Using visualisation to teach grade 12 heights and distances: A study in one secondary school in Zululand district of KwaZulu-Natal province

by

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DECLARATION

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DEDICATION

This thesis is dedicated to the memory of my beloved mother, Bawinile Thembelihle Nzuza, whose love, sacrifices, and unwavering belief in me continue to guide my path. The last words I heard from her as I was walking out of the hospital ward were, “ufunde njalo,” meaning I should continue studying. Those words have remained with me, providing strength during the most challenging moments of this journey. Though she is no longer here to witness this achievement, I carry her love and wisdom in my heart, and I hope this work makes her proud.

Not forgetting my grandfather, Lymon Nzuza, my uncle, Thami Nzuza, and my grandmother, Jabhile Zulu, for raising me after I lost my mother. I am grateful for their love and the strong foundation they have given me. Lastly, I encourage my sisters, Velile Nzuza, Talent Nzuza, and Nontobeko Mkhize, to continue studying, pursue their dreams with determination, and never stop believing in themselves.

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LIST OF ABBREVIATIONS AND ACRONYMS

2D	Two dimensional shape
3D	Three dimensional shapes
ATP	Annual Teaching Plan
CAPS	Curriculum and Assessment Policy
CAST	Cosine, All, Sine and Tan
CTML	Cognitive Theory of Multimedia Learning
DBE	Department of Basic Education
FET	Further Education and Training
SACMEQ	Southern Africa Consortium for the Measurement of Educational Quality
SPSS	Statistical Package for the Social Sciences
TIMSS	Trends in International Mathematics and Science Study

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ABSTRACT

Visualisation has been widely used to enhance learners' conceptual understanding of problem solving in mathematics. The purpose of this study was to explore the effectiveness of visualisation in teaching and learning heights and distances in Grade 12 learners and to determine how visualisation may influence learners' understanding and performance when solving height and distance problems. This study adopted a mixed-methods approach, utilising both quantitative and qualitative methods. A pre-test-intervention-post-test design was employed to assess the effectiveness of visualisation in teaching heights and distances in trigonometry. A pre-test and post-test were administered to 60 Grade 12 learners, divided into experimental and control groups. Semi-structured interviews were conducted with the two teachers from the experimental and control groups, along with lesson observations before, during, and after the intervention. Thematic analysis was applied to analyse the semi-structured interviews and the lesson observation data. The researcher utilised Excel and SPSS tools for statistical data analysis. Microsoft Excel was used to record pre- and post-test scores for the experimental and control group results. This study proposed potential methods for using visualisation to teach and learn about heights and distances in trigonometry. The findings revealed that, prior to the intervention, the control group outperformed the experimental group in most questions on the pre-test. Following the intervention, learners' performance in the experimental group improved, resulting in better outcomes than the control group in the post-test. This indicates that the implementation of visualisation positively impacted the performance of the experimental group.

Keywords: height and distance; mathematics education; trigonometry; visualisation, intervention, performance

CHAPTER 1: ORIENTATION TO THE STUDY

1.1 INTRODUCTION AND BACKGROUND

Low learner achievement in mathematics is a problem in South Africa (SA). According to Dean (2019), learners are not performing well in both science and mathematics despite the 30% pass mark in both these subjects. Reports show that poor performance in mathematics occurs across all levels of schooling in SA, which means that learners struggle with mathematics from primary school to higher education. Consequently, SA has continued to be ranked among the lowest in the mathematics and science assessment performance in the Southern Africa region.

Results from Trends in International Mathematics and Science Study (TIMSS, 2023) indicate that South African learners' performance in mathematics is below their equals globally. At the same time, the Southern and Eastern African Consortium for Monitoring Education Quality (SACMEQ) (2016) ranked SA position eight (8) out of fifteen (15) countries. South Africa's average score was below the average of all developing countries, such as the Philippines, Malaysia, Tunisia and Indonesia. TIMSS (2023) further reports that SA had an average score of 397 points out of 800, below the international average of 478 points. According to SACMEQ (2016), learners in SA are not only struggling in mathematics but also with reading and writing, making it difficult for them to achieve in other subjects. This may be due to teachers' content knowledge, teaching methodology or lack of necessary tools for teaching and learning (Copur- Gencturk et al., 2022).

Department of Basic Education (DBE) (2022) state that teachers' lack of content knowledge and lack of productive teaching strategies from the teachers significantly impact learner's performance in mathematics. Makgakga (2016) indicates that other contributing factors to poor performance include learners' non-attendance, deprived entry marks, reduced assessment techniques and learners' attitudes towards mathematics. In SA, particularly in the Kwa-Zulu Natal (KZN) province, the poor performance of learners is evident in matric results. DBE Diagnostic Report (2022, p320) points out that KZN has had many candidates writing mathematics in matric for the past three (3) years. Table 1.1 below is a breakdown of candidates' performance in mathematics in KZN and the level of achievement from 2022 to 2024.

Table 1.1: ***A breakdown of candidates' performance in mathematics in KZN and the level of achievement from 2022 to 2024***

Year	Total wrote	Total achieved at 30% and above	Total achieved at 40% and above	Total achieved at 50% and above
2022	63 259	34 539 (54.6%)	22 131 (35.0%)	13 097 (20.7%)
2023	61 162	39 266 (64.2%)	27 053 (44.2%)	16 614 (27.2%)
2024	59 184	40 718 (68.8%)	28 630 (48.4%)	17 959 (30.3%)

Table 1 shows a general decline in the number of candidates that achieve 50% and above compared to candidates achieved at 30% and above, which is the minimum mark recognised at most universities. Most learners achieve 30% in mathematics. Although this mark is regarded as a pass, it is not adequate for learners to get points for university entrance, and it is unlikely that those learners will perform well in mathematics at higher educational levels. This shows that there is more quantity than quality in mathematics results. A large number of learners perform poorly in mathematics in KZN, and the province needs close attention since it has not been performing well with such a large number of candidates compared to other provinces. Trigonometry was identified as one of the subjects that learners find difficult, which has an impact on their performance, according to the diagnostic result report (2022). I'll go into further detail on trigonometry for the sake of this study.

In KZN, the Zululand district has many registered learners for mathematics each year. Table 1.2 below breaks down candidates' performance in mathematics in the Zululand district and the level of achievement that learners achieved from 2022 to 2024.

Table 1.2: ***A breakdown of candidates' performance in mathematics in Zululand and the level of achievement from 2022 to 2024***

Year	Total wrote	Total received less than 30%	Total achieved at 30% and above
2022	7 401	3 493 (47.2%)	3 908 (52.8%)
2023	7 164	2 780 (38.8%)	4 384 (61.2%)
2024	6 994	2 595 (37.1%)	4 399 (62.9%)

As shown in the table above, learners' performance in the Zululand district has raised concerns about teaching mathematics in schools. The table shows a worrying trend: There was little difference in learners' performance from 2022 to 2024. Each year, almost half of the learners registered for mathematics fail. The DBE moderators' report (2024) also indicated a decrease in learners registering for mathematics and a concomitant increase in the number registering for mathematics literacy. Mathematics is losing a large number of learners to mathematical literacy.

Trigonometry is a mathematics topic based on the relationships of sides and angles (Maphutha et al., 2022). Nambeye and Samuel (2019) state that trigonometry is a vital part of Grade 10 - 12 mathematics; it contains a higher percentage in paper two and allows learners to understand the real life applications of triangles, angles, distances and heights. Trigonometry is challenging to most learners, and this has been observed in learners' performance over the past three years (2022, 2023 and 2024) (DBE, 2022). Furthermore, the 2023 exam guidelines show that trigonometry carries 50 marks out of 150 in Mathematics Paper 2. Therefore, learners should be taught effectively so that they perform well.

As reported by DBE (2024), learners have performed poorly in trigonometry for three (3) consecutive years, that is, 2022, 2023 and 2024. In 2022, 45.43% of learners were able to answer a sketch and identity question where they were being tested a skill of using Pythagoras' Theorem and using identities. According to the DBE Moderators' Report (2023), learners still have a problem with interpreting the Cartesian plane and simplifying trigonometric ratios. The (ibid) further states that learners could not use identities or interpret trigonometric functions, and only 45.8% of learners were able to answer questions based on Graphs. Only 39.67% of learners were able to answer the question involving heights and distances (2D and 3D problem solving). Moreover, in 2023, only 37.27% of learners were able to answer questions based on heights and distances trigonometry. It is further explained that many candidates skipped the question, and others showed a lack of concept of the angle of elevation and angle of depression as well as their meaning (DBE Moderators' Report, 2024).

Arhin and Hokor (2021) state that poor performance may be caused by the pedagogical competencies of teachers or learners who do not understand the benefits of studying 2D and 3D trigonometry, their historical usages, or their applications in daily life. Madonsela, Ndlovu, and Brijlall (2020) further suggest that learning 2D and 3D may be achieved if the content is delivered in a way that relates to learners' lives and allows them to use it in their everyday experiences.

Teaching and learning three dimensional trigonometry has been described as one of the most challenging topic in the field of mathematics; hence, learners often have difficulty understanding the real life application of the topic (Baidoo et al., 2024). Due to its historical past and ongoing relevance in mathematics, trigonometry has piqued researchers' interest. Most learners who meet the sine, cosine, tangent and cotangent notions for the first time in high school cannot link them to real-life circumstances and have no idea where they came from (Siyepu, 2015). Learners may only learn trigonometric principles if they are shown real life applications and the necessity of trigonometry in their lives (Baidoo et al., 2024).

Gür (2009) in Turkey, Malambo (2016) in Zambia, Tatira (2020), and Khuzwayo (2019) in SA investigated learners' misconceptions and blunders and mathematics instructors' knowledge of trigonometry. Their research focused on subject matter content knowledge, pedagogical content knowledge, and envisioned pedagogy. The most common issues include misapplication of equations, misunderstanding of terminology, logically faulty inferences, distorted definitions, and misunderstanding of the definitions of sine, cosine, and tangent (Gür, 2009).

Tarira (2020) indicate that teachers' knowledge of trigonometry is limited, and their teaching methods are based on traditional teaching. Research done before on teaching and learning trigonometry suggests that various factors may impact learners' performance in mathematics. The leading factor is the lack of effective teaching strategies that some teachers still use in classrooms (Khuzwayo, 2019). DBE (2022) moderators' report states that the poor performance of SA in mathematics may be caused by teachers' lack of interest in applying conceptual and deliberate teaching methodologies in their lesson plans.

Madonsela et al., (2020), in a study that employed the action, process, objects and schema (APOS) theory, explored learners' solving three-dimensional (3-D) problems using trigonometry. The study looked at the mental models used by Grade 12 learners while solving triangles in trigonometry. According to the results, learners only mastered topic information while solving 2D triangles. When solving triangles involving 3D and non-right-angled triangles, almost all learners demonstrated a lack of even action conceptualisation. Action conception suggests that learners were unable to form the appropriate mental structures for the solution of triangles due to gaps in their knowledge construction of preceding ideas (Madonsela et al., 2020). The (ibid) also found that learners could not even recognise the hypotenuse side of a right-angled triangle, making it impossible to grasp the language and structure of questions when several triangles are linked to form three-dimensional objects.

Therefore, recommendations were that teachers should come up with effective ways of teaching heights and distance in trigonometry, and the use of rote learning was discouraged. The authors further suggest that learners should be encouraged to construct knowledge mentally in problem solving and should apply their minds in real life situations. Rote learning leaves the gap in finding effective strategies (that will trigger the mental construction of knowledge) for teaching height and distance problems using trigonometry. It is evident that learners are able to solve 2D problems, but they lack the understanding of solving 3D (Madonsela et al., 2020).

Gerhana et al., (2017) investigated the efficacy of project-based learning in trigonometry in well-funded urban schools in Indonesia. Learners were working in groups and were required to calculate the height of the Wi-Fi tower using a clinometer and meter. The findings demonstrated that a project-based learning paradigm is more effective at improving learners' arithmetic skills. Gerhana et al. (2017) define project-based learning as an instructional method that enables learners to apply information and skills in an interesting way. It is evident that their reach was only based on 2D shapes. They did not apply project-based learning to calculate heights and distances in 3D shapes.

While learners’ understanding of calculating the height of objects using trigonometry has been previously researched in other countries such as Turkey, Indonesia and Zambia, not enough research that is project based and involves the use of visualisation in heights and distances has not been done in SA, particularly in rural areas. From the above discussions, it is evident that not much research has been done on learners’ skills in using trigonometry to calculate heights and distances, especially in the context of rural based learners. Hence, this research is aimed at filling that gap.

1.2 PROBLEM STATEMENT

According to Khofifah et al., (2024), trigonometry is one of the most significant concepts in high school mathematics since it connects to other topics such as algebra and geometry. It is then important that learners are taught the procedural ways of answering the questions based on trigonometry and that they should be taught in a way that will enable them to apply trigonometry outside their classroom setting. Gür (2015) states that trigonometry is one of the mathematical disciplines that learners do not enjoy and struggle with. (ibid) further states that trigonometry is a subject that learners view as being particularly difficult and esoteric when compared to other mathematical disciplines.

The DBE moderators' report of 2022 to 2024 shows that learners are underperforming in trigonometry, particularly in questions based on height and distance. The DBE Reports (2022, 2023) mention that some learners tend to skip this section on the examination and do not even attempt to answer this question. This may indicate that learners do not understand what is expected of them or that the teaching strategy teachers use is ineffective for learners to understand the questions.

Learner's comprehension of trigonometry could be improved if teachers employ innovative and flexible teaching and learning resources (Prabowo et al, 2018). Hence, this research focused on learners being given visuals that are accessible to them, something that they can touch, feel and analyse. Looking at how the questions are usually asked, I have realised that examiners still use ancient structures where learners are expected to calculate the heights and distances. For example, in 2019, learners were given an Egyptian pyramid where they had to calculate the height and distances of a pyramid. My concern is for learners who have not seen an Egyptian pyramid before; therefore, they may struggle to visualise it by just looking at the structure on paper.

Researchers such as Gür (2009) in Turkey, Malambo (2016) in Zambia, Tatira (2020), and Khuzwayo (2019) in South Africa reported that the most common errors made by trigonometry learners are improper use of trig ratios, order of operations, and value and position of sine and cosine, as well as misused data. Furthermore, there are misconstrued terminology, logically flawed inferences, twisted definitions, and technical mechanical flaws. The researcher's experience when conducting the June, September and November examinations (2022) was that learners were unable to answer simple trigonometric questions correctly. For example, in November 2022, learners were required to write $30° = \frac{\sqrt{3}r}{QS}$, but instead, some wrote $\tan 30° = \frac{QS}{\sqrt{3}r}$. This indicated that learners were still struggling with identifying opposite, adjacent and hypotenuse sides in a right-angled triangle and they cannot recall $SOH\ CAH\ TOA$ when calculating heights and distances. Some learners struggled with using trigonometric ratios and Pythagoras theorem in non-right-angled triangles. Learners were unable to identify a right-angled triangle in a 3-D shape, especially when $90°$ was not indicated.

Most research focuses more on urban based schools (Gür, 2009; Malambo, 2016; Tatira, 2020; Gerhana at el., 2017). Schools in rural locations are frequently disadvantaged since some do not have learning resources, such as overhead projectors, smart boards, computers, etc., available for learners in urban areas. Learners in urban areas may have the equipment mentioned above, which can assist in seeing and analysing the shapes of 2D and 3D. This study, therefore, sought to investigate the effectiveness of using visualisation in Grade 12 to teach trigonometry in particular heights and distances. The researcher saw the need for learners to be exposed to visualisation when learning 2D and 3D problem solving.

1.3 THE RESEARCH QUESTIONS

The study aimed to address the following research questions:

1.3.1 The primary research question:

What is the effectiveness of using visualisation on teaching and learning heights and distances in trigonometry to improve academic performance?

1.3.2 The research sub-questions:

- What types of teaching methods do teachers use when teaching heights and distances?
- What are the benefits and challenges (if any) of visualisation in the teaching and learning of trigonometry?
- What are possible ways of implementing effective visualisation in the teaching and learning of heights and distances to improve learner performance?

1.4 PURPOSE, AIMS AND OBJECTIVES

The purpose of this study was to explore the effectiveness of visualisation in teaching and learning of Grade 12 heights and distances to improve academic achievement. The focus was on how the visualisation of heights and distances can improve learners' academic performances. The study examined how learners are currently taught in a focused school prior to suggesting teaching trigonometry using visual models. The study also aimed to improve teachers' approaches to teaching heights and distances using visual models. Through lesson observations and interviews with participants, the researcher investigated the benefits and challenges of visualisation in teaching and learning heights and distances in trigonometry.

1.4.1 The objectives of the study

The objectives of this study were as follows:

- To explore teachers' teaching strategies of height and distances in trigonometry

- To understand the benefits and challenges (if any) of visualisation in the teaching and learning of heights and distances
- To suggest possible ways of implementing effective visualisation in the teaching and learning of height and distances to improve learner performance.

1.4.2 Significance of the study

Trigonometry is one of the topics in Further Education and Training (FET) that learners struggle with, notably in Grade 12 exams (Prabowo et al., 2018). The literature and international and local reports reveal that the methodologies teachers employ to teach trigonometry have a bearing on how learners perform when tested on the topic. It has also been established that learners tend to skip the 2D and 3D problem solving questions, while some learners show no understanding of the question. Consequently, the aim of this research study is to determine the effectiveness of visualisation on learners' performance in 2D and 3D problem solving.

1.5 RATIONALE FOR THE STUDY

The aim of the study was to contribute to the development of learners' understanding of heights and distances in trigonometry since it is evident that learners do not perform well in this aspect (DBE Reports, 2022, 2023). Aside from strengthening my profession as a mathematics educator, the findings of this study will give information for other stakeholders, including teachers, textbook writers, and curriculum developers. The aim of the study was to assist educators in successfully implementing visual models in teaching heights and distances. This study will inform educators about the effectiveness of visualisation in their lessons to improve learners' understanding of trigonometry, heights, and distances.

The study will also inform curriculum developers and examiners on the types of questions that may be incorporated into the examination instead of asking learners to calculate distances and heights of ancient buildings that learners are unfamiliar with. This study encourages teachers, examiners, and curriculum developers to conduct hands-on investigations, where learners can be asked to calculate everyday objects at their disposal, such as the height of a tree or a school gate. This may allow learners to understand trigonometry and relate it to their everyday lives.

1.6 RESEARCH METHODOLOGY AND DESIGN

According to Mishra and Alok (2022), methodology is defined as a process that summarises research proceedings. Therefore, this section aims to give an overview of the methods used by the researcher to collect data and answer research questions. The elements of research methodology discussed include the research approach, research design, population and sampling, instrumentation and data collection techniques, and data analysis and interpretation.

1.6.1 Research approach

Research approach refers to the study plan, strategy, or approach that directs the selection and application of research methodologies that are used in connection with the intended research questions and results (Pregoner, 2024). Research approach is a plan that informs the choice of methodology used in the study. The study used a mixed method approach, which combines quantitative and qualitative techniques. “Researchers adopting a mixed method approach are not restricted to using methods associated with conventional designs, either qualitative or quantitative” (McMillan & Schumacher, 2014, p.33). The reason for using a mixed method design is that methods may have biases and weaknesses. Hence, collecting both quantitative and qualitative data compensates for the shortcomings of each type of data.

1.6.2 Research design

This research study was based on mixed methods experimental design embedded explanatory sequential mixed method. The purpose of mixed methods experimental design is to test the cause and effect relationship using experiment where the qualitative data is used to gain additional insights into factors that influence experimental outcomes (Fetters, 2020). According to Creswell (2018), in a two-part project called the explanatory sequential mixed method, the researcher firstly collects the collects quantitative data, analyses the findings, and then applies a qualitative phase to help explain the quantitative results. In this study, the researcher administered a pre-test and then analysed the results using quantitative analysis. To establish how learners were taught heights and distances and how learners responded to the pre-test, semi-structured interviews were then conducted. The researcher then proposed the use of visualisation as an intervention. During the intervention, the researcher made use of the interactive software Geogebra. However, due to the lack of resources at the chosen school, the researcher had to devise other ways of visualisation. The researcher used the resources accessible to learners, such as tents or cutting out a corner of cardboard to make a 3D shape with heights and distances. A post intervention test was then administered to measure the effectiveness of using visualisation in heights and distances.

1.6.3 Population and sampling

A population is the total number of a defined class of people, events or objects (Hossan et al., 2023). This study was conducted in the Zululand District of the KZN province. Zululand district consists of 26 circuits and 207 secondary schools. Participants were sampled from two of these secondary schools. The study was conducted in two rural secondary schools, experimental and control, with two teachers teaching, both males, Grade 12 in these schools

This study employed sequential mixed method sampling. Since the research is conducted in two phases, quantitative and qualitative, the researcher used population to sample participants for both phases. In quantitative sampling, the researcher employed random sampling where every individual in the target population had an equal chance of being chosen for the study (Stratton, 2021). The population consisted of 61 learners from the experimental group and 144 learners in control group, while my sample was set to have 30 participants from each group. All the 61 and 144 learners from both groups were interested to participate when the researcher approached them. The researcher had to create a sample frame by having a list of all learners in the population using the mark lists obtained from Grade 12 teachers. Each learner was then assigned a unique number from 1 to 61 in experimental and 1 to 144 in control group. The researcher then used a random selection method by making use of drawing lots where the numbers representing learners were placed in a container and drawn one by one randomly until the desired sample size of 30 was acquired. The experimental and control groups' schools comprises only black African learners and educators. The experimental group's school had underperformed in mathematics for the past four (4) years. Most of the learners are from disadvantaged homes with low socio-economic backgrounds.

1.7 INSTRUMENTATION AND DATA COLLECTION TECHNIQUES

The data collection tool is an instrument used to gather data (Karunarathna et al., 2024).This study collected data using four (4) data collection tools: lesson observation, pre-test, semi-structured interviews and post-test. The objective of the study was to explore the effectiveness of using visualisation in teaching height and distances in Grade 12 trigonometry. Along with examining how learners are now taught height and distance, the study also sought to educate teachers by introducing them to the new methods. The results of the two (2) tests that were given, as well as the application of the tactics discussed during the intervention process, were evaluated using pre-test and post-test, classroom observations, and semi-structured interviews. To address the research topics, the study incorporated both qualitative and quantitative data. Thus, the research investigation was suitable for both positivist and interpretivist paradigms.

In light of the aforementioned, the current study is pragmatic; it was founded on a mixed method approach, which employed both qualitative and quantitative techniques to address the research objectives. The study agreed with Creswell's (2018) assertion that pragmatism allows researchers to employ a variety of techniques, worldviews, and presumptions, including various kinds of data collection and analysis, hence enabling mixed methods research.

1.7.1 Pre-test and Post-test

The researchers used sixty (60) learners from the experimental group (30) and control group (30) to conduct pre-and post-tests to measure learners' performance in trigonometry. Pre-tests are frequently used to create a baseline before an intervention, stratify participants based on pre-test results (known as blocking solutions), offer variables in a quasi-experimental design, or show learners' existing proficiency (Kim & Willson, 2010). A pre-test was performed before the intervention in both groups to measure significant differences between the two (2) groups. The researcher conducted a pilot study before conducting a preliminary test in two (2) research groups. A pilot project was used to determine the possibility of testing errors before a large study (Khuzwayo, 2019). The researcher also used a trial to see if the questions were clear and understandable and if enough time was allowed to complete the test. A post-test was then administered after the intervention to determine whether visualisation impacted learners’ performance in heights and distances in trigonometry.

1.7.2 Semi-structured interviews

Semi-structured interviews were conducted with two teachers from both experimental and control group after analysing the pre-test results to understand the challenges learners may experience when solving heights and distances in trigonometry. Semi-structured interviews were conducted after the post-test to understand whether the intervention influenced the results. Teachers were interviewed individually to understand their thoughts and understanding of the teaching heights and distances. A few interview questions were based on teachers teaching math experiences, while the remainder were based on their learners' experiences when solving problems based on heights and distances in trigonometry.

1.7.3 Lesson observations

According to Creswell (2018), when researchers conduct a qualitative observation, they take field notes on people's behaviour and activities in the study area. The researcher records events at the research site in unstructured or semi-structured field notes.

The researcher conducted the baseline observation to understand how teachers teach heights and distances. After the introduction of visualisation, the researcher conducted post observations of the lessons to see if the teacher continues using visual models when teaching heights and distances. McMillan and Schumacher (2014) state that qualitative observers can take on various roles, from observer to participant. The researchers typically ask open-ended questions to the participants, allowing them to express their opinions freely. In this study, the researcher did not participate in the lesson observation. The role of the researcher was to observe the lesson and take field notes.

1.8 DATA ANALYSIS AND INTERPRETATION

The data collected from the pre-test and post-test was grouped into categories: correct answer, incorrect answer, unanswered answer, etc. To get a general view of learner performance in height and distance, category percentages were calculated using the absolute number of learners, not the average percentage. All incomplete or incomplete responses were classified as incomplete, even if used correctly, and blank categories were considered completely unresponsive or not attempted. All incomplete replies, even when used properly, were labelled as such and blank categories were interpreted as not answered. Statistical tests were used to explain statistical significance, confidence intervals and effect sizes (Creswell & Creswell, 2023) to organise qualitative data. The researcher made use of Excel and SPSS tools to statistically analyse data. Microsoft Excel was used to capture pre- and post-test marks from both experimental and control groups. The SPSS tool was then used to do the analysis. T-test was also used to compare the performance of both the experimental and the control groups. Results acquired from pre-test and post-test were declared significant if the p-value was less than 0.05 and insignificant if the p-value was greater or equal to 0.05. Hence, results were interpreted at a 95% confidence limit (Sambo, 2022).

The data was organised, transcribed and typed to interpret the significance of the semi-structured interviews and classroom observations. The data was coded, following Creswell (2018). Coding is the process of grouping words (or paragraphs) or images into categories and identifying those groups with a term, usually based on the participants' natural language. After that, researchers use coding to create descriptions of the persons and categories or themes to analyse. In this research study, themes were developed and linked (Creswell, 2018).

1.9 TRUSTWORTHINESS

Enworo (2023) define trustworthiness as the extent to which a researcher is able to persuade readers that the findings are credible, reliable and can be trusted. The researcher utilised three (3) strategies to verify the trustworthiness of this research study, which are credibility, triangulation, and reliability.

1.9.1 Credibility

When conducting research, researchers must ensure the trustworthiness of their findings (Khuzwayo, 2019). Validity and authenticity are important factors in establishing credibility. The ability of others to duplicate research methods is known as validity or authenticity. Credibility must be established as the initial element or criterion (Creswell, 2018). It's the most crucial element or standard for judging someone's credibility. This is because the researcher has to show that the results are credible by directly connecting them to reality. In this study, the researcher employed extensive descriptive data to ensure credibility. The researcher also remained objective during data analysis to guarantee objectivity.

To ensure that the participants support the research findings, they have to be given an opportunity to validate their veracity; in this study, the researcher presented the findings to them. To provide the study with credibility, sensible questions were employed to obtain the data. Maree (2016) state that the researcher can demonstrate credibility by showing the participants the transcriptions or field notes and asking them to correct errors. In this study, participants were able to articulate and expound on their experiences in learning heights and distances through visualisation through open-ended questions during interviews.

1.9.2 Triangulation

According to Bans-Akutey et al., (2021), triangulation refers to employing many methodological approaches, data sources, analysis methods, or theoretical perspectives in a single research to improve the study's validity. This study enhanced data triangulation by utilising four (4) data gathering techniques, namely observation, pre-test, semi-structured interviews and post-test, to increase the validity.

1.9.3 Validity and reliability

To guarantee validity, the researcher employed member checking, in which the final report was returned to the participants to determine whether they believed the qualitative findings were correct (Creswell, 2018). This allowed individuals to comment on the findings.

Researchers also spent prolonged time in the field. According to Creswell (2018), if the researcher spends more time with the participants, they develop an in-depth understanding of their study and become more experienced with participants in their settings. Their findings become more accurate or valid. The researcher additionally adhered to the guidelines Gibbs (2007) recommended in order to guarantee the validity of this investigation. The researcher made sure that the definitions of the codes were consistent throughout the qualitative data analysis and reviewed every transcript to look for any potential errors. By comparing the independently derived results, the researcher also cross-checked the codes created within the body of existing literature.

1.10 ETHICAL CONSIDERATIONS

The researcher applied for an ethical clearance certificate from the University of South Africa, as well as authorization from the Department of Basic Education in KwaZulu Natal, Zululand District, to undertake school research. Before beginning the study, the researcher obtained the informed consent of teachers and learners. The researcher approached both the Grade 12 teacher and the school principal.

The rights and roles of participants were explained before conducting the research. Participants were assured that their participation was voluntary, confidentiality and that their details would not be disclosed. According to Lewis (2023), one of the most important aspects of autonomy is that participants must participate in the study voluntarily and have the option to withdraw at any moment. The issue of autonomy persisted throughout. Participants received a letter of consent outlining the study's objectives and what was expected of them.

1.11 SCOPE AND LIMITATION

A sample of thirty (30) learners and a teacher from an experimental school was used in this investigation. The results of this study cannot be generalized to all Grade 12 learners and teachers in South Africa, even though the sample size would have been adequate to answer the given research objectives. By limiting the study to written tests, semi-structured questions, and observations rather than open-ended responses, which may have enabled some respondents to contribute more in-depth information, the researcher constrained the study's scope. The study only looked at trigonometric heights and distances; it didn't go into other branches of mathematics.

1.12 DEFINITION OF KEY CONCEPTS

Visualisation – is the technique of creating images or mental representations and efficiently utilising them for mathematics discovery and understanding- is usually supported using diagrams, pictures and visual tools (Hamidreza, Nor, Mohamad, Osman & Suhaimi, 2015).

Trigonometry - is a discipline of mathematics that studies the relationships between triangles' sides and angles, as well as the functions that apply to any angle (Siyepu, 2015).

1.13 CHAPTER LAYOUT

Chapter 1: The first chapter offered an overview of the research and its context. It covered the research statement and questions and presented the aim and importance. This chapter also gave an overview of the methodology employed, ethical considerations, and definitions of the terms.

Chapter 2: The theoretical foundation that guided the research is presented in this chapter.

Chapter 3: The third chapter discusses and reviews the literature relevant to the study and introduces the conceptual understanding of heights and distances.

Chapter 4: This chapter describes the research methodology used for the study, with a focus on the population, sampling, data collection method, methodology selection, research strategy, research approach, research location, study design, and theme analysis.

Chapter 5: The fifth chapter analyses and presents the study's results.

Chapter 6: This chapter presents a discussion of the results.

Chapter 7: The last chapter presents the conclusion and recommendations.

1.14 CONCLUSION

This research study aimed to understand the impact of visualisation on learning heights and distances in Grade 12. This chapter gave an overview of the state of the learning of trigonometry learners' performance in mathematics, especially in Grade 12. Previous research done in teaching and learning of trigonometry was also discussed. This chapter also covered the research statement, research questions, and the aim and importance. It also gave an overview of the methodology employed, ethical considerations, definition of the terms and chapter outline.

CHAPTER 2: THEORETICAL AND CONCEPTUAL FRAMEWORK

2.1 INTRODUCTION

The problem statement, the purpose of the study, its justification, its importance, and an overview of the chapters are discussed in Chapter 1. A brief explanation of how the visualisation of heights and distances is implemented in schools was provided. The chapter also provided the structure of the investigation. Chapter 2 presents the theory that informed the study of using visualisation in the teaching and learning of heights and distances in trigonometry and the impact on learners' understanding. The theory presented is guided by Mayer's (2005) cognitive theory of multimedia learning. The said theory is influenced by several cognitive theories, such as Paivio's (1971) dual coding theory, Sweller's (1988) idea of cognitive load and Baddeley's (1974) working memory model.

Additionally, a constructivist learning philosophy is related to Mayer's (2005) thesis. According to constructivism, learners must actively seek an understanding of the world rather than waiting for it to be supplied. Learners should actively participate in class. This study applied this theory when introducing visual models to the learners. In light of this, the opening section of Chapter 2 discusses the function of theories in both the general framework of this research and in an educational setting. It highlights the significance of applying theories to research and educational activities.

2.2 ROLE OF THEORIES IN EDUCATIONAL RESEARCH

A theory is a significant piece of thought that organizes a wide range of concepts with a high level of explanatory power. Resnick and Stieff (2024) claim that the theory of approach or methodology provides guidance on how to choose which approaches will be most effective in addressing the search queries. In a broad sense, a conceptual framework is a map of how the literature functions in a certain research. "The use of a theory or theories that simultaneously express the researchers' core beliefs and provide a really expressed guide or lens through which they interpret new information is known as a theoretical framework" (Collins & Stockton, 2018, p.2). The aforementioned theoretical framework helped the researcher locate prior research and well-informed concepts regarding the use of visualization in height and distance. Research that lacks a theoretical framework demonstrates that the investigator did not carry out the necessary steps to uncover their deepest guiding ideas and assumptions regarding their investigation (Calder & Sternthal, 2023). As a result, the theory is discussed in this study in connection with the issue statement, the purpose, the relevance, the research questions, and the structure and support for the rationale. Theory served as the foundation for the methods used in this investigation.

2.3 COGNITIVE THEORY OF MULTIMEDIA LEARNING

The cognitive theory of multimodal learning (Mayer, 2005) served as the foundation for this study. The cognitive learning theory of multimedia learning, which was developed by the (ibid), focuses on organizing multimedia teaching practices and using more effective cognitive techniques to support effective learning. It makes three (3) assumptions about how people absorb information: the dual-channel assumption, the limited-capacity assumption, and the active-processing assumption. Multimedia learning is the result of creating mental representations from words and images (Sorden, 2012).

2.3.1 The dual-channel assumption

Mayer departs from the dual point of visual and auditory learning in his cognitive theory of multimedia learning. Mayer (2024) defines multimedia learning as the process of creating mental images using spoken or written material and visuals like drawings and photos. This theory is built around two (2) essential terms: multimedia and learning. The (ibid) further states that multimedia is the presentation of words and pictures, which requires verbal and pictorial presentations. These may include teachers and slides, pictures or visual models in an educational setting. On the other hand, learning is defined as learners' construction of knowledge. Multimedia does not only mean the use of technology during the lesson, but multimedia includes the use of pictures, words, auditory, text, diagrams and visual models (Moreno et al., 2000). Since this research is focused on rural schools that do not have technological equipment such as projectors, computers, etc., it supports using visual models accessible to learners, such as tents and corners of cardboard, to visualise height and distances in 2D and 3D models.

According to Mnguni (2014), learning through visual and auditory models is a cognitive process that includes a variety of mental processes. The theory aims to comprehend mental activities like thinking, remembering, and comprehending because cognitive refers to perceiving and knowing (Sorden, 2005). (Ibid) maintains that multimedia enhances how the human brain functions in the cognitive paradigm of multimedia learning. At the same time, Mayer (2024) asserts that multimedia learning occurs when we build mental representations from these pictures and words. During this research, learners were not only presented with visual models, but the teacher was needed to present and communicate what was presented to the learners.

Pupils do better with visual (image) and verbal (words) input in an educational setting than with only words or pictures (Mayer, 2005). According to AlShaikh et al., (2024), when presenting an instruction or lesson to learners, it is important to take advantage of the visual and auditory channels. This is because every class has learners that learn differently. Some learners may have auditory impairment and rely more on their visual processing channel and vice versa. At the same time, some learners rely on both these channels to process information.

2.3.2 The Limited-Capacity Assumption

The limited capacity assumption indicates that people have difficulty restraining the amount of data or information they can process at any moment (Mayer, 2005). When learners are presented with illustrations or visuals, they are able to hold a few images in their working memory at the given time. Still, learners are unable to remember every detail of the visual or illustration at the same time. Sweller (2024) state that teachers should be aware of the cognitive load or mental activity imposed on learners' memory. Giving learners too many visuals and information may cause an unnecessary cognitive load on their working memory, confusing and preventing them from learning.

During this research, learners were simultaneously presented with various visuals, and they were given enough time to work with one visual until they were comfortable with it (or they could sketch it without referring to the actual illustration). Until they understand how to identify height and distances, measure them, determine the number of vertices and faces, and calculate the height and distance of the given 3D illustration.

2.3.3 The Active-Processing Assumption

In the active-processing hypothesis, humans do not simply learn by passively taking in information. However, they should create mental representations of their own experiences and actively engage in cognitive processes (Mariano, 2014). This happens as incoming information is attended to, organised and integrated with other knowledge systems (Mayer, 2024). Using the appropriate cognitive methods, such as focusing on the lesson's relevant content, mentally organizing it into a cohesive cognitive representation, and mentally integrating it with their existing knowledge, improves learning (Sorden, 2005).

In multimedia learning, there are three (3) types of memory: sensory memory, working memory and long-term memory. According to cognitive theory, during the learning process, external visuals and words from the outside world first enter the sensory memory through the eyes (pictures) and hearing (words). The learner then pays attention to some aspects of the auditory and visual models, which causes the construction of a working memory mental pictorial representation. After forming successive mental images, the learner then blends the collection of images and words into a coherent mental representation, and this is termed Visuospatial thinking, which encompasses the choice, arrangement and integration of pictures (Sorden, 2005). Figure 2.1 summarizes the three (3) types of memories in cognitive theory of multimedia learning.

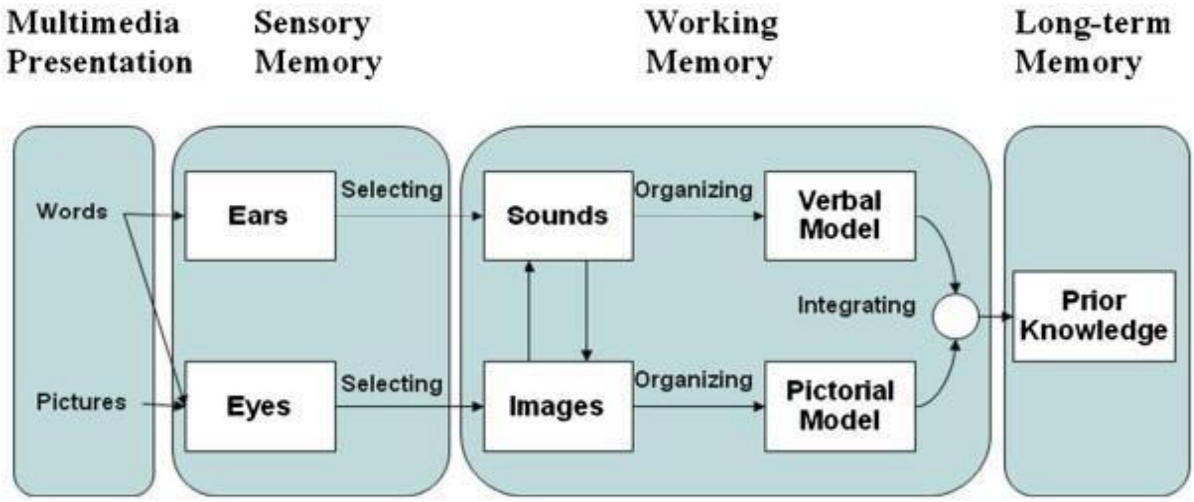


Figure 2.1*: The cognitive theory of multimedia learning* *(Source: Mayer, 2024)*

This study presented learners with visual models representing 3D shapes, such as a cardboard corner. Learners were then requested to sketch the 3D shape without referring to the actual model to see if the image of the visual model was stored in their working memory. If learners are were to sketch the visual model without seeing it, it meant that they have acquired prior knowledge in their long-term memory. Once learners have prior knowledge, they can use it to visualise the problem they may be presented with when solving heights and distances.

2.4 THREE MEMORY STORES IN MULTIMEDIA LEARNING

In multimedia learning, there are three (3) memory stores: sensory memory, working memory, and long-term memory, as shown in the accompanying figure modified by Mayer (2024). According to Mnguni (2014), the human information-processing system occurs when learners are presented with a visual model. This includes visual images or models and words from the outside world. The teacher presents a 3D visual model to calculate the heights and distances in trigonometry and explains how the model looks. As a result, the sensory memory of the eyes and ears receives the supplied information. The identical visual images are stored in sensory memory for a relatively short time because of the sensory motor (Mayer, 2005). At the same time, the precise uttered words are also temporarily stored in the auditory sensory memory.

Working memory is where the majority of multimedia learning takes place (Cavanagh & Kiersch, 2023). As noted by Schweppe et al., (2014), working memory operations necessitate attention and correlate the distribution of working memory with the distribution of attention. Knowledge in the active consciousness is momentarily stored and manipulated by the working memory, claims Mayer (2024). For example, learners might be able to actively concentrate, or they might be able to focus solely on visual images in their imaginations or certain words the teacher says.

After learners have received external images (a corner of a cardboard or a tent) and words (explanation from the teacher) through the eyes (pictures) and ears (words), information is then selected and moved to working memory, where it can be maintained and processed. Each of these specific working memory stores is limited in ability, meaning information is stored for a short period in the working memory. In processing a picture and a verbal model of the incoming information, knowledge is built in working memory and with prior knowledge. Once this combination has happened, the goal of learning has been accomplished or gained (Mayer, 2024).

2.5 FIVE PROCESSES IN A COGNITIVE THEORY OF MULTIMEDIA LEARNING

As stated by Schweppe et al., (2014), for meaningful learning to occur through multimedia, learners must be engaged in Mayer's five (5) processes of cognitive theory. The processes are selecting words**,** selecting images**,** organizing words**,** organizing images and integrating verbal and pictorial models. Table 2.1 below describes Mayer's five processes of cognitive theory that learners encounter when learning through the use of multimedia.

Table 2.1: ***Five Cognitive Processes in the Cognitive Theory of Multimedia Learning*** *(Source: Mayer 2005)*

Process	**Description**
Selecting words	Learners pay attention to relevant words in multimedia message to create sounds in working memory.
Selecting images	Learners pay attention to relevant pictures in a multimedia message to create images in working memory
Organising words	Learners build connection among selected words to create a coherent verbal model in working memory

Organising images	Learners build connection among selected images to create a coherent pictorial model in working memory.
Integrating	Learners build connection between verbal and pictorial models with prior knowledge

To build sounds in their working memory when learning about heights and distances in trigonometry, learners choose or pay attention to words that they think are meaningful. A similar procedure is followed by learners while using photographs; they focus on pertinent images to form images in their working memory. To establish a consistent verbal and graphical model in working memory, learners make connections between selected images and selected phrases.

2.5.1 Mayer's principles of multimedia learning

Mayer (2005) lists twelve (12) principles the multimedia lesson instructor should consider when creating multimedia presentations. These principles should be organized according to their theoretical purpose, which includes eliminating unnecessary processing, regulating necessary processing and encouraging generative processing. These rules apply when learners lack prior knowledge and the pace and complexity of the presented content are too much for them to understand.

2.5.2 Principles for reducing extraneous processing

1. *Coherence principle*

People learn better when unnecessary words, images, and noises are removed rather than added, claims Mayer (2005). Only information that is directly related to the learner's needs should be used by the instructor while creating a multimedia lesson. The following inquiries have to be on the instructor's mind:

- Is the visual material 100% necessary to help with comprehension, or could I find a better and more relevant one?
- Is the language or explanation used simple enough for the learners to understand, or should I trim down a few words?

2. *Signaling principle*

Learners learn better when they are only shown the necessary or exact information to pay attention to. When the teacher uses the chalkboard during the lesson, there should not be much information on the board. When teaching heights and distances, arrows that point out significant information on the given statement should be used. The sketches should be clear/ precise so that learners can identify different shapes in the 3D shapes (Bechtold, 2023).

3. *The redundancy principle*

According to this precept, teachers should evaluate quality using narration and graphics rather than text, graphics, and narration. The underlying idea is that additional text is unnecessary information if there is already narration and pictures, and for learners, this can be too much. This technique is typically applied when lessons contain videos or eLearning programs that include audio. The lesson should only have text or images, not both (Mayer, 2022).

4. *The spatial contiguity principle*

Humans learn best when the relevant text and visual image are physically closer together. There should not be a space between the visual images and the instruction or the information given when learners calculate heights and distances in trigonometry. According to Mayer (2005), this makes it easy for learners to process the information. The instruction and the visual image should be structured as depicted in Figure 2.2.

Question 2: March/ Feb 2014

In the diagram below, RS is the height of a vertical tower. T and Q are two points in the same horizontal plane as the foot S of the tower. From point T the angle of elevation to the top of the tower is 60°. $R\hat{T}Q = \theta$, $R\hat{Q}T = 60°$ and TQ = k metres.

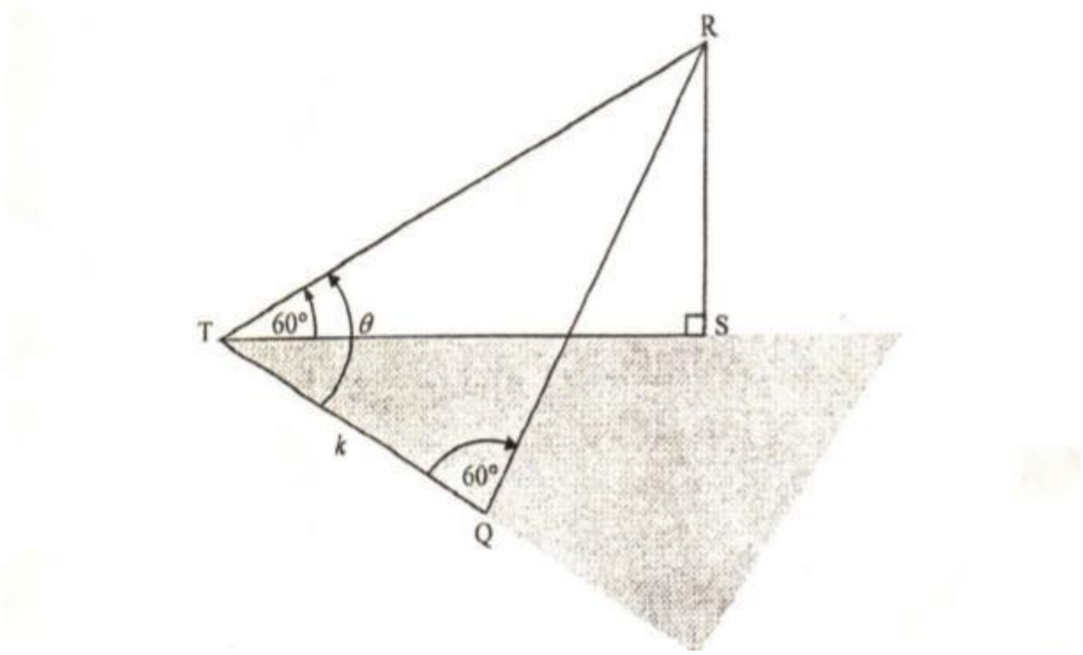


Figure 2.2: ***An illustration showing how to depict the instruction and the visual image*** *(Source: Mathematics p2 February/March 2014 question paper)*

5. *The temporal contiguity principle*

When the visual image is introduced to learners for the first time, the teacher should also explain the visual image. The teacher should not explain before showing the learners what s/he is talking about. This means that the visual material and explanation should be introduced simultaneously (Cavanagh & Kiersch, 2023).

2.5.3 Three principles for managing essential processing

1. *Segmenting principle*

Humans learn better when visual material is presented in learner-paced segments rather than continuously (Mayer, 2022). This means that the lesson should be delivered in segments. The teacher should not use sine and cosine rules when calculating heights and distances simultaneously. The first lesson should focus on the sine rule only, and the following can then focus on the cosine rule. This principle also states that learners should be in control of the lesson.

2. *Pre-training principle*

When humans are familiar with the names and traits of key components, they can learn from visual material more effectively. Before starting the learning process, learners should be introduced to and given a solid understanding of fundamental definitions, terminology, and concepts. This implies that learners should be able to recognise the shapes' names, the angles inside and the hypotenuse, and the opposite and adjacent sides of each angle.

3. *Modality principle*

Humans learn better from graphics with spoken text than with printed text. The teacher should limit the amount of text and rely on visuals.

2.5.4 Four principles for fostering generative processing

1. *Multimedia principle*

Words alone do not provide the same level of learning as words and images. Learners can connect the verbal and graphical models more easily. The cornerstone of Mayer's beliefs is that words and images work better together than just words or images. For learners' visual images should make the information clear.

2. Personalisation principle

Humans learn better from a multimedia lesson when words are conversational rather than formal. This means the lesson should have a conversational voice rather than being overly formal, as this improves the learning experience. Language should be kept simple and casual.

3. Embodiment or voice principle

This principle supports the use of a human voice rather than a computer voice during the lesson. So, during the lesson on heights and distances, the teacher should not use computer voiceovers to deliver the content to learners. Instead, the teacher should use their own voice to interact with the learners.

4. Image principle

The image principle states that humans do not necessarily learn better from a talking head video. This means that during the lesson, the teacher should not record herself teaching privately or download a video of others teaching heights and distances and use those videos as an intervention. Instead, the teacher should present the lesson to learners so that they can interact, get input, and ask questions during the lesson.

2.6 THE SIGNIFICANCE OF MAYER'S COGNITIVE THEORY OF MULTIMEDIA LEARNING

The purpose of using the cognitive theory of multimedia learning as a framework for this study was to explore visualisation in heights and distances in Grade 12 trigonometry to assess learners' mathematics achievement before and after introducing visualisation. The theory assisted the researcher in understanding the mental processes that occur when learners are introduced to visual tools. It also directed how the intervention should take place, giving the researcher direction on planning the lesson during the intervention so that learners could understand multimedia instructions.

2.7 LIMITATIONS AND CRITICISM OF THEORY OF MULTIMEDIA LEARNING

Rasch and Schnotz (2009) conclude that they could not understand whether learners learn better from text and pictures or text content alone, calling the multimedia precept into query. The (ibid) referred to the fact that Mayer's idea doesn't point out whether or not learners understand better from interactive or non-interactive images.

Çeken and Taşkın (2022) mention that Mayer's theory focuses only on understanding physical or didactic learning environments where learners are active participants and the teacher aims to reach a specific objective. This prompts the query of how transferable the findings are to nondidactic learning contexts. This criticism questions the theory's applicability in dynamic classrooms and learning environments. These accusations of lack of application to real-world learning and instruction have frequently been proven true.

According to Sorden (2012), Mayer's cognitive theory of multimedia learning ignores learner stress and motivation, which might increase cognitive load. Stress may prevent learners from focusing on the words and images throughout the teaching session, which prevents knowledge from being stored in working memory. Non-narrative audio, such as background music or outside noise, may distract learners from the multimedia lesson, and the theory does not consider this factor (Sorden, 2012).

Depending on how learners' processing has been customised, the dual channel approach may also work well for some and poorly for others (Çeken & Taşkın, 2022). With adult learners, some may still be stuck in their habits, and their brains have likely been rewired to process, consider and retain information in this fashion. Many conversations might overwhelm learners regarding overload, but having polite conversations can also make other learners tune entirely out. Using a general teaching method to instruct a sizable group of learners could also be challenging. Mayer (2005) avoided asserting that research should be the last word in teaching and learning. Instead, it is clear from the development of the cognitive theory of multimedia learning that they merely attempt to identify what seems to matter in learning circumstances, speculate about it and then continue to search for more comprehensive explanations and hypotheses. The theory is dynamic, and the literature regularly mentions that it will continue to develop, adapt and alter.

2.8 CONCLUSION

In this Chapter, the theoretical and conceptual frameworks that underpin this study has been presented and discussed. Mayer's cognitive theory of multimedia learning provided a foundation by drawing on established theories that explain the key concepts and relationship relevant to the study. This theory serves as a lens through which the research problem is examined, ensuring that the study is grounded in existing knowledge and scholarly discourse. In the following chapter, the frameworks will be applied while discussing the literature review.

CHAPTER 3: LITERATURE REVIEW

3.1 INTRODUCTION

The preceding chapter discussed the theoretical framework for the investigation. It covered the theory that underlies the study's discussion of the effects of using visual aids in teaching and learning about heights and distances in trigonometry on learners' comprehension of the subject. The cognitive theory of multimedia learning by Mayer (2005) is the hypothesis discussed in the preceding chapter. This chapter presents the literature review that guided this study. It discusses what a literature review is, how it is used in research, the rationale for using one in a study, visualisation in mathematics teaching, its benefits, and its glitches. Finally, it discusses performance in mathematics and trigonometry.

3.1.1 Rationale for using literature review in the study

In addition to choosing between a quantitative, qualitative, or mixed methodology approach, Creswell (2018) states that the proposal or study designer must also undertake a literature review. This literature assessment helped determine whether the topic merits additional investigation and offers suggestions for the researcher's focus on a particular area of interest. In addition to informing the reader about the results of studies closely linked to the one being considered, the literature review serves several other goals (Creswell, 2018). Bridging gaps and extending prior research links a study to a larger, ongoing conversation in the literature Creswell, 2018).

3.2 VISUALISATION IN MATHEMATICS TEACHING

“Visualisation is defined as a technique, a product, and a method of creation and interpretation, a reflection of a diagram, representations, and pictures in our thoughts” (Makgakga, 2016, p.61). According to Muzangwa (2018), visual learning refers to the process through which learners make meaning of what they can see. Visualization is important in mathematical problem solving (Osman et al., 2018). Visualisation stimulates learners' imaginations and deepens understanding. It not only helps learners visualise the question, but it also helps improve their thinking skills. By learning to visualise a question, learners can apply this technique to various problem solving topics at any level.

Patahuddin et al., (2020) contend that teaching through visuals should be used in mathematics classes, since maths is related to visual elements. Additionally, learners should master transitions between the numerical, symbolic, and pictorial representations of mathematical topics during the mathematics lessons. As a result, learning through visualisation allows learners to fully grasp the concepts being taught and develop original, creative solutions to challenges. As a result, producing and modifying visual mental images and any spatial inscriptions that may be required to complete mathematics is referred to as visualisation (Presmeg, 2014).

According to Makgakga (2016), visualisation plays an important role, and its importance in mathematical learning has been researched previously. Visualisation in mathematics has a long history, and many math teachers actively promote it (Rosken & Rolka, 2006). Learners' conceptual comprehension of solving mathematical problems has been helped by using visualisation techniques in the classroom (Mnguni, 2014). As noted by to Presmeg (2020), mathematical visualisation is creating images or mental models and using them effectively for mathematical research and comprehension. Visualization occurs when an individual creates spatial arrangement; during this process, a visual image is created in a person's mind, guiding this creation (Presmeg, 2014). According to Mnguni (2014, p.274), "visualisation is crucial when solving arithmetic issues". Mnguni further explains that visualisation sparks learners' imaginations and enhances their comprehension. Visualisation not only aids learners in visualising the question but also helps them develop critical thinking skills. Once they understand how to visualise the question, learners may apply the approach to any topic and solve problems at any level.

Visualisation has been used in various articles, so it is important to explain how it will be used in this research. Visualisation forms or builds intellectual representations and efficiently uses such photos for mathematical discovery and understanding. Diagrams, images, and visible tools commonly support visualisation (According to Hamidreza et al., 2018). According to Hamidreza et al. (2015, p.803), "visualisation is an integral part of problem solving in mathematics". Learners' imaginations are sparked, and their studying is deepened once they use visualisation (ibid). It does more than best help learners visualise the question; it additionally assists them in enhancing their wondering skills. Learners can use this method for any subject matter with problem-fixing to any degree if they apprehend a way to visualise the question. According to Osman et al., (2018, p.88), "a diagram or image that the pupils use or assemble to beautify their know-how will robotically generate a massive image of their thoughts to dig up the answer of the problem".

3.2.1 Benefits of visualisation

Visualisation techniques have been utilised in the classroom to help learners increase their conceptual knowledge of mathematical problem solving (Mnguni, 2014). Osman et al. (2018) assert that learners' grasp of the questions will improve when visualising them. Learners can utilise a visual depiction (drawing) to help them comprehend new topics (Bakar et al., 2016).

They can perceive the connection between the information provided and the requirements of the inquiry. The visualise approach aims to improve the mental grasp of the subject.

Previous research on visualisation shows the benefits of using visualisation in learning. Osman et al. (2018) in Malaysia studied or investigated learners’ achievement in mathematical problem solving after applying the Bar Model visualisation technique. The methodology used entailed pre-test and post-test questions and a semi-structured interview. The results of this study revealed that when visualisation is used in mathematics problem solving, learners perform better. The research suggested that teachers can use this study as an alternative or guide to help learners improve their mathematical problem solving abilities through visualisation.

Makina (2010) investigated the function of visualisation in the growth of mathematical critical thinking. The author investigated how visualising information might help Grade 9 learners in SA strengthen their critical thinking skills and better mathematics comprehension. Visualisation improves our grasp of data handling by encouraging critical thinking (Makina, 2010). According to their research, visualising ideas is essential for developing critical thinking skills and formulating meaning and comprehension. Understanding how learners think when solving problems (visualisation) might help teachers improve how they teach critical thinking to their learners. This research suggested that if teachers help learners improve their arithmetic skills, they are more likely to succeed.

Kashefi et al., (2015) explored the use of visualisation when solving mathematical problems (meta-analysis research). The study aimed to research concepts and methods for visualising difficulties to advance and enhance mathematics education, particularly in Malaysia. The study discovered that visualisation, which combines problem solving and interpretation of drawing skills, can increase a learner's confidence. This approach pushes learners to use critical and inventive thinking to address all of the teacher's inquiries.

3.2.2 Challenges of visualisation

Kashefi et al. (2015) identified difficulties concerning the use of visualisation in their research based on visualisation (meta-analysis research). (ibid) state that learners believe that the visualisation approach does not help them because it requires them to understand how to use a tool such as a sketching diagram or picture to solve a problem. Furthermore, they must gather and sum up all information from various sources to comprehend the entire situation worldwide (Elia & Philippou, 2004). They hesitate to employ them in problem solving because they believe it is tough to find out (Uesaka et al., 2007). Even though they were dealing with a complex problem, they used an algebraic technique to solve it.

As noted by Makgakga (2016), the limitation and difficulty with visualisation is that learners are reluctant to visualise when solving problems. According to their findings, Mudaly et al., (2010), learners had poor visual representations while solving problems and relied on the diagrams provided in the classroom by the teachers. This indicates that learners could not visualise the situation mentally. (ibid) investigated the role of visualisation in learners' conceptual understanding of the graph- functional relationship of a problem. In problem solving, learners interpret graph visualisations and investigate the relationships between graphs and the learner's knowledge of those graphs. Their research found that learners had poor visual comprehension and relied on the diagrams teachers used in class.

3.2.3 Review of previous research on the use of visualisation in the classroom setting

The study on teachers' use of visual representations in science classrooms was done by Cook (2011). The study's primary goals were to learn more about how teachers employ visualisation in their science lessons and how they feel about it. In terms of sampling and population, the researcher chose seven (7) science teachers from seven (7) schools to participate in the study. The study adopted a case study methodology and employed a layered design for the case study. An overarching case study was created to examine science teachers' overall selection and use of graphics.

The seven (7) teachers selected serve in the southern region of the US school, with approximately 1,000 learners, and each educator teaches a different science education course. For six (6) weeks, the researcher observed classes in four (4) science subjects; Earth Science, Physics, Chemistry and Biology each taught by a different professor. Except for physics, two (2) professors instructed each science subject, and each teacher was observed six (6) times (three times) for 21 observations. Teachers used visual representation while being observed in the classroom, while the researcher gathered field notes. Following the observations, recorded class recordings were examined to supplement the initial field notes. Additionally, copies of the visual aids utilised during class were gathered.

Semi-structured interviews were then conducted, in which the researcher interviewed all seven educators to understand their use of visual representation during the lessons. At the beginning and end of the semester, interviews with each educator took place twice.

Teachers were questioned about their usage of visual aids throughout the semester during the final interviews. Some of the queries posed were as follows: "Why do you incorporate pictures into your lessons? What qualities distinguish good graphics from bad ones? What factors affect the type or quantity of visuals used based on the learner? What sources do you utilise to find images to use? How do the resources you have access to or lack thereof affect the images you use? What are any visual restrictions you could have come across? What would you say about the graphic quality of your textbook? What do you imagine the learners do with the pictures in their textbooks? How were the images chosen for the classroom decorations?" (Cook, 2011, p.178)

Cook (2011) states that during this research, the information gathered from various sources (field notes, visuals, and interview replies) was analysed to find themes that arose from the information. Through open coding, the qualitative data were inductively analysed. The results of this study suggest that in the effort to create a set of guidelines for graphic design, it is important to consider how teachers use these visual aids when instructing learners. If design concepts are not studied in the classroom context, they will not have much meaning. Cook (2011) recommended that future studies on instructors' use of visual aids in science classes be broadened to include more conventional classroom settings.

Shabiralyani et al., (2015) conducted a case study on the effectiveness of visual aids in boosting the learning process in District Dera Ghazi Khan aimed to discover how teachers felt about using visualisation in their lessons. The researchers focused on the instructors and pupils of both public and private schools. 200 people (teachers and learners) were included in the study. The influence of visual aids in aiding the learning process of the Dera Ghazi Khan learners was measured using a variety of closed-ended questionnaires. Regression and correlation analysis were used to analyse the data in SPSS. The research was based on quantitative research, using random sampling techniques to collect information from individuals with specific knowledge.

Pie and line graphs were used to analyse the data gathered. In schools, colleges, and universities in Dera Ghazi Khan, the results of the questionnaire suggest that 70% of learners and teachers concur that visual aids help with motivation. In comparison, 30% of learners and teachers disagree. The dispersed plot reveals that 82% of learners and teachers felt using visual aids during class preparation saves time. According to the statistics gathered, 71% of teachers and pupils concur that using visual aids helps prevent boredom. However, 29% don't.

The finding revealed that several schools did not employ visual aids in their classes because they lacked sufficient teaching tools or materials. Additionally, the study showed that the high schools that were the subject of the study did not have enough money to buy teaching and learning materials like globes, maps, and textbooks for teachers to use in the classroom. This was the case during my research since I focused on rural schools. Hence, I introduced accessible material to learners that does not necessarily need funding. The findings gave teachers guidelines to follow when utilizing visual aids in the classroom because they want their learners to fully focus on the lesson and understand the standards and requirements for teaching mathematics. When teachers know how to get the learners' attention, they can make the classroom a friendly and stimulating place for learning. In order to learn more about the subject, the learners were encouraged to read what they had learned instead of just listening to and recording what their teachers said, did, and provided in class. Additionally, teaching with visual aids takes less time. The educators will have more time to organize interesting classes and manage a successful teaching and learning process.

Kashefi et al., (2015) studied visualisation in mathematics problem solving (meta-analysis research). The study investigated concepts and approaches for visualising problems to improve mathematics education, particularly in Malaysia. Researchers chose test questions from the past that addressed visualising a solution to a problem. Initially, 52 previous exam papers were obtained. Still, only 26 were ultimately chosen because the remaining 26 studies had to be disregarded because they did not meet the criteria for the meta-analysis. These test questions were downloaded from the Wiley Online Library, SpringerLink, Google Scholar, Taylor & Francis, SAGE, and ScienceDirect databases.

Twenty-six visualisation studies were picked to be presented in the meta-analysis. The researchers pinpointed numerous elements that can support visualisation in problem solving through the meta-analysis they conducted. They selected the following themes from the 26 studies: external visual representation, nonvisual vs. visual, performance learners, and learning problems utilising diagrams. According to their research, visualisation benefits children with learning difficulties. As a result, some researchers employ visual imaging as a helpful tool for solving mathematical problems using words.

The results also demonstrate that using visualisation during the problem solving was beneficial. Some learners significantly preferred to employ the visual method when solving challenging tasks. According to Victoria (2021), visual aids to solutions are six times more effective than verbal ones and help people develop strong problem solving skills. They also discovered that although preschool learners are frequently exposed to creative visualisation in problem solving, once they enter primary school, they stop doing so since teachers at that time hardly ever teach using visualisation techniques to solve mathematical issues. The researchers concluded that visualisation, which combines the skill interpretation of drawing and problem solving, can boost credibility and potential among the learner. Learners must use creativity and critical thinking to respond to all of the questions the teacher posts when using this method. As discussed in this study, few attempts to thoroughly integrate visualisation in problem solving have been made in Malaysian research. Therefore, it is still unknown how well the combo will increase learners' comprehension.

Makina (2010) investigated how visualization might foster mathematical critical thinking. Makina studied how Grade 9 learners in Pretoria, South Africa, used visualization to improve their critical thinking skills and comprehend mathematics. In terms of population and sampling, the researcher worked with a high school math instructor to pick twelve (12) Grade 9 learners using a non-probability selection technique. Four learners were above average in mathematics, another four (4) were below average, and four (4) were average. This was based on their performance in Grade 9 mathematics. The data was collected during the planned one-hour session with learners. Throughout the session, learners responded in writing to the assigned work. Individual audio-taped interviews were conducted to supplement the written responses' data. The activities chosen were picked because they could be easily modified for use in mathematics classes and have previously been used in other research to increase visualising. The four (4) steps of visualisation image generation, image inspection, image transformation, and image helped the researched to study the written responses and the interviews (Kosslyn, 2005).

As stated by Makina (2010), visualisation is vital in teaching and learning mathematics since it focuses on critical thinking and determines the quality of the understanding. Makina (2010) says that while some teachers prefer to concentrate on teaching the fundamentals of mathematics, most learners do not develop the necessary thinking abilities to create meaningful theoretical differences. Therefore, the development of critical thinking skills, as well as the creation of meaning and comprehension, depends heavily on visualisation. Understanding how learners solve problems (visualise) might help teachers better foster critical thinking in their learners. The conclusion drawn from this paper is that teachers who remain aware of visualisation, one of the fundamental processes to promote critical thinking in the classroom, would likely help learners improve their performance in mathematics.

3.3 TRIGONOMETRY CONCEPT

According to Rosjanuardi et al., (2022), trigonometry is a discipline of mathematics that examines how triangles' sides, angles, and other functions relate. The application of geometric and algebraic knowledge and reasoning are combined in trigonometry. Trigonometry is a crucial part of math education because it deals with real-world issues that frequently arise in engineering, building, design, and physics (Simons, 2016). Learners must comprehend the material and be able to use trigonometry to solve problems in the real world. Trigonometry gives learners a thorough comprehension of the relevant mathematical ideas, making it essential for addressing advanced mathematics problems.

Research based on trigonometry has been done before, and the discussion below focuses on previous research done on trigonometry. Gur (2009) conducted a Trigonometry Learning study, describing errors, potential misunderstandings, and obstacles in trigonometry courses. Six math teachers and 140 learners made up the sample. The most frequent errors made by pupils in answering the questions were chosen. Gur (2009) states that several issues have been noted, including incorrect equation usage, errors in the order of operations, errors in the value and location of sin and cosines, misuse of data, misunderstandings of language, illogical inferences, distorted definitions, and technical-mechanical mistakes.

Other research also focused on learners' misconceptions, errors, and mathematics teachers' knowledge of trigonometry. Researchers such as Malambo (2016) in Zambia, Tatira (2020) and Khuzwayo (2019) in SA did research based on learners' misconceptions, errors, and mathematics teachers' knowledge of trigonometry (subject matter content knowledge, pedagogical content knowledge, and envisioned pedagogy). Their findings were similar to those of Gur (2009). Tarira (2020) also reported that teachers' knowledge of trigonometry was limited, and their teaching methods were based on traditional teaching.

Furthermore, Khuzwayo (2019) state that these misconceptions and errors emerge from learning complexities. Previous research in trigonometry teaching and learning has revealed that various factors influence learners' mathematical performance. The primary factor is the ineffectiveness of instructional strategies used by some mathematics teachers in their classrooms (Khuzwayo, 2019). According to a report by an internal moderator of the DBE (2022), poor math performance in SA is due to teachers' lack of interest in incorporating strategic and conceptual teaching methods into their lesson plans.

3.3.1 The history of Trigonometry

According to Sampaio et al., (2018), the Greek words trigonon, which imply triangles, and -metria, which indicate measures, are the origins of the word trigonometry. Therefore, the study of trigonometry is the measurement of a triangle's sides and angles and the identification of the relationship between them. Bartholomaeus Pitiscus, a German mathematician and astronomer, is credited with developing trigonometry (Figueiredoet et al., 2018). The Egyptians and the Babylonians devised theorems on the side ratios of related triangles. Trigonometry was formerly applicable to the Egyptians in land surveys and constructing the pyramids. Babylonian astronomers connected circular arcs and the lengths of chords enclosing those arcs to trigonometric functions (Figueiredo & Batista, 2018).

3.3.2 Benefits of learning Trigonometry

As noted by Sampaio et al., (2018), mathematics is an established field of knowledge with internal coherence. In the past, trigonometry was primarily employed to address problems in physics and other sciences, allowing for the mathematisation of nature, as seen in studies of periodic phenomena and astronomy, which similarly convey their cognitive values today.

The usefulness of trigonometry in the real world, in surveying, land measurement, mensuration, and navigation, ensured the importance of the subject in colonial times (Sampaio and Batista, 2018). Most mathematicians who studied astronomy devised or utilised trigonometry in some way to make their calculations. Trigonometry was created about a thousand years ago and was still being improved before it was even acknowledged as a field of study. Although much of the research at the time was inaccurate, the ideas that emerged from it are still relevant and useful today. Little would have been accomplished in navigation, land measurement, mensuration, and surveying without the study of trigonometry. Although trigonometry is a major cause of irritation for many people, it is a crucial component of the mathematics core curriculum and must be introduced to pupils. Even though not much has developed with trigonometry since the eighteenth century, perhaps the future will hold some bright minds that will make remarkable discoveries.

3.3.3 Challenges of learning Trigonometry

According to Adhikari et al., (2021), trigonometry is often a source of difficulty and grief for high school and collegiate learners. From my experience as an educator in Grades 10, 11 and 12, few learners understand the concept and application of trigonometry in real life. Learners struggle with trigonometry in general, not just heights and distances. Most of my learners do not understand how to choose a correct trigonometric ratio to calculate heights and distances in 2D shapes, and it becomes too complicated for them when they are presented with 3D shapes. Some struggle to even identify the triangles in a 3D shape. Hence, they cannot apply trigonometry to calculate the required sides and angles.

The issue, according to Makina (2010), is that even when learners are obliged to utilise trigonometric ratio principles to measure triangle angles, many learners fail to make links to everyday life or comprehend the origins of these ideas. In mathematics, trigonometry is frequently introduced as a distinct concept rather than being combined with the concepts of algebra and geometry, which were its antecedents. Due to the lack of a relationship to previously learned mathematics, learners are perplexed by the subject's complexity and wonder what function it serves in mathematics and daily life.

3.3.4 Review of previous research on Trigonometry

Madonsela et al., (2020) researched 3D problems using trigonometry. The research used the theory of action, process, objects and schema (APOS). Their study explored learners' solving three-dimensional problems using Trigonometry. Their study examined the mental models that learners in Grade 12 employed when attempting to solve trigonometric triangles. The investigation was conducted at KwaZulu-Natal, South Africa. A prepared exercise sheet containing questions about three-dimensional difficulties was given to the learners. In order to comprehend the responses on the structured activity sheet, the researchers selected a number of learners for interviews. The information was gathered through interviews and the written answers provided by learners on a structured activity sheet. This indicates the use of the sequential explanatory mixed technique, which enables researchers to gather qualitative data subsequent to quantitative data. Researchers were able to explain and understand the findings of the activity-based task (quantitative information) using the mixed approach and the written response interview (qualitative analysis).

The data were analysed using APOS theory. In their findings, the nature of learners' mental construction shows that learners only exhibited mastery of topic information while solving 2D triangles. The majority of learners showed a lack of even action conceptualization when solving triangles that involved 3D and non-right-angled triangles. According to action conception, learners' inadequacies in their knowledge building of prior ideas may have prevented them from forming the proper mental structures for solving triangles (Madonsela et al., 2020). According to this study, learners have not developed knowledge of real-world problem solving techniques, and this has been noted in their operations. Additionally, when multiple triangles are connected to make three-dimensional objects, learners are unable to understand the language and structure of the questions since they are unable to even identify the hypotenuse side of a right-angled triangle.

Therefore, the recommendations were that teachers should come up with effective ways of teaching heights and distance in trigonometry and discourage the use of rote learning. The authors further suggest that learners should be encouraged to construct mental knowledge in problem solving and apply their minds in real life situations (Madonsela et al., 2020). Rote learning leaves the gap in finding effective strategies (that will trigger the mental construction of knowledge) for teaching height and distance problems using trigonometry since it is evident from Madonsela et al. (2020) research that learners can solve 2D problems. Still, they lack the understanding of solving 3D.

Looking at the research done by Madonsela et al. (2020) does not explain if learners have been taught three-dimensional before. The researchers did not observe the lesson being taught to learners, which did not assure them they were taught it before they could conduct their study. The study did not include any intervention to change learners' performance in three-dimensional problems. Their aim was only to research how learners understand these types of questions and how they answer them. They then came up with a recommendation that other researchers and teachers can use to teach 2D and 3D trigonometry.

Gerhana et al., (2017) studied the effectiveness of project based learning in trigonometry in well-resourced urban schools (Indonesia). Learners were working in groups and were required to calculate the height of the Wi-Fi tower using a clinometer and meter. This study uses a quantitative approach as its methodology. The research methods used in this study were quasi-experimental. Learning model as an independent variable and learner learning achievement in mathematics as a dependent variable are the variables that will be observed. Tests and an observation sheet were the research tools employed. The author utilises a multiple choice instrument with up to 25 questions drawn from previously used trigonometric material to gather data on mathematics learning achievement. Observation sheets assess how well the research fits into the predetermined strategies.

The study's findings revealed that a project-based learning paradigm more successfully increases learners' arithmetic learning achievement. According to Gerhana et al. (2017), project-based learning is an instructional approach that promotes learners to use knowledge and abilities in a fun way. Their reach was only based on 2D shapes. They did not apply project-based learning to calculate heights and distances in 3D shapes.

While learners' understanding of calculating the height of objects using trigonometry has been previously researched in other countries such as Turkey, Indonesia and Zambia, the research that is project based and involves the use of visualisation in heights and distances has not been done in SA. From the research discussed above, it is evident that not much research has been done in the context of rural based learners. Learners' skills in using trigonometry to calculate heights and distances have not been researched. Hence, this research is aimed at filling that gap.

On a didactic sequence (Teaching trigonometry), Sampaio and Batista (2018) studied the historical development of mathematics and its cognitive values. Their research sought to assess the efficacy of Lacey's (1998) historical-philosophical method of teaching trigonometry to identify cognitive values. A study was carried out in a Brazilian high school where learners were the population of the study. The researcher made use of qualitative research. This study aimed to show the learners, teachers, and readers that content is not systematised. Still, it was there in ancient times, past years, or centuries to achieve systematisation in current textbooks. This method enables us to see the content as systematised and ready, as well as the process of studying it as something that has been developed through millennia to reach the systematisation found in modern textbooks. Sampaio and Batista (2018) assert that in order for learners to comprehend how the teaching methodology of the historical components of the material being learned contributes to overcoming obstacles, recognizing and comprehending, and aiding in the teaching and learning process, it is crucial to take this into account. However, the history of mathematics should be viewed as a pedagogical strategy to be used in the classroom rather than as a straightforward methodological tool.

Three (3) mathematics teachers were interviewed since they taught trigonometry at a targeted high school. The questions were based on the difficulties that learners face when presented with a trigonometric question. Some of their responses indicated that learners could not visualise trigonometric functions; they struggled and performed poorly in geometry and algebra. Trigonometry at the triangle, triangle, and trigonometric cycle were all indistinguishable to the learners. Both the functions and the study of the period were not understood by learners. Researchers employed a survey to find out what learners already knew about the Grade 8 triangle-rectangle curriculum and trigonometry in general. They noticed that the majority of learners were having trouble answering these kinds of questions. This demonstrated the necessity for teachers to reintroduce this Grade 8 material through exercises. They recommended that before introducing learners to new material, teachers should review their past understanding of the learners.

3.4 CONCLUSION

This chapter has provided a comprehensive review of existing literature relevant to this study. Key theory, concepts and empirical finding in the field were highlighted. Through the analysis of the previous research, gaps in knowledge have been identified, establishing the need for this study and its contribution to the existing body of work. The review has demonstrated that while significant progress has been made in understanding the role of visualisation in the teaching and learning of heights and distances, little work have been done in rural schools that have no access to technology. By synthesizing insights from the previous research, this chapter laid the groundwork for the study's methodology, ensuring that the research design is informed by established knowledge while addressing identified gaps. The next chapter will detail the research methodology, outlining approach and data collection methods employed to investigate the research problem.

CHAPTER 4: RESEARCH METHODOLOGY

4.1 INTRODUCTION

This chapter explains the mixed-method research employed in a study of one secondary school in Kwa-Zulu Natal Province. The main research paradigm, which is pragmatic, is discussed in the first section of this chapter. The descriptions of the various research methodologies, including qualitative, quantitative and mixed methods, are also discussed. Both the methods for data analysis and the data collection tools used in this study are covered. The chapter also covers discussions on sampling, validity and reliability, followed by a description and justification of the instruments employed in this study. This chapter also discusses ethical concerns and the ethical attitude that this investigation has adopted.

This study hypothesised that using visualisation when teaching heights and distances impacts learners' performance. Therefore, this study had the following hypothesis:

The null hypothesis

The use of visualisation has no effect on learners' performance in terms of heights and distances.

The alternative Hypothesis

The use of visualisation has a positive effect on learners' performance in terms of heights and distances.

As noted earlier, this study intended to answer the following research questions:

The primary research question:

What is the effectiveness of using visualisation on teaching and learning heights and distances in trigonometry to improve academic performance?

The research sub-questions:

- What types of teaching methods do teachers use when teaching heights and distances?
- What are the benefits and challenges (if any) of visualisation in the teaching and learning of trigonometry?
- What are possible ways of implementing effective visualisation in the teaching and learning of heights and distances to improve learner performance?

4.2 RESEARCH METHODOLOGY

The term research methodology describes the methodical, theoretical review of the approaches used in a field of study. It includes a researcher's methods, approaches, and resources for gathering, analysing, and interpreting data. Research methodology offers a structure for conducting research and guidelines for designing the study, collecting data, and analysing the findings. According to Creswell (2020), the research methodology should be aligned with the research questions and objectives.

4.2.1 Research approaches

Research approach refers to the study plan, strategy, or framework that guides the selection and application of research methodologies in relation to the intended research questions and outcomes (Pregoner, 2024). The research approach serves as a blueprint for the methodology employed in the study. This study utilised a mixed methods approach, which integrates both quantitative and qualitative techniques. The three types of research approaches- qualitative, quantitative, and mixed methods research are discussed in the following section.

4.2.1.1 Mixed methods research approach

This study adopted a mixed-methods approach, employing both qualitative and quantitative arms. Schoonenboom (2023) reported that most researchers define mixed methods research as the combination and integration of both quantitative and qualitative data collection and analysis. This meaningful connection or integration produces both quantitative and qualitative data. Makgakga (2016) asserts that employing mixed methods in a single study is based on the belief that neither qualitative nor quantitative methods alone suffice to compare the trends and details of a particular situation when used independently. Creswell (2018) argues that these two methodologies complement one another when employed in a single research study, allowing for a more robust analysis by leveraging the strengths of each method. Mixed methods enabled the researcher to test and justify the research hypothesis, answer all research questions, and enrich the adaptability of the research design. Had the researcher concentrated solely on either qualitative or quantitative approaches, some research questions would have remained unanswered.

Following the collection and analysis of quantitative data, qualitative data was acquired through lesson observations and semi-structured interviews. The quantitative results were interpreted using qualitative data. This study utilised components from both quantitative and qualitative methodologies to address its research questions. Three data collection techniques employed in this study are pre- and post-tests, semi-structured interviews, and classroom observations.

This study aimed to explore the effectiveness of visualisation in teaching and learning heights and distances in Grade 12 in order to improve academic achievement. The focus was on how visualising heights and distances enhances learners' academic performance. The study examined how learners are currently taught at a focused school before suggesting the teaching of trigonometry using visual models. Additionally, the study sought to enhance teachers' approaches to instructing heights and distances with visual models. Through this investigation, the researcher gained insights into how learners are presently taught about height and distance and how visualisation impacts their understanding. A mixed methods approach was employed, utilising pre- and post-tests, semi-structured interviews, and classroom observations to collect both quantitative and qualitative data, thereby addressing the study's stated aim.

The researcher did not employ the quantitative approach alone because it addresses only one part of the study: data collected through pre- and post-tests, not data from observations and interviews. The qualitative approach assisted in obtaining data through observations to better understand how teachers teach and how learners respond to those teaching methodologies. Conducting interviews was aimed at exploring teachers' experiences with teaching heights and distance before and after the intervention. Using both these methods allowed the researcher to answer all research questions and justify the study's hypothesis.

4.3 RESEARCH PARADIGM

Creswell (2018, p.78) defines research as a "series of actions undertaken to gather and analyse data in order to gain a deeper understanding of a subject or problem". The author further states that three primary processes are involved in conducting research: identifying the issue, gathering the necessary data to address it, and presenting the solution. According to Kumatongo and Muzata (2021), a 'paradigm', which represents the researcher's mental perspective, can drive research. The term "paradigm" refers to a pattern, model, or typical example, encompassing cultural themes, worldviews, ideologies, and mindsets (Kankam, 2019). A research paradigm comprises the collection of shared assumptions and principles that scientists possess regarding how issues should be acknowledged and resolved (Anand et al., 2020). A paradigm determines what ought to be examined in a research context, what research questions should be addressed in the study, and how those questions will be analysed and interpreted using the gathered data (Khatri, 2020). Figure 4.1 below describes how researchers should tackle the three fundamental problems of ontological, epistemological, and methodological issues differentiate research paradigms.

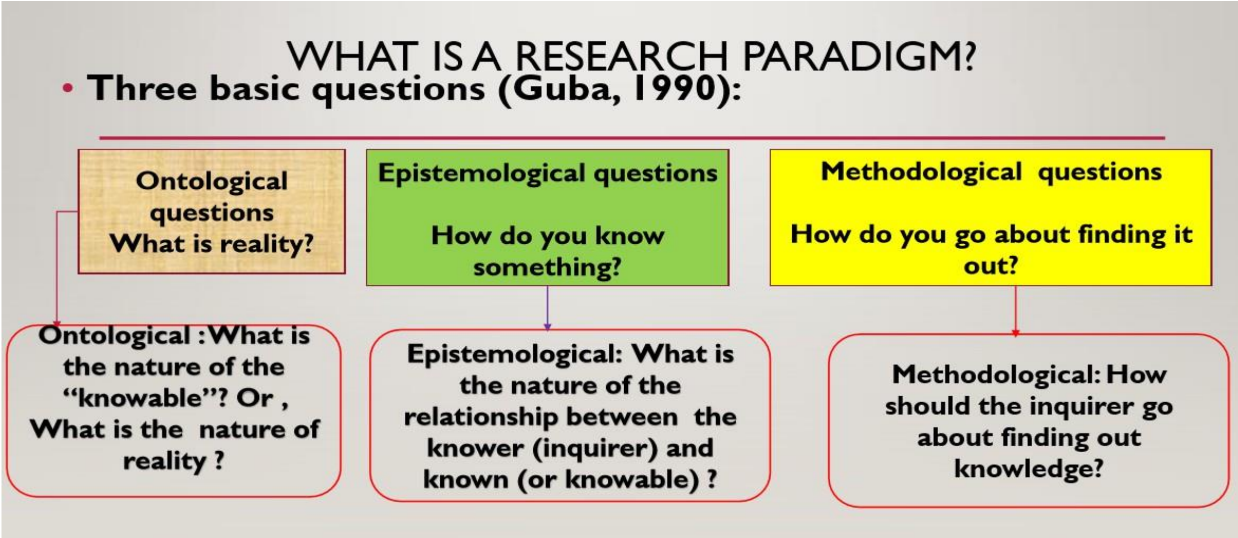


Figure 4.1: ***Three basic questions on research paradigm*** *(Source: Perera, 2018)*

Ontology is the study of beings and their existence. Ontology explores the nature of the knowable and the essence of reality (Perera, 2018), It pertains to the question of what can be known. In this research, ontology explains the teachers' assumptions about reality regarding teaching heights and distances in Grade 12. It aided the researcher in discerning the differences in teachers' comprehension of teaching heights and distances in trigonometry. Ontologically, the researcher gained insights into the teaching strategies employed by the teachers.

Epistemology is the study of knowledge, focusing on how we learn and the relationship between the knower and the known (Al-Ababneh, 2020). The researcher gathered and comprehended the participants' individual experiences through lesson observations and open-ended interviews. This also aided the researcher in understanding how educators employed various approaches to support learners' learning of heights and distances.

A methodology is a plan of action or strategy that influences the methods employed to collect information (Al-Ababneh, 2020). Methodological questions concentrate on how the researcher or inquirer goes about acquiring knowledge. These aspects of the research paradigm help define the delimitations and boundaries of the study. They inform how the researcher should act and carry out successful research.

4.3.1 Pragmatic paradigm

This study espoused a pragmatic paradigm with the intent of using both qualitative and quantitative arms.

4.3.3.1 Origin of pragmatic paradigm

A philosophical movement known as pragmatism emerged in the late 19th and early 20th centuries, focusing on the applications of social reality (Kelly & Cordeiro, 2020). According to Creswell (2018), pragmatism arises from the work of renowned writers such as Peirce, James, Mead, and Dewey (1990), Patton (1990), and Rorty (1990). The worldview of pragmatist researchers is rooted in actions, situations, and consequences rather than past conditions. Table 4.2 below shows the summary of pragmatic paradigm.

Figure 4.2: ***A summary of pragmatic paradigm*** *(Source: Perera, 2018)*

The study adopted a pragmatic viewpoint. It aimed to comprehend the research problem using the various accessible methods, including mixed methods approaches, which combine qualitative and quantitative methodologies simultaneously (Christensen, 2022). This study acknowledges Creswell's (2014) assertion that pragmatism allows researchers to use many methods, multiple realities, diverse worldviews, and assumptions, including the various types of data collection and analysis that enable mixed methods research.

The pragmatism paradigm incorporates elements of both positivism and constructivism. To gain further insights into the subject under study, various data collection tools may be employed alongside other techniques (Sambo, 2022). By combining elements of the interpretative and post-positivist paradigms, the researcher can more effectively tackle the study's issues and hypotheses and bridge any gaps that might have arisen from focusing on a single paradigm alone.

"The foundation of pragmatism is that research can avoid philosophical discussions about the nature of truth and reality, and instead concentrate on practical understandings of specific, real-world problems" (Kelly & Cordeiro, 2020, p.3). Rather than focusing on methodologies for collecting data, the researchers centred their attention on their research questions and problems, employing all available approaches to comprehend the issue at hand (Creswell, 2018). Consequently, this research is characterised as pragmatist, as it employed mixed methods by incorporating both qualitative and quantitative assumptions during the research process. This enabled the researcher to engage with various methodologies to effectively address all their research questions. This actively demonstrates that the pragmatic paradigm allows researchers to select techniques, methodologies, and procedures that align with their research needs (Sambo, 2022).

The researcher relied on participants to develop information through interviews and lesson observations, while maintaining independence from the knowledge obtained from the tests administered to learners. To understand the experiences of the participants (teachers) and derive meaning, the researcher utilised open-ended questions (Creswell & Creswell, 2023). Through one-on-one interactions with participants in their schools, the researcher fostered her own understanding and awareness of the diverse experiences related to the topic under investigation. The meaning expressed by the participants was influenced by their social relationship with the researcher, their individual experiences, and their personal understanding (Creswell & Plano Clark, 2023). Engaging with participants was essential for the researcher to gain insight into their experiences and provide context for the challenges they encounter when teaching heights and distances.

This study explored the effective ways of teaching heights and distances using visualisation in Grade 12. Pragmatism was used in this study because the researcher accepts numerous realities, which are essential for this kind of research (Creswell & Creswell, 2023). The researcher also used a variety of research instruments in an effort to produce knowledge. Since using just one paradigm would have excluded certain aspects of the other, the researcher did not independently use the interpretivist or post-positivist paradigms.

Post-positivist research primarily focuses on the causes that influence outcomes (Creswell, 2018). The shortcomings of the quantitative component of the research study were addressed through the application of post-positivism. However, even the emphasis on observation by post-positivists was insufficient to address all of the study's research questions on its own. Conversely, the researcher employed the constructivist paradigm through interaction with the participants to understand and share their perspectives about their world and deeply comprehend their experiences. The researcher conducted interviews with open-ended questions to gain deeper insight into the participants' views and experiences. As the study utilises mixed methods research, both interpretivism and constructivism are paradigms that can be applied to bridge the gap between the observed phenomena and explanations (Cohen et al., 2018).

4.4 RESEARCH DESIGN

A research design is defined by MacMillan and Schumacher (2014, p.28) as the "strategy that outlines how the research study will be conducted, including the time frame, participants, and method of data collection". In terms of the type of data gathered, the techniques used for data collection, and ultimately the ability to obtain tangible information for the research study, the researcher was guided by the research design.

4.4.1 Mixed methods experimental design

This study employed a mixed-method experimental design within an explanatory sequential mixed-method framework. A mixed-method experimental design aims to test the cause-and-effect relationship using experiments along with qualitative data to gain additional insights into the factors influencing experimental outcomes (Creswell and Creswell, 2023). According to Creswell (2018), explanatory sequential mixed methods comprise a two-part project where the researcher first collects quantitative data, analyses the findings, and then employs a qualitative phase to help elucidate the quantitative results. The experiment is the core component of this study; qualitative methods were utilised before, during, and after the experiment. In this study, the researcher administered a pre-test and analysed the results. Classroom observations followed to examine how the teacher instructed on heights and distances and how learners responded to those teaching methodologies. Semi-structured interviews were conducted to understand teachers' experiences with teaching heights and distances prior to the intervention. The researcher then proposed the use of visualisation as an intervention. During the intervention, the teacher commenced the first cycle by employing the interactive software, Geogebra. The researcher served as an observer while the teacher conducted the lesson. Due to the lack of resources in the selected school, there was no improvement in learners' performance. In the second cycle, the teacher utilised recyclable resources accessible to learners, such as tents and cutting a cardboard corner to create a 3D shape representing heights and distances. A post-test was then administered to assess the effectiveness of using visualisation in teaching heights and distances. Follow-up interviews with the experimental teacher were conducted to understand their experiences and explore whether visualisation was effective.

The researcher employed explanatory sequential design instead of exploratory sequential and convergent concurrent designs. Exploratory sequential design is a three-phase mixed methods approach in which the researcher collects and analyses qualitative data, followed by quantitative data. This design was irrelevant to the study as the researcher initially gathered quantitative data through a pre-test and then utilised the pre-test findings to formulate the questions to obtain qualitative data via baseline interviews. Conversely, convergent concurrent design is defined by Creswell and Plano Clark (2023) as a method for simultaneously collecting quantitative and qualitative data while researchers strive to validate, authenticate, and verify the results. This study did not benefit from a convergent concurrent design since quantitative and qualitative data were not gathered concurrently.

4.5 POPULATION

A population refers to the total number of individuals, events, or objects with a common characteristic relevant to the study (Thacker, 2020). This study was conducted in the Zululand District of the KwaZulu Natal Province, comprising 26 circuits and 207 secondary schools. Participants were sampled from two of these secondary schools. The research involved two rural secondary schools, one experimental and one control, with two male teachers instructing Grade 12 learners at these institutions. Inclusion criteria stipulated that participants must be enrolled in these secondary schools and registered for Grade 12. The study included 60 participants (learners); 30 were in the experimental group and another 30 in the control group from different schools. Two teachers, Teacher A from the experimental group and Teacher B from the control group, were also part of the study. The schools consist exclusively of black African learners and educators. The experimental school has underperformed in mathematics for the past four years, with most of the learners hailing from disadvantaged homes with low socio-economic backgrounds.

4.5.1 Sampling

According to Fink (2024), a sample is a subset of the population selected from a study location to represent the entire population. This study utilised sequential mixed methods sampling. Given that the research is conducted in both quantitative and qualitative phases, the researcher employed the population to sample participants for each phase. In the quantitative sampling phase, the researcher applied random sampling, ensuring that every individual in the target population had an equal chance of being selected for the study (Stratton, 2021). The population comprised 61 learners from the experimental group and 144 learners from the control group, with the sample targeted to include 30 participants from each group. All 61 and 144 learners in both groups expressed interest in participating when approached by the researcher. To create a sample frame, the researcher compiled a list of all learners in the population using the mark lists obtained from Grade 12 teachers. Each learner was assigned a unique number ranging from 1 to 61 for the experimental group and 1 to 144 for the control group. The researcher then employed a random selection method by drawing lots, where the numbers representing learners were placed in a container and drawn one by one until the desired sample size of 30 was reached.

Qualitative research participants are regarded as capable individuals who can reflect on and express their experiences, values, beliefs, and opinions (Akkaş & Meydan, 2024). Non-probability sampling is a technique used in qualitative research to choose groups to study a specific subject selectively or when the entire population is unknown or unavailable (Akkaş & Meydan, 2024). Consequently, the researcher employed convenient sampling, interviewing two teachers after classroom observation and one teacher after the intervention to gain insights into their experiences with teaching heights and distances before and after the intervention. The researcher contacted these teachers from specific schools that were easily accessible and willing to participate. They were assigned to experimental and control groups based on their consent to participate. According to Staller (2021), the number of participants depends on the available resources and the research purpose; a researcher may use one or two participants for narrative analysis.

4.5.2 School profile

The school profile provides an overview of the school(s) used in the study; it focuses on characteristics such as location, learner demographics, size, and other relevant information (Jay-Cen, 2020). The table below summarises the school profile for the experimental and control groups. Both schools are in rural areas within the Zululand District of KwaZulu-Natal Province. They are classified as public schools, which mean they are governed and supported by the Government. All the teachers and learners at these schools are African. The experimental school comprises 855 learners enrolled from Grades 8 to 12 and 23 teachers. The school features two buildings containing 10 classrooms, where learners are overcrowded. This is further evidenced by the teacher-learner ratio of 1:61 in Grade 12. Learners share desks and textbooks in groups of three, which may suggest a lack of furniture and teaching materials. Additionally, learners at the experimental school have no access to technology. Thus, the teacher relies on a chalkboard and textbooks. Table 4.1 below describes the profiles of experimental and control group

Table 4.1: ***Profile of experimental group school and control group school***

	EXPERIMENTAL GROUP SCHOOL	**CONTROL GROUP SCHOOL**
TYPE OF SCHOOL	Public school	Public school
Learner enrolment	61 learners. 39 girls and 22 boys	144 learners. 76 girls and 68 boys
Number of classes in Grade 12	1	4
Teacher-learner ratio	1:61	1:35
Technology integration and learning resources	No computer labs and no laptops available in school. Learners share textbooks in groups	There is a computer lab and enough textbooks

4.5.3 Teacher profile

The teacher profile includes details about the participating teachers, such as their educational background, teaching experience, and relevant qualifications in the subjects they teach (Sugiyarta & Prabowo, 2021). The table below presents a summary of teacher profiles from both the experimental and control groups. The teacher in the experimental group obtained his Bachelor of Education qualification in 2020 and registered for Honours while working, completing the qualification in 2022. He majored in mathematics and physical sciences and is currently teaching both subjects. The teacher does not use technology in his classes due to a scarcity of teaching resources. The teacher from the control group holds a qualification in electrical engineering and, due to his job, registered for a postgraduate certificate in education (PGCE) for one year in 2015. He majored in mathematics, physical sciences, and technology, teaching mathematics exclusively. As the control group school has access to technology, he incorporates it regularly into his mathematics lessons. Table 4.2 below shows the profile of experimental and control group teachers.

Table 4.2: ***Profile of the two sampled teachers***

	EXPERIMENTAL GROUP TEACHER	CONTROL GROUP TEACHER
GENDER	Male	Male
AGE-GROUP	25-30	30-35
QUALIFICATION	B.Ed. Hons in Mathematics	PGCE
SUBJECTS	Mathematics and Physical sciences	Mathematics, Physical sciences and Technology
TEACHING EXPERIENCE	3 years	9 years
TECHNOLOGY USE	No access to technology in school hence relies on textbooks.	Regularly integrates online resources and interactive tools in mathematics lessons.

4.6 THE DATA COLLECTION INSTRUMENT

The data collection instrument employed in this research aimed to address the primary research question. Pre- and post-tests were utilised to gather quantitative data. Both qualitative and quantitative data were collected using a timetable for classroom observations. Teachers from both the experimental and control groups participated in semi-structured interviews, which yielded qualitative data. Following the completion of pre-tests by the learners, the researcher developed the interview questions. These instruments also addressed the secondary research questions supported by the original research question. Figure 4.3 below summarizes the data collection instruments employed in this study.

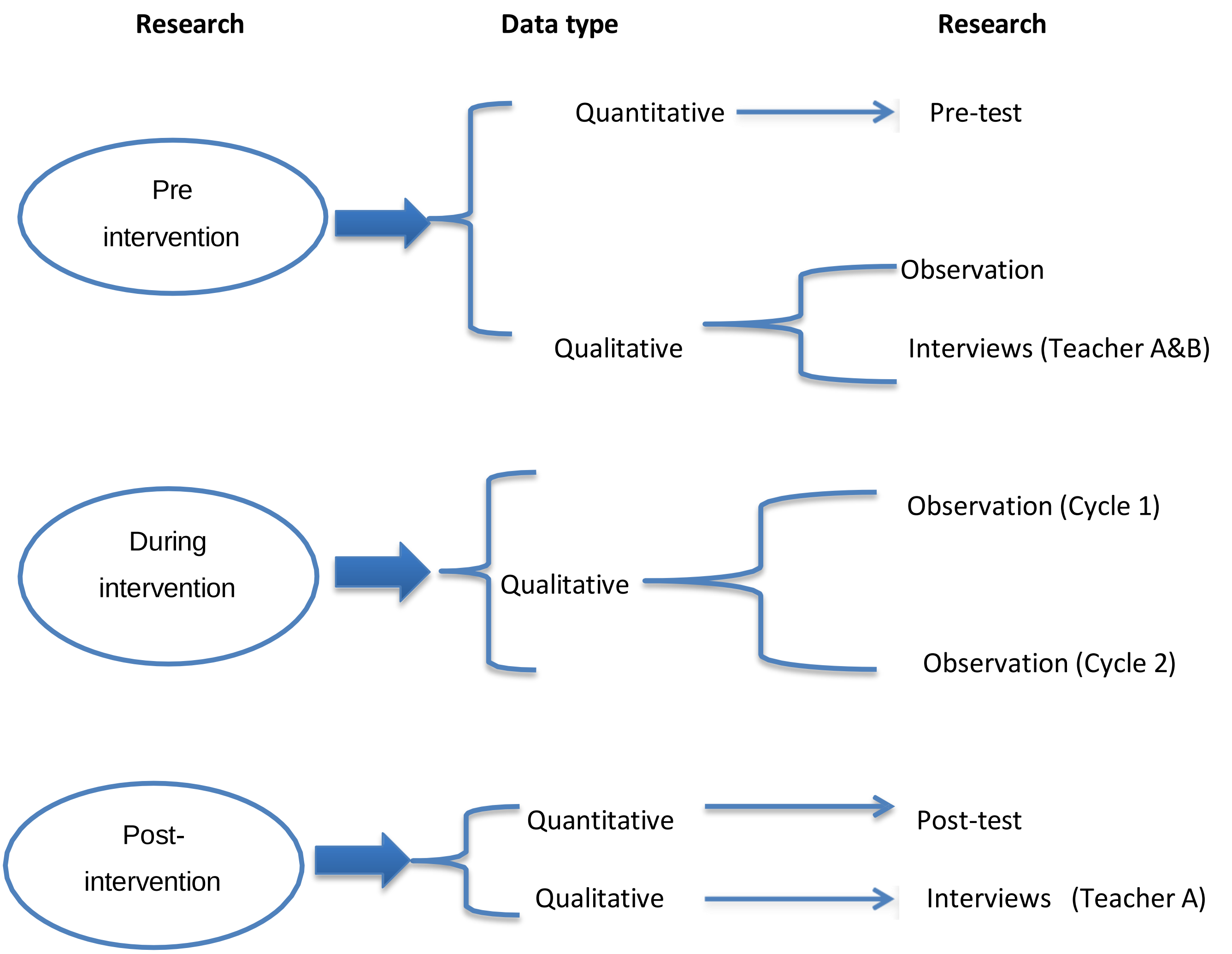


Figure 4.3: ***Mixed Methods: Quan – Qual (Summary)*** *(Source: Makgakga, 2016, p.114)*

Prior to data collection, the researcher conducted a pilot study with ten learners from the third school, separate from the experimental and control group schools. The primary purpose of this pilot study was to assess whether the pre-test instrument would be suitable for a more extensive analysis. The pilot quantitative data was gathered using previous examination papers on trigonometric heights and distances. The questions were moderated by three mathematics teachers from different schools and were structured to align with cognitive levels. The pilot study resulted in additional writing time, as the duration was adjusted since learners were unable to complete their writing, and repetitions of questions were removed.

4.6.1 Pre-and post-test.

The researchers used 60 learners, 30 from the experimental group and 30 from the control group, to conduct pre- and post-tests to measure learners' trigonometry performance. Pre-tests are commonly employed to establish a baseline prior to an intervention, stratify individuals based on their pre-test scores, and assess learners' current levels (Roth, 2022). A pre-test was administered in both groups before the intervention to identify significant differences. The researcher began with a pilot study prior to conducting a preliminary test in the two research groups. A pilot project helped identify potential errors before the extensive study (Khuzwayo, 2019).

The pre-test and post-test for this study were administered to both the experimental and control groups on the same days and in the same order. The researcher obtained valid and reliable data as the pre-test and post-test were conducted on the same day, ensuring the results remained uncontaminated. Scripts were collected simultaneously, and the class teachers assisted in invigilating both groups. Figure 4.4 below shows a summary of how quantitative data was collected from experimental group and control group.

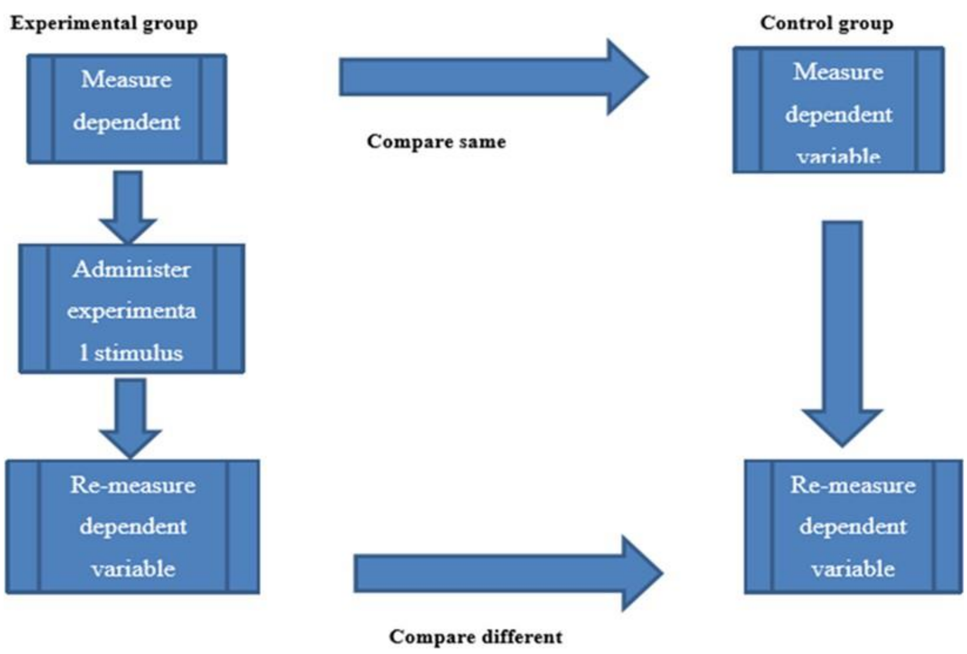


Figure 4.4: ***Diagram of the basic experimental comparison design*** *(Source: Barbie, 2007)*

The figure above illustrates that the pre-test was administered to both the experimental and control groups simultaneously and in the same order. The researcher consulted two Grade

12 teachers from the experimental and control schools to assist with invigilating the tests. The tests were conducted after school for one hour and thirty minutes to prevent disruption to the schools' daily routines. The scripts were collected from both groups at the same time and marked by the researcher. The results were then analysed and compared by the researcher to identify similarities and differences in the performance of these groups. The experimental group subsequently received the treatment or intervention through the use of visualisation. The control group was not exposed to this intervention. Following the intervention given to only the experimental group, both groups completed a post-test simultaneously and in the same order. The researcher then marked the post-test and compared the performance of both groups to determine whether the intervention had aided the experimental group's performance.

4.6.2 Classroom observation

Creswell and Creswell (2018) state that when a researcher conducts a qualitative observation, they take field notes on people's behavior and activities in the study area. In unstructured or semi-structured field notes, the researcher records events at the research site using the prior questions they wish to explore. The researchers began by carrying out the baseline observation conducted solely with the experimental teacher. This phase took four days, with each day allotted an hour. The aim was to understand how the teacher instructs on heights and distances and how learners respond to those teaching strategies. Following the introduction of the intervention, the researcher performed post-observations of the lessons to establish whether the teacher continued to use visual models for teaching heights and distances and if the intervention enhanced learners' performance. Eight lesson observations were conducted with the experimental group for six weeks, during which three hours were allocated for each session. The observations took place weekly on Fridays and Saturdays. McMillan and Schumacher (2014) further state that qualitative observers can take on various roles, from observer to participant. During the lesson observation, the researcher did not participate. The researcher's role was to observe the lesson and take field notes. The researcher only interacted with the learners during interviews and the introduction of the intervention.

4.6.3 Semi-structured interviews

Karatsareas (2022) defines semi-structured interviews as a qualitative research method that offers flexibility through open-ended questions, allowing the researcher to explore topics in depth based on participants' responses. Teachers A and B from the experimental and control groups were interviewed before and after the intervention. Interviewing teachers prior to the intervention served the primary purpose of gaining insight into their experiences instructing learners on heights and distances in trigonometry, as well as understanding their viewpoints on the subject's performance. Post-intervention interviews provided the researcher with insights into teachers' experiences teaching heights and distances through visualisation. Prior to the interview, consent to record it was obtained. Each interview lasted approximately ten minutes. When compiling the analysis, the researcher was able to gather data from the recordings. Two days after the learners completed the pre-test, the semi-structured interviews were conducted with the teachers.

Some of the questions were as follows

Mathematical challenges: In your opinion, are challenges faced by learners when answering questions on heights and distances that may result in low achievement in Grade 12 heights and distances in trigonometry?

Benefits of learning heights and distances: What are the benefits of being taught heights and distances as a Grade 12 learner?

Classroom interaction: How do you engage with your learners while teaching and learning heights and distances in trigonometry?

Resources for teaching and learning: Which resources are used in your lessons when teaching heights and distances?

Assessment in the classroom: When assessing learners, do they know the difference between when to use Pythagoras theorem, sine rule and cosine rule?

4.7 ACTION RESEARCH

Action research is an iterative, collaborative process of inquiry that seeks to address real-world problems or challenges in a specific context by engaging both participants and researchers actively (Bradbury, 2024). It involves a cyclical process of planning, acting, observing, and reflecting to facilitate change while simultaneously generating knowledge. In an educational setting, action research is the methodology teachers employ to investigate a specific topic within the classroom, directly involving them in resolving issues that arise during the teaching-learning process (Lufungulo et al., 2021).

4.7.1 Action Research Model

This study employed a cyclical action research model derived from Kemmis and McTaggart's (1988) cycle of planning, action, observation, and reflection, facilitating continuous refinement and adjustment throughout the research process. The research was carried out over two cycles, with modifications based on reflections and feedback from each cycle. Figure 4.5 illustrated below is grounded in the notion that change and improvement in practice occur through a dynamic, iterative process (Nazari, 2022).

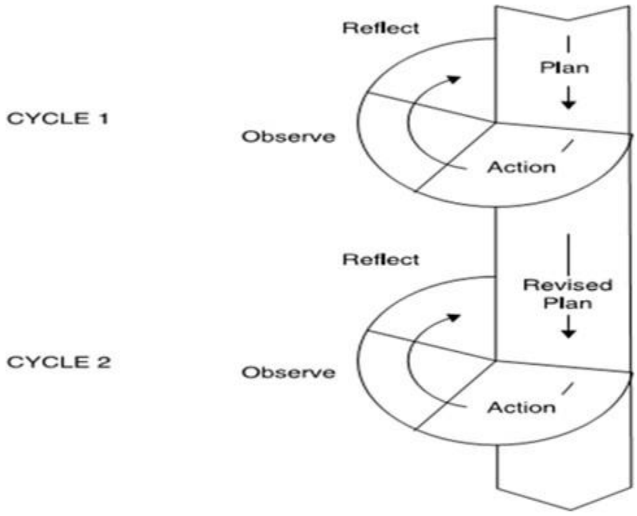


Figure 4.5*: Cyclical AR model based on Kemmis and McTaggart’s (1988)* *(Source Burns, 2010)*

Cycle 1: Introducing visualisation

The planning stage

The researcher noticed that learners struggle with trigonometry, particularly in the context of heights and distances, especially when it comes to three-dimensional shapes. After analysing the moderators’ report and reflecting on her experiences as a Grade 12 mathematics teacher, she conducted action research to address this challenge. The researcher explored strategies to enhance learners' performance in heights and distances. Additionally, she consulted educational theories, including Mayer’s cognitive theory of multimedia learning, to gain insight into how visualisation can aid comprehension.

Action stage

Based on the findings from the planning stage, the researcher developed an action plan where the visual tool was introduced in the form of interactive geometry software (Geogebra) along with play tents and a school gate for demonstration. During the first cycle, Geogebra was introduced as a form of visualisation to learners to see if they could construct the 3D shapes and use them as visuals to answer trigonometric questions. Learners were randomly assigned in groups of five (5) because of the lack of laptops. The school had no computers, so the researcher and the teacher had to find six (6) laptops to use. Laptops, school gates, kids’ play tents, and previous question papers were used during the first cycle.

Observation stage

The researcher kept a record of how the teacher and learners interacted during the first cycle. Some participants could not effectively engage because they worked in large groups with only one laptop per group and a play tent. The school gate obstructed learners from seeing all the aspects of the three-dimensional shape. It only allowed learners to identify right-angled triangles and the opposite, adjacent, and hypotenuse sides. Using GeoGebra was time-consuming, as learners had to be instructed on how to use the software, and not all learners in a group had the opportunity to explore it while working in large groups. Consequently, there was a decline in the number of actively participating learners.

Reflection stage

The researcher reviewed her notes during classroom observations to analyse how learners participated. From their interactions with the teacher in the classroom, she noted that most learners showed less improvement in understanding heights and distances in trigonometry. The researcher then had a workshop and discussion with the teacher in preparation for the second cycle.

Cycle 2

The planning stage

After reflecting on the outcomes of the first cycle, the researcher refined the problem based on the insights gathered. During the planning phase, the researcher and teacher determined the focus area and devised a strategy to enhance learner engagement in the classroom. The challenges of the first cycle included learners working in large groups due to a lack of laptops, which resulted in decreased participation and involvement in group activities. The researcher then proposed using cardboard, where learners constructed their own three-dimensional shapes to visualise and calculate heights and distances. Learners were instructed to work in pairs rather than in large groups.

Action stage

The second cycle encouraged learners to express their creativity by making 3D shapes and employing them as visual aids to answer questions from previous exam papers. This cycle enabled participants to engage in hands-on learning. It motivated them to actively participate while deepening their trigonometry comprehension of heights and distances. Learners collaborated in pairs instead of large groups, allowing the teacher to identify those who required additional support.

Observation stage

The researcher maintained a record of the interactions between the teacher and learners during the second cycle. The lessons had become more learner-centred; learners were provided with opportunities to answer questions posed by the teacher and to share their prior knowledge. They worked in pairs instead of large groups, which facilitated interaction with the constructed visual tool for each learner. In pairs, learners communicated and collaborated to find answers to the questions. The use of visuals during the intervention empowered learners to learn from each other, thereby boosting their confidence and potential. In this cycle, learners were encouraged to express their creativity by creating their own 3D shapes and using them as visual aids to respond to questions from previous papers. Participants were able to engage in learning by doing during this round. This cycle promoted learning and active participation while enhancing participants' understanding of trigonometric heights and distances.

Reflection stage

The researcher reviewed her notes during classroom observations to analyse how learners participated. The second cycle resulted in a more targeted and refined approach, in which the researcher addressed specific challenges from the first cycle while continuing to build on the strength of the original intervention.

4.8 DATA ANALYSIS

Data analysis can be defined as the process of converting the gathered data to meaningful information (Taherdoost, 2022). During data analysis, data is inspected, cleaned, transformed, and modelled to discover useful information and, hence, reach conclusions (Pallant, 2020). The researcher transforms the raw data collected to make sense of what was gathered during data collection.

4.8.1 Data preparation

As stated by Taherdoost (2022), data must be prepared before it can be analysed. Data can be designed through data coding, data entry, missing data, and transformation.

4.8.1.1 Data coding

As reported by Creswell (2018), grouping words (or paragraphs) or images into categories and identifying those groups with a term is usually based on the participant's natural language. The researcher used coding to describe the teachers; teachers from the experimental group were coded as Teacher A, and teachers from the control group were coded as Teacher B. The researcher also used themes to analyse interviews and classroom observations.

4.8.1.2 Data entry

According to Taherdoost (2022), data entry involves inputting coded data into text files or spreadsheets. Stata Release 18 was employed for statistical data analysis in this research study, while Excel served for data management. The collected data from the pre-test and post-test was categorised. For instance, regarding correct, incorrect, and unanswered responses, Creswell (2018) notes that organising words (or paragraphs) or images into categories and labelling those groups with a term is a part of data coding.

4.8.1.3 Missing data

As some participants may not answer all the questions in the pre- and post-tests for various reasons, the researcher should develop a method to record the missing values. Researchers may use -1 or 999 to indicate that the scores for a particular question are absent (Taherdoost, 2022). In this research study, blank or missing responses were considered entirely unresponsive or were not attempted to address the question posed to the learner. All incomplete responses were labelled as such.

4.8.1.4 Data transformation

Data transformation is necessary before interpreting the data (Taherdoost, 2022). It is a process in which the researcher converts, cleans, and structures data into a usable format suitable for analysis. For instance, after marking the pre- and post-tests, the researcher recorded the scores in Excel to facilitate statistical analysis and comparisons between the experimental and control groups.

4.8.2 Analysis of the quantitative data

The data collected from the pre-test and post-test was categorised into correct, incorrect, and unanswered responses. Category percentages were calculated based on the absolute number of learners rather than the average percentage to provide a general overview of learner performance in height and distance. Blank categories were regarded as entirely unresponsive or not attempted in response to the question presented to the learner. These blank categories were understood as instances of neither answering nor trying to answer the questions posed to the learner. Statistical tests were employed to determine statistical significance, confidence intervals, and effect sizes to organise qualitative data (Creswell, 2018).

The quantitative data were examined and organised according to the categories in the test items. Learners' responses were coded as correct, incorrect, blank, or incomplete. Each learner’s written responses to each question were initially coded as correct, incorrect, blank, or incomplete. This process aimed to understand and visualise learners' solutions to questions involving heights and distances. The researcher also analysed the responses to identify common mistakes and misconceptions that affect learners' low performance in heights and distances. The data obtained from the pre-test and post-test were descriptively analysed. Stata Release 18 was used for statistical data analysis, and Excel was employed for data management. To compare the two study groups—the experimental and control groups—a t-test was performed. A two-sided 95% confidence interval was used to interpret the data. In other words, a p-value of less than 0.05 indicated that the results were significant.

4.8.3 Analysis of qualitative data

To analyse the qualitative data for this study, the researcher collected open-ended data from semi-structured interviews with the participants by posing general and targeted questions. This study employed multiple levels of analysis to examine the qualitative data, following Creswell's (2018) phased approach, progressing from specific to general. Figure 4.6 below summarises how qualitative data was analysed.

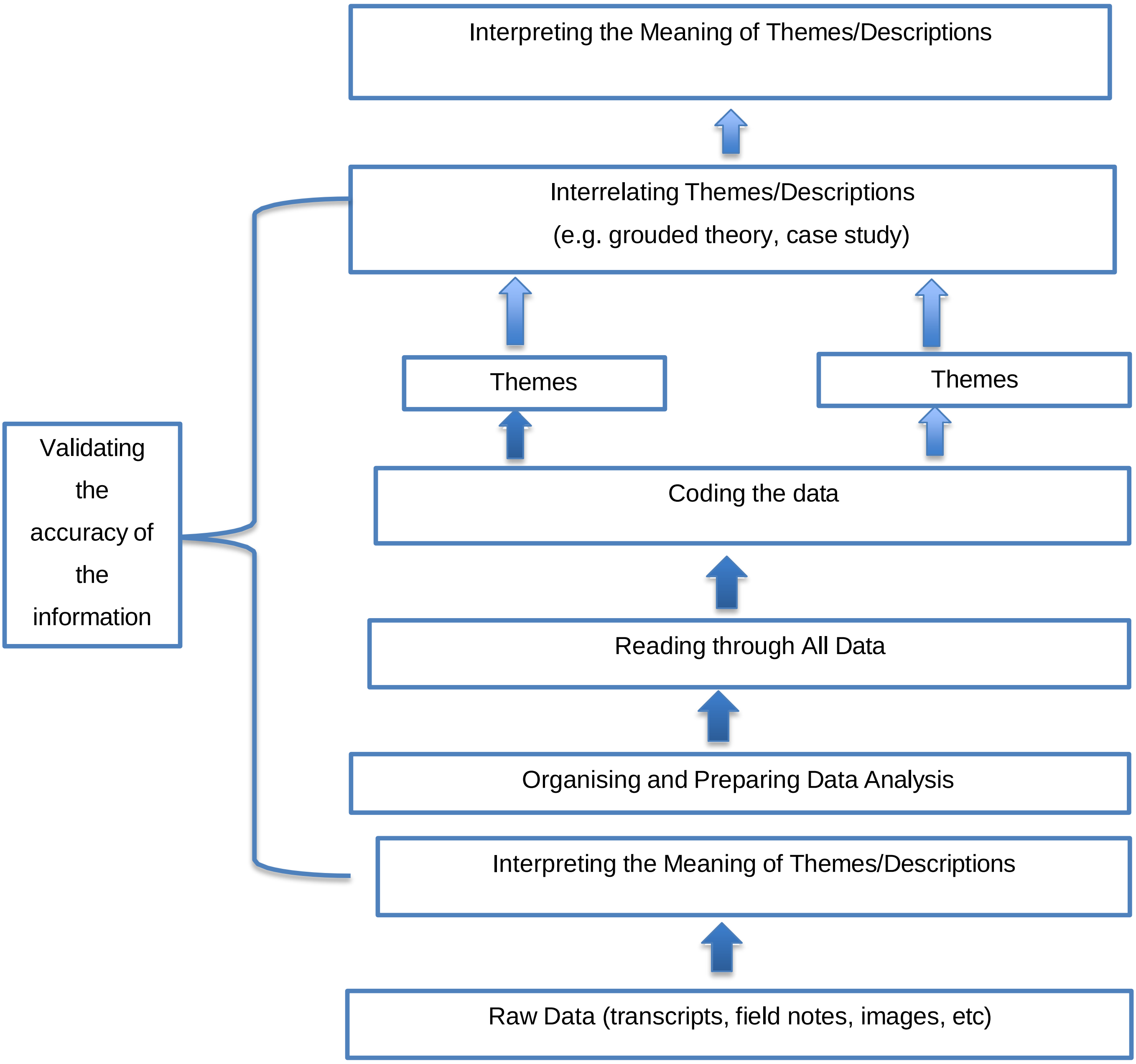


Figure 4.6: ***Data analysis in qualitative research*** *(Creswell, 2018, p. 309)*

To analyse the qualitative data, the researcher followed the steps outlined by Creswell (2018) for analysing qualitative data. The researcher commenced by organising and preparing the raw data for analysis. The data for this study comprised interviews, which were transcribed verbatim, and the field notes were typed up. The collected data was then organised by type and participant group. For instance, the pre-test from the control group was filed separately from that of the experimental group.

Once the data had been organised, the transcripts and field notes were reviewed multiple times to gain an understanding of the participants' experiences. The researcher made notes during this process to document emerging ideas and overarching themes. For instance, while examining the notes from classroom observations and interviews, if the teacher mentioned something during the interview that aligned with the classroom observations, the researcher would take note of it. The data was then broken down into segments during coding. Each segment was given a descriptive label. For example, when analysing interviews with teachers, the codes *Mathematical challenges*, *benefits of learning heights and distances, classroom interaction*, *resources for teaching and learning,* and *assessment* emerged from the initial coding. More initial codes were generated and refined through rounds of coding.

After coding the data, the researcher grouped related codes into categories while identifying key themes that reflected participants' experiences. For instance, learners' motivation, enthusiasm, and participation were consolidated under a broader theme, 'classroom interaction'. In total, five themes emerged from the interviews and observations. The identified themes were then interpreted in light of the existing literature. For instance, the theme 'mathematics challenges' aligned with prior research on the teaching methodologies employed in mathematics instruction. To represent the data, the researcher included direct quotations from participants; for instance, to illustrate the theme of 'classroom interaction', alongside quotes from teachers detailing how they interact with learners when teaching heights and distances.

To validate findings, the researcher used member-checking, triangulation, and peer debriefing (see 4.12 and 4.13.1). Through systematic coding and analysis, the study identified the challenges and benefits of using visualisation in heights and distances. The findings suggest that while teachers may face challenges such as limited teaching resources, overcrowded classrooms, and limited learner engagement when teaching heights and distances, the use of visual models enhances learner engagement and participation, positively impacting learners' performance.

4.9 VALIDITY AND RELIABILITY

The crucial components that enable research to yield valuable results are the validity and reliability of the instruments used. Therefore, it is beneficial to understand how researchers accurately assess the validity and reliability of these instruments (Sürücü & Maslakci, 2020).

4.9.1 Validity

Quintão et al., (2020) define validity as the extent to which a study measures what it is intended to measure. To ensure the validity of the research, the researcher employed various types of validity, such as internal and external, to assess the cause-and-effect relationship of validity. External validity illustrates how closely the results resemble the reference sample or population from which the study sample was drawn. In contrast, internal validity describes how effectively the results measure what they were intended to evaluate (Ahmed & Ishtiaq, 2021).

To ensure strong internal validity, the researcher employed randomisation, whereby participants were randomly assigned to experimental and control groups, which helped minimise differences between the groups. The researcher ensured that participants were similar in terms of prior knowledge, as they were all in grade 12. The use of a control group also facilitated comparison and helped isolate the effect of visualisation. Furthermore, the researcher ensured that the pre- and post-tests were administered to both groups simultaneously to maintain consistency and reduce bias (Trafimow, 2023).

External validity is the extent to which the findings of the study can be generalised to other groups outside the study sample (Trafimow, 2023). To ensure external validity, the researcher randomly selected learners from two schools' experimental and control groups. The intervention was also conducted in the classroom, where learners usually have mathematics lessons, to ensure that the learning conditions match the typical educational environment.

To ensure content validity, the assessment must encompass all aspects of the concept being assessed (Ahmed & Ishtiaq, 2021). In this instance, the researcher ensured that the tests included all relevant areas of heights and distances in trigonometry, such as trigonometric ratios, the sine rule, the cosine rule, and the Pythagorean theorem. The pre-test and post-test featured similar questions but employed different numerical values to prevent memorisation. The pre-test was piloted with ten learners to ascertain whether the questions were straightforward and aligned with the research objectives. The pilot study helped identify any repetitive questions.

Furthermore, the researcher validated the tests by submitting the pre- and post-tests to three qualified and experienced teachers from the Zululand District for moderation. To ensure validity, the researcher employed member checking, whereby the final report was returned to the participants to ascertain whether they felt the qualitative findings were accurate (Creswell, 2018). This provided participants with the opportunity to comment on the findings. Maree (2016) agrees that participants should have the chance to reflect on how closely the explanations provided relate to the unique experiences they shared during the interviews. Sharing the data with participants to validate their perspectives assisted in clarifying any misunderstandings regarding its interpretation (Thomas, 2017).

4.9.2 Reliability

Ensuring the reliability of the research study on heights and distances in trigonometry necessitates that the researcher guarantees the tools and procedures used to measure learners' understanding produce consistent results over time and under varying conditions (Ahmed & Ishtiaq, 2021). To ensure reliability, the researcher utilised established and dependable tests by compiling the pre- and post-tests from previous examination papers. These papers are set at the provincial and district levels and have been validated to be written by Grade 12 learners. The questions were also categorised into different sections; for example, question one focused on the Pythagorean theorem, question two centred on trigonometric ratios, question three concerned the sine rule, and question four addressed the cosine rule. This was done to ensure internal consistency, which refers to how well the test items measure the same concept or skill. The researcher also used inter-rater reliability, in which two qualified mathematics teachers were asked to moderate the tests to ensure that they were correctly marked. Inter-rater reliability refers to the consistency of a measure across observers (Ahmed & Ishtiaq, 2021). No deviation of marks was recorded, which indicates that the marks were consistent. A pilot study was also used to ensure the reliability of the test. One question was eliminated since it was a repetition, and this was done to ensure that the final version of the test was more reliable. The researcher followed the procedures outlined below, as suggested by Gibbs (2007), to ensure the study's reliability: The researcher checked all transcripts for any potential errors. The researcher ensured that the definitions of the codes were consistent throughout the qualitative data analysis, which was performed during the coding process. Additionally, the researcher cross-checked the codes developed against the existing literature by comparing the independently derived results.

4.9.3 Validity and reliability of the data collection instruments

4.9.3.1 Pre-test and post-test

To ensure the reliability and credibility of both tests, the final versions of the set tests were discussed with three experienced mathematics teachers; these teachers moderated the tests before administering them to learners. Discussion and moderation aim to ensure accuracy and pedagogical alignment in mathematics. The reliability of the pre- and post-tests was determined by whether the pre-test results matched or differed from those of the post-test. The tests were piloted at another school to assess learners' performance and to evaluate whether the test results aligned similarly with the post-test.

4.9.3.2 The classroom observation schedule

The researcher spent an extended period in the field. As noted by Creswell (2018), when the researcher engages more deeply with the participants, they gain a comprehensive understanding of their study. They become more familiar with the participants in their settings, resulting in findings that are more accurate and valid.

4.9.3.3 Interviews with teachers

During the interview sessions, the researcher probed unclear responses to gain an in-depth understanding of the teachers' interpretations, ensuring that learners' responses were accurately represented during data reporting and analysis (Sambo, 2022).

4.10 TRUSTWORTHINESS

Enworo (2023) defines trustworthiness as the extent to which a researcher can persuade readers that the findings are credible, reliable, and trustworthy. It further explains that researchers use trustworthiness to ensure that the research processes are ethical and transparent. To evaluate trustworthiness, the researchers employ the following criteria:

4.10.1 Credibility

Credibility relies on the value of truth and examines whether the researcher has established and conveyed a particular degree of trust in the findings based on the phenomenon under study (Lemon & Hayes, 2020). To ensure credibility, the researcher spent extended time in the research field to understand how teachers teach heights and distances. An approach called prolonged engagement assists the researcher in spending sufficient time to become familiar with the study is culture and develop trust with participants (Lemon & Hayes, 2020). The researcher spent three months in the field from April to June 2023. Spending more time in the field allowed the researcher to cultivate a close relationship with participants, leading to a richer understanding of their experiences teaching heights and distances in trigonometry. During the first week, data was collected through a pre-test, followed by four classroom observations conducted over the next four days. The third week was dedicated to interviews with both teachers from the experimental and control groups. In the following six weeks, the use of visualisation was introduced, accompanied by classroom observation interactions on Fridays and Saturdays each week. Field observations were conducted for three hours per day, enabling in-depth and consistent data collection. This was followed by the learners from both groups writing a post-test, and the researcher conducted follow-up interviews with teachers from both experimental groups.

The researcher also utilised persistent observation to ensure credibility. As noted by Ahmed (2024), persistent observation involves maintaining an open mind, acknowledging one's own biases, and engaging in self reflection. As a mathematics teacher, the researcher’s own experiences may influence assumptions about how teachers instruct on height and distance, as well as how learners respond to these teaching strategies. Through reflexive journaling, the researcher became aware of their biases and subsequently modified the data collection techniques, choosing to conduct interviews with the teachers instead of the learners. Teacher interviews assisted the researcher in confirming findings during classroom observations.

Another method employed by the researcher to ensure credibility is triangulation. Creswell (2018) defines triangulation as using various methods to collect data on the same subject. In this study, triangulation was achieved by employing multiple data collection methods, including pre- and post-test observations and interviews. Investigator triangulation was also utilised, wherein the researcher independently gathered, compared, and combined data from other studies to enhance confidence. The data collected from this study was compared to previous studies to identify literature gaps (Perera, 2018). Methodological triangulation was also applied to cross validate the findings, where the results of the qualitative method reinforced those of the quantitative method.

4.10.2 Transferability

The degree to which the results of this study can be applied to different situations or environments with other participants is referred to as transformability (Enworo, 2023). This study was conducted in a rural secondary school with limited teaching resources and no technology. The techniques employed during the intervention, where cardboard was used to create visual models illustrating heights and distances, significantly enhanced learners’ engagement and understanding of these concepts. Given the rural context with a lack of teaching resources, these findings may be relevant to other secondary schools with similar demographic characteristics. Additionally, comparable teaching methods could be assessed in suburban or urban high schools to evaluate their effectiveness in different environments.

However, the transferability of these findings may be constrained by factors such as the availability of teaching resources, including technology and learners’ backgrounds.

4.10.3 Dependability

Dependability is the quality of consistent and durable research findings over time (Ahmed, 2024). It emphasises whether the research processes are logical, well-documented, and traceable. The researcher maintained a comprehensive audit trail throughout the research process to ensure dependability. An audit trail refers to the process of documenting and record-keeping that researchers undertake to uphold transparency, reliability, and credibility (Carcary, 2020). In creating an audit trail, the researcher provides a detailed account of all research decisions and actions undertaken during the study (McGrath, 2021). The audit trails include thorough records of data collection procedures, such as observation logs and test administration dates, ensuring consistent data collection. Any alterations to the research design, such as modifications to interview questions, were documented.

To ensure the study's dependability, detailed methodological documentation was maintained throughout the research process. Data was collected using tests administered at the beginning and end of the study, along with classroom observations and teacher interviews. This thorough documentation supports the study’s dependability by providing a transparent account of research methods and procedures, facilitating replication, and ensuring the reliability of the findings. In addition, data interpretation was also submitted to other researchers and supervisors to minimize personal bias and subjective interpretation.

4.10.4 Confirmability

Confirmability is the degree to which the results are unbiased and objective, ensuring they are not influenced by the researchers' personal prejudices or inclinations (Ahmed, 2024). To ensure confirmability, the researcher utilised various approaches to minimise bias and guarantee that the findings were grounded in the data. Member checking was employed when the final report was returned to the participants to ascertain whether they felt the qualitative findings were accurate (Creswell, 2018). Direct quotations from the interviews assured that the findings were rooted in participants’ responses. The teachers validated the report to confirm that their responses were accurately captured. The researcher also utilised recordings that allowed her to replay the audio when compiling the results.

Furthermore, themes were developed from the data while reporting classroom observations, which were confirmed with the participants. In peer debriefing, tutorial lessons and conferences were used to present the study. The researcher sought feedback from colleagues and supervisors to validate interpretation. Peer debriefing increased objectivity and validated the accuracy of the results by introducing different viewpoints and reducing personal bias (Ahmed, 2024).

4.11 ETHICAL ISSUES

Researchers have a duty and obligation to adhere to the code of conduct that governs most professions (Drolet et al., 2023),. Researchers must ensure the privacy, welfare, and rights of their participants. In this research, the researcher applied for an ethical clearance certificate from the University of South Africa, which the ethics committee approved in January 2023. Permission to conduct research in schools was sought from the KZN DBE, Zululand district. The informed consent of teachers and learners was requested prior to conducting the study. The researcher approached the grade 12 teachers and the school’s principal. The rights and roles of participants were explained to them before the research commenced. They were assured that their participation was voluntary, and that confidentiality would be maintained, with their details not disclosed. One of the key elements of autonomy is that participants must engage in the study willingly and be free to withdraw at any time (Bertram et al, 2014),. The principle of independence was observed throughout the research, and participants were provided with a letter of consent that clearly outlined the study's objectives and their expected involvement. Both the teachers and participants signed these consent forms.

4.12 CONCLUSION

This chapter discusses the mixed-method approach utilised in a study of a secondary school and the methodological tools employed. It highlights key research paradigms, descriptions of research methodologies and approaches, as well as tools for data gathering and analysis methods. It also provides discussions and explanations regarding sampling, validity and reliability issues, trustworthiness, and ethical concerns. The exploration of the study's qualitative and quantitative data will be the focus of the following chapter. For the purposes of triangulation, the data collected from qualitative and quantitative sources were integrated.

CHAPTER 5: RESULTS

5.1 INTRODUCTION

The research approach used in this study was covered in the preceding chapter. The information gathered using both qualitative and quantitative techniques is presented in this chapter. Lesson observations, interviews, and pre-test and post-test data form the basis of the presentation. The analysis of the pilot study, the analysis of the pre-test, the interviews with teachers to understand their experiences with teaching heights and distances, the classroom observations before and during the use of visualisation in teaching heights and distances, the analysis of the post-test (quantitative data), the statistical analysis, and the conclusion comprise the chapter's five (5) subsections.

The study hypothesised that using visualisation when teaching heights and distances impacts learners' performance. Therefore, the following hypotheses were formulated:

The null hypothesis

The use of visualisation has no significant effect on learners' performance in teaching heights and distances.

The alternative Hypothesis

The use of visualisation has a significant effect on learners' performance in teaching heights and distances.

The results are analysed, discussed, and detailed to answer all the research questions:

- What types of teaching methods do teachers use when teaching heights and distances?
- What are the benefits and challenges (if any) of visualisation in the teaching and learning of trigonometry?
- What are possible ways of implementing effective visualisation in the teaching and learning of heights and distances to improve learner performance?

This research study employed triangulation, a mixed-method design (see 4.5.1.1), to collect data. The researcher utilised both qualitative and quantitative data interchangeably to balance the strengths of each data collection tool. The study commenced with quantitative data, where the researcher conducted a pre-test to measure learners' trigonometry performance. Qualitative data was obtained through interviews, allowing the researcher to understand teachers' experiences with teaching heights and distances. The researcher also conducted classroom observations to gain insight into how learners were taught heights and distances in a targeted school. Further classroom observations were conducted during the intervention to assess how learners responded to the use of visualisation in heights and distances lessons. After the intervention, learners were given a post-test to measure its effectiveness. The following section describes the pilot study that informed the instrumentation of the main study.

5.2 ANALYSIS OF DATA FROM THE PILOT STUDY

Pilot study was conducted with an aim of testing the effectiveness of the data collection instruments. According to Creswell (2018), the objective of conducting pilot study was to test whether results on an instrument are content valid, to give a preliminary assessment of the items' internal consistency, and to enhance the questions, format, and instructions. The pre-test instrument was piloted using ten (10) participants from a third school different from the two (2) schools the researcher used to collect data. The pilot quantitative data were collected using previous question papers on trigonometric heights and distances. Three mathematics teachers from different schools moderated the questions, and they were structured to follow the cognitive levels.

Pilot testing allowed the researcher to evaluate the duration of the test and consider potential learner fatigue (Creswell, 2018). The researcher set the duration as 45 minutes for learners to complete the pre-test. The pilot study indicated that 45 minutes may require adjustment, as some participants skipped questions, suggesting they needed additional time to complete the test. As a result, the allocated time was extended to 1 hour. After the pilot study, one question was removed due to its similarity to another question in the test, both targeting the same cognitive level. This question was substituted with one that required learners to calculate the area of a non-right-angled triangle, thereby enhancing the structural diversity of the test. Following the supervisor’s review of the pre-test, it was recommended that two-dimensional questions be included as the first and second questions before learners tackle those based on three-dimensional shapes. This adjustment helped the researcher determine whether learners understand when to apply trigonometric ratios, the Pythagorean theorem, the sine rule, and the cosine rule for calculating heights and distances. Consequently, questions one and two were incorporated, and the total marks increased from 25 to 35. The final version of the test was then discussed with one experienced high school mathematics teacher and my supervisor to establish the validity of the test.

To analyse the pilot study, the researcher used the DBE achievement rating in conjunction with Bloom’s taxonomy levels. The DBE achievement rating consists of levels (codes), percentages, and descriptions of each level. The taxonomy levels measure questions from easy to complex. This made it easy for the researcher to analyse the results from the pilot test.

The outcomes of the pilot study were shared before presenting the results of the main study. From the pilot study table, it is evident that few learners were able to answer questions 2 and 3. These questions required learners to apply what they had learned from lower grades to grade 12. Most learners were unable even to indicate on their diagrams that they could identify isosceles triangles and use their properties to attempt to answer the question. Regarding question 1, about 4 or 5 out of 10 (40%-50%) learners were able to answer it correctly. This question required learners to apply grade 9 work, where they needed to use the Pythagorean theorem to calculate the third side. The gap identified when analysing the results indicates that question 2 required to be separated into triangles to determine whether learners can differentiate between when to use the sine rule and the cosine rule.

Most learners skipped questions 2, 3, and 4. This aligns with the reports from the DBE moderators in 2024, which indicated that some learners omitted this section in the examination and did not even attempt to answer questions on heights and distances. This may suggest a lack of understanding regarding what is expected of them, and it also informs me, as a researcher, that I should adjust the duration since it was 45 minutes (for 35 marks); this may indicate that learners did not complete the paper due to time constraints. The duration of the pre-test was increased to 1 hour. The piloted test above reveals that learners struggled to answer most questions and apply prior knowledge to address the questions on heights and distances.

5.3 ANALYSIS OF THE MAIN STUDY

In this section, the researcher presented the data analysis from the main research study.

5.3.1 Pre-test results quantitative analysis

The researcher administered a pre-test to the experimental and control groups, assisted by the mathematics teachers in their respective schools during regular class time, and the participants were given 60 minutes to complete it. To analyse the pre-test, the researcher employed coding, where learners' responses were classified as correct, incorrect, blank, or incomplete. This method was adapted from Didis and Erbas's (2015) categories for analysing learners' responses. Responses not attempted by learners were categorised as blank (BL). Responses that were attempted but not fully completed, even if the learners followed a correct mathematical process, were classified as incomplete (InC). Correct responses (CR) refer to answers that learners completed with 100% accuracy in their steps or reasoning. Incorrect responses (IR) denote answers that do not address the question, in which incorrect methods were used. The primary objective of using coding was to provide a descriptive analysis of learners' responses. This approach allowed the researcher to identify common errors and misconceptions, ensuring these issues would be addressed during the intervention.

5.3.1.1 Learner responses to question 1 items

Table 5.1 analyses question 1 of the pre-test for both the experimental and control groups. Question 1 assessed knowledge of the CAST diagram in a Cartesian plane. The CAST rule enables learners to determine the sign of the trigonometric ratios in any of the four quadrants. Furthermore, question 1 evaluated the understanding of the definitions of sine, cosine, and tangent and calculated height and distances using Pythagoras' theorem. Table 5.1 below reveals that control group performed well compared to the experimental group in questions 1.1 and 1.2. The highest percentage for the experimental group in question 1.1 was 76.67%, whereas in the control group, it was found to be 80%. Question 1 consisted of two items, coded as 1.1 and 1.2, with question 1.1 assessing learners on the CAST diagram and question 1.2 assessing learners on the definitions of trigonometric ratios. Table 5.1 below shows pre-test results of experimental and control group in question 1 items.

Table 5.1 ***Distribution of learners' absolute numbers (with percentage) of Correct, Incorrect, Blank, and Incomplete answers for the pre-test on question 1 for experimental and control groups***

QUESTION	GROUP	PRE-TEST			
1.1	Experiment	CR	IR	InC	BL
		18 (60%)	6 (20%)	2 (6.67%)	4 (13.3%)
	Control	23 (76.67%)	2 (6.67%)	2 (6.67%)	3 (10%)
1.2	Experimental	22 (73.3%)	4 (13.3%)	0 (0%)	4 (13.3%)
	Control	24 (80%)	2 (6.67%)	4 (13.3%)	0 (0%)

The above table shows that the experimental group in category CA (in question 1.1) was 60% and 76.6 in the experimental and control groups, respectively. Learners were required to draw a sketch on a Cartesian plane and find the missing side using Pythagoras' theorem. This question required skills in isolating the trigonometric ratio and fixing the definition of cosΘ = $\frac{12}{13}$, and they were also required to mark the quadrants included in $180° \leq \theta \leq 360°$. Figure 5.1 below shows the summary of trigonometric ratios of the four quadrants.

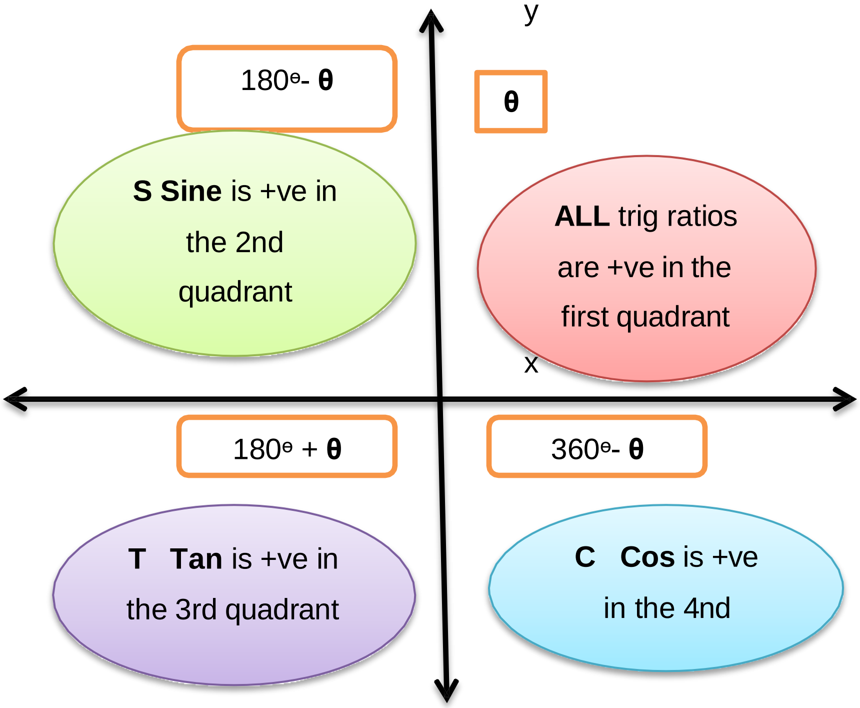
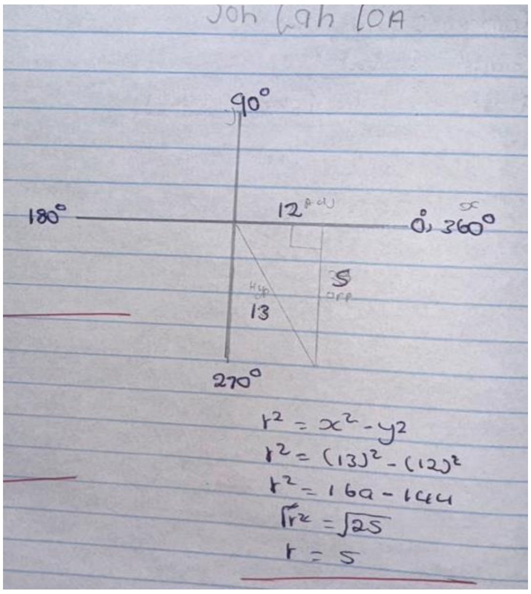
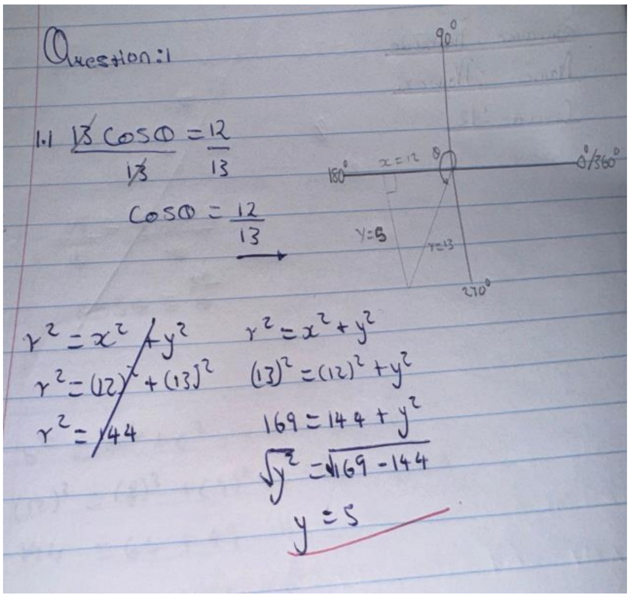


Figure 5.1: ***CAST diagram showing the trigonometric ratio of the four quadrants***

Learners require knowledge from the above CAST diagram to obtain the sign of the trigonometric ratio in any of the four quadrants. According to Maphutha et al., (2023), learners have difficulty identifying quadrants in the CAST diagram and linking quadrants with their intervals. Since cosΘ = $\frac{12}{13}$ is positive, learners are expected to draw a rough sketch in a quadrant where cosine is positive, using the given angle and labelling two of the sides. The third side can then be determined using Pythagoras' theorem.

The results show that some control and experimental group learners answered 1.1 and 1.2 incorrectly. This revealed that these learners could not use the knowledge from the above diagram to make a sketch. Some learners did not draw the sketch, so they calculated the wrong side using Pythagoras' theorem. This resulted in them not getting question 1 correct since they were supposed to use the sketch to write the definitions of sinΘ and cosΘ and substitute them for given expressions. Sample 1 and sample 2 below are learners' responses on questions based on Pythagoras theorem.

Learner’s response: Sample 1	Learner’s response: Sample 2
	

The samples above indicate that learners do not consider the signs of the sides in a Cartesian plane. In Sample 1, the learner could utilise the given interval to create a sketch in the correct quadrant; however, the learner incorrectly wrote the Pythagorean theorem formula, substituting hypotenuse side 13 instead of 12 for x. The learners struggle to connect the values of x, y, and r with the adjacent, opposite, and hypotenuse sides. Conversely, in Sample 2, the learner sketched the incorrect quadrant but provided a correct formula with accurate substitution. The value of y in the chosen quadrant is negative, yet the learner did not consider that. This indicates that learners need to be exposed to the concepts of quadrants, their intervals, and how to choose the appropriate quadrant.

Most learners responded with blank answers, while incomplete responses came from fewer learners. Incomplete responses account for 6.67% of learners in the experimental and control groups, whereas blank responses make up 13.3% in the experimental and 10% in the control groups. Learners who were able to answer question 1.1 also used their correct answers to respond to question 1.2.

5.3.1.2 Learner Responses to Question 2 items

Table 5.2 below analyses question 2 results of the pre-test for both experimental and control groups. Question 2 assessed the use of Pythagoras to calculate the distance and trigonometric ratios to calculate the angle.

Table 5.2 ***Distribution of learners' absolute numbers (with percentage) of Correct, Incorrect, Blank, and Incomplete answers for the pre-test on question 2 for experimental and control groups***

QUESTION	GROUP	PRE-TEST			
2.1	Experiment	CR	IR	InC	BL
		15 (50%)	9 (30%)	6 (20%)	0 (0%)
	Control	15 (50%)	7 (23.3%)	2 (6.67%)	6 (20%)
2.2	Experimenta l	4 (13.3%)	13(43.3%)	8 (26.67%)	5(16.67%)
	Control	6 (20%)	6(23.3%)	10 (33.3%)	8 (23.3%)

Table 5.2 shows that the experimental group's correct response category is 50% and 13.3% in questions 2.1 and 2.2, respectively. This indicates that 50% of learners could find the third side in a right-angled triangle; they understood how Pythagoras theorem works. Most learners in the experimental group used the distance formula used in coordinate geometry, which may indicate that they cannot differentiate between mathematics topics. They might not know how the distance formula works. On the contrary, in the control group, 50% and 20% had correct responses in questions 2.1 and 2.2, respectively. This indicates that learners in both groups performed the same. Of the 50% of learners that answered question 2.1item in the experimental group, only 13.3% of learners were able to answer 2.2, where they had to use trigonometric ratio, specifically sinϴ, to find the required angle. Some learners used the cosine rule instead, which is the wrong methodology since no three sides are given.

If learners wanted to use the cosine rule, they had first to find the third side using Pythagoras theorem. Most learners used the cosϴ or tanϴ ratio, which shows that they might be unable to differentiate between the opposite hypotenuse and the adjacent sides of the required angle. Most learners who gave incorrect and incomplete categories used the incorrect formulae and procedure to calculate the required side in 2.1 and the required angle in 2.2. As reported by Taamneh et al., (2024), learners tend to use incorrect rules in trigonometry because they lack understanding of the application of these rules of trigonometric comparisons and their lack of mastery of the trigonometric formulas that must be used when calculating angles and distances. All experimental group participants attempted question 2.1, while 16.6% left blank spaces in 2.2. Learners performed the same in the CR category in both the experimental and control groups for the 2.1 question. However, the control group performed better than the experimental group in the CR category in 2.2.

5.3.1.3 Learner Responses to Question 3 items

Table 5.3 below analyses question 3 results of the pre-test for the experimental and control groups. Question 3 assessed whether learners can differentiate between when to use the cosine rule and the sine rule if they are not given a right-angled triangle.

Table 5.3 ***Distribution of learners' absolute numbers (with percentage) of Correct, Incorrect, Blank, and Incomplete answers for the pre-test on question 3 for experimental and control groups***

QUESTION	GROUP	PRE-TEST			
3.1	Experiment	**CR**	**IR**	**InC**	**BL**
		12 (40%)	10 (33.3%)	8(26.7%)	0 (0%)
	Control	13 (43.33%)	7 (23.33%)	9 (30%)	1(3.33%)
3.2	Experimental	5 (16.7%)	12(40%)	5 (16.7%)	8 (26.67%)
	Control	3 (10%)	6(20%)	14 (46.7%)	7 (23.3%)
3.3	Experiment	1 (3.33%)	13 (43.33%)	7 (23.33%)	9 (30%)
	Control	5 (16.7%)	6 (20%)	12 (40%)	5 (16.7%)
3.4	Experiment	2 (6.7%)	9 (30%)	5 (16.7%)	14 (46.6%)
	Control	9 (30%)	11 (36.7%)	2 (6.7%)	8 (26.7%)

In question 3.1 item, learners were given a triangle with two sides and an included angle (SAS), which requires them to use the cosine rule to find the required side. The above table indicates that the experimental group in category CA (in question 3.1) was 40% and 43.3% in experimental and control groups, respectively. These learners demonstrated that they understood the requirements to use the cosine rule (cosA = …) 26.7% and 30% of learners from experimental and control groups had incomplete answers. Most of these learners understand that they are required to use the cosine rule, but they struggled with solving the value of the distance algebraically; some wrote a correct formula but struggled with correct substitution. No learner from the experimental group left a blank space, and only one in the control group left a blank space. Learners that fall under IR in both groups used the wrong methodology. Some used the sine rule, and others applied Pythagoras theorem.

In question 3.2 item, learners were given a triangle with two interior angles and one side. They were given a pair of opposite sides and angles, which required them to use the sine rule to find the required side. Before using the sine rule, they had to find the third angle using the sum of angles in a triangle. The above table indicates that the experimental group in category CA (in question 3.2 item) was 16.7% and 10% in the experimental and control groups, respectively. In the InC category, 16.7% of learners were able to find the third angle using the sum of angles in a triangle from the experimental group, but they could not continue finding the required side; some of them did not continue after finding the third angle. A higher percentage (46,7%) in the control group had incomplete answers. Compared to the question 3.1 item in the BL category, more learners did not attempt this question.

In question 3.3, learners were given a triangle with three given sides (SSS) and ϴ as a required angle, which required them to apply cosine rule to find the required angle. The above table indicates that the experimental group in category CA (in question 3.3 item) was 3.33% and 16.7% in the experimental and control groups, respectively. Learners from both groups in the IR Category made the same error as in question 3.1, where they used incorrect formulae, and those using the cosine rule struggled with substitution. According to Taamneh et al., (2024), learners use incorrect formulas because they do not know the proper completion steps; learners also struggle with solving trigonometric equations using algebra. Hence, a higher percentage of learners in the experimental group had IR. In question 3.4 item, learners were given a triangle with three given sides (SSS) and ϴ as a required angle, which required them to use the cosine rule to find the required angle. The above table indicates that the experimental group in category CA (in question 3.3) was 3.33% and 16.7% in the experimental and control groups, respectively. A higher percentage of learners from both these groups left a blank space in this question.

5.3.1.4 Learner Responses to Question 4 items

Question 4 features a 3D shape where learners are required to calculate heights and distances. This question comprises a right-angled triangle and a non–right–angled triangle. It, therefore, assesses whether learners can calculate the following:

a) Finding the sum interior angles of a triangle.
b) Use the sine rule, cosine rule, Pythagoras theorem or appropriate trigonometric ratios to solve for the triangle to find either a length or an angle.
c) Use correct formulae to find the area of the non-right-angled triangle.
d) Apply algebraic knowledge to solve equations.

Table 5.4 below analyses pre-test results of Question 4 for the experimental and control groups.

Table 5.4 ***Distribution of learners' absolute numbers (with percentage) of Correct, Incorrect, Blank, and Incomplete answers for the pre-test on question 3 for experimental and control groups***

Question	Group	PRE-TEST			
4.1.1	Experimental	CR	IR	InC	BL
		7 (23.3%)	8 (26.7%)	13 (43.3%)	2(6.7%)
	Control	16 (53.33%)	5 (16.67%)	9 (30%)	0 (0%)
4.1.2	Experimental	4 (13.3%)	16 (53.3%)	7 (23.33%)	3 (10%)
	Control	17 (56.7%)	10 (33.3%)	3 (10%)	0 (0%)
4.1.3	Experimental	3 (10%)	20 (66.7%)	3 (10%)	4 (13.3%)
	Control	6 (20%)	12 (40%)	9 (30%)	3 (10%)
4.2	Experimental	7 (23.3%)	14 (46.7%)	5 (16.67%)	4 (13.33%)
	Control	10 (33.33%)	9 (30%)	7 (23.34%)	4 (13.33%)
4.3	Experimental	1(3.3%)	14 (46.7%)	5 (16.67%)	10 (33.33%)
	Control	3 (10%)	16 (53.3%)	4 (13.3%)	7 (23.4%)
4.4	Experimental	1 (3.33%)	9 (30%)	6 (20%)	14 (46.7%)
	Control	2 (6,7%)	16 (53.3%)	2 (6.7%)	10 (33.3%)

Since question 4.1.1 was divided into three steps, hence three (3) marks, most learners in both groups were able to find the sum of angles in the triangle, which is basic knowledge from grade 8 and 9 work. The control group outperformed the experimental group. Only 1 participant skipped this question from the experimental group, and no learner skipped the question from the control group.

The second step required participants to find the distance of BC using the sine rule, a routine procedure and basic knowledge from grade 11 work. The third step required participants to find the required distance of AB using trig ratios in a right-angled triangle, which is basic knowledge from grade 10 work. The control group outperformed the experimental group in all the above steps. Most of the learners did not skip this question in both groups. Participants attempted the question but got it wrong and were able to see that they needed to use the sine rule, but they did not know how to apply it correctly. The overall performance of learners from Table 5.4 indicates that the control group performed better than the experimental group since the CR category indicates 23.3% and 53.3%. Most learners that fall into InC category did the first and second steps correctly, but they did not use the correct methodology to find the distance of AB.

Analysis of question 4.1.2

The second question had only one step. Participants were required to use Pythagoras' theorem in a right angled triangle to find one missing side in triangle ABC or use the trig ratio. This question is classified as basic knowledge from grade 9 work. The question is the continuation from 4.1.1. Most learners could not answer this question correctly because they did not find the answer in 4.1.1. 56.7% and 13.33% of learners in both the control and experimental groups, respectively, were able to answer this question correctly. Some learners who answered 4.1.1 could not use their correct answers to calculate 4.1.2. 53.3% and 33.3% of experimental and control group participants used the wrong methodology to calculate the required side. One of the common mistakes made by learners in the experimental group was to use Pythagoras' theorem incorrectly when they made the required side of the subject of the formula. This error was also made in question 2, where learners cannot differentiate between opposite, hypotenuse and adjacent sides of the required angles.

Analysis of question 4.1.3

This question required two steps: first, participants were required to find side CD in triangle ABD using the sine rule. This question is classified as a routine procedure and is a Grade 11 work. **Finding side CD in triangle ABD using sine rule (routine procedure, grade 11 work) and Using Pythagoras theorem in triangle ACD to find side AD (grade 9 work)**

10% and 36.7% of participants in the experimental and control groups, respectively, were able to answer this question correctly. This question required the AC length calculated in the previous question. The learners who answered correctly are some of those who were able to answer the previous question. Most learners who were able to answer 4.1.2 were unable to use their correct answer to calculate side CD. 66.7% and 40% of participants in the control and experimental groups, respectively, could not answer this question correctly. The remaining percentage of learners left a blank space; they did not attempt to answer this question.

The overall performance of learners from Table 5.4 indicates that the control group performed better than the experimental group since the CR category suggests 10% and 20%. Most learners in the InC category did the first step correctly, but they did not use the correct methodology to find the distance of AD. Ten percent and thirty percent of learners in the experimental and control groups, respectively, had incomplete answers, and some learners had the correct first step but did not complete the second step.

Analysis of question 4.2

This question required learners to calculate angle α using triangle ADC, a right-angled triangle, using trig ratio (grade 10 work). 23.3% and 33.3% of participants in the experimental and control groups, respectively, were able to answer this question correctly. Some learners could not differentiate between a side opposite and adjacent to α. This led to 46.7% and 30% of participants in the experimental and control groups, respectively, being unable to answer this question correctly. Since learners could not even recognise the hypotenuse side of a right-angled triangle, it made it impossible for them to grasp the language and structure of questions when several triangles are linked to form three-dimensional objects (Madonsela et al., 2020).

Analysis of question 4.3

Learners were required to calculate an angle using the cosine rule. It required knowledge acquired from grade 11. 3.3% and 10% of participants in the experimental and control groups, respectively, were able to answer this question correctly. The control group performed better than the experimental group. Some learners are unable to differentiate between the conditions required to apply the sine or cosine rule. Some learners used trigonometric ratios to calculate the angle. This is incorrect because the given triangle is not right-angled. Maklina (2010) reported that these participants lacked a focused mental scanning of the qualities of the image within working memory; hence, they could not notice the similarities and differences in shapes triangle is not a right-angled triangle.

Analysis of question 4.4

This question required learners to calculate the area of a non-right-angled triangle, grade 11 work. 3.3% and 6.7% of participants in the experimental and control groups, respectively, were able to answer this question correctly. This is one of the questions that both groups responded to poorly. The most common error made by learners was using the incorrect area formula. Some learners used A=LENGTH x BREATH. This formula does not apply to this question since the triangle is non-right-angled. Most of the learners from both groups skipped this question.

5.4 ANALYSIS OF PRE-INTERVENTION CLASSROOM OBSERVATION OUTCOMES (QUALITATIVE DATA)

The data collection of this study began with pre-tests, interviews, and classroom observation to understand the type of teaching method that the teacher currently uses and how learners respond to that method during the lesson (O'Leary, 2020). The main aim was to answer the first research question of the study: What types of teaching methods do teachers use when teaching heights and distances? The answer to this question was also strengthened by the pre-test and interviews, which are discussed later in this chapter. To find the answer to the first research question, the researcher investigated the classroom environment and observed the lessons where the teacher was teaching heights and distances.

The observations were not meant to criticise or evaluate the teachers but to observe their teaching styles, the material used, and how learners responded to the lesson. Makgakga (2016) argues that it is vital that the researcher builds trust with participants and the teacher during classroom observation; thus, the teacher's voice and opinions were prioritised to make him comfortable with being observed while teaching. The observation occurred during the second week of April 2023, after the school reopened, to understand the teaching methods the grade 12 teacher used to teach heights and distances in trigonometry. As mentioned earlier, the classroom observation aimed to understand how the experimental group was taught heights and distances. The focus was on how participants interacted with their teacher, the teaching methods, the introduction to the lesson, the type of assessments or classroom activities used, and the support given to struggling learners.

5.4.1 The experimental group

The four-day classroom observation, each lasting one hour, took place in the school where the research was collected. The two weeks of classroom observations, which consisted of two lesson observations per week, identified the following themes, which address the goal of comprehending the pedagogical practices in the classroom.

Summary of outcomes of classroom observation

5.4.1.1 Styles, approaches and teaching methods

During the first lesson about heights and distances, there was no baseline assessment or questions to assess learners' prior knowledge. The teacher started the lesson by summarising on a chalkboard the work of Grades 10 and 11: how and when to use Pythagoras' theorem and cosine and sine rules. Each day after the first day, the teacher will start the lesson with the *statement, “We are continuing for yesterday’s lesson; today, we are going to discuss…”* This was done without recapping and reminding learners of what was done in the previous lessons.

In the view of Fatima (2022), teachers are key players in learning when teacher-centred learning. Learners are perceived as passive information consumers, yet teachers serve as information providers or evaluators, keeping an eye on them to ensure they get the answers. Hence, the lessons observed were teacher-centred; learners were there to collect information without a chance to discuss the topic or link it with their everyday experiences. The teacher used a lot of telling methods and made a lot of examples on a chalkboard. He would explain step by step how the questions should be answered. Learners were not given time to discuss the content and try to solve the questions independently. After each example, he would give learners two minutes to copy the example.

Learning entails making sense of new information by drawing on prior knowledge. Learners are more prone to misunderstand, misinterpret, or even ignore new material when their past knowledge is erroneous (Chew & Cerbin, 2021). Learners’ prior knowledge was not considered because they were not asked any questions; the question the teacher kept asking was, “*Do you understand?*” and the whole class would say, “*Yes,*”. If a question is posed to learners, some raise their hands, and only those learners are picked to answer. There was no encouragement for learners who did not raise their hands to show they knew the answer to the posed question. When answering questions on 3D, the teacher used the separation method, where learners had to separate the 3D triangles to see the number of faces, vertices and angles within the solid 3D shape.

5.4.1.2 Learners’ interaction with the teacher and with each other

According to Ayuwanti and Siswoyo (2021), teacher-learner interaction is one of the most vital interactions since it affects learners’ understanding. For learners to understand mathematics, they need the teachers' attention, stimulation and guidance. During the lesson observation, the teacher used questions most of the time to interact with the learners, and for most of these questions, he provided the answers for them. For example, in the second lesson, he asked, “*When do we use Pythagoras’ theorem in a triangle?”* and never gave learners an opportunity. Immediately after asking that question, he answered himself, “*We use Pythagoras’ theorem in a right-angled triangle if we are given two sides and we want the third side*”. This type of questioning method did not allow learners to provide their pre-knowledge from the previous grades and thus did not support learners’ development. When they were given a chance to respond, the teacher would pose a question, and learners raised their hands. Only those learners that raised their hands were interacting with the teacher. There was no encouragement for those learners who did not raise their hands.

When the teacher establishes a classroom setting where learners actively participate in class discussions, learner interaction is improved. Within this kind of setting, educators foster the growth of knowledge and meaningful discourse by asking learners to defend their positions, clarify their viewpoints, and assess those of others (Worku & Alemu, 2020). The teacher did not allow learners to interact with one another or discuss the content. This means that learners were deprived of an opportunity to share mathematical ideas and learn from one another. Even when learners were given classwork, they wrote it individually; they were not sharing ideas. I observed that few learners wanted to discuss the work being done, but they were afraid to do so openly. I also observed that some learners were disengaged with the lesson; they showed no interest in what was being taught. Not much was expected from the learners during the lessons as learners were the passive recipients of information. The teachers expected them to note down the work he wrote on the chalkboard, and after giving examples, learners were expected to write their classwork individually. Once they were done writing (they were rushed to finish writing because of time), the teacher would pick any learner to come and write remedial on a chalkboard.

The lessons were rushed because the teacher constantly worried about covering the days stipulated in the annual teaching plan. The teacher continually encouraged learners to finish their classwork because term two work has a lot of topics that need to be covered. The teacher will constantly say to learners, *"…I'm giving you a few minutes to finish your classwork because we have to finish a lot of topics according to the pacesetter"*. Struggling learners were not given extra support because the teacher continued teaching to finish the annual teaching plan even during morning and afternoon classes.

5.4.1.3 Teaching and learning resources

Tachie and Chirese (2013, p.67) state that "resources are vital and should be used in lessons to support teaching and learning". No object for the demonstration was used; the teacher relied on the grade 12 mind action series textbook for questions, and a few previous question papers were used for examples, class works and homework. Learners were sharing textbooks in groups of three and four; this may indicate that the school might have a shortage of these textbooks. The lesson was mainly based on the use of board and chalk. No posters, pictures or visuals were used to demonstrate how to calculate heights and distances.

5.4.1.4 Classroom assessment

Wilson (2018) argues that classroom assessment is vital in the education system. It allows learners to show what they know, understand, or can do after being taught particular concepts. Assessment also enables teachers to receive feedback about their lessons, allowing them to modify their teaching styles and the teaching and learning activities that learners engage in.

CAPS (2011) argues that Assessment is integral to teaching and learning in the National Curriculum Statement. Therefore, it should be part of every lesson teachers use to assess learners' understanding of what is being taught. The DBE requires mathematics teachers to have classwork written daily (five classwork per week), and learners are expected to be given homework (five homework per week). From my observation, the teacher gave learners classwork and homework each day. His challenge was that most learners were not writing homework, so he spent much time checking the homework and trying to discipline those who did not. Most questions were adapted from the textbook, and some were adapted from previous question papers. Due to the scarcity of textbooks, the teacher wrote the questions on the chalkboard since some groups did not have a book.

5.5 PRE-INTERVENTION SEMI-STRUCTURED TEACHER INTERVIEWS

Two teachers from the experimental and control groups were interviewed. The main aim of the interviews was to gain insights into teachers' experience teaching heights and distances and understand their perspectives on learners' performance in heights and distances in trigonometry. Before the interviews, permission to record them was obtained, and each interview lasted about 10 minutes. Recordings allowed the researcher to recall the information when compiling the analysis. The semi-structured interviews were collected two days after the learners administered the pre-test.

To find out what difficulties the teachers could be having during the teaching and learning in heights and distances in grade 12, the same interview questions were posed to each of them in the same sequence. Teachers were assigned codes, teacher A and B, where teacher A is the experimental group teacher and teacher B is the control group teacher. All responses are transcribed verbatim.

5.5.1 Analysis of semi-structured interviews with teachers after administering pre-test

The questions in the general categories that are printed in italics were posed to the two teachers as follows:

Mathematical challenges: In your opinion, are challenges faced by learners when answering questions on heights and distances that may result in low achievement in grade 12 heights and distances in trigonometry?

Benefits of learning heights and distances: What are the benefits of being taught heights and distances as a grade 12 learner?

Classroom interaction: How do you engage with your learners while teaching and learning heights and distances in trigonometry?

Resources for teaching and learning: Which resources are used in your lessons when teaching heights and distances?

Assessment in the classroom: when assessing learners, do they know the difference between when to use Pythagoras theorem, sine rule and cosine rule?

Transcript 1: What, in your opinion, are challenges learners face when answering questions on heights and distances that result in low achievement in grade 12 heights and distances in trigonometry?

To fully understand learners' difficulties when solving heights and distances in trigonometry, the first interview question was asked to probe teachers' perspectives regarding the perceived obstacles in the participating classrooms. Teacher A's answers to Transcript 1's interview question are shown in Extract 1.1 below.

Extract 1.1

Teacher A **:** *Learners are not doing well in heights and distances because they do not have prior knowledge of the work done in grade 10 and grade 11. When they attempt the questions, they cannot show an understanding of the basic trigonometric ratios. It is even difficult for them to calculate the height and distances when given two-dimensional shapes, which makes it even harder for them when given three-dimensional shapes. Some of them are not serious about their work, no matter how much you try as a teacher to explain trigonometric concepts; they just do not get it. Even if you give them homework, they do not attempt it at home. It is difficult to monitor if all learners write homework because of the overcrowded classes. Another challenge when teaching heights and distances is that my learners use the wrong formulae even if provided with the formula sheet.*

Researcher **:** *What support do you give your learners so that they understand work done in grades 10 and 11?*

Teacher A **:** *I try to re-teach the content done in grades 10 and 11, but I summarise that work because of time.*

Teacher A's perception is that learners do not have prior knowledge of the concepts learnt in previous grades, making it difficult for them to understand heights and distances in trigonometry. The teachers also mentioned that learners could not solve problems when given two-dimensional problems, which supports pre-test results. When learners were presented with two-dimensional problems, they could not show an understanding of trigonometric ratios. According to Van Brummelen (2020), trigonometric ratios are only defined for non-right angled angles of right triangles. This is supported by Alfitri et al., (2023); in their research, they reported that one of the common errors that are made by learners understanding the concepts, namely, learners' inaccuracy when writing the idea of trigonometric ratios.

Teacher A also mentioned that learners use incorrect formulae when calculating heights and distances. Taamneh et al., (2024) reported that learners misuse the formula because they are unsure of the proper steps to complete the process. Some learners even apply Pythagoras's rules in a non-right-angled triangle. The reason for using this may be that learners are not proficient with the necessary trigonometric formulas and do not comprehend the rules of trigonometric comparisons.

Extract 1.2

Teacher B **:** *Regarding trigonometry, some learners just shut down. They do not show any interest in teaching and learning. Their attitude towards trigonometry affects their performance in height and distances; they tend to skip these questions in an examination*. It is a *problem because trigonometry contains more marks in an examination. In my class, most learners do well in mathematics and try their best regarding height and distance. Those learners are able to use knowledge from previous grades to answer questions based on two and three dimensions.*

Researcher : *What support do you give struggling learners, and how do you deal with their negative attitude towards heights and distances?*

Teacher B: *I conduct morning and afternoon classes for struggling learners. I also give them extra work, which motivates them to work hard. They do not enjoy staying behind after school while others leave. I also try to encourage and speak to them as their parents, which helps change their attitude towards this topic.*

Teacher B stated that one of the challenges he encountered when teaching heights and distances was some learners' negative attitudes towards trigonometry. Learners with negative attitudes and beliefs are more likely to shy away from mathematics. Süren et al., (2020). One could argue that learners' diminished confidence and unfavourable views about mathematics contribute to their increased mathematics anxiety; thus, they do not perform well. Makgakga (2016) also state that low academic achievement in Mathematics may be caused by factors such as entry behaviour, motivation and attitude. So teachers need to address this behaviour and come up with ways to win learners and change their bad attitude towards heights and distances in trigonometry.

Teacher A conducts extra classes to support learners and motivates them to address their negative attitudes toward heights and distances. He also mentioned that not all his learners struggle to answer questions about heights and distances. Most are doing well, and they are able to recall what they learnt in previous grades and apply that knowledge when calculating heights and distances.

Transcript 2: What are the benefits of being taught heights and distances to grade 12 learners?

EXTRACT 1.3

Teacher A : *I think the knowledge learners get from learning heights and distances will assist them if maybe they want to build something like a roof of the house, especially the kind of roof that has sides and angles that require a builder to measure using instruments and since some of them are doing Engineering, Graphics and Design (EGD), I think that the knowledge they get from studying heights and distances in mathematics will help them to answer questions in EGD where they are required to calculate the area of the house or a roof. They will then apply the knowledge from mathematics to answer.*

EXTRACT 1.4

Teacher B : *For learners who want to study mechanical engineering, for example, this will assist them when constructing bridges. They will know how to make or* *calculate the correct angles and heights of the bridges they construct. They will use the knowledge acquired at the high school level in their workplace.*

Responses from both teachers indicated that they are aware of the real life applications of heights and distances in trigonometry and where their learners will apply the knowledge learnt. Teachers mentioned that trigonometry is required for numerous tasks, including measuring fields, lots, and areas; building heights, widths, and lengths; installing ceramic tiles; calculating roof inclination; and many more tasks. According to Husain, Rana and Pandey (2021), numerous mathematical concepts are frequently applied in practical settings. One such concept widely used in many jobs, such as dams, bridges and other construction structures, is trigonometry. Hence, heights and distances in trigonometry have proven to be dependable and helpful in solving real-world problems.

Transcript 3: How do you engage with your learners while teaching and learning heights and distances in trigonometry?

EXTRACT 1.5

Teacher A : *Engaging learners in a topic such as this can be challenging, as I have explained that learners in my class find this topic difficult. Even if you ask them simple questions, only a few learners raise their hands and respond; most keep quiet. Even if I initiate group work, it becomes chaotic; hence, I do not use that method often. I prefer that they work individually when writing classwork.*

Researcher : *How do you engage with learners who do not raise their hands when you ask questions?*

Teacher A : Since they do not raise their hand, it indicates they do not know the answer, so I engage with those who raise their hands.

Teacher A responses indicate that he only pays attention to the learners who raise their hands when they are asked questions. This was also confirmed during the classroom observation; this method does not allow learners to interact with one another and share ideas. He also mentioned that when learners interact with one another through group work, the class becomes chaotic. Hence, he avoids the interaction between learners. By using words such as "*Since they do not raise the hands, it indicates that they do not know the answer, so I engage with those that raised their hands",* he assumed that learners do not know the answer without taking into consideration that learners may be shy to speak in front of other learners.

EXTRACT 1.6

Teacher B **:** *I start my lessons with short questions to recall learners' understanding of the work done in previous grades. These questions are designed in such a way that they address conceptual understanding of mathematics. Learners need to know the basics of trigonometry, such as trigonometric ratios and their definitions, quadrants, Pythagoras theorem and so on. This practice has become the standard procedure, and my learners look forward to it. I have often been asked, "Where is the warm-up today?" referring to the baseline assessment. I continue to give learners engaging activities and include real life applications of heights and distances. Learners often work in pairs to share ideas because when we work collectively, some learners are shy about sharing answers with the whole class.*

Researcher : *How do you engage with learners who are struggling?*

Teacher B : *The effective method is individual practice, where learners work individually when writing classwork. This allows me to oversee and discuss the concepts with learners individually. Fortunately, our school doesn't have many learners in class, so engaging with every learner in the classroom is easy. The most important thing for me is to create a classroom environment where learners are in charge of their learning.*

Teacher B, in Extract 1.6, stated that his lessons begin with testing learners' prior knowledge; this is done through questioning and writing baseline assessments. As stated by Dong et al., (2020), prior knowledge assists learners in making sense of new information and connecting it to what they already know; prior knowledge also promotes learners' engagement. Since the teacher begins with prior knowledge, it increases interest and motivation; hence, learners are most likely to engage with questions based on heights and distances successfully. The teacher also mentioned that he continuously gives learners questions based on real life situations. Learners need to apply mathematics to solve real life problems since it allows them to attach meaning to the mathematical construct they develop while solving problems (Van den Heuvel-Panhuizen and Drijvers, 2020).

Teacher B also mentioned that to engage with struggling learners, he uses a method of moving around the class while learners write individually. This allows him to discuss the work with each learner since he has a few learners in each class.

Transcript 3: Which resources are used in your lessons when teaching heights and distances?

EXTRACT 1.7

Teacher A : *I use the mind action series mathematics textbooks; learners share the textbooks in groups of three or four since there is a shortage of textbooks. The problem is that it is challenging to track textbooks because learners do not return them when school closes, and some learners take textbooks at the beginning of the year and drop out before returning them. I also use previous question papers for learners' practice. Although I was trained to use Geogebra during our content workshop, I cannot use it to teach heights and distances because the school does not have computers, and learners are not familiar with using computers. I rely only on the chalkboard, coloured chalk and textbooks.*

Researcher ***:*** *Do you cope without enough textbooks?*

Teacher A : *Not at all, because when learners write classwork, they move around to find a desk with a textbook. This wastes teaching and learning time. Some learners end up not writing or finishing the work given and claim not to find the textbook to use. It is also difficult to monitor if learners write because of overcrowded classes.*

Teacher A's perception that "learners share the textbooks in groups of three or four" indicates a shortage of mathematics textbooks in the school. The most frequent issue influencing learners' performance is the absence of teaching and learning resources, such as textbooks (Ndjangala et al., 2021). The assertion said that an ample supply of reference materials, mainly textbooks, enhances learning. This may mean that learners' performance in heights and distances may be affected by the shortage of textbooks. Teacher A also asserted that "*learners end up not writing or finishing the work given and claim that they could not find the textbook to use".* This corresponds with the research conducted by Ndjangala et al., (2021). They reported that having a lack of textbooks has a detrimental impact on learners' performance. For instance, if a learner brings their textbook home for homework, the other learners won't get the chance to use it, negatively impacting their performance on heights and distances.

EXTRACT 1.8

Teacher B : I use textbooks, and learners share in pairs. We use textbooks for classwork and homework. The best way for learners to understand heights and distances is by incorporating technology during teaching. I used PowerPoint presentations and the smart board to sketch 3D shapes, allowing them to see all the sides of the figure. Learners also make use of the computer lab to explore these shapes.

Researcher : How do they explore using computers?

Teacher B : They use the Tinkercad app installed on the school computers. It allows them to design 3D shapes. Learners are familiar with the tool because we usually use it from grade 10 upwards, especially when introducing topics such as Euclidean and analytical geometry.

Researcher : Are computers enough, or do learners share?

Teacher B : Learners do not share the computers because we have few learners in each class.

According to Makgakga (2016), The Curriculum and Assessment Policy Statement of 2012 states that Mathematics teachers must use computers and technology when teaching. This allows learners to engage in problem solving. Teacher B's perception that "*the best way for learners to understand heights and distances is by incorporating technology"* indicates that the teacher is aware that technology assists learners in understanding the lesson. He further mentioned that the class has "smart boards"; this may indicate that the school has advanced learning materials since learners can access the computer lab.

Transcript 4: when assessing learners, do they know the difference between when to use Pythagoras theorem, sine rule and cosine rule?

EXTRACT 1.9

Teacher A : *Most learners showed that they understand Pythagoras' theorem, sine, and cosine rules and when to use them when asked. They mention that it is used when a triangle is right-angled and if two sides are known. As much as they* *knew this concept, the problem was that most learners could not apply that knowledge when given the problems to solve. Some learners use the distance formula when they are supposed to use Pythagoras' theorem. I think the main problem is that learners do not analyse the diagrams given to them and do not read the written information they are usually provided. Most of them use any rule they feel like using without considering the information presented.*

Researcher **:** How do you assess your learners when teaching heights and distances?

Teacher A *: I usually assess them through one classwork and one homework per day in a week; I also give them informal tests after I finish teaching the whole topic. Assignments and external exams are used as assessment methods.*

EXTRACT 1.10

Teacher B **:** *When assessing learners' prior knowledge through questions and baseline assessment, learners show an understanding of all three concepts and know when and how to use them. Even struggling learners show an understanding of these concepts, although some may struggle to use them correctly. The extra classes assist me to attend to them and correct their misunderstandings.*

Researcher **:** How do you assess your learners when teaching heights and distances?

Teacher B **:** Follow the mathematics subject policy regarding classwork and homework. I give them homework and classwork daily, but on Fridays, I usually give them more homework so that they can study on weekends. Our school has weekly tests, so every Thursday, they write a short test and mark it in class. I also use the Programme of Assessment (POA), where I plan formal assessments at the beginning of the year. The POA has a list of assignments, investigations, and formal examinations.

Both Teacher A and Teacher B state that when they ask learners about these three concepts, Pythagoras theorem, sine rule and cosine rule, learners show an understanding of how they are used. Teacher A stated that although most know these concepts, they struggle to use them correctly. The teacher indicated that many learners cannot use these concepts correctly using the word "most". On the other side, Teacher B used the word "some" to indicate that few learners are unable to use these concepts correctly when solving heights and distances. Teacher A said that one of the reasons why learners are not doing well is because they do not thoroughly analyse the diagram and rush to answer questions.

Both teachers, A and B, mentioned that classwork and home are given to learners daily. Teacher B mentioned that he gives his learners extra work on Fridays so that they practice during the week; on the other hand, Teacher A did not mention anything about the extra work given to learners. Teacher A explained that he gives his learners an informal test once he is done teaching heights and distances to see if they understand the topic. Teacher B assesses his learners weekly so he can track weekly if learners understand the work covered each week.

5.6 IMPLEMENTATION OF THE USE OF VISUALISATION AS AN INTERVENTION STRATEGY

Visualisation was implemented as an intervention strategy in Phase 2 of this study. The use of visualisation in heights and distances was then introduced to the Grade 12 teacher of the experimental group. This strategy aimed at achieving the following objectives:

- To understand the benefits and challenges (if any) of visualisation in the teaching and learning of heights and distances
- To suggest possible ways of implementing effective visualisation in the teaching and learning of height and distances to improve learner performance.

5.6.1 Classroom observations during the intervention

Eight lesson observations were conducted in four (4) weeks on Fridays and Saturdays weekly, for three hours per week, with the experimental group during the intervention. The teacher prepared eight (8) lessons beforehand, which were broken into different topics such as *Pythagoras theorem, trigonometric ratios, sine rule, and cosine rule.* The observations occurred on Fridays and Saturdays weekly. The teacher and the researcher discussed visualisation as an intervention, and the teacher then planned the lesson based on their discussion. The researcher served as the observer while the teacher conducted the classes. Data gathered during the lessons were then used to achieve the study's objectives.

5.6.1.1 Teacher-learner interaction

To ascertain what the learners understood about the heights and distances concepts, a question-and-answer format was employed to connect their prior knowledge to the new knowledge (Sambo, 2022). During the first week of the intervention, learners did not participate as anticipated; few were active during the learning process. Based on the work of Mayer’s theory of multimedia learning, teachers should help learners optimize germane load, which is the work learners put into comprehending and learning where intrinsic movement is the driving force. The teacher motivated learners by asking questions based on their learning in the previous grades to boost their confidence. For example, during the third observation, the teacher asked learners what trigonometric ratios are, and they raised their hands to answer. Even learners who were not raising their hands were encouraged to give answers. Before the intervention, the teacher had not used questions and responses to help learners grow their trigonometry knowledge of heights and distances. Learners would respond with an answer; if correct, the teacher would move on without possibly challenging the learner's comprehension or thinking abilities.

The lessons were more learner-centred; learners were given an opportunity to answer questions the teacher posed and provide their prior knowledge. The premise of cognitive processing holds that learning should involve human participation rather than passively absorbing information, identifying and selecting relevant material, organising it into visual and/or verbal models, and integrating those new models with prior knowledge (Mayer, 2024).

Mayer (2022) states that the brain processes information through two channels. One is for the visual channel, and the other is for the auditory channel. During the lessons, learners were not only presented with visual models, but the teacher was present and explained what was presented to the learners. Learners were shown the pictorial model (from the previous question paper) and the visual models they made using cardboard. The teacher took advantage of both these channels to cater to some learners who may have an auditory impairment and rely more on their visual processing channel and vice versa, as well as learners who may depend on both these channels to process the information. Based on the work of Mayer (2005), Information enters sensory memory when learning begins. It eventually moves to working memory, where it can be processed. Learners' retention is improved when they gain knowledge through two different routes. The conclusion is that to enhance and promote learning, teachers should use dual channels.

Mayer's' theory of multimedia learning assisted the teacher in understanding the mental processes that take place when learners are introduced to visual tools. It also served how the intervention should take place. It gave the teacher direction on how to plan the lesson during the interventions so that learners understand the use of multimedia instruction.

During the second week of observations, learners were first shown a sketch from the previous question paper and asked to make their own visual models using cardboard. They were then asked to draw that visual image without referring to the actual model to see if the image of the visual model was stored in their working memory. Results from the sketches showed that learners were able to sketch the visual model without seeing it, which meant that they had acquired prior knowledge in their long-term memory. According to Mayer (2005), once learners have prior knowledge, they can use it to visualise the problem they may be presented with when solving heights and distances. Participants were able to recall visual mental representations from long term memory and placed that image in a central location within working memory (Makina, 2010).

According to Mayer, the human working memory can only retain five to seven chunks of data (Mnguni, 2014). As a result, teachers need to be cautious to balance the amount of knowledge offered and the capacity of their learners to learn. Hence, the lessons were broken into topics such as *Pythagoras theorem, trigonometric ratios, sine rule and cosine rule.* The aim was to ensure learners were not overwhelmed with too much information in one lesson. The teacher gave brief, concise, and clear instructions to minimise the strain on learners' cognitive processes.

5.6.1.2 Learner-learner interaction

The teacher gave learners an opportunity to interact among themselves. This allowed learners to share ideas and solve height and distance questions. Learners were working in pairs to minimise the large number of groups used prior to the intervention. During the third lesson, learners were given a task to make their own 3Dshapes using cardboard (see 5.5.1.3). Learners used these visuals and question papers to answer questions about heights and distances. Learners were able to come up with more effective ways of visualising the 3D shapes besides the ones that came with the teacher. Learners communicated and worked in pairs to find solutions to the questions at hand. Using visuals during the intervention gave learners autonomy to learn from one another, hence boosting learners' confidence and potential. The use of visualisation inspired critical thinking, which further led to a better understanding of heights and distances

Learners were given an opportunity to share their solutions with the class through the use of presentations. This allowed the whole class to interact with the presenters and ask them questions. The teacher also gave learners feedback on what they had presented. During the feedback, most learners indicated they had gained momentum. Most learners were getting correct answers when working in pairs, and they were able to address their peers' questions successfully. For example, as one pair of learners presented a solution to questions based on Pythagoras' theorem, one learner asked, "*Is it possible to use Pythagoras' theorem if the triangle is a non-right-angled triangle*?" The presenters' responses indicated that they could differentiate between when to use Pythagoras' theorem and sine and cosine rules. The response was, *"It is not possible; if the triangle is not a right-angled triangle, then the sine rule or cosine rule should be applied, depending on what is given in a triangle and what is required".*

5.6.1.3 Teaching and learning resources during the intervention

Makgakga (2016) states that teaching and learning resources are essential in the mathematics class. Learners did not participate as expected during the first three lessons using visualization. Hence, it resulted in two cycles of the study. During the first cycle, Geogebra was introduced as a form of visualization to learners to see if they were able to construct the 3D shapes and use them as visuals to answer trigonometric questions. Learners were randomly grouped into five (5) groups because of the lack of laptops. The school had no computers, so the researcher and the teacher had to find six (6) laptops to use. Laptops, school gates, kids' play tents, and previous question papers were used during the first cycle. Some participants could not effectively participate because they worked in large groups and had one laptop per group and a play tent. School gates did not allow learners to see all the dimensions of the 3D shape. The school gate only allowed learners to identify right-angled triangles and opposite, adjacent and hypotenuse sides.

According to Mayer's (2005) cognitive theory of multimedia learning, the first type of cognitive load is extraneous load, which refers to the cognitive effort wasted on resources that do not promote successful learning. Extraneous load is facilitated by the use of resources that are improper in terms of substance and complexity. It also alludes to superfluous extraneous information that hinders comprehension. It is best to reduce this kind of load as much as possible. Hence, the researcher and the teacher decided to change the teaching resources since the first cycle was time consuming. Learners had to be taught how to use Geogebra, sketch 3D shapes using Geogebra and explore those shapes before answering questions from previous question papers.

The researcher then had a workshop and discussion with the teacher in preparation for the second cycle. The researcher then proposed using cardboards, where learners built their 3D shapes and used them to calculate heights and distances. Therefore, the second cycle focused on introducing visuals that will allow learners not to share, visuals that they can make themselves using resources around them. Sample 1 and sample 2 below shows visual tools learners used during the second cycle.

SAMPLE 1	**SAMPLE 2**

Sample 1 shows three visual tools made by learners using cardboard during the second cycle. This cycle allowed learners to be creative by creating their own 3D shapes and using them as visuals to answer questions from previous question papers. It also allowed participants to learn by being hands-on, motivating them to learn and participate actively while enhancing their understanding of heights and distances in trigonometry. Learners worked in pairs rather than in large groups, allowing the teacher to identify learners who needed more attention. Sample 2 shows a group that volunteered to demonstrate using rulers. The picture was taken during the demonstration with the learners' and the experimental teacher's consent. Participants were able to devise more effective demonstrations than the one introduced by the teacher.

5.6.1.4 Assessment during the intervention

Learners' progress was tracked using classwork and homework during the assessment process. To track the learners' development and degree of comprehension of the content covered in class, the teacher favoured to assign an exercise to the learners following each session during the intervention. Since there were eight (8) lessons observed, each of these lessons had both classwork and homework. Classwork and homework were given to monitor learners' understanding and misconceptions. This allowed the teacher to correct learners' misconceptions before the next lesson. Classwork and homework were taken from previous question papers, grade 12 revision materials, and books compiled by subject advisors each year. The teacher also made copies from the textbook for learners without textbooks.

5.7 TEACHER A SEMI-STRUCTURED INTERVIEW

A semi-structured interview was conducted with the experimental teacher (Teacher A) immediately after implementing the intervention and classroom observations. The aim was to establish his perspective on the benefits of visualisation, if any.

Transcript 1: What did you gain in this intervention and why?

Extract 1.10

Teacher A ***:*** *I have gained a lot from this intervention. It was quite an experience. I have learnt a new way of approaching heights and distances without spending money buying visuals I could use for demonstration. I have also learnt how to engage with my learners when given real life problems to solve. I was sceptical about allowing learners to take control of their learning process. Still, I have learnt that learners learn better when they are given an opportunity to share ideas and work together in problem solving.*

The teacher mentioned that he has learnt a new way of approaching heights and distances. From the baseline observation, the teacher did not use any form of demonstration. From his explanation, the teacher had an idea that to use visual models in class, he has to buy visuals to use for demonstration. This might be why he only relied on the chalkboard and textbook during baseline observation. This study focused on using visuals that are accessible to learners; hence, the use of cardboard was introduced (Roberts, 2022).

The teacher also mentioned that he learnt how to interact with learners when teaching heights and distances through the use of visual models. The teacher plays a vital role in using visuals in heights and distances to balance two modes of thinking: verbal-logical and visual-pictorial (Mayer, 2005). Through visualisation, learners can investigate various mathematical concepts and verify their theories and solutions by exploring the generated objects, offering them an interactive learning environment (Stein et al., 2020). Therefore, learners must be given an opportunity to take control of their learning and investigate mathematical concepts.

Transcript 2: Are there any challenges that need attention to successfully implement visualisation in teaching heights and distances?

Extract1.11

Teacher A **:** *I think the challenge is time. The use of visualisation requires time for it to be implemented successfully, and it does not happen overnight. Another challenge is having a large number of learners in one class; it makes it difficult to assist struggling learners.*

Researcher : *What do you think could be done to address the challenges?*

Teacher A*:* *Learners should not be working in large groups and share visual tools because that can be time consuming, and some learners shy away from* *participating because they are working in large groups. The teacher also needs to seek help from other mathematics teachers in school to monitor learners when making their 3D shapes.*

Visualisation when teaching heights and distances requires time. Learners have to start by making their 3D shape, and the teacher needs to monitor them during that process (Stein et al., 2020). Teachers need to be aware that visualisation takes time, and learners can spend too much time creating and presenting their visuals alone (Vale and Barbosa, 2021).

Teacher A also mentioned that having an overcrowded class makes it difficult for learners to work in large groups because the teacher cannot assist struggling learners. He suggested that when the teacher decides to use groups, learners should not be allocated in large groups so that the learning through visual models is effective.

Transcript 3: **What are the most important benefits of visualisation in teaching heights and distances?**

Extract 1.12

Teacher A **:** *I have been teaching these learners since Grade 10, but I have never seen them so confident and willing to share ideas. Some learners excited me because I have seen a different side of them. Learners were very active during problem solving. I have also seen a positive shift from the lessons I conducted before intervention and during intervention. Learners' conceptual understanding and its application improved.*

Researcher : Do you think visualisation was implemented successfully and why?

Teacher A **:** *Judging from learner participation during the intervention, it was implemented successfully. Still, it will also be verified if there was a change in marks when analysing pre-and post-tests.*

The use of visualisation in heights and distances encouraged learner participation and interaction (Ruthven, 2018). The teacher mentioned that learners were participating actively; this was also observed during the classroom observation. When learners were introduced to the use of visualisation, especially during the second cycle, they participated and were able to come up with different ways of visualisation. Learners worked well in pairs and shared their solutions; hence, the teacher mentioned a difference in learner performance during post-intervention compared to pre-intervention. The intervention positively impacted learners' conceptual understanding; learners could differentiate between when to use Pythagoras' theorem, trigonometric ratios, sine and cosine rules after the intervention.

Transcript 4: Do you think visualisation can be used in the future teaching of heights and distances, and why?

Extract 1.13

Teacher A **:** *Yes, it can be used because it allows learners to use what they have to solve real life problems. It will enable learners to interact with one another and learn from each other.*

Researcher : How can the use of visualisation be improved, especially in rural schools, in the future?

Teacher A : *Lack of technology and technology infrastructure, such as insufficient computers and lack of suitable educational software in schools, should be addressed because the world is in the fourth industrial revolution. Learners should be exposed to the use of technology in mathematics.*

The teacher mentioned that since the experimental school did not have computers that could be used for visualisation, it is important that the issue is raised to the school and the department that these learners need to be supported by being provided with an opportunity to use computers in the future. This will minimise the gap between learners in rural and urban schools. Using technology in mathematics allows teachers to address their professional needs and promote learner learning (Stein et al., 2020).

5.8 POST-TEST RESULTS: QUANTITATIVE DATA

To help make sense of the Grade 12 heights and distances visualisation technique, the descriptive statistics derived from the posttest data are discussed below in the context of the research goal. The post-test data was categorised using the same criteria for the learners' replies as those used in the pre-test data analysis, correct responses (CR), incomplete responses (InC), incorrect responses (IR), and blank responses (BL) (Didis & Erbas, 2015:1141).

5.8.1 Learner responses to Question 1 items

Question 1 assessed the knowledge of the definition of sine, cosine, and tangent, the use of the CAST diagram, and the calculation of height and distances using the Pythagoras theorem. The table below indicates the experimental group's performance improved compared to the pre-test. The highest percentage for the experimental group in CR was 76.67%, whereas in the control group, it was 80. This shows an improvement in experimental group responses in CR compared to their performance in the Q1 pre-test. Table 5.5 below shows post-test results of experimental and control group in question 1 items.

Table 5.5 ***Absolute numbers (as a percentage) of learners' correct, incorrect, blank, and incomplete responses for question 1 of the post-test for the experimental and control groups***

QUESTION	**GROUP**	**PRE-TEST**			
1.1	Experiment	**CR**	**IR**	**InC**	BL
		25 (83.3%)	2 (6.7%)	3 (10%)	0 (0%)
	Control	12 (40%)	4 (13.3%)	9 (30%)	5 (16.7%)
1.2	Experimental	19 (73.3%)	0 (13.3%)	11 (0%)	0 (0%)
	Control	14 (46.7%)	3 (10%)	13 (43.3%)	(0%)

The table above shows that in question 1 item, the percentage ranges from 73.3% to 83.3% and 40% to 46.7% in CR for both the experimental and control groups, respectively. This indicates that the experimental group performed better in both questions 1.1 and 1.2 than the control group. When these percentages are compared to the pre-test, the experiment shows improvement. However, not all learners were able to arrive at the correct answer; the majority of those learners had an incomplete answer, which led them not to get full marks. Hence, those learners were categorised as InC. Compared to the pre-test findings, the experimental groups' percentage changes in the post-test for InC, IR, and BL indicated that the learners had improved their use of the CAST diagram and ability to calculate height and distance using Pythagoras' theorems. The control groups' CR percentage dropped in questions 1.1 and 1.2. Overall, the results indicated that the experimental group learners' performance in Q1 had improved, indicating that the learners had connected the concepts and the strategies to answer the question items.

5.8.2 Learner responses to Question 2 items

Table 5.2 below analyses question 2 results of the post-test for both experimental and control groups. Question 2 assessed the use of Pythagoras to calculate height and distance and the use of trigonometric ratios to calculate the angle.

Table 5.6 ***Absolute numbers (as a percentage) of learners' correct, incorrect, blank, and incomplete responses for question 2 of the post-test for the experimental and control groups***

QUESTION	GROUP	PRE-TEST			
2.1	Experiment	CR	IR	InC	BL
		23 (76.7%)	4 (13.3%)	3 (10%)	0 (0%)
	Control	10 (33.3%)	6 (20%)	9 (30%)	5 (16.7%)
2.2	Experimental	21 (70%)	5 (16.7%)	2 (6.7%)	2 (6.7%)
	Control	4 (13.3%)	6 (20%)	11 (36.7%)	9 (30%)

According to the results displayed in the above table, the CR demonstrated that learners' percentages ranged from 70% to 76.7% and from 13.3% to 33.3% for the experimental and control groups, respectively. This improved the experimental group's understanding of using Pythagoras to calculate distance and trigonometric ratios to calculate angles. 10% and 6.7% of the experimental group's responses to questions 2.1 and 2.2, which requested knowledge of how to apply Pythagoras to calculate distance and trigonometric ratios to compute angle, demonstrated that the learners lacked the necessary understanding. According to the IR results in 2.1, 13.3% of the learners had an inadequate understanding of how to use trigonometric ratios to calculate the magnitude of an angle.

5.8.3 Learner responses to Question 3 items

Table 5.3 below analyses question 3 results of the post-test for both the experimental and control groups. Question 3 assessed if learners can differentiate between when to use the cosine rule and the sine rule if they are not given a right-angled triangle.

Table 5.7 Absolute numbers (as a percentage) of learners' correct, incorrect, blank, and incomplete responses for question 3 of the post-test for the experimental and control groups

QUESTION	GROUP	PRE-TEST			
3.1	Experiment	**CR**	**IR**	**InC**	**BL**
		30 (100%)	0(0%)	0 (0%)	0 (0%)
	Control	27 (90%)	2 (6.7%)	1 (3.3%)	0(0%)
3.2	Experimental	26 (86.7%)	3(10%)	1 (3.3%)	0(0%)
	Control	20 (66.7%)	3(10%)	3 (10%)	4 (13.3%)
3.3	Experiment	18 (60%)	6 (20%)	4 (13.3%)	2 (6.7%)
	Control	5 (16.7%)	6 (20%)	12 (40%)	5 (16.7%)
3.4	Experiment	14 (46.7%)	9 (30%)	6 (20%)	1 (3.3%)
	Control	9 (30%)	11 (36.7%)	2 (6.7%)	8 (26.7%)

The results of the experimental and control groups are shown in Table 5.3 above. The CR percentages ranged from 46.7% to 100% and 16.7% to 90%, respectively. This suggests that both groups' learners were able to use the cosine and sine rules correctly, primarily in 3.1 and 3.2. The experimental group's 30% IR percentage was the highest, indicating that the learners lacked the necessary knowledge to compute an angle using the cosine rule. Learners employed the cosine rule, although they found the algebraic process difficult. The experimental group's maximum percentages of BL and InC were 6.7% and 13.3%, respectively, indicating an improvement over the pre-test. A small percentage of learners left blanks or provided incomplete responses even though these learners lacked conceptual understanding of the sine and cosine rules.

5.8.4 Learner responses to Question 4 items

Table 5.8 below analyses question 4 results of the post-test for both the experimental and control groups. Question 4 consists of a 3D shape in which learners were required to calculate heights and distances. It consists of a right-angled triangle and a non-right-angled triangle.

Table 5.8 ***Absolute numbers (as a percentage) of learners' correct, incorrect, blank, and incomplete responses for question 4 of the post-test for the experimental and control groups***

Question	Group	PRE-TEST			
		CR	IR	InC	BL
4.1.1	Experimental	7 (23.3%)	5 (16.7%)	18 (60%)	0 (0%)
	Control	3 (10%)	4 (13.3%)	23 (76.7%)	0 (0%)
4.1.2	Experimental	26 (86.6%)	1 (3.3%)	3 (10%)	0 (0%)
	Control	2 (6.7%)	6 (20%)	19 (63.3%)	3 (10%)
4.1.3	Experimental	3 (10%)	3 (10%)	22 (73.3%)	2 (6.7%)
	Control	6 (20%)	11 (36.7%)	10 (33.3%)	3 (10%)
4.2	Experimental	14 (46.7%)	4 (13.3%)	8 (26.7%)	0 (0%)
	Control	1 (3.3%)	12 (40%)	7 (23.3%)	10 (33.3%)
4.3	Experimental	8 (26.7%)	12 (40%)	7 (23.4%)	3 (10%)
	Control	1 (3.3%)	16 (53.3%)	0 (0%)	13 (43.3%)
4.4	Experimental	10 (33.3%)	7(23.3%)	10 (33.3%)	3 (10%)
	Control	1 (3.3%)	20 (66.7%)	2 (6.7%)	7 (23.3%)

Table 5.8 shows that, for Q1 items, the percentages for the experimental and control groups ranged from 10% to 86.6% and 3.3% to 20% for CR, respectively. The experimental group outperformed the control group in all questions, with the exception of 4.1.1. The CR category was satisfactory, but the learners' performance was better in Q4.1.3 items. The experimental group's high percentage of InC was generated in Q4.1, Q4.2, and Q4.4. The majority of learners comprehended the requirements in question 4, but their answers were incomplete, resulting in incomplete marks.

5.9 STATISTICAL ANALYSIS

The data acquired from the pre-test and post-test are descriptively analysed and provided below in accordance with the study's objectives. Stata Release 18 was used for statistical data analysis, and Excel was used for data management. A t-test was employed to compare the two study groups; the experimental and control groups. A 2-sided 95% confidence interval was used to interpret the data. Unless otherwise stated, a p-value of less than 0.05 indicated that the results were significant. **Note: Results are significant if $p < 0.05$, and If $p \geq 0.05$, results are not significant.**

5.9.1 Analysis of pre- and post-test results

Each question, from Q1.1 to Q4.4, has its own pre- and post-test analysis. The p-value is compared to determine the statistical significance of the two groups. Table 5.9 below shows analysis of pre-test and post-test results for question 1.1.

Table 5.9: Analysis of the pre-and post-test results for Q1.1 The Wilcoxon Rank Sum (Mann-Whitney) Test

PRE-TEST				POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	838	915	30	1066	915
Control	30	992	915	30	746	915
Combined	60	1830	1830	60	1830	1830
p-value = 0.2014				**p-value = 0.0021**		

The analysis from Q1.1 revealed no significant difference between the groups in the pre-test (p-value=0.2014), which is greater than 0.05 at a 91% confidence limit. The control group recorded a higher score compared to the experimental group. The difference between groups is slight, although Q1 yielded no significant difference.

The post-test analysis reveals a significant difference between the groups, a p-value of 0.0021, which is less than 0.05 at a 91% confidence limit. In the post-test, the experimental group outperformed the control group. In contrast to the pre-test results, the experimental group showed a more significant improvement in scores while recording low scores in the post-test. Therefore, introducing visual models raised the experimental group's scores since the learners did better on the post-test than on the pre-test. Table 5.10 below shows analysis of pre-test and post-test results for question 1.2.

Table 5.10: Analysis of the pre-and post-test results for Q1.2: The Wilcoxon Rank-Sum (Mann-Whitney) Test

PRE-TEST				POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	889	915	30	1001	915
Control	30	941	915	30	829	915
Combined	60	1830	1830	60	1830	1830

p-value =0.7125	**p-value = 0.1805**

There was no discernible difference between the groups in the pre-test and post-test, according to the analysis from Q1.2 ($p - values = 0.7125$ and $p - values = 0.1805$, respectively), which is higher than 0.05 at a 91% confidence limit. The control group outperformed the experimental group in the pre-test, indicating subpar performance prior to the use of visual aids in height and distance instruction. Additionally, the experimental group did better on the post-test after the intervention, suggesting that the learners' comprehension of the CAST diagram and the application of Pythagoras' theorem to determine the third side of a right-angled triangle was enhanced. Table 5.11 below shows analysis of pre-test and post-test results for question 2.1.

Table 5.11: Analysis of the pre-and post-test results for Q2.1: The Wilcoxon Rank-Sum (Mann-Whitney) Test

	PRE-TEST			**POST-TEST**		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	945	915	30	1118.5	915
Control	30	885	915	30	711.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.6765				**p-value =0.0009**		

The analysis from Q2.1 revealed no significant difference between the groups in the pre-test ($p - value = 0.6765$), which is greater than 0.05 at a 91% confidence limit. The control group outperformed the experimental group in the pre-test, indicating inadequate achievement prior to intervention.

The two groups had a significant difference, according to the post-test analysis of these data for Q2.1 ($p - value$ = 0.009), which was less than $p - value$ 0.05. In the post-test following the intervention, the experimental group outperformed the control group with a significantly higher score. The post-intervention test results indicated that visualisation positively affected the learners' performance in trigonometric ratios in grade 12.Table 5.12 below shows analysis of pre-test and post-test results for question 2.2.

Table 5.12: Analysis of the pre- and post-test results for Q2.2: The Wilcoxon Rank-Sum (Mann-Whitney) Test

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	851	915	30	1226.5	915
Control	30	979	915	30	603.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.3361				**p-value =0.0000**		

There was no discernible difference between the groups in the pre-test, according to the analysis from Q2.2 ($p - value = 0.3361$), which is higher than 0.05 at a 91% confidence level. In comparison to the experimental group, the control group scored higher. Although there was no significant difference in the pre-test results from Q1, there is a modest difference between the groups. The post-test examination of these data suggested that there was a significant difference between the two groups for Q2.2 ($p - value$ = 0.0000), which was less than $p - value$ 0.05. The experimental group fared much better than the control group on the post-test that was administered after the intervention. Table 5.13 below shows analysis of pre-test and post-test results for question 3.1.

Table 5.13: *Analysis of the pre-and post-test results for Q3.1: The Wilcoxon Rank-Sum (Mann-Whitney) Test*

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	851	915	30	1226.5	915
Control	30	979	915	30	603.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.4211				**p-value =0.2373**		

The analysis from Q3.1 revealed no significant difference between the groups in the pre-test ($p - value = 0.4211$), which is greater than 0.05 at a 91% confidence limit. The control group outperformed the experimental group in the pre-test, indicating inadequate achievement prior to intervention. In the post-test, the experimental group showed improvement compared to the pre-test; the experimental group outperformed the control group. However, there was no significant difference between the groups in the post-test ($p - value = 0.2373$), which is greater than 0.05 at a 91% confidence limit. Table 5.14 below shows analysis of pre-test and post-test results for question 3.2.

Table 5.14: ***Analysis of the pre-and post-test results for Q3.2: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	863	915	30	987.5	915
Control	30	967	915	30	842.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.2760				**p-value =0.2078**		

The analysis from Q3.2 revealed no significant difference between the groups in pre-test and post-test ($p - value = 0.7125$ and $p - value = 0.1805$, respectively), which is greater than 0.05 at a 91% confidence limit. In the pre-test, the control group performed better than the experimental group, suggesting poor performance before the introduction of visualisation in teaching heights and distances. However, the rank sum in the post-test indicates an improvement in the performance of the experimental group compared to the pre-test. The control group performed better in the pre-test compared to the post-test. Table 5.15 below shows analysis of pre-test and post-test results for question 3.3.

Table 5.15: ***Analysis of the pre-and post-test results for Q3.3: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	929.5	915	30	1047	915
Control	30	900.5	915	30	783	915
Combined	60	1830	1830	60	1830	1830
p-value =1.0000				**p-value =0.0153**		

The data in the Table above depicts the analysis of the Q3.3 pre-test results, indicating no significant difference scores between the two study groups ($p - value$ = 1.0000), greater than p − $value$ 0.05 at a 95% confidence limit. The results show that the performance of the experimental and control groups is slightly similar; the difference in the rank sum is 29. When compared in the post-test, the results indicate that there were significantly different scores between the two study groups ($p - value$ = 0.0153), less than p − $value$ 0.05 at a 95% confidence limit. The experimental group outperformed the control group in the post-test. When comparing the pre-and-post-tests, the experimental group performed higher in the post-test compared to the post-test, whereas the control group's performance declined compared to the pre-test. Table 5.16 below shows analysis of pre-test and post-test results for question 4.1.1.

Table 5.16: ***Analysis of the pre-and post-test results for Q4.1.1: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	749	915	30	1190.5	915
Control	30	1081	915	30	639.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.0102				**p-value =0.0000**		

The data in the table above depicts the analysis of the Q 4.1.1 pre-test results, indicating significantly different scores between the two study groups ($p - value$ = 1.0102), less than p – $value$ 0.05 at a 95% confidence limit. The post-test results indicated a significant difference between the two study groups ($p - value$ = 0.0000), less than p − $value$ 0.05 at a 95% confidence limit. The experimental group outperformed the control group in both pre and post-tests. Table 5.17 below shows analysis of pre-test and post-test results for question 4.1.2.

Table 5.17*: Analysis of the pre-and post-test results for Q4.1.2: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	784.5	915	30	1151.5	915
Control	30	1045.5	915	30	678.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.0370				**p-value =0.0000**		

The analysis from Q4.1.2 revealed a significant difference between the groups in pre-test and post-test ($p - value = 0.0370$ and $p - value = \ 0.000$, respectively), which is less than 0.05 at a 91% confidence limit. In the pre-test, the control group performed better than the experimental group, suggesting poor performance before the introduction of visualisation in teaching heights and distances. However, the rank sum in the post-test indicates an improvement in the performance of the experimental group compared to the pre-test. The control group performed better in the pre-test compared to the post-test. Table 5.18 below shows analysis of pre-test and post-test results for question 4.1.3.

Table 5.18: ***Analysis of the pre- and post-test results for Q4.1.3: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

PRE-TEST				POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	770	915	30	1186.5	915
Control	30	1060	915	30	583.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.0106				**p-value =0.0000**		

The analysis from Q4.1.3 revealed a significant difference between the groups in the pre-test ($p - value = 0.0106$), which is less than 0.05 at a 91% confidence limit. The control group recorded higher scores than the experimental group, with a difference of 290 in their rank sum. The post-test analysis revealed a difference between the groups with a p-value of 0.0000, which is less than 0.05 at a 91% confidence limit in the post-test; the experimental group outperformed the control group. In contrast to the pre-test results, the experimental group showed a more significant improvement in scores while recording low scores in the post-test. The introduction of visuals before and the models raised the group's scores since the learners did better on the post-test than on the pre-test**.** Table 5.19 below shows analysis of pre-test and post-test results for question 4.2.

Table 5.19: ***Analysis of the pre-and post-test results for Q4.2: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

PRE-TEST				POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	822	915	30	1172.5	915
Control	30	1008	915	30	557.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.0955				**p-value =0.0000**		

The Q4.2 pre-test results are analysed in the data above, and the results show that there were no significant differences in scores between the two study groups (p − $value$ = 0.0955), with a 95% confidence level of less than p − $value$ 0.05. According to the findings, the control group scored considerably higher than the experimental group before the intervention. As a result, the experimental group underperformed on the pre-test compared to the control group.

Compared to the post-test, the table above revealed a significant difference between the two study groups ($p - value$ = 0.0000), less than $p - value$ 0.05 at a 95% confidence limit. The experimental group's performance improved compared to the pre-test, and the experimental group outperformed the control group in the post-test. The post-intervention test findings showed that visualisation helped learners perform better in Grade 12 trigonometry problems involving heights and distances. Table 5.20 below shows analysis of pre-test and post-test results for question 4.3.

Table 5.20: ***Analysis of the pre-and post-test results for Q4.3: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	883.5	915	30	1172.5	915
Control	30	946.5	915	30	557.5	915
Combined	60	1830	1830	60	1830	1830
p-value =0.3533				**p-value =0.0000**		

Prior to the intervention, the data suggested that, for Q4.3 (p $- value$ = 0.3533), greater than 0.05 at a 91% confidence limit, there was no statistically significant difference between the two study groups. The experimental group scored lower than the control group; however, the difference between the two groups was insignificant and did not provide a statistically significant difference.

Following the intervention, a significant difference was found between the two groups for Q4.3 of the post-test (p$- value$ = 0.0000), with a 95% confidence level falling below 0.05. The control group's post-test score was lower than that of the experimental group. Visualisation was effective based on the experimental group's higher post-test performance compared to the pre-test. Table 5.21 below shows analysis of pre-test and post-test results for question 4.4.

Table 5.21: ***Analysis of the pre- and post-test results for Q4.4: The Wilcoxon Rank-Sum (Mann-Whitney) Test***

	PRE-TEST			POST-TEST		
Group	Obs	Rank sum	Expected	Obs	Rank sum	Expected
Experimental	30	914	915	30	1175	915
Control	30	916	915	30	655	915
Combined	60	1830	1830	60	1830	1830
p-value =1.0000				**p-value =0.0000**		

The analysis from Q4.4 indicates no significant difference between the groups in the pre-test ($p - value = 1.0000$), which is greater than 0.05 at a 91% confidence limit. However, the control group outperformed the experimental group in the pre-test, indicating inadequate achievement prior to the experimental group's intervention.

The two groups had a significant difference, according to the post-test analysis of these data for Q2.1 ($p - value$ = 0.0000), which was less than $p - value$ 0.05. In the post-test following the intervention, the experimental group outperformed the control group with a significantly higher score. The post-intervention test results indicated that the use of visualisation had a positive effect on learners' performance in trigonometric ratios in grade 12.

5.9.1 Summary of the pre- and post-test results

Table 5.18 below summarises the pre-test and post-test results administered in the experimental and control groups. Both within and between groups, the p values and rank sum scores determine the significance, and no significance between the study groups from the pre-test and post-test is compared.

Table 5.22: An overview of the experimental and control groups' rank sum scores for the pre- and post-tests: The Mann-Whitney (Wilcoxon Rank-Sum) Test

EXPERIMENTAL				**CONTROL**		
QUESTION	**PRE-TEST**	**POST-TEST**	**Pre vs Post P-VALUE**	**PRE-TEST**	**POST-TEST**	**Pre vs Post P-VALUE**
1.1	**945.0**	**1066.0**	0.0017	**885.5**	**746.0**	0.9453
1.2	**851.0**	**1001.0**	0.0034	**979.0**	**829.0**	0.9935
2.1	**945.0**	**1118.5**	0.5982*	**885.0**	**711.5**	0.7095
2.2	**851.0**	**1226.5**	0.0245	**979.0**	**603.5**	0.6855
3.1	**960.0**	**960.0**	< 0.0001	**870.0**	**870.0**	0.0001*
3.2	**863.0**	**987.5**	< 0.0001	**967.0**	**842.5**	< 0.0001*
3.3	**929.5**	**1047.0**	< 0.0001	**900.5**	**783.0**	< 0.0001*
3.4	**900.0**	**1103.3**	< 0.0001	**930.0**	**727.0**	0.0005*
4.1.1	**749.0**	**1190.5**	0.669*	**1081.0**	**639.5**	1.0000
4.1.2	**748.5**	**1151.1**	0.0001	**1045.5**	**678.5**	0.5927
4.1.3	**770.0**	**1186.5**	< 0.0001	**1060.0**	**583.5**	0.9846
4.2	**822.0**	**1172.5**	0.0013	**1008.0**	**657.5**	0.9888
4.3	**883.5**	**1155.5**	0.0001	**946.5**	**674.5**	1.0000
4.4	**914.0**	**1175.5**	< 0.0001	**916.0**	**655.0**	0.7500
TOTAL RANK	**771.5**	**1264.5**		**1058.5**	**565.5**	
Group comparisons in the pre-test p-value= 0.0332 **Group comparisons in the pre-test p-value= 0.0000** **Pre vs post-test experimental group' p-value =< 0.0001** **Pre vs post-test control group's p-value = 0.2858**						

The results in Table 5.18 indicate the analysis of the experimental and control study groups, revealing different rank sum scores between the pre-test and post-test for Q1 to Q4. The experimental group and control groups' results showed a significantly different score in the pre-test ($p - value$ = 0.0332), less than 0.05 at a 95% confidence limit. This indicates a significant difference in learners' performance when comparing groups. The experimental group had a total rank sum of 772.5. In contrast, the control group had a rank sum score of 1058.5 in the pre-test, indicating that the control group outperformed the experimental group in the pre-test. Table 5.18 indicates that the control group scored significantly higher in total marks ($p = 0.0332$), as well as on Q4.1.3 ($p = 0.0106$), Q4.1.2 ($p = 0.0370$), and Q4.1.1 ($p = 0.0102$) compared to the Experimental group. The differences between the two study groups were insignificant for all other items.

The experimental and control groups' results showed a significantly different score in the post-test ($p - value$ = 0.0000), less than 0.05 at a 95% confidence limit. The total rank sum indicates that, unlike the pre-test, the experimental group ($rank\ sum = 1264.5$) outperformed the control group ($rank\ sum = 565.5$) when post-tests are compared. Table 5.18 presents the results of group comparisons in the post-test, showing that the Experimental group scored significantly higher on all items than the Control group. Exceptions were Q1.2 ($p = 0.1805$), Q3.1 ($p = 0.2373$), and Q3.2 ($p = 0.2078$), where the differences between the groups were not significant.

When comparing the pre and post-test of each group, table 5.18 shows that post-test results ($rank\ sum = 1264.5$) were significantly higher than pre-test results ($rank\ sum = 771.5$) for the experimental group with a p-value of <0.0001, indicating that learners' performance in post-test improved in most questions except for Q2.1 and Q4.1.1. In these cases, the p-values were not below the nominal 0.05, indicating no significant difference between the pre- and post-test performance. The reason for no significant difference in these questions is that the experimental group's learners performed well in both these questions in the pre-test and post-test.

On the other hand, table 5.18 indicates that the pre-test and post-test scores were statistically similar in the control group ($p - values = 0.2858$), indicating no significant difference in pre and post-test, except for questions 3.1, 3.2, 3.3, and 3.4, where the pre-test scores were significantly lower than the post-test scores. In these cases, the pre-test scores were substantially lower than the post-test scores since p-values were below the nominal 0.05, indicating a significant difference between the pre-and post-test. Since this is a controlled group, the researcher does not have a clue why there was a slight improvement in these questions since teachers use many teaching strategies to teach heights and distances. The rest of the questions' p-values are above the nominal 0.05, indicating no significant difference between the pre-and post-test observations. This suggests that learners' performance in questions 1, 2 and 4 did not change in pre and post-tests. Overall, the total rank sum demonstrates that there was a decline in learner performance in the post-test ($rank - sum = 565.5$) compared to the pre-test ($rank - sum = 1058.5$)

In conclusion, the table above indicates that the intervention improved the performance of the experimental group’s learners since most of the question items showed a positive difference between the pre-test and post-test.

5.10 CONCLUSION

In the pre-test using the Wilcoxon Rank Sum Test, the experimental group did not perform as well as the control group ($p - value < 0.05$), indicating significantly different scores, according to the study's results presented in this chapter. However, following the intervention, the experimental group's ($rank - sum = 1264.5$) improved significantly compared to the control group's ($rank - sum = 565.5$), suggesting that the intervention positively impacted the experimental group's understanding of heights and distances in trigonometry. The post-test improvement of the two study groups was compared using the rank-sum scores, with the experimental group showing improvement in most questions. In contrast, the control group exhibited similar outcomes in most questions. The qualitative and quantitative findings are discussed in the following chapter, framed by the ideas and literature employed in this investigation.

CHAPTER 6: DISCUSSION OF THE FINDINGS

6.1 INTRODUCTION

This chapter discusses the findings drawn from the quantitative and qualitative data presented in Chapter 5. It outlines the analyses of lesson observations conducted before and during the intervention, alongside a semi-structured interview prior to the intervention with two teachers from both the experimental and control groups, as well as a semi-structured interview after the intervention with the experimental teacher. The chapter details the results of the quantitative data, followed by the qualitative data interpretation. The quantitative findings of the study from the pre-test and post-test are assessed using the previously mentioned analytical approach. The theoretical framework and literature evaluation were also utilised to interpret the data to answer the research questions.

6.2 THE QUANTITATIVE RESULTS

Both experimental and control groups participated in the pre-test and post-test, which produced quantitative data results. Stata Release 18 was utilised for statistical data analysis, and Excel was used for data management. The experimental and control groups were compared using the P-value and rank sum scores. The improvement in the rank sum scores was regarded as occurring both within and between the experimental and control groups before and after the intervention was introduced.

6.2.1 The pre-test and post-test results

Pre-tests and post-tests were conducted to compare learners' performance in calculating heights and distances in trigonometry before and after the use of visualisation as an intervention. Samosa et al., (2021) define visualisation as a learning method that is a crucial component of representation which facilitates comprehension, memory and recall, and problem solving.

The findings of this study were gathered from the collected data through pre-test and post-test, and a statistical software package named Wilcoxon test was used. The Wilcoxon or rank sum test is a nonparametric statistical method used to compare the Control and Experimental groups (Dao, 2022). Wilcoxon test was used to assess whether the use of visualisation has an effect or no effect on learners' performance on heights and distances. Hence, this allowed the researcher to adopt either a null or alternative hypothesis for this research study. The error rate of $p = 0.05$ was used to interpret the results where if $p < 0.05$, the results were declared significant and if $p > 0.05$ in the results were not significant.

Learners' performance was analysed per item before and after the use of visualisation as an intervention. The results are discussed below.

6.2.1.1 QUESTION 1 ITEM RESULTS BEFORE AND AFTER THE INTERVENTION

The results from Q1, which has two sub-questions (Q1.1 and Q1.2), revealed a noticeable difference in the performance of the experimental and control groups in both the pre-test and post-test. The p-values between the pre-test and post-test for the experimental group ($p - value = 0.0017$) and control group ($p - value = 0.9453$) suggest that the intervention positively affected the experimental groups' performance using Pythagoras' theorem. The results indicated that the experimental group's performance $(rank - sum = 945.0)$ was slightly higher than the control group$(rank - sum = 885)$ in question 1.1 in the pre-test. Some learners in the experimental group were unable to use the knowledge from the CAST diagram to make a sketch. This concurs with Amedume et al., (2022); that some learners did not draw the sketch; hence, they calculated the wrong side using Pythagoras theorem. Learners were unable to solve the root of the results; they often forgot formulae and ways of answering the questions given, and they were not careful when solving problems (Retnawati, 2020). This resulted in them not getting question 1 correct since they were supposed to use the sketch to write the definition of $sin\theta$ and $cos\theta$ and substitute on a given expression. In contrast, results from the post-test revealed that in Q1.1, the experimental group ($rank - sum = 1001.0$) outperformed control group ($rank - sum = 829.0$). This indicates that the experimental group's performance increased when the pre-test and post-test rank sum were compared.

Similarly, in sub-question 1.2, the experimental group showed a significantly different score between the pre-test and post-test $(p - value = 0.0034)$, hence outperformed the control group $(p - value = 0.9935)$. In the pre-test, control group $(rank - sum = 979.0)$ and performed better than the experimental group $(rank - sum = 851.0)$, suggesting poor performance before the introduction of visualisation in teaching heights and distances in the experimental group. After the intervention, the learners' ability to interpret the CAST diagram was improved, and the Pythagoras theorem was used to calculate the third side in a right-angled triangle. The rank sum scores between the pre-test and post-test from the experimental group indicate the improvement in learners' performance as the difference between the scores is 150. On the other hand, the rank sum scores from the control group revealed a decline in learners' performance from the pre-test to the post-test.

The experimental group's results showed a significantly different score between the pre-test and post-testtest $(p - value = 0.0017)$ in question 1.1. In the post-test, learners showed an understanding of quadrants and the use of Pythagoras theorem and how they are applied when answering the question posed. The control group, on the other hand, showed no significant difference in the scores between pre-test and post-test in Q1.1 $(p - value = 0.9453\ )$ greater than the p − $value$ 0.05. When comparing the rank sum scores of the control group in pre-test $(rank - sum\ = 885.5)$ and post-test$(rank - sum\ = 746.0)$, there was a decline in learner performance. The rank sum scores indicate that in the experimental group, there was an improvement in the learner performance when comparing the pre-test $(rank\ sum\ score = 945.0)$ and post-test $(rank - sum = 1066.0)$.

In summary, both sub-questions from question 1 indicate that before the intervention, the control group performed better than the experimental group. However, in sub-question 1.1, the performance in both groups was slightly different. After the intervention, the experimental group showed improvement and outperformed the control group.

6.2.1.2 QUESTION 2 ITEM RESULTS BEFORE AND AFTER THE INTERVENTION

Question 2 had two sub-questions, Q2.1 AND Q2.2. The results from Q2.1 revealed that there was no significant difference between the groups in pre-test$(p - value = 0.6765)$are greater than 0.05 at a 91% confidence limit. The results indicate that in Q2.1 pre-test, the experimental group performed slightly higher $(rank\ sum\ = 945.0)$ when compared to the control group$(rank\ sum = 885)$. There was a significant difference between the two groups, according to the post-test analysis of these data for Q2.1 $(p\ - value\ =\ 0.009)$, which is less than the p-value, indicating a significantly higher score. The post-intervention test results revealed that the use of visualisation positively impacted learners' performance in trigonometric ratios in Grade 12. This suggests that since this is a control group, the researcher may not be aware of the reason for this decline, as this group did not receive the intervention. The control group might underperform due to the lack of structured learning, whereas the experimental group benefits from the treatment (Zach, 2020).

On the other hand, in Q2.2, the control group scored significantly higher in total marks $(rank\ sum = 979.0)$ compared to the experimental group $(rank\ sum = 851.0)$ on the pre-test. The post-test rank sum scores indicate that the intervention positively affected the experimental group $(rank\ -\ sum\ =\ 1226.5)$. Q2.2 showed an improvement in learners' performance compared to control group$(rank - sum = 603.5)$. This significant difference in performance is also demonstrated by the group's $(p - value =\ 0.0245)$, which is less than 0.05 at a 91% confidence limit, indicating that there was improvement when the pre-test and post-test were compared. The rank sum score of the control group changed from $(ranksum =\ 979.0)$ in pre-test to $(rank - sum = 603.5)$ in the post-test with the p-value of

($p - value = 0.6855$), indicating no significant difference in learners' performance when the pre-test and post-test were compared.

In summary, the intervention positively affected the experimental group as their performance improved when the pre-test ($ranksum = 851.0$) and post-test ($rank - sum = 1226.5$) were compared. In question 2.1, learners used trigonometric ratio specifically $sin\theta$, to find the required angle. Some learners used cosine rule instead in the pre-test in question 2.2, which is the wrong methodology since there is no three sides given. If the learner wanted to use cosine rule, they had to first find the third side using Pythagoras theorem. Most learners used either $cos\theta$ or $tan\theta$ ratio, which shows that they might not be able to differentiate between opposite, hypotenuse and adjacent sides of the required angle. Most learners who gave incorrect and incomplete categories used the incorrect formulae and procedure to calculate the required side in 2.1 and the required angle in 2.2. In the post-test, learners were now able to use trigonometric ratios correctly and their definitions, and they were able to differentiate when to use the sine rule and cosine rule.

6.2.1.3 QUESTION 3 ITEM RESULTS BEFORE AND AFTER THE INTERVENTION

Question 3 had four sub-questions (3.1 to 3.4). The results from Q3 indicate that the experimental group ($rank - sum = 960.0$) that performance was slightly higher than the control group ($rank - sum = 870.0$) in question 3.1 pre-test. The rank sum score of the experimental group in 3.1 remained at ($rank - sum = 960$) in both the pre-test and post-test. This shows that in both of these tests, learners in the experimental group demonstrated that they understood that the question required them to use the cosine rule and that they could use it correctly.

Control group ($rank - sum = 863.0$) outperformed experimental group ($rank - sum = 967.0$) in question 3.2 pre-test. The rank sum score of the experimental group in 3.2 changed from ($rank - sum = 863.0$) to ($rank - sum = 987.5$) with the p-value of ($p < 0.0001$), indicating that visualisation positively affected learner performance, and the results were declared significant. In the pre-test, most learners skipped this question, and others were only able to find the third angle using the sum of angles in a triangle, but they were unable to use that answer to find the required length. In the post-test, learners were able to use the sine rule correctly to find the required side, hence the improvement in their rank sum score. This concurs with Boakye and Nabie (2024), who state that for learners to improve their understanding of the sine rule, they need to be exposed to a method that fosters an interactive, practical and engaging learning environment, which increases motivation and retention compared to traditional teaching methods. Since the experimental group was exposed to this model, they outperformed the control group; this demonstrated the model's effectiveness in enhancing comprehension and application of trigonometric concepts.

The experimental group $(rank - sum = 929.5)$ performed slightly better than the control group (rank sum= 870.0) in question 3.3 pre-test. The rank in question 3.3 changed from $(rank - sum = 929.5)$ in pre-test to $(rank - sum = 1047.0)$ the post-test showed a p-value <0.0001, indicating a significant difference in learners' performance, and the intervention positively affected learners' performance. Learners were presented with a triangle with three given sides (SSS) and Ɵ as a required angle, necessitating using the cosine rule to determine the required angle. In the post-test, they successfully applied the cosine rule to find the angle and demonstrated the correct mathematical steps for calculating it. The Asian Journal of Advanced Research Report (2023) noted common challenges, such as a fragmented understanding of the rules and difficulties applying them in real life contexts. The study recommended that teachers incorporate effective interventions such as practical applications, in this case, visualisation, to bridge these gaps. This aligns with Adu and Folson (2023), that the use of visuals minimises cognitive load and enables learners to focus on exploration and connections between concepts.

Control group $(rank - sum = 930.0)$ outperformed experimental group $(rank - sum = 900.0)$ in question 3.4 pre-test. The rank sum of the experimental group in question 3.4 changed from $(rank - sum = 900.0)$ to $(rank - sum = 1103.3)$ with the $p - value < 0.0001$ indicated that learners performed better in the post-test compared to the pre-test; thus, the results were declared significant. This means that the intervention positively affected learners' performance in question 3.4. Overall, the results show a significant improvement in the performance of learners in post-test $(p - value < 0.0001)$ in all sub-questions on question 3. This indicates that the use of visualisation had a positive impact on learners' performance since learners were now able to differentiate between when to use the cosine rule and the sine rule if they were not given a right-angled triangle.

The results from the control group indicates that the pre-test and post-test were statistically similar, except for questions 3.1 $(p - value\ 0.0001)$, 3.2 $(p - value < 0.0001)$, 3.3 $(p - value < 0.0001)$, and 3.4 $(p - value\ 0.0005)$ where the pre-test scores were significantly lower than the post-test scores since p-values were below the nominal 0.05, indicating a significant difference between the pre-and post-test. Since this is a controlled group, the researcher does not have a clue why there was a slight improvement in these questions since teachers use many teaching strategies to teach heights and distances. The rest of the questions' p-values are above the nominal 0.05, indicating no significant difference between the pre-and post-test results. This suggests that learners' performance in questions 3.1, 3.2 and 3.44 did not change from pre-test to post-tests.

6.2.1.4 QUESTION 4 ITEM RESULTS BEFORE AND AFTER THE INTERVENTION

Question 4 had four sub-questions (4.1 to 4.4), where 4.1 consisted of 4.1.1 to 4.1.3. Question 4 consisted of 3D shapes requiring learners to calculate heights and distances. This question was made up of a right-angled triangle and a non-right-angled triangle. It, therefore, assesses if learners can find the sum interior angles of a triangle, use the sine rule, cosine rule, Pythagoras theorem or appropriate trigonometric ratios to solve for the triangle to find either a length or an angle and use correct formulae to find the area of the non-right-angled triangle. In question 4.1.1, the control group $(rank - sum = 1081.0)$ outperformed the experimental group $(rank - sum = 749.0)$ in the pre-test. This question assessed three trigonometric concepts: learners' understanding of calculating the sum of angles in the triangle, using that answer to find the distance of BC using the sine rule, and the required distance of AB using trig ratios. In both groups, learners were able to find the sum of angles in a triangle, but most learners in the experimental group could not use that correct answer to calculate the required side further. Some of them skipped this question. In the post-test, the experimental group $(rank - sum = 1190.5)$ outperformed the control group $(rank - sum = 639.5)$. This indicates that the use of visualisation had a positive impact on learners' performance in the use of sine rule and trigonometric ratios.

In question 4.1.2 the control group $(rank - sum = 1045.5)$ outperformed the experimental group $(rank - sum = 748.5)$ in the pre-test. Learners were required to use Pythagoras' theorem in a right-angled triangle to find one missing side in triangle ABC or use the trig ratio. Most learners from the experimental group could not answer this question correctly because they did not find the correct answer in 4.1.1. As noted by Rudi et al., (2020), learners memorise formulae without understanding their derivation, which leads to errors such as misapplying subtraction instead of addition or neglecting to square the sides in the formula. These procedural memory deficits are common among learners with learning difficulties. In the post test, the experimental group $(rank - sum = 1151.1)$ outperformed the control group $(rank - sum = 678.5)$. Learners in the experimental group showed an understanding of how Pythagoras theorem and trigonometric ratios work. They were now able to identify adjacent, opposite and hypotenuse sides in a right-angled triangle. Hence, their substitution of trigonometric ratios was correct. Hadjerrouit (2020) concurs that for effective learning, combining strategies such as visual tools, memory aids and applied exercises can support memorisation and a deeper understanding of Pythagoras' theorem.

In question 4.1.3 the control group $(rank - sum = 1060.0)$ outperformed the experimental group $(rank - sum = 770.0)$ in the pre-test. This question required the AC length calculated in the previous question. Some of the learners who answered correctly were able to answer the previous question. Most learners who answered 4.1.2 were unable to use their correct answer to calculate side CD, especially from the control group. Most learners in the experimental group could not answer this question correctly, and some left a blank space; they did not attempt to answer it. In the post test, the experimental group $(rank - sum = 1186.5)$ had an increase in the number of learners who performed well in question 4.1.3, on the other hand, the control group $(rank - sum = 583.5)$ There was a decline in learners' performance.

In question 4.2 the control group $(rank - sum = 1008.0)$ outperformed the experimental group $(rank - sum = 822.0)$ in the pre-test. This question required learners to calculate angle α using triangle ADC, a right-angled triangle, using trig ratio. Few learners in the experimental and control groups were able to answer this question correctly. Some learners could not differentiate between a side opposite and adjacent to α. In the post test, the experimental group $(rank - sum = 1172.5)$ outperformed the control group $(rank - sum = 657.5)$. Practical scenarios are an effective methodology for learners to understand the use of trigonometric ratios, for example, calculating the angle of elevation using tangent or determining the slope of a ladder with sine (Wikantari et al., 2022). These problems integrate visualizations with real life applications, making the theory relatable and improving retention. Hence, using a model made up of cardboard as a visual tool positively impacted how learners performed in question 4.2.

In question 4.3 the control group $(rank - sum = 946.5)$ outperformed the experimental group (rank sum=883.5) in the pre-test. Learners were required to calculate an angle using the cosine rule. Some learners in both groups were unable to differentiate between the conditions required to apply the sine or cosine rule. Some learners used trigonometric ratios to calculate the angle. This was incorrect because the given triangle was not a right-angled triangle. In the post-test, the experimental group $(rank - sum = 1155.5)$ outperformed the control group $(rank - sum = 674.5)$. The cardboard-based- hands-on visualisation allowed the experimental group to grasp better the geometric relationship between the sides and angles in the cosine rule. Tajuddin et al., (2023) asserts that the use of visualisation to teach cosine rule is an effective way to enhance learner understanding, more especially for non-right-angled triangles; visualisation allows learners to connect abstract formulas to tangible concepts.

In question 4.4, the experimental group $(rank - sum = 914.0)$ had similar performance as the control group $(rank - sum = 916.0)$ in the pre-test, although the rank sum of the control group is higher by two points. This question was based on learners applying the calculated heights and distances in the previous question to calculate the area of the three-dimensional shape. This was one of the poorly answered questions from both groups. The most common error made by learners was using the incorrect area formula. Some learners used $A = lxb$. This formula is not applicable to this question since the triangle is non-right-angled. Most of the learners from both groups skipped this question. In the post- test, the experimental group $(rank - sum = 1175.5)$ outperformed the control group $(rank - sum = 655.0)$. Learners in the experimental group were now able to use height and distances to calculate the area of a three-dimensional shape.

The results from question 4 indicate that the intervention positively affected learners' performance in calculating heights and distances when presented with three-dimensional shapes. The p-values in question 4.1.2 (0.0001), 4.1.3 (< 0.0001), 4.2(0.0013), 4.3 (0.0001) and 4.4 (< 0.0001) indicating that there was significant difference as the p-values were above the nominal 0.05. Except for 4.1.1 $(p - value = 0.669)$, in this case, the p-value was not below the nominal 0.05, indicating no significant difference between the pre-and post-test observations. In other words, the intervention in the experimental group did not significantly alter how learners understood the correct way of using the sine rule. In question 4.1.1, most learners were able to find the sum of angles in the triangles, which is basic knowledge from Grade 8 and 9 work. Of those learners who got the first step correct, they also acquired marks by using a correct sine rule formulae, but they could not get the calculation correctly in the pre-test; hence, they could not get a total in question 4.1.1. In conclusion, the intervention improved the performance of the experimental group's learners since most of the question items showed a positive difference between the pre-test and post-test.

6.3 SUMMARY OF THE PRE-TEST AND POST-TEST RESULTS

The summary of the pre-test and post-test was created using the rank sums from the experimental and control groups. It is shown in the table below.

TABLE 6.1: SUMMARY OF THE PRE-TEST RESULTS

Two-sample Wilcoxon rank sum (Mannâ€“Whitney) test

Group	Obs	Rank sum	Expected
CONTROL	30	1058.5	915
EXP	30	771.5	915
Combined	60	1830	1830

$p - value$ = 0.0332

Table 6.1 shows the 95% confidence limit in the pre-test. This indicates that the control group had conceptual knowledge of heights and distances in trigonometry and was able to use that knowledge to answer questions correctly in the pre-test. On the other hand, the experimental group did not perform well in the pre-test compared to the control group.

TABLE 6.2: SUMMARY OF THE POST-TEST RESULTS

Two-sample Wilcoxon rank sum (Mannâ€“Whitney) test

Group	Obs	Rank sum	Expected
CONTROL	30	565.5	915
EXP	30	1264.5	915
Combined	60	1830	1830

$p - value$ = 0.0000

Table 6.2 indicates that the experimental group ($rank - sum = 1264.5$) outperformed the control group ($rank - sum = 565.5$) in the post-test. There was a significant difference in the performance of these groups ($p - value = 0.0000$) 95% confidence limits. The rank sum of the experimental group improved in the post-test compared to that of the pre-test, suggesting that the use of visualisation in teaching heights and distances positively affected learners' performance in the post-test. The experimental group gained more conceptual knowledge from the intervention.

6.4 QUALITATIVE FINDINGS

This section discusses the findings from the lesson observations and semi-structured interviews conducted with experimental and control group teachers. The lesson observations were used to establish whether the intervention impacted how learners and teachers interact with one another and impacted learners' performance. Furthermore, the interviews were used to understand how teachers taught heights and distances before the intervention and their perspective on visualisation when teaching heights and distances.

6.4.1 Lesson observations

The lesson observations were gathered before and after the intervention. The baseline lesson observations were conducted in the experimental school to understand how the teacher taught heights and distances and how learners responded to those teaching methodologies. The strategies used by the teacher during the baseline observation indicated that the lessons were teacher centred since learners were perceived as passive information consumers. Yet, teachers serve as information providers or evaluators, keeping an eye on them to ensure they get the answers. Hence, the lessons observed were teacher centred; learners were there to collect information and were not given a chance to discuss the topic or link it with their everyday experiences. This suggests that learners were not given an opportunity to provide their prior knowledge and link it with the new information provided by the teacher. Mayer and Rothermel (2023) highlighted that in subjects like science and mathematics, a teacher centred approach could be beneficial for introducing complex concepts as it allows for clear, structured explanations. Nguyen (2023) also found that while teacher centred approaches are still widely used, they tend to be less effective in fostering long term retention and critical thinking, while learner centred approaches lead to better learner engagement and academic achievement.

The results during the baseline observation revealed that learners were not given an opportunity to interact with one another and discuss the activities they were given. Learners were not exposed to working in groups to learn from one another. Abramczyk and Jurkowski

(2020) state that group work provides learners with an opportunity to tackle complex mathematics problems collaboratively and allows learners to share problem solving strategies, receive feedback from peers and explore alternative methods. Working in groups gives learners a platform to express their mathematics thinking and build their self confidence in mathematics classrooms. The teacher instructed learners to write their activities individually, which may helped other learners to process and internalise what they have learnt. However, underperforming and underperforming learners were disadvantaged because the teacher only paid attention to participating learners during the lessons. During baseline observation, the teacher focused mainly on learners who were participating actively in class, and the under-achieving learners were not receiving any attention, which may have demotivated them to try participating in class.

Mid-intervention findings

Learners who were not actively involved in the learning process during the baseline observation were now encouraged to participate and share their thoughts with peers through group work. This suggests that the intervention encouraged the learner centred learning environment.

Post-intervention findings

The findings after the intervention revealed that visualisation allowed learners to actively participate during the lessons and share ideas as they were working in small groups. What they were seeing

As lessons became more learner centred, learners were given the opportunity to answer the questions posed by the teacher and were encouraged to share their prior knowledge. According to Mayer (2005), the premise of cognitive processing asserts that learning should involve human participation rather than passive absorption of information. This involves identifying and selecting relevant material, organising it into visual and verbal models, and integrating those new models with prior knowledge. Learners were provided with the chance to work in groups, which is a crucial component of active learning in educational settings and aligns with Mayer’s cognitive theory of multimedia learning. The teacher ensured that group members discussed and agreed on the essential elements before they began their multimedia project of designing three-dimensional shapes. While learners engaged in discussions within their small groups, the teacher moved around the desks to check whether they were completing the activities and ensured that all learners were accommodated in their allocated groups.

The collected data before, during, and after the intervention strategy was envisioned to respond to the following research objectives:

- What types of teaching methods do teachers use when teaching heights and distances?
- What are the benefits and challenges (if any) of visualisation in the teaching and learning of trigonometry?
- What are possible ways of implementing effective visualisation in the teaching and learning of heights and distances to improve learner performance?

The before and during the intervention lesson observations are discussed below under the following themes.

6.4.2 Teaching strategies and learner participation

Observations before the intervention showed that learner engagement varied significantly as the lessons were mainly teacher centred. The teacher's role was central to the learning process; he delivered lessons and provided instruction to learners while learners were perceived as receivers of information from the teacher. While few learners actively participated in questioning and taking notes of what the teacher was explaining, many appeared disengaged during the lessons. They seemed distracted and disinterested during long periods of direct instruction. Learners were not motivated to share their thoughts and ideas during the lesson, and the teacher only paid attention to those learners who were actively involved in the learning process. Learners were not given an opportunity to share ideas as they were instructed to work individually. The classroom was organised in rows facing the front where the teacher delivered content.

Observations during the intervention with the experimental group revealed a significant increase in learner participation, particularly during collaborative activities where learners worked in groups. The lessons were primarily learner centred, and the teacher was present to guide the learners. The teacher's role shifted notably from that of a content expert to that of a facilitator. Rather than providing direct answers, the teacher guided learners through questioning, offered resources, and provided feedback on their progress. This teaching approach allowed for more personalised support, albeit requiring a higher level of preparation and flexibility. Learners appeared more motivated to engage in group discussions and were eager to share their thoughts and findings with their peers. Consequently, learners' academic performance improved in both group activities and individual assessments (post-test). Learners demonstrated a deeper understanding of the application of trigonometric ratios, Pythagoras' theorem, the sine rule, and the cosine rule in three-dimensional shapes.

6.4.3 Classroom interaction

Hattie (2023) states classroom interactions are dynamic and can impact learner learning and engagement. Prior to the intervention, the classroom interaction was characterised by teacher centred communication. The teacher frequently led discussions, asked questions, and provided answers to learners. Learners were not encouraged to lead classroom discussions, resulting in limited input, especially from less confident learners. Classroom interactions were predominantly structured, with the teacher promoting rote memorisation. During the intervention, learners began to take more initiative in discussions, asking questions and offering answers. The teacher also encouraged participation by incorporating open-ended questions that facilitated critical thinking and exploration. Peer-to-peer interaction became more common as learners worked in small groups, fostering peer feedback.

The classroom atmosphere before the intervention was quiet and passive, with few opportunities for spontaneous discussion. Most learners were hesitant to engage and ask questions during the lessons. The teacher focused mainly on compliance and task completion rather than collaboration and critical thinking. During the intervention, the classroom atmosphere was more dynamic and interactive. Learners encouraged one another and built on each other’s ideas. Their focus was on exploration and discovery rather than rote learning.

6.4.4 Teaching and learning resources

When analysing teaching and learning resources before and after the intervention, the researcher looks at how the materials and tools used in the classroom evolved due to changes in teaching methods or strategies (Ong and Quek, 2023). The intervention involved a new teaching method in which learners worked collaboratively to create three-dimensional shapes that they used to visualise heights and distances in trigonometry.

Before the intervention in the experimental group, no object for the demonstration was used; the teacher relied on the Grade 12 mind action series textbook for questions, and a few previous question papers were used, for example, for class work and homework. These textbooks were the primary resource of content and instruction. Learners shared textbooks in groups of three and four; the findings indicate that the school had a shortage of these textbooks. The lesson was primarily based on the use of blackboard and chalk. The chalkboard was mainly used for writing down key concepts and diagrams during the lessons. The classroom environment had no posters, pictures or visuals to demonstrate how to calculate heights and distances. The experimental school had limited technology; hence, the lessons could not incorporate laptops for demonstration. Varaidzai et al., (2020) reported that the issue of low academic achievement in mathematics is worsened by a lack of valuable resources for math teachers to utilise. Varaidzai et al., (2020) discovered that 60% of the learners they interviewed said the lack of suitable textbooks in their schools was why they did poorly in mathematics. Similarly Cassibba et al., (2020), pointed out that teachers must have resources and that learners and teachers should reference a range of textbooks because they offer a variety of viewpoints.

During and after the intervention, teaching resources will likely become increasingly diverse, interactive, and learner centred to foster engagement, collaboration, and active learning (McTigue et al, 2024). The experimental teacher incorporated visualisation using cardboard to create three-dimensional shapes that complemented his usual teaching materials, such as textbooks and the blackboard. He presented real-world problems through the three-dimensional models created by the learners; for instance, they calculated the height of the model using the angle of elevation. Learners were also encouraged to draw and visualise how they would approach the questions. This method bridged the gap between theoretical knowledge and practical application. The findings confirmed that the effectiveness of using visualisation in heights and distances in trigonometry lies in its ability to enhance conceptual understanding, increase learner engagement, and improve problem solving skills. Blanco, Camargo, and Sequeiros (2024), confirm that visual tools, whether through simple diagrams or advanced technology, support learners in better grasping the geometrical relationships involved in these problems. By providing clear, intuitive representations of abstract concepts, visualisation reduces mistakes and misconceptions, making it an indispensable component of teaching trigonometry (Mkhwanazi et al., 2023).

6.4.5 Classroom assessment

Assessing learners effectively is crucial for understanding their progress, identifying their strengths and areas for improvement, and informing instructional decisions (Granberg et al., 2021). The data collected during classroom observation before the intervention with the experimental group confirms that the teacher gave learners classwork and homework each day. His challenge was that most learners did not write homework, so he spent a lot of time checking the homework and trying to discipline those who did not.

The teacher of the experimental group did not allocate sufficient time for providing feedback to learners. In class, the teacher did not implement corrections effectively, and no discussion of answers took place. Instead, the teacher merely provided answers, which did not benefit the learners. The teacher failed to utilise the feedback session to understand the difficulties faced by learners in solving the assigned problems. Moreover, the teacher struggled to keep track of his learners' progress in their mathematics learning and their problem solving capabilities. He used the chalkboard to present answers and explanations.

During the intervention, the teacher continued using homework and classwork as assessment forms. The difference was that, since learners were working in pairs, they were encouraged to share ideas, utilise peer feedback, and compare their answers. Consequently, learners were motivated to communicate with one another and the teacher, and they could ask questions if they needed clarity. The teacher also incorporated pre-assessment through questioning to understand and identify learners' existing knowledge, skills, and gaps. While learners were discussing questions, the teacher employed observational assessment, observing learners during the lessons to assess their behaviour, skills, and understanding. The main reason for conducting observational assessment was to observe how learners collaborate during group work and to monitor learners' participation in discussions or their approach to problem solving.

6.5 SEMI-STRUCTURED INTERVIEWS WITH TEACHERS

As mentioned in Chapter Four's methodology, the semi-structured interviews used to collect data from teachers allowed the interviewer to clarify and rephrase the questions for the teachers. The semi-structured interviews were also used to determine the significance of the two teachers' teaching methodologies prior to the intervention and the reasons behind the differences in mathematical performance between the experimental and control groups. The two teachers were interviewed prior to the interventions, and the experimental group teacher was interviewed after the intervention.

The purpose of the post-intervention interview sessions was to assess the impact of the use of visualisation in the experimental group and to address the following study objectives.

- To understand the benefits and challenges (if any) of visualization in the teaching and learning of heights and distances
- To suggest possible ways of implementing effective visualization in the teaching and learning of height and distances to improve learner performance.

The following themes emerged from the semi-structured interviews.

6.5.1 Grade 12 learners' performances in heights and distances trigonometry

Poor performance in heights and distances can be attributed to several conceptual and instructional factors (Mkhwanazi et al., 2023). The data collected during semi-structured interviews with the teachers of both the experimental and control groups indicated that learners struggled with trigonometric ratios (sine, cosine, and tangent). The experimental teacher explained that learners found it difficult to apply these ratios, essential for solving problems related to heights and distances. This aligns with the findings from the pre-test, where learners who struggled with trigonometric ratios exhibited no understanding of the properties of right-angled triangles or the relationship between angles and sides.

The experimental teacher explained that learners struggled to visualise problems before the intervention. Solving heights and distances requires learners to visualise or draw triangles based on word problems or real life scenarios. Consequently, learners who struggle with spatial reasoning or translating word problems into diagrams find identifying the relevant sides and angles challenging. One of the most common mistakes that learners made was confusing the angles of elevation and depression, which led to incorrect equation setting and solving for unknown variables. Since heights and distances necessitate setting up equations, the findings from semi-structured interviews revealed that learners lacked strategies for problem solving; hence, they made errors in their calculations. The experimental teacher noted that questions involving heights and distances, especially when learners are presented with three-dimensional shapes, entail multiple steps, such as applying trigonometric ratios, using the Pythagorean theorem, and working with various geometric concepts. Mayer (2005) reported that this resulted in cognitive overload, where learners felt overwhelmed by the complexity, leading to forgetting key steps.

6.5.2 The learners' knowledge of heights and distances trigonometry

The experimental group's lack of knowledge concerning heights and distances indicated that learners struggle with conceptual understanding (Blanco et al., 2024). The teacher from the experimental group observed that the learners displayed a lack of conceptual understanding, as they were not actively participating before the intervention. This aligns with findings from classroom observations, which suggested that learners were listening to the teacher. When the teacher posed questions related to trigonometric concepts, the learners failed to provide answers; consequently, the teacher repeatedly supplied answers.

There was a positive change during the intervention as learners actively participated and shared ideas. The teacher's strategy supported learners in gaining a conceptual understanding of heights and distances. According to Mkhwanazi et al. (2023), learners' knowledge of heights and distances is built upon an understanding of trigonometric ratios, the ability to visualise and solve trigonometric problems, and the application of the concepts to real life situations. Therefore, learners' understanding and performance can be enhanced by focusing on foundational skills, improving problem solving abilities, and employing visualisation techniques. During and after the intervention, the experimental group's engagement increased, suggesting that the intervention was successful in assisting learners to acquire a conceptual understanding of heights and distances.

6.5.3 Learner support in height and distance trigonometry

The findings from the semi-structured interview with the experimental teacher, supported by classroom observation, confirm that the teacher of the experimental group struggled to guide and motivate learners who could not provide correct answers to questions. Teachers should be adept at designing an inquiry-based mathematics classroom and promoting active participation in mathematics instruction (Sambo, 2022). When it came to supporting or accommodating each learner's unique learning style during instruction, teachers from the experimental group appeared to face difficulties.

Data collected from semi-structured interviews reveals that the teacher preferred learners to work individually, which could hinder them from obtaining information or assistance from their peers. The control group teacher explained, "*The effective method is individual practice, where learners work individually to complete classwork. This gives me a chance to oversee and discuss the concepts with learners one-on-one.*" This method did not work for the experimental group, as they were demotivated and did not participate when questioned.

Data from semi-structured interviews with the experimental teacher after the intervention revealed that the teacher successfully applied the use of visualisation, as learners were given the opportunity to share ideas in small groups and were encouraged to present their answers on the chalkboard to receive peer feedback. The teacher also mentioned that he supported learners by moving around the groups to check whether they were answering the questions correctly, and he guided them as they engaged with the questions.

6.5.4 Pedagogical content knowledge

Pedagogical content knowledge (PCK) refers to teachers' understanding and teaching of subject matter. It involves how teachers present and explain concepts in a manner that is beneficial and meaningful to learners (Sakaria et al., 2023). Data from baseline semi-structured interviews reveals a limited use of active teaching strategies prior to the intervention. The experimental teacher noted that he employed a lecture style where learners passively received information, resulting in minimal participation and a lack of development in the problem solving skills necessary for mastering heights and distances. Classroom observations confirmed that lessons devoid of hands-on activities and interactive tools to help deepen learners' understanding. Additionally, the teacher struggled to provide learners with feedback; when mistakes were made, he did not promptly correct them, as he focused on those learners who were actively engaged during the teaching and learning process. Furthermore, the lessons were characterised by a minimal connection to the real world; heights and distances were taught as abstract mathematical subjects without sufficiently linking them to real-world applications, which could make the content feel relevant and engaging for learners.

During the implementation of the use of visualisation as an intervention, the teacher incorporated active learning where interactive, learner centred activities such as problem solving exercises, group work and peer feedback were used. These encouraged learners to engage deeply with the material and develop problem solving skills. The lessons incorporated hands-on learning, where hands-on activities such as real-world examples, interactive simulation, and visual tools and diagrams were used to help learners understand and visualise the relationship between sides and angles. The teacher's PCK was significantly enhanced after the intervention, resulting in more effective and engaging instruction in trigonometry in heights and distances. The intervention helped the teacher to move from a more traditional, content-focused approach to one that actively engages learners, addressing their misconceptions and making subject matter more relevant and accessible.

6.5.5 Teachers' efficacy

According to Perera and John (2020), teachers' efficacy refers to teachers' belief in their ability to impact learners' learning. It encompasses teachers' confidence in their teaching skills, ability to manage the classroom environment, and capacity to promote learner engagement. During the implementation of the intervention, the teacher was initially reluctant to adopt new teaching strategies. This demonstrated limited confidence in adopting the new strategy, which involved introducing visualisation to learners. The teacher was constantly concerned that visualisation would take time and learners would not respond positively. This indicated that the teacher had a limited belief in learners' potential for improvement. The teacher expressed frustration with learners' struggles when he stated, "*Engaging learners in a topic such as this can be challenging, as I have explained that learners in my class find this topic difficult. Even if you ask them simple questions, only a few learners raise their hands and respond; most remain silent".* Suggesting that the teacher felt ineffective due to learners' difficulties with key concepts and problem solving. This also implies that the teacher believed learners were not interested in the subject, particularly in abstract topics such as heights and distances. Consequently, this led to a sense of helplessness in fostering learners’ interest.

The teacher also expressed a lack of confidence in his classroom management: “*Even if I initiate group work, it becomes chaotic, so I do not often use that method. I prefer that they work individually when writing class work.”* This lack of confidence negatively affected his sense of efficacy and willingness to try new teaching strategies.

6.6 IMPLEMENTATION OF VISUALISATION AS AN INTERVENTION STRATEGY

The implementation of visualisation as an intervention strategy took the form of a continuous interactive learning process. The researcher started by consulting educational theories, such as Mayer’s cognitive theory of multimedia learning, to understand how visualisation can aid the understanding of heights and distances. The intervention resulted in two cycles, which ensured that every learner in the experimental group understood heights and distances through visualisation.

The researcher then developed an action plan where visual tools were introduced, such as interactive geometry software (Geogebra) and play tents and school gates for demonstration. During the first cycle, Geogebra was introduced as a form of visualization to learners to see if they could construct the 3D shapes and use them as visuals to answer trigonometric questions. Learners were randomly grouped in groups of five (5) because of the lack of laptops. The school had no computers, so the researcher and the teacher had to find six (6) laptops to use. Laptops, school gates, kids’ play tents, and previous question papers were used during the first cycle. The first cycle did not produce good results as not all learners were able to use laptops since they were sharing them in larger groups. The first cycle required a lot of time since learners had to be taught how to use Geogebra.

The second cycle was developed to enable all learners to participate and gain a conceptual understanding of heights and distances. It encouraged creativity, allowing learners to create their 3D shapes and use them as visual aids to answer questions from previous papers. This cycle fostered hands-on learning and motivated participants to engage actively, enhancing their understanding of trigonometric heights and distances. Learners worked in pairs rather than large groups, facilitating easier interaction with the constructed visual tool, which helped the teacher identify those needing additional support. The lessons became more learner centred; learners had the opportunity to respond to the teacher’s questions and share their prior knowledge. They communicated with one another, collaborating in pairs to solve the tasks at hand. The use of visuals during the intervention granted learners the autonomy to learn from each other, boosting their confidence and potential. In this second cycle, learners expressed their creativity by developing 3D shapes and employing them as visual aids to address questions from earlier papers. Participants learned by doing, as this cycle promoted active participation while enhancing their comprehension of trigonometric heights and distances.

6.7 ANSWERING THE RESEARCH QUESTION

As indicated in Chapter 1, this study aimed to answer the following primary research question:

What is the effectiveness of using visualisation on teaching and learning heights and distances in trigonometry to improve academic performance?

The secondary research questions were formed to answer the primary research question; they are answered in the following sections.

6.7.1 Pedagogical approaches

The data from classroom observations conducted prior to the intervention revealed that the experimental teacher did not utilise baseline assessment to evaluate learners' prior knowledge. The lessons were teacher centred; learners did not contribute any information during the teaching and learning process. They were expected to listen and absorb any information that the teacher provided. When learning is teacher centred, instructors play a crucial role in the process, claims Fatima (2022). Although learners are perceived as passive information consumers, teachers act as information producers or assessors, monitoring learners to ensure they comprehend the material. Thus, the observed lessons were teacher centred; learners were not given the opportunity to discuss the subject or relate it to their own experiences; instead, they were there to gather information. In addition to creating numerous examples on the chalkboard, the instructor employed a great deal of telling. He would guide learners through the process of answering questions step by step.

The experimental teacher discouraged learners from working in groups, as he mentioned during the semi-structured interviews that the class would become chaotic if he allowed learners to discuss. Hence, during the classroom observation, learners worked individually on writing classwork. This means that learners were deprived of an opportunity to share mathematical ideas with one another and learn from one another.

During the implementation of visualisation as an intervention strategy with the experimental group, teaching and learning took the form of action research. In the first cycle, objectives were set collaboratively by the researcher and the teacher; the first aim was to introduce group work where learners shared the visual tools (laptop), given the limited number of laptops available. Another goal of working in groups was to encourage learners to discuss the content and to make the lessons more learner centred. Laptops, the school gate, play tents, and previous question papers were utilized during the first cycle. Observational data revealed that some participants struggled to engage effectively due to the large group sizes and having only one laptop and play tent per group. The school gate hindered learners' ability to perceive all dimensions of the three-dimensional shape. It only allowed learners to identify right-angled triangles and the relationships between opposite, adjacent, and hypotenuse sides. The limited availability of laptops indicated that the researcher should integrate technology into the lessons provided the school has computers and learners are accustomed to using them to avoid spending excessive time teaching them how to navigate computers and to prevent grouping learners into large groups.

The first cycle was not as productive as planned by the teacher and researcher; therefore, the researcher held a workshop and discussion with the teacher in preparation for the second cycle. The researcher subsequently proposed using cardboard, allowing learners to create their 3D shapes and use them to calculate heights and distances. Thus, the second cycle focused on introducing visuals that would enable learners to work in smaller groups, utilizing readily available resources. The objective of the second cycle was to reduce the number of learners in groups, facilitate pair work, and ensure they had opportunities to gather information from the rest of the class.

The second cycle allowed learners to be creative by making their 3D shapes and using them as visuals to answer questions from previous papers. This cycle enabled participants to learn through hands-on experience. It motivated them to learn, engage actively, and enhance their understanding of heights and distances in trigonometry. Learners worked in pairs instead of large groups, which enabled the teacher to identify those needing more attention. The teacher could reach out to struggling learners, who were encouraged to share their solutions with the class and receive peer feedback after their presentations. Participants devised more effective demonstrations beyond those introduced by the teacher. One group volunteered to demonstrate using rulers and the floor. Participants successfully created additional practical demonstrations beyond the teacher's introduction.

6.7.2 Requirements for successful use of visualization as an intervention strategy

The data collected during the intervention revealed that using visualisation in the context of heights and distances as an intervention requires the researcher to be careful and considerate in their planning. Firstly, it is essential to ensure the learners' understanding of concepts; the teacher must verify that learners grasp basic trigonometric concepts such as sides, angles, sine, cosine, and tangent. During the intervention, the teacher made certain that terms such as angle of elevation and angle of depression were clearly defined, and he also provided real life examples to illustrate these terms.

Another requirement is for the teacher to incorporate a step-by-step breakdown in which three-dimensional problems are divided into smaller parts before teaching complex concepts. The lessons should begin with simple setups, focusing solely on the use of Pythagoras' theorem to minimise excessive workload in one lesson. The subsequent lessons should concentrate on trigonometric ratios, enabling learners to calculate heights, distances, and angles. The sine and cosine rules can then be taught on different days to ensure learners fully grasp these concepts. Both the teacher and learners should demonstrate the process of visualising the problem by setting up trigonometric equations and solving them step by step.

Creating their own visual tools is another requirement that helps learners understand heights and distances. The data collected during classroom observation revealed that a learner-centric approach is essential for the successful use of visualisation as an intervention strategy. In this study, learners were involved in creating their visual tools using cardboard and drawing diagrams. The teacher ensured that the diagrams provided to learners were clearly labelled with the data. The teacher also ensured that different learning styles were catered for. The teacher followed Mayer's (2005) theory of multimedia learning by employing dual channels. The teacher offered a variety of visual, verbal, and hands-on activities.

Another requirement for successful implementation is giving learners feedback so that their misconceptions are corrected. The data collected through observation during intervention revealed that feedback and assessment are vital in ensuring the successful implementation of visualisation. Learners should be provided with a variety of practice problems to test their understanding of trigonometric concepts and their application. As they worked, the teacher should move around the desks to review and discuss common errors, such as incorrectly applying trigonometric ratios.

To ensure the effective use of visualisation in understanding heights and distances, the teacher should encourage learners to communicate and discuss the content covered during the lesson. Collaborative learning was introduced throughout the intervention, which aided learners in exploring various visualisation techniques, as one group demonstrated using rulers and the floor.

6.7.3 Potential barriers

The use of visualisation in teaching and learning heights and distances can potentially face several barriers. Therefore, the researcher must identify and address these barriers. The first barrier identified during classroom observation is learners' lack of conceptual understanding. When learners lack basic trigonometric knowledge, it becomes difficult for them to connect with visual representations. Hence, it was important for the teacher to conduct preliminary lessons on basic trigonometric concepts before introducing visualisation to learners.

Another barrier observed was learners' poor visualisation skills. Some learners struggled to interpret and create visual representations using cardboard. The teacher had to use step-by-step guides, scaffolded information, and hands-on practice to develop visualisation skills. Another barrier was time constraints. Visualisation requires a lot of time, and teachers may not have enough time on the curriculum to focus intensely on it.

Since inadequate resources existed in an experimental school, limited access to laptops hindered the use of advanced visualisation, in this case, Geogebra. This extended time in the field meant that the teacher and researcher had to move to the second cycle. The solution to the problem was to use a no-cost alternative, where learners created handmade models and used them as visual tools.

6.7.4 Benefits of the use of visualisation in the grade 12 classroom

The use of visualisation in teaching concepts such as heights and distances offers numerous benefits to learners. These benefits are measured by the change in learner performance before and after the intervention.

6.7.4.1 Change in learner performance

The data gathered from pre-tests, post-tests, classroom observations, and semi-structured interviews with the teachers revealed a positive change in learner performance following the introduction of visualisation as an intervention strategy. The findings indicated that the use of visual representation helped learners enhance their conceptual understanding. Visualisation assisted learners in grasping mathematical concepts by making them more concrete and relatable. For example, when learners utilised visual tools, they could see the relationship between distances, heights, and angles, which facilitated comprehension of trigonometric principles.

The classroom observation during and after the intervention indicated improved learner participation within the experimental group. Prior to the intervention, only a few learners were engaging, with the teacher focusing solely on these individuals. Following the intervention, learners were encouraged to participate actively, demonstrating their interest in learning. The use of visualisation made learning dynamic and enjoyable. Learners were able to engage with one another and the teacher. The findings revealed that learners could explain their solutions and share them with the rest of the class. This fostered collaborative learning, as learners created visual tools in pairs and solved real-world problems, promoting teamwork and peer learning.

The study's results in Chapter 5 indicate that the experimental group did not perform well in the pre-test, as assessed using the Wilcoxon Rank Sum Test (p-value < 0.05), which revealed significantly different scores. However, after the intervention, the experimental group's rank sum scores (rank sum = 1264.5) significantly improved compared to the control group (rank sum = 565.5), indicating that the intervention positively affected the experimental group's understanding of heights and distances in trigonometry. The post-test improvement of the two study groups was evaluated using the rank sum score, which demonstrated that the experimental group excelled in most questions. In contrast, the control group achieved similar results across most questions.

6.7.4.2 Change in teaching practice

The data from classroom observations before and after the intervention revealed a noticeable change in the teaching practices of the experimental group’s teacher. Prior to the intervention, the teacher relied on traditional teaching methods, where explanations centred heavily on verbal and textual communication. The lessons were lecture-based, and problem solving was achieved solely through the use of formulae, without any visual aids. Following the intervention, the teacher employed interactive visual tools to enhance learners’ understanding and incorporated real-world examples with the aim of contextualising learning.

Before the intervention, the experimental teacher preferred that learners work individually. The lessons were teacher centred, with limited opportunities for learners to share input. The teacher provided minimal feedback on individual learning challenges. After the intervention, the classroom was interactive, with learners actively participating in creating and interpreting visual tools. As learners were working in pairs, the teacher provided personalised feedback.

Furthermore, the intervention equipped the teacher with the knowledge of how to use questioning to assess learners' prior knowledge. The teacher also employed prompt questions to explore learners' thinking as they responded to questions about heights and distances. The questioning style of the experimental group's teacher also improved as he shifted his focus from obtaining correct answers to understanding learners' prior knowledge and addressing their misconceptions.

Due to a textbook shortage, the teacher used previous question papers, as there were sufficient copies for the learners. Rather than grouping them into large pairs to share a textbook, the teacher made copies of the relevant pages he intended to use and relied on the previous question papers.

6.7.4.3 Developing effective visual models

The results of using visualisation to teach heights and distances indicate that learners should be actively involved in the learning process. Lessons should be designed in a learner-centred manner. Teachers should not merely observe; they should monitor the learning process to ensure that learners are fulfilling their responsibilities and provide feedback.

Developing effective visual models requires the researcher and the teacher to define what learners should learn. For example, when the teacher was designing lesson plans, he mentioned that learners should understand angles of elevation and depression, know how to apply trigonometric ratios and solve real life problems. The visual models should be aligned with the lesson objectives.

Visual models should be accompanied by verbal explanations. Mayer (2005) defines multimedia learning as forming mental images through spoken or written materials and visuals such as drawings and photographs. To facilitate understanding of multimedia learning, the teacher paired visual models with step-by-step verbal explanations to accommodate different learning styles. To effectively address all learning styles, the teacher needs to be aware of the learners' strengths and weaknesses, their learning levels, and how they learn (Sambo, 2022). Learners were also continually encouraged to articulate what they observed in their models to reinforce understanding.

Developing effective visual models requires teachers to utilise appropriate tools and resources. The findings from the first cycle of implementing visualisation revealed that the visual tools used should consist of the materials available in the school and be sufficient for every participant. Suppose the teacher uses software such as Geogebra. In that case, they should ensure that there are enough laptops or computers for learners to use and that learners are computer literate to avoid spending excessive time teaching them how to operate computers. The second cycle of this study focused on using physical models, where learners created tangible aids such as cardboard triangles and came up with the idea of using rulers for demonstration. This suggests that when learners create visual models using accessible materials, they can become more creative.

6.8 DEFICIENCY IN MAYER'S COGNITIVE THEORY OF MULTIMEDIA LEARNING

Mayer's cognitive theory of multimedia learning (CTML) was the theoretical framework underpinning this study. While Mayer's theory is a well established model for effective multimedia educational tools, it also has deficiencies and limitations (Sweller, 2020). Clark et al., (2023) state that the theory offers limited consideration of learner variability. CTML overlooks the differences among learners and does not account for their prior knowledge, learning styles, and cognitive abilities. Ignoring these factors may result in a less effective learning experience for a diverse classroom.

CTML focuses on dual channels where the teacher must utilise visual and auditory models to explain the content being learned through multimedia. According to Sweller (2020), this theory overlooks the role of other sensory modalities, such as kinaesthetic or tactile learning, where learners best understand and absorb information through hands-on experiences, physical activities, and movement. While the emphasis is on dual channels, CTML lacks guidance on how learners should be assessed to determine whether the dual channels are effective. Consequently, the teacher may struggle to evaluate whether the use of multimedia has met the learning objectives.

The CTML emphasises the use of technology, which may not be available or accessible in all educational settings. The multimedia tools highlighted in this theory include presentation software, interactive simulation tools, and video and animation platforms. Learners in resource-limited schools, such as the experimental group, did not fully benefit from these multimedia tools. Consequently, this study involved two cycles: the first cycle followed the tool suggested by CTML (Geogebra), but due to the limited number of laptops and the learners' exposure to technology, the second cycle had to incorporate resources that are accessible to learners.

According to Mayer (2022), CTML inadequately considers emotional and motivational factors. The theory overlooks the impact of motivation and engagement on the learning process. Therefore, if a teacher designs lessons solely based on CTML, they may struggle to engage or inspire learners during the teaching and learning process.

6.9 CONCLUSION

This chapter discusses the quantitative and qualitative data generated by this study. The pre-test and post-test findings indicated that the control group performed better before the intervention than the experimental group in most of the questions. Learners in the experimental group struggled primarily with the application of the cosine and sine rules. Although most learners were familiar with the Pythagorean theorem and trigonometric ratios, they faced challenges when applying these concepts to solve problems. After the intervention, the experimental group outperformed the control group in all the questions set, indicating that the use of visualisation positively impacted learners' understanding of heights and distances in trigonometry.

The qualitative data generated indicated that the classroom observations revealed that before the intervention, the lessons were teacher centred, with learners merely absorbing information and not fully participating during the lessons. After the intervention, there was an improvement in learner participation; learners began to ask questions for clarity, worked in pairs to discuss the content, interacted with one another, and presented their findings to the whole class to receive feedback from both the teacher and their peers. Before the intervention, learners were discouraged from communicating with each other, as the experimental teacher explained during the interview that they created noise when discussing, which made him comfortable with them working individually. This strategy was not beneficial for learners, as struggling learners were deprived of the opportunity to ask questions or seek clarity from their peers.

CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

7.1 INTRODUCTION

According to Mayer (2020), visualisation in education refers to using visual tools to represent information, making both concrete and abstract concepts easier to understand. These visual tools may include pictures, diagrams, and animations. The use of visualisation during the intervention provided clarity to learners and simplified complex concepts (Blanco et al., 2024). It also improved learner engagement; learners were motivated and focused when information was presented in an appealing format. Additionally, visualisation aided in retention, as the visual nature of the content enhanced learners' memory recall.

This study introduced visualisation in teaching heights and distances to investigate its impact on learner performance. It intended to change the teaching practices of the experimental group's teacher, who used a traditional teaching style. The study was conducted in two schools, the experimental and control groups, to compare the effectiveness of teaching heights and distances using visualisation and compare learners' performance before and after the intervention.

The results from the pre-test and post-test indicated that learners in the experimental and control groups performed differently. The test analysis indicates that the control group outperformed the experimental group before the intervention. After the intervention, the experimental group outperformed the control group, indicating that the intervention positively impacted learners' performance. This chapter concludes the benefits of visualisation by reflecting on the rationale and research design. It also discusses this study's limitations and recommends further research studies. However, drawing participants from a high performing and an underperforming school introduced contextual differences that complicated thematic coding and cross group comparison. The researcher acknowledges this limitation hence, future research will use of random sampling to prevent similar non-equivalence and to strengthen the internal validity of qualitative comparison.

7.2 IMPORTANCE AND DESIGN OF THE STUDY

Samosa et al., (2021) define visualisation as a learning method that is a crucial component of representation, facilitating comprehension, memory, recall, and problem solving. Teaching through visualisation requires the teacher to utilise visual tools that learners can reference while solving problems. Previous studies have researched the use of visualisation in teaching heights and distances. Bekene Bedada and Machaba (2022) investigated the effectiveness of Geogebra in Grade 12 trigonometry. Their study involved Science, Technology, Engineering, and Mathematics learners from various Hadiya Zone schools. These learners were exposed to technology, as the schools had computers to use. The results indicated that the use of Geogebra positively impacted learners' performance. This methodology was found to be effective only for learners with access to technology; thus, those in rural schools with limited or no access to technology, such as during the first cycle of this study, did not benefit from Geogebra. Consequently, this study was designed to introduce visualisation using recyclable materials that learners can utilise to visualise heights and distances in trigonometry within the experimental group. Furthermore, the study aimed to investigate the effectiveness of visualisation at the learner level by employing pre-tests.

A pre-test was administered to investigate Grade 12 learners' performance in heights and distances with both experimental and control groups prior to the intervention. The pre-test revealed that the control group outperformed the experimental group in most questions. Interviews were conducted with two teachers from the experimental and control schools to understand the difficulties learners face when solving problems in heights and distances in trigonometry, as well as the teaching methodologies employed by the teachers before the intervention. Classroom observations were conducted in the experimental group both before and during the intervention.

7.3 MAIN FINDINGS

The results from the pre-test revealed that the experimental group performed poorly compared to the control group. The statistical analysis in Chapter 6 indicated a significant difference in the performance of these two study groups. The pre-test results indicated that learners in the experimental group struggled with CAST diagrams, trigonometric ratios, and the use of sine and cosine rules. These challenges were addressed through the use of visualisation as an intervention. Conversely, the results from the post-test revealed that a significant difference occurred, as the experimental group showed improvement in performance and outperformed the control group. This improvement suggests that the use of visualisation in heights and distances had a positive effect on learners' performance. The changes in the experimental teacher's teaching practices and the learners' exposure to a new approach to teaching heights and distances using cardboards resulted in improvement.

Data gathered prior to the intervention suggests that the teaching strategies employed by the experimental teacher were ineffective, as learners displayed a lack of mathematical knowledge in trigonometry. Learners demonstrate their mathematical understanding by applying logical reasoning, solving mathematical problems, and selecting appropriate solving strategies (LibreTexts, 2023). Before the intervention, learners were unable to communicate their reasoning and discuss their solutions because they were instructed to work individually. Furthermore, learners did not engage in self-assessment to reflect on their mathematical knowledge. During the classroom observations, the lessons were teacher centred; the teacher acted as a narrator while learners merely accumulated information. Participation was minimal, with few learners able to answer the questions posed by the teacher. The teacher failed to consider the learners' prior knowledge, as no baseline assessments or questions were posed to evaluate their mathematical knowledge from previous grades.

The results from the post-test revealed that there was an improvement in the experimental group from question one to question four. The rank sum for all questions increased, indicating that the intervention had a positive effect on learners' performance. The p-value comparing the pre-test and post-test of the experimental group was found to be < 0.0001, suggesting a significant difference in learners' performance across all questions in the post-test. To support this claim, the rank sum total score also improved, showing 771.5 in the pre-test and 1264.5 in the post-test. Consequently, the rank sum total difference was calculated to be 493, resulting from the use of visualisation as an intervention.

During the intervention, the use of accessible resources, such as cardboard to create three-dimensional shapes, textbooks, and previous examination papers, led to positive improvements in learners' understanding of height and distance. According to Sambo (2022), the availability of teaching and learning resources, along with their appropriate application, results in positive learning outcomes, thereby promoting improvements in learners' mathematics performance. The results from this study suggest that the resources used when teaching height and distance should be relevant to the learners' environment. If the school has computers and learners are familiar with them, the teacher may incorporate technology into the lessons. However, if the school is disadvantaged, the teacher should devise creative ways to visualise the problems using available resources.

The qualitative data collected during the intervention revealed improved learner participation when visualisation was implemented, compared to their participation with the traditional teaching approach. Stanciuscu et al., (2022) indicates that visualisation tools positively impact learner participation. They further found that learner engagement and learning outcomes are enhanced when teachers employ group problem-based learning activities. Similarly, Al-Murtadha (2019) found that using visual tools motivates learners; consequently, they participate more actively in a collaborative learning environment. These findings align with those from this research; as learners were working individually prior to the intervention, there was no participation. Learners were merely observing the teacher and following instructions. The teacher then proposed group work during the first cycle due to a lack of learning resources. Subsequently, learner participation was observed as they began to discuss content and assist one another. However, some learners still did not actively participate because they were working in large groups and had insufficient resources. As the study progressed to the second cycle, learner participation became more evident; the learners worked in pairs and engaged hands-on while creating their three-dimensional shapes. They began to interact with the teacher, asking questions for clarity. They were also given the opportunity to demonstrate their solutions on the chalkboard and received feedback from both the teacher and their peers.

The results of this study suggest that the study's objectives were achieved. The objectives were to explore teachers' strategies for teaching heights and distances in trigonometry. This goal was accomplished through data collected from classroom observations prior to the intervention. The methodology employed by the teacher was traditional, with lessons being teacher centred and learners receiving the information provided. Another objective was to understand the benefits and challenges, if any, of using visualisation in the teaching and learning of heights and distances. Data collected through pre-tests and post-tests indicated that the use of visualisation benefited learners, as their performance in heights and distances improved following the intervention. The results also suggested that when using visualisation, the teacher should ensure that the visual tools employed are relevant and familiar to the learners to avoid spending unnecessary time acclimatising them. The final objective was to propose potential methods for implementing effective visualisation in the teaching and learning of heights and distances to enhance learner performance.

The key elements for the effective implementation of visualisation in teaching heights and distances were identified as follows: 1) the use of visualisation for heights and distances requires sufficient time; 2) visual tools used should be adaptable to cater to different learning styles, and these tools should be accessible to learners; 3) visual tools must be clear, concise, and easy to understand for learners; 4) the use of visualisation should complement other teaching methodologies, such as group discussions; 5) learners' feedback on the effectiveness of the visual tools should be taken into consideration for future implementation.

Other logistical issues may disrupt the execution of the intervention, such as activities and academic programmes (Makgakga, 2016). It is vital that the researcher notes these issues, as they may minimise the benefits of using visualisation for learners. A well-thought-out plan should be established to overcome these logistical issues so that learners and participating teachers benefit from the study.

7.4 THE RESEARCHER’S VOICE AND THEORETICAL FRAME

The study was underpinned by Mayer’s cognitive theory of multimedia learning. The results obtained from this study provided insights into how visualisation may be implemented to achieve positive outcomes. The theory suggested that visualisation could be employed through the use of dual channels, where both visual and verbal channels are essential. The results from the qualitative data indicated that the use of visualisation benefited the experimental teacher by introducing him to a new teaching methodology, thus enhancing learner participation and performance.

The experimental group demonstrated significant improvement in the post-test compared to the pre-test, indicating that visualisation in understanding heights and distances enhanced learner performance. However, for the teacher to successfully implement visualisation, they must consider the resources available in the school and the learners' ability to utilise them. Involvement during teaching and learning is crucial to motivate and encourage participation among learners. Therefore, the lessons should be learner centred. Additionally, collaboration among learners should be fostered, as visualisation promotes group discussions that enhance participation and shared understanding.

One of the key elements of this study was the positive collaboration between the teacher in the experimental group and the researcher. There was mutual respect and understanding, indicating a strong partnership that facilitated the effective implementation of the strategies, contributing to the study’s success. For the use of visualisation to be successful, both parties shared a common goal, aligning with the study’s objectives, which ensured clarity and desired outcomes. Communication was open, featuring regular discussions to exchange ideas, progress, and feedback. The teacher also actively developed lesson plans, which helped ensure that the visualisation strategies discussed with the researcher were practical. The results suggested that using visualisation necessitates training and support for teachers. In this study, the researcher provided comprehensive training to the teacher on utilising visual tools to foster confidence during implementation. The findings revealed that visualisation can transform teachers’ methodologies, enhancing learners’ performance in measuring heights and distances.

Mayer’s (2005) cognitive theory of learning is guided by a combination of words and images to explain concepts. This study ensured the application of this dual channel, where the teacher used visual tools alongside verbal explanations to prevent overloading learners by presenting only essential information and minimising irrelevant elements. This approach is known as the coherence principle (Mayer, 2005). The results from this study reveal that if the researcher and the teacher adhere to CTML principles, visualisation tools can enhance learner engagement; they can also support memory retention by applying real life problems and improve overall learning outcomes.

7.5 PROPOSED EFFECTIVE USE OF VISUALISATION

The proposed effective use of visualization as an intervention strategy can be determined from the results of this study, the theoretical framework, and existing literature. Visualization is a powerful tool that enhances learners’ understanding of abstract concepts by transforming them into concrete visual representations (Mayer, 2021). The use of visualization can be used for various purposes, such as improving teachers’ methodologies to enhance learners’ performance in understanding heights and distances. Furthermore, the use of visualization fosters critical thinking, problem solving skills, and active learning, whereby learners take control of the learning process.

For this study, visualization was implemented in a rural school that lacks teaching resources, with the aim of using the school's available resources. The Curriculum and Assessment Policy Statement proposed using visualization in teaching mathematics (DBE, 2011). The (ibid) emphasizes the use of diagrams, models, and visual tools. CAPS states that visual models help learners understand scientific and mathematical concepts, particularly trigonometry and geometry.

To implement visualisation effectively in schools, teachers should encourage learners to create physical models of triangles using cardboard or similar materials to visualise the angle of elevation and depression. These physical models can help learners manipulate and better understand trigonometric relationships (Chatima, 2021). If the school has computers, teachers may use tools such as Geogebra, allowing learners to manipulate and observe changes in triangles, angles, and distances using such geometric software.

The findings from this study suggest that it is vital for the researcher and the experimental group’s teacher to choose the right visualisation tool. Visual tools used in teaching should be accessible to learners and must produce high-quality visualisations. The first cycle of this study indicated that when learners are presented with visual tools that they are unfamiliar with, it may hinder their participation effectively, consequently adversely affecting their performance in heights and distances. When learners are working with visual tools, they need to receive guidance. Consistent guidance helps learners stay on track and become aware of how to utilise the visual tool properly. The results from this study also suggest that learners should receive constructive feedback as they engage with the visual tools to solve mathematical problems. Positive and negative feedback are essential for supporting learners in their learning process.

7.6 LIMITATIONS OF THE STUDY

Although the results of this study indicate that the use of visualisation was effective in enhancing learners' performance, there are limitations regarding the nature of the results that must be acknowledged. A sample of thirty (30) learners and one teacher from an experimental school were employed in this study. While the sample size may have been adequate to address the research questions posed, the findings cannot be generalised to all Grade 12 learners and teachers in South Africa. The researcher confined the scope of this study by including only pre-tests, post-tests, semi-structured interviews, and observations instead of open-ended responses, which might have allowed some respondents to provide more detailed information. Furthermore, the research was restricted to heights and distances in trigonometry and did not explore other areas of mathematics.

A key limitation of this study is pre-test in-equivalence between the experimental group and control groups. This is resulted from drawing participants from two different schools where experimental school is an underperforming school for the past three years and control school is a high performing school. These pre-existing differences may be resulted from accesses to resources, quality of teaching and learning environment which are likely to contribute to variation in pre-test scores. Hence, this in-equivalence limits the generalizability of the results. The results after the intervention may not translate to settings where learners' populations and schools are performing on the same level. In future research, the researcher will aim to incorporate more rigorous randomization such as random sampling where where every individual in the target population had an equal chance of being chosen for the study (Stratton, 2021). These sampling procedures will enhance group equivalence and strengthen validity of the findings.

SUGGESTIONS FOR FUTURE RESEARCH

As the world moves towards the fourth industrial revolution, schools in both urban and rural areas must engage with technology-driven visualisation. Consequently, further studies are needed to explore and investigate the benefits of using technology-driven visualisation.

Future research may examine the impact of tools such as Desmos, Geogebra, and virtual reality on enhancing learners' understanding of heights and distances. Given that some teachers, including the experimental teacher, still favour traditional teaching styles, future studies could investigate the integration of technology-based visualisation tools with conventional teaching methods to create a blended learning experience for both teachers and learners. Additionally, future research may focus on accessibility and inclusion, exploring how technology can facilitate accessible visualisation for learners with disabilities to ensure equity in education.

Future research studies may investigate heights and distances across different populations, lower and upper grades, and both advantaged and disadvantaged schools to enhance the generalizability of the findings. The study may also be conducted in the long term to investigate the sustainable impact and changes over time. Spending long term time in the field may assist the researcher in observing changes in the teacher's teaching methods and learners' performance after the intervention.

7.7 CONCLUSION

This study investigated the benefits of using visualisation to teach heights and distances in trigonometry. The use of visualisation appeared to have a positive impact on both teachers' teaching strategies and learners' performance. The study demonstrated that integrating visual tools significantly enhanced learners' conceptual understanding and engagement. By employing cardboard to create three-dimensional shapes, learners were able to grasp abstract trigonometric concepts and apply them to solve problems from textbooks and previous examination papers. Hence, the results from this study supported the alternative hypothesis that the use of visualisation significantly affects learners' academic performance in heights and distances in trigonometry. The study revealed that the implementation of using visual tools that are accessible to learners positively impacted learners' performance.

The use of visualisation not only bridged the gap between theoretical knowledge and real life problems but also addressed diverse learning styles, particularly benefiting learners who excel with both visual and verbal channels. Given that South Africa still has schools lacking technology, this study should encourage teachers to adopt this strategy for teaching heights and distances. Additionally, the findings of the study should help school administrators, math teachers, and other educators better understand how visualisation might improve instruction in their classrooms. The outcomes should also grant the participating schools the advantages of this approach, improving the learners' overall academic performance.

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APPENDICES

APPENDIX A: APPLICATION FORM TO CONDUCT RESEARCH IN KWAZULU NATAL DEPARTMENT OF EDUCATION INSTITUTIONS

kzn education
Department:
Education
KWAZULU-NATAL

Application for Permission to Conduct Research in KwaZulu Natal Department of Education Institutions

1. Applicants Details

Title: Prof / Dr / Rev / Mr / Mrs / Miss   Surname: Nzuza

Name(s) Of Applicant(s): Philasande Zamokuhle   Email: zamonzuza1@gmail.com

Tel No: 0847979706   Fax: N/A   Cell: 0731752348

Postal Address: P.O BOX 746, NONGOMA, 3950

2. Proposed Research Title:
Visualization in Grade 12 Height and Distances in Trigonometry: A Case Study of Zululand District in Kwa-Zulu Natal Province

3. Have you applied for permission to conduct this research or any other research within the KZNDoE institutions?   Yes ~~No~~

If "yes", please state reference Number: ______________

4. Is the proposed research part of a tertiary qualification?   ~~Yes~~ No

If "yes"
Name of tertiary institution: UNIVERSITY OF SOUTH AFRICA

Faculty and or School: Unisa College of Education

Qualification: Masters in Mathematics Education

Name of Supervisor: DR TP MAKGAKGA   **Supervisors Signature** ______________

If "no", state purpose of research: ______________

5. Briefly state the Research Background

In South Africa, low learner achievement in mathematics is a problem; learners are not performing in both science and mathematics despite the 30% pass rate in both these subjects (Dean, 2019). A number of educational policies such as TIMSS and SACMEQ show that poor performance in mathematics is across all levels of schooling in South Africa, which means that learners struggle with mathematics from primary school level to higher education level (Nambeye and Samuel, 2019). In questions involving heights and distances (2D and 3D problem solving), in 2018, only 39.67% learners were able to answer the question Moreover, in 2019, 37.27% of learners were able to answer questions based on heights and distances trigonometry. DBE Moderators' Report (2019) further explained that many candidates skipped the question and others show a lack of concept of angle of elevation and angle of depression as well as their meaning. Madonsela, Ndlovu and Brijlall (2020) state that poor performance may either be caused by the pedagogical competencies of teachers or learners do not understand the benefits of studying 2D and 3D trigonometry, their historical usages or their applications in daily lives. According to Makgakga (2016), other contributing factors to poor performance include students' non-attendance, deprived entry marks, reduced assessment techniques as well as attitudes of learners towards mathematics. The authors further suggest that learning 2D and 3D may be achieved if the content is delivered in such a way that it relates to learners' lives and they are allowed to use it in their own experiences (Madonsela et al, 2020)

6. What is the main research question(s) : ____

Main Research question

What is the effectiveness of using visualization on teaching and learning heights and distances in trigonometry to improve academic performances?

Sub questions

- What types of teaching methodologies or strategies do teachers use when teaching heights and distances?
- What are the benefits and challenges (if any) of visualization in the teaching and learning of trigonometry?
- What are possible ways of implementing effective visualization in the teaching and learning of heights and distances to improve learner's performance?

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- What are the benefits and challenges (if any) of visualization in the teaching and learning of trigonometry?
- What are possible ways of implementing effective visualization in the teaching and learning of heights and distances to improve learner's performance?

15. Declaration

I hereby agree to comply with the relevant ethical conduct to ensure that participants' privacy and the confidentiality of records and other critical information.

I Philasande Zamokuhle Nzuza declare that the above information is

True and correct

15 march 2023

Signature of Applicant **Date**

16. Agreement to provide and to grant the KwaZulu Natal Department of Education the right to publish a summary of the report.

I/We agree to provide the KwaZulu Natal Department of Education with a copy of any report or dissertation written on the basis of information gained through the research activities described in this application.

I/We grant the KwaZulu Natal Department of Education the right to publish an edited summary of this report or dissertation using the print or electronic media.

15 march 2023

Signature of Applicant(s) **Date**

APPENDIX B: PERMISSION LETTER FROM THE HEAD OFFICE

KWAZU U NA AL C
CA ION
O L C OF SO

OFFICE OF THE HEAD OF DEPARTMENT

Private Bag X9137, PIETERMARITZBURG, 3200
Anton Lembede Building, 247 Burger Street, Pietermaritzburg, 3201
Tel: 033 392 1051

Em ail. Phindile.duma@kzndoe.gov.za

Enquiries. Mrs B.T. Ntuli Ref.:2/4/B/7443

Miss Philasande Zarrloktih1e N zu za
PLC]. lBox 74G
ON O

Dewar M iss Nz.uxo

PERMISSION TO CONDUCT RESEARCH JN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: "VISUALIZATION IN GRADE 12 HEIGHT AND DISTANCES IN TRIGONOMETRY: A CASE STUDY OF ZULULAND DISTRICT IN KWAZULU-NATAL PROVINCE.", in the KwaZuIu-Natal Department of Education institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not ident4iable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Pnncipals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 20*^ March 2023 to 31•° December 2025.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Mrs Buyi Ntuli at the contact numbers above.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full reporUdissertation/theSis must be submitted to the research office of the Department. Please address it to The OMce of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note th at your research and interviews will be limited to school s and institutions in KwaZulu-Natal Department of Education.

Mr GN Ngcobo
Head of Department: Education
Oate: 20'^ March 2022

GROWING KWAZULU-NATAL TOGETHER

APPENDIX C: ETHICS CLEARANCE CERTIFICATE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2022/11/09

Ref: **2022/11/09/63260840/39/AM**
Name: Ms PZ Nzuza
Student No.:63260840

Dear Ms PZ Nzuza

Decision: Ethics Approval from 2022/11/09 to 2025/11/09

Researcher(s): Name: Ms PZ Nzuza
E-mail address: 63260840@mylife.unisa.ac.za
Telephone: 073 175 2348

Supervisor(s): Name: Dr. Tšhegofatšo Makgakga
E-mail address: makgatp@unisa.ac.za
Telephone: 012 429 4293

Title of research:

Visualization in Grade 12 Height and Distances in Trigonometry: A Case Study of Zululand District in Kwa-Zulu Natal Province.

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2022/11/09 to 2025/11/09.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2022/11/09 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2025/11/09**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2022/11/09/63260840/39/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof Mpine Makoe
ACTING EXECUTIVE DEAN
qakisme@unisa.ac.za

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APPENDIX D: LETTER REQUESTING PERMISSION TO CONDUCT RESEARCH

Request for permission to conduct research at participating high schools
12 October 2022
Title: Visualization in Grade 12 Height and Distances in Trigonometry: A Case Study of Zululand District in Kwa-Zulu Natal Province.

Contact person
Philasande Zamokuhle Nzuza
46 Osborne road, Eshowe
0731752348
Zamonzuza1@gmail.com

Dear Sir and Grade 12 learners
I, Philasande Zamokuhle Nzuza, am doing research with Dr Makgakga, a senior lecturer in the Department of Mathematics towards M Ed. at the University of South Africa. We have funding from UNISA Masters & Doctoral Bursary for materials required to complete the research study. We are inviting you to participate in a study entitled Visualization in Grade 12 Height and Distances in Trigonometry: A Case Study of Zululand District in Kwa-Zulu Natal Province

The aim of the study is to explore the effectiveness of visualisation in teaching and learning of Grade 12 heights and distances to improve academic achievement. The focus will be based on how visualization in heights and distances improve learners' academic performances. The study will address how learners are currently taught in a focused school prior suggesting teaching trigonometry using visual models. The study also aimed at improving teachers' approaches in teaching heights and distances using visual models. The researcher will investigate the benefits and challenges of introducing visualisation in teaching and learning of heights and distances in trigonometry. Your institution has been selected because it falls under Zululand District and it is accessible to me as a researcher.

The study will entail observations where I will be observing how learners are currently being taught heights and distances by their teacher, after observation, I would like to conduct a pre-test to understand if the teaching strategies used were effective for learners' understanding. I will then conduct interviews to understand learners' responses given on pre-test. I would then introduce an intervention where learners will be exposed to visuals like cardboards, tents etc. we will use these visuals to calculate heights and distances. Post-test will then be conducted to understand if the intervention made a difference or not.

There are no foreseeable risks to learners by participating in the study. learners will receive no direct benefit from participating in the study; however, the possible benefits to education are the understanding of the application of trigonometry in real life problems. Neither learners or the school will receive any type of payment for participating in this study. I will provide a link to a website or social media page at the end of my study and post the findings on that website once the study has been completed. In this way, all the stakeholders will have access to the findings. I will summarize my findings in simple terms so that they can be understood by a non-academic audience.

Yours sincerely

Philasande Zamokuhle Nzuza
Educator/ Researcher

APPENDIX E: LETTER REQUESTING PARENTAL CONSENT FOR PARTICIPATION OF MINORS IN A RESEARCH PROJECT

Dear Parent

Your child is invited to participate in a study entitled Visualization in Grade 12 Height and Distances in Trigonometry: A Case Study of Zululand District in Kwa-Zulu Natal Province. I am undertaking this study as part of my master's research at the University of South Africa. The purpose of this study is to explore how learners are currently taught heights and distances in trigonometry and how those teaching strategies affect learners' performance. The study also aims at exploring the effect of visualisation in teaching and learning of grade 12 heights and distances and to determine how visualisation may affect learners' understanding and performance in solving heights and distances questions. The possible benefits of the study are the improvement of learners understanding of trigonometry. I am asking permission to include your child in this study. I expect to have 20 other children participating in the study.

If you allow your child to participate, I shall request him/her to:

- Take part in a survey
- Take part in an interview
- Complete a pre and post-test

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His or her responses will not be linked to his or her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are the understanding of the application of trigonometry in real life problems. Neither your child nor you will receive any type of payment for participating in this study.

Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities in the second term of 2023 school calendar with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available.

In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

If you have questions about this study please ask me or my study supervisor, Dr Makgakga, Department of mathematics, College of Education, University of South Africa. My contact number is 0731752348 and my e-mail is 63260840@mylife.unisa.ac.za. The e-mail of my supervisor is makgatp@unisa.ac.za. Permission for the study has already been given by the principal and the Ethics Committee of the College of Education, UNISA.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child: ....

Sincerely

______________________________ ______________________________

____

Parent/guardian's name (print) Parent/guardian's signature:

Date:

______________________________ ______________________________

____

Researcher's name (print) Researcher's signature

Date

APPENDIX F: A LETTER REQUESTING ASSENT FROM LEARNERS TO PARTICIPATE IN A RESEARCH

Title of study: Visualization in Grade 12 Height and Distances in Trigonometry: A Case Study of Zululand District in Kwa-Zulu Natal Province

Dear potential participant

I am doing a study on the effect of introducing Visualization in Grade 12 Height and Distances in Trigonometry as part of my studies at the University of South Africa. Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that teachers can use to teach height and distances effectively better. This will help you and many other learners of your age in different schools.

This letter is to explain to you what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study. I would like to observe how you are currently being taught heights and distances by your teacher, after observation, I would like to conduct a pre-test to understand if the teaching strategies used were effective for your understanding. I will then conduct interviews to understand your responses given on pre-test. I would then introduce an intervention where you will be exposed to visuals like cardboards, tents etc. we will use these visuals to calculate heights and distances. Post-test will then be conducted to understand if the intervention made a difference or not.

I will write a report on the study but I will not use your name in the report or say anything that will let other people know who you are. You do not have to be part of this study if you do not want to take part. If you choose to be in the study, you may stop taking part at any time. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk.

If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at:0731752348 Do not sign the form until you have all your questions answered and understand what I would like you to do.

Do not sign written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study and I know what I will be asked to do. I am willing to be in the study.

Learner's name (print).. Learner's signature .... Date: ....

Witness's name (print) ....Witness's signature .... Date: ....

(The witness should be over 18 years old and present when signed.)

Parent/guardian's name (print) ....Parent/guardian's signature:....................................... Date: ....

Researcher"s name (print)...Researcher"s signature:............................Date:.............................

APPENDIX G: PRE-TEST AND POST-TEST

Pre-test

Grade : 12
Marks 35

1) Show all calculations. **2) You may use an approved calculator.** **3) All answers must be rounded to one decimal p\ace unless stated otherwise.** **MATHEMATICS**

Question 1

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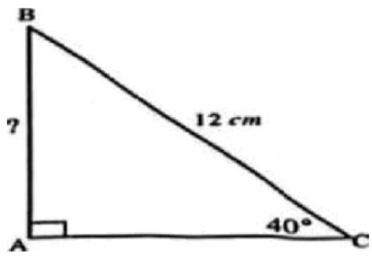
(a) $\tan\theta$ (b) $(\sin\theta + \cos\theta)^2$

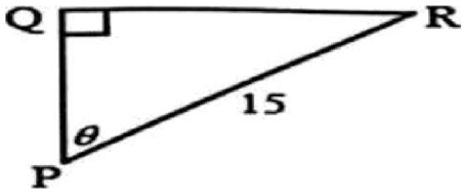
Question 2

ΔABC is given with $\hat{A} = 90°$, $\hat{C} = 40°$ and $BC = 12cm$. Determine the length of the side AB.



(2)

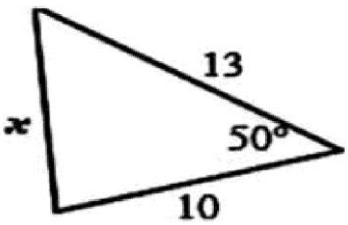
2.2. AQPR is given with QP= 12 units and PR= IS units. Determine the length pf QP and size of angle P



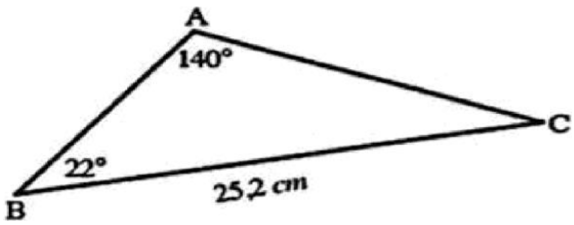
(2}

Question 3

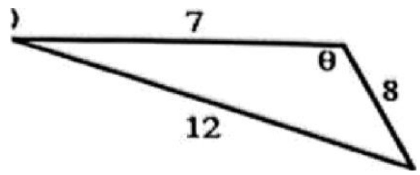
3.a. oetermłne the valua of x



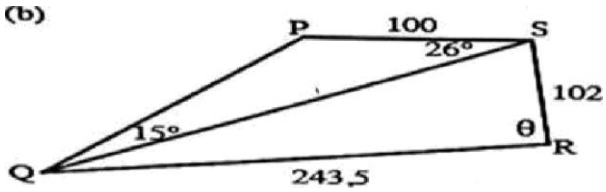
In the diagram below we have ΔABC with $BC = 25,2cm$, $\hat{A} = 140°$ and



3.3. FInd the size •t e



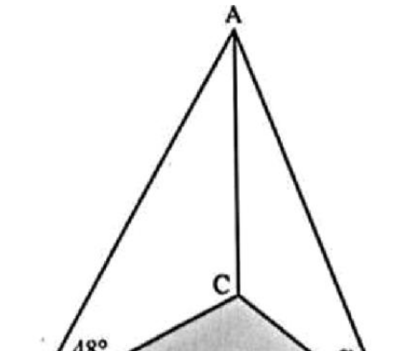
3.4. Find the length of PQ



(2)

QUESTION 4

In the following sketch, AC is a venical line, aBCD lies in a horizontal plane. BD = 10 m. CBD - 40° and CDB-6D°.Thc angle r‹u•tionofA from B iS 48° 3nJ the azigle of ele¥etion of A from D is c‹.



Calculate

4.1. **The length of:**

4.1.1.	**AB**	**(4)**
4.1.2.	**AC**	**(2)**
4.1.3.	**AD**	**(3)**

4.2. **Calculate the size of α** **(3)**

4.3. **Calculate the size of angle ABD** **(3)**

4.4. **Calculate the area of triangle ABD** **(3)**

THANK YOU FOR PARTICIPATING!!!!

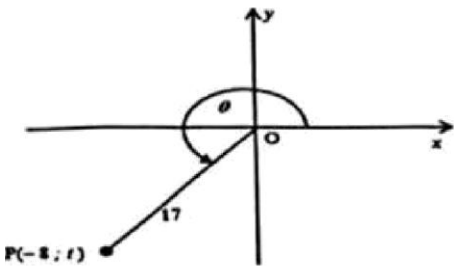
POsT TEST

Grade **:1Z**

Marks **: 35**

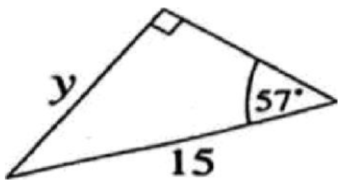
1) **Show all calculations.**
3/ **You may use an approvedcalculator.**
3) **All answers must be rounded to TWO decimal place unless stated otherwise.**

Question 1

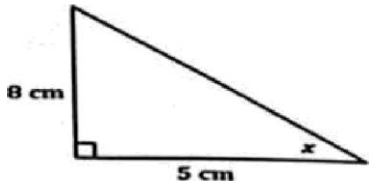


Question 2

2.1.Determine the length y

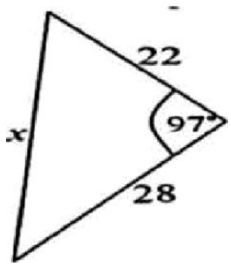


2.7.determine the size of angle x



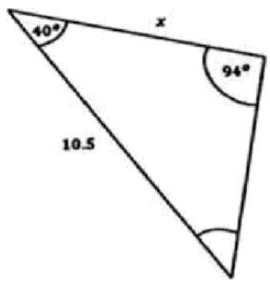
Question 3

Find the unknown side or angle in each of the following diagrams:



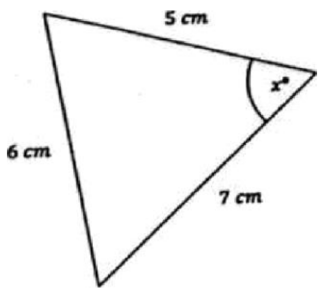
(z1

3.T

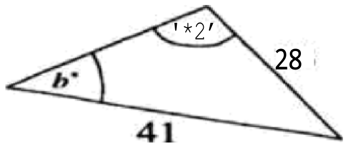


13)

3.3

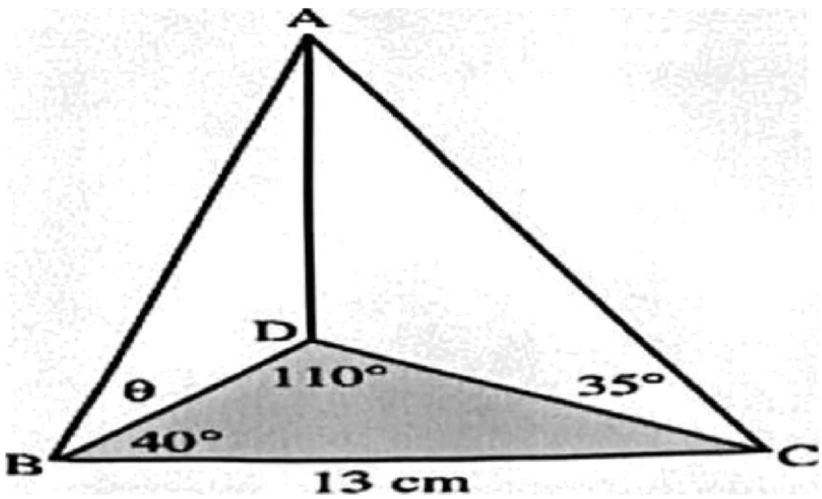


3.4



QUESTION 4

in the diagram below, AD is the vertiE a) line. Triangle BDC lies ah hori‹ontat ptane. BC=13cm, anB!• D8C=4O", angle BDC= 110". the angle of elevation of A from C Is 35" and the angle of elevation of A to B is €s



Calculate:

4.17he length of

4.a 1. AC **(4)**

(2)

4.1.5. AB **(3)**

4,2 The magnitude of 0 **(3)**

4.3 The magnitude of angle BAC **(3!**

4,4 The area of triangle ABC

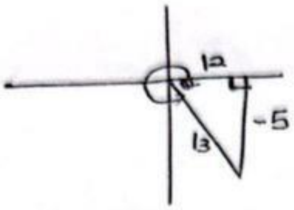
THANK YOU FOR PARTICIPATING IIIII

APPENDIX H: PRE-TEST AND POST-TEST MEMORANDUM

PRE-TEST MEMO

Question 1

1.1.



$r^2 = x^2 + y^2$

$(12)^2 + y^2 = (13)^2$

$y^2 = (13)^2 - (12)^2$

$y^2 = 25$

$y = \pm 5$

$\therefore y = -5$

$\rightarrow \tan\theta = -5/13$ 2

1.2. $(\sin\theta + \cos\theta)^2$

$= \left(\frac{-5}{13} + \frac{12}{13}\right)^2$

$= \frac{49}{169}$ 2

Question 2

2.1. $\sin\theta = \frac{AB}{BC}$

$\sin 40^\circ = \frac{AB}{12}$

$AB = 7{,}71$ 2

2.2. $\sin\theta = \frac{12}{15}$

$\theta = \sin^{-1}(12/15)$

$= 53{,}13$

$\cos\theta = \frac{QP}{PR}$

$\cos(53{,}13) = \frac{QP}{15}$

$QP = 9$ units 2

Question 3

3.1. $a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$x^2 = (13)^2 + (10)^2 - 2(13)(10)\cos 50°$ ✓

$\sqrt{x^2} = \sqrt{101,8752215}$ (2)

$x = 10,09$ ✓

3.2. $\hat{C} = 18$ (sum ∠ in Δ) ✓

$\frac{AB}{\sin\hat{C}} = \frac{BC}{\sin\hat{A}}$

$\frac{AB}{\sin 18°} = \frac{25,2}{\sin 140°}$ ✓ (3)

$AB = 12,11$ ✓

3.3 $a^2 = b^2 + c^2 - 2bc\cos\hat{A}$

$(12)^2 = (7)^2 + (8)^2 - 2(7)(8)\cos\theta$ ✓

$\theta = \cos^{-1}\left(\frac{(12)^2 - (7)^2 - (8)^2}{-2(7)(8)}\right)$

$\theta = 106,07°$ ✓ (2)

3.4. $\frac{PQ}{\sin\hat{S}} = \frac{PS}{\sin\hat{Q}}$

$\frac{PQ}{\sin(26)} = \frac{100}{\sin(15)}$ ✓

$PQ = 169,37$ ✓ (2)

Question 4

4.1.1. $B\hat{C}D = 80°$ (sum ∠ Δ) ✓

$\frac{\sin\hat{C}}{c} = \frac{\sin\hat{D}}{d}$

$\frac{\sin 80°}{10} = \frac{\sin 60°}{d}$ ✓

$d = 8,79$ ✓

$\cos\hat{B} = \frac{BC}{AB}$

$\cos 48 = \frac{8,79}{AB}$

$AB = 13,14$ ✓ (4)

4.1.2. $\tan 48 = \frac{AC}{8,79}$ ✓

$AC = 9,76$ ✓ (2)

4.1.3. $\frac{\sin \hat{C}}{c} = \frac{\sin \hat{B}}{b}$

$\frac{\sin 80}{10} = \frac{\sin 40}{CD}$

$CD = 6,53$ ✓

$CD^2 + AC^2 = AD^2$

$(6,53)^2 + (9,76)^2 = AD^2$ ✓

$AD = 11,74$ ✓ (3)

4.2. $\tan\alpha = \frac{9,76}{6,53}$ ✓

$\alpha = \tan^{-1}\left(\frac{9,76}{6,53}\right)$ ✓

$\alpha = 56,22$ ✓ (3)

4.3. $\cos\hat{B} = \frac{d^2 - a^2 - b^2}{-2(d)(a)}$

$= \frac{(13,14)^2 - (10)^2 - (11,74)^2}{-2(13,14)(10)}$ ✓

$\hat{B} = \cos^{-1}\left(\frac{(13,14)^2 - (10)^2 - (11,74)^2}{-2(13,14)(10)}\right)$ ✓

$\hat{B} = 75,64^\circ$ ✓ (3)

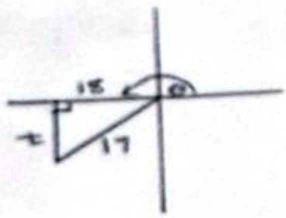
4.4. Area $= \frac{1}{2} AB.BD \sin A\hat{B}D$ ✓

$= \frac{1}{2}(13,14)(10) \sin 75,64^\circ$ ✓

$= 63,65$ ✓ (3)

POST-TEST MEMORANDUM

Question 1



1.1. $x^2+y^2=r^2$

$(-8)^2+(t)^2=(17)^2$ ✓

$t^2=(17)^2-(-8)^2$

$\sqrt{t^2}=\sqrt{225}$

$t=\pm 5$

$\therefore t=-5$ ✓ (2)

1.2. $\cos(-\theta)$

$=\cos\theta$ ✓

$=-8/17$ ✓ (2)

Question 2

2.1. $\sin 67^\circ = \frac{y}{15}$ ✓

$y=12,58$ ✓ (2)

2.2. $\tan x = \frac{8}{5}$ ✓

$x=\tan^{-1}(8/5)$

$x = 57,99$ ✓ (2)

Question 3

3.1. $x^2 = (22)^2 + (28)^2 - 2(22)(28)\cos 97°$

$\sqrt{x^2} = \sqrt{1418,143031}$

$x = 37,66°$ ②

3.2. $\hat{C} = 46°$

$\frac{x}{\sin 46°} = \frac{10,5}{\sin 94°}$

$x = 7,57$ ③

3.3. $(6)^2 = (5)^2 + (7)^2 - 2(5)(7)\cos x$

$x = \cos^{-1}\left(\frac{(6)^2-(5)^2-(7)^2}{-2(5)(7)}\right)$

$x = 57,12°$ ②

3.4 $\frac{\sin \hat{b}}{28} = \frac{\sin 92°}{41}$

$\hat{b} = \sin^{-1}\left(\frac{\sin 92°}{41} \times 28\right)$ ②

$= 43,04°$

Question 4

4.1.1. $\frac{DC}{\sin 40°} = \frac{13}{\sin 110°}$

$DC = 8,89$ ④

$\cos 35 = \frac{8,89}{AC}$

$AC = 7,28$

4.1.2. $\tan 35° = \frac{AD}{DC}$

$\tan 35 = \frac{AD}{8,89}$

$AD = 6,22$ ②

4.1.3. $\hat{C} = 50^\circ$ (sum $\angle$ Δ)

$$\frac{\quad}{\sin 50^\circ} = \frac{13}{\cancel{10,60}\ \sin 110^\circ}$$

Ę io,fin›)” †§6›**Ś”= A6R

$$AB^2 = 151,0484$$

$$\theta = \tan^{-1}\left(\frac{6,22}{10,60}\right)$$

$$\theta =$$

4.3. $x^2 + y^2 = r^2$

$$(8,89)^2 + (6,22)^2 = AC^2$$

$$AC = 10,85$$

$$13^2 = 12,29^2 + 10,85^2 - 2(12,29)(10,85)\cos \hat{A}$$

$$\hat{A} \quad \cos\left(\frac{(13)^2 - (12,29)^2 - (10,85)^2}{-2(12,29)(10,85)}\right)$$

4.4. a ^®’ '*"”"^°

„ C,c„zn3 Ćio,s s‘f**s‹•> 4cz•› o*)

APPENDIX I: SAMPLE OF LEANER'S SCRIPT (PRE-TEST)

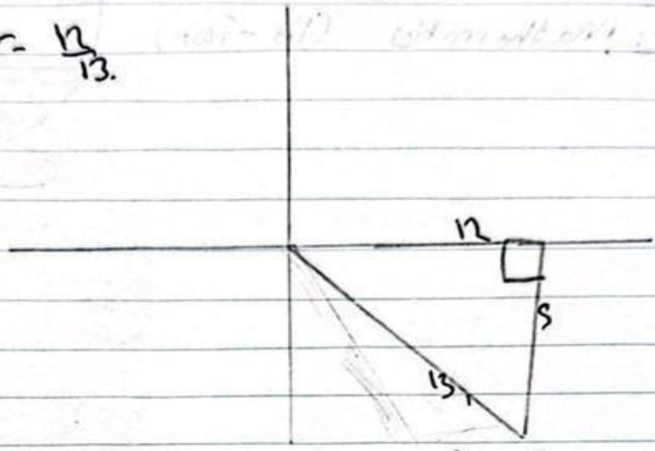
Name: Mvelo.
Surname: Ndwandwe
Grade: 12
Subject: Mathematics (Pre-Test)

$\frac{05}{35}$

Question 1.

1.1 $13 \cos \theta = \frac{12}{13}$ $180° < \theta < 360°$

$\cos \theta = \frac{12}{13}$.



a. $\tan \theta = \frac{a}{r}$

$= \frac{5}{13}$

$r^2 = y\ x^2 + y^2$

$= (13)^2\ x^2\ (12)^2$

$x^2 = 169 - 144$

$x^2 = \sqrt{25}$

$= 5$

1

b. $(\sin \theta + \cos \theta)^2$

$\frac{a}{h} = \frac{a}{h}$

$= \frac{5}{12} + \frac{13}{12}$,

$= \frac{18}{12}$,

$\frac{y}{r} + \frac{x}{r}$

$= \frac{13}{13} + \frac{12}{13}$

$= \frac{289}{169}$.

0

Question 2.

2.1 $AB = \sqrt{(x_2 - x_1)^2 (y_2 - y_1)^2}$

$= (90 - 0)^2 (40 - 0)^2$

$= 3600$

2.2 $PR^2 = QR$
$= \sqrt{144 - 22,3}$
$= 2$

Question 3

3.1 $x^2 =$
$= (10)^2 + (13)^2$
$=$

$= x^2 =$
$\sqrt{(13)^2 - (10)^2 + y^2}$
$= \sqrt{169 - 100 + y^2}$
$= 3$

3.2 $AB = (x_2 - x_1)(y_2 - y_1)$
$\sin\theta = \frac{AC}{BC}$

$25,2 \sin\theta = \frac{AC}{25,2}$ 25,2

$= 2$

Question 4

4.1.1 $\hat{B} + \hat{C} + \hat{D} = 180$ Sum of ∠'s in △
$40 + C + 60 = 180°$
$\hat{C} = 80°$

$\hat{A} + \hat{C} + B = 180$
$A + 90 + 48 = 180$
$\hat{A} = 42°$

$\frac{\sin D}{d} = \frac{\sin C}{c}$
$\frac{\sin 60}{d} = \frac{\sin(80)}{10}$
$\frac{10 \sin 60}{\sin(80)} = \frac{d \sin(80)}{\sin(80)}$
$d = 8,80$

$\cos 48 = \frac{BC}{AB}$
$AB \cos 48 = \frac{8,80}{AB} \cdot AB$
$\frac{AB \cos 48}{\cos 48} = \frac{8,80}{\cos 48}$
$AB = 13,15$ m

4

4.1.2 $AC^2 = AB^2 + CB^2$

$AC^2 = (13,14)^2 + (8,80)^2$

$\sqrt{AC^2} = \sqrt{250}$

$AC = 15,\cancel{88}\ 82$ m

4.1.3 $\sin = \frac{AC}{AD}$

$AD \sin(90) = \frac{15,82}{\cancel{AD}} \cancel{AD}$

$\frac{AD \sin(90)}{\sin(90)} = \frac{15,82}{\sin(90)}$

$AD = 15,82$ m

4.2 $\hat{A} + \hat{D} + \hat{C} = 180°$ Sum of $\angle$'s of a $\triangle$

$\hat{A} + \hat{a} + 90 = 180°$

$q + \hat{A} = 180 - 90$

$A = 90 - \hat{a}$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{90 - a}{a} = \frac{\sin(90)}{15,82}$

$a\frac{\sin(90)}{\sin(90)} = \frac{1423,8 - a}{\sin 90}$

$a =$

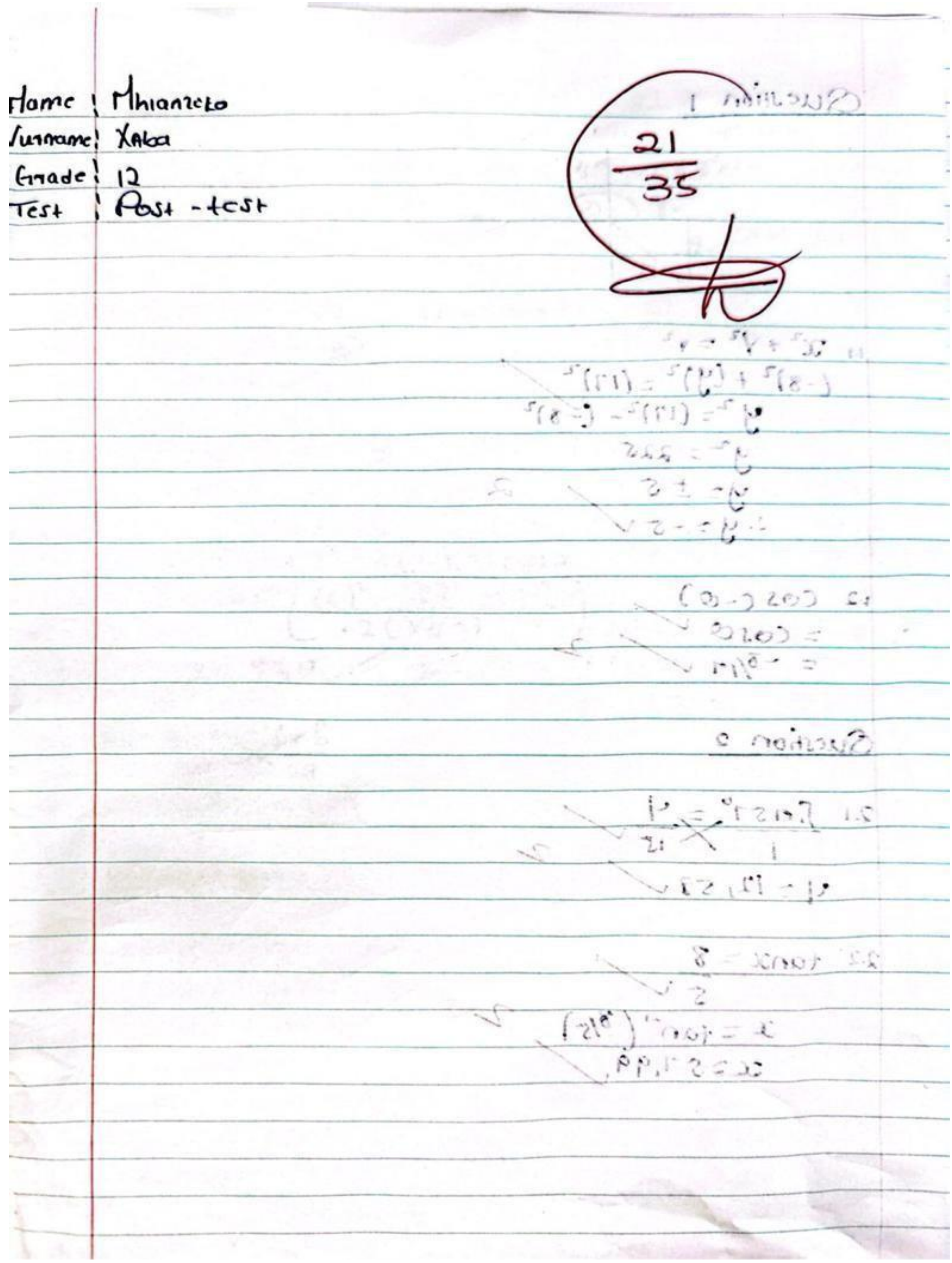
4.3

4.4 Area of $\triangle ABD = \frac{1}{2} AB \times AD$

$= \frac{1}{2}(13,15)(15$

Area of $\triangle ABD = 104$ m

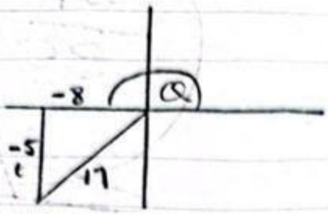
APPENDIX J: SAMPLE OF LEANER'S SCRIPT (POST-TEST)



Name : Mhlanzeko
Surname : Xaba
Grade : 12
Test : Post - test

21/35

Question 1



1.1 $x^2 + y^2 = r^2$

$(-8)^2 + (y)^2 = (17)^2$

$y^2 = (17)^2 - (-8)^2$

$y^2 = 225$

$y = \pm 5$

$\therefore y = -5$ 2

1.2 $\cos(-\theta)$

$= \cos\theta$

$= -8/17$ 2

Question 2

2.1 $\frac{\sin 57^\circ}{1} = \frac{y}{15}$ 2

$y = 12,57$

2.2 $\tan x = \frac{8}{5}$

$x = \tan^{-1}(8/5)$ 2

$x = 57,99$

Question 3

3.1 $x^2 = (22)^2 + (28)^2 - 2(22)(28)\cos 97^\circ$

$\sqrt{x^2} = \sqrt{1418,14303}$

$x = 37,66^\circ$ 2

3.2 $\hat{A} + \hat{B} + \hat{C} = 180^\circ$ (sum ∠ Δ)

$40^\circ + 94^\circ + \hat{C} = 180^\circ$

$\hat{C} = 46$

$\frac{x}{\sin 46} = \frac{10,5}{\sin 94}$ 3

$x = 7,57$

3.3 $(6)^2 = 5^2 + 7^2 - 2(5)(7)\cos x$

$x = \cos^{-1}\left(\frac{(6)^2 - (5)^2 - (7)^2}{-2(5)(7)}\right)$

$x = 57,12$ 2

3.4 $\frac{\sin 92}{41} = \frac{\sin \hat{b}}{28}$

$\sin \hat{b} = 27,98$

Question 4

4.1.1 $\frac{Dc}{\sin 40} = \frac{13}{\sin 110}$

$Dc = 8,89$ 2

4.1.2 $\tan 35^\circ = \frac{AD}{Dc}$

$\tan 35 = \frac{AD}{8,89}$ 1

$AD = 5,22$

4.1.3 $\frac{BD}{\sin 50} = \frac{13}{\sin 110°}$ ✓

$BD = 10,60$ ✓ 2

incomplete!

4.2 $\sin \theta = \frac{10,60}{6,22}$

$\theta = \sin^{-1}\left(\frac{10,60}{6,22}\right)$

$\theta =$ 0

4.3 $x^2 + y^2 = r^2$

$(8,89)^2 + (6,22)^2 = AC^2$

$\sqrt{AC^2} = \sqrt{117,7205}$

$AC = 10,85$

$BC = AC = 10,85$ ✓ 1

Incomplete!

4.4 Area $= \frac{1}{2}$ ~~b x h~~

$= \frac{1}{2}(10,85)(10,60)$

$= 57,51$ 0

APPENDIX K: SEMI-STRUCTURED INTERVIEW BEFORE INTERVENTION

Mathematical challenges: What, in your opinion, are challenges faced by learners when answering questions on heights and distances, that may result to low achievement in grade 12 heights and distances in trigonometry?

Benefits of learning heights and distances: What do you think are the benefits of being taught heights and distances as a grade 12 learner?

Classroom interaction: How do you engage with your learners during the teaching and learning of heights and distances in trigonometry?

Resources for teaching and learning: Which resources are used in your lessons when teaching heights and distances?

Assessment in the classroom: when assessing learners, do they know the difference between when to use Pythagoras theorem, sine rule and cosine rule?

APPENDIX L: SEMI-STRUCTURED INTERVIEW AFTER THE INTERVENTION

Transcript 1: What did you gain in this intervention and why?

Transcript 2: Are there any challenges that need attention to successfully implement visualisation in teaching heights and distances?

Transcript 3: What are the most important benefits of visualisation in teaching heights and distances?

Transcript 4: Do you think visualisation can be used in the future teaching of heights and distances, and why?

APPENDIX M: OBSERVATION SCHEDULE

OB5ERVATIOn **DATt OF OBSERVANON:** kEssON/ SUBJECT VcNUE **OBSERVER**	
Introduction of the Ie4son:	
Te4chers inpUt styles, approaches and teaching mafhods	
Notes of instructions given	
What are learners expected to do	
Support given	
Learners 'interaction with teacher and with each other	
Learners' response to given tasks	
Material or object used for demonstration	
Summary of strength/ difficulties	
Teachers'response if the learner answered In correctiY	

Types of responses from teamers	
1, Leamers ask for Instruclfons to be repeated	
. Responds approprlately	
3. Carries out part of Insttuction	
q. Lpng pause before Earrying put instruction	
s. Observe other learners and fol1ow thelr lead	
6. Appears to be confused	
7. No response	
8. Non-compliantresponse	
9. No response	

APPENDIX N: STATISTICAL ANALYSIS

21 May 2024, 08:07:11

The Wilcoxon test, also known as the rank sum test, is a nonparametric statistical method used to compare the Control and Experimental groups.

Table 1 indicates that the Control group scored significantly higher in total marks (p = 0.0332), as well as on Q4.1.3 (p = 0.0106), Q4.1.2 (p = 0.0370), and Q4.1.1 (p = 0.0102) compared to the Experimental group. For all other items, the differences between the two study groups were not significant.

Table 1: Group comparisons in the pre-test

Group	Rank sum	p-value	Item
Control	885.0	0.6765	Q2.1
Exp	945.0		
Control	979.0	0.3361	Q1.2
Exp	851.0		
Control	885.0	0.6765	Q2.1
Exp	945.0		
Control	979.0	0.3361	Q1.2
Exp	851.0		
Control	870.0	0.4211	Q3.1
Exp	960.0		
Control	967.0	0.2760	Q3.2
Exp	863.0		
Control	900.5	1.0000	Q3.3
Exp	929.5		
Control	930.0	1.0000	Q3.4
Exp	900.0		
Control	1081.0	0.0102*	Q4.1.1
Exp	749.0		
Control	1045.5	0.0370*	Q4.1.2
Exp	748.5		
Control	1060.0	0.0106*	Q4.1.3
Exp	770.0		
Control	1008.0	0.0955	Q4.2
Exp	822.0		
Control	946.5	0.3533	Q4.3
Exp	883.5		
Control	916.0	1.0000	Q4.4
Exp	914.0		
Control	1058.5	0.0332	Total (Pre-test)
Exp	771.5		

Table 2 presents the results of group comparisons in the post-test, showing that the Experimental group scored significantly higher on all items compared to the Control group. Exceptions were Q1.2 (p = 0.1805), Q3.1 (p = 0.2373), and Q3.2 (p = 0.2078), where the differences between the groups were not significant. Further details are provided in the Appendix.

Table 2: Group comparisons in the post-test

Group	Rank sum	p-value	Item
Control	764.0	0.0021*	Q1.1
Exp	1066.0		
Control	829.0	0.1805	Q1.2
Exp	1001.0		
Control	711.5	0.0009*	Q2.1
Exp	1118.5		
Control	603.5	0.0000*	Q2.2
Exp	1226.5		
Control	870.0	0.2373	Q3.1
Exp	960.0		
Control	842.5	0.2078	Q3.2
Exp	987.5		
Control	783.0	0.0153*	Q3.3
Exp	1047.0		
Control	727.0	0.0006*	Q3.4
Exp	1103.0		
Control	639.5	0.0000*	Q4.1.1
Exp	1190.5		
Control	678.5	0.0000*	Q4.1.2
Exp	1151.5		
Control	583.5	0.0000*	Q4.1.3
Exp	186.5		
Control	657.5	0.0000*	Q4.2
Exp	1172.5		
Control	674.5	0.0000*	Q4.3
Exp	1155.5		
Control	655.0	0.0000*	Q4.4
Exp	1175.0		
Control	565.5	0.0000*	Total (Post-test)
Exp	1264.5		

APPENDIX

```
. for var q11_pre- total_post : ranksum X, by( group)

->  ranksum q11_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         992         915
         EXP |       30         838         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1510.04
                     ----------
Adjusted variance       3064.96

H0: q11_pre(group==CONTROL) = q11_pre(group==EXP)
         z =  1.391
Prob > |z| = 0.1643
Exact prob = 0.2014

->  ranksum q12_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         941         915
         EXP |       30         889         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -2076.36
                     ----------
Adjusted variance       2498.64

H0: q12_pre(group==CONTROL) = q12_pre(group==EXP)
         z =  0.520
Prob > |z| = 0.6030
Exact prob = 0.7125

->  ranksum q21_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         885         915
         EXP |       30         945         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -807.20
                     ----------
Adjusted variance       3767.80

H0: q21_pre(group==CONTROL) = q21_pre(group==EXP)
         z = -0.489
Prob > |z| = 0.6250
Exact prob = 0.6765
```

```
->  ranksum q22_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         979         915
         EXP |       30         851         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -837.71
                     ----------
Adjusted variance       3737.29

H0: q22_pre(group==CONTROL) = q22_pre(group==EXP)
         z =  1.047
Prob > |z| = 0.2951
Exact prob = 0.3361

->  ranksum q31_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         870         915
         EXP |       30         960         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -650.85
                     ----------
Adjusted variance       3924.15

H0: q31_pre(group==CONTROL) = q31_pre(group==EXP)
         z = -0.718
Prob > |z| = 0.4725
Exact prob = 0.4211

->  ranksum q32_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         967         915
         EXP |       30         863         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -2209.96
                     ----------
Adjusted variance       2365.04

H0: q32_pre(group==CONTROL) = q32_pre(group==EXP)
         z =  1.069
Prob > |z| = 0.2850
Exact prob = 0.2760
```

```
->  ranksum q33_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€"Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30       900.5         915
          EXP |       30       929.5         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -3922.50
                     ----------
Adjusted variance        652.50

H0: q33_pre(group==CONTROL) = q33_pre(group==EXP)
         z = -0.568
Prob > |z| = 0.5703
Exact prob = 1.0000

->  ranksum q34_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€"Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30         930         915
          EXP |       30         900         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -3526.27
                     ----------
Adjusted variance       1048.73

H0: q34_pre(group==CONTROL) = q34_pre(group==EXP)
         z =  0.463
Prob > |z| = 0.6432
Exact prob = 1.0000

->  ranksum q411_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€"Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30        1081         915
          EXP |       30         749         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -353.39
                     ----------
Adjusted variance       4221.61

H0: q411_pre(group==CONTROL) = q411_pre(group==EXP)
         z =  2.555
Prob > |z| = 0.0106
Exact prob = 0.0102
```

```
->  ranksum q412_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30      1045.5         915
         EXP |       30       784.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1132.50
                     ----------
Adjusted variance       3442.50

H0: q412_pre(group==CONTROL) = q412_pre(group==EXP)
         z =  2.224
Prob > |z| = 0.0261
Exact prob = 0.0370

->  ranksum q413_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30        1060         915
         EXP |       30         770         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1381.53
                     ----------
Adjusted variance       3193.47

H0: q413_pre(group==CONTROL) = q413_pre(group==EXP)
         z =  2.566
Prob > |z| = 0.0103
Exact prob = 0.0106

->  ranksum q42_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30        1008         915
         EXP |       30         822         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1861.78
                     ----------
Adjusted variance       2713.22

H0: q42_pre(group==CONTROL) = q42_pre(group==EXP)
         z =  1.785
Prob > |z| = 0.0742
Exact prob = 0.0955
```

```
->  ranksum q43_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30       946.5         915
         EXP |       30       883.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -3336.23
                     ----------
Adjusted variance       1238.77

H0: q43_pre(group==CONTROL) = q43_pre(group==EXP)
         z =  0.895
Prob > |z| = 0.3708
Exact prob = 0.3533

->  ranksum q44_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         916         915
         EXP |       30         914         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -3720.00
                     ----------
Adjusted variance        855.00

H0: q44_pre(group==CONTROL) = q44_pre(group==EXP)
         z =  0.034
Prob > |z| = 0.9727
Exact prob = 1.0000

->  ranksum total_pre, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30      1058.5         915
         EXP |       30       771.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties      -16.02
                     ----------
Adjusted variance       4558.98

H0: total_~e(group==CONTROL) = total_~e(group==EXP)
         z =  2.125
Prob > |z| = 0.0336
Exact prob = 0.0332
```

```
->  ranksum q11_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€"Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30         764         915
          EXP |       30        1066         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -2363.14
                     ----------
Adjusted variance       2211.86

H0: q11_post(group==CONTROL) = q11_post(group==EXP)
         z = -3.211
Prob > |z| = 0.0013
Exact prob = 0.0021

->  ranksum q12_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€"Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30         829         915
          EXP |       30        1001         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1091.31
                     ----------
Adjusted variance       3483.69

H0: q12_post(group==CONTROL) = q12_post(group==EXP)
         z = -1.457
Prob > |z| = 0.1451
Exact prob = 0.1805

->  ranksum q21_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€"Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30       711.5         915
          EXP |       30      1118.5         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -864.79
                     ----------
Adjusted variance       3710.21

H0: q21_post(group==CONTROL) = q21_post(group==EXP)
         z = -3.341
Prob > |z| = 0.0008
Exact prob = 0.0009
```

```
->  ranksum q22_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30       603.5         915
         EXP |       30      1226.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -606.61
                     ----------
Adjusted variance       3968.39

H0: q22_post(group==CONTROL) = q22_post(group==EXP)
         z = -4.945
Prob > |z| = 0.0000
Exact prob = 0.0000

->  ranksum q31_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         870         915
         EXP |       30         960         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -3922.88
                     ----------
Adjusted variance        652.12

H0: q31_post(group==CONTROL) = q31_post(group==EXP)
         z = -1.762
Prob > |z| = 0.0780
Exact prob = 0.2373

->  ranksum q32_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30       842.5         915
         EXP |       30       987.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1958.90
                     ----------
Adjusted variance       2616.10

H0: q32_post(group==CONTROL) = q32_post(group==EXP)
         z = -1.417
Prob > |z| = 0.1563
Exact prob = 0.2078
```

```
->  ranksum q33_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         783         915
         EXP |       30        1047         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1976.06
                     ----------
Adjusted variance       2598.94

H0: q33_post(group==CONTROL) = q33_post(group==EXP)
         z = -2.589
Prob > |z| = 0.0096
Exact prob = 0.0153

->  ranksum q34_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         727         915
         EXP |       30        1103         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1500.00
                     ----------
Adjusted variance       3075.00

H0: q34_post(group==CONTROL) = q34_post(group==EXP)
         z = -3.390
Prob > |z| = 0.0007
Exact prob = 0.0006

->  ranksum q411_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30       639.5         915
         EXP |       30      1190.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -368.64
                     ----------
Adjusted variance       4206.36

H0: q411_p~t(group==CONTROL) = q411_p~t(group==EXP)
         z = -4.248
Prob > |z| = 0.0000
Exact prob = 0.0000
```

```
->  ranksum q412_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30       678.5         915
          EXP |       30      1151.5         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -852.97
                     ----------
Adjusted variance       3722.03

H0: q412_p~t(group==CONTROL) = q412_p~t(group==EXP)
         z = -3.877
Prob > |z| = 0.0001
Exact prob = 0.0000

->  ranksum q413_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       29       583.5         870
          EXP |       30      1186.5         900
-------------+---------------------------------
     Combined |       59        1770        1770

Unadjusted variance     4350.00
Adjustment for ties     -522.33
                     ----------
Adjusted variance       3827.67

H0: q413_p~t(group==CONTROL) = q413_p~t(group==EXP)
         z = -4.631
Prob > |z| = 0.0000
Exact prob = 0.0000

->  ranksum q42_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

        group |      Obs    Rank sum    Expected
-------------+---------------------------------
      CONTROL |       30       657.5         915
          EXP |       30      1172.5         915
-------------+---------------------------------
     Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties     -666.10
                     ----------
Adjusted variance       3908.90

H0: q42_post(group==CONTROL) = q42_post(group==EXP)
         z = -4.119
Prob > |z| = 0.0000
Exact prob = 0.0000
```

```
->  ranksum q43_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30       674.5         915
         EXP |       30      1155.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1587.08
                     ----------
Adjusted variance       2987.92

H0: q43_post(group==CONTROL) = q43_post(group==EXP)
         z = -4.400
Prob > |z| = 0.0000
Exact prob = 0.0000

->  ranksum q44_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30         655         915
         EXP |       30        1175         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties    -1109.24
                     ----------
Adjusted variance       3465.76

H0: q44_post(group==CONTROL) = q44_post(group==EXP)
         z = -4.416
Prob > |z| = 0.0000
Exact prob = 0.0000

->  ranksum total_post, by( group)

Two-sample Wilcoxon rank-sum (Mannâ€“Whitney) test

       group |      Obs    Rank sum    Expected
-------------+---------------------------------
     CONTROL |       30       565.5         915
         EXP |       30      1264.5         915
-------------+---------------------------------
    Combined |       60        1830        1830

Unadjusted variance     4575.00
Adjustment for ties      -15.25
                     ----------
Adjusted variance       4559.75

H0: total_~t(group==CONTROL) = total_~t(group==EXP)
         z = -5.176
Prob > |z| = 0.0000
Exact prob = 0.0000
```

The data analysis was performed using Stata Release 18. The Wilcoxon signed-rank sum test, a non-parametric alternative to the paired samples t-test, was used when it was preferable not to assume the difference between the two variables was interval and normally distributed, but rather ordinal. Specifically, for paired data, the `ranksign` command in Stata determined the direction of the difference after using the `ranktest` command to compare the two conditions. The results were interpreted with an error rate of 0.05.

Table 0a: Variable description

Name	Variable
id	Learner ID
q11exp_pre	Q1.1 EXP_PRE
q12exp_pre	Q1.2 EXP_PRE
q21exp_pre	Q2.1 EXP_PRE
q22exp_pre	Q2.2 EXP_PRE
q31exp_pre	Q3.1 EXP_PRE
q32exp_pre	Q3.2 EXP_PRE
q33exp_pre	Q3.3 EXP_PRE
q34exp_pre	Q3.4 EXP_PRE
q411exp_pre	Q4.1.1 EXP_PRE
q412exp_pre	Q4.1.2 EXP_PRE
q413exp_pre	Q4.1.3 EXP_PRE
q42exp_pre	Q4.2 EXP_PRE
q43exp_pre	Q4.3 EXP_PRE
q44exp_pre	Q4.4 EXP_PRE
totalexp_pre	TOTAL EXP_PRE
q11exp_post	Q1.1EXP_POST
q12exp_post	Q1.2 EXP_POST
q21exp_post	Q2.1 EXP_POST
q22exp_post	Q2.2 EXP_POST
q31exp_post	Q3.1 EXP_POST
q32exp_post	Q3.2 EXP_POST
q33exp_post	Q3.3 EXP_POST
q34exp_post	Q3.4 EXP_POST
q411exp_post	Q4.1.1EXP_POST
q412exp_post	Q4.1.2 EXP_POST
q413exp_post	Q4.1.3 EXP_POST
q42exp_post	Q4.2 EXP_POST
q43exp_post	Q4.3 EXP_POST
q44exp_post	Q4.4 EXP_POST
totalexp_post	TOTAL EXP_POST

Table 1: Experimental Group

Pre vs. Post	P-value
Q1.1	0.0017
Q1.2	0.0034
Q2.1	0.5982*
Q2.2	0.0245
Q3.1	< 0.0001
Q3.2	< 0.0001
Q3.3	< 0.0001
Q3.4	< 0.0001
Q4.1.1	0.669*
Q4.1.2	0.0001
Q4.1.3	< 0.0001
Q4.2	0.0013
Q4.3	0.0001
Q4.4	< 0.0001
Total	< 0.0001

*Indicates insignificant difference at a 0.05 error rate

Table 1 shows that post-observations were significantly higher than pre-observations for the Experiential group, except for Q2.1 and Q4.1.1. In these cases, the p-values were not below the nominal 0.05, indicating no significant difference between the pre-and post-test observations.

Table 2: Control Group

Pre vs. Post	P-value
Q1.1	0.9453
Q1.2	0.9935
Q2.1	0.7095
Q2.2	0.6855
Q3.1	0.0001*
Q3.2	< 0.0001*
Q3.3	< 0.0001*
Q3.4	0.0005*
Q4.1.1	1.0000
Q4.1.2	0.5927
Q4.1.3	0.9846
Q4.2	0.9888
Q4.3	1.0000
Q4.4	0.7500
Total	0.2858

*Indicates a significant difference at a 0.05 error rate

Table 2 indicates that the pre-test and post-test observations were statistically similar, except for questions 3.1, 3.2, 3.3, and 3.4, where the pre-test scores were significantly lower than the post-test scores

APPENDIX O: EDITING CERTIFICATE

Unit 3 West Square Business Park
407 West Avenue
Randburg
2t 94

6 February 2025

TO WHO3•t JT PtAY CONCERN

This serves to confirm that I have edited and made the necessary corrections and tmendatonstothtnudp

USING VgSIJALISATION TO TEACH GRADE IN H EIGXTS AND DISTANCES: A STUDY EN ONE SECONDARY SCHOOL OF ZULULAND DJSTR3CF OF XWAZULU - **KATAL**

by

NZUZA PI-£ILASANDE ZAMOKUH LE

Since rely

J Musi
Editor

APPENDIX P: TURNITIN REPORT

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