelling evidence that WMCBPSI is an effective pedagogical strategy for enhancing calculus problem-solving skills. Key conclusions include:

1. Integration of CLT Principles: Incorporating working memory contexts in instruction significantly reduced cognitive load, enabling students to develop and automate effective problem-solving schemas. The quantitative improvements in students’ problem-solving performance, together with qualitative evidence of improved engagement and strategy use, demonstrate that instructional designs grounded in Cognitive Load Theory can effectively support calculus learning.

2. Effectiveness of Worked Examples**:** The use of variable worked examples equipped students with diverse solution strategies, reducing the novelty effect and supporting deeper understanding of problem-solving processes. Students also demonstrated greater flexibility in applying these strategies to unfamiliar problems, indicating improved schema acquisition and transfer of learning.
3. Methodological Strength: The researcher‘s direct involvement in delivering both instructional methods ensured consistency and minimized instructor-related variability, enabling a clearer comparison of outcomes. This strengthened the internal validity of the study by ensuring that differences in students' performance could be attributed primarily to the instructional approaches rather than differences in teaching style or instructor expertise.
4. Pedagogical Impact: WMCBPSI not only improved problem-solving performance but also enhanced students‘ engagement and confidence in calculus, demonstrating its value as a transformative instructional approach. The integration of collaborative learning, structured guidance, and cognitive load management created a more supportive learning environment than conventional problem-solving instruction.
5. Theoretical Contribution: The study contributes empirical evidence supporting the application of Cognitive Load Theory to calculus problem-solving instruction. Specifically, it demonstrates how instructional strategies such as chunking, variable worked examples, split-attention reduction, and collaborative learning can be integrated into mathematics instruction to enhance students' problem-solving skills and conceptual understanding.

In summary, WMCBPSI emerges as a highly effective instructional strategy for improving calculus problem-solving skills, affirming the value of applying Cognitive Load Theory in mathematics instruction. The convergence of quantitative and qualitative findings provides strong evidence that WMCBPSI offers a practical and theoretically grounded approach for improving calculus teaching and learning in higher education.

9.3. RECOMMENDATIONS

Based on the findings and conclusions of this study, the following recommendations are proposed to enhance calculus problem-solving instruction:

Instructional Recommendations

1. Instructional Design: Calculus instruction should be purposefully designed to foster problem-solving skills by integrating strategies that reduce cognitive load and connect mathematical concepts to students‘ working memory. Incorporating principles such as chunking, worked examples, and split-attention reduction should be prioritized. This recommendation is supported by the significant improvements observed in students' problem-solving performance and the positive perceptions reported by students exposed to WMCBPSI.
2. Review of Methodologies: Educational institutions should review and align their instructional methodologies with Cognitive Load Theory (CLT) principles to ensure the effective development of problem-solving schemas. This alignment could enhance learning efficiency and deepen conceptual understanding in mathematics education. Professional development opportunities should also be provided to enable instructors to effectively implement CLT-informed instructional strategies.
3. Manageable Class Sizes: Class sizes should be maintained at a manageable level to facilitate active group work, meaningful peer interaction, and individualized guidance, all of which are critical for effective problem-solving instruction. The qualitative findings showed that effective collaboration, continuous scaffolding, and instructor support were important contributors to the success of WMCBPSI, all of which become more difficult to implement in overcrowded classrooms.
4. Integration of Working Memory Contexts: Instructors should deliberately integrate working memory contexts into calculus instruction to manage cognitive load effectively. This will allow students to allocate cognitive resources to higher-order problem-solving processes rather than unnecessary processing demands. The improved performance of the experimental group demonstrates the practical value of systematically incorporating these principles into mathematics instruction.
5. Maintain Academic Rigor: While implementing cognitive load–based strategies, care should be taken to preserve the depth and complexity of calculus content. A balance should be maintained between innovative teaching methods and the academic rigor necessary for high-quality mathematics education. Managing cognitive load should therefore be viewed as a means of enhancing conceptual understanding and problem-solving competence rather than simplifying mathematical content.

Institutional Recommendation

Educational institutions should support the implementation of WMCBPSI by providing appropriate instructional resources, encouraging collaborative learning environments, and promoting professional development programmes that enable mathematics instructors to apply Cognitive Load Theory principles effectively.

Research Recommendation

Future studies should build on the findings of the present study by investigating the implementation of WMCBPSI in different educational contexts, larger populations, and other mathematical and STEM disciplines. Such research will help establish the broader applicability and sustainability of the instructional approach.

9.4. POSSIBLE FURTHER STUDIES

This study examined the effect of WMCBPSI on first-year university students‘ calculus problem-solving skills. While the findings provide valuable insights, the study limitations and findings identify several areas that warrant further investigation. Future research should extend and refine the current findings by examining the effectiveness of WMCBPSI in broader educational contexts and under different instructional conditions.

Several promising avenues for further research emerge:

1. Longitudinal Studies on Retention and Transfer: Examine the long-term effects of WMCBPSI on the retention and transfer of calculus problem-solving skills, including application in other mathematical domains or real-life contexts. This recommendation arises from the relatively short intervention period used in the present study.
2. Expanding to Other Educational Levels: Conduct similar studies across secondary or postgraduate education to assess whether WMCBPSI yields comparable benefits. Such studies would determine whether the effectiveness of WMCBPSI extends beyond first-year university mathematics students.
3. Exploration of Cognitive Load Factors: Investigate how different cognitive load factors interact with working memory-based instructional approaches, particularly for complex calculus concepts.
4. Adaptation to Digital and Blended Learning Environments: Assess the effectiveness of WMCBPSI delivered through digital or hybrid platforms in remote or blended learning settings.

5. Diverse Contextual Applications: Examine the application of WMCBPSI in other STEM disciplines—such as physics, engineering, or computer science—to evaluate adaptability and effectiveness.
6. Cross-Cultural Comparative Studies: Explore WMCBPSI's effectiveness in different cultural or educational contexts to inform tailored interventions for diverse student populations.
7. Exploring Individual Differences: Investigate how individual differences—such as prior knowledge, working memory capacity, or learning preferences—mediate WMCBPSI's effectiveness.
8. Intervention on Non-Cognitive Skills: Assess how WMCBPSI influences motivation, anxiety, and self-efficacy alongside problem-solving skills.

Collectively, these research directions will contribute to a deeper understanding of how WMCBPSI can be refined, adapted, and implemented across diverse educational settings while extending the theoretical and practical contributions of the present study. Future applications of WMCBPSI should continue to align with emerging cognitive research, ensuring that instructional practices remain adaptive, evidence-based, and responsive to evolving educational contexts.

9.5. LIMITATIONS OF THE STUDY

While this study contributes valuable insights into calculus problem-solving instruction, it is important to acknowledge its limitations:

1. Sample Size and Group Assignment

The inability to randomly assign participants to experimental and comparison groups, along with the relatively small sample size, limits the generalizability of the findings. Future studies should employ random assignment and larger sample sizes. Furthermore, the use of a static-group comparison design limits the strength of causal inferences that can be drawn when compared with randomized experimental designs. Nevertheless, the convergence of quantitative and qualitative evidence strengthens confidence in the study's conclusions.

2. Duration of Intervention

The two-month intervention period may not have been sufficient to produce lasting changes in students' problem-solving schemas. Longer-term interventions could yield more significant and enduring results. Consequently, the findings should be interpreted as evidence of the short-term effectiveness of WMCBPSI, while its long-term influence on knowledge retention, transfer of learning, and schema automation requires further investigation.

3. Sample Scope

This study focused on first-year mathematics students at a single Ethiopian university and addressed calculus topics related to functions of one variable. Findings may not generalize to other departments, universities, or multi-variable calculus topics. Accordingly, the interpretation of the findings should remain within these contextual boundaries, and caution should be exercised when applying the results to different educational settings or student populations.

4. Access to Facilities

Differences in access to university facilities—such as libraries, computer laboratories, and learning resources—between groups may have influenced intervention effectiveness. Future studies should ensure equal access to facilities. Variations in institutional resources may have influenced students' opportunities for independent learning and practice, thereby affecting the implementation and outcomes of WMCBPSI. Future studies should control such contextual factors to improve the comparability and generalizability of findings.

These limitations highlight areas for refinement in future research, pointing to the need for continued investigation to strengthen and broaden the applicability of the findings.

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APPENDICES

APPENDIX A: ACHIEVEMENT TEST

ACHIEVEMENT TEST

SUBJECT : Calculus of functions of one variable

LEVEL : First year at university

TOPICS COVERED : Limits, Derivatives, and Integration

QUESTION TYPE : Long questions

TIME ALLOWED : 2:00 hours

INSTRUCTION : Start every question on a new page

1. Find numbers a and b such that $\lim_{x\to 0}\frac{\sqrt[3]{ax+b}-2}{x}=1$.
2. Is there a number that is exactly 1 more than its cube? Justify your answer.
3. Suppose *f* is a function with the property that $|f(x)| \leq x^2$ for all *x*. Find f' (0).
4. Find the maximum area that can be enclosed by a rectangle whose perimeter is p meters long.
5. If x, y and z are positive numbers, prove that

   $$\frac{(x^2+1)(y^2+1)(z^2+1)}{xyz} \geq 8$$

6. Car A is travelling west at $50\frac{km}{h}$ and car B is travelling north at $60\frac{km}{h}$. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other

when car A is 3 km and car B is 4km from the intersection?

7. The figure shows a horizontal line y=c intersecting the curve $y=8x-27x^2$. Find the number *c* such that the areas of the shaded regions are equal.

8. A solid is generated by rotating about the X-axis the region under the curve y=f(x), where f is a positive function and x≥0. The volume generated by the part of the curve from x=0 to x=b is b^2, for all b>0. Find the function f.

9. Suppose f(1)=f'(1)=0 , f'' is continuous on [0,1] and |f''(x)|≤3 for all x. Show that $|\int_0^1 f(x)\,dx| \le \frac{1}{2}$

**The test problems were derived from Thomson Learning, Inc 2008.*

APPENDIX B: INTERVIEW SCHEDULE WITH STUDENTS'

1. Do you think there is a need to teach problem solving in a calculus class? Why?
2. Do you think you have benefitted from the problem solving instruction implemented in your class? Why?
3. Do you think the problem solving instruction implemented in your class can improve students' problem solving skills in calculus if it were to be implemented in other classes? Why?
4. Do you think there were challenges in implementing the problem solving instruction in your class to influence your learning and problem solving skill in calculus? What are that challenges? How can we overcome these challenges?
5. What are the views of other students on the implementation of the problem solving instruction in your class?
6. How can you advise someone, maybe your teacher, to implement problem solving instruction effectively?

APPENDIX C: CLASSROOM OBSERVATION SCHEDULE

Observation focus variables
1. Students' reaction to the instruction implemented in their class. 2. Students adaptation to the instruction implemented in their class. 3. Influence of the instruction implemented on students' problem solving skills. 4. Challenges posed to students by their exposure to the instruction implemented in their class. 5. Students experiences in learning new strategies for problem solving during problem solving activities in class. 6. Students level of involvement and contribution during problem solving activities in class. 7. Students' problem solving approaches and strategies during problem solving activities in class. 8. Students' level of exposition to previously acquired knowledge of problem solving activities in class. 9. Students' ability to connect previously acquired knowledge and experiences to novel problem solving activities in class. 10. Students level of dependence to the researcher during problem solving activities in class. 11. Challenges related to students problem solving approach during problem solving activities in class.

APPENDIX D: TOOL TO EVALUATE STUDENTS PROBLEM SOLVING SKILL LEVELS

Table: TIMSS Scale based on Polya's Model

TIMSS Level	Interpretation	Descriptions
1	Very low	Very minimal attempt has been made to solve the problem (None of the steps in Polya's model were successfully executed).
2	Low	Although solutions are evidently erroneous, there is minimal understanding of the problem (Is the student able to identify the information given in problem and the goal of the problem?)
3	Intermediate	Can a student devise a plan to find a connection between the given information and the unknown that will enable him/her to calculate the unknown/to solve the problem? (Can a student devise a plan and recalling from memory some knowledge that links what is given and what is wanted.)
4	High	Reasoning through the problem and executing plan (Is the student able to carry out that plan and check each stage of the plan and write the details that prove that each stage is correct?)
5	Advanced	Can look back and check the validity of their solutions (evaluate the solution to see if it makes sense/reasonable, if error has been made in the solution, and/or if there is an easier way to solve the problem.)

APPENDIX E: A TOOL TO EVALUATE STUDENTS PROBLEM SOLVING STRATEGIES

Table: The problem-solving strategy model

Strategy code	Strategy category	Descriptions
1	Arithmetic method	It is used when the subject writes down a mathematical statement involving one or more operations on the numbers given in the problem.
2	Algebraic method	It is used when the subject chooses one or more unknowns as variables and equation(s) is (are) set up.
3	Model drawing Method	It is used when the solution is suggested by or follows a model or a diagram as in the Singapore bar model strategy.
4	Guess-and-check	It is used when the subject guess a solution to the problem and then check whether it satisfy the constraints in the problem or not. It involves the following steps: (a) Make a guess of a certain answer or the unknown in the problem based on an estimation; (b) Check if the constraints given in the question or implied from some of the question statements are satisfied. If all the constraints are satisfied, the guess is correct; the answer has been obtained or can be worked out. All processes will end at this point. If the constraints are not satisfied, the guess will be refined or adjusted, and another guess will be made, and then another round of guess-and-check will begin.
5	Looking for a pattern	It is used when the subject recognizes that some kind of pattern is occurring in the problem. The pattern could be geometric, or numerical, or algebraic. If a student can see regularity or repetition in a problem, he/she might be able to guess what the continuing pattern is and then prove it. It involves the following three steps:

		(a) Several specific instances/special cases/particular examples of a problem are explored and listed; (b) A pattern (or a conjecture, a generalisation, a hypothesis, or a common property among those special cases) is determined by investigating the special cases explored in step (a); and (c) A solution to the entire problem is found by applying the generalised result in Step (b) and then proving it.
6	Logical reasoning	Logical reasoning strategy is used when some forms of 'if-then' reasoning are used.
7	Indirect Reasoning	It is used when the subject attack a problem indirectly. In using proof by contradiction to prove that P implies Q (a) assume that P is true and Q is false; and (b) try to see why this can't happen. Somehow we have to use this information and arrive at a contradiction to what we absolutely know is true.
8	Work backwards	It is used when the subject imagines that the problem is solved and work backwards step-by-step, until he/she arrives at the given data. Then he reverses his steps and thereby constructs a solution to the original problem.
9	Draw a picture	It is used when the subject draws a picture and trace/visualise the problem situation pictorially until he/she finds an answer that satisfies the constraints of the problem.
10	Usefamiliar knowledge	It is used when the subject tries to recognize something familiar. Relate the given situation to previous knowledge. Look at the unknown and try to recall a more familiar problem that has a similar unknown.
11	Use an auxiliary aid	It is used when the subject introduce something new, extra, an auxiliary aid, to help make the connection between the given and the unknown. For instance, in a problem where a diagram is useful, the auxiliary aid could be a new line drawn in a diagram. In a more algebraic problem it could be a new unknown that is related to

		the original unknown.
12	Take cases	It is used when the subject splits a problem into several cases and gives a different argument for each of the cases. For instance, we often have to use this strategy in dealing with absolute value.
13	Use an analogue	It is used when the subject tries to think of an analogous problem, that is, a similar problem, a related problem, but one that is easier than the original problem. If a student can solve the similar, simpler problem, then it might give him the clues that are needed to solve the original, more difficult problem. For instance, if a problem involves very large numbers, you could first try a similar problem with smaller numbers. Or if the problem involves three-dimensional geometry, you could look for a similar problem in two-dimensional geometry. Or if the problem you start with is a general one, you could first try a special case.
14	Establish Sub goals	It is used when the subject sets sub goals (in which the desired situation is only partially fulfilled) to solve a problem. If the subject can first reach these sub goals, then he may be able to build on them to reach the final goal. We often use this strategy to solve complex problems.
15	Mathematical Induction	It is used when the subject use the following principle in proving statements that involve a positive integer *n*. PRINCIPLE OF MATHEMATICAL INDUCTION Let $S_n$be a statement about the positive integer n. Suppose that 1.S_1 True. 2.$S_{k+1}$is true wheneverS_k is true. Then is $S_n$true for all positive integers n.

APPENDIX F: SUMMARY OF STUDENTS TOTAL POST-TEST SCORES IN WRITING THE CALCULUS PROBLEMS OF THE ACHIEVEMENT TEST

Group	STUDENT CODE	Score (100%)
Experimental	1	54
	2	72
	3	29
	4	59
	5	81
	6	73
	7	73
	8	52
	9	19
	10	73
	11	53
	12	62
	13	44
	14	81
	15	55
	16	80
	17	73
	18	55
	19	30

20	21
21	53
22	68
23	80
24	59
25	52
26	53
27	71
28	75
29	60
30	38
31	67
32	38
33	76
34	71
35	71
36	60
37	73
38	53
39	45
40	73
41	75

Comparison	1	23
	2	67
	3	48
	4	71
	5	80
	6	28
	7	6
	8	25
	9	55
	10	64
	11	34
	12	87
	13	52
	14	10
	15	40
	16	58
	17	23
	18	48
	19	30
	20	63
	21	35
	22	21

23	14
24	25
25	62
26	75
27	63
28	25
29	71
30	34
31	14
32	42
33	22
34	54
35	56
36	56
37	30
38	40

APPENDIX G: SUMMARY OF STUDENTS PROBLEM SOLVING SKILLS IN WRITING THE CALCULUS PROBLEMS OF THE ACHIEVEMENT TEST

Group	STUDENT CODE	TEST QUESTIONS								
		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
Experimental	1	3	2	3	2	3	3	3	4	2
	2	4	4	2	3	4	4	1	4	3
	3	1	2	2	2	1	1	1	1	3
	4	4	3	3	5	4	5	2	5	3
	5	5	4	4	5	4	3	3	4	4
	6	5	4	4	4	3	5	3	5	3
	7	4	4	2	3	4	3	1	4	4
	8	3	3	3	1	3	1	2	4	4
	9	1	2	2	1	1	1	3	1	3
	10	5	4	4	4	4	4	5	5	3
	11	4	3	4	5	4	5	2	3	4
	12	4	4	1	3	1	3	3	2	4
	13	1	3	2	3	1	3	2	2	4
	14	4	4	4	4	4	4	2	3	4
	15	4	3	4	5	4	4	2	5	4
	16	5	4	4	5	4	5	2	4	3
	17	5	4	4	4	3	5	4	5	3
	18	3	2	3	3	3	3	3	1	4

	19	1	2	2	2	1	3	2	2	3
	20	1	2	2	2	1	1	5	1	3
	21	3	2	3	2	3	1	3	4	1
	22	4	4	2	4	1	4	2	3	3
	23	4	4	4	4	4	3	2	3	1
	24	3	2	3	3	3	4	3	1	4
	25	4	3	4	5	4	4	2	5	4
	26	4	3	4	5	4	4	2	5	4
	27	4	3	3	4	4	5	2	5	2
	28	5	4	4	4	4	4	2	5	4
	29	4	3	3	5	4	5	2	5	3
	30	4	3	4	4	4	4	2	5	4
	31	4	4	2	4	1	3	3	2	3
	32	1	3	2	3	1	3	2	2	4
	33	4	4	4	4	4	4	2	4	2
	34	4	3	3	5	4	5	2	5	3
	35	4	4	2	4	3	4	2	3	3
	36	4	4	3	3	1	4	3	1	4
	37	5	4	4	4	4	4	2	5	4
	38	4	3	4	5	4	5	2	5	4
	39	3	3	2	1	3	1	2	3	4
	40	4	3	4	4	4	5	2	5	3

	41	4	4	2	4	4	4	1	4	2
Comparison	1	1	2	2	2	1	1	2	1	3
	2	4	3	4	3	3	3	2	4	3
	3	3	3	2	2	1	1	2	2	3
	4	5	4	4	5	4	5	5	5	3
	5	4	3	4	4	4	5	2	2	1
	6	1	2	2	2	1	1	2	2	4
	7	1	2	2	1	1	1	1	1	1
	8	4	4	4	2	1	3	3	3	3
	9	3	2	3	2	1	1	2	1	1
	10	4	3	4	3	1	4	2	4	1
	11	1	3	2	2	1	3	2	2	4
	12	4	4	2	4	4	3	2	4	1
	13	3	2	2	1	1	1	2	2	2
	14	1	2	2	1	1	1	1	1	2
	15	3	3	2	1	1	1	2	2	4
	16	3	2	3	2	1	1	2	2	1
	17	4	4	4	4	4	3	3	3	3
	18	4	4	4	5	4	4	3	3	3
	19	4	3	4	3	1	3	3	4	3
	20	4	2	4	2	1	3	2	4	4
	21	1	3	2	1	1	3	2	2	4

	22	1	2	2	1	1	1	1	1	3
	23	1	2	2	1	1	1	1	1	3
	24	1	2	2	2	1	1	2	2	4
	25	4	2	3	2	1	3	2	2	1
	26	4	3	4	4	4	4	2	2	3
	27	5	4	4	4	4	5	4	5	4
	28	4	4	4	3	1	3	3	3	3
	29	4	3	4	3	3	4	2	5	3
	30	4	3	4	4	1	4	3	5	3
	31	4	4	2	4	4	3	2	5	2
	32	3	3	2	2	1	1	2	2	3
	33	1	2	2	2	1	1	2	1	3
	34	3	2	2	1	1	1	2	3	2
	35	5	4	4	4	4	5	3	4	3
	36	3	2	3	2	1	1	2	1	1
	37	1	3	2	2	1	1	2	2	4
	38	4	3	4	5	4	4	3	5	3

** 1=Very low; 2=Low;3=Intermediate;4=High;5=Advanced*

APPENDIX H: SUMMARY OF STUDENTS' PROBLEM SOLVING STRATEGIES IN WRITING THE CALCULUS PROBLEMS OF THE ACHIEVEMENT TEST

Group	STUDENT CODE	TEST QUESTIONS								
		Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
Experimental	1	14	2	2	2	7	9	2	6	4
	2	6	6	2	14	6	9	2	6	14
	3	1	6	8	2	2	1	2	14	14
	4	12	9	4	13	8	9	11	2	4
	5	14	9	12	4	14	9	11	14	14
	6	6	6	2	14	13	10	11	8	13
	7	6	7	2	14	6	9	2	14	6
	8	6	6	2	2	6	1	2	2	6
	9	1	4	2	2	2	1	11	8	6
	10	7	6	8	14	13	10	11	8	6
	11	14	6	12	9	6	9	2	6	14
	12	6	6	4	13	1	10	11	8	6
	13	1	6	11	9	2	9	2	2	6
	14	6	9	11	14	6	9	2	8	14
	15	7	9	12	14	8	10	11	2	14
	16	14	9	12	4	14	9	2	14	6
	17	6	6	8	14	13	10	11	6	13
	18	12	2	4	14	8	10	2	2	6

	19	1	7	8	4	2	9	2	2	12
	20	1	6	8	2	2	1	11	14	4
	21	6	2	2	2	7	1	2	2	8
	22	12	6	2	9	1	9	2	8	13
	23	6	9	11	14	6	10	2	6	8
	24	12	2	4	14	8	9	2	6	6
	25	6	6	12	9	6	9	2	14	14
	26	7	7	12	14	7	10	11	2	12
	27	12	9	11	9	13	9	2	6	13
	28	12	9	12	14	13	10	11	14	12
	29	12	9	4	13	9	9	11	2	4
	30	6	6	12	13	6	9	2	14	6
	31	12	6	2	9	1	10	11	8	13
	32	1	6	8	9	2	9	2	2	6
	33	6	7	8	14	6	9	2	6	13
	34	12	9	8	4	9	10	11	2	13
	35	14	6	2	9	14	9	2	8	6
	36	6	6	4	13	1	9	2	6	6
	37	12	6	12	14	13	10	11	14	6
	38	14	7	12	9	7	9	11	6	14
	39	6	6	11	2	6	1	2	6	6
	40	12	9	2	14	13	9	11	6	4

	41	6	7	2	14	6	9	2	14	4
Comparison	1	1	6	2	2	1	1	2	14	12
	2	6	9	2	14	6	10	11	14	4
	3	6	6	2	4	1	1	2	8	4
	4	12	9	12	4	13	9	11	14	13
	5	6	9	12	14	6	9	11	8	8
	6	1	7	8	2	2	1	2	2	6
	7	1	2	2	2	1	1	2	6	4
	8	12	6	12	4	2	10	2	8	4
	9	12	2	2	2	2	1	2	2	4
	10	6	9	2	13	2	9	11	6	8
	11	1	6	2	2	2	9	2	2	6
	12	6	6	8	14	6	9	11	14	8
	13	6	2	2	2	2	1	2	8	13
	14	1	2	2	2	1	1	2	6	4
	15	6	6	2	2	1	1	2	2	6
	16	12	2	8	2	2	1	2	2	8
	17	6	6	12	14	6	9	2	6	6
	18	12	6	12	4	13	10	11	2	13
	19	12	6	12	9	2	10	2	2	4
	20	6	2	2	2	2	10	11	6	14
	21	1	6	2	2	1	9	2	2	12

	22	1	4	2	2	1	1	2	14	14
	23	1	4	2	2	1	1	2	6	6
	24	1	6	2	2	2	1	2	2	6
	25	6	2	8	2	2	10	11	6	8
	26	6	9	12	9	6	10	11	8	6
	27	12	9	12	14	13	9	11	14	12
	28	12	6	12	9	2	10	2	8	4
	29	6	9	8	14	13	9	11	2	13
	30	12	6	12	14	2	9	2	6	13
	31	6	6	8	14	6	9	11	6	4
	32	6	6	2	4	1	1	2	6	13
	33	1	4	2	2	1	1	2	14	12
	34	6	2	8	2	2	1	2	6	13
	35	6	6	12	14	6	10	11	14	6
	36	12	2	2	2	2	1	2	6	13
	37	1	6	8	2	2	1	2	2	6
	38	12	7	12	13	13	9	11	14	13

**1=Arithmetic method;2=Algebraic method; 3=Model drawing method; 4=Guess-and-check;5=Looking for a pattern;6=Logical reasoning; 7=Indirect reasoning;8=Work backwards;9=Draw a picture;10=Use familiar knowledge;11=Use an auxiliary aid;12=Take cases; 13=Use an analogue; 14=Establish sub goals;15=Mathematical induction.*

APPENDIX I: SUMMARY OF STUDENTS PROBLEM SOLVING SKILL LEVELS AND THEIR SOLUTION STRATEGIES IN WRITTING EACH ACHIEVEMENT TEST QUESTION

Solution	Test questions		
Strategy	Q1	Q2	Q3
Arithmetic	$E^{1}_{3,21,36,7,41,34}$ $C^{1}_{7,1,24,21,3,20,10,2,29,30,27}$		
Algebraic		$E^{2}_{9,2,33,30}$ $C^{2}_{24,37,32,3,34,25,26,38,35}$	$E^{2}_{20,39,21,23,29,6,5}$ $E^{3}_{9,8,33}$ $E^{4}_{13,26}$ $C^{2}_{7,23,24,21,32,3,13,20,10,2,29,19,30}$ $C^{3}_{37,35}$ $C^{4}_{14,11,25}$
Model drawing			
Guess-and-check		E^{2}_{21} $C^{2}_{10,2,30}$	E^{1}_{24} $E^{3}_{19,2,30,15,17}$
Pattern			
Logical reasoning	$E^{3}_{8,33,28}$ $E^{4}_{20,39,24,12,14,25,4,27,17,5}$ $E^{5}_{13,35}$	$E^{2}_{3,41}$ $E^{3}_{8,18,36,25,4,34,28}$ $E^{4}_{20,13,1,24,35,23,29,6,17,10}$	

	$C^3_{23,32,13,19,38}$ $C^4_{14,33,11,15,9,25,26,5,17,28}$ C^5_{18}	$C^2_{7,29}$ $C^3_{23,21,13,16,20,8,19,27}$ $C^4_{6,15,9,36,31,28,18}$	
Indirect Reasoning	$E^4_{31,11}$ E^5_1	E^2_7 $E^3_{11,37}$ $E^4_{39,27,5}$ C^2_1 C^3_4	
Work backwards			$E^2_{3,7,41,34}$ E^3_{40} $E^4_{1,35,27}$ $C^2_{1,15,28,38,27}$ $C^3_{34,26}$ C^4_{17}
Draw a picture		$E^3_{19,31,38,15,40,26}$ $E^4_{32,12,22,14,26}$ $C^3_{14,33,11,5,17}$ $C^4_{22,12}$	

Use familiar knowledge			
Use an auxiliary aid			$E^{2}_{36,28}$ E^{3}_{38} $E^{4}_{12,14}$
Take cases	$E^{3}_{2,30}$ $E^{4}_{19,23,38,15,29,40,26}$ $E^{5}_{26,10}$ $C^{3}_{37,34,35}$ $C^{4}_{6,36,16,31,8,4}$ $C^{5}_{22,12}$		$E^{4}_{32,18,31,22,25,11,26,4,10,37}$ $C^{4}_{22,33,6,9,36,16,5,12,31,8,18,4}$
Use an analogue			
Establish Sub goals	E^{3}_{9} $E^{4}_{18,6,37}$ $E^{5}_{32,22}$		
Mathematical Induction			

$E^k_{i,j,\dots}$=a student in experimental group whose student code is i,j,...is at a problem solving skill level of k ;

$C^k_{i,j,\dots}$=a student in comparison group whose student code is i, j, ... is at a problem solving skill level of k

Continued...

Solution Strategy	Test questions Q4	Q5	Q6
Arithmetic		$E^1_{24,23,29,17}$ $C^1_{7,23,24,3,13,20,10,2,19,30}$	$E^1_{3,8,21,41,33,28}$ $C^1_{7,23,1,24,37,32,3,13,34,10,2,29,19,30,38,35,}$
Algebraic	$E^1_{8,21,28}$ $E^2_{9,3,41,33}$ $C^1_{24,32,3,13,20,10,2,38}$ $C^2_{7,1,37,21,34,25,29,26,30,35,}$	$E^1_{3,21,36,7,41,34}$ $C^1_{1,6,37,11,21,32,34,16,25,29,26,31,8,38,3}$	
Model drawing			
Guess-and-check	E^2_{7} $E^5_{32,22,40}$ $C^2_{23,6,19}$ $C^5_{22,36}$		

Pattern			
Logical reasoning		$E^{3}_{8,28}$ $E^{4}_{20,39,18,12,14,25,4,27,5}$ C^{3}_{14} $C^{4}_{33,15,9,5,28,18}$	
Indirect Reasoning		$E^{3}_{9,33}$ $E^{4}_{11,37}$	
Work backwards		$E^{3}_{2,30}$ $E^{4}_{19,31}$	
Draw a picture	$E^{3}_{36,34}$ $E^{4}_{23,38,29,6}$ $E^{5}_{18,25,37}$ $C^{3}_{16,31}$ C^{4}_{5}	$E^{4}_{15,40}$	$E^{3}_{9,32,39,36,7,34}$ $E^{4}_{20,12,23,30,25,4,27,6,17,5}$ $E^{5}_{19,18,22,38,15,37,26}$ $C^{3}_{21,15,9,20,28}$ $C^{4}_{11,17,8,4}$ $C^{5}_{22,33,12}$

Use familiar knowledge			$E^{3}_{24,2,14,29}$ $E^{4}_{1,31,11,26,10}$ $E^{5}_{13,35,40}$ $C^{3}_{14,6,16,25,26,31}$ $C^{4}_{36,5}$ C^{5}_{18}
Use an auxiliary aid			
Take cases			
Use an analogue	$E^{3}_{24,17}$ E^{4}_{4} $E^{5}_{19,15}$ C^{3}_{11} C^{5}_{4}	$E^{3}_{13,35}$ $E^{4}_{1,38,26,10,26}$ C^{3}_{17} $C^{4}_{22,36,12,4}$	
Establish Sub goals	$E^{3}_{20,39,2,30}$ $E^{4}_{13,1,12,35,14,26,27,10,26,5}$ $E^{5}_{31,11}$	E^{3}_{6} $E^{4}_{32,22}$	

	$C^{3}_{14,17}$ $C^{4}_{33,15,9,12,8,28,18}$		
Mathematical Induction			

$E^{k}_{i,j,\ldots}$=a student in experimental group whose student code is i,j,...is at a problem solving skill level of k ;

$C^{k}_{i,j,\ldots}$=a student in comparison group whose student code is i, j, ... is at a problem solving skill level of k

Continued...

Solution Strategy	Test questions Q7	Q8	Q9
Arithmetic			
Algebraic	$E^{1}_{20,3,39,5}$ $E^{2}_{8,18,36,12,22,7,23,14,25,38,4,34,27,6,28}$ $E^{3}_{9,2,33,30,17}$ $C^{1}_{24,3,10,2}$ $C^{2}_{7,23,1,37,21,32,13,34,20,29,19,30,38,35,}$ $C^{3}_{6,9,16,31,8}$	E^{1}_{2} $E^{2}_{36,7,34}$ $E^{4}_{8,33}$ $E^{5}_{19,31,11,15,40}$ C^{1}_{37} $C^{2}_{1,21,13,34,20,29,27}$ C^{3}_{36}	

		C^4_{16} C^5_{17}	
Model drawing			
Guess-and-check			$E^2_{9,5}$ $E^3_{19,41,15,26}$ $C^1_{24,37}$ $C^2_{3,28}$ $C^3_{14,23,6,16,31}$
Pattern			
Logical reasoning		$E^1_{30,17}$ $E^3_{18,14,28}$ $E^4_{9,20,27}$ $E^5_{35,38,37,26}$ $C^1_{24,3,2,35}$ $C^2_{26,19}$ $C^3_{9,38}$ $C^4_{11,25}$ $C^5_{8,28}$	$E^3_{21,1,22,6}$ $E^4_{39,8,24,36,2,30,4,34,17,10,28}$ $C^3_{9,2,5,18}$ $C^4_{1,21,13,29,27}$
Indirect Reasoning			
		E^1_{21} $E^2_{24,29}$	$E^1_{33,14}$ $C^1_{33,11,15,34,26}$

Work backwards	$E^{3}_{12,23,6}$ $E^{5}_{13,1}$ $C^{2}_{23,33,32,5}$ $C^{3}_{6,31}$
Draw a picture	
Use familiar knowledge	
Use an auxiliary aid	$E^{2}_{19,31,11,26,15,40,10,37,26}$ $E^{3}_{32,13,21,24,29}$ E^{4}_{35} $E^{5}_{1,41}$ $C^{2}_{14,33,11,15,25,26,5,17,28}$ $C^{3}_{36,18,4}$ C^{4}_{12} C^{5}_{22}
Take cases	E^{3}_{7} $E^{4}_{11,26}$ $C^{3}_{7,30}$ $C^{4}_{20,12}$
Use an analogue	$E^{2}_{38,27}$ $E^{3}_{13,35,23,29,40}$

			C^{1}_{35} $C^{2}_{32,38}$ $C^{3}_{22,36,17,8,19,4}$
Establish Sub goals		$E^{1}_{3,41}$ $E^{4}_{32,39,22,5}$ $E^{5}_{25,26,4,10}$ $C^{1}_{7,10,30}$ $C^{4}_{14,15,18}$ $C^{5}_{22,12,4}$	$E^{3}_{20,3}$ $E^{4}_{32,18,12,31,25,37}$ C^{3}_{10} C^{4}_{25}
Mathematical Induction			

$E^{k}_{i,j,\ldots}$=a student in experimental group whose student code is i,j,…is at a problem solving skill level of k ;

$C^{k}_{i,j,\ldots}$=a student in comparison group whose student code is i, j, … is at a problem solving skill level of k

APPENDIX J

WMCBPSI LESSON PLAN

Course: Calculus of Functions of One Variable
Topics:

1. Optimization Problems Using Derivatives
2. Related Rates as an Application of Derivatives

Duration: 100 minutes
Target group: First-year university students
Context: Ethiopian agriculture and irrigation

Learning Objectives

By the end of the lesson, students will be able to:

1. identify the objective and constraints in an optimization problem;
2. formulate a one-variable objective function;
3. use derivatives to determine maximum or minimum values;
4. interpret optimization results in a real-life agricultural context;
5. identify changing quantities in a related-rates problem;
6. formulate an equation relating the changing quantities;
7. use implicit differentiation to determine an unknown rate of change; and
8. interpret and verify the solution in a real-life irrigation context.

Prior Knowledge

Students are expected to know:

- functions of one variable;
- algebraic manipulation;
- differentiation;
- critical points;
- implicit differentiation; and
- basic geometric formulas.

Instructional Materials

Board/markers, contextual problem sheets, diagrams, worked examples, group worksheets, and individual assessment sheets.

Lesson Procedure

Pólya's Stage	Time	Topic and Main Activities	WMCBPSI Features
1. Understand the Problem	10 min	Optimization: Present an Ethiopian agricultural fencing problem. Students identify the known quantities, unknown variables, objective, and constraint.	Contextual learning, chunking, self-explanation
2. Devise a Plan	8 min	Optimization: Students determine the constraint, express one variable in terms of the other, and formulate the one-variable area function.	Worked-example effect, chunking, reduced split attention
3. Carry Out the Plan	22 min	Optimization: Students differentiate the objective function, find the critical point, determine the maximum, and complete a short group practice problem.	Worked examples, collective working memory, self-explanation

4. Look Back	10 min	Optimization: Students verify the constraint, confirm the maximum, and interpret the result in the agricultural context.	Self-explanation, schema consolidation
5. Understand the Problem	10 min	Related Rates: Present an irrigation-tank problem. Students identify the quantities that are changing, the given rate, and the rate that must be determined.	Contextual learning, chunking, reduced split attention
6. Devise a Plan	8 min	Related Rates: Students identify the geometric relationship between the variables and determine which equation should be differentiated.	Worked-example effect, chunking, self-explanation
7. Carry Out the Plan	22 min	Related Rates: Students differentiate the relationship with respect to time, substitute the known values and rates, and solve the problem collaboratively.	Collective working memory, worked examples, variable examples
8. Look Back	10 min	Related Rates: Students check units, signs, and reasonableness of the answer and interpret the rate in the irrigation context.	Self-explanation, schema consolidation, independent application
Total	100 min		

TOPIC 1: OPTIMIZATION PROBLEMS USING DERIVATIVES

Pólya Stage 1: Understand the Problem

Contextual Worked Example

A farmer has 120 m of fencing to construct a rectangular vegetable plot beside an irrigation canal. The canal forms one side, so only three sides require fencing. Find the dimensions that maximize the area.

Let x be the width and y the length.

Students identify:

Known information:
The farmer has 120 m of fencing.

Unknown:
The width x and length y.

Objective:
Maximize the area:

$A=xy.$

Constraint:
Only three sides require fencing:

$2x+y=120.$

Students explain in their own words what the problem is asking and why the canal is not included in the fencing constraint.

Pólya Stage 2: Devise a Plan

Students determine how to reduce the problem to a one-variable optimization problem.

From the constraint,

$2x+y=120.$

Express y in terms of x:

$y=120-2x.$

Substitute into the area formula:

$A=xy$.

Therefore,

$A(x)=x(120-2x)$

and

$A(x)=120x-2x2$.

Students explain why expressing one variable in terms of the other is necessary.

Pólya Stage 3: Carry Out the Plan

Differentiate:

$A'(x)=120-4x$.

Set the derivative equal to zero:

$120-4x=0$.

Thus,

$x=30$.

Substitute $x=30$ into the constraint:

$y=120-2(30)=60$.

Therefore, the dimensions are:

$30\ \text{m}\times 60\ \text{m}$.

The maximum area is:

$A=30\text{m}(60\text{m})=1800\ \text{m}^2$.

To verify that this is a maximum:

$A''(x)=-4<0$.

Therefore, the critical point gives a maximum.

Short Group Practice

A farmer has 100 m of fencing and an existing wall forms one side of a rectangular storage yard. Determine the dimensions that maximize the area.

Students formulate:

$2x+y=100,$

$y=100-2x,$

$A(x)=x(100-2x).$

They then use

$A'(x)=0$

to determine the dimensions.

Students explain the reasoning behind each step rather than simply giving the answer.

Pólya Stage 4: Look Back

Students verify:

$2(30)+60=120.$

They confirm that the available fencing is fully used and that

$A''(30)<0$

confirms a maximum.

Students interpret the result:

A rectangular plot measuring 30 m by 60 m gives the farmer the largest possible cultivated area, 1800 m^2, under the given fencing constraint.

Students briefly discuss whether the same method could be used for other agricultural optimization problems.

TOPIC 2: RELATED RATES AS AN APPLICATION OF DERIVATIVES

Pólya Stage 1: Understand the Problem

Contextual Worked Example

Water is being pumped into a cylindrical irrigation tank at a rate of 0.5 m3/min.

The tank has a radius of 2 m. At what rate is the water level rising when the water depth is 3 m?

Let:

- V = volume of water in the tank;
- h = height of the water;
- r=2 m = radius of the tank.

Students identify:

Given:

$\frac{dV}{dt}$=0.5 m3/min,

r=2 m.

Unknown:

$\frac{dh}{dt}$.

Relationship:

The volume of a cylinder is related to its height by

V=πr2h.

Students explain what quantity is changing and what rate must be found.

Pólya Stage 2: Devise a Plan

Students identify the equation relating volume and height:

V=πr^2h.

Since the radius is constant, differentiate both sides with respect to time t:

$\frac{dV}{dt}=\pi r^2\frac{dh}{dt}$.

The plan is therefore:

1. Write the geometric relationship.
2. Differentiate with respect to time.
3. Substitute the known radius and volume rate.
4. Solve for the rate at which the water level is rising.
5. Check the units and interpret the result.

Pólya Stage 3: Carry Out the Plan

Starting with

$V=\pi r^2 h$,

differentiate with respect to time:

$\frac{dV}{dt}=\pi r^2\frac{dh}{dt}$.

Substitute

$\frac{dV}{dt}=0.5$

and

$r=2$.

Therefore,

$0.5=\pi(2)^2\frac{dh}{dt}$.

Thus,

$0.5=4\pi\frac{dh}{dt}$.

Therefore,

$\frac{dh}{dt}$=0.54π=18π m/min.

Hence,

$\frac{dh}{dt}$=18π m/min

or approximately

0.0398 m/min.

The water level is therefore rising at approximately 0.0398 m/min, or about
3.98 cm/min.

Group Practice

Students solve a similar irrigation problem in groups:

Water enters a cylindrical irrigation tank with a radius of 3 m at a rate of 0.8 m3/min. Determine the rate at which the water level rises.

Students identify the variables, formulate the volume relationship, differentiate with respect to time, and solve for the unknown rate.

Pólya Stage 4: Look Back

Students check:

1. Are the units correct?
   The answer should be in metres per minute.
2. Is the sign reasonable?
   Since water is entering the tank, the water level should be increasing, so the rate should be positive.
3. Does the answer make sense in context?
   The water level rises gradually because the tank has a relatively large cross-sectional area.

4. Can the same strategy be used elsewhere?
   Students identify other situations involving changing quantities, such as irrigation tanks, water channels, crop-storage containers, and filling reservoirs.

Variable Examples for Related Rates

Groups solve similar problems using different values and contexts:

- Irrigation tank: changing water volume and water depth.
- Water channel: changing water level and cross-sectional dimensions.
- Agricultural storage container: changing volume and height.

Students identify the common mathematical structure:

Relationship between quantities→Differentiate with respect to time→Substitute known rates →Find unknown rate.

The emphasis is on forming a schema for related-rates problems rather than memorizing individual solutions.

Individual Performance Task

Optimization

A cooperative has 240 m of fencing to construct a rectangular irrigation holding area beside an existing boundary. Determine the dimensions that maximize the area and interpret the result.

Students independently apply Pólya's four stages.

Related Rates

An irrigation tank is cylindrical with radius 2.5 m. Water enters the tank at a rate of 1 m3/min. Determine the rate at which the water level rises.

Students independently:

1. understand the problem;

2. devise a plan;
3. carry out the mathematical procedure; and
4. look back by checking units, signs, and reasonableness.

WMCBPSI Features Embedded in the Lesson

Feature	Application
Chunking	Each problem is organized into Pólya's four manageable stages.
Collective working memory	Students solve selected optimization and related-rates problems collaboratively.
Worked examples	A complete example is provided before students solve partially worked and independent problems.
Reduced split attention	Context, diagrams, variables, equations, and mathematical information are presented together.
Variable examples	Similar problems are presented with different numbers and agricultural/irrigation contexts.
Self-explanation	Students explain why each equation, differentiation step, and solution is appropriate.
Schema formation	Students identify the common structure of optimization and related-rates problems.
Independent schema application	Students independently apply Pólya's four-stage process to novel problems.

Time Summary

Topic	Understand	Devise	Carry Out	Look Back	Total
Optimization Problems Using Derivatives	10 min	8 min	22 min	10 min	50 min
Related Rates	10 min	8 min	22 min	10 min	50 min
Total	20 min	16 min	44 min	20 min	100 min

APPENDIX K: ETHICAL CLEARANCE

College of Science, Engineering and Technology_ School of Science_ERC

NHREC Registration # : (if applicable)
Ref #: 6147
Name: Mr KUMLACHEW TADESSE DERSEH
Student #: 49026356

Date: 28/03/2025

Dear: Mr KUMLACHEW TADESSE DERSEH

Decision: Ethics Approval from 28/03/2025 to 29/03/2030

Researcher: Mr KUMLACHEW TADESSE DERSEH

School of Science

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Supervisor: Prof SJ Johnston johnssj@unisa.ac.za

EFFECT OF WORKING MEMORY-CONTEXT-BASED PROBLEM SOLVING INSTRUCTION ON THE CALCULUS PROBLEM SOLVING SKILLS OF FIRST YEAR STUDENTS AT AN ETHIOPIAN UNIVERSITY

Qualification: DOCTOR OF PHILOSOPHY IN MATHEMATICS, SCIENCE AND TECHNOLOGY EDUCATION

Thank you for the application for research ethics clearance by the College of Science, Engineering and Technology_ School of Science_ERC for the above mentioned research study Ethics approval is granted for five years.

The **low risk application** was **reviewed** by College of Science, Engineering and Technology_ School of Science_ERC on 28/03/2025 in compliance with the Unisa Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the College of Science, Engineering and Technology_ School of Science_ERC .
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing, accompanied by a progress report.

5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.

6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.

7. No field work activities may continue after the expiry date (29/03/2030). Submission of a completed research ethics progress report will constitute an application for renewal, for Ethics Research Committee approval.

Additional Conditions

1. Disclosure of data to third parties is prohibited without explicit consent from Unisa.
2. De-identified data must be safely stored on password protected PCs.
3. Care should be taken by the researcher when publishing the results to protect the confidentiality and privacy of the university.
4. Adherence to the National Statement on Ethical Research and Publication practices, principle 7 referring to Social awareness, must be ensured: "Researchers and institutions must be sensitive to the potential impact of their research on society, marginal groups or individuals, and must consider these when weighing the benefits of the research against any harmful effects, with a view to minimising or avoiding the latter where possible." Unisa will not be liable for any failure to comply with this principle.
5. Kindly note that the College of Science, Engineering and Technology_ School of Science_ERC requires the submission of regular progress reports to be submitted quarterly. Inline with section 7.2 of the Unisa Policy on Research Ethics (2024).

Note

The reference number 6147 should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.

Kind regards,

Prof L.L Noto
Chair of College of Science, Engineering and Technology_ School of Science_ERC
E-mail: notoll@unisa.ac.za

Prof. ZL Linganiso
By delegation from the Executive Dean of College of Science, Engineering and Technology_ School of Science_ERC
E-mail: lingazk@unisa.ac.za

APPENDIX L: CONSENT LETTERS

(1) Request to validate the achievement test

Kumlachew Tadesse Derseh
Debre Berhan University
P.O.box 445
Cell: 0918020136
E-mail: 49026356@mylife.unisa.ac.za

Dear __________________
Re: Permission to assist in the validation of an achievement test for first year students

My name is Kumlachew Tadesse Derseh. I am studying a PhD in Mathematics Education, at the University of South Africa (UNISA). The purpose of my study is to develop a working memory-context-based problem solving model to enhance students' problem solving skills in calculus. As part of the research I need to collect data from students'. The collection of data will involve the administration of an achievement test to students and interviews with these students, and observing students during instruction. The results from this study will inform both policy and practice. I have already discussed my research with some colleagues who have provided in-practice support. As a requirement, the data collection instruments must be validated prior to administration to the participants. I therefore request you to assist me with the validation of the achievement test. A questionnaire has been designed for this purpose, and is attached with this request.
After reading this letter, please tick the appropriate option: agree or not to agree. Participation is strictly voluntary. Should you wish to get more information, my contact details are on the top of this request letter.

Thank You,

Yours sincerely
Kumlachew Tadesse Derseh

(2) Request for permission to use the university as experimental site

Kumlachew Tadesse Derseh
Debre Berhan University
P.o.box 445

To: Research and Community Service V/President
Debre Berhan University
Debre Berhan

Dear sir,

First and above all I would like to present my salutation. I think this letter will address you very well. As you know I am a member of the College of Natural and Computational Science at Debre Berhan University and I am studying my PHD at UNISA, with specialisation in mathematics education. I am requesting to get permission so as to carry out my research studies at the university using a group of first year students enrolled to physics department and taught calculus by me since 2011, and to use data for my research studies. It is my pleasure if you could please understand why the research is being done and what it will involve before you decide to accept my request.

Today there is a huge need for Ethiopia to produce skilled enough students to cop and successfully manages as posed by modern technology which is used in the workplace and have necessary problem solving skills. Therefore, there has to be a means that enables students to develop these skills in all subjects in general and mathematics in particular. Problem of quality education is hot issue of our time here in Ethiopia. This research may contribute some input for solving the problem. Having this in mind I want to see effect of working memory-context- based problem solving instruction on the calculus problem solving skills of students' at DBU.

In the study I want to teach concepts in calculus to a group of first year students enrolled to physics department using working memory-context-based problem solving instruction and see the effect on the calculus problem solving skills of students. I will make every effort to protect students' privacy. I will not use their name in any of the information I get from this study or in any of the research reports. Any information that I get in the study will be recorded with a code number that will let me know who the student is. When the study is finalized, the key that shows which code number goes with student's name will be destroyed. Any information that I get from this research will be used in any way I think is best for publication.

If you have any questions about the research or any related matters, please contact the researcher at:

Mr. KumlachewTadesse Derseh

College of natural and computational science

Debre Berban University

P.o.box 445

Cell: +251918020136

Email: 49026356@mylife.unisa.ac.za

Or

The researcher supervisor at

Prof L. David Mogari

Institute for Science & Technology Education

University of South Africa

P.O.Box 392, UNISA, 0003, South Africa

Tel: +27 12 429 3904, Fax: +27 12 429 8690

Email: mogarld@unisa.ac.za

Or

UNISA College of Graduate Studies Ethics Committee at

Prof A.Minnaar

Graduate Studies Ethics Committee Chairperson

Email: minnaa@unisa.ac.za

Consent:

By signing this consent form, I confirm that I have read and understood the information and have had the opportunity to ask questions about the research studies. I understand that the study is important and have accepted the researcher's ethical consideration with reference to participant students and the institution. I understand that the study will not cause any physical and/ or emotional harm on participant students. I have accepted the researcher's request to conduct his studies.

Name:__________________________ Signature_____________ Date________________

Research and Community Service V/President

Debre Berhan University

. P.o. box 445

Tel: +2519....................., Fax: +251..........................., Email:.............................

Researcher

Name______________________________Signature______________Date________

(3) Request for student participation

Kumlachew Tadesse Derseh
Debre Berhan University
P.o.box 445
Cell : 0918020136
E-mail : 49026356@mylife.unisa.ac.za

Dear student,

Re: Request for your participation in research

My name is Kumlachew Tadesse. I am currently doing a PhD degree in mathematics education at the University of South Africa. As a student for the PhD programme I will be conducting an educational research with first year university students, at DBU. The topic of my research is "*Investigating the effect of implementing a working memory-context-based problem solving instruction on students' calculus problem solving skills*", and the purpose of my research is to improve students' calculus problem solving skills. If you agree to participate in this research, you will participate in a series of problem solving activities. I will also be administering a post-test in order to track progress. Results from these tests will simply be used to track problem solving progress.

During lessons, you will also be observed on your progress. At the end of the project you might be interviewed in order to give opinion of a working memory-context-based problem solving instruction. The benefits of this research study consist of improving calculus problem solving skills and greater preparation for the future. Participation is completely voluntary. Your name and program results will not be released. I am only interested in seeing how to provide you with the best education.
If you have any questions please feel free to contact me. My telephone number is: 0918020136.
Hoping to hear from you soon.

Mr. Kumlachew Tadesse

Please sign and return the bottom portion of this consent form as soon as you have read the letter above.

I,.., acknowledge that the researcher has explained to me the need for this research, explained what is involved and offered to answer any questions. I freely and voluntarily consent to participate in this research. I understand all information gathered during the research will be completely confidential.

Name of student: ....
Signature: ....
Date: ....