Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions. A case of selected high school in Mopani West District, Limpopo Province

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Declaration

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I declare that the above dissertation is my own work, and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the dissertation to originality checking software, and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.

13 February 2025

Signature Date

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Dedication

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Abstract

Poor mathematics performance is a global issue, including in South Africa, with a reported decline in skills worldwide Berisha et al. (2024). Learners struggle with understanding algebraic fractions, which are essential in mathematics. Inadequate algebraic knowledge can impact learners' confidence and success in Further Education and Training, limiting their career prospects in Science and Engineering. The purpose of this study was therefore to explore the misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions. Thirty-six (36) learners from Mopani West District in Limpopo studying mathematics were purposively sampled to participate in the study. The qualitative case study method used included data collection techniques such as a written test and semi-structured interviews. This dissertation draws upon Newman's theory to provide a framework for understanding and explaining the phenomenon. Findings reveal that learners manipulate rules they learned in whole numbers and inappropriately apply them in algebraic fractions. This misapplication is evident in errors related to comprehension, procedural skills, transformation, and encoding. Based on the findings, the study recommends that learners should work on problems that foster connections in mathematics. A further recommendation is that learners need to have a deep understanding of the underlying principles governing the operations with fractions. Further research is necessary to explore alternative learner-centered strategies, such as inquiry-based learning and active notetaking, to assess their potential impact on enhancing learner performance in algebraic fractions. Investigating the efficacy of these approaches will provide valuable insights into their effectiveness in fostering deeper understanding and improving learner performance in mathematics education.

Keywords: Algebraic fractions, misconceptions, errors, mathematics performance, manifest, Grade 10 learners, comprehension, process skill, transformation, encoding.

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List of Tables and Figures

List of Acronyms

ATP	Annual Teaching Plan
BCAE	Both Correct Answer and Explanation
BIAE	Both Incorrect Answer and Explanation
CAIE	Correct Answer and Incorrect Explanation
CAPS	Curriculum and Assessment Policy Statement
CE	Comprehension Error
CPCA	Correct Procedure and Correct Answer
DBE	Department of Basic Education
DH	Departmental Head
DME	Department of Mathematics Education
EE	Encoding Error
FET	Further Education and Training
GET	General Education and Training
HCF	Highest Common Factor
HL	Home Language
LCD	Lowest Common Denominator
MCQ	Multiple Choice Questions
MPIA	Moderate Procedure and Incorrect Answer
NSC	National Senior Certificate
OECD	Organisation for Economic Co-operation and Development
PCPIA	Partially Correct Procedure and Incorrect Answer
PISA	Programme for International Students Association
PSE	Process Skill Error
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SES	Subject Education Specialist
SMT	School Management Team
SGB	School Governing Body
STEM	Science, Technology, Engineering, and Mathematics
TIMSS	Trends in International Mathematics and Science Study
TE	Transformation Error
UNISA	University of South Africa

Chapter 1
Introduction

1.1 Background of the Study

In South Africa, the 10th grade serves as the beginning of the phase for further education and training (FET) band. Learners in FET are expected to focus on either pure mathematics, mathematical literacy, or technical mathematics. For Grade 10 pure mathematics, algebraic fractions are covered in the curriculum. In addition, learners must be familiar with algebra to enrol in courses like Science, Technology, Engineering, and Mathematics (STEM) to understand algebraic fractions. Moreover, knowledge of rational and irrational numbers is essential for careers in STEM industries. A career in the sciences is difficult for an undergraduate student who is incapable of understanding anything beyond whole numbers and basic rational numbers. In an era in which future competitiveness relies on how effectively learners are prepared for the 21st century STEM careers, learners need to be equipped early with the resources that will enable them to acquire the capacity for critical thinking, analysis, and integration of innovative and sophisticated concepts (AlAli & Wardat, 2024).

Mathematics is a subject that is required in many nations, even developing ones, for entry to college or university and for employment in specific positions, particularly in the public services (Varaidzaimakondo & Makondo, 2020). In contrast to working with whole numbers, dealing with fractions provides mathematical growth and boosts learners' understanding of mathematics (Jarrah et al., 2022). Its applications can be divided into two categories, namely: i) Conceptual understanding, and ii) Procedural fluency. Conceptual understanding enables learners to categorise fractions based on their dimensions, whereas procedural understanding requires the use of mathematical rules. These mathematical rules include the use of addition, multiplication, division, and subtraction operations (Saban et al., 2021). Koh et al. (2021) note that the difficulty that learners encounter when solving tasks related to algebraic fractions is due to erroneous linkages between real-world situations and mathematical problems on the subject. Different kinds of misconceptions exhibited lead to errors[1] in calculations. It is also noted that most educators find it difficult to accept the crucial reality that comprehension of

[1] Misconceptions and errors are often interconnected. They cannot be separated because the application of flawed (incorrect conception) understanding lead to incorrect solutions (errors). Misconceptions can develop from making multiple errors and gaining confidence out of those errors (Brodie, 2014).

algebraic fraction concepts provides a crucial foundation for studying mathematics, and it is essential to use this as the basis for instructing learners on the subject (Karp & Karp, 2021).

Algebraic fractions can be defined as quotients of two algebraic expressions, for example, $\frac{x^2-6x+9}{x-3}$ (Pettit et al., 2016). Furthermore, algebraic fractions consist of phrases with variables, constants, and algebraic operations like addition and subtraction. In the given algebraic fraction, x is identified as a variable, where 9 is a constant. Although algebraic fractions are frequently among the topics that are problematic for learners to comprehend (Lee & Dreyfus, 2020), numerous operations, including factorisation, multiplication, and cancellation of common components are necessary to simplify these algebraic fractions. The Curriculum and Assessment Policy Statement (CAPS) for mathematics stresses that learners should be able to effectively solve such problems with confidence (Department of Basic Education [DBE], 2013).

Stemele and Jina Asvat (2024) report that learners' errors and misconceptions in algebraic fractions are often attributed to learners failing to manipulate algebraic concepts. In addition, studies by Deringol (2019) and Mulwa (2015) discover that learners' poor performance in fraction calculation may be mostly related to an insufficient understanding of the fraction notion. An inadequate grasp of this context makes it difficult for learners to comprehend the idea of fractions. Wiest and Amankonah (2019) mention that conceptual understanding entails the ability to apply mathematical principles to several topics and perceive the links between the concepts and methods. The two authors noted that learners memorise rules, formulas, words, and algorithms rather than attempting to comprehend the reasoning underlying fractional operations. This finding is consistent with research by Teh et al. (2020) which reveal that learners had challenge acquiring conceptual and procedural knowledge in algebra, which affects their capacity to retain and recall the concepts taught.

In the same vein, numerous studies, including the Trends in Mathematics and Science St6udy (TIMSS), the Programme for International Students Assessments (PISA), and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), have identified language as one of the factors that contributes to deficient performance in the learning of Algebra, albeit not the only factor at play (Reddy et al., 2016). In the TIMSS 2015 cohort, only 31% of South African learners were reported to speak the language of instruction and learning at home constantly or almost constantly (Isdale et al., 2017). Those learners outperformed those whose home languages were different from the ones they were learning in school. These elements were discovered as the primary causes of the errors and misconceptions that learners make as they solve tasks involving Algebra. Due to their weak knowledge of foundational ideas, South African learners' arithmetic performance is of a low standard (Reddy et al., 2016; Spaull, 2019). In the same breadth, the TIMSS (2019) study indicates that 61% of Grade 5 learners are unable to do fundamental mathematical operations including addition and subtraction of whole numbers, multiplication of whole numbers by one-digit whole numbers, or solving straightforward word problems (Spaull, 2019). The preceding discussions suggest that proficiency in algebraic fractions is crucial for success in mathematics (Viegut, 2024). This study sought to gain insight into the misconceptions and errors exhibited by Grade 10 learners when simplifying tasks involving algebraic fractions.

1.2 Rationale of the Study

Most learners in South Africa do extremely poorly in algebra (OECD, 2020a), and this has an impact on the performance of mathematics in general. The main aim of this study was to explore errors and misconceptions that manifest when learners simplify tasks involving algebraic fractions. It was important to undertake this study because Algebra is a fundamental component of nearly every mathematics programme. Although educators occasionally analyse learners' responses, it is rare to find instances where they integrate the findings into their teaching (Hennessy et al., 2023). Educators frequently remark how they must complete the syllabus and worry that they will not have enough time to thoroughly address the errors and misconceptions detected.

If learners are not given the opportunity to learn from their mistakes, this might impede their understanding of the concept. Furthermore, learners might experience challenges in connecting algebraic fractions to related concepts. If learners can use mathematical symbols and rules utilising algebraic operations, they might perform better in mathematics. When faced with issues related to algebraic fractions, learners frequently struggle to advance in other branches of mathematics. Most importantly, learners who can recognise their own pattern of errors from their solution can benefit from this study. Since errors and misconceptions have an impact on learners' subsequent studies, being able to learn from mistakes is likely to enhance learner performance and learner attainment in mathematics. Learners can improve their performance in mathematics if they can manipulate mathematical symbols and rules using algebraic operations.

The findings, implications, and recommendations in this study offer valuable insights and practical suggestions on how learners can develop a better understanding of the concept of algebraic fractions. This understanding will help them enhance their skills in the simplification of algebraic fractions. Furthermore, this research topic will aid in suggesting ways of enhancing learners' understanding of algebraic fractions. The study will lay a solid foundation for future research of this kind that must be conducted to advance and address numerous problems and challenges related to learners' comprehension of algebraic fractions in the domain of mathematics.

1.3 Problem Statement

One of the most challenging aspects of mathematics lessons in the school curriculum is learning algebraic fractions because simplifying them is difficult. With 30% of the mathematics curriculum focused on Algebra (DBE, 2013), it underscores the significance of this component in building foundational knowledge and skills for higher-level studies and practical applications. This emphasis makes Algebra a core part of the overall curriculum, where learners will spend significant time mastering concepts like variables, constants, basic operations, and problem-solving with algebraic fractions.

According to the DBE (2013), 17% of learners in Grade 12 find the assessment activities for simplifying algebraic fractions challenging. To further elaborate on this finding, the National Senior Certificate (NSC) diagnostic report (DBE, 2018) highlights that some candidates had difficulty in applying fractional concepts to selling and discount prices. These kinds of challenges may suggest that candidates exhibited some errors and that there may be some gaps in their understanding of algebraic principles. The diagnostic report recommended that educators regularly revise concepts such as percentages, proper fractions, and decimal fractions with Grade 12 learners as a suggestion for improvement. According to these findings and suggestions made, I would add that frequent reinforcement may not be sufficient unless fractions are taught for conceptual comprehension at the primary school level.

According to Makonye (2012) and Van de Walle et al. (2019), inadequate knowledge of the steps needed to simplify a problem result in procedural errors. The following example illustrates how a learner committed a procedural error. In addition, simplification of the problem like $\frac{6xy^2-9xy}{-3xy}$ causes learners to commit errors. In the second example, the learner simplified the problem as follows: $\frac{6xy^2-9xy}{-3xy} = \frac{2(3xy)^2-9xy}{-3xy} = \frac{(6xy)^2}{3xy} - \frac{9xy}{3xy} = \frac{12}{3} - 3 =$ 4-3=1.

In this instance, the learner incorrectly expressed the first term in the numerator as the product of two factors. The learner also erroneously omitted the negative sign in the denominator. This might be a careless mistake, or the learner did not know what to do to remove the negative sign from the denominator. The procedure used by the learner was incorrect.

The following examples are from a study by Baidoo et al. (2020) illustrating the examples of learners' answers where learners were required to simplify the tasks: Example i) shows a case where a learner exhibits a procedural error.

$\boldsymbol{i}$) $\quad \frac{\mathbf{2x-5}}{\mathbf{3}} - \frac{\mathbf{3x-2}}{\mathbf{4}}$

$= 12(\frac{\mathbf{2x-5}}{\mathbf{3}}) - \mathbf{12}(\frac{\mathbf{3x-2}}{\mathbf{4}})$

$= 6x - 20 - 9x - 6$

$= 14 - 3x$

According to Baidoo et al. (2020), from the learner's solution in question i), the learner used the Lowest Common Denominator (LCD) to multiply each term in the expression and cancel out the outer term by the denominator. Quotients were interpreted as a pair of whole numbers. Hence the category of an error exhibited by the learner is procedural. The learner demonstrated a lack of comprehension of rules required to simplify.

Another learner's example illustrates a situation in which a learner manifests a language error. The learner's answer is presented as example ii) as follows:

ii) $\quad 36\boldsymbol{y^2} - \frac{\boldsymbol{x^2}}{\mathbf{25}}$

$= \mathbf{36}\boldsymbol{y^2} - \boldsymbol{x^2} + \mathbf{25} - \boldsymbol{x^2}$

$= \mathbf{36}\boldsymbol{y^2} - \boldsymbol{x^2} + \mathbf{25}$

$= 11\boldsymbol{y^2}\boldsymbol{x^2}$

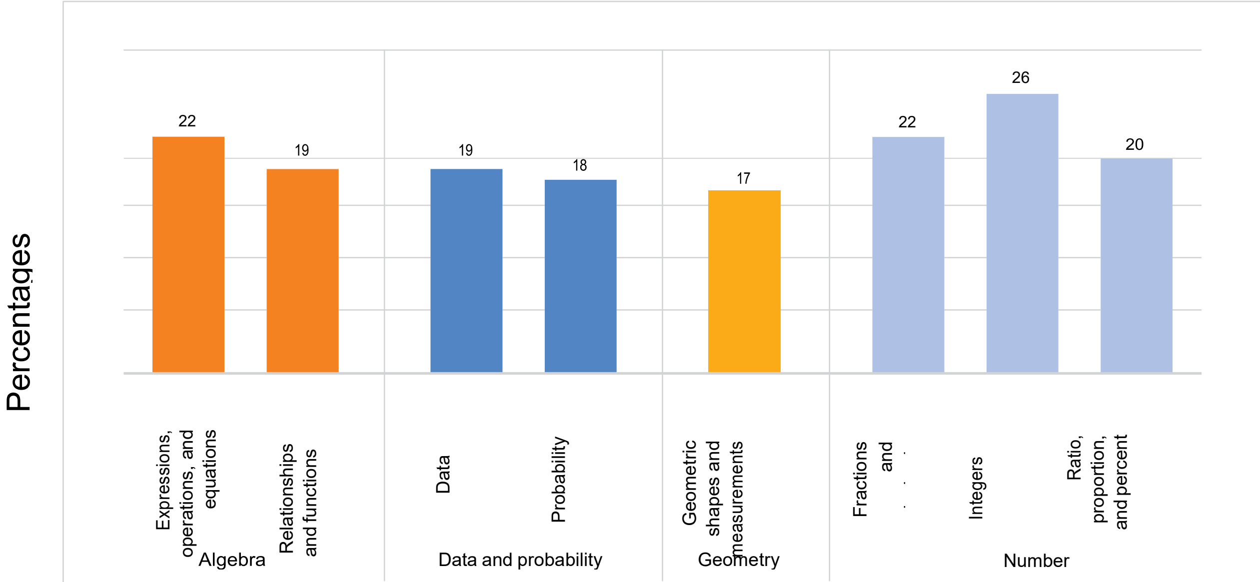
The learner fails to provide the answer since they did not comprehend mathematical concepts and integers. The second term was written as: $-x^2 + 25$, subtracted unlike terms and the terms were multiplied in the final step (Baidoo et al., 2020). Im and Jitendra (2020) provide evidence in support of this, demonstrating that persistent misconceptions can hinder learners' comprehension of mathematical ideas and lead to frequent repetition of errors.

To determine whether learners are equipped with the basic knowledge of rational numbers, they must be able to use equivalence to represent the same value in different forms (Van de Walle et al., 2019). Most learners are incompetent in fundamental and basic mathematical competencies that belong in the lower grades, due to inadequate algebraic skills (DBE, 2023). A study conducted within the context of mathematics reveals that a significant number of learners struggle with the understanding of algebraic fractions (Baidoo et al., 2020). For instance, typical errors are encountered when simplifying problems such as $\frac{6x^2+12x}{6x}$, where the learner writes the answer as: $\frac{x^2+12x}{x} = \frac{12x^3}{x} = 12x^2$. In this case, the learner has a misunderstanding, believing that only variables are added, and has confused the rules for multiplying terms with those for adding terms. In this case, an application error was committed.

Application errors are brought about by learners' failure to correctly apply the knowledge to particular or varied problems (Ornstein & Hunkins, 2020). The presented example illustrates how the learner used the rules incorrectly to simplify the algebraic fraction. Due to a lack of comprehension of the concepts and abilities learned in the process of simplifying algebraic expressions, the learner added exponents to the numerator and further cancelled out 6 on the numerator and denominator. This observation confirms Bergman et al. (2024)'s view that some learners would be persuaded to take out the common factor whenever they encounter similar variables in the denominators and numerators. In addition, Bergman et al. (2024) emphasise the importance of factoring in enhancing problem-solving skills among learners. In addition, Mbatha and Bansilal (2024) mention that an error analysis often exposes learners' misunderstandings in mathematics. The two authors demonstrated in their research that systematic error analysis can uncover specific misconceptions held by learners, which can inform targeted instructional strategies. Errors made by learners can sometimes produce an insight into learners' understanding regarding a particular concept or skills (Angraini et al., 2023). Wrong interpretation of a notion will therefore lead to different types of errors. **Figure 1** presents the average percentage on performance of mathematics topics from TIMSS (2019) South African Item Diagnostic Report for Mathematics learners' performance.

Figure 1

Mathematics Topics Covered in Grade 9



Source: TIMSS 2019 South African Grade 9 dataset by Mullis et al. (2020, p. 38). Copyright: 2022 by the researcher.

Figure 1 shows a notable trend in the performance of Algebra for South African Grade 9 learners. The report indicates that although Algebra ranked joint second, with a performance of 22%, the average performance in this topic is below 50%. I believe that analysing and exposing the errors and misconceptions that learners encounter when attempting to simplify algebraic fractions tasks will improve learner performance in mathematics.

My interest in learners' understanding of the algebraic experience was sparked by observation and practical teaching expertise, my collaboration with other educators, and my Master of Education research. This interest prompted me to explore ways in which I could recommend methods for learners to enhance their learning. Such methods include the identification of errors and misconceptions that manifest when simplifying problems related to the topic. To identify the challenges faced by learners, I analysed their responses in a test designed to assess their understanding of mathematical concepts, procedures, and relationships in Algebra (Durkin et al., 2023). Understanding a subject essentially means to exploit it and implement strategies that exceed expertise and ordinary competency (Aldemlr & Uysal, 2024). Moreover, it is crucial to think and demonstrate creativity when confronted with new information.

Misconceptions form part of teaching and learning. Fujii (2020) alerts researchers to the fact that only a few of the misconceptions that have been identified at the elementary and secondary levels are taken into consideration for inclusion in actual teaching situations. **Table 1** presents a content area achievement comparison between South African and international learners.

Table 1

Content Area Achievement (%) Comparison Between South African and International Learners

	Number	Algebra	Geometry	Data and chance
South Africa	21	20	19	26
International	44	37	37	47

Source: Mosimege et al. (2017, p. 11).

The content area achievement comparison between Grade 9 South African and international learners is presented in **Table 1**. According to the statistics, South African learners are not doing well in algebra; they received a score of 20%, but international learners received a higher score of 37%. This may be the root cause of South African learners' poor performance in the FET phase of algebra, which indicates a lack of prior knowledge (Copur-Gencturk, 2021; & Chand et al., 2021). Therefore, educators in lower grades should ensure that learners grasp the fundamental concepts and skills required to solve algebraic tasks and simplify expressions in algebra.

Drawing on my experience as a Departmental Head (DH) and the in-service training sessions I have attended, errors and misconceptions may emerge due to the following factors.

i) **Nature of the concepts**: Because some concepts are counter-intuitive, learners must be taught to pause and reflect before coming up with the correct answer. The nature of the mathematical task might lead to errors in the way the learner works and, as a result, the learner will not comprehend what is needed to fix it (Baidoo, 2019).

(ii) **Pedagogical knowledge factors**: Educators emphasise procedural fluency over conceptual comprehension in their instruction. These variables include the expertise and abilities developed throughout the learning process regarding how learners learn. The focus of pedagogical knowledge factors is on how to convey and formulate the subject so that learners can understand it (Gerhard et al., 2023). Gerhard et al. (2023) further point out that it aims to provide insight into what makes knowing certain subjects simple or complex.

(iii) **Curriculum factors:** Learning from educators on a subject that is not a precondition for the next topic. For example, while utilising a round cake as a model to determine the portion of the whole is useful, it is not part of the Grade 10 curriculum. Gericke et al. (2023) found that educators rarely give learners structured learning experiences to aid in the acquisition of conceptual and procedural information.

(iv) **Curriculum content coverage:** Educators who rush to the next topic in the Annual Teaching Plan (ATP) by skipping some subtopics can impede learners' progress because some learners will not be able to manipulate other algebraic expressions related topics. Weber and Mofield (2023) state that content knowledge focuses on understanding the fundamental ideas of the subject matter to be mastered.

(v) **Cognitive factors:** There is a content gap among learners. For instance, learners lack fundamental algebraic skills, making it impossible for them to divide, add, or subtract fractions. Numerical results might cause confusion and misinterpretations according to the problem's logical level (Baidoo, 2019). In addition, Baidoo (2019) indicates that the mistake might have been brought on by a challenge that was too complex for the learner to handle. Baidoo (2019) presents the explanation of examples of misconceptions that learners have simplified as follows:

1) $\frac{4x^3}{5} + \frac{x^2}{4} + \frac{x}{3} = \frac{6x^6}{12} = \frac{1}{2}x^6$

Example 1 shows the misconceptions of adding algebraic fractions; unlike terms were added on the numerator and the first law of exponents was applied. The learner lacks knowledge of principles required to simplify algebraic fractions involving exponents. The learner failed to find the LCD since denominators were added, thus leading to misconceptions.

2) $\frac{5}{x+3} - \frac{3}{x-2} = \frac{2}{2x} = \frac{1}{x}$

Example 2 shows the lack of procedure in simplifying fractions. This was the misconception of adding and subtracting numerators and denominators which relates to operations and mathematical symbols. The rule of finding the LCD was misunderstood.

3) $49x^2 - \frac{y^2}{36}$ = 49-36 ($x^2 - y^2$) = 23($x^2 - y^2$) = $23x^2 - 23y^2$

In example 3 the learner assumed that the negative sign between the two terms means subtraction of constants and variables. The learner could not find the LCD and lacks prior knowledge of difference of two squares. This is attested by Smith (2021a) who describes a misconception as an incorrect application of the law, an excessive or inadequate generalisation, or an alternate interpretation of the circumstances. Pritchett (2021) argues that errors may be the consequence of negligence or incorrect sign or text interpretation. These misconceptions manifest errors that can affect learners' ability to grasp concepts accurately. Hence misconceptions and errors are two different concepts which are related but different. In addition, in their examinations, learners show misconceptions while dealing with algebraic fractions and find the simplification of such fractions to be difficult (Lamon, 2020; Mhakure et al., 2014). This occurs when learners do not conceptualise algebraic fractions.

1.4 Purpose of the Research

The purpose of this study was to explore Grade 10 learners' misconceptions and errors manifested when simplifying algebraic fractions.

The findings of this research provide empirical evidence which might assist learners to navigate the wide range of learning strategies to simplify algebraic fractions.

1.5 Research Questions and Objectives

1.5.1 Main research question

What misconceptions and errors manifest when Grade 10 learners simplify algebraic fractions?

1.5.2 Secondary Questions

1. What categories of errors do learners manifest when simplifying algebraic fractions?
2. What misconceptions cause the manifested errors?
3. What are ways of reinforcing learners' learning of algebraic fractions?

1.5.3 Aim and Objectives of the Study

1.5.3.1 Aim of the Study

The aim of this investigation was to outline and describe categories of errors and misconceptions that manifest when Grade 10 learners simplify algebraic fractions. The aim was achieved through the following objectives:

1.5.3.2 Objectives of the study

- Establish the categories of errors that learners manifest when simplifying algebraic fractions.
- To uncover the root cause of misconceptions.
- To suggest ways in which learners' learning of algebraic fractions could be reinforced.

In other words, it sought to gain an understanding of:

- The categories of errors and misconceptions that manifest when learners attempt to simplify algebraic fractions.
- How learners work out algebraic fractions.

1.6 Methodological Considerations

1.6.1 Research Approach

This study employed a qualitative approach to explore Grade 10 learners' misconceptions and errors when simplifying algebraic fractions. Qualitative research comprises collecting, analysing, and interpreting non-numerical data (Susanto et al., 2024). It is a framework that includes stages and procedures for gathering, analysing, and interpreting data (Lim, 2024).

1.6.2 Research Design

Research design is the practice of gathering significant research findings in accordance with a particular design (Polgar & Thomas, 2020). I adopted a case study design to accomplish the research's goals and objectives. The research design assisted in analysing trends for comprehension of its behaviours and situations (Tomaszewski et al., 2020).

1.6.3 Research Site, Population and Sampling

The population of this study constituted learners from one high school in Mawa circuit, Mopani West District in Limpopo province. From a population of 113 Grade 10 learners, 36 learners who were studying mathematics were purposively sampled to participate in the study. The study employed a qualitative research method.

1.6.4 Data Collection Instrument and Process

I used the test (Appendix O) and interviews (Appendix Q) to collect data in the study. Learners wrote a test on algebraic fractions. From 36 learners who wrote the test, nine learners were further selected for interviews. The selection was based on the findings from the test. These participants were selected to check how well they understood algebraic fractions and probe their solutions. The selection was done as follows: a) Learners whose test results indicate the knowledge and skills required, b) Learners who missed few steps in solving the algebraic fraction problems, and c) Those who could not answer the questions correctly.

1.6.5 Data Analysis and Interpretation

The analysis of data was led by the study's research questions, literature that deals with the study's topic (misconceptions and errors in algebraic fractions) and themes that emerged from the study. The themes that emerged from the study were: (i) Incorrect ways of applying rules - addition and subtraction of fractions; (ii) Minimal competence in dividing algebraic fractions, and iii) Difficulty in analysing word problems. Data were analysed using categories of error that manifested during the test and the interviews after data were collected. The study's findings were used to suggest strategies to improve learners' learning of algebraic fractions and recommendations for future work.

1.7 Ethical Considerations

I obtained permission to conduct research from the Research Ethics Committee at the University of South Africa (UNISA) (Appendix B), Limpopo Provincial Department of Education (Appendix D), Mopani West District Director (Appendix E), learners (Appendix L), and the school principal (Appendix G) where the study was conducted. Permission letters explained my role, the data collection process and the analysis of data. Parents of learners who participated in the study were also requested to permit their children to participate in the study in writing by completing the consent form (Appendix J). After completing that process, I went into the field and gathered data based on the conditions specified in the ethics approval letter. I assigned pseudonyms to the participating learners like Learner 25B, Learner 1F etc. The aim of pseudonyms was to protect learners. I collected data during normal school hours and the interviews were collected three days after the I had analysed the learners' scripts.

1.8 Definitions of Terms

Many definitions have been proposed for the concepts of errors and misconceptions; however, for the purposes of this study, an error is defined as a perceived erroneous procedure which employs incorrect rules and procedures, and a misconception as a mathematical claim that a learner perceives about a specific concept and contends to be true. A study by Khalid et al. (2020) and Cholily et al. (2020) reveal that learners' poor comprehension of the algebraic concepts lead to negative experiences of learning the topic. The following section defines phrases used in the study and are written in bold. They are explained and clarified as follows:

a. Roberts and Allen (2022) indicate that **misconceptions** related to fractions arise when learners have erroneous conceptual understandings of fractions, such as failing to recognise fractions as parts of a whole or using procedures incorrectly. In addition, **misconceptions** in fractions are defined by Zheng and Geller (2021) as cognitive errors arising when learners misunderstand the relationship between the numerator and denominator, hindering their ability to add, subtract, or compare fractions correctly. Furthermore, **misconceptions** about fractions are defined by Thompson and Seitz (2020) as an inaccurate knowledge or belief regarding the characteristics of fractional numbers, which frequently results in errors committed when performing calculations and tackling mathematical tasks. The authors emphasise that addressing misconceptions is essential to the learning process since they can significantly hinder the ability of learners to comprehend more advanced mathematical concepts. Misconceptions often occur due to learners' prior knowledge, experience or incorrect reasoning.

b. Jones and Smith (2020) define an **error** in mathematics as a mistake committed during the problem-solving process, which frequently emerges from incorrect applications of mathematical concepts, misunderstandings of problems or miscalculations. In addition, Brown and Patel (2021) define it as differences in mathematics between anticipated and executed outcomes, which may arise due to various reasons, including misconceptions, computational errors, or logical fallacies in reasoning. Furthermore, Kim and Lee (2022) maintain that mathematical **errors** can be categorised into procedural errors, conceptual errors, and computational errors, each impacting learning and comprehension of mathematical concepts differently. Moreover, Cai and Nie (2020) define an error as the misinterpretation or incorrect implementation of mathematical concepts leading to incorrect solutions or answers. Although the term has been defined in a variety of ways, this study did not ignore the idea that errors are regarded as a fundamental aspect of learning as they provide constructive feedback and opportunities for improvement (Stiggins, 2020). As Prince (2020) argues, important ways in which learners form a conceptual structure of knowledge arises from active learning. Learners can develop their knowledge by participating in active activities that encourage analysis, synthesis, and assessment of the concepts taught in class.

c. According to Cohen and Lee (2020), an **algebraic fraction** is one that has algebraic expressions in both the numerator and denominator, frequently containing variables and constant coefficients. Both authors additionally acknowledge that these fractions may be regarded as numerical fractions and serve as essential for clarifying relationships in algebra. In addition, Torres and Gupta (2021) define an **algebraic fraction** as the quotient of two polynomial expressions, which enables operations that are comparable to those performed on rational numbers and allows for the inclusion of variables and constants. Furthermore, Wilson and Harris (2022) define **algebraic fractions** as mathematical expressions derived from the division of one polynomial by another, often employed in algebraic equations to denote relationships and simplify complicated expressions.

1.9 Limitations of the Study

Creswell and Poth (2022) define limitations as constraints and restrictions that might affect the scope, validity, or generalisability of research findings or hypotheses. Theofanidis and Fountouki (2019) maintain that to help the reader comprehend how applicable and reliable the study's conclusions are, the study's limitations should be highlighted. The study employed a qualitative research methodology; hence, it required a smaller sample size. The study was conducted at one high school with just one Grade 10 class doing mathematics due to the chosen research methodology and resource limitations.

The main objective in conducting this study was to acquire greater insight into the misconceptions and errors which learners in Grade 10 manifest when attempting to simplify tasks involving algebraic fractions. However, considering the following limitations, the study's conclusions cannot be applied generally. For instance, although many other mathematical concepts were covered in the curriculum, the research focus was limited to algebraic fractions.

1.10 Significance of the Study

The study has the potential to improve learning outcomes and enhance the quality of mathematics education for Grade 10 learners and beyond, and the following sectors of the society, namely:

i) **Learners:** By clearing up frequent misconceptions and errors, educators may assist learners better grasp algebraic fractions and provide them with access to more enlightening examples and explanations. In addition, the study can encourage learners to focus on laying a solid algebraic foundation for future study in mathematics and related subjects like STEM.

ii) **Educators:** The study may assist educators in designing better assessments that go beyond testing procedural knowledge to assess learners' comprehension of algebraic fractions. In addition, educators can also better educate learners for the mathematics requirements of college and future employment.

iii) **Department of Basic Education:** To improve the accessibility and understandability of the algebra curriculum for learners, the DBE can provide curriculum developers with information about areas that may require additional attention or clarification. By providing useful data to stakeholders and policymakers in the education sector, learning tools like workbooks, textbooks, and online materials can be improved.

iv) **Educational psychology**: By examining the fundamental causes of misconceptions and errors, researchers can acquire understanding of how learners process and absorb concepts related to mathematics. This information can influence theories of learning and help create more successful teaching strategies for mathematics education.

1.11 Organisation of Dissertation

An outline of the study is provided as follows:

Chapter 1: Introduction to Study

This chapter begun by introducing the background of the study by providing a quick summary of its justification, issue description, purpose, relevance, and research questions. The concepts of the study have also been made clearer.

Chapter 2: Literature Review

To delve into the issues with mathematical errors and misconceptions and how they could be employed to improve education, I describe some of this study's literature research. The theoretical background of the investigation is described.

Chapter 3: Research Design and Methodology

In this chapter, I discuss the methodology employed in this research design. The chapter also addresses data collection, data processing, and the sampling technique. In addition, the relevant ethical issues are covered.

Chapter 4: Presentation of Data

In Chapter 4, I present the findings of the data collected. The test results are also presented in more detail.

Chapter 5: Discussion, Summary, and Conclusion of the Study

In this chapter I discuss the conclusions after looking at the gathered data. The research questions are addressed in this chapter and examine the study's conclusions. The theoretical framework's utility is also considered. Towards the end of the chapter, the limitations of the study are examined.

1.12 Chapter Summary

This chapter provided background information, discussed problem statement, explained the study purpose and outlined the research questions. It also presented how the chapters are outlined. The next chapter discusses the literature review and theoretical framework.

Chapter 2
Literature Review

2.1 Introduction

A review of the literature incorporates the findings of previous studies to provide an in-depth analysis of the literature pertinent to a particular research issue, hence strengthening the foundation of knowledge within the current studies (Paul & Criado, 2020). To enable comparisons and contrasts, pertinent literature from previous research is carefully chosen and incorporated into the current research (Paul & Criado, 2020). Hart (2018) describes a literature review as an efficient assessment of chosen works on a particular research topic. It serves as a reference for researchers on the subject's prior study (Khanyile, 2016). Moreover, it helps the researchers by educating them on the methodology, reasoning, research questions, and conclusions of related previous studies. In other words, the gap identified from literature assists in modifying the research question.

This chapter investigates research on the misconceptions and errors that arise when simplifying algebraic fractions tasks. The first section focuses on the concept of algebraic fractions, the second examines the nature of misconceptions and errors, followed by categories of errors and misconceptions, complexities in learning algebraic fractions, the root cause of errors and misconceptions, and the last section presents the gap identified in the literature. The chapter also covers the theoretical framework upon which the study is based. The purpose of the review is to explore the misconceptions and errors that Grade 10 learners manifest when simplifying algebraic fractions tasks. The next section describes the concept of algebraic fractions.

2.2 The Concept of Algebraic Fractions

Lamon (2020) defines algebraic fractions as quotients of two terms. These terms consist of numbers and variables which can be monomials, binomials or polynomials. Epstein (2024) indicate that if the degree of the polynomial in the numerator is smaller than the degree of the polynomial in the denominator, the fraction is said to be properly algebraic. {e.g., $\frac{x-1}{x^2-1}$ }; if the numerator is of a higher degree, it is improperly algebraic. $\{\text{e.g.}, \frac{x^2-y^2}{x-y}\}$. In the study conducted by Jackson and Peterson (2021), it is argued that algebraic fractions are difficult for learners to master since they demand comprehension of mathematical ideas including exponents, division, multiplication, equations, perfect squares, and rational numbers. This argument seems to suggest that learner performance in algebraic fractions could be hindered if learners do not have a grasp of the mentioned concepts.

In terms of categorising algebraic fractions, Maphini (2018) describes different types as follows: 1) *Rational algebraic fractions* have rational polynomials as both the numerator and the denominator. For example: $\frac{x^2-2x+1}{x-1}$, where the denominator will never be zero. 2) *Complex algebraic fraction* is described as a fraction which has more than one fraction on both the numerator and denominator. For example: $\frac{\frac{5}{x+2}-\frac{3}{x+3}}{\frac{2}{x+3}-\frac{1}{x+2}}$ and 3) Irrational algebraic fractions are expressions that involve a polynomial in the denominator, where at least one of the variables or terms under the root is irrational (Kumar, 2021). For example: $\frac{x^{\frac{1}{2}}-\frac{1}{3}a}{x^{\frac{1}{3}}-x^{\frac{1}{2}}}$.

Several researchers have studied difficulties learners face in learning algebraic fractions. For instance, Mangwende (2021) conducted an empirical study on the relationship between learners' proficiency in common and algebraic fractions in Grade 10 and realised that learners who had mastered the arithmetic of common fractions were more capable when dealing with the arithmetic of algebraic fractions. According to Smith (2022), arithmetic algebraic fractions are expressions with a polynomial numerator and denominator. Wang and Lee (2021) define arithmetic algebraic fractions as an expression created by dividing two polynomials, in which learners must use their comprehension of algebraic manipulation, such as factorisation, common denominators, determining LCD, and simplification processes. Maphini (2018) focused on implementing an international teaching model to investigate Grade 9 learners' ways of working with rational algebraic fractions and concluded that learners rely on their comprehension of laws of simplifying exponents when they simplify algebraic fractions as they struggle to add and subtract fractions. Baidoo (2012) also conducted a study on algebraic fraction simplification focusing on mistakes committed by Grade 10 mathematics learners and found that learners were unable to simplify algebraic fractions due to conceptual errors. Although the main finding is that learners rely on the learnt rules to simplify fractions, it is notable that these studies emphasise the significance of focusing on misconceptions and errors to improve learners' comprehension and performance when simplifying algebraic fractions, which eventually contributes to enhancing outcomes in algebra learning.

For this study, I focused only on investigating misconceptions and errors exhibited when learners simplify tasks involving rational algebraic fractions. These fractions use operational signs like addition, subtraction, multiplication and division to express the relationships between numbers and variables. They also incorporate the LCD, Highest Common Factor (HCF), exponents and variables. By examining errors and misconceptions together, learners and educators can develop a comprehensive view or holistic understanding of the learning process. The next section provides an overview of the concepts: errors and misconceptions.

2.3 Misconceptions Versus Errors

For Brown and Smith (2020), an error is a sort of divergence from what is correct or a type of variance from what has already been discovered. This definition emphasises the idea that errors reflect the misunderstanding or misapplication of rules. In other words, an error can be defined as a range of discrepancies that support something that is thought to be true, as previously observed. Errors can result from learners misunderstanding a deceptive concept for a genuine one, which leads to misconceptions when it is repeated and ingrained in learners' understanding (Smith & Wiser, 2021). Jansen and van der Meijden (2020) define misconception as the incorrect application of a rule, an overgeneralisation or under generalisation that leads to alternative interpretations of mathematical situations.

For Biney et al. (2023), a misconception refers to an incorrect belief or perspective that results from an error of understanding. Misconceptions lead to significant learning challenges in mathematics when learners attempt to utilise their insufficient prior instruction, flawed reasoning, and false memories when solving tasks (Rittle-Johnson & Schneider, 2020). A further definition is given by Pillow and Davis (2021), who describe mathematical misconceptions as parts of a learner's structure that are not mathematically valid and cause them to convey false information. In a similar vein, Im and Jitendra (2020) argue that misconceptions may stem from prior knowledge that people have wrongly generalised.

It appears that these definitions and arguments imply that misconceptions and errors are two concepts which are related but different (Lamon, 2020). Misconceptions often emerge because of inappropriate learners' prior learning experiences that is often incorrect or insufficient (Karpudewan et al., 2020). Learners usually perceive what they are doing to be correct or are uncertain about what they are doing (Neidorf et al., 2020; Rushtons, 2014). What I have picked up from the definitions in the previous paragraph is that such mathematical errors occur when the person who commits them thinks that what was done is correct, which is a sign of faulty logic. In addition, learners make these errors due to a lack of knowledge learnt from lower or previous grades. To simplify tasks involving algebraic fractions, learners should master basic concepts such as exponents, variables, and coefficients.

All the ideas shared in this section suggest that to explore more on the concepts of errors and misconceptions, I need to look for patterns of errors exhibited when learners simplify algebraic fractions. Another argument brought by Indraswari et al. (2018) is that learners struggle to use algebraic reasoning to analyse and construct logical arguments. This study supports the findings of Smith and Wiser (2021) who contend that learners may make errors by mistaking a false notion for an actual idea. This can cause misconceptions when the concept is repeated and becomes embedded in the learner's thought. Furthermore, Smith et al.(2022) study emphasise that procedural errors frequently result from a combination of views based on previous mathematical concepts and knowledge gaps. Rosenbaum et al. (2024) back up the claim that mastering methods for solving tasks requires a combination of knowledge, critical thinking, and process review. For example, learners make errors because they misinterpret the concepts in problem-solving activities. Williams (2022) argues that a lack of fundamental conceptual knowledge causes many learners to struggle. Failure to understand the meaning of the concept by the learner will result in the learner making errors repeatedly.

Considering all that has been discussed in this section, it is possible that learners themselves, educators, textbooks, the context, and classroom practices could be potential sources of the misconceptions. To support this view, Smith (2021d) reveals that the textbook is the main source of information in classrooms, and this causes learners to have serious misconceptions. This understanding may arise from the author's viewpoint on the concept, which may not be evident to the reader. If there are misconceptions in their early conceptualisation, those misconceptions will be reflected in subsequent products and will cause errors.

There are two different kinds of errors, namely: i) Systematic, and ii) Unsystematic errors (Thompson & Lee, 2021). Unsystematic errors are usually simpler to rectify and are caused by misconceptions or errors, such as incorrect computations or misunderstanding of the task (Thompson & Lee, 2021). The two authors proceed, contending that systemic errors can seriously impede a learner's development and are caused by inaccurate formulae, rigid conformity to incorrect procedures, or flaws in problem-solving tasks. Hattie and Donoghue (2021a) point out that learners should be able to remedy these blunders if given a second chance. These errors may be the result of working memory overflow, haste, or irresponsibility.

Errors include miscalculations, mistakes, and poor judgement which can affect the overall learning outcomes (Garcia & Patel, 2022).

Although differences of opinion still exist, these misconceptions relate to important aspects that learners exhibit in their comprehension and problem-solving processes (Suseelan et al., 2023). The two authors assert that misconceptions play a crucial role in determining how learners' approach mathematical concepts and can impede their progress if they are not promptly and correctly addressed. Misconceptions are misinterpretations and assumptions that stem from erroneous information, significantly affecting learners' comprehension (Ngu & Phan, 2023). Through my observation and teaching expertise, I agree with Ngu and Phan that these misconceptions inhibit learners' learning abilities and prevent them from fully understanding algebraic fractional ideas during the learning experience.

Contrary to errors, misconceptions signify a lack of comprehension and an improper application of mathematical generalisation (Basar et al., 2021). Rismayanis et al. (2022) argue that lack of comprehension of concepts in mathematics will result in it being challenging for learners to master the subject, solve tasks, and apply it in everyday life. In addition, the capacity to develop strategies for solving tasks, do basic computations, employ symbols to convey concepts, and modify the form when learning mathematics are all examples of comprehending mathematical concepts (Rismayanis et al., 2022). Although one may classify all misconceptions as errors, not all errors may be misconceptions because misconceptions are a specific type or category of errors (Gosh et al., 2022). This supports findings by Khan and Ahmed (2022) that learners find it difficult to relate algebraic concepts since they do not have a conceptual knowledge of it. This is corroborated by Nguyen and Pruitt (2022) who highlight how learners' difficulties with mathematical language led to inaccurate task comprehension. In other words, a misconception leads to an error, or a misconception is a certain kind of systematic view that leads to an error. On the other hand, errors can be described as incorrect answers or mistakes that are consistent, intentional, and repeated (Baidoo et al., 2020).

Deficits in conceptual understanding, erroneous reasoning processes, and erroneous conceptual generalisations lead to numerous kinds of errors and misconceptions (Biney et al., 2023). Binney et al. (2023) note that learners often prioritise memorisation of theorems, formulas, and algorithms, which might result in errors and misconceptions. Important to mention, Ernest (2018) suggests that making errors and having misconceptions is a normal aspect of learning mathematics. As per Rahman and Smith's (2022) study, learners encounter difficulties in accurately representing algebraic expressions because of calculation errors, which can lead to encoding errors. Key to grasping concepts, as highlighted by Von Glaserfeld's (1995) theory, a child learns through the interactions between previous assumptions and recently taught concepts, which is consistent with constructivist theory. In other words, acquisition of knowledge does not imply memorisation of rules. Consequently, instrumental learning may constitute the primary root cause of errors and misconceptions. This idea is reinforced by Turner (2022) who highlights that immediate feedback rectifies misunderstandings and promotes progress in learning outcomes. Furthermore, a children's thoughtless memory and recall of mathematical formulas may be caused by their incapacity to generate and recreate ideas. The next section discusses an overview of various categories of errors and misconceptions.

2.4 Categories of Errors and Misconceptions

Table 2 presents eight categories of errors with explanations of how learners solve algebraic problems (Makonye & Khanyile, 2015, p. 62).

Table 1

Eight Categories of Errors with Examples and Explanations. (Learners were expected to simplify the algebraic fractions fully)

Category of error	Example(s): Simplify	Learners' solution and explanation
1. Cancellation error	1.1 $\frac{12+6x}{3}$ 1.2 $\frac{4y-8}{2y}$	= 4+2x Each term on the numerator was divided by 3 and cancelled out. The variable y was cancelled and each term on the numerator was divided by 2, then 2-4 = -2 was written as the answer.
2. Confusing error	2.1 $\frac{3a^2+27}{a^2+9}$ 2.2 $\frac{2x^2+8}{x^2+16}$	$= \frac{3(a+3)(a+3)}{(a+3)(a+3)}$ =3 Two binomials were written and cancelled, and the common factor 3 was taken out as the answer. $= \frac{(2x+4)(2x+4)}{(x+4)(x+4)} = 4$: multiplying 2 x 2 was left on then numerator by cancelling out $x+4$ on both numerator and denominator.
3. Common factor error	3.1 $\frac{4x^2+8x}{8x}$	= $4x^2$ 8x was eliminated from the numerator and denominator.
4. Lowest Common Denominator error	4.1 $\frac{4}{x^2y^2}+\frac{3}{xy}-\frac{2}{x}$	LCD = x $\frac{4}{x^2y^2}.\frac{x}{x}+\frac{3}{xy}.\frac{x}{x}-\frac{2}{x}.\frac{x}{x} = \frac{4x}{x^3y^2}+\frac{3x}{x^2y}-\frac{2x}{x^2}$ Each term on the numerator and denominator was multiplied by x.
5. Trinomial factorisation error	5.1 $\frac{4}{x^2-8x+16}+\frac{2}{x^2-4}$	The learner was unable to factorise the trinomial and wrote: $=\frac{4}{(x-8)(x-2)}+\frac{2}{(x-2)(x+2)}$
6. Careless error	6.1 $\frac{2x^2}{x^2}+\frac{1}{x}$	$= 2+\frac{1}{x}$. x^2 on the numerator and denominator in the first term were cancelled.
7. Procedural error	7.1 $\frac{x^2-9}{x^2-5x+6}$	$=\frac{(x+3)(x+3)}{(x+2)(x+3)} = \frac{3}{2}$, the learner eliminated terms on the numerator and denominator due to lack of prior knowledge.
8. Dropping denominator Error	8.1 $\frac{3}{x+2}-\frac{2}{(x+2)^2}$	$= \frac{3}{x+2}-\frac{2}{(x+2)(x+2)}$ $= \frac{1}{(x+2)}$ Numerators and denominators were subtracted, and denominator x+2 was dropped.

Source: Makonye, J., & Khanyile, D. (2015, p. 62). V (2). Copyright: 2022 by researcher

Table 2 presented solutions from learners along with examples for each of the eight categories of errors(Makonye & Khanyile, 2015). The research findings demonstrate that most learners struggle with algebraic fraction simplification. Prior to proceeding on to simplification, learners were supposed to factorise in Questions 1.1, 1.2 and 3.1 by determining the common factor first. Yet, learners were unable to work it out as they were unable to comprehend the concepts they had learned in previous grades. To simplify the algebraic fraction for Questions 4.1, 5.1, 6.1 and 8.1, learners had to first determine the LCD, but they struggled to do so. For instance, in Question 8.1 the learner subtracted the numerator and the denominator, by ignoring the brackets. In support of their claim that brackets are not necessary, Papadopoulos and Gunnarsson (2020) emphasise that "the avoidance of brackets when writing arithmetic expressions does not always necessarily mean lack of structural understanding" (p. 206) and suggest that this is an implicit use of "mental" brackets. Similarly, for Question 7.1, learners had to factorise the numerator and denominator, but they were unable to determine the factors on both the numerator and the denominator. For instance, in Question 7.1, the learner eliminated terms inside the brackets without realising that terms inside the brackets are treated as a single entity. This notion is supported by Stemele and Jina Asvat (2024) who state that the terms enclosed in parenthesis ought to be regarded as a single unit. These misconceptions hindered learners from fully simplifying the fraction. This seems to suggest that learners should work in groups or in pairs to assist each other to grasp the fundamental concepts of fractions, especially fractions that involve variables and numerals, and how to determine the LCD. Jones and Carter's review from 2022 highlights the value of collaborative learning models and suggests that learners gain from group discussions which assist them to clarify their understanding and eliminate misconceptions. Addressing these errors and misunderstandings can help learners become more confident and enhance their competence with algebraic fractions. **Table 3** further delineates the categories of errors and the nature of learners' mistakes Baidoo et al., (2020, pp. 105-108).

Table 2

Categorisation of Errors Using some Examples of Learners' Answers

Question and learners' answer	Analysis of learners' solution	Category of answer
1. Simplify		
1.1 $\frac{2x-5}{3} - \frac{3x-2}{4}$ $= \frac{(4\,.\,3)^{x+1}}{2^{2x}\,.3^{x}}$ $= \frac{2^{2(x+1)}\,.\,3^{x+1}}{2^{2x}\,.3^{x}}$ $= \frac{2^{2x+2}\,.\,3^{x+1}}{2^{2x}\,.3^{x}}$ $= \frac{2^{2}.3^{1}}{2.3}$ $= \frac{12}{6}$ $= 2$	Many rules were incorrectly applied which resulted from a lack of understanding on how to apply the skills and concepts learned.	**Application error**
1.2 $\frac{10^{x-1}.12^{x+1}}{8^{x}.15^{x-1}}$ $= \frac{2^{-1}.\,5^{x}.4^{x}.\,3^{1}}{2^{1}.\,4^{x}.5^{x}.3^{-1}}$ $= 2^{-1-1}.5^{x-x}.4^{x-x}.3^{1+1}$ $= 2^{-2}.5^{0}.4^{0}.3^{2}$ $= 2^{-2}.1.1.9$ $= \frac{9}{4}$	Each base was attached with an exponent which caused a misconception. That showed a lack of properties to simplify the question.	**Conceptual error**

Source: Baidoo, J., Adane, M., & Luneta, K. (2020, pp.105-108). Vol (2). Copyright: 2022 by the researcher.

Tables 2 and 3 are good illustrations of how learners' errors in algebraic fractions are frequently attributed to misconceptions and errors learners commit. As a such, it is necessary to explore misconceptions and common errors learners make when simplifying algebraic fractions tasks. In line with the categories of errors presented in **Tables 2 and 3**, the following are other examples of categories of errors. For instance:

Procedural error: Learners tend to mix up rules and formulas due to the content gap and lack of procedural knowledge to simplify the question. The learner added the exponents and multiplied the bases. Example: Simplify $\frac{10^{n-1}.12^{n+1}}{8^{n+1}.15^{n-1}} = \frac{120^{2n}}{120^{2n}} = 1$

Careless error: Learners have a required knowledge of solving algebraic fractions but slip and write incorrect answers by just cancelling the numerator and denominator in the second term. Example: Simplify $\frac{1}{x} + \frac{\cancel{x}}{\cancel{x}} = \frac{1}{x}$. The learner committed the error because they cancelled the numerator and the denominator. The learner assumed that the solution for the second term is zero and wrote the wrong solution. Kaminski et al. (2023) reinforce this notion by pointing out that high school learners frequently have misconceptions about equivalent forms of fractions.

Conceptual error: Due to their inability to connect prior information to current knowledge, learners struggle to understand the topic which underpins the topic concern. In simplifying the example $\frac{4x^2-16}{x^2-x-4}$, the learner wrote the solution as: $\frac{4(x^2-4)}{(x-2)(x-2)} = \frac{4(x-2(x-2)}{(x-2)(x-2)} = 4$.

The learner could find the HCF on the numerator but was unable to find the difference of two squares and to factorise the trinomial on the denominator because of lack of conceptual knowledge. Ahmad and Nasir's (2023) analysis of the literature emphasise how common errors are made by learners who try to factor or expand algebraic expressions in their basic knowledge. A review by Fitria and Susanto (2023) support this claim by emphasising the value of clear instructions regarding the relationship between expanding and factoring, stressing that misconceptions in these areas result in more significant difficulties with simplifying tasks. Wilson and Cheng's (2022) literature analysis continue to address misconceptions through encouraging more profound conceptual understanding in learners.

Application error: Learner knows and understands how to work with factorisation, division of fractions, and exponents but lacks understanding of how to apply them. In simplifying the following example: $\frac{16x^4y}{27z^3} \div \frac{4xy^2}{3z^2}$,the learner wrote the solution as: $\frac{4x^3y}{9z}$. The learner applied the second law of exponents by dividing the numerators and denominators, which is the wrong procedure for simplifying the task.

These errors suggest the likelihood that learners have conceptual gaps and tend to overgeneralise rules. More challenges and complexities that learners normally encounter are presented in the next section. These complexities can arise in various areas.

2.5 Complexities in Learning Algebraic Fractions

Most learners are not able to factorise trinomials, sum and difference of two squares as well as factorising by grouping. Learners find it difficult to find the LCD in simplifying tasks. The challenges experienced by learners to manipulate problems exist because it is necessary for learners to comprehend mathematical ideas including division, variables, exponents, perfect square factorisation, and rational numbers (Zulfa et al., 2020). This difficulty learners have in manipulating these tasks has a negative impact on arithmetic performance, which has an impact on learner attainment.

Gayu et al. (2021) indicate that there are other factors that lead to learning issues with fractions that are not inherent to common fractions but are rather influenced by cultural norms and educational system attributes. The authors also stress the fact that these cultural influences apply to all of mathematics, not only rational number arithmetic. A deep approach to learning and comprehending mathematics is taken by learners who have a good conceptual understanding of fractions, whereas a shallow approach is taken by learners who have a weak mental understanding of fractions (Cai & Wang, 2020).

The knowledge gap arises from the fact that algebraic fractions are frequently expressed using symbolic and mathematical language (Kaur et al., 2021). In addition, studies on misconceptions revealed that learners who have acquired a different perception cannot be assisted by strategies like redoing a lesson to highlight a point or misconceived concepts in mathematics (Zuza et al., 2022). My expertise as a mathematics educator, giving learners more similar tasks did not help to reduce the misconception and errors they were making during task solving. What they normally do is just to memorise rules and facts (Naseer & Hassan, 2014).

Learners lack knowledge pertaining to fundamentals of mathematics like simplification and thus could not reach a particular solution (Adams et al., 2021). The learners could not simplify by using identities and had problems pertaining to simplification with factorisation. Each learner should respond to a question or task in mathematics by finding the solution, which requires more in-depth analysis and reasoning (Hansen, 2022). One strategy to enhance a learner's capacity for mathematical problem-solving is to provide problems frequently. According to a common belief, every problem has an enabling circumstance. Finding connections between current circumstances and prior experiences, as well as taking appropriate action to address the situation at hand, constitute the process of problem-solving (Polya, 2021).

Moreover, Polya (1946) defines problem solving as the endeavour to find or look for a solution to any challenge that arises in achieving a goal that cannot be attained right away. One of a learner's core mathematical abilities is the capacity for problem-solving. A learner is anticipated to be capable of enhancing their capacity for simplifying mathematical issues through a problem-solving type test. According to Bransford et al. (2020), a learner cannot be regarded to have learned anything meaningful unless they have the competence to use the knowledge they have acquired to solve problems. Similarly, Muchoko et al. (2019) argue that the general difficulties that learners face while solving algebraic expressions are that of simplification and factorisation. In addition, Ahmad and Nasir (2023) emphasise the prevalence of errors in their foundational knowledge when learners attempt to expand or factor algebraic expressions. To support this argument, Fitria and Susanto (2023) stress the importance of explicit instructions on the connection between expanding and factoring, noting that misunderstanding in these areas leads to broader challenges in problem solving. The findings demonstrated that learners struggle to understand the distributive and associativity properties. Thus, the current study aimed to explore how Grade 10 learners simplify tasks involving algebraic fractions.

My experience as the DH of mathematics helped me observe that learners commit these errors because they lack basic knowledge from lower grades and have a content gap because of curriculum content not covered by educators. For example, in simplifying the problem: $\frac{13x^2y}{20b^3} \div \frac{4xy^2}{5b^2}$, the learner wrote the solution as: $\frac{13x^2y \div 4xy^2}{20b} = \frac{3{,}25xy}{20b}$. The concept of division as it relates to calculating algebraic fractions was not understood by the learner. Despite knowing the rule, the wrong denominator was chosen. It is clear that numerous researchers have put a lot of effort into finding solutions to the issue of errors and misconceptions made by learners when simplifying algebraic fraction tasks; my obligation at this point is to contribute to both what has been accomplished and what has not.

Jones and Lee (2020) state that learners' learning difficulties often arise from their inability to understand the foundational concepts that underpin various procedures. I agree with Jones and Lee since learners find it difficult to simplify tasks involving algebraic fractions. Drawing from my experience as an educator, when simplifying the algebraic fraction $\frac{10^{2x-3}.4^{1-x}.8^x}{25^{2+x}}$, the learner would often present the solution as follows:

$$\frac{10^{2x-3}.4^{1-x}.8^x}{25^{2+x}} = \frac{5^{2x}.2^{-3}.4^1.4^{-x}.4^x.2}{5^{2x}.5^2} = 5^{2x-2x-2}.2^{-3+1}.4^{1-x+x} = 5^{-2}.2^{-2}.4^1 = \frac{1}{25}.\frac{1}{4}.4 = \frac{1}{25}$$

The learner did not attach each base with the binomial exponent, each base was attached to a single term of the exponent which led to misconception. This showed learners’ lack of understanding in changing of bases to its simplified form required to simplify the algebraic fraction. The learner committed a conceptual error because of absence of principles underpinning the topic of algebraic fractions. It is apparent from the literature that misconceptions cause learners to make errors (Baidoo et al., 2020). In simplifying the following example:

$$\frac{5}{(y+5)(y+5)} - \frac{3}{(y+5)(y-5)} - \frac{2}{(y+5)(y-5)}$$

the learner wrote LCD = (y+5), and the solution as follows:

$$= \cancel{(y+5)}\text{x } \frac{5}{(y+5)\cancel{(x+5)}} - \cancel{(y+5)} \text{ x} \frac{3}{\cancel{(y+5)}(y-5)} - \cancel{(y+5)} \text{ x} \frac{2}{\cancel{(y+5)}(y-5)} = \frac{5-3-2}{(y+5)(y-5)(y-5)} = 0$$

In the example, the learner knew that they must first find the LCD when simplifying rational algebraic fractions and multiplied each term by the LCD. The learner subtracted the numerator and cancelled the common factors. Hence the LCD error was identified. The misconception on simplifying algebraic fraction led the learner to a cancellation error.

Thompson et al. (2021) assert that it requires cognitive connections throughout several cognitive domains to grasp rational numbers, allowing for deeper comprehension and applications. Given the intricacy of the situation, several constructivist-based solutions that emphasise meaningful instructions would be advantageous for overcoming misconceptions and errors. In addition, Ali (2021) claims that algebra's long history makes studying it challenging and that it took 3000 years for it to emerge. Furthermore, a lengthy period was required for the development of algebra (Wang & Lee, 2021). As a result, the transition from learning fractional concepts to learning algebraic concepts is a tough process that encompasses a lot of sub-domains that learners find challenging. Each of the important subjects and related research findings must be considered to comprehend this intricacy. Cavanagh (2021) points out that algebra is necessary for the full development of real numbers because arithmetic appears to require them for trouble-free operation.

Moreover, Cokley and McClain (2020) note that most learners in Grades 8 and 9 have difficulties comprehending concepts in mathematics. Most learners stop learning mathematics at higher levels for this reason. If learners do stop learning mathematics, then misconceptions are not cleared up, learners will persist in making errors even at university (Baker & Leach, 2021). By outlining algebra's fundamental premise when working with arithmetic before learning formal algebra, these errors may be avoided (Booth & Newton, 2020). As a result, algebra may not be wholly bound to arithmetic and is shown to be not limited to the realm of numbers.

Kumar and Gupta (2021) suggest that when simplifying algebraic fractions and rational algebraic fractions, learners use rote learning. If algebraic fractions are accessible to all, a bridge connecting arithmetic and algebraic fractions must be constructed. Conceptual knowledge and the capacity to perform arithmetic operations with algebraic fractions serve as the foundation (Johnson & Lee, 2022). To foster a deeper understanding of the challenges identified, and to develop strategies for sustainable improvements, the next section outlines the root causes of errors and misconceptions, contributing to my personal and professional growth.

2.6 Root Causes of Errors and Misconceptions

Focus issues, fast thinking, memory overload, or a refusal to acknowledge crucial aspects of the problem are all contributing factors of errors manifested by learners (Smith & Brown, 2021). Misconceptions occur when an educator assumes that a learner understands a concept, yet the learner does not fully grasp all of it (Jones & Miller, 2021). A detailed analysis of learner's responses to problems involving algebraic fractions may reveal misconceptions, and it may provide the researcher with information on the underlying reasons for such misconceptions and errors.

Because mathematics is a language of symbols and notation, the study explored how learners use the algebraic signs and notation to construct knowledge. Misconception entails purposely misrepresenting a principle, making too many or too few assumptions or having a different perspective on the situation (Thompson, 2023). In addition, errors are misconceptions that learners manifest when simplifying tasks that may result from negligence, incorrect symbol or text interpretation, insufficient expertise or comprehension of that mathematical topic or concept, or an inability to properly check the answer (Lee & Parker, 2021). Furthermore, Robinson and Lee (2021) argue that constructivism promotes a teaching style where learners receive little to no guidance during learning, which causes learners to get lost and dissatisfied. Moreover, Mathaba et al. (2024) emphasise that errors may also arise from a lack of attention during problem-solving, which can cause learners to make careless mistakes due to rushed calculations. This seems to suggest that learners should be encouraged to doublecheck their steps before concluding their final answer or check the reasonableness of their answers (Muslim et al., 2024).

2.7 Challenges Experienced by Learners in Learning Algebraic Fractions

2.7.1 Gaps Identified in the Literature

The gap identified in the literature is that many researchers have studied the difficulties faced by educators in the teaching of algebraic fractions, not learners. For instance, Chauraya (2017) has highlighted that in-service teachers' impressions and interpretations of math learners' errors was the subject of research. Mtumtum (2020) conducted research on the effects of teachers' comprehension of learners' mathematical blunders. In contrast, Baidoo (2019) conducted research on learners' misconceptions and errors when simplifying algebraic fractions. Mangwende (2021) conducted research on the relationship between learners, and their proficiency in common algebraic fractions in Grade 10. Moreover, Biney et al. (2023) conducted research on errors and misconceptions in solving linear inequalities in one variable; but their study was conducted among student teachers and not learners.

Drawing from literature, many of these current studies on algebraic fractions pay particular attention to educators teaching the learners and not learners learning the concepts of algebraic fractions. This in turn developed educators and left learners displaying more errors and misconceptions during simplification of algebraic tasks. Although the study by Baidoo (2019) and Mangwende (2021) focused on learners' misconceptions and errors, additional categories of errors remain to be discovered. Secondly, the Mopani West District, the location of the study's research, has not yet addressed this research deficit. Considering learners are assigned writing tasks, errors and misconceptions emerging during writing tasks should be taken into consideration.

In addition, if learners are intelligent enough to recognise and analyse the errors and misconceptions manifested by themselves and can probe into their own mistakes and simplify tasks correctly, performance in mathematics can improve across GET and FET phases. Furthermore, if learners can grasp the aspect of algebraic fractions in Grades 8, 9 and 10, they can proceed to Grade 11 and Grade 12 with good knowledge and skills of algebraic fractions. This will help learners in drawing connections between algebra and other mathematical concepts like geometry, finance, trigonometry, probability, and statistics.

Learners should have knowledge and understand the skills and concepts essential to solve tasks involving algebraic fractions. Nearly every area of mathematics is impacted by algebra (Gelfand & Shen, 2021). To ensure that learners advance to higher grades without experiencing major issues, the early secondary school stages must be handled properly. To highlight the importance of misconceptions forming part of teaching and learning, Fujii (2020) brought to the scholars' attention that "so far many misconceptions have been identified at the elementary and secondary levels, however only a few of them are considered for inclusion in actual teaching situations" (p. 625). It is against this background that I decided to embark on the project to explore the errors and misconceptions that learners in Grade 10 manifest when attempting to simplify tasks involving algebraic fractions. Furthermore, I posit that addressing errors and misconceptions is one of the fundamental practices of the process by which learners construct knowledge. Williams (2022) emphasises that many learners struggle due to lack of foundational understanding. In addition, Jones and Carter (2022) emphasise the effectiveness of collaborative learning structures, suggesting that learners benefit from collaborative discussions that allow them to articulate their understanding and challenge their misconceptions. To guide the research process in understanding the problem studied, the following theoretical perspectives were used.

2.7.2 How to Overcome such Challenges

Learners must be able to designate fractions using their correspondence to prove that they are prepared for manipulations with rational numbers (Van de Walle et al., 2019). For example, if learners believe that the sum of two fractions is always less than one (Lamon, 2020), then they should be introduced to situations where the sum is bigger than one early on. In addition, Van de Walle (2004) asserts that concrete materials may be used to improve awareness of addition and subtraction. I employed Newman's error analysis to categorise learners' errors and analyse them to determine their frequency and possible causes. The error analysis results helped in designing the intervention targeting the errors. Some of the questions aimed at assessing learners' understanding on the use of algebraic language, categorisation of concepts and creating scenarios from the given problems. In addition, Robinson and Branden (2020) argue that constructivism promotes a teaching style where learners receive little to no guidance during learning, which causes learners to get lost and dissatisfied.

2.8 Theoretical Framework

2.8.1 Introduction

The frameworks that guided this study was drawn from socio-cultural theory and Newman's error analysis theory, emphasising that knowledge is uniquely and individually constructed through various strategies used in problem-solving. Hansson (2020) supports this view, noting that misconceptions and errors are common in both teaching and learning processes. In other words, making errors and having misconceptions is a normal aspect of learning and can be a vital component of the learning process if they are handled through diagnostic approaches (Kumar & Singh, 2023). By applying constructivist principles in the analysis of test results, the researcher gains a more comprehensive understanding of how learners construct knowledge and how best to support their cognitive development. Additionally, Newman's error analysis model was employed to describe and interpret the types of errors and misconceptions exhibited by learners. The first part of this section discusses social constructivism, and the second part focuses on Newman's theory of error analysis.

2.8.2 Social Constructivism

This study was guided by Vygotsky's Sociocultural Theory (1978). Vygotsky (1978) considered the following: a) The child is an active participant in their own learning; b) Language plays a major role; c) Children are involved in their own learning and in discovering and developing new understandings; and d) Social interaction collaborates interaction. Constructivism in the context of learning is that learning involves constructing meaning and systems of meaning. The following was drawn from the core concepts of Vygotsky' s theory: a) Learning is an active process; b) Learning is a social activity; c) Learning is contextual; d) Learning is personal and exists in the mind of the learner. This involves thinking deeply about what has been learnt and to reflect and gain insight into their thoughts (Madrakhimovich, 2023).

Phillips and Soltis (2020) assert that social constructivism is concerned with development, building, and how society operates in social groupings that share a culture, traditions, and significance generated via discussion and collaboration among participants. Bruner (1990) describes the importance of comprehending how knowledge is actively generated by incorporating valuable learning and that knowledge acquisition is an active process. All these ideas point to the fact that learners generate knowledge in their own way and learning is influenced by prior knowledge. Moreover, experience and prior knowledge are the foundations upon which meaningful understanding is built. Reflective learning encourages learners to apply their developing knowledge to real-world situations, which deepens understanding and reinforces its relevance (Sitopu et al., 2024).

Deficits in conceptual understanding, erroneous reasoning processes, and erroneous conceptual generalisations lead to numerous kinds of errors and misconceptions (Biney et al., 2023). Binney et al. (2023) mention that learners often prioritise retention of theorems, formulas, and algorithms, which might result in errors and misconceptions. However, Ernest (2018) suggests that making errors and having misconceptions is a normal aspect of learning mathematics. Von Glaserfeld's (1995) theory suggests that a child learns through the interactions between previous assumptions and recently taught concepts, which is consistent with constructivist theory. Based to this notion, a child's thoughtless memory and recall of mathematical formulas may be caused by their incapacity to generate and recreate ideas. Consequently, instrumental learning may constitute the primary root cause of errors and misconceptions. The constructivism hypothesis was applicable to this study because it considers the possibility that misconceptions may emerge as learners construct their own knowledge. Therefore, misconceptions are a natural aspect of learning.

2.8.3 Proponents of the Theory

Dewey (1859–1952), who is frequently credited as the father of constructivism, is the originator of constructivism's foundations (Akpan et al., 2020), whilst Bruner (1915) and Piaget (1896–1980) are perceived as the main founders of cognitive constructivists (Metsämuuronen & Räsänen, 2018). Furthermore, a lengthy period was required for the development of algebra (Wang & Lee, 2021). Vygotsky (1896–1934) is thus perceived as the great founder of social constructivism (Bruning et al., 1995). These proponents and founders consider knowledge to be created by individuals rather than something that is passed down to them (Burman et al., 2024). In addition, one of the many educational ideologies is portrayed by constructivism due to its extensive recommendations on effective teaching methods and how instructors can be prepared (Pugh, 2022). According to constructivist approach, to improve on knowledge acquisition, learners incorporate what they currently possess (Akpan et al., 2020). Von Glasersfeld (1995) asserts that constructivist teachers frequently delve into how learners understand the issue and why their approach to identifying a solution offers great potential (Sergei et al., 2019).

Muis and Franco (2021) believe that misconceptions emerge when a fresh notion has no relevance to existing information, making assimilation or accommodation impossible. In addition, Kriukow (2020) contends that constructivism is a spawn of interpretivism and that it is the notion that knowledge comprises numerous facets and is flexible, enabling a researcher to arbitrarily construct a reality out of participants' experiences. Interpretivism is an extension or epistemological branch of constructivism (Schneider & Preckel, 2021). Kramp (2021) argues that interpretivists prioritise subjectivity over objectivity and regard the researcher as an integral part of the phenomena they are studying. Furthermore, Tashakkori and Teddlie (2021) emphasise that rather than focusing on quantity or theory testing, the data collected using this method represent quality and the theory development of an experience. Based on the justification, I thought constructivism would be the best philosophy to be employed as the foundation for this study because the aim was to investigate misconceptions and errors manifested by Grade 10 learners in mathematics.

Constructivists contend that misconceptions arise when a novel concept lacks a connection to previously held beliefs, making integration or adaptation difficult (Vosniadou, 2021). This view shows that constructivism begins its analysis at how learners use their experience to make learning meaningful. When learners consider a novel experience and understand that it does not correspond with their pre-existing paradigm, accommodations are made (Piaget, 1964). To incorporate the novel concept, people modify their conceptual model. In contrast to assimilation, which happens when learners arrange their cognitive abilities to include the novel concept into their existing schema, this process is called making meaning of the new idea by connecting it to background experience. Once the process of assimilation and accommodation is complete, the person will be at an equilibrium where the problem has provided guidance on how to effectively employ the knowledge provided to solve algebraic fraction tasks.

Because learners may foster a productive learning environment by exchanging ideas with one another, Vygotsky (1978) believes that children learn through engagement with their culture and social interactions. In addition, Vygotsky (1978) also stresses the value of studying the way culture influences how cognitive development occurs. A child's system of memory adapts as the youngster engages with the environment (Vygotsky, 1978). Dewey (1968) places a significant emphasis on inquiry-based learning and the provision of learning opportunities outside of the classroom. Kshetree et al. (2021) emphasise that classroom practices enable learners to face their misconceptions and errors, thus making learning productive and enjoyable. This framework describes what each learner should bring to the classroom. Because misconceptions could arise, learners must be able to pick up new information and connect it to previously learned material.

To anchor constructivism, this investigation made use of Ohlsson's (1996) theory which argues that learning from mistakes involves two stages: (a) error detection and (b) error rectification. The theory further emphasises that it is counterintuitive to assume that learners can identify errors using prior information. In other words, errors and misconceptions can be minimised if learners employ prior knowledge while learning algebraic fraction concepts. Declarative knowledge and procedural knowledge are the two essential elements in learning from performance failures. Declarative knowledge, which is the foundation of learning, is defined as knowledge that has not been transformed into procedural knowledge (Hattie & Donoghue, 2021a). Errors occur when procedural knowledge is either absent or flawed (Rittle-Jonhson et al., 2021). According to Abdullah (2023), learners could reduce calculation mistakes if they could learn to organise their thoughts.

2.8.4 Classroom Implications of the Theory

Constructivism is a theory of learning that emphasises an active engagement in learners being actively involved with their own expertise and comprehension through reflective practice and experiential learning. Constructivist analysis of misconceptions and errors provides a comprehensive framework for improving the comprehension of learners. This approach emphasises the significance of exploring errors and misconceptions as learning opportunities, thereby encouraging reflective, adaptive learning practices. By focusing on the constructive aspects of error analysis, learners could critically reflect on their thought processes, reconstruct their understanding in a meaningful manner, and then execute the required adjustments.

The meaning of each dimension in the context of learning algebraic fractions is that: a) Learners bring unique prior knowledge and experience when questions are posed in such a way that learners draw from experience and previous grades; b) Some of the questions need learners to apply what they have learnt in real life situations; c) Knowledge is constructed uniquely and individually in multiple ways, through using a variety of strategies in solving algebraic fractions; d) Learning is both an active and reflective process where learners are interviewed to probe them on how they simplify tasks, the reasoning behind that, and how they reflect on their responses; e) Learning is a developmental process if learners use different representations in simplifying algebraic fractions tasks; and f) Learning is internally controlled and mediated by the learner and how the learner reflects on their responses.

The assumption in this study was that misconceptions may emerge as learners constructed their own knowledge. This is supported by Hansson (2020) who indicates that misconceptions and errors made by learners can be a regular occurrence in both teaching and learning. The constructivist method focuses on reorganising the learning process through the reconstruction of conceptual links. Children do not necessarily internalise the rules and procedures prescribed by a third party to help them master mathematics (Piaget, 1970).

The properties of the world are created by society members collectively (Kim, 2001). Social constructivism argues that knowledge is constructed socially through interaction, and language usage plays a fundamental role (Jaworski, 1994). Naylor et al.'s review from 2022 supports this by demonstrating how learners' performance can be significantly impacted by misconceptions in algebraic language, reinforcing the need for educators to clearly explain the proper usage of algebraic language. The misconceptions that learners encounter while solving numerals are based on language, specialisation, notation, and generalisation (Nurkamilah & Afriansyah, 2021). Nofrianto et al. (2022) emphasise that interventions that focused on language usage significantly improved learner understanding and error reduction in algebraic tasks. The teaching of algebraic fractions can be more effectively done when learners construct knowledge socially and make connections from their previous knowledge to explore solutions.

2.8.5 How the Proposed Study was Guided by Constructivism and Newman's Theory

According to Piaget (1964), cognitive constructivism places the assumption that learners engage autonomously to comprehend things, execute tasks, organise through the material at hand, and either accommodate or assimilate the current experience into their pre-existing mental schemata. The underlying tenet of constructivism is that learners build their knowledge by reflecting on their experiences. To reflect on their experiences, learners need to: (a) use the appropriate language; (b) apply new knowledge and skills to real-life situations; (c) be creative, analytic, and problem solvers; (d) and have an inquiring mind. As a result, individuals can solve difficulties by using their past knowledge and connecting it to the new information.

In analysing errors and misconceptions exhibited by learners and from the constructivist point of view, the following approach has been considered:

- **Contextual Understanding**: Bear in mind the context in which learners responded to questions. Consider their prior knowledge and experiences that may have influenced their answers, providing insight into their learning journey.

- **Evaluate Problem-Solving Strategies**: Assess the strategies learners used to arrive at their answers, especially in open-ended or application-based questions. This helps to gauge their understanding of concepts and their ability to apply what they have learned.

- **Reflective Feedback**: Encourage learners to reflect on their test results. Ask them to articulate their thinking behind specific answers, promoting metacognition and self-assessment (particularly during the interview).

- **Identify Misconceptions**: Examine patterns in incorrect answers to uncover common misconceptions. Analysing these misconceptions can reveal learners' thought processes and understanding.

By applying constructivist principles in the analysis of test results, I gained a deeper understanding into how learners construct knowledge and how to effectively support their growth.

Australian math teacher Anne Newman initially presented the Newman error analysis method in 1977. In numerous nations, including Australia, Malaysia, Thailand, India, and others, the Newman approach is frequently used to identify various sorts of math errors made by learners (Suyitno & Suyitno, 2015). Newman's error analysis model, as presented in Table 4, assists in identifying, categorising and analysing misconceptions and errors to determine their frequency and possible causes.

Several researchers define these categories of errors as follows: Process skill errors arise when learners do not understand the steps involved and make mistakes in calculation (Asriyani et al., 2020). In addition, learners could reduce calculation mistakes if they could learn to organise their thoughts (Abdullah, 2023). Salmeron et al. (2024) describe a comprehension error as an error occurring when learners can read questions yet struggle with comprehending what is expected of them. To support this view, Sumule et al. (2018) point out that comprehension errors arise when learners do not thoroughly read the questions and do not comprehend the significant terms in that subject matter. For Sumule et al. (2018), a transformation error occurs when learners are unable to formulate or prepare a solution to the problem. Verzosa-Quinto and Mabansag (2023) claim that transformation errors occur when information from a problem is translated into a mathematical equation or solution incorrectly. Magfirah and Suryawati (2019) emphasise that learners' inability to identify or select an appropriate method or concept for the task leads to the transformation error. The encoding error arises when a learner completes the mathematical task successfully but fails to provide their response in writing (Wardhani &

Argaswari, 2022). Priliawati et al. (2019) suggest that learners frequently do not review their work before collecting it, they are unable to present their answers and conclusions, and they fail to finish writing them. Rahman and Smith (2022) argue that learners struggle with precise representation of algebraic expressions due to mistakes made during calculations, contributing to encoding errors. The four types of errors, as explained by Newman's error analysis, are depicted within the framework and are described in **Table 4.**

Table 3

Newman's Error Theory Model Analysis

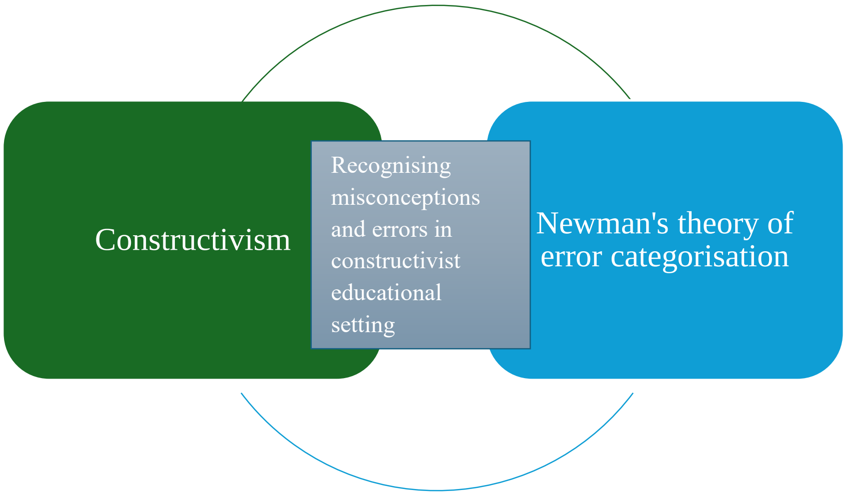
Types of errors	**Pointers according to Newman's theory**	**What the theory means in the context of this study. The following serve as indicators:**
Comprehension error (CE)	Learners are unable to understand the problems that have been presented to them, leading to incorrect approaches.	• How well learners comprehend the algebraic concepts and instructions required for the task.
Transformation error (TE)	Learners are unable to transform the problem into a mathematical representation such as selecting the correct formula solve to a given task.	• Learners' capacity to appropriately manipulate expressions might be impeded by incorrect application of the approaches.
Process skill error (PSE)	Learners apply incorrect procedures or rules, leading to mistakes in the computation or calculation.	• This refers to specific difficulties that learners encounter when attempting to execute the mathematical procedure required for task simplification.
Encoding error (EE)	Learners are unable to demonstrate the authenticity or fail to write down the conclusion or capture the final answer correctly.	• Focus is more on how learners translate a verbal or graphic problem statement into a mathematical form and their strategic competence in solving problems.

Source: Paut et al. (2023, pp 399-402.)

In conclusion, **Figure 2** reflects my perspective on the importance of placing constructivism at the core of the learning process, highlighting that the identification of errors is also an integral part of this process.

Figure 2

Representation of Conceptual Framework



Source: Bada & Olusegun, (2015).

Constructivism asserts that knowledge is acquired by learners through experiences and reflections, as is demonstrated in **Figure 2**. This approach emphasises comprehending learners' thought processes and any misconceptions. In constructivist education, recognising misconceptions and errors is essential as it enables educators to fill in knowledge gaps, promote deeper learning, and assist learners in rebuilding their knowledge. Through recognising and rectifying these misconceptions, educators may promote a more efficient learning process (Sims & Fletcher-Wood, 2021) that corresponds with the unique cognitive growth of every pupil (Kim & Lundberg, 2016). I argue that identifying errors and misconceptions should be done in a manner that enables learners to successfully develop knowledge. This perspective emphasises the significance of constructively addressing misconceptions to enable learners to engage fully with the content and take responsibility for their own learning. Novak et al. (2023) argue that if learners are unable to get the correct answer, they might get frustrated and demotivated. This seems to suggest that by focusing on exposing these common errors and misconceptions, educators may assist learners in developing a more comprehensive and coherent understanding of the subject. Moreover, in an environment of constructivist learning, addressing errors and misconceptions is vital for assisting learners to successfully reconstruct their knowledge.

2.9 Chapter Summary

The consequences of this study are positioned within the arguments on encouraging the acquisition of algebraic fractions to be engaging for learners learning mathematics, since algebra was highlighted as needing more attention in the outcomes of various international benchmark examinations, such as TIMSS (Mullis et al., 2020). The categories of errors that learners manifest when simplifying algebraic fractions were reviewed in the literature, along with the tactics for minimising the errors. There is a theoretical justification for why some errors and misconceptions are unavoidable. Avoiding errors and misconceptions among learners may hinder their acquisition of new knowledge and perpetuate conceptual misunderstanding. It becomes difficult for learners to grasp fractions in the FET phase if they are not grasped in the GET phase. As a result, the study's findings support the need for efforts to be made to help learners understand algebraic fractions in a way that is adaptable and accessible. This technique will provide learners with the understanding and abilities they need to simplify algebraic fractions tasks during the learning process. By minimising errors and misconceptions during simplifying algebraic fractions task, it will also assist learners in developing conceptual comprehension. I will discuss the research design and methodology for this study in more detail in the following chapter.

Chapter 3
Research Methodology

3.1 Introduction

Creswell and Poth (2018) define research methodology as the approach utilised to conduct a study, which comprises choosing the population, the sample, the data collection equipment, techniques, and analytic frameworks that will be used by the study. Enhancing the definition, Leedy and Ormrod (2018) emphasise that these methods aim to enhance the comprehension of a topic of interest or concern. Building on these definitions, Saunders et al. (2019) highlight that the procedures used in research depend on the nature of data, be it non-numerical (qualitative) or consisting of numerals (quantitative), and its intention. Drawing from these descriptions, a planned approach is critical to allow the researcher to select appropriate tools that will keep the research focused and provide meaningful analysis and conclusions. The research's objectives, the amount of time and resources available, and the study's philosophical underpinnings and methodology are all reflected in the method adopted (Saunders et al., 2022). The research paradigm, research approach, and study design, sampling techniques, along with considerations related to quality and ethics are among the topics discussed in this chapter.

3.2 Research Paradigm

A research paradigm is as a framework that directs the researcher and a set of standards for how the world is seen (Guba & Lincoln, 2018). With similar understanding, Creswell and Poth (2022) emphasise that a research paradigm is a perspective that a researcher utilises to describe the world and establish suitable research methods. In this study, I aligned myself with interpretivism philosophy. This philosophical stance reveals a world in which reality is socially created, complicated, and ever-changing, and prominently used in qualitative methods (Taylor & Medina, 2022). Interpretivists have their own interpretations of reality to embrace their point of view, looking at the world through an array of distinct lenses (Creswell & Poth, 2022). This aligns with the perspective of Alharahsheh and Pius (2020) who assert that interpretivism is a method of study trend that prioritises the use of qualitative methods for data collection. A variety of methods can be employed by interpretivists to access reality through social construction such as language and shared meaning and, amongst them, interviews are the most typical (Alharahsheh & Pius, 2020).

Researchers that use qualitative approaches and the interpretivist paradigm often look for unique perspectives and personal experiences in their data. To be able to acquire a deeper understanding of the event and identify the complicated obstacles, interpretivists also accept a relativist ontology in which an event may have multiple interpretations (Kaplan, 2021). In other words, the idea of having a single objective truth is challenged. From the perspectives of ontological and epistemological philosophies, social narrators' experiences shape the meaning of reality, which may make contextualised events less devastating (Omodan, 2022). The study, carried out within an interpretivist paradigm, focused on the significance concealed in verbal and recorded observations to comprehend certain events or phenomena (Flick, 2022). As pointed out by Maree (2022), this paradigm generally entails analysing records, oral histories, and narratives obtained through data collection approaches like interviews. I adopted this paradigm to understand the subjective experiences of how learners make sense of algebraic fractions (Schwandt, 2021).

The ontological assumption of interpretivism posits that reality is multifaceted (Walker, 2019) and that there are various interpretations for every situation (Yanow, 2017). In other words, reality is multifaceted in the existence of various situations. To understand the phenomenon from different angles, I utilised various methods for data collection such as conducting a test and interviews (Creswell & Poth, 2018). I also immersed myself in the research setting (Creswell & Poth, 2021). For example, I invigilated the test written by the learners and personally interviewed the learners. The aim was to get first-hand information and to better understand the context in which data are generated. According to the epistemological perspective of interpretivism, knowledge is constructed through social processes, emphasising subjective meanings and the importance of context (Taylor & Francis, 2022). The notion stresses that knowledge is not universally applicable but transformed by individual and collective experiences. I acknowledge the interconnectedness of key elements in the research process, including that the nature of reality (learners' understanding of algebraic fractions), the methods used (tests, and interviews), and the values guiding the research (openness to new ideas and critical engagement with learners' responses) are linked.

3.3 Research Approach

I followed the qualitative research approach in this study. According to Maduna (2023), qualitative research comprises collecting, analysing, and interpreting non-numerical data through interviews, focus groups or observations to gather insights into participants' perspectives. Echoing the thoughts of qualitative research is the study of nature, comprising its qualities, numerous manifestations, the environment in which it appears, or the viewpoints from which it might be observed (Denzin & Lincoln, 2022). Anderson and Brown (2023) characterise the qualitative research approach as a framework that includes stages and procedures for gathering, analysing, and interpreting data. These strategies, which combine empirical and interpretative approaches to a range of subject matter, aim to resolve societies' scientific and practical controversies (Busetto et al., 2020).

To conduct qualitative research, one must inquire about people's opinions, experiences, motivations, values, attitudes, and beliefs about social phenomena that occur in their natural surroundings (Chandra & Shang, 2019). This approach utilises descriptive and interpretive data collection and analysis techniques (Roberts & Graham, 2023). It also analyses and exposes problems related to the issue due to the lack of information regarding the topic (James & Lee, 2023). Qualitative research operates under the premise that people create their own meanings and interpretations of social reality, which are often contextual and temporary in nature (Willis, 2020). Most of the time, qualitative research uses narrative data obtained through techniques like focus groups, on-site observations, and interviews rather than numerical data. To analyse such data, themes and patterns have been sought. It entails reading the data again and then investigating it. The findings' validity and efficiency of analysis will be significantly impacted by the way the data are collected.

Qualitative methods were employed in this study where the goal was to analyse, explain, and characterise how learners understand algebraic fractions (Audu, 2022). The aim was to explore errors and misconceptions manifested by Grade 10 learners when simplifying algebraic fractions tasks. These processes happened in a natural setting which is a classroom situation (Hattie, 2021). The qualitative approach and interpretivist design aligned well in this study because I was able to understand and interpret errors and misconceptions exhibited by Grade 10 learners in simplifying algebraic fractions. In other words, I was able to gather information on participants' comprehension of the concept, as well as their opinions and experiences (Fitzgerald, 2021).

3.4 Research Design

Sharma et al. (2023) define a research design, often called a research strategy, as a plan to respond to a collection of questions. The research strategy is selected after providing thoughtful consideration to the kind of resources needed, the tools and resources that are available, and the type of data that must be collected (Alharbi et al., 2021). To emphasise the importance of careful consideration of methodological processes, Alturki (2021) argues that the research plan has a significant impact on the methodologies and procedures employed. With the same understanding, Creswell and Poth (2022) describe it as the overarching plan chosen, ensuring that many of the study's components are incorporated in a coherent and reasonable manner, thus successfully addressing the research topic. Furthermore, Babbie (2022) enhances the definition of the research plan to consider the significance of designing research activities in ways that are most likely to accomplish the research objectives. O'Leary (2022) asserts that it is genuinely the blueprint for collecting, measuring, and analysing data. In this study, the research processes followed ensured that the methodologies used were compatible with the kind of research question being posed and the kind of data collected which would ultimately lead to reliable and valid results (Flick, 2022).

Moreover, a good research design starts with a question and moves on to the goals and objectives of the study (Murphy & Sanders, 2022). Basically, the study design outlines the methodology the researcher uses to investigate the main research question. Therefore, it is crucial that the researcher adopt a design that is likely to accomplish the research's goals and objectives. In the current study, I adopted the case study research design that was likely to accomplish the research's goals and objectives. Although the case study design may be described in multiple ways, one common definition describes it as a comprehensive look into an individual, a community, a group, or any other unit which the researcher carefully examines through data related to multiple aspects of the entity under investigation (Gustafson, 2017). Clarifying the meaning further, Heale and Twycross (2017) assert that case studies allow researchers to better comprehend complicated situations by examining them in the context of nature.

A case study, according to Stake (2022), is an examination of a specific event or circumstance that makes use of a variety of data sources to provide a thorough insight into data collection. This understanding is further unpacked in the study by Yin (2018) who explains a case study as an empirical investigation of a phenomena conducted inside its actual settings, especially when it is challenging to distinguish the limits between the phenomenon and its surroundings. Building on this definition, Yin (2022) describes it as a qualitative method wherein the researcher explores a bounded system progressively through a comprehensive collection of data that incorporates multiple sources of information. Further considerations were made by Tomaszewski et al. (2020) who add the aspect of considering the analysis of a social trend for comprehension of its behaviours and situations. In the same breadth, Merriam and Tisdell (2022) describe a case study as an approach for qualitatively researching social phenomena that consider all aspects of a unique being.

The purpose of a case study is to enable researchers to thoroughly examine complex subjects, providing insights that would be overlooked in larger surveys or exploratory studies. It is also essential to note that it enables the execution of various data collection methods, including document/content analysis, observations, and interviews, making it rich in data collection as well. Furthermore, a case study emphasises the subject's operating setting, which is crucial for comprehending the dynamics that influence findings. The study utilised the case study approach to capture a variety of viewpoints as opposed to the singular perspective I obtained from an interview. It offered me the chance to understand the study better and it also lessened the possibility of bias by obscuring one person's personal goal. In this study, I was immersed in the situation to get deeper into the context surrounding learners’ understanding of algebraic expressions to obtain a more thorough picture of the situation (Guetterman & Fetters, 2018). The design was used to derive reliable inferences from the research questions' responses. The study's research design included the strategy for choosing the participants as well as the methods for gathering and analysing data (Creswell & Poth, 2022). The case study design is employed for different purposes. For instance, some scholars employ it as an instrument to assist practitioners implement best practices based on actual experiences (Anderson, 2021). Others employ it to examine the effects of practices and policies in particular contexts (Geller & Pomeranz, 2021), providing insights into what functions well, what does not function well and why this is the case? The aim of the case study was to gain an in-depth understanding of how well Grade 10 learners comprehend algebraic fractions, including the misconceptions and errors that arise, to ensure that the findings were informed by a broad range of contexts.

A specific occurrence under study is referred to as a case (Creswell & Poth, 2018; Yin, 2018). It might be individuals, groups, events, or situations. This was a single case study since it focused on one specific group of Grade 10 learners simplifying algebraic fractions (Yin, 2018). The decision was made because concentrating on learners allows the researcher to see and analyse misconceptions and errors that learners manifest as they are learning. In addition, by focusing on learners, the researcher can categorise errors and pinpoint specific misconceptions that learners encounter, which may inform future interventions and teaching and learning strategies.

To be explicit, in this study, a *case* represents a group of sampled learners involved in the study. It represents the subject of exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions. This case's selection demonstrates how pertinent it is to the main research question: What misconceptions and errors manifest when Grade 10 learners simplify tasks involving algebraic fractions? On the other hand, *bound* establishes the scope of the study (Creswell & Poth, 2018; Yin, 2018). In this study, bound entailed exploring a case of a selected high school in Mopani West District, the demographic characteristics of the school, and the time frame of data collection. Limitations aid in keeping the study's focus narrow and keep it from getting too wide.

3.5 Data Collection Methods, Instruments, and Procedures

3.5.1 Data Collection

Data collection is the systematic method of obtaining and analysing data from several sources for the purpose of acquiring a comprehensive understanding of a specific phenomenon (Polgar & Thomas, 2020). According to Polgar and Thomas (2020), it involves selecting appropriate methods and tools for collecting qualitative data, ensuring validity and reliability, and adhering to ethical guidelines to protect participants' rights and privacy. The data collection methods deemed relevant for this study included a test, document analysis and semi-structured interviews.

i) Test

Learners' errors and misconceptions were investigated using a standardised school-based assessment task (test) on algebraic fractions. The test was also used to assess learners' mathematical skills, which revealed both their knowledge and any misconceptions and errors they had. I designed questions depending on the work that learners had already completed. I validated the questions by extracting them from Maths 911 workbook for Grade 10 which was developed by Bubble-gum Team and Liberty Life, and past examination question papers from (2019-2022) NSC Grade 10. In addition, I also adapted questions from the textbook, school-based assessment tasks, and international assessments. I further considered the levels of

Bloom's Taxonomy when developing questions and ensured that the test measured what was supposed to be measured. Questions were designed to determine how well learners comprehended algebraic fractions conceptually, and how well they could do fundamental calculations involving fractions. The procedurally oriented activities were designed to test learners' ability to use both simple and sophisticated algebraic fractions techniques. Moreover, the learners were required to analyse, apply, and transfer their mastery of algebraic fractions concepts and skills to a variety of contexts. The goal of the test was to identify the difficulties learners encounter when learning algebraic fractions in a classroom setting.

ii) **Interview Guide**

An interview guide was employed to gather data by exploring assumptions regarding errors and misconceptions that Grade 10 learners manifest when attempting to simplify algebraic fractions tasks. I generated data from interviews specially designed for identifying misconceptions and errors as well as establishing categories of errors that Grade 10 learners manifest when simplifying algebraic fractions tasks. I collected data on the pattens of errors and misconceptions that manifested.

Interviews are described as a dialogue for acquiring information and involve both the interviewee and the interviewer, who coordinate the progress of the talk and ask questions (Easwaramoorthy & Zarinpoush, 2006). This approach is suitable for gathering extensive information in situations that are flexible and complex, where questions must eventually be modified to accommodate different people, to examine feelings, beliefs, emotions, and experiences, and inquire into delicate social issues (Madill, 2023). DeJonckheere and Vaughn (2019) define semi-structured interviews as a technique for gathering qualitative data that entails a discussion or dialogue between the researcher and participant under the guidance of an adaptable, predetermined interview procedure supplemented by further inquiries, probes, and remarks. It permitted me to alter the questions being asked in relation to the interview's pace and the answers provided by the participant (Alamri, 2019). Busetto et al. (2020) assert that semi-structured interviews use open-ended questions and that using an interview guide aids in determining the main topics of interest, often with the addition of sub-questions. Qualitative interviews record a person's viewpoints, experiences, feelings, and tales while being guided and facilitated by the interviewer (Billups, 2019).

For this study, I was able to collect data using semi-structured interviews. Learners were given a test on algebraic fractions to write. The interview was conducted after I had completed marking the test scripts, depending on how learners had answered the questions. The interview was flexible, where I started with a clear plan of questions but further probed learners to get more understanding of their ideas and thoughts. With an individual interview, I was able to regulate the procedures while also giving the participants the opportunity to express themselves. The goal was to gain a variety of viewpoints on the subject being studied. However telephonic interviews were taken into consideration if the participant was not physically accessible. With the participants' consent, I also utilised a voice recorder throughout the interview so that I could listen to the recording afterwards in case there was a crucial point I had missed or forgotten. Whatever the participant said during our conversation remained confidential.

From a purposive sample of 36 learners who wrote the test, I further purposively sampled nine learners who were interviewed based on the errors that manifested in the written test. These participants were selected for one-on-one interviews to check how well they understood algebraic fractions. The interview data also assisted me in answering the research sub-question: How can learners' learning of algebraic fractions be enhanced? By selecting participants that represented different contexts within the phenomenon of interest, I was able to gain a more comprehensive understanding of the research topic. This selection criterion also enhanced the quality and richness of the information collected.

I was able to interact with respondents differently during the individual interview based on their individual responses and how well they understood the algebraic concepts that led to the errors. An interview with the nine selected learners was conducted to understand learners' thought processes during simplifying tasks involving algebraic fractions. I was aware that some of the learners might be introverts, however I was able to pick up the common challenges experienced by all learners. During the interview, learners were asked open-ended questions to elicit personal and theoretical background knowledge and answer in the manner that suited them. Three broad categories of questions were part of the interview: (1) Errors and misconceptions in the domain of knowledge, (2) Errors and misconceptions in comprehension, and (3) Errors and misconceptions manifested in the application area. The interviews lasted for 15 minutes per learner and were administered in a classroom set aside after school hours. Since the goal was to get useful information for the research, an interview was the most appropriate strategy.

The interview offered the most pertinent data regarding the study's subject. Other questions were a follow-up of issues picked up in the written test. The goal was to gain a variety of viewpoints on the subject being studied. There were no telephonic interviews taken since all the participants were physically accessible. My questions were incorporated into my interview questions, allowing me to quickly return to participants even if their talk deviated from the topic due to unanticipated issues.

My underlying assumptions, informed by a constructivist philosophy, guided my approach to inquiry and knowledge production. This perspective posits that learning is a contextualised process of constructing knowledge rather than merely acquiring it (DeVries, 2022). The development of questions was aimed at assessing learners' understanding of algebraic language, categorisation of concepts and creating scenarios from the given problems (Hammer & Wildavsky, 2018). To address any errors or misconceptions, learners justified their responses with reasons. Learners' algebraic reasoning skills were explored through interviews. To ensure that their actions and ideas were understood, learners conveyed their ideas in words. Deficiencies were identified during these logical procedures. As already mentioned, I audio recorded the interviews and then transcribed the recordings.

3.5.2 Pilot Study

A pilot study offers the qualitative researcher a variety of special opportunities, including conducting in-depth interviews, interacting with participants, selecting the ideal interview location, and taking the chance to elicit emergent interview themes (Kpadding & Sloane, 2022). This suggests that doing pilot research prior to the main investigation establishes the necessary framework for ensuring participant comfort throughout data collection. Martinez and Lopez (2023) further note that the pilot study may also allow the researcher to practise qualitative inquiry, so enhancing the credibility of their qualitative research, in addition to providing a foundation for self-evaluation of their preparedness and competency.

A pilot study was conducted to determine if the test and interview questions I had constructed were reliable enough to be employed in the study and evaluate if the items would elicit the anticipated responses. I had telephone conversations with the pilot participants who were: i) Mathematics Grade 10 educator, and ii) Subject Education Specialist (SES) for mathematics in the FET phase. These participants matched the targeted Grade 10 learners in the sense that the participant who was the educator was teaching the same subject and grade at another school. In a similar vein, the SES for mathematics had 20 years teaching experience in mathematics in the FET phase and had 4 years' experience as the SES. The participants were requested to provide comments regarding the components in the instruments' clarity, paying particular focus to the phrasing of the questions. The interview questions were responded by same participants. The findings of the pilot study demonstrated that the test was appropriate for the algebraic fraction topic, and both participants agreed with all the interview questions. The test instrument and the semi-structured interview questions remained unchanged in response to the findings. **Table 5** presents the participant profiles from the pilot study:

Table 4

Participants' Profile

Participant's designation	Gender	Qualifications	Teaching Experience	Grades taught	Support provided
Educator	Female	Bachelor of Science (Mathematics and Physical Science) Postgraduate Certificate in Education (PGCE) Mathematics FET phase and Natural Sciences	16 years teaching 2 years DH	Currently teaching Grades 10-12	Teaching mathematics in Grades 10-12, FET phase
SES	Male	Bachelor of Education (Honours in Mathematics) FET phase Masters in Mathematics Education	20 years teaching 4 years SES	Currently working as SES for Mathematics FET	Assisting Mathematics educators at FET phase

It is clear from **Table 5** that I had the added advantage of obtaining an experienced and exceptionally qualified educator and SES for Mathematics as the starting point to assist with ensuring the test and the interview questions were of the highest standard.

3.5.3 Instrumentation

The instrument was aimed at gathering information on learners' understanding of algebraic fractions. I pre-visited the school before conducting research. The purpose of the pre-visit was to explain the main aim of conducting the research, which was clearly spelled out in the application letters and consent forms. The visit further helped the participants with what to expect during the actual visit to the school. Going to the class and breaking the ice before the actual data collection allayed learners' emotional discomfort of seeing me for the first time during data collection, because by the time I collected the data, learners were familiar with me. I maintained the focus by also using the time well. Outlining the purpose of the research reduced ambiguity, stress, and uncertainties for participants (Taylor et al., 2024). I ensured that the school-based support team was functional by assessing the following: i) Ensuring that the team included social workers, parents, and educators; ii) Outlining each team member's duties and responsibilities assisted to ensure that everyone understood their part in assisting learners and improving research. iii) I made sure that everyone involved—parents, educators, learners, and the broader community—was aware of the role of the school-based support team and how it impacts learner support. Educators strived to create a comfortable and supportive learning environment where learners felt safe. I ensured that consent forms were completed for the implementation of systems that respect individuals' privacy (Khan & Ahmed, 2022).

I also used the assessment task (test) and interview questions that were appropriate for the age group being assessed. The age-appropriate assessments reduced the chance of psychological distress and eased uncertainty among young participants (Harris & Thompson, 2022. A pleasant, calm, and distraction-free environment was provided by doing the following: i) The principal of the school gave me permission to administer a test in the Grade 10 mathematics class. ii) I encouraged a respectful and calm setting along with outlining expectations for learners' conduct and engagement during the study. iii) Before the research commenced, learners were instructed to put their smartphones and other electronic devices on silent mode to reduce interruptions. iv) Nine selected learners participated in interviews that took place under a tree during school afternoon study.

I made learners feel more at ease during the testing procedure, where I fostered an atmosphere of trust, transparency, and confidentiality by doing the following: i) Learners were provided with clear explanations of the study's objectives and the procedures involved; ii) The research's ground rules for confidentiality and polite interaction were clearly set out at the outset; and iii)

I promoted open communication by paying attention to the input of the learners. iv) I displayed ethical research practices through complying with ethical guidelines regarding informed consent, data protection, and confidentiality; v) During the interview, I offered opportunities for learners to share their thoughts and knowledge regarding algebraic fractions at an independent pace; and iv) I allowed them to ask questions and raise their concerns before the writing of the test.

3.5.4 Content Analysis

The process of extracting themes, patterns, or biases from the written text of sources such as books, music, magazines, television, human interaction, and conversion transcripts describes **content analysis** (Hsieh & Shannon, 2021). Content analysis of written tests submitted by learners, and transcribed interviews constituted data analysis. It was not enough to simply describe the responses provided by learners on the written test, and interview transcripts; critical analysis and interpretation of the data were also required. To comprehend the trends and patterns that emerge from data, content analysis is applied in qualitative research. Krippendorff (2021) suggests that content analysis enables the researcher to identify patterns and classifications of learners' errors and misconceptions from their work.

I used content analysis to analyse data obtained from learners’ scripts and their journals. I also used thematic analysis to analyse data collected through the interviews and the test. The collected data were categorised, making it easier to identify patterns, relationships, and insights (Braun & Clarke, 2019). However, the sub-categories underpinning each category emerged from learners’ experiences. Hence inductive analysis was utilised in this regard. I collected learners' responses and categorised the errors that manifested. Without a predefined hypothesis, I uncovered common misconceptions exhibited when solving algebraic fraction tasks. Newman’s theory of error analysis assisted in interpreting and analysing data.

3.6 Population and Sample

3.6.1 Target Population

The people, organisations, or groups to which the study's conclusions may be applied generally or universally make up the study's population (Casteel & Bridier, 2021). Creswell and Poth (2021) define it as the total number of participants in the study. Building on this definition, Creswell and Poth (2022) define it as a collection of people who share certain traits. For instance, the population in this study were all Grade 10 learners in the selected school. From this population, only learners who enrolled for mathematics were targeted for the study**.** Worth noting is the fact that the school was selected based on its poor performance for the past three years. There were three classes in Grade 10 at the school, and each class had an average of 36 learners. Two classes were doing the general stream, and the third one was the Science stream. The targeted population was used to draw a sample. The research was conducted at a different school, where I was not teaching. I needed sufficient data to analyse, hence I sampled all 36 learners in the mathematics class (Science stream) to participate in the study.

3.6.2 Sampling

Sampling is defined as the selection of participants from the group of individuals from which data are collected (Etikan et al., 2021). Turner (2020) defines it as the procedure of objectively and methodically picking a smaller percentage of the target population to act as study participants. Purposive sampling, sometimes referred to as judgmental sampling, is the process whereby the researcher purposefully chooses participants or components according to predetermined standards (DePoy & Gitlin, 2020), finding participants who meet the requirements and have the traits and qualities the researchers is looking for (Mason, 2002; & Patton, 2002). I selected Grade 10 learners who provided relevant information for the research. This sampling method also allowed me to explore and test specific concepts of the phenomenon under study. A sample of 36 participants (all the learners from the science stream) were selected from a population of 113 Grade 10 learners (two general stream classes). For the one-on-one interview, learners were selected based on their performance on the written test. I acknowledge that the sample made did not accurately depict the general population or the pertinent population. I employed the purposive sampling method which is an approach commonly employed in qualitative research to choose individuals from the target population who demonstrate the required traits (Mwita, 2022). In addition, when compared to other sample strategies, purposeful sampling is less expensive and takes less time (Etikan & Bala, 2021). Furthermore, a purposive sample's findings cannot be extended to encompass a broader population (Andrade, 2021). That in turn led to a deeper understanding of the research topic being investigated.

To prevent repetition in the sampling, the selection process ceased when no more unique information was predicted (DeMarco, 2020). The learners' responses enabled me to make sense of how they understood algebraic fractions.

By employing purposive sampling, I enhanced the quality and relevance of the data collected and contributed to the overall rigour of the research topic (Fuchsia et al., 2021). Purposive sampling provided greater depth of the information from a smaller number of units. Due to my close vicinity and for convenience (O'Reilly & Parker, 2021), the school was selected from the same District and Circuit where I was working.

3.7 Data analysis

3.7.1 Thematic Analysis

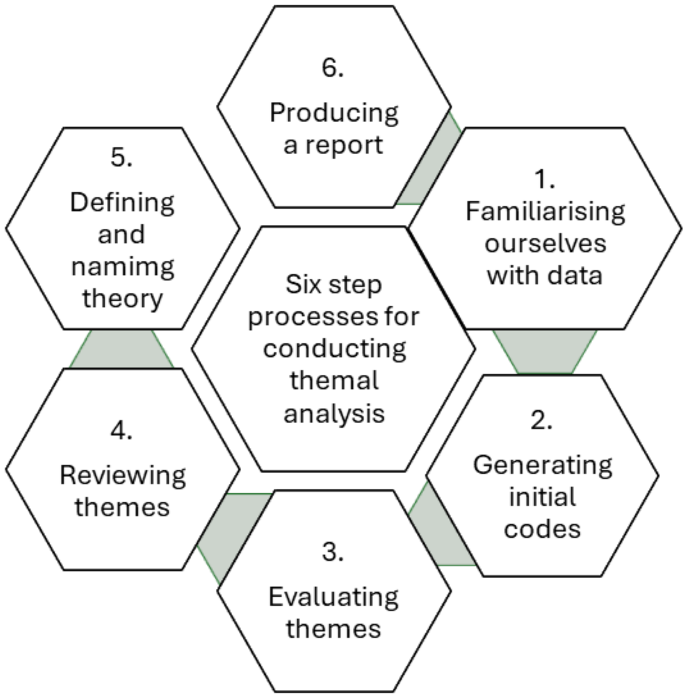
Thematic analysis is a qualitative research method used to identify, describe, analyse, and report themes within data (Karashiali et al., 2023). According to Constantinou et al. (2023), the purpose of thematic analysis is to: i) allow researchers to identify themes and interpret recurring themes within qualitative data; ii) enhance understanding of complex data by breaking it down into manageable themes; and iii) facilitate insights through which researchers can uncover insights that may not be immediately evident, informing further research or practical applications. Through in-depth data analysis, thematic analysis enables a thorough examination of participants' opinions and personal experiences (Braun & Clarke, 2022). Braun and Clarke (2021a) argue that it is typically used in conjunction with a collection of texts, notably transcripts or interviews. Analysing qualitative data can be done through thematic analysis (Mihas, 2023).

Thematic analysis is suitable for researchers interested in qualitative methods (Braun & Clarke, 2021b) and is beneficial for both novice and experienced researchers due to its flexibility and practical nature. Terry et al. (2021) further describe the importance of thematic analysis as follow: i) Due to its flexibility, it can be applied across various disciplines, making it adaptable to different contexts, and ii) Its richness of data provides a detailed, nuanced, understanding of participants' experiences, contributing to the depth of qualitative research. I inductively analysed data, and the analysis was interpreted within the framework of the theoretical perspective (Newman's theory of error analysis). The sub-themes that emerged from the tests and interviews were analysed inductively and I provided detailed accounts of the meanings, context and interpretations of the practices.

The thematic analysis process was conducted in an organised and methodical manner by following these six essential phases, which guarantee that the analysis remains connected to the data and accurately portrays the breadth and depth of the qualitative data for study. For the analysis of data, I was guided by the model of Braun and Clarke (2006) that demonstrates the importance of conducting thematic analysis and involved a six-step process (cited in Kiger & Varpio, 2020). With this approach, non-numerical data might be analysed for patterns, themes, and meanings. Figure 3 presents the six-step process for conducting thematic analysis (Kiger & Varpio, 2020).

Figure 3

Six-Step Process for Conducting Thematic Analysis



Source: Braun & Clarke (2006), as cited in Kiger and Varpio (2020, p.4).

3.7.1.1 Step 1: Familiarising Ourselves with Data

Becoming familiar with data entails immersing oneself in it, reading it in repeating cycles, and being comfortable with the data, each of which yields new insights (Mihas, 2023). To gain a thorough grasp of the data's complexities requires studying the information several times over, making thorough notes, and making preliminary remarks. Through review of the data, I discovered essential themes and trends that would guide the analysis in future stages.

3.7.1.2 Step 2: Generating Initial Codes

Creating codes entails applying the code to a contextual segment rather than just a single word and coding for the greatest number of themes (Kiger & Varpio, 2020). In the subsequent phase of thematic analysis, I rigorously categorised intriguing traits, themes, and concepts emerging from the qualitative data (Braun & Clark, 2022). To attempt to gather the core of each code, this approach entailed finding significant components of data and employing codes that were descriptive. I initiated the process of categorising and organising the data through the development of these initial codes, establishing the basis for further research and theme development. Reviewing codes and related text extracts, combining, or reducing codes into broader and more important sequences, and then analysing the results are the steps involved in developing a theme (Nowell et al., 2022). Coding is the systematic method that applies meaningful classifications to data portions that are pertinent to the study's concerns (Creswell & Creswell, 2021). Moreover, coding facilitates the division of large amounts of qualitative data sets into more manageable components (Maguire & Delahunt, 2017).

Axial coding is the second stage of the coding analysis technique which identifies the relationships between various codes (Charmaz, (2021). Axial coding examines how the categories are connected to one another and links the categories to their subcategories. Nguyen (2021) defines selective coding as the process that combines all the categories created during the axial coding step and turns them into core categories or primary phenomena. During the selective coding procedure, the central theme was developed by compiling all the categories and notes made during the initial coding phase. It enabled me to choose and include ordered data categories from axial coding in cogent and meaningful terms. I categorised patterns emanating from the test. Probing questions for the interview were also developed from these patterns. These patterns were further categorised into themes.

3.7.1.3 Step 3: Evaluating Themes

The third stage of thematic analysis required me to organise and collect the generated codes into potential themes to discover trends and connections. The process of developing themes involved classifying the codes into more advanced topics. To group themes into these more general categories, I used tables (Braun & Clarke, 2021a). Recurring themes that stretched across several codes were pursued, and similarities, differences, and connections between codes to create significant sets were analysed. I recognised more advanced themes using thematic analysis, which matched with conclusions from the data.

3.7.1.4 Step 4: Reviewing Themes

Reviewing the themes entails going over the data coded to the component codes again and questioning the possible themes. Subsequently, I thoroughly investigated and refined the themes to ensure that they accurately represented the data at hand. This involved thoroughly analysing each theme to look for repetition, inconsistencies, or repetition as well as exploring the relationships between them. I improved the thematic analysis's validity and reliability by carrying out this thorough review, ensuring that the identified themes precisely reflected the qualitative data.

3.7.1.5 Step 5: Defining and Renaming Themes

The codes that emerged were categorised according to themes. Kiger and Varpio (2020) define a theme as a patterned response or interpretation that emerges from the data to answer the research question. In addition, a theme is defined as an expression that expresses developing patterns, similarities, recurring ideas, or experiences in coded portions of text (Lopez, 2021). In this phase of thematic analysis, I engaged in identifying the basis and extent of every theme. To determine the key characteristics and limitations of each theme, I carefully analysed the related codes and data. Furthermore, I designated the themes brief, precise titles to effectively express their meaning and preserve their essence. Thus, by examining the data I identified recurring themes.

3.7.1.6 Step 6: Producing a Report

Producing the report entails narrating a complicated story of the themes, possibly beginning with a description of each theme's significance supported by examples, and then perhaps analysing other themes to identify general trends or meta-themes (Braun & Clarke, 2021b). I worked to determine the origins and scope of each theme during this stage of thematic analysis. The pertinent codes and data were thoroughly analysed to ascertain the key characteristics and limitations of each theme. In addition, the themes were given brief, comprehensive sub-titles to convey their significance and preserve their fundamental essence. For the study to properly address the research objectives, the concluded report should go beyond merely describing codes and themes to develop a narrative that offers a clear, concise, and logical explanation of the data (Kiger & Varpio, 2020). A test on how to simplify algebraic fractions was completed by 36 participants and took one hour. I analysed the solutions in the marked test scripts to see where the learners made the most errors and had the greatest misconceptions about the content.

Using the identified errors and misconceptions as a guide, I conducted semi-structured interviews. I analysed and categorised the errors that appeared in the marked test. In addition, errors that frequently emerged in the responses provided by the learners were verified. The learners' responses were independently tabulated, and I then analysed them to generate categories. Following a comparison of the two data sets, the discrepancies were deliberated, with the objective of reaching consensus. Braun and Clarke (2021c) assert that using both narrative descriptions and representative data extracts, such as participant quotes and document excerpts, the analysis should make sense of the results and support the claim that the researcher's explanation fully addresses the research question. For readers to fully understand the meaning of any direct data extraction, according to Braun and Clarke (2021b), it must be supplemented by a textual explanation outlining its significance and providing context.

3.8 Quality Criteria

There are five important concepts to evaluate the quality of qualitative research: credibility; transferability; dependability; confirmability; and authenticity. I took the following steps to meet these five criteria, which helped improve the quality of my study:

3.8.1 Trustworthiness

Trustworthiness describes the degree of confidence in the validity, applicability, reliability, and confirmability of the data, as well as the processes involved in its interpretation and gathering (Patel, 2021). However, since qualitative research uses non-numerical data and small, non-randomised samples, it is not feasible or relevant to measure validity and reliability indices (Ahmad et al., 2019). Persistent engagement and observation to enhance the quality of the narrative, triangulation using multiple data sources and thick description Smith (2021b) to offer a thorough contextual understanding, and member checking to verify the accuracy of interpretations were all employed to ensure trustworthiness (Ali, 2019). I also carried out member checking by doing the following: i) I identified the learners who had been selected to take part in the study, ii) I presented the nine selected learners with their interview scripts so they could probe their answers, iii) The learners were given the opportunity to express their thoughts about their interpretations by asking if they agreed or disagreed with the questions, iv) I asked the learners to address any misconceptions that they felt had not been fully comprehended or represented, and v) I kept the transcripts of the interviews in my final report., which were essential for confirming the validity of results (Birt et al., 2021) and forms the basis of outstanding qualitative research. In addition, readers are usually required to establish their own assertions regarding the researchers' thinking as qualitative research uses a range of methodologies that may not adhere to a regular pattern (Stahl & King, 2020). As stated by Nowell et al. (2017), researchers can utilise trustworthiness to convince both themselves and their readers of the significance of their study findings. The criteria for maintaining trustworthiness in qualitative research are described in the following subsections together with their significance and the procedures this study conducted to comply with such.

3.8.1.1 Credibility

Lincoln and Guba (2020) maintain that credibility focuses on the truthfulness of the findings, about how well the study reflects the participants’ reality which includes techniques such as triangulation, member checking, and prolonged engagement. Yin (2021) argues that by employing known research techniques, research designs pertinent to addressing the research objectives, and the theoretical underpinnings of the study, credibility can be ensured. Stahl and King (2020) define credibility as a person's subjective verdict on the extent to which a study's conclusions line up with reality. I was immersed in the study and interacted with the participants. I analysed the test and learners’ reflective journals and used the interview to validate what had been found in the test and to enrich the overall analysis of the test. Furthermore, I constantly reviewed the data to ensure credibility and trustworthiness

3.8.1.2 Transferability

Transferability describes the way results can be applied to different situations or settings (Thompson, 2022). Cohen et al. (2021) define transferability as the ability of study findings to be meaningfully associated with people who were not directly involved. Exploring, comprehending, or offering a thorough description of the phenomenon being studied is the fundamental objective of qualitative research, not generalising the findings (Cresswell & Poth, 2018; Ritter et al., 2023). The results were only relevant where the study was conducted because of the demographics of the individual school. However, the findings were validated with educators who were not part of the main study to see how closely their interpretation of the data collected and the learners' errors were compared to ensure that the results were transferable (Smith, 2022). Moreover, rather than being a prescription, in qualitative research, a transfer is a concept that must be investigated to determine whether it can be used in another context (Stahl & King, 2020). I have provided rich descriptions to allow others to determine the applicability of the findings to their own contexts. For instance, other scholars might review the methodologies used in this study and use them in the context if they are applicable to their situations.

3.8.1.3 Dependability

Dependability involves the stability of findings over time (Elo & Kyngäs, 2021). Smith (2021a) defines dependability as the degree to which research findings can be easily replicated with same participants and settings. In their study, Hadi and De Boer (2021) argue that dependability can be attained by ensuring that decisions and justifications regarding the research approach, research design, data collection, data analysis, and interpretation methods are clarified in detail. The description of participants using specific inclusion and exclusion criteria improves dependability (Noble & Smith, 2021). To ensure dependability, a qualitative case study design was used with Grade 10 learners in Limpopo province, Mopani West District. Since the goal was to explore the errors and misconceptions that learners in Grade 10 have about algebraic fractions, the test and one-on-one interview were employed as data sources. An analysis of the data was carried using emerged patterns. Furthermore, I provided a detailed description of the study processes to strengthen reliability and facilitate replication and verification by other researchers (Korstjens & Moser, 2021).

3.8.1.4 Confirmability

Moser and Korstjens (2018) define confirmability as how much other researchers can depend on one's findings. Houghton et al. (2021) state that confirmability is the process of minimising the biases of researchers when analysing data. From the same viewpoint, Hsieh and Shannon (2019) understand it as the degree to which results can be confirmed or corroborated by others. In other words, this involves maintaining a clear audit trail and documenting decisions throughout the research process and maintaining the objectivity of the research results. When dependability, transferability, and credibility are attained, confirmability is consequently established (Lincoln & Guba, 2021). For other individuals to comprehend how and why those decisions took place, researchers should offer indicators such as justifications for theoretical, methodological, and analytical choices made throughout the study (Nowell et al., 2017). By avoiding errors caused by the spread of false information, fabrication, and inaccurate interpretation of information, it is possible to promote the acquisition of accurate knowledge (Ioannidis, 2021). This procedure is necessary to preserve the integrity of the study and to ensure the results constitute a significant contribution to the domain of knowledge. Attaining reliable information, it summarised the findings by including insightful statements from participants that illustrated each new theme. Where applicable, participants' quotes were provided to display the data's outcomes without risking confidentiality and anonymity. This is line with Wong's (2020) argument which indicated that the concept of authenticity relates to the fairness of representation of the participants' voices and ensures that different perspectives are accurately portrayed, promoting a deeper understanding of the researched phenomenon.

3.9 Ethical Considerations

The University of South Africa's Research Ethics Committee was consulted for approval of the research proposal and research instruments and ethical clearance was acquired (Appendix B). Furthermore, approval to include one selected high school in this study was obtained from the Limpopo Provincial Department of Education (Appendix D). In addition, ethical considerations were given to the study's participants as well as those who offered direct and indirect contributions to the study. To ensure that this study adhered to appropriate ethical standards, I implemented several safety measures. The first step was to make sure that this research was unique and not a reprise of previously completed work. This was done by thoroughly exploring the research topic to discover knowledge gaps. Since research involves people, it is important to be aware of legal and ethical obligations. Research should be safe and protect the rights and wellbeing of participants in any study (Resnik, 2022). The following moral principles, according to McMillan and Schumacher (2014), were followed in this study. The specifics are provided in the following sections.

3.9.1 Informed Consent

The participating school was requested to sign the consent form to confirm their consent to take part in the study (Appendix G). Participants were required to read and comprehend the terms of the study as well as how their rights would be protected before signing the consent forms (Appendix L). To allow their children to participate, parents of the learners who took part in the study also received consent forms (Appendix J), and the participants received the assent forms (Appendix K). In addition, participants were informed of their decision to decline the opportunity to take part and their right to discontinue their involvement at any time and without explanation (Lynch, 2020). Informed consent occurs when a potential participant opts to take part in a study after receiving adequate details about the nature of the research (Beauchamp, 2021).

3.9.2 No Harm or Risk to Participants

I ensured that none of the participants in this study was harmed emotionally or physically, particularly during the interview session. In addition, I examined the risks and potential harm to the participants, the institution, the greater community and myself before commencing with the study (Hafferty & Franks, 2020). Moreover, when making decisions that are ethical, the researcher ought to strive to exclude, isolate, or mitigate threats to all participants and stakeholders (Israel & Hay, 2020). I did not anticipate major psychological risks; however, it was normal for learners to experience anxiety associated with writing a mathematics test. As a result, some learners experienced fear of being judged by others and worried about being labelled as good or bad. I minimised learners' anxieties by conducting a pre-visit to inform the learners about the benefits of participating in this research. For example, i) it provided an opportunity to expand learners' knowledge and develop resilience in solving algebraic fractions, and ii) it identified their knowledge gap, and they realised that they needed practice and additional study to strengthen their algebraic skills, and iii) it motivated learners to study harder.

In terms of learners' psychological risks related to being tested, I made learners aware that the test was not going to contribute any percentage to their year mark or final mark at the end of the year. However, I explained that it was important for them to take part in this test as I would gain insights into how learners came to understand the concepts. Learners were given a standardised test to write (Appendix O), the purpose thereof being to assess their understanding of algebraic fractions. I encouraged participants to be as honest as possible to assist others (researchers, educators, and policy makers) on how learners come to know and understand learning material on algebraic fractions.

3.9.3 Voluntary Participation

Flicker and Guta (2020) define voluntary participation as the concept that individuals should be allowed to decide whether to participate in studies without being exposed to excessive coercion. The research participants were provided with comprehensive information on the nature, aims, risks, and potential benefits of the study, permitting them to make an independent decision about whether to participate (Flicker & Guta, 2020). Participants had no obligation to participate, nor were they subjected to coercion or intimidation. Participants had no obligation to engage in the information-gathering process and could withdraw at any moment, according to the application letter (Appendix K) (Lidz & Appelbaum, 2021).

Prior to participating in the study, the participants submitted their informed consent (Appendix L). I thoroughly informed the participants about the goals, methods, risks, and advantages of taking part in the study. In addition, I made the participants aware that their participation was entirely voluntary and that they could withdraw from the study at any time without any negative consequence or loss of benefits. Furthermore, I ensured that parents or guardians were fully informed and agreed to their child's participation, since participants were minors (Appendix J). Moreover, I ensured that participants had the autonomy to make their own decisions regarding participation, reinforced by a clear, respectful communication of their rights.

3.9.4 Full Disclosure or Deception

I informed participants and their parents or guardians about the nature, purpose and potential risk of the study to enable them to make an informed choice about participation (Smith & Jones, 2023; Turner & Fields, 2022). To ensure compliance with ethical standards pertaining to disclosure and the use of deception, I submitted the study for review by an ethics committee, and approval was granted (Appendix B). Participants were identified by using pseudonyms. I ensured that the participants were at ease enough to pose questions and provide honest responses. In addition, I assured participants that they would get a debriefing following the study if any deception had been committed (Turner & Fields, 2022). This included letting them comprehend what the main purpose of the research was, letting them know about their rights, and attending to any potential concerns brought on by deception (Geier et al., 2021).

3.9.5 Confidentiality

In research, confidentiality refers to the ethical and legal obligation to safeguard participants' privacy by not sharing their personal information without their consent (Beauchamp & Childress, 2019). It ensures that any personal information acquired throughout the course of the study will be kept safe and available to authorised people only. Participants are protected from any harm or privacy violations by confidentiality, which ensures that any sensitive data or identifying information supplied cannot be revealed to unauthorised parties (Liamputtong, 2019). Liamputtong (2019) further suggests that it fosters trust between participants and researchers, enabling open and honest communication throughout the study. I provided the participants with information on the intended use of their data and ensured the confidentiality of their names in any publications or presentations that emerged from the research (Brown & Lee, 2022; Smith & Walker, 2021). I ensured that no one would have access to any of the personal data that were collected from learners, including audio recordings, transcripts from interviews, and test answer sheets (Brown & Lee, 2022). All the data would be stored securely at the University of South Africa.

3.9.6 Anonymity

O'Leary (2022) defines anonymity as the process of ensuring that participants' identities are kept confidential from researchers and cannot be documented in a way that would identify them. Although comments in an anonymous survey cannot be traced back to specific participants Roberts and Stone (2022), it protects their privacy and promotes more sincere and open communication (O'Leary, 2022). According to Liamputtong (2019), maintaining participant anonymity while sharing personal information for research purposes is a fundamental ethical guideline. Participants were explicitly informed about the study's anonymity and that their participation would not compromise their identity when giving their permission (Appendix K). In addition, I ensured that their own experiences or responses from the test and interview were not identified in the data they reported (Barker & Muir, 2023). Considering I used pseudonyms to hide identifying information such the participants' names and places of employment, the study's participants remained anonymous. To conceal their identity, I instead assigned them codes such as Learner 1F, Learner 25B, and so forth.

3.9.7 Applying to the UNISA Ethics Committee for Ethical Approval

To ensure that my research was ethical, I applied for ethics approval from the Research Ethics Committee at UNISA (Appendix B). I requested permission to do data collection from the following stakeholders: Limpopo Department of Education (Appendix C); The district office that oversees the school (Appendix E) the parents (Appendix J) and from the learners (Appendix L). After completing that process, I went into the field and gathered data based on the conditions specified in the ethics approval letter.

3.10 Chapter Summary

This chapter explained the selection of the research methods and demonstrated how they contribute to the reliability and validity of the findings. It also addressed the ethical considerations inherent in the research process. In addition, the chapter outlined the methodological framework, providing a comprehensive explanation of how the study was designed and executed to achieve its objectives. The target population and sampling procedures were also considered. The methods and resources used for data gathering were also described, and comprehensive descriptions of the interviewing techniques were provided. The gathering and analysis of data were addressed. Quality criteria was investigated and considered. The ethical guidelines that controlled my conduct were outlined. The study's limitations were discussed. The findings from the semi-structured interviews and written test are presented in the next chapter.

Chapter 4
Presentation of Data

4.1 Introduction

This chapter presents and interprets the findings of the data collected from 36 learners who wrote a test on algebraic fractions. The purpose of this study was to identify and describe errors and misconceptions that manifest when Grade 10 learners simplify algebraic fraction tasks. For data collection, learners wrote a test and participated in the interviews. I conducted semi-structured interviews to explore the reason behind the solutions provided by learners, aiming to uncover errors and misconceptions. I report the findings directly as observed. The findings revealed that learners experience challenges when solving problems involving algebraic fractions and these challenges were categorised under the following themes: (i) Incorrect ways of applying rules - addition and subtraction of fractions; (ii) Minimal competence in simplifying algebraic fractions and (iii) Difficulty in analysing word problems.

The themes emerged during the coding of data (Williams & Moser, 2019). These themes were conceptualised within Braun and Clarke's (2016) model of constructivist theory. The primary objective was therefore to outline and describe the types of errors and related misconceptions that manifest when learners simplify tasks involving algebraic fractions. Although it is acknowledged that tables and graphs are primarily utilised in quantitative research, their purpose in this chapter is limited to providing organisation, clarity, and highlighting significant patterns. Above all, they have been used to engage various audiences and provide visual representation. Newman's Error Analysis categories were used to interpret arguments presented in this chapter. As discussed in Chapter 2, the principles of social constructivism assisted me to consider how learners create knowledge as they engage with information through experience, prior knowledge and reflection. The test and interviews were aimed at answering the following research questions.

Main research question

What misconceptions and errors manifest when Grade 10 learners simplify algebraic fractions?

Secondary questions

1. What categories of errors do learners manifest when simplifying algebraic fractions?
2. What misconceptions cause the manifested errors?
3. What are ways of reinforcing learners' learning of algebraic fractions?

The next section presents the demographics and context of the school. This helps me to recommend or suggest interventions that are likely to be effective in that setting.

4.2 Demographics and Context of the School

I collected data from a single high school in the Mopani West District, Limpopo province. The school has two classrooms for Grade 10. One classroom is for the science stream where the data were collected, and the other one is for the general stream. Both learners and educators at the school use pit toilets and the learners speak two different languages, that is, Sepedi and Xitsonga, but only Sepedi HL (Home Language) is taught at the school. The school has only two educators for Mathematics, one teaches Grade 8-10, and the DH teaches Grades 11 and 12. The school underperformed in 2021 and 2022, obtaining 45.2% in 2021 and 55.3% in 2022 in the NSC examinations. For a school to be considered performing, the DBE stipulates it must attain 70% or higher. Considering the school achieved 94.4% in 2023, it was able to overcome its underperformance. Despite achieving well in 2023 and underperforming in 2021 and 2022, the school has continued to underperform in mathematics for the past three years in the NSC examinations, achieving 29.4% in 2021, 15.6% in 2022, and 31.3% in 2023.

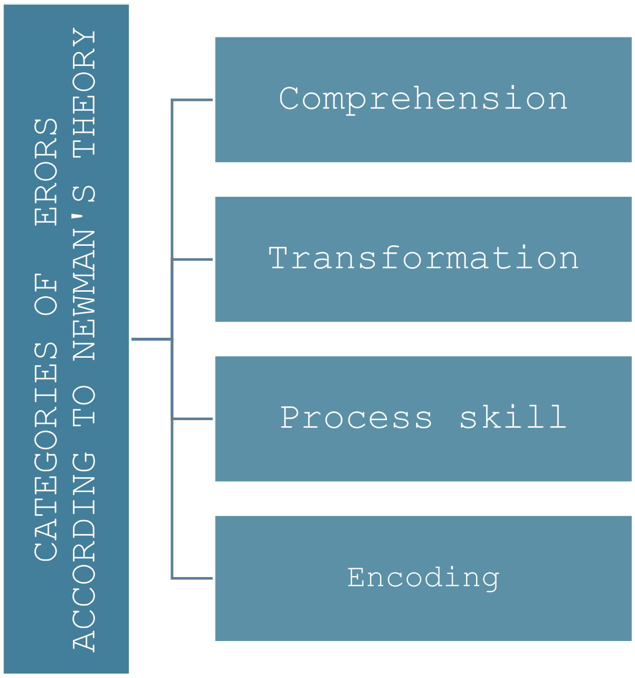
Due to the high failure rate in mathematics, when learners fall below the 40% pass criterion, several learners repeat the grade in the GET phase. Learners may fail mathematics in Terms 1-3, however because the DBE policy prevents repeating a phase for over three years, learners are progressed in mathematics by the end of the year due to age. When learners have progressed to the next grade without having mastered the concepts or knowledge necessary for mathematics, this has an impact on their achievement in the subject. Consequently, it can be challenging for educators and administrators to support learners at diverse developmental stages socially and academically. The demographic analysis offers a basis for making well-informed analyses that provide benchmarks for measuring educational effectiveness and learning practices. Taking the demographics of schools into account has also shaped the way recommendations in this study were presented.

4.2.1 Key Insights Enhancing the Understanding of the Research Topic

Figure 4 and **Table 6** present the framework of how findings and insights derived from the study were communicated. In other words, the patterns identified from data and the arguments presented in this chapter are substantiated by Newman's theory of error analysis. **Figure 4** illustrates the various categories and outlines the specific elements encompassed by each.

Figure 4

Categories of Errors According to Newman's Theory



According to Newman, the error category is determined by the process of problem-solving (Paut et al., 2023) and consists of five types, namely: i) Reading errors, ii) Comprehension, iii) Transformation, iv) Process skills and v) Encoding errors. I did not present reading errors in the study since it never manifested in the answers during the test and the interviews. I presented pointers for the other four categories of errors according to Newman's error analysis and these are described in **Table 6** (Paut et al., 2023).

Table 5

Newman's Error Categories and their Pointers

Categories of errors	Pointers according to Newman's theory
Comprehension error (CE)	Learners are unable to understand the problems that have been presented to them, leading to incorrect approaches.
Transformation error (TE)	Learners are unable to transform the problem into a mathematical representation, such as selecting the correct formula to solve a given task.
Process skill error (PSE)	Learners apply incorrect procedures or rules leading to mistakes in the computation or calculation.
Encoding error (EE)	Learners are unable to demonstrate the authenticity or fail to write down the conclusion in expressing the final answer.

Source: Paut et al. (2023, pp 399-402).

4.3 Presentation of Data

The data in this chapter are presented into two sections for clarity and analytical rigour. Section A focuses on Multiple Choice Questions (MCQs), while Section B presents and interprets learners' solutions for Questions 2 and 3, which required learners to demonstrate their problem-solving processes explicitly. In Section A, the analysis prioritises cases of incorrect answers and the accompanying explanations provided by learners. The aim in this case is to analyse how each question performed statistically and to closely look at what informs the selected option. I also acknowledge that learners might have resorted to guessing or alternating answers, which may not genuinely represent their true understanding.

This focus has facilitated the identification of common errors and misconceptions, thereby highlighting areas where learners may struggle with algebraic fractions. Section B, on the other hand, assesses learners' ability to articulate their reasoning and problem-solving strategies, offering insights into their comprehension and application of concepts. This dual-section approach allows for a comprehensive understanding of both performance and qualitative reasoning among learners.

4.3.1 Section A: Presentation of Data for Question 1 (MCQs)

4.3.1.1 Analysis of Question 1

Question 1 comprised multiple-choice questions with four possible options, from which learners were required to select the answer that corresponded to the question. Learners were also asked to explain the reasoning behind their selected answers. The samples of these explanations are provided in Chapters 4 and 5 from **Figure 14** to **Figure 49** respectively**.** This question was divided into five sub-questions. I marked all 36 scripts and analysed the answers of all the learners. **Table 7** presents the analysis of marked scripts for Question 1.

Table 6

Analysis of Multiple-Choice Questions (MCQ) for all 36 Learners in Question 1

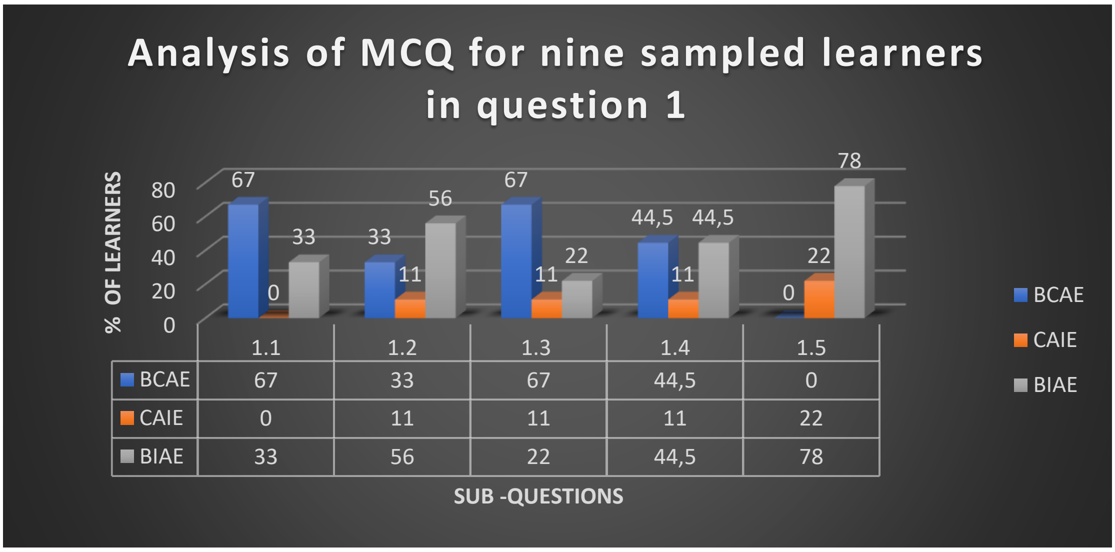
Question 1 Simplify the following expressions	Learners' Correct and Incorrect Answers: Numbers and Percentage		
	Both correct answer and explanation (BCAE)	Correct answer and incorrect explanation (CAIE)	Both incorrect answer and explanation (BIAE)
1.1 $\frac{3-6x}{3}$	16 (44 %)	2 (6 %)	18 (50 %)
1.2. $\frac{4}{x}+\frac{3}{y}$	3 (8 %)	4 (11 %)	29 (81 %)
1.3 $\frac{4x^3-6x}{2x}$	15 (42%)	6 (16 %)	15 (42 %)
1.4 $\frac{a}{2xy}+\frac{2b}{3yz}+\frac{3c}{4zp}$	8 (22%)	2 (6 %)	26 (72 %)
1.5 $\frac{3^x+3^{x+1}}{2.3^x}$	2 (6%)	4 (11%)	30 (83 %)

Source: Researcher

A notably trend in the test scores of the MCQ for all 36 learners in Question 1 is presented in **Table 7**. According to the findings, 44% of learners answered Both Correct Answer and Explanation (BCAE) on Question 1.1. Although it is less than 50%, or less than half of the class, this demonstrates that learners did their best on this question. Sixteen percent of learners got a Correct Answer and Incorrect Explanation (CAIE) for Question 1.3. The implication might be that they indulged in guesses since they could not justify what they had chosen. Since over 80% of learners come under Both Incorrect Answer and Explanation (BIAE), most learners had a serious challenge in answering Questions 1.2. and 1.5. It might be inferred that they were not competent in adding fractions that involved variables and numerals for Question 1.2 and exponential laws for Question 1.5. From 36 learners, I selected nine learners for interviews and analysed their test scripts**.** **Figure 5** presents analysis of these selected learners.

Figure 5

Analysis of Marked Scripts for Nine Sampled Learners in Question 1 MCQ



	1.1	1.2	1.3	1.4	1.5
BCAE	67	33	67	44,5	0
CAIE	0	11	11	11	22
BIAE	33	56	22	44,5	78

Source: Researcher

Figure 5 presents the analysis of the nine learners' scripts for the MCQs who were selected for the interview to share specific difficulties they encountered, and to reflect on their learning to get a more holistic view of their experiences in making sense of algebraic fractions. This was a targeted selection in the sense that these learners had a notably performance and enabled me to capture a wide range of insights. What follows is an account of how selected participants performed in the first question. Analysis of Question 1 revealed that most learners struggled to justify their choices. Sometimes the description was not clear.

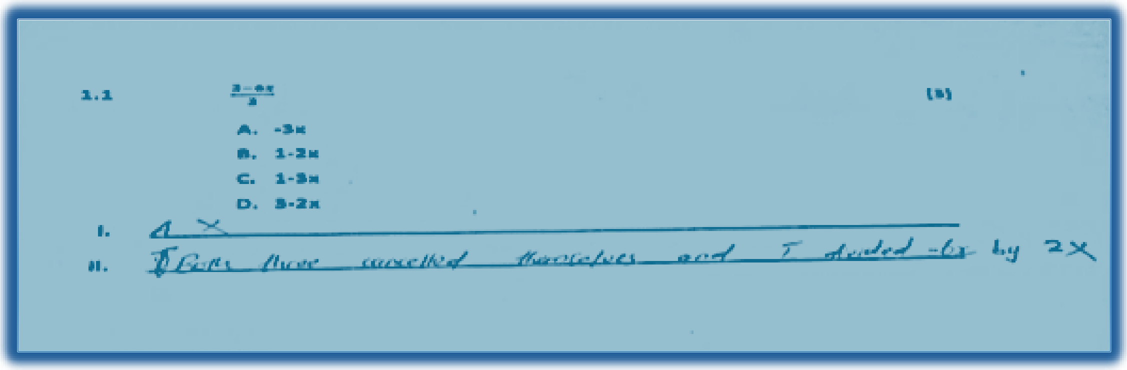
In the same vein, the responses for nine selected learners were classified as: i) Both Correct Answer and Explanation (BCAE), ii) Correct Answer and Incorrect Explanation (CAIE), and iii) Both Incorrect Answer and Explanation (BIAE). As shown in **Figure 5**, 67% of the learners who were in the BCAE group did the best on Question 1.1. This seems to suggest that these learners had no trouble answering Question 1.1. About 22% of learners scored Question 1.5 correctly, but their explanation was incorrect which might mean that they were unsure about their option. Considering more than 60 % of learners fall within BCAE in Question 1.1, no learner (0%) received CAIE. More than three quarters (78%) of the selected learners for Question 1.5 satisfied the BIAE, which is less close to the analysis for all 36 learners. Similarly, more than 50% of learners are included by the BIAE for Question 1.2.

It also appeared that the learners used the rules incorrectly. The option chosen by Learner 1F suggests that the learner subtracted unlike terms on the numerators and just cancelled out the 3s on the numerator and denominator. To attest to this view, Learner 1F responded as follows:

Figure 6

Snippet for completion of Question 1.1

Learner 1F: Comprehension Error

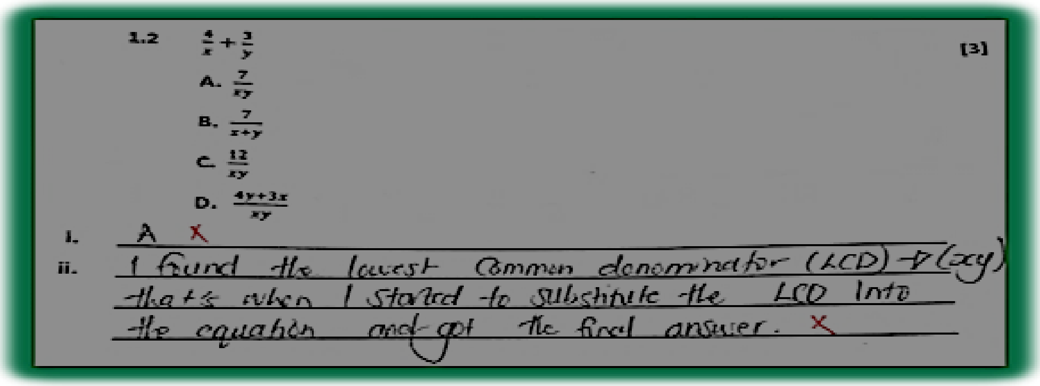


The reason for the learner's response seems to have some important evidence to indicate that the learner does not know how to factor algebraic expressions. The misconception that manifests in this case is the incorrect way of cancelling the common factor. This challenge also suggests that the learner does not understand the meaning of cancellation in the context of this problem. Also, it is not clear why the learner divided -6x by 2x. Although, I acknowledge that this is not clear, the learner also exhibits the challenge of finding the quotient of like terms. Hence the learner manifested comprehension error. The purpose of this question was to see how learners simplify an algebraic fraction where the denominator is a constant and the numerator is an expression. In the same vein, Learner 9I failed to apply the rule correctly and presented the solution as follows:

Figure 7

Snippet for completion of Question 1.2

Learner 9I: Comprehension Error and Transformation Error


1.2 $\frac{4}{x}+\frac{3}{y}$ [3]

A. $\frac{7}{xy}$

B. $\frac{7}{x+y}$

C. $\frac{12}{xy}$

D. $\frac{4y+3x}{xy}$

i. A ✗

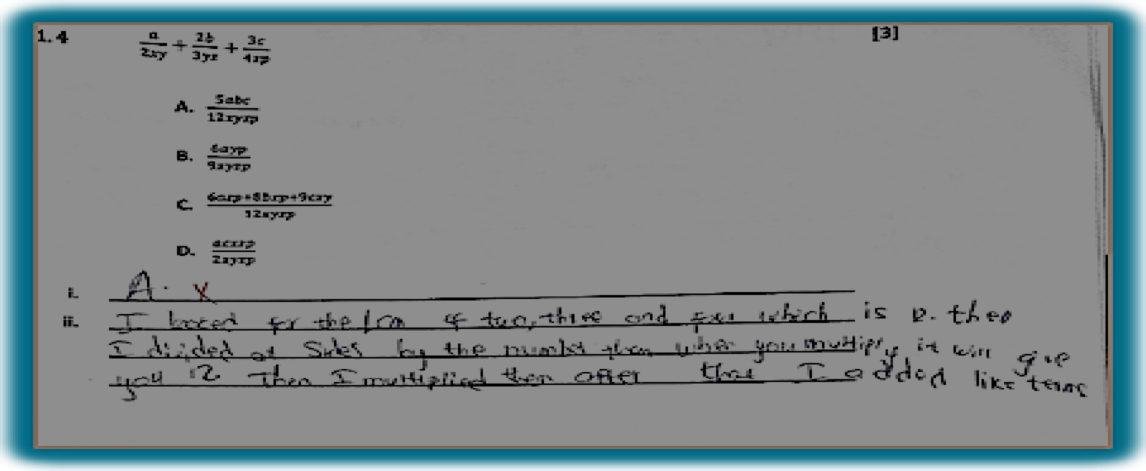
ii. I fund the lowest Common denominator (LCD) → (xy) that's when I started to substitute the LCD into the equation and got the final answer. ✗

Although Learner 9I managed to determine the LCD for the given fraction, it is not clear why she said she substituted the LCD into the equation. The learner confused simplification with substitution. It seems the learner does not know the difference between an expression and an equation. Examining the responses closely, the learner seemed to lack the underlying principles of using the denominator. One might also ask a question as to how the substitution was done in this instance. Secondly, it appears that the learner is not aware that the aim of determining the LCD is to maintain equivalence; in other words, to make sure that their values remain unchanged. Rewriting the algebraic fraction with a common denominator or incorrectly combining the numbers is a challenge in this case which might be a calculation mistake. In addition, learner 9I made a comprehension error as she confused the meaning of mathematical symbols and numbers used in algebra. The explanation provided by the learner suggests a difficulty in articulating thoughts clearly. The learner exhibits a limited comprehension of the instructions, and the meaning of key concepts presented in the question. Likewise, it appears that Learner 17A had the same challenge of adding unlike terms.

Figure 8

Snippet for completion of Question 1.4

Learner 17A: Comprehension Error and Process Skill Error



1.4 $\frac{a}{2xy}+\frac{2b}{3yz}+\frac{3c}{4zp}$ [3]

A. $\frac{5abc}{12xyzp}$

B. $\frac{6ayp}{9xyzp}$

C. $\frac{6azp+8bxp+9cxy}{12xyzp}$

D. $\frac{4cxzp}{2xyzp}$

i. A.

ii. I looked for the LCM of two, three and four which is 12. then I divided all sides by the number that when you multiply it will give you 12 then I multiplied them after that I added like terms

In **Question 1.4**, Learner 17A also showed comprehension error due to incorrectly adding unlike terms on the numerator. The learner does not even know that the coefficient of a is 1 since she added 2 and 3 only on the numerator and leaving out 1 as the coefficient of a. The learner in this case has difficulty with symbolic representation that leads to the confusion of identifying different components of an expression. This challenge is an example of a comprehension error. The purpose of this question was to find out if learners could perform the operation of adding fractions with different denominators using variables and numbers by converting them to an equivalent form.

Learner 29R was not among the nine learners selected for an interview, but I used Learner 29R's script to compare the reason behind the selection of the incorrect option with Learner 19C. Looking at Learner 29R and Learner 19C, they selected the same option, but their reasoning was different.

Figure 9

Snippet for completion of Question 1.5

Learner 19C : Comprehension and Process Skill Errors

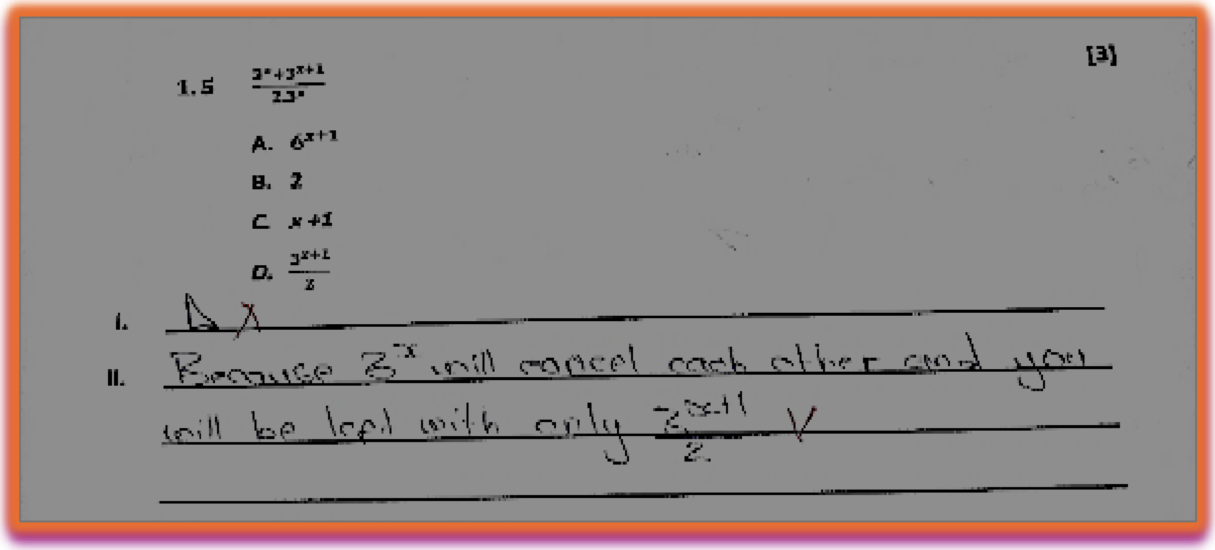
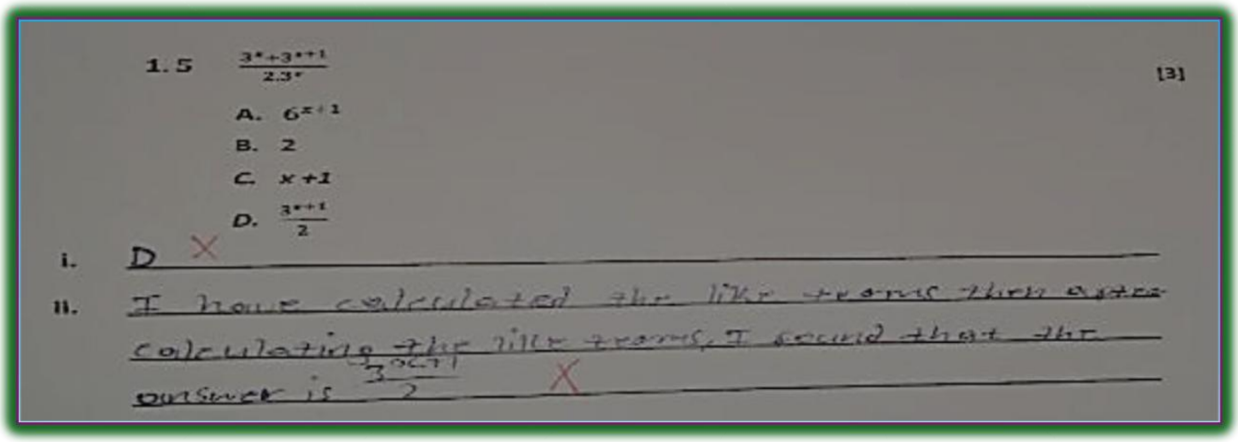


Figure 10

Snippet for completion of Question 1.5

Learner 29R: Transformation Error



Interpreting Learner 19 C's explanation, she mistakenly cancelled terms that are not products. For Question 1.5, Learner 19C had the misconception of incorrectly applying the rules of operations without considering the common factor first. The learner simplified by cancelling the common factor on the numerator and denominator. Hence, the learner demonstrated a comprehension error due to incorrect use of algebraic procedure. A process skill error was exhibited by Learner 19C due to the inability to apply exponential rules since the learner had a misconception from the previous step.

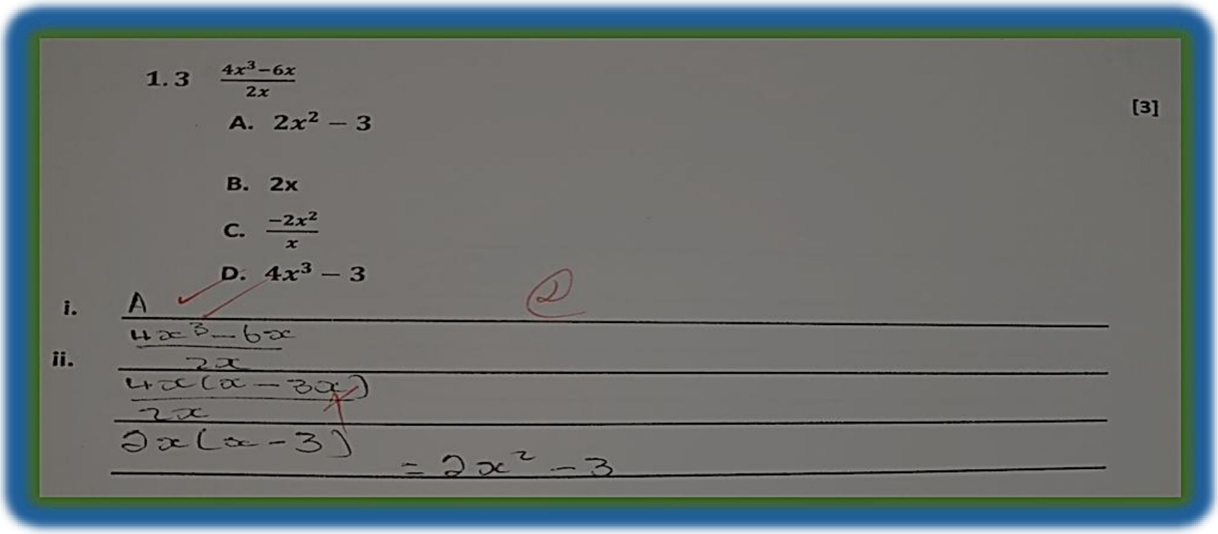
Drawing from the Learner 29R's response, it seems to suggest that the learner lacks prior knowledge of exponents learned from the previous Grades 8 and 9. For instance, when looking closely at the explanation and the option selected, the learner appears to have a challenge with decomposing fractions. I observed that learners experience challenges when it comes to the use of mathematical notations. Moreover, it is also difficult to understand what it means when the learner wrote, "I have calculated the like terms". This type of response prompted me to select the learner as one of the interviewees to probe their answers. From his response, it looks like the learner has just cancelled the 3x on both the numerator and the denominator and disregarded the operational sign on the numerator. This might also be a memory retrieval issue in the sense that the learner may struggle to recall the process accurately. Hence the learner manifested a transformation error which is a discrepancy that occurred during the process of taking out the common factor.

Learner 15M selected the correct letter A in sub question 1.3 and responded as follows:

Figure 11

Snippet for completion of Question 1.3

Learner 15M: Comprehension Error



Although Learner 15M selected the correct letter A and further showed how she arrived at selecting the correct letter, the learner did not understand the instructions because she did not describe in words how she solved the problem. Despite being unable to understand the instructions, the learner seems to comprehend the rules and procedures needed to simplify the question as she provided the correct steps. This could be attributed to several issues. First, it might be that the learner did not attend to details due to rushing. Second, it could be a habitual pattern in the sense that she is used to always simplifying the task, failing to provide an explanation of how to make it easier. The latter might be categorised as a systematic error due to the incorrect usage of instructions. As a result of not grasping the instructions or the meaning of key concepts in the question, the learner manifested a systematic error. A systematic error is when a learner's habit leads them to consistently misunderstand or apply instructions incorrectly. Helping learners see these patterns and teaching them how to modify their responses to fit various contexts are ways to address systematic errors. This is also another type of error I observed in the MCQ. I did not use the themes for Question 1 MCQ considering that there were no steps on how learners arrived at the answer.

The next section presents the analysis of Questions 2 and 3 to gain deeper insights into their thought patterns and knowledge. The data is presented in accordance with the results of the test and the interviews. The next subsection presents an analysis of Questions 2 and 3.

4.3.1.2 Analysis of Questions 2 and 3

I marked all 36 scripts of the learners and purposively selected a sample of nine learners for the interviews. Table 8 presents the analysis of test solutions for sub-questions 2.1–2.5 including 3.1–3.3 for the nine sampled learners. **Table 8** presents the total score for each sub-question, the average mark as a percentage per question, as well as the marks achieved by each learner.

Table 7

Analysis of Questions 2 and 3

Question	2.1	2.2	2.3	2.4	2.5	3.1	3.2	3.3
Total marks per question	4	4	4	4	5	5	5	5
Learners	Marks obtained by each Learner							
L17A	0	4	2	1	1	0	0	3
L25B	0	0	0	0	0	0	0	0
L19C	0	1	0	0	0	0	0	0
L18D	3	4	4	2	0	4	2	1
L11E	1	2	4	0	0	3	0	2
L1F	0	1	0	1	0	0	0	0
L21G	1	4	4	2	4	4	0	5
L7H	1	1	0	1	4	0	0	0
L9I	4	4	4	4	1	2	5	0
Total	10	21	18	11	10	13	7	11
Average %	28	58	50	31	22	29	16	24

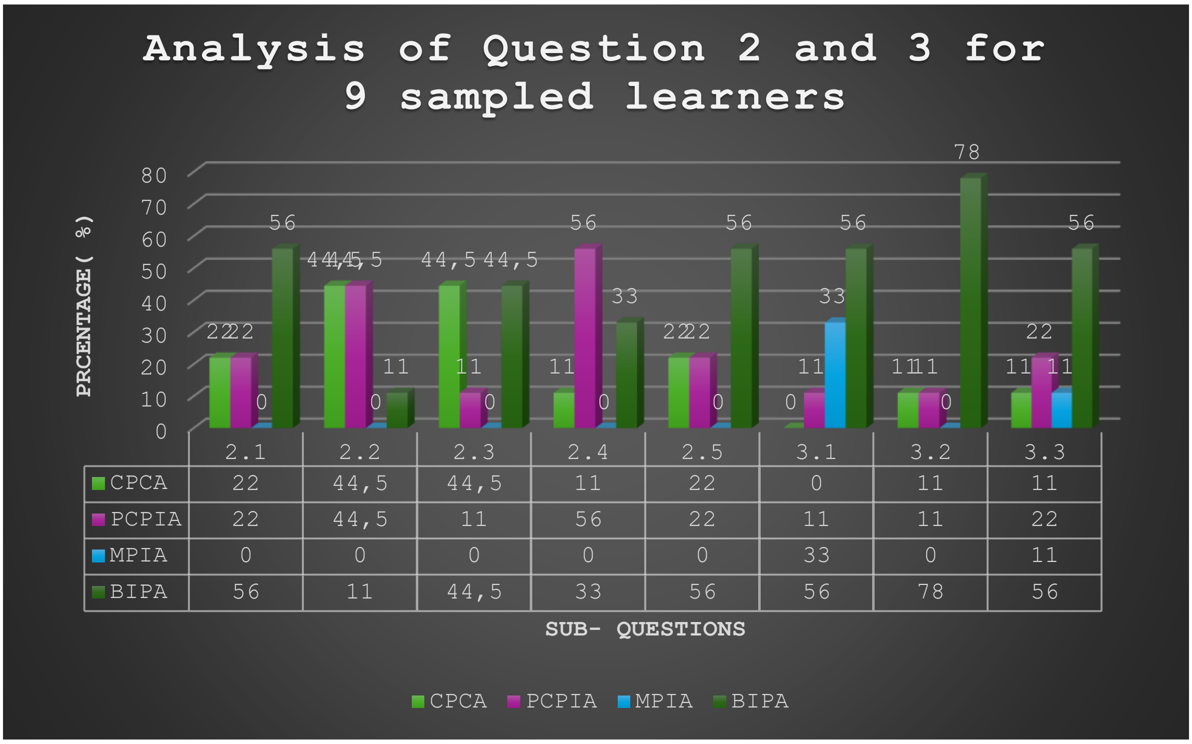
Note: Synopsis of question-by-question analysis of marked scripts for Questions 2 and 3. Source: Researcher.

As evident in **Table 8,** learners demonstrated stronger performance in Questions 2.2 and 2.3, as indicated by average percentages of 50 or higher. In contrast, many learners encountered greater difficulties with Question 3.2. with the lowest percentage of 16. The highest performing learners were L9I, L18D and L21G, while L25B and L1F faced the most challenges.

Figure 12 presents the analysis of Questions 2 and 3, where learners must fully simplify the algebraic fractions. Similarly, as in Question 1, the analysis revealed that most learners are experiencing problems when simplifying algebraic fractions tasks. I classified the learners' responses as: i) Correct Procedure and Correct Answer (CPCA), ii) Partially Correct Procedure and Incorrect Answer (PCPIA), and iii) Moderate Procedure and Incorrect Answer (MPIA), and iv) Both Incorrect Procedure and Answer (BIPA). The difference between partial and moderate in the context of mathematics for Questions 2 and 3 is that: i) Partial refers to incomplete solutions that demonstrate that some correct steps are not fully understood by the learner, and ii) Moderate refers to a greater comprehension of the concepts that have been learned, with fewer understanding gaps that could prevent the learner from fully simplifying the task.

Figure 12

Analysis of Marked Scripts for Questions 2 and 3



Source: Researcher

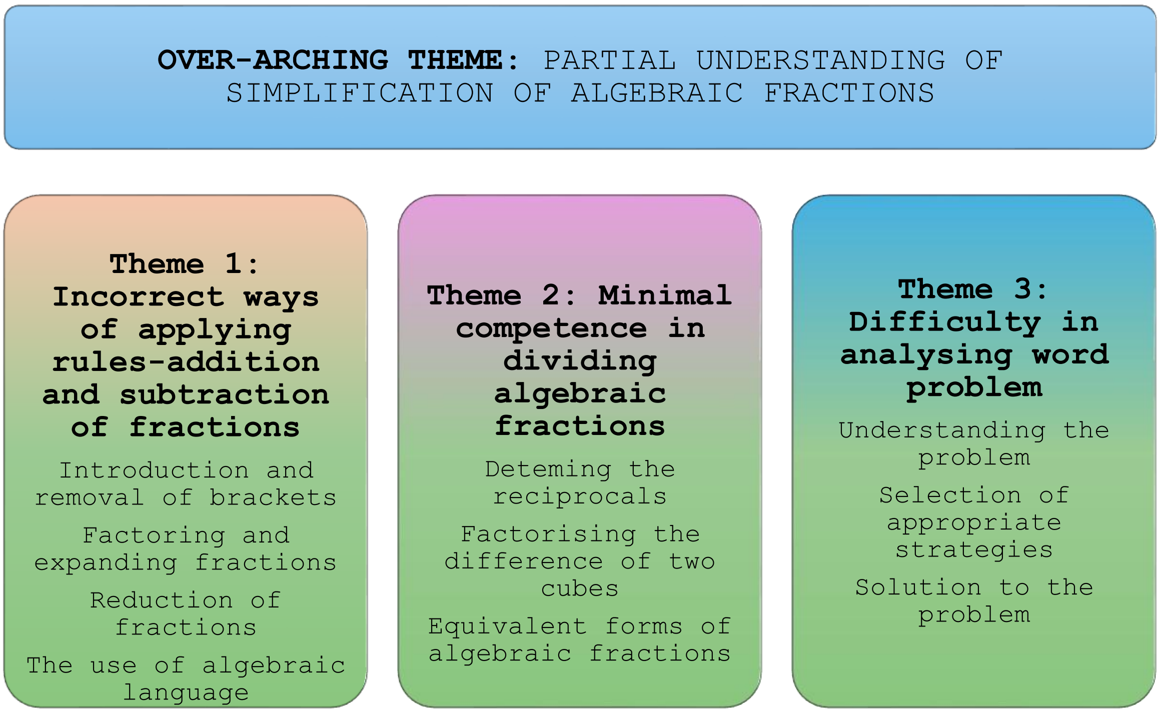
From **Figure 12**, since most learners got a score of 44,5% for Correct Procedure and Correct Answer (CPCA), which is the highest score among all questions, learners did better on Questions 2.2 and 2.3. Over 50% of the learners answered Partially Correct Procedure and Incorrect Answer (PCPIA) on Question 2.4. This seems to indicate that although most learners were able to recognise some pertinent information from the test, they failed to make the connections between all the components required to arrive at the correct answer. No learner received a Moderate Procedure and Incorrect Answer (MPIA) for Questions 2.1–2.5, which seems to suggest that this was the most challenging question for learners because most of them did not attempt to get some steps correct. Approximately 33% of learners fall under the MPIA category in Question 3.1, reflecting that only a third of the learners comprehended the topic better since they were able to apply the concepts that they learned and had fewer comprehension gaps. In Question 3.2, 78% of learners fall under Both Incorrect Procedure and Answer category, indicating that this question is the source of a significant number of misconceptions. Questions 2.1, 2.5, 3.1, and 3.3 follow, in which over 50% of the selected learners are classified as BIPA. The following section presents the overarching theme with themes that emerged from the tests and interview for Questions 2 and 3.

4.4 Over-arching Theme: Partial Understanding of Simplifying Algebraic Fractions

The over-arching theme arose from discussions of the following themes: i) Incorrect ways of applying rules - addition and subtraction of fractions; (ii) Minimal competence in dividing algebraic fractions, and (iii) Difficulty in analysing word problem. The categories of each theme are presented in **Figure 13.** The categorised data is presented in an integrated way, with the illustrations structured around individual questions, each serving a specific purpose.

Figure 13

Categories of Themes Emerging from Test and Interviews



The next section presents Theme 1, which emerged from the test and interviews for Question 2.

4.4.1 *Theme 1: Incorrect ways of Applying Rules- Addition and Subtraction of Fractions*

This refers to instances where learners fail to fully simplify the algebraic fractions due to incorrect conceptions leading to errors in problem solving. The identified criteria are presented in an integrated manner to provide a comprehensive overview of misconceptions and errors exhibited when learners simplify tasks involving algebraic fractions. The purpose of Questions 2 and 3 were for learners to simplify the algebraic fractions fully. In Question 2.1 learners were asked to simplify the fraction fully, where the numerators contain single variables, and the denominators contain binomial variables. Figure 14 presents a snippet of Learner 21G's answer for Question 2.1.

Figure 14

Snippet for Completing Question 2.1

Learner 21G: Process skill error and Comprehension error

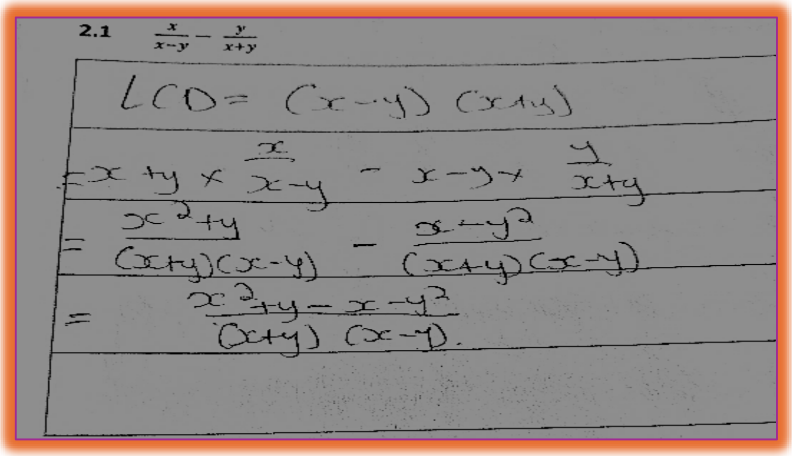


Figure 14 reveals that although Learner 21G managed to determine the LCM, she had a challenge multiplying a monomial by a binomial. It is evident on the first term in the algebraic fraction, she only multiplied x by x and did not consider y. The same reasoning applied to the second term of the same algebraic fraction. The error might be attributed to the fact that the learner cannot group the terms that should be treated as a unit. Secondly, failing to group the terms made the learner unaware that it is not correct to confuse the operational sign minus and the negative sign. The last two terms of the solution provide a clear explanation of the point being discussed. Another observation is that Learner 21G seems to miss a point that determining the LCD or using the equivalent fraction involves rewriting a fraction. All these attempts in solving the problem suggest that the learner exhibits the process skill and

comprehension errors. In other words, errors occur either due to mistakes in the steps or procedures followed, or because of a fundamental misunderstanding of the underlying concepts or principles. What follows is an account of how Learner 21G explained the steps in her solution to Question 2.1 during the interview

> ***Researcher****: In question 2.1, provide a brief explanation of each step of your answer.*
> ***Learner 21G****: In 2.1. You first find the LCD which was (x+y)(x-y) and then you multiply the x+y with this fraction so that the LCD can be the same and you also do it here and that is how you will arrive to* $x^2 + y - x - y^2$ *. And then here this was our final answer which is* $x^2 + y - x - y^2$ *over our LCD (x+y)(x-y).*

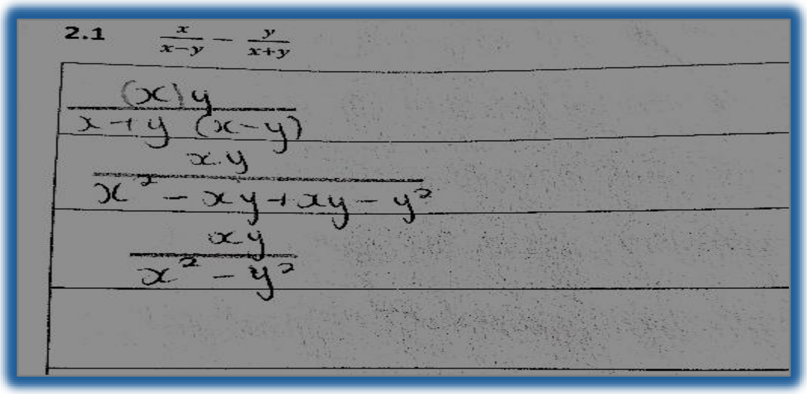
What emerged from this interview is that although **Learner 21G** was able to determine the LCD, her response suggests that she has a limited knowledge of how equivalent fractions are related. This means that the learner might be missing the fact that two algebraic fractions are equivalent if they simplify to the same expression or can be transformed into one another through factoring or cancelling common factors. Perhaps the reason behind this might be the fact that she already knew what the LCD was, and she wrote it down and forgot about the rules and procedures for simplifying fractions. Still the learner does not realise that she committed an error when multiplying a binomial by a monomial. Important concepts like the introduction of brackets, equivalent fractions and how the negative sign is applied did not come out clearly during the interview.

It has also emerged that **Learner 11E** was also able to determine the LCD. However, despite being able to write the correct LCD, she omitted to introduce the brackets in the denominator for Step 1. Although this is the case, the learner managed to simplify the expression in the denominator correctly. It then suggests that the learner did the calculations mentally to simplify the expression in the denominator. Most importantly, it was incorrect for the learner to multiply x and y in the numerator, and this seems to indicate that the learner does not understand the fact that the LCD is determined to minimise the use of complex calculations. That is why sometimes there is a need to express the mathematical statement into equivalent forms that are easier to manage. **Figure 15** substantiates this argument.

Figure 15

Snippet for Completion of 2.1

Learner 11E: Comprehension error



This solution also points to the fact that Learner 11E has knowledge of i) additive inverses, and ii) simplifying expressions by first removing the brackets. When asked to provide clarity on how the problem was solved, Learner 11E had some difficulty in providing the reason behind multiplying x and y. To confirm this, she responded as follows:

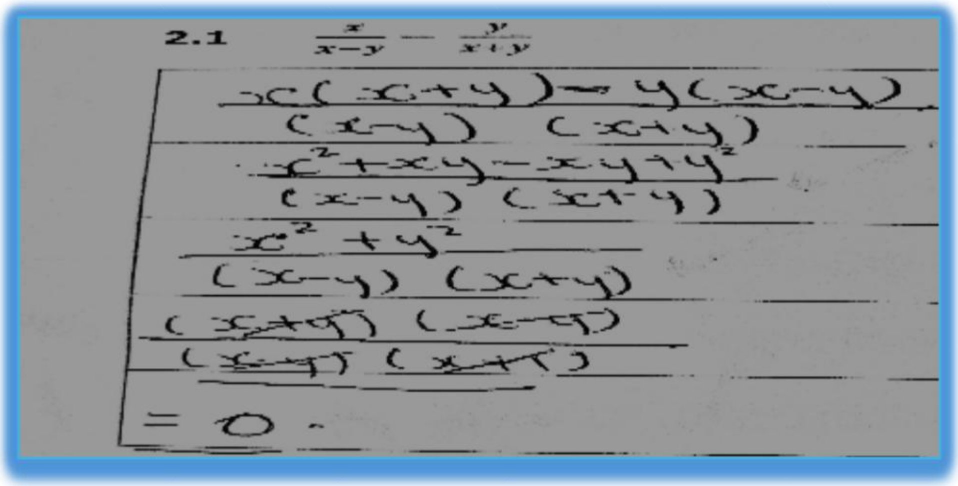
> ***Researcher:*** *Provide a brief explanation of each step of your solution to Question 2.1*
>
> ***Learner 11E:*** For *2.1 in the numerators, I multiplied x with y, and it gave me xy, then at denominators I multiplied it, (..., paused), the expression and it was* x^2*- xy+ xy -* y^2*, and I added the like terms. And my answer was xy divided by* x^2*-*y^2*.*

Another finding is that the learner omitted the negative sign. It might be because she calculated the product in the denominator, it is also correct to do the same for the numerator. This finding seems to hint at two issues. Firstly, finding the LCD is always associated with finding the product. Secondly, omitting the negative sign might suggest that the learner feels that they complicate simple problems or fail to connect the concept of negative numbers to tangible examples. The comprehension error manifested in this case exhibits a lack of solid comprehension of the underlying concepts. Another challenge was identified in the solution provided by L18D, as illustrated in **Figure 16.**

Figure 16

Snippet for Completion of Question 2.1

Learner 18D: Encoding error



As evident in **Figure 16**, the learner had a misconception of assuming that $x^2 + y^2$ can be simplified further as the difference of two squares. The learner proceeded to simplify by cancelling out binomials on the numerator and denominator terms which gave her 0 as the answer. Learner 18D did not realise that the expression $x^2 + y^2$ cannot be simplified further as these are unlike terms. Hence the learner exhibited an encoding error due to misapplying calculations by writing zero as the answer. During the interview, the learner was probed to explain how she came up with the final answer and she responded as follows:

> **Researcher:** *Can you explain how you came up with the final answer in Question 2.1?*
>
> ***Learner:*** *How I arrived at the answer?*
>
> **Researcher**: Yes
>
> ***Learner:*** *Here?*
>
> **Researcher:** Yes, explain the steps that you have written.
>
> ***Learner:*** *I…(pause) I cross multiplied. I did cross multiplication, then they cancelled each other then I was left with zero(0).*

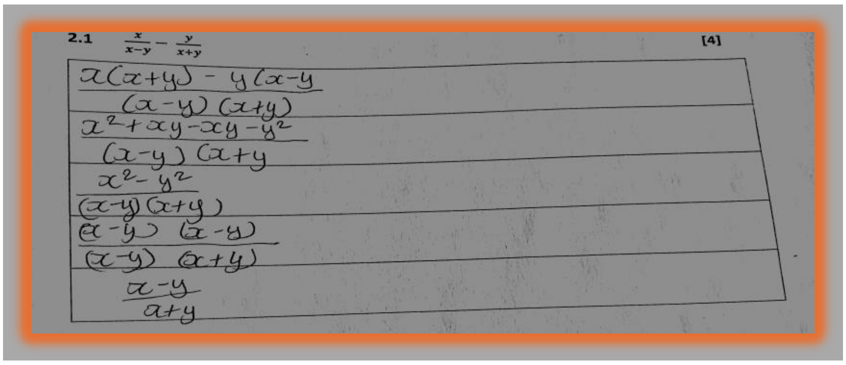
Despite being able to simplify the problem in Question 2.1, the learner seemed confused in explaining the steps that she did as she could not explain all the steps that she wrote. She only explained the part of cross multiplication to remain with zero, which was incorrect. More challenges associated with the use of a negative sign, the inability to use correct mathematical terminology when providing explanations also emerged. In explaining how she arrived at the answer, Learner 7H responded as follows:

> ***Researcher:*** *Explain how you arrived at the final answer in question 2.1*
>
> ***Learner 7H****: Ok. On Question 2.1, I took the denominator, and it became the numerator, then this one became that one. And I multiplied it with the numerator again then I found the LCD. After that I added the like terms and cancelled, and that is how I found my final answer.*

Figure 17

Snippet for Completion of Question 2.1

Learner 7H: Process skill error



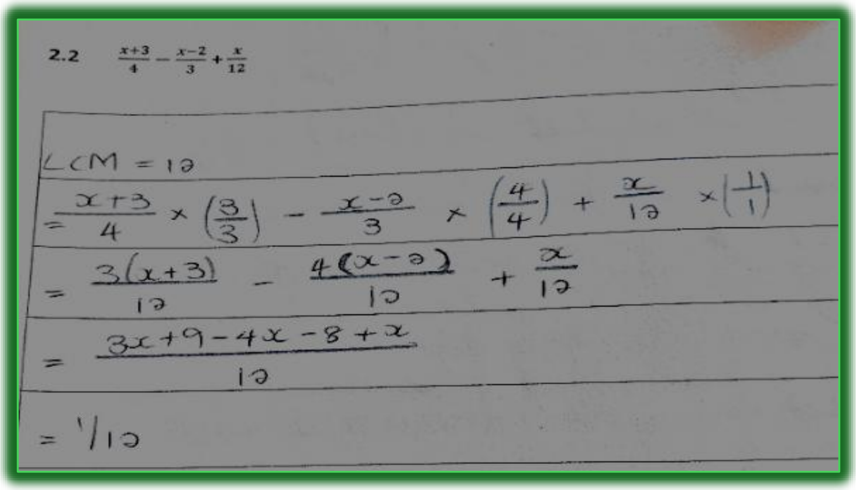
From the learner's response, taking the denominator to become the numerator might not be a good explanation because from the denominator only one factor was considered. Another concern is that: What does it mean algebraically to say the denominator becomes the numerator? The learner seems not to understand that the numerator and the denominator are two distinct components of a fraction. Secondly, the learner might not realise that the numerator and the denominator are not changeable unless specified by the operation. Evidently, it suggests that the learner is not aware that 'moving the number to the numerator' implies the application of the specific operation, which in this case is multiplication (multiplying the numerator by the LCD). This challenge reflects a deeper issue with grasping the core algebraic principles. This is also not a rule that part of the denominator should move to the numerator. Another observation is that Learner 7H might mistakenly think that, in removing the brackets, only the first term is affected.. The learner also, for instance in Step 3, does not realise that the denominator is like having expressed the numerator as the product of two factors. The learner exhibited a process skill error in this case.

Focusing on Question 2.2, the expectation was that learners would simplify the expression fully. The numerators of the expression contain variables and numerals, and the denominators contain numerals only. I found that similarly, Learner 11E also exhibited the misconception of confusing the negative sign and minus operational sign when solving problems. **Figure 18** is a case in this point.

Figure 18

Snippet for Completion of Question 2.2

Learner 11E: Transformation error



Despite being able to determine the LCD and introduce the brackets as presented in **Figure 18,** Learner 11E had a challenge using the negative sign when removing the brackets. The learner, in Step 2 multiplied negative 4 (-4) by negative two (-2) to get negative eight (-8) instead of positive eight (+8). This might be either a mistake or the fact that the learner misses the key idea that a negative sign must be applied to each term inside the brackets. In other words, the learner might not have the understanding that removing brackets, in essence, is about rewriting the expression in an equivalent form. Hence the importance of considering all the terms in brackets. Transforming mathematical expressions in this instance, is a challenge. The learner manifested the transformation error. However, the learner's solution is notably better compared to other learners as she was able to add and subtract like terms on the numerator of the third step. Justifying her answer, Learner 11E said:

Researcher: In question 2.2, explain how you arrived at the final answer.

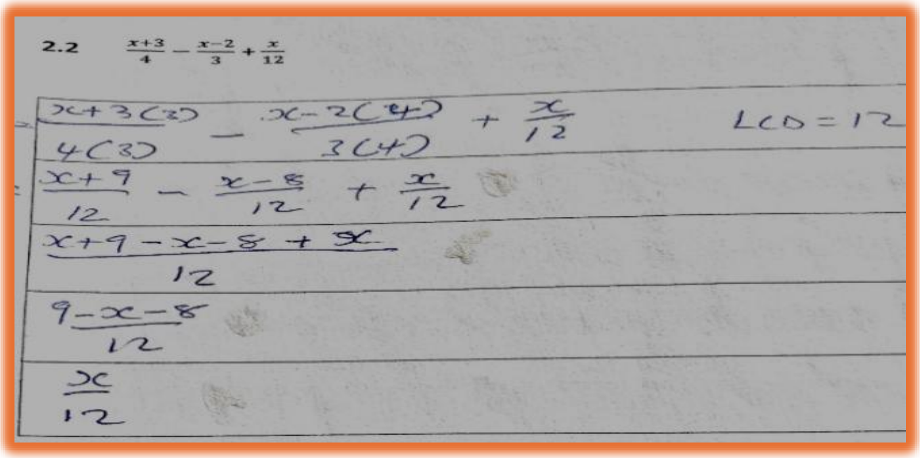
Learner 11E*: 2.2, firstly I checked if there is, …(paused) I checked, I looked for LCD which gave me 12. Then I divided 12 by the denominator then multiplied it by each expression. Then I multiplied the numerators by the number I got when dividing the LCD with the denominator. Then after I have same denominator, I used one and added all the numerators. Then it gave me 1/12.*

From Learner 11E's response, she does understand how to determine the LCD. However, the phrase: "*Then I divided 12 by the denominator then multiplied it by each expression"* is confusing because it is not clear what it means. What emerges once again is the importance of understanding the underlying principle behind obtaining 12 and multiplying each term by it, as well as the ability to clearly explain this process. These kinds of explanations suggest that the learner might know what she meant but saying it can be a challenge. It is also acknowledged that the learner used the correct mathematical terminology while providing her explanation. Likewise, Learner 1F had a challenge distributing the multiplier across the terms which are units. **Figure 19** supports the argument.

Figure 19

Snippet for Completion of Question 2.2

Learner 1F: Process skill error



2.2 $\frac{x+3}{4} - \frac{x-2}{3} + \frac{x}{12}$

$\frac{x+3(3)}{4(3)} - \frac{x-2(4)}{3(4)} + \frac{x}{12}$ LCD = 12

$\frac{x+9}{12} - \frac{x-8}{12} + \frac{x}{12}$

$\frac{x+9-x-8+x}{12}$

$\frac{9-x-8}{12}$

$\frac{x}{12}$

Learner 1F was able to determine the LCD, however looking at Step 2, he had a challenge that he did not consider multiplying x by 3 on the first term. The same applies to the second term where he multiplied x by -4. The reason might be that he considered x as a unit. It might also be that the learner thinks if the LCD is a constant, you do not consider variables. This could be another misconception. When probed he said:

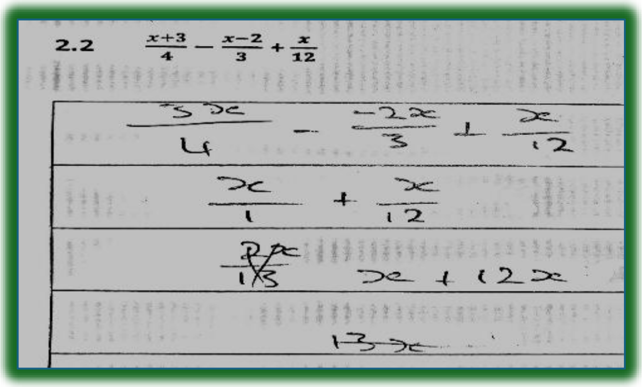
> ***Researcher:*** *Explain how you arrived at the final answer in question 2.2*
>
> **Learner 1F***: Hmmm…. I found the LCD which was 12. I then multiplied 4 by 3, it gave me 12, and that is where I got* $\frac{x}{12}$*. So, I took the denominator and multiplied it by the…(paused) by the first eh… question and then when I multiply them, I found x+9 the first one and the second one I found x-8 and then the LCD was 12. So, the last one was* $\frac{x}{12}$*. I then took all the numerators, I added them together, and then the third step I added and subtracted the like terms, and then the answer I found was* $\frac{x}{12}$.

The issue of "*taking the denominator and multiplying it by the…"* (learner paused), does not come out clearly. In other words, it is not clear what it means algebraically. Hence the learner manifested a process skill error. **Figure 20** presents challenges that the learner experienced when determining the LCD given denominators that contain numerals only.

Figure 20

Snippet for Completion of Question 2.2

Learner 25B: Comprehension error



As evident from **Figure 20,** Learner 25B in Question 2.2 struggles with the procedural steps needed to determine the LCD in simplifying the algebraic fraction, which led the learner to exhibit a comprehension error. For instance, Learner 25B had the misconception of treating fractions like whole numbers. For instance, the learner subtracted the first term from the numerator of the second term. Similarly, he subtracted the second term's denominator from the first terms to obtain 1(one). He then added the two x's and obtained 2x. He continued to cross multiply in his second step, where he multiplied x and 1 to get x, he then multiplied x and 12 to get 12x. In addition, this learner noticed the numerators and denominators are like terms and decided to add them to get 2x divided by 13 as the solution, but he decided to cancel this solution as he was not certain about it. He further noticed that these are like terms and decided to add them to get 13x even though he was not sure since he decided to cancel 13x. When asked to elaborate on his solution, he responded as follows:

> ***Researcher:*** *Elaborate how you came up with the solution for question 2.2*
>
> ***Learner 25B:*** *2.2, equation says x+ 3 so I saw that x is x+2 not x times 3. So, it was 3x over 4 minus one... minus 2x over 3 plus x over 12. So, I combined like terms and I the like terms it was x over 1 plus x over 12. So, it gave me x +12 and it was the answer.*

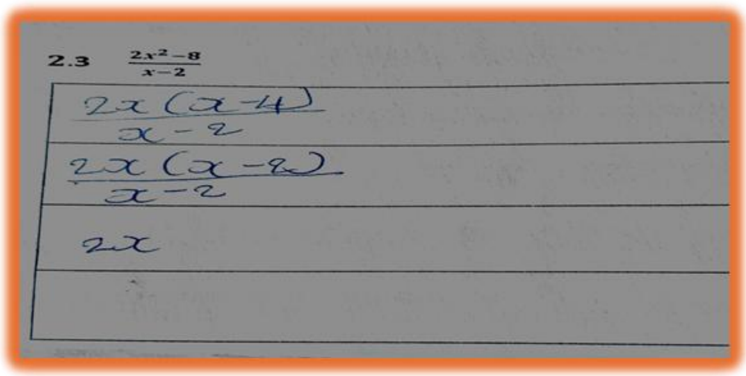
The first challenge is that the learner talks about equations instead of expressions. The learner does not have the conceptual understanding of algebraic fractions, hence exhibited the comprehension error, particularly on the use of algebraic language.

For example, in Question 2.3, learners were expected to simplify the fraction fully by applying the knowledge of determining the common factor and factorising the difference between two squares. **Figure 21** presents the snippet for completion of Question 2.3.

Figure 21

Snippet for Completion of Question 2.3

Learner 7H: Process skill error



When asked to probe on her answer, Learner 7H responded as follows:

Researcher*: Explain how you arrived at your final answer in question 2.3.*

Learner 7H: *2.3. I removed the common factor. Then after removing the common factor, I was left with* $\frac{2x(x-2)}{x-2}$. *Then I cancelled the x-2 and the x-2 that is when I came to my final answer.*

Although Learner 7H had an idea that she had to start by factorising the numerator, it seems that she was unaware that the numerator could be factored to result in the difference of two squares, which is easier to simplify. This appears to indicate a gap in prior knowledge regarding the simplification of binomials. The sudden change from $x - 4$ to $x - 2$ might have been prompted by the fact that the learner wanted to apply the cancellation rule. Hence the learner manifested a process skill error. The next figure presents the snippet for completion of Question 2.3 for Learner 17A.

Figure 22

Snippet for Completion of Question 2.3

Learner 17A: Process skill error and Transformation error

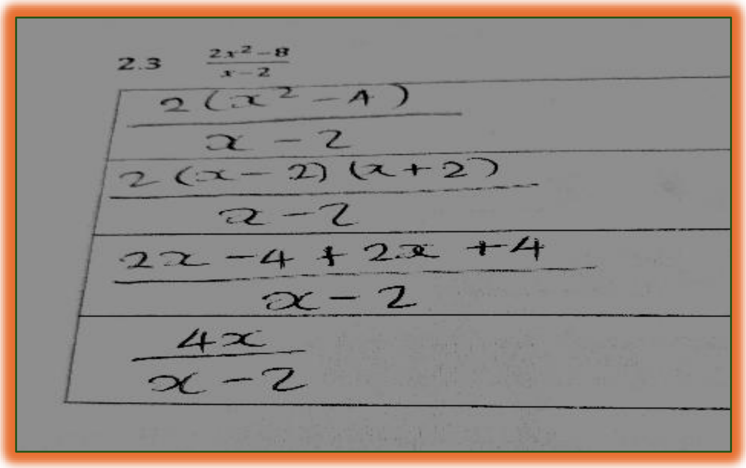


Figure 22 illustrates that Learner 17A managed to factorise the expression on the numerator, however she failed to recognise that $x - 2$ on both numerator and denominator could be cancelled out, which would simplify the algebraic fraction and make it easier to solve. Step 3 illustrates the process skill error in the sense that the learner was unable to simplify the numerator using the correct procedure. In her response, the learner did not explain how she expanded the numerator in Step 2. The response below demonstrates this point.

Researcher: ***In question 2.3, explain how you came up with the final answer.***

Learner 17A*: On 2.3 I took out the common factor which was 2 and then factorised the expression and then I added the like terms, and then I was left with the final answer.*

Despite being able to determine the common factor, the learner's response during the interview demonstrates that she failed to transform accurately as she confuses simplification with factorisation. This is supported by Baidoo (2019) who suggests that challenges with algebraic fractions are triggered by an insufficient comprehension of the concepts. Hence the learner exhibited a transformation error.

For Question 2.4, learners were required to simplify the expression that contains numerals and variables, where they were supposed to determine the LCD first, then simplify the expression. **Figure 23** provides the snippet for the completion of Question 2.4.

Figure 23

Snippet for Completion of Question 2.4

Learner 11E: Process skill error

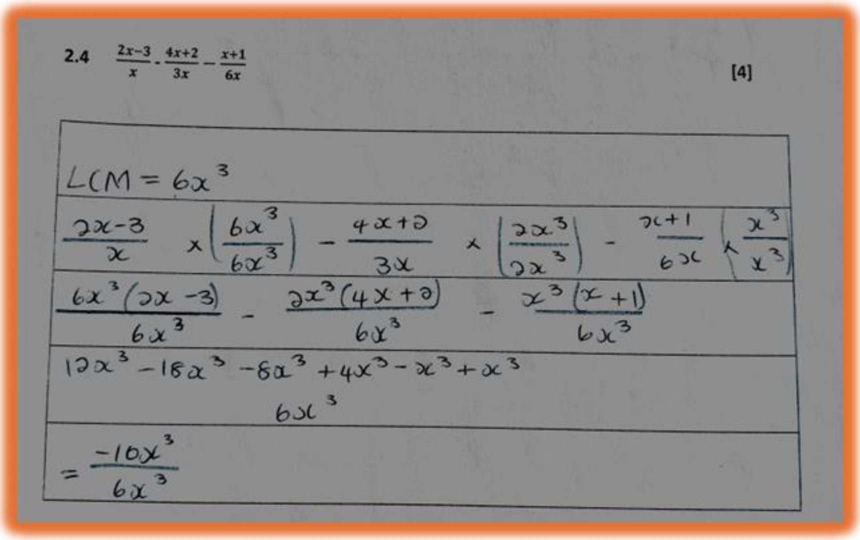


Figure 23 shows that Learner 11E knows that the lowest common multiple is 6, however, she incorrectly applied the rule to determine the LCD when the variables are the same. The misconception that is exhibited is that because there are three x's, it means the LCD is $6x^3$.

This appears to be a comprehension error. Distributing a negative sign across the terms in brackets is a challenge. The mistake that the learner committed is to write $12x^3$ instead of $12x^4$ and $8x^3$ instead of $8x^4$. This might show a lack of understanding of the laws of exponents dealt with in Grades 8 and 9. Outlining the process used, Learner 11E explained:

> ***Researcher:*** *Explain how you arrived at the final answer in question 2.4.*
>
> ***Learner 11E****: 2.4 I used the method of looking for the LCM. It gave me $6x^2$ and I divided the LCM by each and every denominator, then I multiplied the answer that I got when dividing and I got the same denominator which was x, $6x^3$ and I added all the numerators and I got the answer as -$10x^3$ divided by $6x^3$.*

These misconceptions led the learner to manifest a process skill error due to incorrect conceptions of rules.

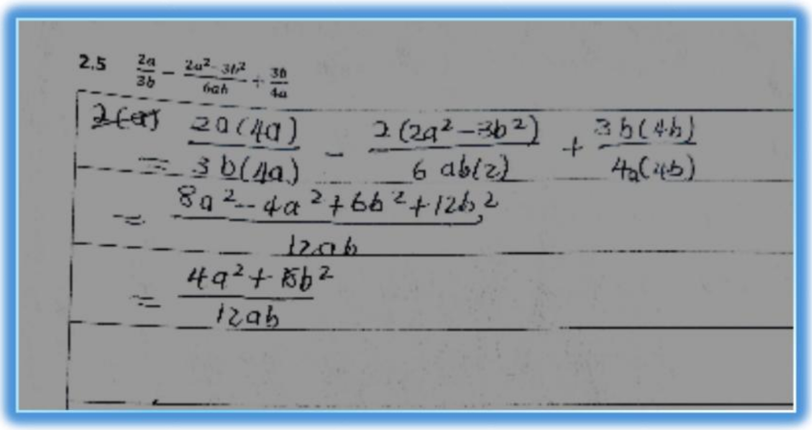
The next section presents the snippets for completion of Question 2.5. Question 2.5 requires learners to simplify an algebraic expression that contains more than one variable on the denominator and more than one term on the numerator of a single term. Learners were supposed to determine the lowest common multiple and simplify.

Figure 24 presents the snippet of completing Question 2.5 where the numerator and denominators of the fraction contain variables and numerals in each term.

Figure 24

Snippet for Completion of Question 2.5

Learner 9I: Encoding error



As evident in **Figure 24,** Learner 9I was aware that the LCD should be 12ab. However, she made a careless mistake when multiplying the last term by $\frac{4b}{4b}$ instead of $\frac{3b}{3b}$. Although the learner made a mistake, she was consistent in writing the correct LCD on the denominator correctly. Despite this challenge, the learner correctly applied the rules until the last step. Clarifying the steps followed, she said:

> ***Researcher:*** *In question 2.5, clarify the steps you have written until the final answer.*
>
> ***Learner 9I****: 2.5 I had to find the Lowest Common Denominator which is 12ab. I also had to multiply both the denominator and the numerator, where I got common factors on the numerators where I had to substitute to get the final answer which is* $4a^2 + 18b^2$ *over 12ab.*

The learner exhibited a strong understanding of algebraic principles and effectively employed precise language to explain the process of solving the problem. However, the learner manifested an encoding error due to inaccuracy in performing arithmetic calculations. The next section presents theme 2, which also emerged from the test and interviews for Questions 3.1–3.3.

4.4.2 Theme 2: Minimal Competence in Dividing Algebraic Fractions

Theme 2 also emerged when data from the test, interviews and were presented for Question 3. The purpose of Question 3.1 was to simplify the algebraic fraction expression that contains division sign, trinomials, common factor, and difference of two squares. To factorise the expression, learners were supposed to invert the multiplication sign into a division sign, then factorise completely. **Figure 25** presents a snippet for completion of Question 3.1

Figure 25

Snippet for Completion of Question 3.1

Learner 11E: Transformation error

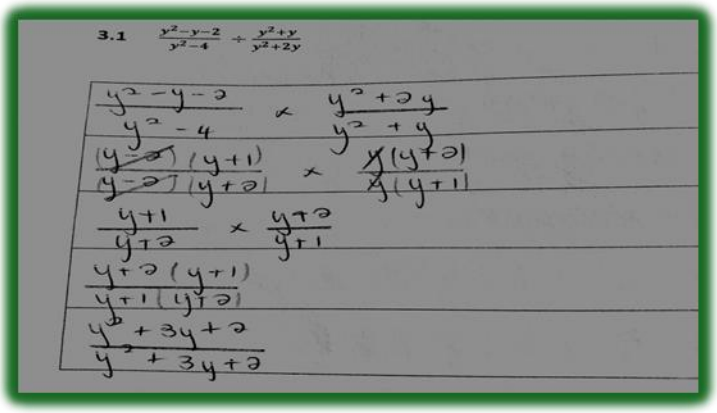


Figure 25 presents the solution of Learner 11E. Learner 11E was able to factorise and invert the fraction; however, she failed, in Step 3, to notice that the numerator and the denominator can be cancelled to remain with 1 as the answer. In this case, the learner might not know that the concept essentially involves determining how many times the number fits into itself. Otherwise, it might mean that the learner has an incorrect conception that when simplifying a fraction with variables, the answer should also have a variable. Responding to justify her argument, she said:

> ***Researcher:*** *Explain how you came up with the final answer in question 3.1*
>
> ***Learner 11E****: Firstly, they gave me a division sign, which means I had to change it into multiplication sign. When changing the division sign into multiplication sign you must know that even the left, the left exp…(paused) hmmm… , the left terms must also change, which means the denominator must become the numerator, the numerator must become the denominator. The next step I expanded the denominator, first term denominator. Then the denominator was difference of two squares, in the numerator I removed the common factor which was y(y+2) divided by… (paused), there was also a common factor y, I removed y, and I was left with y+1. I cancelled the terms that were the same and I was left with y+1 divided by y+2 multiplied by y+2 divided by y+1. Then I multiplied the terms, the numerators and the denominators, the final answer was* y^2 *+3y+2 divided by* y^2*+3y+2.*

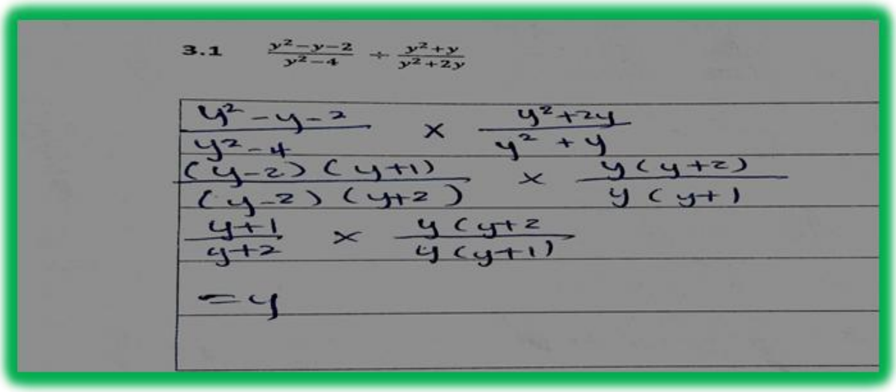
Learner 11E explained how she simplified the problem correctly but still failed to realise that the quotient in the final step is 1. Despite having the correct processes in mind, the misconception of not recognising that factors on the numerators and denominators can be cancelled impacted the learner's final answer. Hence the learner exhibited a transformation error due to being able to get close to arriving at the final answer but failing to accurately write the final answer.

Figure 26 demonstrates in detail how Learner 18D got y as the answer. Likewise, the learner might have a misconception that when dealing with algebraic fractions, the answer should always have a variable.

Figure 26

Snippet for Completion of Question 3.1

Learner 18D: Transformation error



For algebraic fractions which are quotients, some misconceptions and errors were identified from the learners' work. When Learner 18D was asked to probe her answer during the interview, she responded as follows:

> ***Researcher****: In question 3.2, how did you arrive at the final answer?*
>
> ***Learner 18D****: Ah, ... this, first I ... firstly I need to factorise, so that denominator may can cancel the nume…(paused) hmm… the numerator can cancel the denominator. I factorised this (y^2-y- 2), because the factors of y^2 is y, and the factors of 2 that when you subtract or add will give you 1, it is, ... it is 2 and 1. Here I looked for the common factor, which is y, here this is like a difference between two squares and I opened two*

brackets also, I looked for the factors of 4, I found out that it is 2 and 2 , so here I also looked for the common factor, which is y, opened two brackets (y+ 1). I forgot to close the brackets here. So, after here, I… this y-2 cancelled this y-2, and I was left with y+1 divide by y+2. I multiple… I did multiplication, then, here I was left with y(y+2) divide by y(y+1). From here, y+1 cancelled this y+1 and y+2 cancelled this y+2, y cancels y, I was left with… Eh… I was left with y.

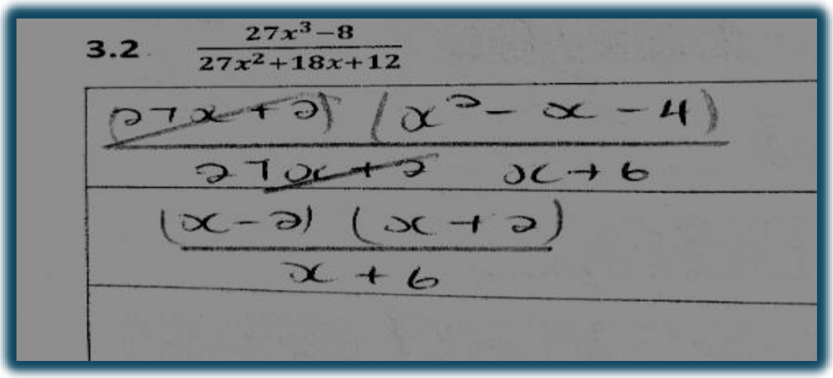
Despite being unable to get the correct answer due to the mistake of cancelling incorrectly, the learner also failed to notice that the y's on the numerator and denominator must cancel to remain with 1 as the answer. The response of Learner 18D demonstrates that she has knowledge of simplifying algebraic fractions; hence the learner manifested a transformation error.

Question 3.2 required participants to simplify by factorising the difference of two cubes on the numerator and determining the common factor first on the denominator. Another inappropriate cancellation of terms arose when Learner 11E wrote incorrect factors on the numerator and denominator of Question 3.2, as presented in **Figure 27.**

Figure 27

Snippet for Completion of Question 3.2

Learner 11E: Comprehension error



It appears the learner did not grasp well how a trinomial is factored. What appears to be in her mind, is to factorise and apply the cancellation rule which was inappropriately applied in this case. Probing how she arrived at the answer, she said:

Researcher: *Explain how you arrived at the final answer in question 3.2*

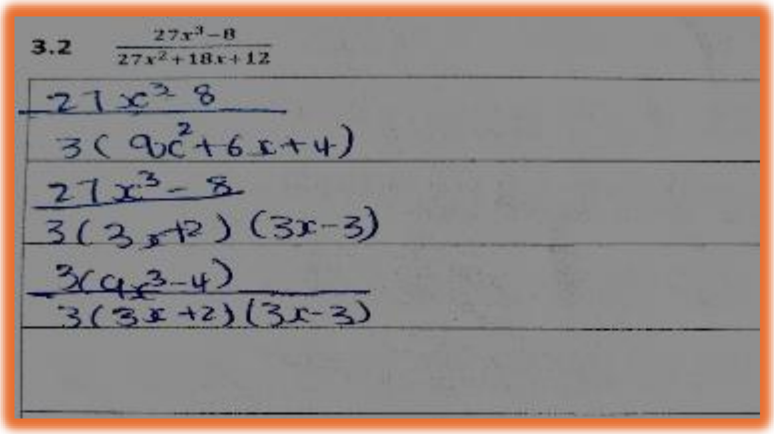
Learner 11E*: 3.2, I ...(paused) hmmm. I removed…hmmm…3.2 in the first bracket I put 27x+ 2 close bracket into x^2-x-4 close bracket divided by (27x+2) (x+6). Then 27x+2 was cancelled with the denominator. Then I expanded the terms of x^2-x-4 into (x-2) (x+2) divided by x+ 6.*

The learner's response demonstrates a comprehension error stemming from an inability to factorise the difference between two cubes. Another learner's snippet for question 3.2 is presented in **Figure 28.**

Figure 28

Snippet for Completion of Question 3.2

Learner 18D: Process skill error



When Learner 18D was asked to probe her solution during the interview, she responded as follows:

Researcher: *Explain how you arrived at the final answer in question 3.2*

Learner 18D*: I did not understand the question because when I looked at 27x^3-8, I looked for the common factor, I did not find it. Then I decided to leave it like this, and deal with the denominator. Here I found out that the common factor is 3, because yah the common factor is 3, I opened 2 brackets, and do like this, then I left the…(paused)I left 27x^3-8 like this because I did not understand it. So, here I started factorising (3x+2) (3x-3). Then I don't know how I wrote what I have written because I did not understand.*

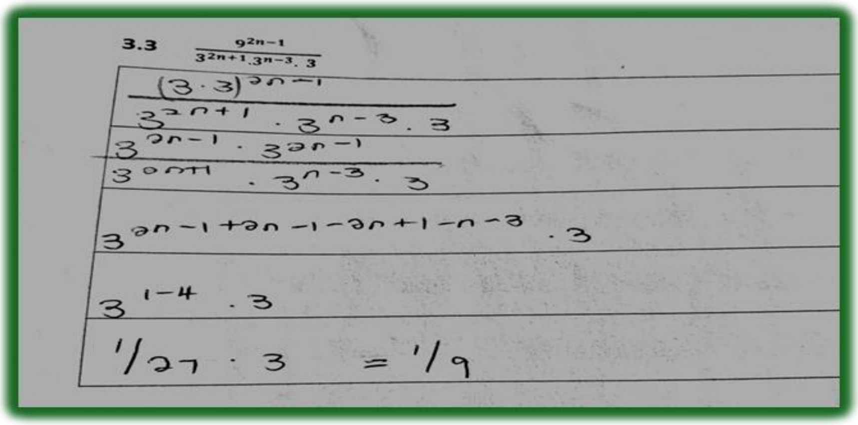
From the learner's answer and the response from the interview, Learner 18D failed to determine the factors of the difference of two cubes on the numerator due to lack of knowledge of determining the factors of cubic expression. Despite being able to determine the common factor on the denominator, the learner had a misconception of writing incorrect factors of the trinomial on the denominator which led the learner to exhibit a process skill error. This is evident from her response from the interview when she said: "*I started factorising (3x+2) (3x-3). Then I don't know how I wrote what I have written because I did not understand*".

Question 3.3 required learners to simplify by applying the laws of exponents by first determining the prime base factor on the numerator. Challenges regarding the application of exponential rules are demonstrated in **Figure 29**.

Figure 29

Snippet for Completion 3.3

Learner 11E: Transformation error



Although Learner 11E had knowledge of determining the prime base factors of 9, it appears that the learner got confused in terms of expressing numbers with negative exponents. The learner experienced challenges in transforming powers to equivalent forms. A transformation error is manifested. This may be due to the learner's lack of understanding of the quotient rule of exponents since the learner did not change the signs of exponents in the third step. When asked to justify her solution, she explained as follows:

Researcher: *Explain how you came up with the answer in question 3.3*

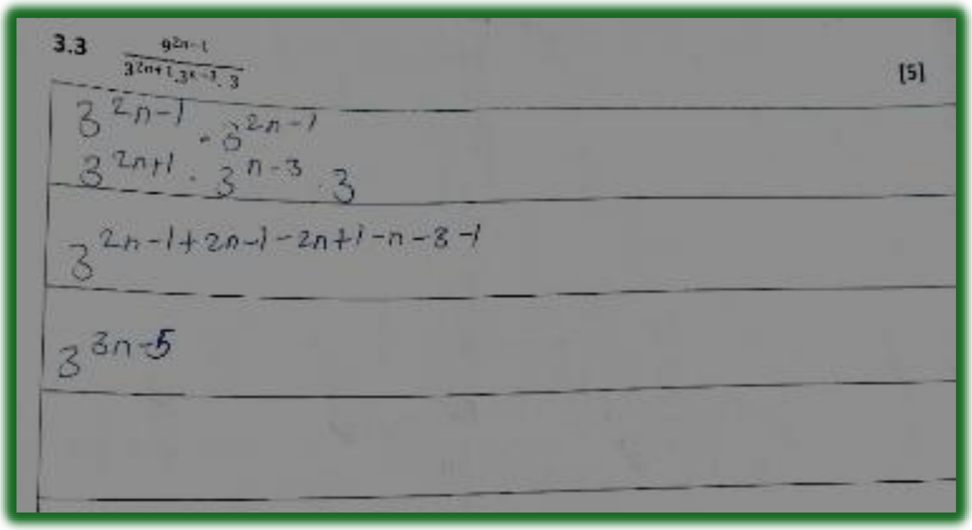
Learner 11E*: 3.3; firstly, I looked for the prime numbers of 9 in the numerator which was 3 and 3. I multiplied the prime numbers by the exponent and divided by 3 to the power 2n+1 multiplied by 3 to the power n-3 multiplied by 3. Then because the bases were the same, I decided to go with one base and then I subtracted where I was supposed to subtract and added where I was supposed to add the exponents, which gave me 3 to the power 1-4 multiplied by 3. And it gave me 1/27 multiplied by 3 which is 1 over 9.*

Learner 11E's response suggests that she finds applying the reciprocal rule challenging. Similarly, Learner 17A was not consistent in applying the quotient rule; she did not maintain the same principle of applying the quotient rule. At some stage, she considers exponents with variables and at some stage only the constants. Refer to **Figure 30**.

Figure 30

Snippet for Completion of Question 3.3

Learner 17A: Transformation error



When probing the learner to justify her argument, she said:

Researcher: *Explain how you came up with the final answer in question 3.3*

Learner 17A*: On 3.3 I looked for the factors of 9, and it was 3 by 3. Ok, then 3 by 3 I wrote it, but I added both, on the both sides of the 3s, I added this 2n-1 and it was over 3 to the power 2n+1 and 3…eh by 3 to the power n-3. I took the common factor, which was 3 and added all the exponents, which were 3 to the power 3n-5.*

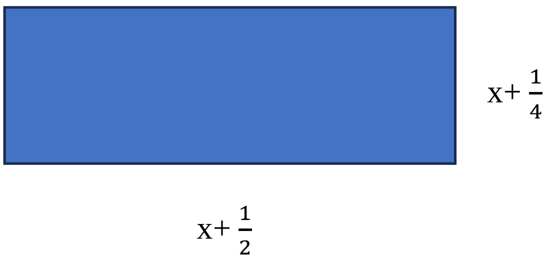
This transformation error seems to stem from the learner's lack of a conceptual understanding of exponential laws.

4.4.3 Theme 3: Difficulty in Analysing Word Problem

This theme arose from the word problem when I interviewed learners. For word problem, learners were given an answer sheet to simplify the given task by writing the answer. In addition, learners were required to probe their solutions by explaining how they arrived at their final answer. I gave learners the word problem to simplify. The word problem reads as follows: ***If you are given the rectangle with the length of*** $x+\frac{1}{2}$ ***and the breadth of*** $x+\frac{1}{4}$***. Determine the area of the rectangle in terms of x***

Figure 31

Word problem



The purpose of the word problem was to check prior knowledge of learners from lower grades. For instance, I wanted to determine if they knew the formula for calculating the area and if they could translate the word problem into a mathematical statement. **Figure 32** presents how Learner 1F solved the problem.

Figure 32

Snippet for Completion of Word Problem

Learner 1F: Encoding error

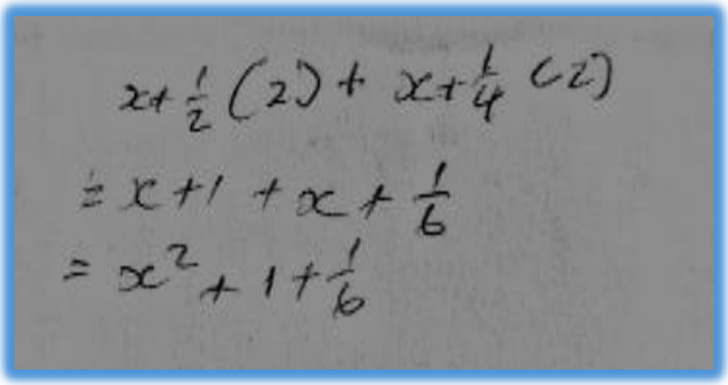


Figure 32 shows the learner used the wrong formula, which is a formula for calculating the perimeter. This could be the result of the learner struggling with the area calculation approach from earlier grades or it might be that this is the only formula that she remembered. Furthermore, the distributive property was incorrectly applied on the first term. A careless mistake was also committed when finding the product of the second term. When I probed the solution during the interview, she responded as follows:

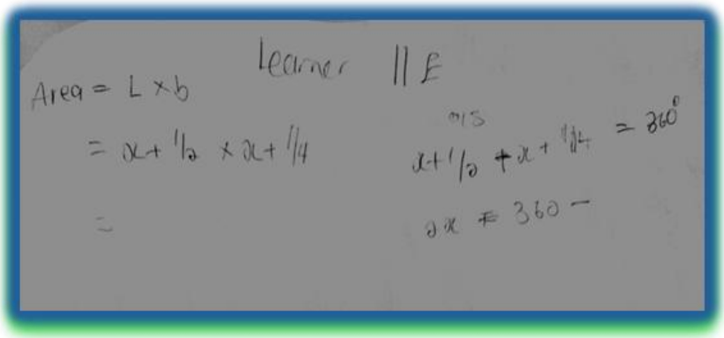
> ***Researcher****: Explain how you determined the area of a rectangle*
>
> ***Learner 1F****: Firstly, I will take x+ ½ as the breadth, and then I will multiply by 2 and x+ ¼ which is the length. I will multiply it by 2. So, the answer to this question will be x+1 because 2 will cancel 2 and then + x + 1/6 and then and then is equal to* x^2*+1+1/6.*

Although the explanation speaks to the solution, the learner did not indicate how he arrived at $\frac{1}{6}$. Hence the learner manifested an encoding error. **Figure 33** illustrates another challenge for Learner 11E:

Figure 33

Snippet for Completion of Word Problem

Learner 11E: Encoding error



Despite being able to determine the formula to calculate the area of a rectangle, the learner seemed confused as she said '*degrees*' in writing her answer, which made her stop simplifying as she was not sure of her solution. When the learner was asked to probe her answer during the interviews, she responded as follows:

> **Researcher:** *Explain how you determined the area of a rectangle.*
>
> **Learner 11E**: *When calculating the area of the rectangle, you multiply the length by the breadth. Which means you are going to say ($x+\frac{1}{2}$) multiply by ($x+\frac{1}{4}$) . Firstly, I am going to look for the values of x. They say a rectangle is equal to 360 degrees, which means each and every corner of a rectangle is equal to 90 degrees. Then you are going to say $x+\frac{1}{2}+x+\frac{1}{4}$ is equal to 360 degrees.*

From the learner's response during the interviews, she correctly substituted the dimensions of the rectangle in the formula; however, she failed to calculate correctly. This might be because she did not introduce the brackets in the first step. The learner was confused as she said that "*I am going to look for the values of x. They say a rectangle is equal to 360 degrees".* Hence the learner exhibited an encoding error. To add to this perspective, Wang (2022) supports the perspective that learners frequently encounter difficulty in understanding the context of the problem, which might hamper their ability to build up and simplify fractions effectively. This indicates that there is a gap in learners' grasp of both algebraic fundamentals and language employed in problem solving, inhibiting learners from completing the task correctly. This seems to suggest that for them to correct their errors and misconceptions, they need to practice these word problems on a regular basis.

4.5 Overview of Open Questions

Learners were asked open questions to check how they responded to tasks involving algebraic fractions. When Learners 21G, 9I, 1F and 11E were asked during the interview what they understand about algebraic fractions, they responded as follows:

> ***Researcher:*** *What do you understand by simplifying the algebraic fraction fully?*
>
> ***Learner 21G:*** *They mean write the expression in its simplest form.*
>
> ***Learner 9I:*** *You must simplify the expression fully where there will be no common factors until you get to the last answer.*
>
> ***Learner 1F:*** *According to my opinion, I think they were trying to tell us simplify in its simplest form using LCD, common factor, prime numbers etcetera.*
>
> ***Learner 11E:*** *I think when it says simplify the expression fully it means you must eh…, simplify all of it. Like you must… shoo…ok. You must eh… fully simplify all the question like eh… taking the LCD and simplifying all of it.*

From the responses provided by the learners, Learners 21G and 9I had an idea of how to simplify the algebraic fraction. Although Learner 1F included the LCD, common factor, and prime numbers, his response is notably better compared to the response of Learner 11E. Learner 11E seemed confused since she kept on posing when responding to the question.

I further asked learners the following open-ended question involving algebraic fractions task and Learner 9I responded as follows:

> ***Researcher:*** *What do you think are the challenges you encounter when attempting to solve algebraic fraction problems*?
> ***Learner 9I:*** *The problems that I encounter are… (paused) is to factorise. Sometimes I don't know how to factorise. Secondly, finding the lowest common denominator can be hard because sometimes we are given those of alphabets. That's all for me.*

From the learner's response, she demonstrated that she had a problem with fractions involving variables (the learner named variables alphabets), factorisation and determining the LCD.

When analysing the test results, many learners had difficulty in determining the LCD, factorising and simplifying algebraic fractions involving variables. This is supported by the research which states that learners had only learned the algebra algorithmically without conceptual understanding of algebraic expressions (Zulfa et al., 2020).

In the same vein, Learner 1F was asked the same question as Learner 9I, he responded as follows:

> ***Learner IF****: Eh… I think the problems we encounter when solving algebraic fractions problems is that we don't really know what the LCD, the first problem that we found is that we don't really know what the LCD is. The second problem, we have a problem when we are solving when it comes to signs. Yah… oh those are the challenges I think we have.*

This seems to suggest that errors in earlier understanding create frustration in learners in mathematics and diminish learners' motivation and confidence. Hence, better teaching strategies to develop a deeper conceptual knowledge prior to introducing procedural tasks is needed from educators to implement it. The next section presents patterns of errors emerging from the test and interviews.

4.6 Summary of Categories of Errors

Based on the findings from the test and the interviews conducted, I have summarised the percentage of learners' errors for each category in **Table 9** and **Figure 34.**

Table 8

Percentage of Learners' Categories of Errors Emerging from Test and Interviews

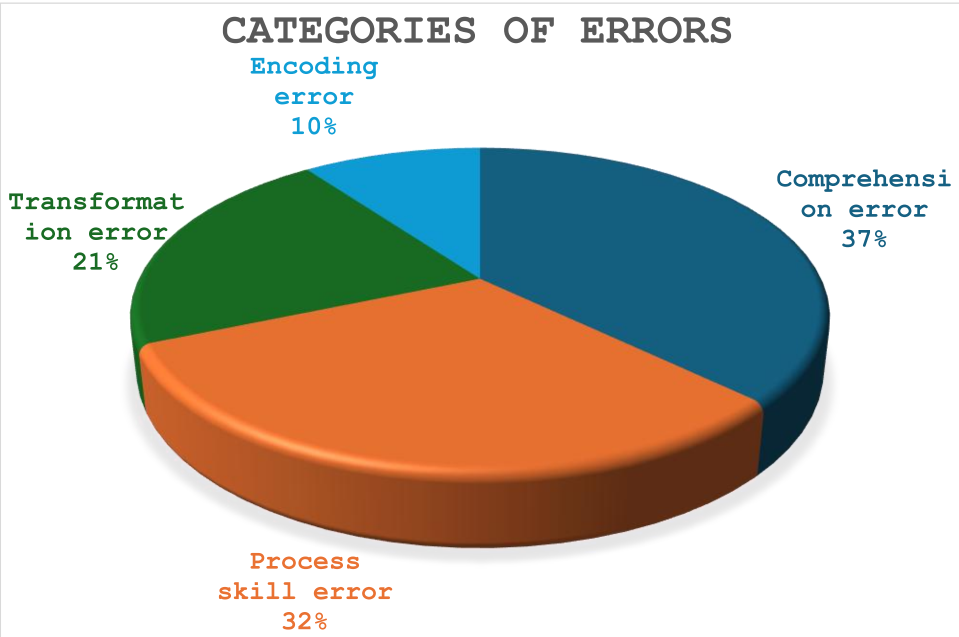
Category	**Frequency of learners' errors**	**Percentage of learners' errors**
Comprehension error	26	37.1
Process skill error	22	31.5
Transformation error	15	21.4
Encoding error	7	10.0
Total	70	100

Source: Researcher

The findings of the test and the interviews showed that there were 70 cases of each category of error. The highest ranking of the four categories, comprehension error, accounts for 26 (37.1%) of the errors made by learners, as seen in Table 9. This seems to suggest that many learners had difficulty comprehending the concepts required to simplify tasks. The process skill error ranks second in the category with 22 cases (31,5%). The implication here is that most learners did not follow the procedure to simplify the tasks because they lacked prior knowledge and skills to do so. Third place went to the transformation error, with 15 cases (21.4%) of learners making errors. Although most learners are aware of the fundamental concepts and procedures required for simplification, they are incapable of properly transforming the work. Most learners understand the concepts and procedures required for simplification, but they were unable to effectively change the task since they had previously made mistakes in the phases involved. With seven cases (10%), the encoding error came in last among all these categories. This might be because of the learners being given a single word problem to complete and being encouraged to go deeper into their response. It demonstrated that learners had a learning barrier that kept them from completely reducing the word problem, even though it was comparatively small. The pie chart showing the percentages of the error categories is shown in **Figure 34**

Figure 34

Percentages of Categories of Errors



Source: Researcher

The most common category of errors was the comprehension error, which was caused by inadequate comprehension of the concepts when answering questions about understanding the concept material and arithmetic processes. This suggests that learners should grasp the fundamental concepts of algebraic fractions to minimise other categories of errors. This would assist in reducing process skill errors, which occur because learners lack procedural knowledge for simplifying fractions. Most learners were unable to convert the word problem into algebraic expression, thus exhibiting transformation errors. This might be because most learners continue working using incorrect steps which caused them to fail to properly work on a problem. Encoding errors occurred as some of the learners were unable to draw a conclusion to the answer. This seems to suggest that learners were unable to draw conclusions from the presented problem. The next section presents a summary of the chapter.

4.7 Chapter Summary

This chapter summarised and interpreted the study's findings about the misconceptions and errors that Grade 10 learners manifest when simplifying algebraic fractions. The analysis of the findings was guided by the aim, research questions and objectives of the study. The snippets of learners' answers from the test and the interviews were used to assist in presenting the findings of the research. A pie chart, tables, and bar graphs were presented to give a picture of how the test results and the interviews were analysed. This seems to suggest that the root cause of these errors might be because learners knew the necessary steps but failed to apply them correctly due to conceptual misunderstanding procedural mistakes. The next chapter presents the results, emphasises the contributions, implications, and recommendations for future research study and the conclusion.

Chapter 5
Discussion and Conclusion of the Study

5.1 Introduction

This chapter presents the summary, recommendations, suggestions for further research and conclusions of the research. The discussion of the results from the test and interviews are the main topic of this chapter. Tables and bar graphs have been utilised to demonstrate and comprehend the data. The purpose of this qualitative case study was to explore misconceptions and errors that manifest when Grade 10 learners simplify algebraic fractions. Specifically, it sought to comprehend the rationale for learners' misconceptions and errors when simplifying algebraic fraction. In this chapter, I have included few excerpts from learners' responses to support my argument. The findings in the preceding chapters were predicated on the themes that emerged from the data and sought to explore the following research questions:

Main research question

What misconceptions and errors manifest when Grade 10 learners simplify algebraic fractions?

Secondary Questions

1. What categories of errors do learners manifest when simplifying algebraic fractions?
2. What misconceptions cause the manifested errors?
3. What are ways of reinforcing learners' learning of algebraic fractions?

5.2 Summary of the Findings in Terms of Research Questions

The research explored the misconceptions and errors manifested by Grade 10 learners in one high school in Mawa Circuit within the Mopani West District. The findings are presented in terms of the research questions as follows:

5.2.1 Research Question 1: What categories of errors do learners manifest when simplifying algebraic fractions?

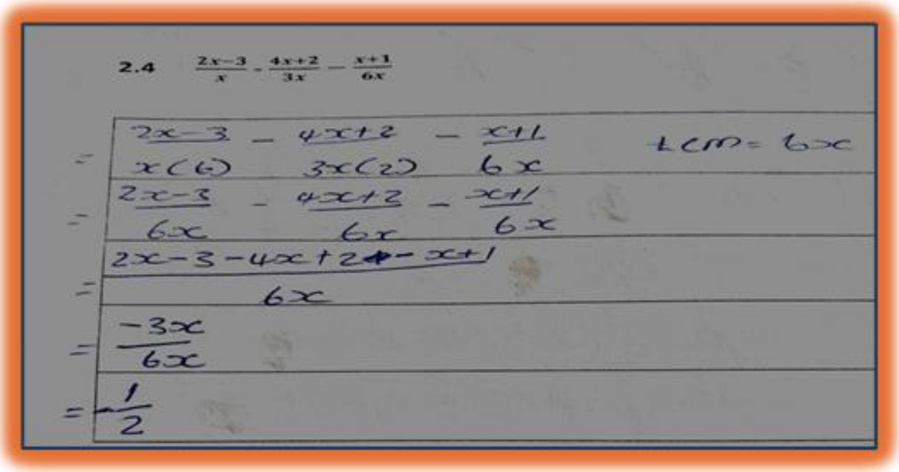
This question focuses on categorising the errors manifested when Grade 10 learners simplify tasks involving algebraic fractions. This study identified errors at various stages of the problem-solving process, ranging from interpreting word problems to simplifying the final answer. These errors emerged following the analysis of the test results and the conducted interviews. The key categories of errors observed in mathematical contexts include a) process skill, b) comprehension, c) transformation, and d) encoding errors. In addressing the research question, I conducted a detailed analysis of these four error categories, which were derived from the findings

a) Process skill error

Process skill errors arise when learners do not understand the steps involved and make mistakes in calculation (Asriyani et al., 2020). To support this perspective, Paut et al. (2023) indicate that errors in process skills arise when learners apply rules or solve tasks appropriately but make calculation and computational mistakes. For instance, in **Figure 35** Question 2.4 Learner 1F answered as follows:

Figure 35

Snippet for completion of Question 2.4



The learner correctly identified the LCD but failed to execute simplification procedures correctly, leading to an incorrect answer, suggesting that the learner lacks understanding of steps required to simplify algebraic fractions. In explaining how he arrived at the answer, Learner 1F responded as follows:

Researcher: *Explain how you arrived at the final answer in Question 2.4.*

Learner 1F*: The LCM was 6x. So, the first step I multiplied the denominators by 6x, and then I found these answers. And then the second step I added all the numerators and then I divided it by 6x and then the third step, when I added and subtracted the numerators, I found* $\frac{-3x}{6x}$ *so the answer was negative* $\frac{1}{2}$.

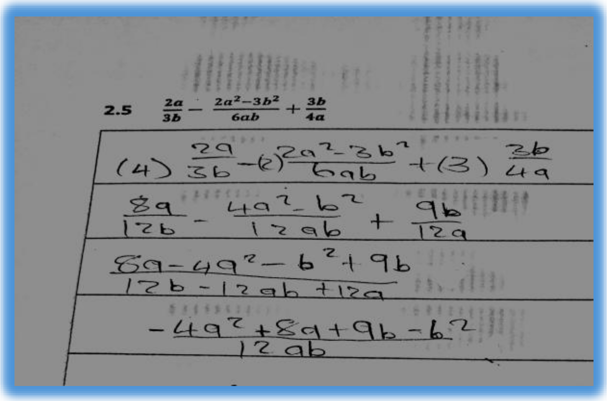
Learner 1F's comment during the interview and his response as presented in Figure 35 clearly indicated that he had an idea on how to determine the LCD. The learner failed to get the correct answer due to misconceptions made in the previous steps. Hence the learner manifested a process skill error which was caused by the lack of knowledge needed to simplify expressions. This finding resonates with Newman's theory, which posits that learners do not know the procedures needed to complete the simplification of the task. This finding adds to Smith and Wiser's (2021) argument that errors can result from learners misunderstanding a deceptive concept for a genuine one, which leads to misconceptions when it is repeated and ingrained in a learner's understanding. From the arguments put forth by scholars, it seems to suggest that either learners have limited, or no understanding of the procedures needed to solve the problems involving algebraic fractions. Put differently, the learner might be lacking the necessary knowledge or understanding of the steps, methods and strategies required to solve the problems correctly. Moreover, the study by Smith et al. (2022) highlights that procedural errors often arise from a combination of knowledge gaps and misconceptions rooted in earlier mathematical concepts. To conclude this section, I support the idea that mastery of problem-solving procedures involves a combination of knowledge, critical thinking and review of methods used (Rosenbaum et al., 2024). This combination facilitates the enhancement of the learning process.

b) Comprehension error

Sumule et al. (2018) explain that comprehension errors arise when learners do not thoroughly read the questions and do not comprehend the significant terms in that subject matter. In addition, according to Salmeron et al. (2024) comprehension errors occur when learners can read questions yet struggle with comprehending what is expected of them. This suggests that learners who make comprehension errors are unable to comprehend the solutions to the task because they only understand the problem and not the information in the question being asked. These errors stem from learners' misunderstanding of mathematical concepts, language or the context of the problem. This might happen because learners are able to read the words but are unable to comprehend the task's broader meaning. To support this argument, Learner 19C responded as follows during the interviews to explain how she answered the question which was to simplify.

Figure 36

Snippet for Completion of Question 2.5



I asked the learner to probe her answer, and she responded as follows:

> **Researcher:** *Explain how you arrived at your answer in Question 2.5.*
>
> ***Learner 19C****: I looked for the LCD, and I multiplied 4 by 2, I got 8 over 12; I also multiplied the denominators, to get 12. I said 4 multiplied by 3, I got 12. Negative…then 2 multiplied by 2, $4a^2$, and 2 multiplied $-3b^2$, I got $-b^2$ over 12ab. And then I multiplied 3 by 3b, I got 9b, then I multiplied 3 by 4, I got 12, and I simplified the whole expression, I said 8a- $4a^2 - b^2$ + 9b over 12b-12ab +12a. I got $-4a^2$+ 8a + 9b- b^2 over 12ab.*

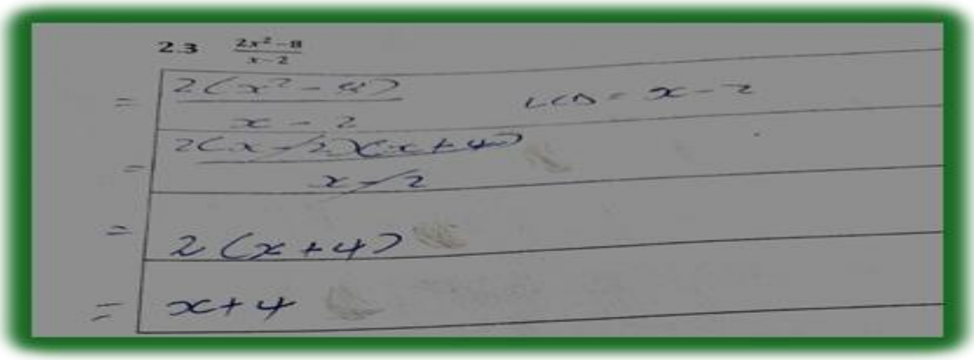
Upon reviewing the response, two issues can be identified: Firstly, finding the LCD is always associated with finding the product. Secondly, omitting the negative sign might suggest that the learner feels that she complicates simple problems or fails to connect the concept of negative numbers to tangible examples. The comprehension error manifested in this case exhibits a lack of solid comprehension of the fundamental concepts. This adds to the research by Khan and Ahmed (2022) which highlights that learners struggle to relate algebraic concepts due to lack of conceptual comprehension. This is supported by Nguyen and Pruitt (2022) who express how difficult learning is for learners with the language of mathematics resulting in misinterpretation of the task. Lack of comprehension of concepts in mathematics will result in it being challenging for learners to master the subject, solve tasks, and apply it in everyday life (Rismayanis et al., 2022). In addition, the capacity to develop strategies for solving problems, do basic computations, employ symbols to convey concepts, and modify the form when learning mathematics are all examples of comprehending mathematical concepts (Rismayanis et al., 2022). Despite this evidence, learners should have knowledge of algebra and comprehend all the concepts and skills needed to complete tasks involving algebraic fractions. This suggests that learners are unable to comprehend the basic characteristics of fractions and the relationships between denominators. This seems to indicate that learners are not able to recognise prime factors in the denominators, and this leads to errors in simplification of fractions. This indicates that learners should begin with simplifying fraction tasks involving numerals only for them to practise determining the LCD, then including variables after they have grasped how to determine the LCD in the first task. This will allow learners to build their confidence and skills before tackling more complex tasks.

c) Transformation error

A transformation error occurs when learners are unable to formulate or prepare a solution to the problem (Sumule et al., 2018). In addition, Paut et al. (2023) maintain that a transformation error occurs when learners lack understanding of the task to convert it into real mathematics. For instance, **Figure 37** presents the solution for Learner 1F:

Figure 37

Snippet for Completion of Question 2.3



From **Figure 37**, the learner confused the monomial with a polynomial as he determined the LCD of the monomial, which is incorrect. Despite being able to determine the common factor, the learner failed to simplify the second term using the common factor in Step 1. In addition, the learner introduced x-2 on the numerator so that he could cancel x-2 on both numerator and denominator. When Learner 1F was asked to probe his solution during the interview, he responded as follows:

Researcher: *Explain how you came up with the response to question 2.3.*

Learner 1F: *The LCD of 2.3 was x-2. So, the 2nd step I firstly ehh…(paused) found the ... I firstly ehh…factorised it and then I took the LCD as the denominator. And then I…. x-2 cancel each other. As then what was left is 2x+ 4 in brackets and then eh…as the answer I found x+4.*

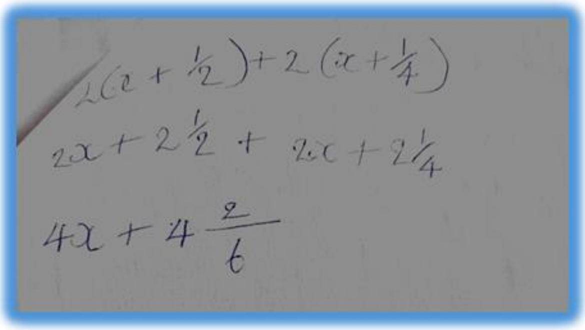
The learner seemed confused in explaining how he arrived at the final answer since he kept on pausing. He said he took the LCD as the denominator to simplify the expression. From the learner's answer and his response from the interview, the learner failed to transform correctly. In addition, the learner was able to determine the common factor but failed to simplify the second term in Step 1. In other words, the learner might not have the understanding that removing brackets, in essence, is about rewriting the expression in an equivalent form. Hence the importance of considering all the terms in brackets. Transforming mathematical expressions, in this instance, is a challenge. The learner manifested the transformation error. Based on the findings from the test and the interviews, learners were uncertain of the cause of the error, even if the previously recorded method or concept was correct. These errors can be addressed by reinforcing the fundamental principles of algebra, ensuring that learners comprehend these principles in the way that they can apply transformation accurately.

d) Encoding error

An encoding error arises when a learner completes the mathematical task successfully but fails to provide their response in writing (Wardhani & Argaswari, 2022). In addition, Paut et al. (2023) define it as errors that occur when learners are unable to appropriately understand the final answer they got. Many learners struggle to finish word problems because they have been unable to determine the formula to calculate the area of a rectangle, which hindered them from understanding the meaning of their answers. The findings revealed that some of the learners were unable to capture the answer correctly. For instance, **Figure 38** presents the solution for Learner 7H.

Figure 38

Snippet for Completion of Word Problem



From **Figure 38** the learner had a misconception of multiplying the length and the breadth by 2 and she also used addition instead of multiplying the dimensions of area of rectangle. During the interviews, Learner 7H was asked to probe her solution to the word problem, and she responded as follows:

> ***Researcher:*** *Explain how you determined the area of the rectangle:*
>
> ***Learner 7H****: The area of the expression of the rectangle. Ok, I can try, because I have forgotten the formula of calculating the area of the rectangle. So, I will just do it as 2(x+* $\frac{1}{2}$ *) because the rectangle has, has ... opposite of its sides are equal. Let me calculate. Hmm…I don't have a calculator.*

The learner seemed confused in explaining appropriately what she had written as the answer. In addition, the learner said she needed a calculator in simplifying the task even though there was no need to use it. The learner manifested an encoding error since she could not draw a conclusion from the answer she provided. This is a clear indication that learners struggle with precise representation of algebraic expressions due to mistakes made during calculations; thus, contributing to encoding errors (Rahman & Smith, 2022). To build on this perspective, Priliawati et al. (2019) mention that learners frequently do not review their work before collecting it and they are unable to present their answers and conclusions, and they fail to finish writing them (Asriyani et al., 2020). Errors may also arise from a lack of attention during problem-solving, which can cause learners to make careless mistakes due to rushed calculations (Mathaba et al., 2024). This seems to suggest that learners should be encouraged to double-check their steps before concluding their final answer or check the reasonableness of their answers (Muslim et al., 2024). Although an encoding error ranked the least of all categories of error, it can also hinder the performance of learners in algebra. If this occurs, learners might at a later stage find it difficult to grasp more advanced material built on this basic information. Being unable to get the correct answer might frustrate and demotivate learners (Novak et al., 2023). Hence learners could reduce calculation mistakes if they can learn to organise their thoughts (Abdullah, 2023) and have a strategy to effectively answer the questions to improve proficiency in simplifying algebraic fractions.

I conclude this section, and respond to the first research question: *What categories of errors do learners manifest when simplifying algebraic fractions*?
In solving algebraic problems, learners demonstrated errors related to comprehending concepts; incorrect procedures in solving problems; inability to transform mathematical tasks, and inability to provide the answer to word problem. Through thorough investigation, I identified that these errors are linked to misconceptions, which will be discussed in the next section. It appears that while learners have been taught the underlying principles, they still require further guidance, which could be implemented by providing them with tasks to analyse these errors, thereby facilitating a deeper understanding of the concepts. Such approaches might strengthen their confidence and ability to comprehend algebraic fractions.

5.2.2 Research Question 2: What misconceptions cause the manifested errors?

This question aims to explore the specific misconceptions and challenges that learners in Grade 10 encounter when attempting to simplify algebraic fractions, leading to the occurrence of errors. Although many studies have focused on the errors and misconceptions that learners commit when simplifying algebraic fraction tasks, some scholars have not been able to identify the misconceptions that cause the errors that manifested. Usually errors are once off episodes, however, when errors are persistent then we have a misconception. The current study sought to validate this claim, from another perspective, by building on the work of Gosh et al. (2022) which emphasise that even though misconceptions may be classified as errors; not all errors can be classified as misconceptions since misconceptions are a specific type or category of errors. Elucidating this perspective, an error reflects systematic conceptual misunderstanding (Elagha et al., 2024). I concur with Gosh et al. (2021) that a misconception is a specific, incorrect understanding that causes errors, and an error may or may not be due to misconceptions. Suparman et al. (2024) further acknowledge that misconceptions are observed alongside errors. The focus in this case was more on describing the causes of misconceptions that directly led to the errors, in other words, to answer the research question: *What* *misconceptions cause the manifested errors?* I classified the types of misconceptions into subcategories. The next subsections address the misconceptions that emerged from the manifested errors, which are the categories of errors found in Research Question 1 emerging from the learners' test and the conducted interview.

5.2.2.1 Misconceptions that Lead to Errors in Process Skills

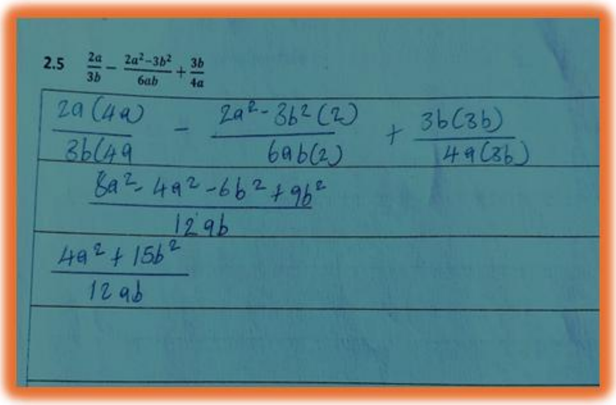
During analysis of the test and the conducted interviews, the findings revealed that misconceptions that caused learners to exhibit process skill errors were as follows: a) introducing and applying the distributing property in removing the parentheses; b) equivalent forms of algebraic fractions; c) factoring and expanding binomials and trinomials; and d) inability to determine the common factor and the LCD. The following subsection presents misconceptions about introducing and applying the distributing property in removing the parentheses.

a) Introducing and applying the distributive property in removing the parentheses

I found that learners experience some challenges with the use of brackets in simplifying algebraic fractions. They either fail to distribute a number across all the terms in brackets or they fail to introduce the brackets to group the terms so that they can properly distribute the LCD. Stemele and Jina Asvat (2024) agree with this idea and indicate that the terms in brackets should be treated as a single entity. Some of the learners, for instance, Learner 7H in **Figure 39,** presented her answer as follows:

Figure 39

Snippet for Completion of Question 2.5



In addition, Learner 7H responded as follows during the interviews:

Researcher: Describe how you came up with the answer to question 2.5.

Learner 7H: *Ok, on 2.5 I needed like common factor, ehh…(pasued) and LCD too. Then I found out that my LCD was 12ab. Then I managed to make all my denominators equals to 12ab, by multiplication. Then as we know that the number that we multiply we must multiply on the numerator. Then I added like terms. Then that is where I came to my final answer which was* $\frac{4a^2+15b^2}{12\mathrm{ab}}$.

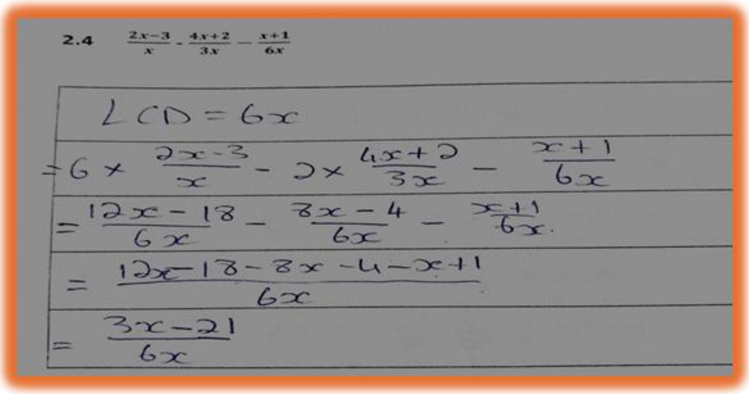
Despite being able to determine the LCD, the learner neglected to introduce the brackets to the binomial of the second term of the expression on the numerator. In accordance with this misconception, the learner only applied brackets to the second term of the binomial, which resulted in an inaccurate term distribution. She also handled the negative sign incorrectly since she neglected to add brackets. Hence the learner exhibited process skill error. Arguing their point of not using the brackets, Papadopoulos and Gunnarsson (2020) emphasise that “the avoidance of brackets when writing arithmetic expressions does not always necessarily mean lack of structural understanding” (p. 206) and indicate that this is an implicit use of ‘mental’ brackets. I express a similar sentiment but still emphasise that there is a need for caution, as learners may otherwise make careless mistakes. The mistakes were evident when learners sometimes distributed the LCD to only one term inside the brackets, ignoring the other terms. Misconceptions about the equivalent form of algebraic fractions are presented in the next subsection.

b) Equivalent forms of algebraic fractions

During the analysis of the test and the interviews, most learners struggled to realise that two fractions are equivalent if they represent the same value, even if they appear different, but learners incorrectly concluded that they are not equivalent because of different numerators and denominators. Consider, for instance, **Figure 40**, Learner 21G’s snippet for Question 2.4.

Figure 40

Snippet for Completion of Question 2.4



Learner 21G responded as follows during the interview.

> ***Researcher:*** Explain how you arrived at the answer in Question 2.4
>
> ***Learner 21G:*** *You first find the LCD which is 6x and then you multiply each of the fraction with the number that will make the LCD to make the LCD the same. And don't forget also to multiply the numerators when you multiply the denominators. And that's how you arrived here and then you group the like terms and that is how you will arrive at 3x-21 over 6x.*

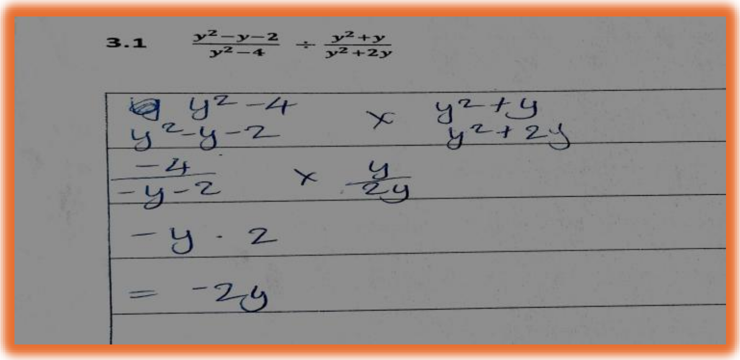
Even though she was able to find the LCD, the learner's response seems to indicate she failed to recognise that two algebraic fractions are equivalent if they factor into one another or simplify to the similar expression. In addition, the learner's interview response was unclear since she did not provide a detailed explanation of her writing. This view is supported by Kaminski et al. (2023) who identify that misconceptions regarding equivalent forms of fractions are common among high school learners. To add to this, a literature review by Wilson and Cheng (2022) discuss the instructional strategies to address the misconceptions by fostering deeper conceptual understanding for learners. The next subsection presents misconceptions in factoring and expanding binomials and trinomials.

c) Factoring and expanding binomials and trinomials

A literature review by Ahmad and Nasir (2023) highlight the prevalence of errors when learners attempt to expand or factor algebraic expressions in their foundational knowledge. To support this argument, a review by Fitria and Susanto (2023) stresses the importance of explicit instructions on the connection between expanding and factoring, noting that misunderstandings in these areas lead to broader challenges in problem solving. This is what emerged from the test and interviews, where learners incorrectly applied factoring formulas in binomials and trinomials due to oversimplification. To support this argument, some learners knew that if an expression had a division sign, the division sign should change into a multiplication sign and then invert the numerator and denominator of the second expression. However, Learner 7H had a misconception of first inverting the numerator and denominator of the expression. In addition, the learner failed to determine that factors of the binomial and the trinomial. The learner just cancelled like terms on the numerator and denominator which led the learner to wrong simplification. This is evident by Learner 7H in **Figure 41**.

Figure 41

Snippet for Completion of Question 3.1



Learner 7H was further asked to probe her answer during the interviews and she responded as follows:

> ***Researcher:*** Explain how you arrived at the answer in Question 3.1
>
> ***Learner 7H:*** *On the first one I cancelled y^2 and y^2, because it was y^2-4 over y^2-y-2. I cancelled it. I was left with -4 over -y-2. Then on the second one I cancelled 2, then I was left with y over 2y. Then it was -y multiplied by 2. The it became equals to -2y.*

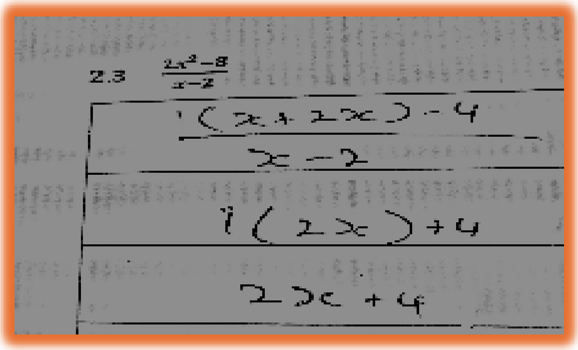
The learner's response during the interviews seems to suggest that she has not yet fully developed the necessary process skills to accurately carry out operations involving polynomials, and she lacks a solid understanding of the fundamental algebraic concepts in factoring and expanding binomials and trinomials, highlighting the critical need for a grasp of polynomial properties. In addition, learners should be given tasks that reinforce the fundamental skills needed for factoring and expanding polynomials. The next section presents the learners' inability to determine the common factor and the LCD.

d) Inability to determine the common factor and the LCD

The findings from the test and the interviews I conducted revealed that learners had a serious challenge in determining the common factor and the LCD. This is evident for Learner 25B in **Figure 42**, who failed to determine the common factor.

Figure 42

Snippet for Completion of Question 2.3



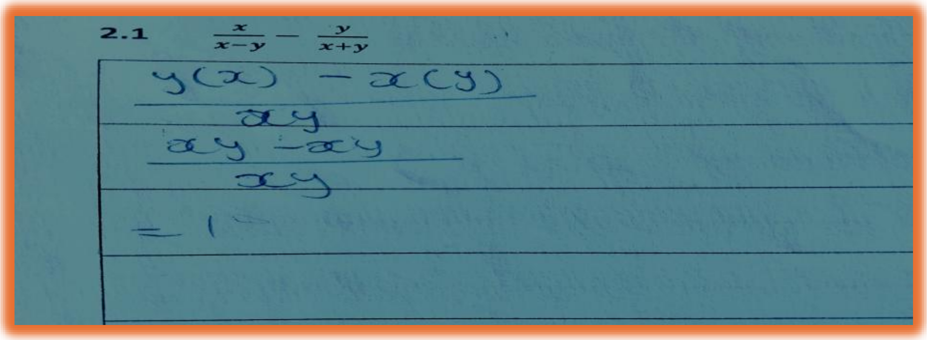
From **Figure 42,** the learner lacks procedures of determining the common factor since he wrote 1 as the outer term and multiplied it by the binomial x+2x and further subtracted 4 on the numerator which is incorrect. When the learner was asked to explain his response, he replied as follows:

> ***Researcher:*** *How did you arrive at the answer in question 2.3?*
>
> ***Learner 25B****: 2.3 I found out I have 2x^2-8 over x-2. I found out the terms when I multiply it by any number it can give me the equation, me the upper equation. So, it was 1 (x+2x) - 4 and then over x-2. Then, I found out there is any like term there I minus it…. I divided it by x-2. So, my answer was 2x+4.*

It is evident from the learner's interview response that he has no prior experience determining the common factor. The findings during the interview test and the conducted interviews also revealed that some learners are not able to determine the common factor during simplification of task, which hinder them from obtaining the correct solution. For instance, **Figure 43** presents the snippet for Learner 17A.

Figure 43

Snippet for Completion of Question 2.1



From **Figure 43**, the learner had a misconception of determining the LCD by multiplying the numerators of the expression. In addition, the learner did not notice that the like terms on the numerator will result in zero in Step 2, the learner decided to cancel each term on the numerator by the denominator to get 1 as the final answer. When probed to justify her answer during the interview, she responded as follows:

> ***Researcher:*** *Explain how you came up with your final answer in question 2.1*
>
> ***Learner 17A:*** *Ok at 2.1…(paused), the way I looked at the expression, I thought that… I knew that I should take out the common factor and then I multiply the numerators. And then I left the denominators into the simplest form. And then I found that some of the values are going to cancel each other and that is how I left with the final answer.*

According to the learner's interview response, she was unaware that she should first determine the LCD, as she mentioned taking out the common factor, which is incorrect. In addition, she also mentioned that she left the denominators in their simplest form, which is also incorrect. To support this view, Wang and Lee (2021) describe arithmetic algebraic fractions in this context as an expression formed by division of two polynomials where learners must apply their knowledge of algebraic manipulation, including factorisation, common denominators, determining the LCD, and simplification techniques. This seems to suggest that learners should have knowledge of determining the common factor and LCD for them to be able to simplify algebraic fractions correctly. The next section presents misconceptions that cause comprehension errors.

5.2.2.2 *Misconceptions that Result in Comprehension errors*

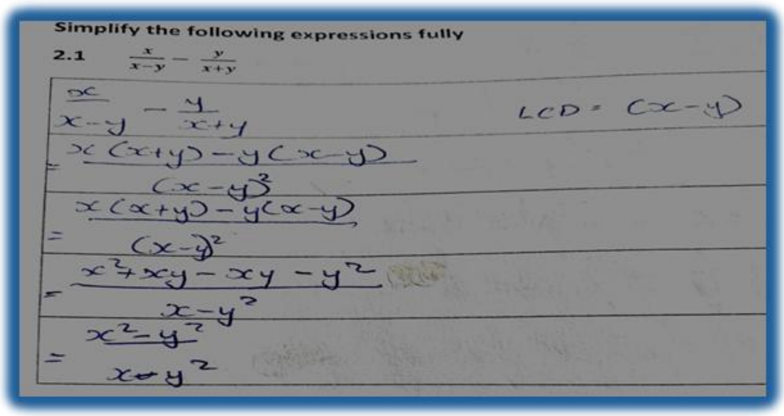
The findings from the test and the interviews revealed the following misconceptions that caused comprehension errors: a) the use of algebraic language, b) lack of prior knowledge, and c) confusion with cancellation of terms. The next subsection presents the use of algebraic language.

a) The use of algebraic language

Based on the interview and the test, the findings revealed that some of the learners failed to use the correct mathematical language when explaining their solutions. For instance, Learner 1F was asked to simplify the following expression as presented in **Figure 44.**

Figure 44

Snippet for Completion of Question 2.1



When asked to explain his solution, he responded as follows:

> ***Researcher:*** Explain how you arrived at the answer in Question 2.1
>
> ***Learner 1F:*** *Firstly, I... I took the…. I took the LCD out and I multiply all the equation with it. So, I …step no.1 I took eh… I took the LCD and multiplied it by x. And then I minus it by minus y. And then I took the LCD as the denominator. And then the second I added all the … all the numerators. And then I also took the denominator again. And then the third step I added them all. I added where I must add and subtracted where I must subtract. And then I found* $\frac{x^2-y^2}{x^2-y^2}$.

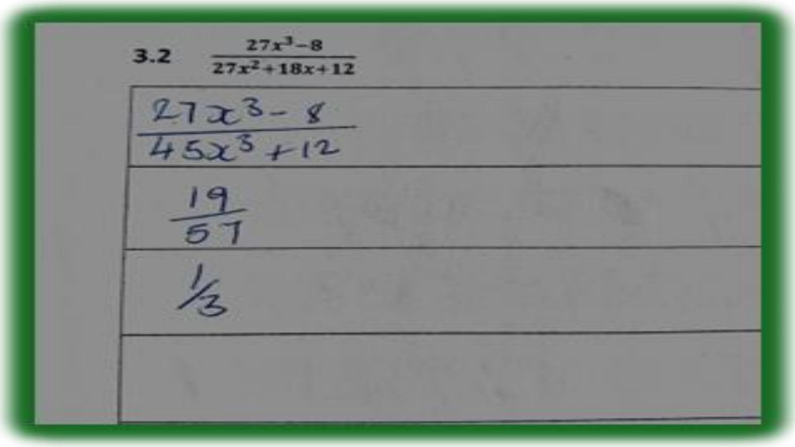
From the learner's response from the interview, it shows that he lacks conceptual understanding of algebraic fractions; for instance, the learner talks about equations instead of expressions. In addition, the learner uses minus instead of subtraction. To build on this understanding, a review by Nofrianto etal. (2022) emphasise that interventions that focused on language usage significantly improved learner understanding and error reduction in algebraic tasks. This is supported by Naylor et al.'s (2022) review highlighting how misconceptions in algebraic language can directly connect to learners' performance, noting that educators must explicitly teach the meaning of appropriate use of algebraic terminology. The misconceptions that learners encounter while solving numerals are based on language, specialisation, notation, and generalisation (Nurkamilah & Afriansyah, 2021). This suggests that learners should actively acquire, and review terms related to algebraic fractions with the aim of ensuring correct comprehension during lessons as well as studying, as learners incorrectly interpret information if they do not fully comprehend the contextual information provided by the language used. The next subsection presents the lack of prior knowledge.

b) Lack of prior knowledge

This misconception focuses on how most learners fail to possess a solid understanding of basic algebraic concepts which are crucial for simplifying fractions during the test and the interviews. The findings revealed that most learners had a challenge determining the factors of cubic expressions and factorising trinomials due to lack of prior knowledge. This is evident in **Figure 45**, which presents the answer for Learner 7H as follows:

Figure 45

Snippet for Completion of Question 3.2



From **Figure 45,** the learner added terms like whole numbers on the numerator and denominator, since she did not consider the fact that unlike terms cannot be added or subtracted. When asked to probe her answer during the interview, she responded as follows:

Researcher: *Explain how you came up with the final answer in question 3.2*

Learner 7H: *3.2 Hmmm…. Ok 3.2 I added on the denominators I added x^2+x it was x^3 then plus 12. On the upper side it was 27x^3 -8 then eh… I cancelled the x^3, then I was left with 27-8, which was 19, and 45 + 12 which was 57. And making…, simplifying it was 1 over 3.*

From the learner's response from the interview, she also lacks prior knowledge of adding and subtracting like terms since she cancelled terms without determining the factors first. To support this argument, Im and Jitendra (2020) argue that misconceptions may stem from prior knowledge that people wrongly generalised. To draw attention to the importance of knowledge stemming from experience, Karpudewan et al. (2020) emphasise that misconceptions often emerge because of learners' inappropriate prior learning experiences that are often incorrect or insufficient. It also indicates that learners are lacking in the fundamental concepts necessary for effective learning when it comes to simplifying algebraic tasks. This seems to suggest that to enhance their algebra skills, learners should take the time to assess and identify their key areas of challenges. With the goal of improving comprehension through cooperative and shared thoughts, learners ought to further divide into study groups and tackle tasks together. The next section presents misconceptions that led to transformation errors.

5.2.2.3 Misconceptions that cause transformation errors

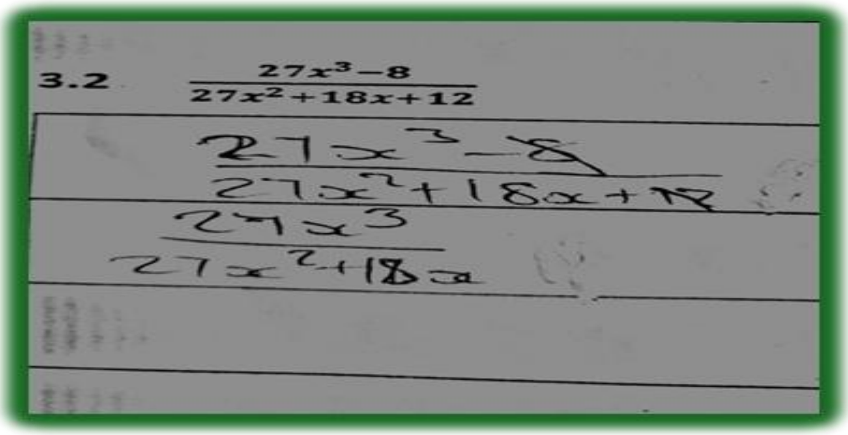
The findings during the analysis of the test and the interviews also revealed that learners had a challenge with the following misconceptions associated with transformation errors: a) confusion with cancellation and combination of terms, and b) application of exponential rules. The next subsection discusses confusion with variable and cancellation terms.

a) Confusion with cancellation and combination of terms

The findings during the test and interviews revealed that many learners had a challenge regarding the cancellation of combination of like terms. Moreover, learners did not cancel the terms on the numerator and denominator correctly, which led them to wrong simplification. For instance, Learner 18D in **Figure 46** presented her answer as follows:

Figure 46

Snippet for Completion of Question 3.2



As evident in **Figure 46**, the learner cancelled the constant terms on the numerator and denominator even though they are not equal. In addition, she further wrote the expression as it is on the first step to show that she did not comprehend the task. When asked to explain how she arrived at the final answer, she responded as follows:

> ***Researcher:*** Explain how you arrived at the answer in Question 3.2
>
> ***Learner 19C****: I added the expression as it was, I said $27x^3$-8 over $27x^2$ + 18x +18. I got $27x^3$ over $27x^2$ + 18x.*

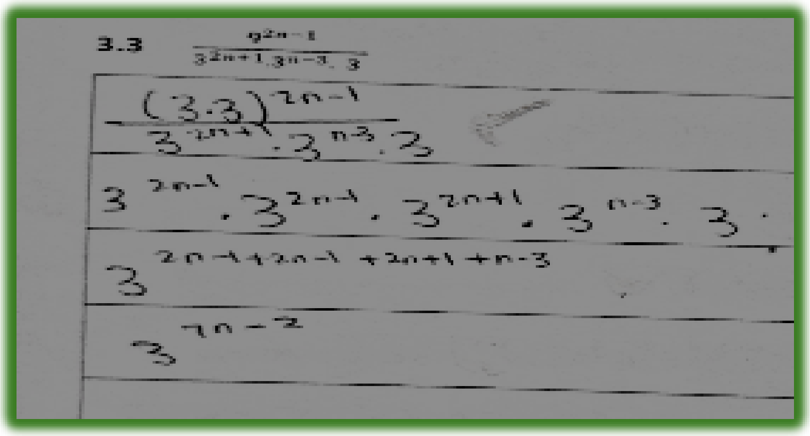
The learner's response from the interview indicates that she was confused with the terms on the numerator and denominator. According to Neidorf et al. (2020) and Rushton (2014), who argue against the improper cancellation of terms, learners often assume they are performing the correct approach or are confused about what they are doing. This suggests that to further enhance their algebraic learning, learners should learn how to identify like terms in an expression and determine common factors. This seems to suggest that learners should work on applying theoretical knowledge to practical tasks involving cancellation. The next subsection presents the application of exponential rules.

b) Application of exponential rules

During the analysis of the test and the interviews, the findings revealed that learners had a significant challenge in simplifying tasks involving exponents. Most learners failed to apply the quotient rule of exponents in simplifying the task. This is evident in **Figure 47** for Learner 18D as follows:

Figure 47

Snippet for Completion of Question 3.3



Despite being able to determine the prime base factors of 9, the learner applied the multiplication rule of exponents instead of the quotient rule. This argument is supported by Jansen and van der Meijden (2020) as they define misconceptions as the improper application of a rule, an overgeneralisation or undergeneralisation that leads to alternative interpretations of mathematical situations. This seems to suggest that learners should grasp fundamental concepts like exponents, variables, and coefficients for the purpose of simplifying these kinds of algebraic fraction tasks. In addition, different rules of exponents, such as bases that are multiplied, divided, and raised to the powers, should also be reviewed and practised by learners. This implies that learners commit these misconceptions due to a lack of knowledge of exponential rules. The next subsections present misconceptions that caused encoding errors.

5.2.2.4 Misconceptions that cause encoding errors

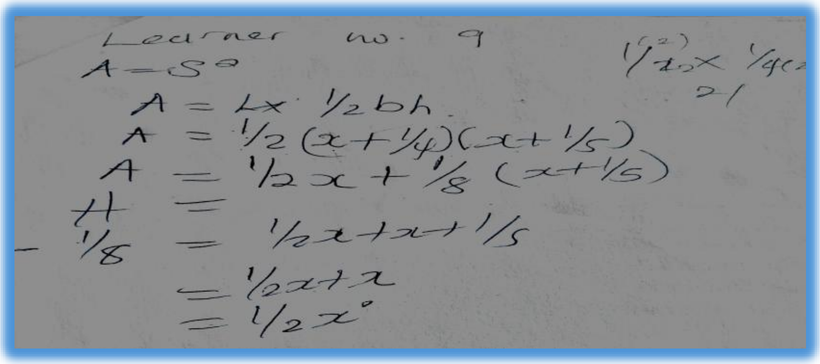
Based on the interviews I conducted for the word problem, the findings revealed that learners had a challenge in selecting appropriate strategies and correctly translating the word problem. The following subsection presents the selection of appropriate strategies to solve the word problem.

a) Selection of appropriate strategies to solve the word problem

A study by Suseelan et al. (2023) emphasises that learners struggle to transition from numerical to verbal problem solving, failing to identify relevant information due to lack of strategic knowledge. The findings from the interviews conducted on the word problem revealed that learners experienced a greater challenge in translating word problems into algebraic expressions. To support this argument, learners were unable to translate the word problem into a mathematical algebraic fraction due to previously made mistakes. This is evident from Learner 9I in **Figure 48** who presented her answer as follows:

Figure 48

Snippet for Completion of Word Problem



Learner 9I applied the incorrect formula to determine the expression representing the area of a rectangle. This appears to suggest that learners learnt about these formulae; however, it might be that they know the shapes but confuse the formulae. A potential gap may lie in the learner's ability to visualise the concept of area effectively. Furthermore, the formulae may have been learned in isolation, without consideration of the connections between different geometric shapes. In particular, the learner may not have fully grasped the connection between the area of a triangle and that of a rectangle. Moreover, the learner confused the length with the breadth and failed to notice that the numerals in the specified dimensions are not equal. After probing the learner for clarity, she indicated that:

> **Researcher:** *Explain how you arrived at the final answer in determining the area of a rectangle*
>
> **Learner 9I**: *First, you write A =* s^2 *which is because it's a… yhoo! It's a rectangle find the area where we must find the area of a rectangle which is length times breadth, half breadth times height. That is when you write you're A = half, where your breadth is x +*$\frac{1}{2}$ *in brackets. Our height is x+*$\frac{1}{5}$ *. You put your numbers in a calculator and get to a final answer by substituting. There is an x which means that I must solve it. We say* $\frac{1}{2}$ *times x, which will give us* $\frac{1}{2}x$ *and* $\frac{1}{2}$ $times\ quarter$*, which is…eh…* $\frac{1}{8}$*. That is when you will take numbers without variables to the other side which is* $\frac{1}{8}$ *. On the other side*

we will say $\frac{1}{2}$ x +x + 5, you will also take 5 to the other side and change the sign. You will be left out with $\frac{1}{2}$ x + x, which will give you $\frac{1}{2}$ x^0.

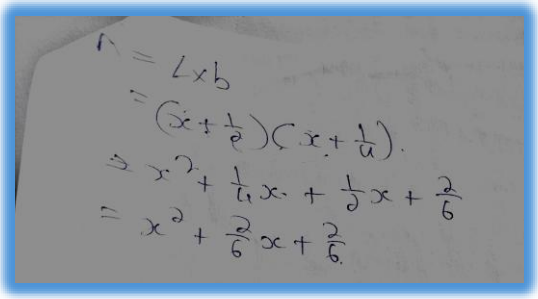
The learner's response suggests that she failed to notice that the numerals in the specified dimensions are not equal. Considering that she was unable to select the correct formula and use appropriate strategies to solve the problem, this implies that learners might have difficulty applying strategies for problem-solving, which could cause them to misunderstand the task. This seems to suggest that they should collaborate with their peers to talk about specific tasks and remedies for the purpose of learning more about different approaches and their prospective success. The next subsection deals with incorrectly translating the word problem.

b) Incorrectly translating the word problem

The findings revealed that learners face a challenge in translating the word problem into an algebraic fraction due to previously made mistakes. This is evident with Learner 21G in **Figure 49** in simplifying the expression for word problem as follows:

Figure 49:

Snippet for Completion of Word Problem



The learner responded as follows during the interview:

Researcher: Explain how you arrived at the answer in solving the word problem

Learner 21G: *The formula for a rectangle is length multiplied by breadth. And then our length is x+ $\frac{1}{2}$, and our breadth is x+ $\frac{1}{4}$. And then we will multiply this by this. And then we will say x multiply by x will be x^2. x multiplied by this is $\frac{1}{4}$ x. And then $\frac{1}{2}$ multiplied by this will be $\frac{1}{2}$ x. And then $\frac{1}{2}$ + $\frac{1}{4}$ it will be + $\frac{2}{6}$.*

This response shows that the learner has a fundamental misunderstanding of fractions, as they have mistakenly applied rules for whole numbers to fractional operations. In particular, the learner added both the numerator and the denominator, which demonstrates a lack of understanding of the appropriate method for adding fractions.

To support this argument, a review brought by Indraswari et al. (2018) emphasise that learners struggle to use algebraic reasoning to analyse and construct logical arguments. For example, learners made errors because they misinterpreted the concepts in problem-solving tasks. This seems to suggest that failure to understand the meaning of the concept will result in the learner making errors repeatedly.

I conclude this section, and directly respond to the second research question: *What misconceptions cause the manifested errors?*

The exploration of misconceptions that cause the manifested errors in the test and interviews when simplifying algebraic fractions provided critical insights into the challenges encountered by learners. This seems to suggest that learners still need to improve in solving complex tasks during simplification; hence, learners must be taught simplified methods and procedures of simplification. In addition, learners should collaborate in groups and pairs when writing classwork to ensure they can reinforce one another's different approaches to simplification of tasks. To demonstrate that this is a beneficial practice from the perspective of learners, a study by Kshetree et al. (2021) emphasise that classroom practices enable learners to face their misconceptions and errors, thus making learning productive and enjoyable. Hence, additional guidance through the implementation of strategies is required to enhance learners' understanding of algebraic fractions.

To answer the Research Question 2: *What misconceptions cause the manifested errors?* The learners' inability to comprehend the question led to process skill errors since they were uninformed of the procedures and rules required to simplify the algebraic fraction task. The most frequently occurring errors were comprehension errors, which were followed by process skill and transformation errors. This is supported by Verzosa-Quinto and Mabansag (2023), who claim that transformation errors occur when information from a problem is translated into a mathematical equation or solution incorrectly, when formulas are used incorrectly, or when procedures are mixed incorrectly because of a lack of knowledge about mathematical concepts or confusion about the most effective strategy to be employed. Yet, an inaccurate strategy based on previous steps, as well as incorrect implementation and computation of the produced calculation, led to errors in writing the answer. This supports the findings of the Solanki's (2021) study, which emphasises that learners should already be familiar with the mathematical concepts and plan from the transformation stage since failing to comprehend and write the solution plan will negatively impact process skills. The study's conclusions also showed that learners' incapacity to fully understand the task was the root cause of encoding errors. Of all four categories of errors, the encoding error was the one that appeared the least. In a similar vein, when it came to the word problem-solving tasks, learners made errors because they misinterpreted the fundamental concepts. The next section presents Research Question 3: *What are ways of reinforcing learners' learning of algebraic fractions?*

5.2.3 Research Question 3: What are ways of reinforcing learners' learning of algebraic fractions?

This question seeks to identify effective methods for reinforcing learner's learning of algebraic fractions in terms of enhancing learners' understanding and skills when simplifying fractions. To answer this research question, I discussed the ways in subsections to understand the importance of each method.

a) **Targeted feedback**: Learners should be given immediate constructive feedback on simplification of tasks for them to immediately learn from their errors. It assists learners in applying their knowledge to various contexts. This view is supported by Turner (2022) who emphasises that immediate feedback corrects misunderstandings and facilitates improvement in learning outcomes.

b) **Explicit instructions on concepts**: Williams (2022) emphasises that many learners struggle due to lack of foundational understanding. To give an example, learners frequently struggle with adding and multiplying fractions. They attempt to add up the numerators and denominators directly, which is incorrect. Learners should be instructed on the usage of a step-by-step methodology which will reduce the simplifying process into manageable steps. This will make it possible for learners to appropriately factorise and cancel throughout the simplification of task.

c) Collaborative learning: Give learners a marked task with an incorrect solution and request them to identify misconceptions that led to wrong simplification of the task. A review by Jones and Carter (2022) emphasise the effectiveness of collaborative learning structures, suggesting that learners benefit from collaborative discussions that allow them to articulate their understanding and challenge their misconceptions. To support this view, learners should work in groups or in pairs to discuss their thoughts and explain their justifications to one another. Each group or pair should present their findings and solutions to the rest of the class. This can expose various simplification strategies and draw attention to frequent errors. In this way, learners will be able to identify where they had incorrect conceptions and be able identify and remember their misconceptions and errors that hinder their performance.

d) **Strategic algebraic tasks.** Learners should practise simplifying algebraic fraction tasks starting from simpler ones, for instance, fractions that involve numerals only to ensure that learners are comfortable with basic concepts. In this way, learners will gain confidence, then they continue solving complex tasks that include variables and numerals for them to gain more confidence in simplifying these tasks.

I conclude this section, and directly respond to research question 3: *What are ways of reinforcing learners' learning of algebraic fractions*? The exploration of strategies to reinforce learners' understanding of algebraic fractions is fostering the performance of learners' learning of algebraic fractions. Future research and practical applications of these strategies are essential in creating supportive learning environments.

The findings in answering research question 3: *What are ways of reinforcing learners' learning of algebraic fractions?* can be summed up as follows: Feedback should be offered to learners right away so that they may review their mistakes and correct them. For learners to gain confidence in their ability to solve problems, they should also learn different approaches to simplify problems from less complicated and complex ones. In addition, learners should create WhatsApp groups, distribute tasks among themselves, and exchange comments using different methods. Furthermore, learners should self-profile when it comes to simplifying work; low achievers should begin with fundamental tasks to simplify, while high achievers should handle challenging tasks, simultaneously assisting the low achievers. There are certain things that learners could do more to ensure that they minimise the misconceptions and errors that they manifest.

Moreover, learners should be allowed the opportunity to correct their own mistakes for them to be able to have confidence in their algebraic learning of fractions. To support this view, learners who are unable to comprehend mathematical concepts will struggle to master the content, struggle to solve problems, and struggle to apply what they learn in their daily lives

(Rismayanis et al., 2022). I identified three key issues from the learners' solutions and the insights gained during the interviews, which are as follows: a) Learners do not use the correct mathematical language when describing the steps; b) Learners struggle with the proper use of the negative sign: c) Learners have difficulty with removing brackets; moving part of the denominator is not a rule (learners should be assisted to comprehend the core algebraic principles instead of adopting their own analysis as a rule). Such analysis might not be applicable in certain cases; hence, it cannot be a rule. Thus, addressing such misconceptions and errors is crucial. This seems to suggest that a proactive approach in identifying and rectifying these misconceptions and errors can foster a more robust mathematical foundation for learners. To add on this, it will enable learners to navigate algebraic fractions with greater proficiency and a deeper understanding.

5.3 Reflecting on the Utility of the Theoretical Framework

The context of this study involved exploring errors and misconceptions exhibited in solving problems involving algebraic fractions. This perspective was understood from a theoretical view; therefore, Newman's theory was found to be useful as it assisted in analysing the misconceptions and errors exhibited. It also helped me to develop a deeper understanding of learners' mathematical misconceptions by guiding the formulation of the research questions. The second key influence complementing Newman's theory, however, was constructivism. As defined by Vygotsky (1978), constructivism in the context of learning is that learning involves constructing meaning and systems of meaning. The interpretation of the data and the recommendations in this study were guided by the principles of knowledge construction. For instance, one of the principles states that knowledge is actively generated by incorporating valuable learning and that knowledge acquisition is an active process (Bruner, 1990). I assert that the strategies employed to address the identified gap are intentionally designed to facilitate the construction of knowledge by learners. Analysing errors has been deemed fit to enhance learning. Just to provide few examples, the principles of social constructivism guided the study in the following ways:

- **Learning is a developmental and reflective process**

This involves thinking deeply about what has been learnt and reflecting and gaining insight into their thoughts (Madrakhimovich, 2023). In this study the interviews were conducted and learners reflected on their learning by explaining how the problems were solved. Secondly, they were assessed on problem solving and, in this case, reflective learning encourages learners to apply their developing knowledge to real-world situations, which deepens understanding and reinforces its relevance (Sitopu et al.,2024).

- **Prior knowledge and experience**

Questions were developed in such a way that they demanded learners to use prior knowledge to solve problems. Learners needed to have knowledge of determining the LCD in addition and subtraction of like and unlike terms which influences how they interpret and integrate new information. This is about using the prior knowledge to build on new knowledge (Hattan et al., 2024).

5.4 Limitations of the Study

The study utilised a relatively small sample size, involving only one class with a maximum of 36 learners, limiting the generalisability of the findings. Some errors and misconceptions may not have been thoroughly analysed owing to these time limitations, particularly during the interview. A longer time frame could permit learners to rewrite the test and use different strategies to solve the problem and thereafter comment on what they had learnt. This would have assisted me in gaining a deeper understanding of how learning develops and becomes a reflective process, as learning is a developmental and reflective process.

5.5 Recommendations Based on the Findings of the Study

Based on the findings of the study, most of the errors recorded were comprehension errors, then process skill errors, followed by and transformation errors and encoding errors. This seems to suggest that learners had a significant challenge in using the correct procedures and comprehending the basic concepts required to simplify the algebraic fraction task. To build on this, a study by Ellis and Martinez (2022) indicates common misconceptions such as misapplication of rules, confusion between multiplication and addition of fractions as well as difficulties in recognising equivalent fractions hinder the performance of learners. Hence, I suggest the following recommendations:

- **Use of technology:** Examine how technology, such as applications and online exercises, might help learners in correctly comprehending and simplifying mathematical fractions. Investigate the impact of several teaching techniques in reducing misconceptions, including direct teaching, inquiry-based learning, and collaborative learning.

- **Curriculum analysis:** I recommend that research be used to identify any curricular gaps or places where the beginning or teaching of concepts could lead to misconceptions.

- **Curriculum development:** Analyse the current curricula to find any gaps that could be influencing learners to misconceptions regarding algebraic fractions. In addition, develop and assess tools or resources intended to clarify specific misconceptions about algebraic fractions.

- **Diagnostic assessment:** Develop an evaluation designed exclusively to identify areas where algebraic fractions need to be simplified and monitor performance over time. In addition, explore how learners' capacity to detect and correct their errors is affected by immediate and targeted feedback.

- **Expanded populations**: Extending the study to multiple schools, different grades, and educational systems makes it possible to compare findings and draw broader conclusions.

- **Longitudinal studies**: Implement long-term research to observe how misconceptions evolve over time and if targeted interventions are successful in enhancing comprehension among learners.

- **Interdisciplinary approaches:** Explore the way comprehension and reducing misconceptions may be accomplished by connecting mathematical fractions to other subjects, such as economics, science, and technology, or real-life situations**.**

- **School Management Team (SMT), School Governing Body (SGB), and Department of Basic Education (DBE):** The SMT, SGB and DBE should recommend competent and more qualified educators to teach learners mathematics.

5.6 Suggestions for Future Research

This scope of the study was limited by the small sample size it employed—36 learners from one high school. The present study could potentially be repeated by future researchers using a larger, random sample of learners. Subsequently, it is also possible to extend the study across additional provinces in South Africa. I suggest expanding the study to include different grades, schools and educational systems to compare results and draw a broader conclusion.

5.7 Reflexivity

I articulate the progression of my research journey, outlining the key stages, challenges, and insights gained throughout the process, starting from the proposal stage and writing up of the dissertation stage. At the time I began crafting the research proposal, I had no idea how to adequately acknowledge my sources or paraphrase a statement that I had taken from other scholars. I am appreciative of my supervisor, who never gave up on me. To make it possible for us to be able to communicate, my supervisor additionally instructed me how to join for a meeting on Teams, demonstrating her extreme patience with me. The supervisor suggested that I participate by attending the UNISA Department of Mathematics Education (DME) and Academic Writing Teams Workshops to gain more experience. Moreover, she encouraged me to present my topic during a DME workshop so that, as a researcher, I would be able to be corrected on mistakes I have made. Above all, she kept on saying to me, "If you want to be successful researcher, you need to read a lot of literature."

I managed to craft my presentation and presented; this really assisted me since I gained a lot from the leadership of the UNISA DME and the participants. This motivated me to join the UNISA Showcase competition which was held on the 26 November 2024. I managed to present my topic on TEAMS virtually, where I was awarded a certificate of being the best oral presenter in the Masters category (Arts and Humanities Discipline) of postgraduate student research and innovation showcase. The DME workshops and the UNISA showcase competition inspired me to continue and conclude my dissertation; thus, they were insightful and informative. It was fascinating research that permitted me to demonstrate my expertise while discovering areas for improvement. Despite all the challenges encountered during the research process, I felt thrilled to be a part of the masters degree course students in mathematics education. I persisted due to the fact I had a desire to successfully complete the research. I could possibly share my research experiences with others, assisting others and those who are novices in the field of research.

5.8 Conclusion

The study discovered a range of frequently occurring misconceptions exhibited by learners, including misapplication of the procedures and rules for manipulating fractions, an inability to determine the lowest common factor and the common factor during simplification, challenges adding up terms, challenges translating word problems, an inability to factorise, incorrect cancellation of terms on the numerator and denominator, and an inability to recognise a negative sign during simplification. These misconceptions caused learners to exhibit errors that prevented them from completing algebraic tasks correctly. Furthermore, this suggests that a better teaching strategy is required, one that develops deeper conceptual knowledge prior to introducing procedural tasks. In addition, the implementation of diagnostic assessments enables educators to detect misconceptions at an early stage of the learning process. Furthermore, persistent assessment and feedback systems are crucial for assisting learners in progressing towards improved proficiency, as they offer opportunities for corrections of errors in real time.

This study also offers a critical evaluation of the two frameworks employed in the research, assessing their effectiveness. The limitations of the study were discussed, and suggestions for future research and implications for policymakers were presented. Lastly, I reflected on my personal beliefs throughout the study, exploring how these may have shaped both the research process and the findings.

5.9 Last Word

As an individual, I have developed through this research. It is essential to address the misconceptions and errors learners in Grade 10 manifest when simplifying algebraic fractions. This is pivotal to enhance their overall academic performance and mathematical comprehension. Educators, as well as district and provincial officials, can assist learners to overcome challenges and strengthen their basic mathematics by promoting enhanced conceptual comprehension and implementation to practice effective teaching techniques. In the long term, this approach will produce a generation of competent and confident mathematicians whilst enhancing individual learning experiences. After giving this some thought, I am convinced that I have reached an important milestone in my journey as a researcher.

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Appendices

Appendix A: Confirmation letter of registration

UNISA university of south africa

0301

MKHARI C MS
P O BOX 2092
TZANEEN
0850

STUDENT NUMBER : 41595149

ENQUIRIES TEL : 0800 001 870

eMAIL : mandd@unisa.ac.za

2024-09-28

Dear Student

I hereby confirm that you have been registered for the current academic year as follows:

Proposed Qualification: MED MATHS EDUC (90061)

CODE	PAPER	S	NAME OF STUDY UNIT	NQF crdts	LANG.	PROVISIONAL EXAM.DATE	EXAMINATION CENTRE(PLACE)
Study units registered without formal exams:							
DFMED96			MEd - Mathematics Education (Dissertation)	**	E		

You are referred to the "MyRegistration" brochure regarding fees that are forfeited on cancellation of any study units.

Your attention is drawn to University rules and regulations (www.unisa.ac.za/register).
Please note the new requirements for reregistration and the number of credits per year which state that students registered for the first time from 2013, must complete 36 NQF credits in the first year of study, and thereafter must complete 48 NQF credits per year.
Students registered for the MBA, MBL and DBL degrees must visit the SBL's ESOnLine for study material and other important information.
Readmission rules for Honours: Note that in terms of the Unisa Admission Policy academic activity must be demonstrated to the satisfaction of the University during each year of study. If you fail to meet this requirement in the first year of study, you will be admitted to another year of study. After a second year of not demonstrating academic activity to the satisfaction of the University, you will not be re-admitted, except with the express approval of the Executive Dean of the College in which you are registered. Note too, that this study programme must be completed within three years. Non-compliance will result in your academic exclusion, and you will therefore not be allowed to re-register for a qualification at the same level on the National Qualifications Framework in the same College for a period of five years after such exclusion, after which you will have to re-apply for admission to any such qualification.
Readmission rules for M&D: Note that in terms of the Unisa Admission Policy, a candidate must complete a Master's qualification within three years. Under exceptional circumstances and on recommendation of the Executive Dean, a candidate may be allowed an extra (fourth) year to complete the qualification. For a Doctoral degree, a candidate must complete the study programme within six years. Under exceptional circumstances, and on recommendation by the Executive Dean, a candidate may be allowed an extra (seventh) year to complete the qualification.

BALANCE ON STUDY ACCOUNT: 0.00

Yours faithfully,

Prof MM Sepota
Acting Registrar

1031 0 00 0

UNIVERSITEIT VAN SUID-AFRIKA PRETORIA UNIVERSITY OF SOUTH AFRICA

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

Appendix B: Ethical Clearence

Date: 2023/07/05

Dear Ms C Mkhari

Decision: Ethics Approval from 2023/07/05 to 2026/07/05

Ref: **2023/07/05/41595149/01/AM**
Name: Ms C Mkhari
Student No.:41595149

Researcher(s): Name: Ms C Mkhari
E-mail address: 41595149@mylife.ac.za
Telephone: 076 775 0828

Supervisor(s): Name: Dr S.M Kodisang
E-mail address: kodissm@unisa.ac.za
Telephone: 012 429 3964

Title of research:

Misconceptions and errors displayed by Grade 10 learners when solving problems involving Algebraic fractions.

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2023/07/05 to 2026/07/05.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2023/07/05 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

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3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2026/07/05.** Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2023/07/05/41595149/01/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof Mpine Makoe
ACTING EXECUTIVE DEAN
qakisme@unisa.ac.za

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Appendix C: Application to conduct research in Limpopo Provincial Department of Education

Enquiries

Name: Ms Cynthia Mkhari Cell : 076 7750 828 Work: 015 004 1626 Email : 41595149@mylife.unisa.ac.za Ethics Ref: 2023/07/05/41595149/01/AM	P.O Box 2092 Tzaneen 0850 25 July 2023

The Head of Department

Limpopo Department of Education

Private Bag X9489

Polokwane

0700

Dear Sir/Madam

APPLICATION FOR PERMISSION TO CONDUCT RESEARCH IN MOPANI WEST DISTRICT

My name is Cynthia Mkhari. I am a Masters student at the University of South Africa (UNISA), at the Department of Mathematics Education. The title of my research is **Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions.** This research will be conducted under the supervision of Doctor S.M Kodisang from the Department of Mathematics Education.

With this request, I am asking for permission to study one high school in the Mopani West District. A copy of the University of South Africa's ethical clearance certificate and copies of the consent forms that will be utilised for the study are attached. I consent to provide a bound copy of the entire study's report to the Department of Basic Education when the study is completed.

Hope to hear from you soon.

Yours faithfully

Cynthia Mkhari

Appendix D: Limpopo Provincial Department of Education Approval Letter

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

CONFIDENTIAL

Ref: 2/2/2 Enq: Makola MC Tel No: 015 290 9448 E-mail: MakolaMC@edu.limpopo.gov.za

Mkhari C
P.O BOX 2092
TZANEEN
0850

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"Misconceptions and errors displayed by Grade 10 learners when solving problems involving Algebraic fractions."**

3.The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

REQUEST FOR PERMISSION TO CONDUCT RESEARCH : MKHARI C Page 1

Cnr 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X 9489, Polokwane, 0700
Tel:015 290 7600/ 7702 Fax 086 218 0560

The heartland of Southern Africa-development is about people

UNISA college of education

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes.

26/09/2023

Mashaba KM
DDG: CORPORATE SERVICES

Date

Appendix E: Letter to the District Director

Request for permission to conduct research at Mopani West District of Limpopo Province

Date: ______________________

Mopani West District Director, Mrs P.P Modika
Mopani West District

Dear Madam

Subject: Request to conduct research in a school from your District

My name is Cynthia Mkhari, and I am student at the University of South Africa. l am presently enrolled for the MEd with a specialisation in Mathematics education. To complete the requirements for the degree, l need to conduct research that is related to the area of my specialization. The purpose of the study is to explore misconceptions and errors manifested by Grade 10 learners when simplifying Algebraic fractions.

My supervisor is Dr Sophy Kodisang, a lecturer in the Department of Mathematics Education. Her office telephone number is 012 429 3964.

Thirty (36) learners who are doing grade 10 Mathematics at Mopani West District will participate in the study. The respondents' answers will be observed and thereafter they will be interviewed. All data collected will be analysed and a report regarding the study will be written.

l believe that the work l am doing could be relevant to mathematics education in Mopani West District. The information collected will be treated with confidence and l hereby give undertaking that:

All participation is voluntary.

- The school's name, as well as teachers participating, will not be revealed in the findings of the research.
- All discussions with participants will be treated with confidentiality.
- The schools can withdraw from the research study at any time.
- If the school is willing to participate, it will be requested to sign consent forms provided.

l would appreciate an early reply to the request.

Thank you for your assistance in this matter.

Yours Sincerely

Mkhari Cynthia

Appendix F: Letter to the Principal

Request to principals for permission to conduct research in schools.

Date: ______________________

The principal of Makobo High School

Mawa Circuit

Dear Sir/Madam

Subject: Request to conduct research in your school

My name is Cynthia Mkhari, and I am student at the University of South Africa. l am presently enrolled for the MEd with a specialization in mathematics education. To complete the requirements for the degree, l need to conduct research that is related to the area of my specialization. My research project is entitled: **Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions.** My supervisor is Dr Sophy Kodisang, a lecturer in the Department of Mathematics Education. Her office telephone number is 012 429 3964.

I will be invigilating learners writing a test on algebraic fractions learned in class. A semi-structured interview will be conducted with the participant after learners have completed test. The participants will therefore be requested to sign a consent form provided. All data collected will be analysed and a report regarding the study will be written. This research study will not only benefit the institution involved but will contribute to Mathematics teaching in the Mopani West District. Please note that if you allow one educator and his/her grade 10 mathematics class to participate in the research study, the following ethical values will apply:

- The learners' participation is voluntary.
- All information will be treated with confidentiality and anonymity to ensure that no harm or bad effect will be caused to participants by the research study
- All scripts and interview recordings will be destroyed at the end of the study
- Learners will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

I will avail to you the summary of the study results at the time of completion if you would wish to have the summary.

l would appreciate an early reply to the request.

Thank you in advance for your support.

Yours Sincerely

Mkhari Cynthia

Appendix G: Consent form for Principal

I, __________________the principal of Makobo High understand the context of the research study and I grant permission that the research study title: **Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions** may be conducted at the school. I am aware that the educator and learners' participation is voluntary; all information will be treated with confidentiality and anonymity to ensure that no harm or bad effect will be caused to participants by the research study; participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

__________________________ ______________________

Principal Date

School Stamp

UNISA college of education

Appendix H: Requesting permission from the School Governing Body (SGB)

Request to the SGB for permission to conduct research in a school

Date: ____________________

Dear Sir/Madam

Subject: **Request to conduct research in your school**

My name is Cynthia Mkhari, I am student at the University of South Africa. I am presently enrolled for the MEd with a specialisation in Mathematics education. To complete the requirements for the degree, I need to conduct research that is related to the area of my specialisation. My research project is entitled: **Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions.** The purpose of the research is to explore the misconceptions and errors that learners manifest during the simplification of tasks.

My supervisor is Dr Sophy Kodisang, a lecturer in the Department of Mathematics Education. Her telephone number is 012 429 3964. Learners will be given a test to write, and I will be the invigilator. A semi-structured interview will be conducted with the learners after the learners have written a test. The interview will be audio recorded for a verbatim transcription. The participants will therefore be requested to sign a consent form provided. All data collected will be analysed and a report regarding the study will be written. This research study will not only benefit the institution involved but will contribute to Mathematics teaching in the district. Please note that if you allow Grade 10 Mathematics learners to participate in the research study, the following ethical values will apply:

- the learners' participation is voluntary
- all information will be treated with confidentiality and anonymity to ensure that
- no harm or bad effect will be caused to participants by the research study
- participants will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

I would appreciate an early reply to the request.

Thank you in advance for your support.

Yours Sincerely

Mkhari Cynthia

Appendix I: Consent form for School Governing Body

I______________________________________ the SGB member of____________________ understand the context of the research study and I grant permission that the research study (Title: **Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions)** may be conducted at the school. I am aware that the learners' participation is voluntary; all information will be treated with confidentiality and anonymity to ensure that no harm or bad effect will be caused to participants by the research study. The test scripts and interview recordings will be destroyed at the end of the study. Learners will be granted the right to withdraw when they so wish, they may also refrain from answering questions when they see it necessary.

____________________________ ______________________

SGB Member Date

School Stamp

UNISA college of education

Appendix J: Consent letter to parent

Dear Parent

Your ______________(son/daughter/child) is invited to participate in a study entitled **Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions.** I am undertaking this study as part of my master's research at the University of South Africa. The purpose of the study is Misconceptions and Errors manifested by Grade 10 learners in solving Algebraic fractions tasks and the possible benefits of the study are the improvement of results in Algebra. I am asking permission to include your child in this study because by your child's participation in the study, more misconceptions and Errors can be resolved. I expect to have 35 other children participating in the study.

If you allow your child to participate, I shall request him/her to:

- **Take part in an interview**. To avoid interfering with the educators' time spent on curriculum material, the interview will be conducted after school hours, during study time. After the pre- and post-test have been completed by all learners, an interview will be conducted. One (1) hour will lapse between the pre- and post-tests.
- **Take part in a group interview**. There will be a focus group interview where learners can respond to questions collectively. Each learner group will have six participants for the group interview, which will be held in class. Thirty (30) minutes will be allocated for the focus group interview.
- **Complete a test**. The test will each take an hour to complete after school hours.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only. There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are gained algebraic expertise and understanding following study. Neither your child nor you will receive any type of payment for participating in this study. Your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be in the study now and change your mind later without any penalty. The study will take place during study time with the prior approval of the school and your child's teacher. However, if you do not want your child to participate, an alternative activity will be available like participating in remedial work for the written test.

In addition to your permission, your child must agree to participate in the study and you, and your child will also be asked to sign the assent form which accompanies this letter. If your child does not wish to participate in the study, he or she will not be included and there will be no penalty. The information gathered from the study and your child's participation in the study will be stored securely on a locked computer in my locked office for five years after the study. Thereafter, records will be erased. The benefit of this study is to significantly enhance learners' learning both inside and outside of the classroom and to give learners the best algebra introduction possible during the FET period of their school years and in higher education.

There are no potential risks. There will be no reimbursement or any incentives for participation in the research. If you have questions about this study, please ask me or my study supervisor, Dr Sophy Kodisang, Department of Mathematics Education, College of Education, University of South Africa. My contact number is 076 775 0828 and my e-mail is 41595149@mylife.unisa.ac.za. The e-mail of my supervisor is kodissm@unisa.ac.za. Permission for the study has already been given by the principal and the Ethics Committee of the College of Education, UNISA. You are deciding about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child: ________________________________

Parent/guardian's name: ____________Parent/guardian's signature: __________Date: ______

Sincerely________________________

Researcher's name: **Mkhari Cynthia** Researcher's signature: ____________Date: ______

Appendix K: Assent letter from a child

Title of research: Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions

Dear ________________________ Date: ________________________

I am doing a study on Misconceptions and Errors manifested by Grade 10 learners when simplifying Algebraic fractions tasks as part of my studies at the University of South Africa. Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that I can find ways that your teachers can use to make you understand the topic on Algebraic fractions better. This may help you and many other learners of your age in different schools.

This letter is to explain to you that I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

I would like you to write a test, complete a questionnaire about the types of misconceptions and errors you are making as a learner, and be interviewed about the types of mistakes you are making in class activities, tests, assignments, and exams. I will include you in a six-person focus group. Your responses for the test will take approximately an hour, the questionnaire and the focus group discussion will take about 30 minutes each.

I will write a report on the study, but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you do not want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I am finished with my study, I shall return to your school to give a short talk about some of the helpful and interesting things I found out in my study. I shall invite you to come and listen to my talk. The benefit of this study is to significantly enhance learners' learning both inside and outside of the classroom and to give learners the best algebra introduction possible during the FET period of their school years and in higher education. There are no potential risks that are involved.

You will not be reimbursed or receive any incentives for your participation in the research. If you decide to be part of my study, you will be asked to sign the form on the next page. If you have any other questions about this study, you can talk to me or you can have your parent, or another adult call me at

076 775 0828. Do not sign the form until you have all your questions answered and understand what I would like you to do.

Do not sign the written assent form if you have any questions. Ask your questions first and ensure that someone answers those questions.

Researcher: **Cynthia Mkhari**   Phone number: 076 775 0828

Appendix L: Consent form for Grade 10 learner

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about my study, and I know what I will be asked to do. I am willing to be in the study.

______________________	______________________	______________________
Learner's name (print):	Learner's signature:	Date:
______________________	______________________	______________________
Witness's name (print)	Witness's signature	Date:

(The witness is over 18 years old and present when signed.)

______________________	______________________	______________________
Parent/guardian's name (print)	Parent/guardian's signature:	Date:

Researcher's name: **Mkhari Cynthia**

Researcher's signature: ____________________ Date: _________________

Appendix M: Consent letter for one-on-one interview

I____ grant consent/assent that the information I share during the one-on-one interview may be used by Mkhari Cynthia for research purposes. I am aware that the group discussions will be digitally recorded and grant consent/assent for these recordings, provided that my privacy will be protected. I undertake not to divulge any information that is shared in the group discussions to any person outside the group to maintain confidentiality.

Participant ‘s Name (Please print): ___________________________________

Participant Signature: ____

Researcher’s Name: **Mkhari Cynthia**

Researcher’s Signature: ____

Date: ____

Appendix N: Cover Letter for Questionnaire

Title: Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions.

Dear respondent

This questionnaire forms part of my master's research entitled: **Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions.** for the degree MEd at the University of South Africa. You have been selected by using a *sampling* strategy from the population of 36 participants. Hence, I invite you to take part in this survey.

The aim of this study is to investigate the types of errors and misconceptions that learners manifest when simplifying Algebraic fractions tasks. The findings of the study may benefit learners who will further their studies doing mathematics.

You are kindly requested to complete this survey questionnaire, comprising of six questions. No foreseeable risks are associated with the completion of the questionnaire which is for research purposes only. The questionnaire will take approximately 30 minutes to complete. You are not required to indicate your name or organisation, and your anonymity will be ensured; however, indication of your age, gender, occupation position etcetera will contribute to a more comprehensive analysis. All information obtained from this questionnaire will be used for research purposes only and will remain confidential. Your participation in this survey is voluntary and you have the right to omit any question if so desired, or to withdraw from answering this survey without penalty at any stage. After the completion of the study, an electronic summary of the findings of the research will be made available to you on request.

Permission to undertake this survey has been granted by the UNISA and the Ethics Committee of the College of Education, UNISA. If you have any research-related enquiries, they can be addressed directly to me or my supervisor. My contact details are:076 775 0828 and my e-mail: 41595149@mylife.unisa.ac.za and my supervisor can be reached at 012 429 3964 Department of Mathematics Education, College of Education, UNISA, e-mail: kodissm@unisa.ac.za.

By completing the questionnaire, you imply that you have agreed to participate in this research. Please return the completed questionnaire to the researcher before ________________

Yours sincerely

Cynthia Mkhari

Appendix O: Algebraic Fractions Test

Learner number: ______________________

Date: ________________________________

Mathematics Test

Grade: 10

Marks: 50

Duration: 1 hour

Instructions:

1. This question paper consists of 3 questions.
2. Clearly show all calculations you have used to determine the solutions.
3. Answers only will not necessarily be awarded full marks.
4. You may use an approved scientific calculator.
5. Answer all questions on the spaces provided in the question paper.
6. Wite neatly and legibly.

Question 1 **[15]**

Four possible answers are given for each Algebraic fraction expression. Each question has only one correct answer. Answer the question as follows:

i. Simplify each question and write the letter of the answer corresponding to the number of the question.
ii. Explain how you arrived at the answer

1.1 [3]

A. -3x
B. 1-2x
C. 1-3x
D. 3-2x

i. ____

ii. ____

1.2 $\frac{4}{x} + \frac{3}{y}$ **[3]**

A. $\frac{7}{xy}$

B. $\frac{7}{x+y}$

C. $\frac{12}{xy}$

D. $\frac{4y+3x}{xy}$

i. ____

ii. ____

____

____

1.3 $\frac{4x^3-6x}{2x}$ **[3]**

A. $2x^2 - 3$

B. 2x

C. $\frac{-2x^2}{x}$

D. $4x^3 - 3$

i. ____

ii. ____

____

____

1.4 $\frac{a}{2xy}+\frac{2b}{3yz}+\frac{3c}{4zp}$ **[3]**

A. $\frac{5abc}{12xyzp}$

B. $\frac{6abc}{9xyzp}$

C. $\frac{6azp+8bxp+9cxy}{12xyzp}$

D. $\frac{acxzp}{2xyzp}$

i. ____

ii. ____

____

____

1.5 $\frac{3^x+3^{x+1}}{2.3^x}$ **[3]**

A. 6^{x+1}

B. 2

C. ***x +1***

D. $\frac{3^{x+1}}{2}$

i. ____

ii. ____

____

____

Question 2 **[20]**

Simplify the following expressions fully

2.1 $\frac{x}{x-y} - \frac{y}{x+y}$ [4]

2.2 $\frac{x+3}{4} - \frac{x-2}{3} + \frac{x}{12}$ [4]

2.3 $\frac{2x^2-8}{x-2}$ [4]

2.4 $\frac{2x-3}{x}$ - $\frac{4x+2}{3x} - \frac{x+1}{6x}$ [4]

2.5 $\frac{2a}{3b} - \frac{2a^2-3b^2}{6ab} + \frac{3b}{4a}$ [4]

Question 3 **[15]**

Simplify the following expressions fully

3.1 $\frac{y^2-y-2}{y^2-4} \div \frac{y^2+y}{y^2+2y}$ [5]

3.2 $\frac{27x^3-8}{27x^2+18x+12}$ [5]

3.3 $\dfrac{9^{2n-1}}{3^{2n+1}.3^{n-3}.\ 3}$ [5]

Appendix P: Algebraic fractions Test Memorandum

QUESTION 1: **[15]**

1. Three (3) marks each, i.e. 2 marks for correct letter and 1 mark for explanation

No.	Answer and explanation	Mark allocation
1.1	B	✓✓✓ [3]
1.2	D	✓✓✓ [3]
1.3	A	✓✓✓ [3]
1.4	C	✓✓✓ [3]
1.5	B	✓✓✓ [3]

QUESTION 2 **[20]**

No	Solution	Explanation of ticks	Mark allocation
2.1	$\frac{x}{x-y} - \frac{y}{x+y}$; LCD= (x-y) (x+y) $= \frac{x(x+y)-y(x-y)}{(x-y)(x+y)}$ $= \frac{x^2+xy-xy+y^2}{(x-y)(x+y)}$ $= \frac{x^2+y^2}{(x-y)(x+y)}$	✓ LCD ✓ Removing brackets ✓ Adding like terms ✓ Answer	[4]
2.2	$\frac{x+3}{4} - \frac{x-2}{3} + \frac{x}{12}$; LCD = 12 $= \frac{3(x+3)-4(x-2)+x}{12}$ $= \frac{3x+9-4x+8+x}{12}$ $= \frac{17}{12}$ $= 1\ {}^{5}/_{12}$	✓ LCD ✓ Removing brackets ✓ Adding like terms ✓ Answer	[4]

2.3	$\frac{2x^2-8}{x-2}$ $=\frac{2(x^2-4)}{x-2}$ $=\frac{2(x-2)(x+2)}{x-2}$ = 2(x+2)	✓ Common factor ✓ Difference of two squares ✓ ✓ Answer	[4]
2.4	$\frac{2x-3}{x}-\frac{4x+2}{3x}-\frac{x+1}{6x}$; LCD= 6x $=\frac{6(2x-3)-2(4x+2)-x+1}{6x}$ $=\frac{12x-18-8x-4-x-1}{6x}$ $=\frac{3x-23}{6x}$	✓ LCD ✓ Removing brackets ✓ Adding like terms ✓ Answer	[4]
2.5	$\frac{2a}{3b}-\frac{2a^2-3b^2}{6ab}+\frac{3b}{4a}$;LCD= 12ab $=\frac{(2a)(4a)-2(2a^2-3b^2)+(3b)(3b)}{12ab}$ $=\frac{8a^2-4a^2+6b^2+9b^2}{12ab}$ $=\frac{4a^2+15a^2}{12ab}$	✓ LCD ✓ Removing brackets ✓ Adding like terms ✓ Answer	[4]

QUESTION 3			[15]
3.1	$\frac{y^2-y-2}{y^2-4} \div \frac{y^2+y}{y^2+2y}$ $= \frac{y^2-y-2}{y^2-4} \, x \, \frac{y^2+2y}{y^2+y}$ $= \frac{(y-2)(y+1)}{(y+2)(y-2)} \text{ X } \frac{y(y+2)}{y(y+1)}$ $= \quad 1$	✓ Factors of trinomial ✓ Inverting ✓ Difference of two squares ✓ Common factor ✓ Answer	[5]
3.2	$\frac{27x^3-8}{27x^2+18x+12}$ $= \frac{(3x-2)(9x^2+6x+4)}{3(9x^2+6x+4)}$ $= \frac{3x-2}{3}$	✓✓Factorising cubes ✓✓Factorising trinomial ✓Answer	[5]
3.3	$\frac{9^{2n-1}}{3^{2n+1}.3^{n-3}.\ 3}$ $= \frac{(3^2)^{2n-1}}{3^{2n+1+n-3+1}}$ $= \quad 3^{4n-2-2n-1-3+3-1}$ $= \quad 3^{n-1}$	✓ Prime bases ✓ Raising powers ✓ Simplification ✓ ✓ Answer	[5]

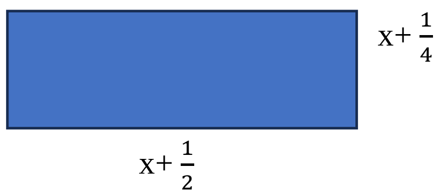
Appendix Q: Interview questionnaire

Instructions:

- Answer all questions
- Answer the word problem number 5 on the answer sheet provided

1. What do you understand about algebraic fractions?
2. Explain how to determine the LCD in question 2.1 from the test?
3. How do you determine the common factor in Question 2.3 from the test that you have written?
4. What do you think are the challenges you encounter when simplifying algebraic fractions tasks?
5. If you are given the rectangle with the length of $x+\frac{1}{2}$ and the breadth of $x+\frac{1}{4}$.

   Determine the expression of the area of the rectangle in terms of x.



Appendix R: Language Editor’s letter

Mrs G C Hannant
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23 January 2025

TO WHOM IT MAY CONCERN

I certify that I have edited the Master’s dissertation

Exploring misconceptions and errors that manifest when Grade 10 learners simplify tasks involving algebraic fractions. A case of selected high school in Mopani West District, Limpopo Province

by

Cynthia Mkhari

However, the correction of all errors/missing information remains the responsibility of the author.

G.C. HANNANT (BA HED)

Appendix S: Turnitin Report