Grade 9 Learners’ Understanding of Algebraic Expressions: A Case of a Selected High School in Frances Baard District in the Northern Cape.

by

EMMANUEL KWANKYE

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SUPERVISOR: DR S M KODISANG

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Declaration

Name: Emmanuel Kwankye

Student Number: 55094589

Degree: Master of Education in Mathematics Education

Grade 9 Learners' Understanding of Algebraic Expressions: A Case of a Selected High School in Frances Baard District in the Northern Cape.

I declare that the above dissertation is my own work, and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references.

I further declare that I submitted the dissertation to originality checking software, and that it falls within the accepted requirements for originality.

I further declare that I have not previously submitted this work, or part of it, for examination at UNISA for another qualification or at any other higher education institution.

________________________ 30 July 2025

Signature. Date

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My gratitude also goes to the Northern Cape Department of Education for allowing me to conduct this study.

Dedication

This study is dedicated to my beloved grandmother, Mary Antwi, for her unconditional love, support and guidance since childhood. I am very grateful. Many thanks to you!

Abstract

Mathematics underachievement is a global challenge, with South Africa also experiencing a decline in skills consistent with international trends. This study aimed to explore the understanding of algebraic expressions amongst Grade 9 learners. The study was inspired by the fact that learners in the Northern Cape usually demonstrate dismal performance in algebraic expressions. When learners lack strong algebraic skills, it can reduce their confidence and achievement in further education and training, restricting access to careers in Science and Engineering.

This study draws on Skemp's theory of understanding to help explain and make sense of the phenomenon of poor performance in algebraic expression. The study adopted a qualitative case study research approach in which 40 learners from a selected school in the Frances Baard District, Northern Cape in South Africa, participated. Test and semi-structured interviews were used to collect data.

The findings of the study revealed that learners lack the fundamental concepts or pre-requisite knowledge in algebraic expression. They also lack relational understanding in algebraic expressions and moreover, their mathematical connection abilities were very low.

The implications of this study include but are not restricted to, the need for teachers to pay special attention, monitor, document and carefully examine learners' misconceptions and conceptual gaps when teaching algebraic expression. There is also a need for learners to be exposed to appropriate, practical and real life-scenarios during the teaching and learning of algebraic expressions to make the content more simple, realistic, engaging and understandable.

These stated implications should be regarded in view of some of the limitations of this research, including the small sample size, limited time and resources.

KEY TERMS: Algebraic expressions; Grade 9 learners; senior phase; mathematical connections; understanding, Skemp's theory.

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List of Acronyms

Abbreviation/Acronym	Description
ANA	Annual National Assessment
CAPS	Curriculum and Assessment Policy Statements
CEDU	College of Education
DBE	Department of Basic Education
FOIL	First, Outside, Inner, Last
GET	General Education and Training
FET	Further Education and Training
GCD	Greatest Common Denominator
STEM	Science Technology Engineering and Mathematics
TIMSS	Trends in International Mathematics and Science Study
UNISA	University of South Africa
2-D	Two-Dimensional

1. CHAPTER 1 BACKGROUND AND INTRODUCTION OF THE STUDY

1.1 Background to the Study

Algebra is widely acknowledged as a fundamental concept (Alam & Mohanty, 2024) that must be accessible to all learners (Jung et al., 2024) and is considered an essential prerequisite for advanced mathematics, as well as a gatekeeper to higher education. As evidenced by Sarimanoğlu (2019), algebra maintains a critical status in mathematics (Essuman et al., 2024; Juraev & Bozorov, 2024). Mathaba and Bayaga (2019) define algebra as a mathematical concept that deals with expressions, equations and inequalities-related rules. It serves as a pivotal cornerstone, equipping learners with essential skills for mathematical reasoning across various fields of mathematics in their lives (Van de Walle et al., 2014). This topic continues to be pivotal in majority of programs in universities and colleges. For instance, it is very crucial for one to have a solid foundation in algebra to understand many key concepts in science, statistics, commerce and modern technology (Essuman et al., 2024). Moreover, algebra is a major area of mathematics that tests learner's ability to solve real-world problems and promotes logical thinking (Essuman et al., 2024). These descriptions demonstrate the significance of algebra across all levels of education.

Learners experience distinct challenges in algebra compared to other mathematics concepts including nations that demonstrate high proficiency in mathematics like Singapore, Hong Kong, Japan, China and Korea (Hill et al. 2016). These unique problems include inadequate comprehension of rational numbers and manipulation of algebraic symbols (Reeder, 2017; Samuel et al., 2016). This understanding suggests that issues and challenges that are algebraically related are quite general. Unfortunately, these challenges put learners at several disadvantages, such as being excluded from advanced courses, which leads to a scarcity of Science, Technology, Engineering, and Mathematics (STEM) skills, and reduces admission rates, which ultimately leads to fewer employment opportunities (Barbieri et al., 2019).

Similarly, the Trends in International Mathematics and Science Study (TIMSS) also highlights a major concern regarding South African learners' performance in international mathematics, with low achievement (Mullis et al., 2020; Mosimege et al., 2017).

As revealed in Table 1.1, out of the 39 nations that took part in the TIMSS of 2019, South Africa was the second lowest (38th position), achieving an average score of 389, with no improvement in the overall score in algebra (Mullis et al., 2020).

Table 1.1

Average achievement in mathematics content domains

Country	Overall Mathematics Average Scale Score	Number (63 Items)		Algebra (61 Items)		Geometry (43 Items)		Data and Probability (39 Items)	
		Average Scale Score	Difference from Overall Mathematics Score	Average Scale Score	Difference from Overall Mathematics Score	Average Scale Score	Difference from Overall Mathematics Score	Average Scale Score	Difference from Overall Mathematics Score
[2] Singapore	616 (4.0)	611 (4.1)	-5 (1.0) ▽	619 (4.6)	3 (1.3) ▲	619 (3.9)	3 (0.8) ▲	620 (4.9)	4 (2.1) ▲
Chinese Taipei	612 (2.7)	613 (2.7)	1 (1.0)	618 (2.6)	6 (1.4) ▲	623 (2.7)	11 (1.3) ▲	593 (2.5)	-19 (1.6) ▽
Korea, Rep. of	607 (2.8)	605 (2.6)	-2 (1.5)	609 (3.5)	2 (1.1) ▲	617 (2.9)	10 (1.0) ▲	598 (2.6)	-9 (1.7) ▽
Japan	594 (2.7)	578 (3.5)	-16 (1.4) ▽	602 (3.2)	8 (1.3) ▲	610 (3.4)	16 (1.9) ▲	594 (2.5)	0 (0.7)
† Hong Kong SAR	578 (4.1)	570 (4.2)	-9 (1.5) ▽	584 (3.9)	5 (1.5) ▲	596 (4.6)	18 (1.6) ▲	563 (5.6)	-16 (3.5) ▽
[2] Russian Federation	543 (4.5)	541 (4.6)	-2 (1.0) ▽	560 (5.0)	16 (1.1) ▲	540 (5.2)	-3 (1.2) ▽	517 (4.7)	-26 (2.1) ▽
Ireland	524 (2.6)	541 (3.0)	17 (2.1) ▲	505 (2.8)	-18 (1.1) ▽	506 (2.8)	-18 (0.9) ▽	541 (3.4)	17 (2.0) ▲
Lithuania	520 (2.9)	514 (3.0)	-6 (1.4) ▽	518 (2.9)	-2 (1.1) ▽	529 (3.0)	9 (1.2) ▲	522 (3.1)	2 (1.5)
[3] Israel	519 (4.3)	519 (4.2)	0 (1.3)	528 (5.0)	9 (1.2) ▲	506 (4.8)	-13 (1.8) ▽	511 (4.9)	-8 (2.3) ▽
Australia	517 (3.8)	522 (3.9)	4 (0.7) ▲	501 (4.1)	-16 (1.1) ▽	513 (4.0)	-4 (1.0) ▽	533 (3.9)	15 (1.4) ▲
Hungary	517 (2.9)	515 (3.1)	-1 (1.4)	509 (3.0)	-8 (1.0) ▽	521 (3.3)	5 (1.9) ▲	521 (3.2)	4 (2.2)
† United States	515 (4.8)	520 (4.5)	4 (0.7) ▲	520 (5.4)	4 (0.9) ▲	499 (4.8)	-16 (1.1) ▽	509 (5.4)	-6 (1.8) ▽
England	515 (5.3)	519 (5.4)	4 (2.1) ▲	504 (5.8)	-11 (1.6) ▽	509 (5.3)	-6 (1.5) ▽	523 (6.2)	9 (1.9) ▲
Finland	509 (2.6)	515 (2.6)	6 (0.9) ▲	489 (2.9)	-20 (1.2) ▽	511 (3.2)	2 (2.0)	514 (3.6)	5 (1.7) ▲
† Norway (9)	503 (2.4)	507 (2.3)	5 (1.0) ▲	477 (3.0)	-26 (1.7) ▽	502 (2.3)	-1 (1.0)	518 (3.0)	15 (1.3) ▲
[2] Sweden	503 (2.5)	502 (2.4)	-1 (1.3)	496 (2.9)	-7 (1.9) ▽	495 (3.1)	-7 (1.4) ▽	513 (3.7)	11 (2.2) ▲
Cyprus	501 (1.6)	499 (2.2)	-2 (1.8)	515 (2.6)	14 (1.7) ▲	490 (2.3)	-11 (1.5) ▽	493 (2.7)	-8 (2.0) ▽
Portugal	500 (3.2)	492 (3.3)	-8 (1.7) ▽	499 (3.3)	-2 (1.2)	509 (3.3)	9 (1.2) ▲	498 (3.2)	-3 (1.6)
Italy	497 (2.7)	495 (2.4)	-3 (1.4)	491 (2.7)	-7 (2.3) ▽	510 (3.7)	12 (2.4) ▲	494 (3.3)	-4 (2.0)
Turkey	496 (4.3)	493 (4.3)	-2 (1.7)	493 (4.6)	-3 (1.2) ▽	490 (4.2)	-6 (1.4) ▽	502 (4.3)	7 (1.1) ▲
[2] Kazakhstan	488 (3.3)	482 (3.4)	-5 (1.4) ▽	504 (3.7)	16 (1.2) ▲	486 (3.8)	-2 (1.6)	463 (3.3)	-25 (1.5) ▽
France	483 (2.5)	477 (2.6)	-6 (1.2) ▽	468 (2.8)	-15 (1.5) ▽	493 (2.7)	11 (1.6) ▲	496 (2.6)	13 (1.4) ▲
† New Zealand	482 (3.4)	483 (3.6)	2 (1.7)	464 (3.5)	-17 (1.8) ▽	477 (3.4)	-5 (1.6) ▽	496 (3.7)	14 (1.6) ▲
Bahrain	481 (1.7)	473 (2.2)	-8 (1.7) ▽	485 (2.1)	4 (1.5) ▲	493 (2.3)	12 (1.9) ▲	465 (2.0)	-16 (1.3) ▽
Romania	479 (4.3)	478 (4.5)	-1 (1.3)	490 (4.6)	11 (1.7) ▲	472 (4.7)	-7 (1.6) ▽	458 (4.5)	-21 (1.8) ▽
United Arab Emirates	473 (1.9)	474 (1.9)	1 (0.7)	486 (2.1)	12 (0.8) ▲	462 (2.1)	-12 (1.0) ▽	451 (2.1)	-22 (1.0) ▽
[1] Georgia	461 (4.3)	466 (4.7)	5 (1.7) ▲	473 (4.3)	12 (2.2) ▲	449 (4.4)	-12 (2.8) ▽	429 (5.1)	-32 (3.9) ▽
Malaysia	461 (3.2)	458 (3.1)	-3 (1.1) ▽	456 (3.3)	-4 (1.6) ▽	466 (3.7)	6 (2.6) ▲	457 (3.5)	-4 (1.2) ▽
Iran, Islamic Rep. of	446 (3.7)	442 (4.2)	-4 (1.6) ▽	450 (3.8)	4 (1.2) ▲	442 (4.4)	-5 (1.7) ▽	435 (4.0)	-11 (1.5) ▽
ψ Qatar	443 (4.0)	441 (4.0)	-2 (1.0)	454 (4.0)	10 (1.5) ▲	435 (4.0)	-8 (1.2) ▽	423 (4.7)	-20 (1.9) ▽
ψ Chile	441 (2.8)	442 (3.2)	1 (1.4)	439 (3.1)	-2 (1.8)	434 (4.3)	-6 (3.2)	434 (3.2)	-6 (1.3) ▽
Lebanon	429 (2.9)	432 (2.7)	2 (1.3)	452 (3.0)	23 (1.3) ▲	422 (3.2)	-7 (2.1) ▽	383 (3.5)	-46 (2.4) ▽
ψ Jordan	420 (4.3)	408 (4.5)	-12 (1.3) ▽	442 (4.8)	22 (1.2) ▲	413 (4.6)	-7 (3.0) ▽	396 (4.2)	-24 (1.8) ▽
[2] ψ Egypt	413 (5.2)	414 (5.4)	1 (2.2)	413 (6.0)	0 (2.0)	417 (5.3)	4 (1.3) ▲	380 (5.4)	-33 (1.4) ▽
ψ Oman	411 (2.8)	392 (3.0)	-19 (1.5) ▽	427 (3.0)	16 (1.4) ▲	418 (3.2)	7 (1.1) ▲	393 (2.9)	-17 (1.4) ▽
ψ Kuwait	403 (5.0)	- -	- -	- -	- -	- -	- -	- -	- -
[2] ψ Saudi Arabia	394 (2.5)	- -	- -	- -	- -	- -	- -	- -	- -
ж South Africa (9)	389 (2.3)	- -	- -	- -	- -	- -	- -	- -	- -
ψ Morocco	388 (2.3)	377 (2.7)	-11 (1.3) ▽	370 (3.1)	-18 (1.6) ▽	413 (2.2)	25 (1.4) ▲	372 (2.4)	-16 (1.3) ▽

Source: Mullis et al., (2020 p. 201)

As shown in Table 1.2, there was a slight improvement in Algebra among South African learners in the TIMSS 2023. However, out of the 44 countries, South Africa's average score (411) in Algebra was among the lowest together with Jordan, Malaysia, Kuwait, Saudi Arabia and Morocco (Von Davier et al., 2024).

Table 1.2

Relative Performance in Mathematical Content by Countries.

2023

Country	Overall Mathematics Average Scale Score	Number (63 Items)		Algebra (58 Items)		Geometry and Measurement (42 Items)		Data and Probability (37 Items)	
		Average Scale Score	Difference from Overall Score	Average Scale Score	Difference from Overall Score	Average Scale Score	Difference from Overall Score	Average Scale Score	Difference from Overall Score
[3] Singapore	605 (6.1)	606 (5.7)	1 (1.6)	604 (6.8)	-1 (1.8)	605 (6.0)	-1 (1.3)	615 (6.8)	10 (1.7) ▲
Chinese Taipei	602 (3.1)	611 (3.4)	9 (1.7) ▲	612 (3.2)	10 (1.4) ▲	600 (3.0)	-2 (1.6)	585 (2.9)	-17 (1.1) ▼
Korea, Rep. of	596 (3.0)	602 (3.1)	6 (1.5) ▲	595 (3.1)	-1 (1.4)	603 (3.6)	7 (1.6) ▲	584 (3.8)	-12 (2.9) ▼
† Japan	595 (3.0)	590 (3.7)	-5 (1.2) ▼	593 (3.5)	-2 (1.3)	600 (3.2)	5 (1.6) ▲	609 (3.2)	15 (2.0) ▲
† Hong Kong SAR	575 (5.0)	576 (5.3)	1 (1.2)	577 (5.5)	3 (1.3)	579 (5.6)	4 (1.3) ▲	562 (5.4)	-13 (1.7) ▼
[2] England	525 (4.5)	532 (4.8)	8 (1.5) ▲	513 (4.8)	-12 (1.6) ▼	519 (4.6)	-6 (1.1) ▼	537 (5.4)	12 (2.2) ▲
Ireland	522 (2.7)	528 (2.9)	6 (1.3) ▲	503 (2.9)	-19 (1.2) ▼	513 (3.3)	-8 (2.4) ▼	546 (3.3)	24 (1.3) ▲
[2] Czech Republic	518 (2.3)	525 (2.4)	7 (0.8) ▲	513 (2.6)	-5 (1.5) ▼	520 (2.5)	2 (1.0)	504 (2.3)	-14 (0.9) ▼
[2] Sweden	517 (2.4)	514 (2.4)	-4 (1.2) ▼	511 (2.8)	-6 (1.7) ▼	516 (3.1)	-1 (2.0)	532 (3.0)	15 (1.2) ▲
[2] Lithuania	514 (3.1)	509 (3.2)	-5 (1.1) ▼	512 (3.3)	-1 (0.8)	526 (3.3)	12 (0.9) ▲	506 (3.5)	-7 (1.6) ▼
[2] Austria	512 (2.3)	511 (2.3)	-1 (1.2)	517 (2.7)	5 (2.1)	519 (2.8)	7 (1.7) ▲	495 (2.5)	-17 (0.8) ▼
Australia	509 (3.5)	505 (3.6)	-4 (1.2) ▼	498 (3.4)	-10 (0.9) ▼	506 (3.5)	-3 (1.6)	532 (3.8)	24 (1.0) ▲
[3] † Türkiye	509 (4.3)	511 (4.6)	2 (1.6)	498 (4.9)	-11 (1.5) ▼	496 (4.4)	-13 (2.1) ▼	529 (4.4)	20 (1.0) ▲
Hungary	506 (3.7)	510 (3.9)	4 (1.0) ▲	504 (3.9)	-2 (1.6)	506 (3.8)	0 (1.6)	498 (3.5)	-8 (0.9) ▼
Finland	504 (2.6)	504 (2.6)	0 (1.3)	490 (2.7)	-13 (0.8) ▼	513 (2.7)	9 (0.9) ▲	508 (2.9)	4 (1.2) ▲
[2] Norway (9)	501 (2.3)	495 (2.4)	-5 (1.2) ▼	485 (2.6)	-15 (1.0) ▼	506 (2.8)	6 (1.4) ▲	521 (3.3)	21 (2.2) ▲
Italy	501 (3.0)	502 (2.8)	2 (1.1)	492 (3.3)	-9 (1.9) ▼	509 (3.8)	8 (1.6) ▲	496 (3.2)	-5 (0.8) ▼
Malta	499 (1.2)	500 (1.3)	0 (0.8)	496 (1.6)	-3 (1.3)	496 (1.5)	-3 (1.3)	505 (1.6)	6 (1.3) ▲
≡ Romania	496 (4.9)	503 (4.7)	7 (1.2) ▲	502 (5.1)	6 (1.7) ▲	492 (5.1)	-4 (1.8)	467 (6.1)	-29 (2.8) ▼
Cyprus	494 (2.7)	487 (2.9)	-7 (1.3) ▼	507 (3.0)	14 (1.4) ▲	492 (3.0)	-2 (1.4)	486 (3.2)	-8 (1.7) ▼
United Arab Emirates	489 (1.7)	493 (1.7)	4 (0.4) ▲	496 (1.8)	8 (0.4) ▲	479 (2.0)	-10 (0.7) ▼	475 (1.9)	-13 (0.7) ▼
≡ United States	488 (4.2)	486 (4.4)	-2 (1.1)	492 (4.6)	4 (0.9) ▲	480 (4.0)	-8 (1.0) ▼	492 (4.1)	4 (1.1) ▲
[3] Israel	487 (3.9)	485 (3.9)	-2 (1.0)	494 (4.1)	7 (1.9) ▲	484 (4.1)	-4 (1.7)	481 (4.4)	-6 (1.9) ▼
France	479 (3.1)	475 (3.2)	-3 (1.2) ▼	466 (3.6)	-13 (1.4) ▼	482 (3.6)	3 (1.8)	491 (3.5)	12 (1.4) ▲
Azerbaijan	479 (3.7)	494 (4.0)	15 (1.8) ▲	489 (4.2)	11 (1.9) ▲	458 (4.1)	-20 (1.9) ▼	442 (4.6)	-37 (2.5) ▼
Portugal	475 (2.7)	468 (2.6)	-8 (1.1) ▼	474 (3.3)	-1 (2.3)	485 (3.0)	10 (1.7) ▲	474 (2.5)	-1 (1.4)
[1] Georgia	467 (3.2)	477 (3.4)	10 (2.0) ▲	476 (3.5)	10 (1.6) ▲	452 (3.2)	-15 (1.7) ▼	433 (3.2)	-33 (2.1) ▼
Kazakhstan	454 (3.5)	454 (3.3)	0 (1.3)	466 (4.1)	12 (1.7) ▲	445 (3.4)	-9 (1.4) ▼	432 (3.4)	-22 (2.4) ▼
Qatar	451 (4.2)	448 (4.5)	-3 (1.7)	459 (4.4)	8 (1.4) ▲	441 (4.5)	-10 (1.2) ▼	443 (4.5)	-9 (1.4) ▼
Bahrain	426 (2.4)	423 (2.4)	-3 (1.7)	434 (2.8)	7 (1.4) ▲	416 (2.7)	-11 (1.7) ▼	421 (2.6)	-5 (1.7) ▼
Ψ Iran, Islamic Rep. of	423 (3.6)	416 (3.7)	-7 (2.1) ▼	424 (3.8)	1 (1.0)	406 (3.7)	-17 (1.3) ▼	423 (3.4)	-1 (1.5)
Uzbekistan	421 (4.3)	433 (4.6)	12 (1.2) ▲	427 (5.1)	6 (2.0) ▲	415 (4.3)	-6 (1.8) ▼	371 (4.8)	-50 (2.5) ▼
† Chile	416 (3.2)	411 (3.0)	-5 (1.6) ▼	414 (3.7)	-2 (2.9)	410 (3.0)	-6 (2.3)	420 (3.9)	4 (1.4) ▲
Oman	411 (2.7)	408 (2.8)	-3 (1.6)	419 (2.8)	8 (0.7) ▲	403 (2.7)	-8 (1.6) ▼	403 (2.7)	-8 (1.6) ▼
Malaysia	411 (3.5)	409 (3.6)	-2 (0.8)	406 (3.7)	-4 (1.1) ▼	417 (3.4)	6 (1.0) ▲	403 (3.5)	-8 (1.4) ▼
Ψ Kuwait	399 (5.2)	399 (5.4)	0 (2.1)	382 (6.1)	-17 (1.6) ▼	386 (6.3)	-13 (2.8) ▼	402 (4.5)	4 (2.2)
[2] Saudi Arabia	397 (3.3)	393 (3.7)	-4 (1.6) ▼	401 (3.5)	5 (1.0) ▲	385 (3.4)	-12 (2.1) ▼	397 (3.4)	0 (1.8)
Ψ South Africa (9)	397 (3.1)	410 (3.2)	13 (1.4) ▲	411 (3.6)	14 (1.8) ▲	358 (3.8)	-39 (1.3) ▼	341 (3.6)	-55 (1.8) ▼
Jordan	388 (3.2)	392 (3.1)	4 (1.4) ▲	405 (3.4)	17 (1.4) ▲	373 (3.6)	-15 (1.6) ▼	359 (4.0)	-29 (2.9) ▼
Morocco	378 (3.0)	382 (3.2)	5 (1.7) ▲	375 (3.8)	-3 (1.5)	380 (3.6)	2 (1.6)	354 (2.8)	-24 (1.3) ▼
Ψ Palestinian Nat'l Auth.	382 (3.0)	- -	- -	- -	- -	- -	- -	- -	- -
Ψ Brazil	378 (2.8)	- -	- -	- -	- -	- -	- -	- -	- -
≡ New Zealand	485 (4.1)	484 (4.2)	-1 (1.5)	472 (4.5)	-13 (1.9) ▼	483 (4.5)	-3 (2.8)	503 (4.0)	18 (1.5) ▲
Ж Cote d'Ivoire	263 (4.9)	- -	- -	- -	- -	- -	- -	- -	- -

Source: Von Davier et al. (2024. Downloaded from https://timss2023.org/results)

Table 1.3 below displays how South African learners performed in Algebra particularly in the constructed-response questions in which the Grade 9 learners obtained 18.3% pass rate as compared to the international pass rate of 35.8 %.

Table 1.3

South African Learners' performance in Algebra

	Constructed-response questions	
	South Africa	International average
Number	33,3	54,1
Algebra	18,3	35,8
Geometry	24,8	44,5
Data and chance	12,5	36,4

Source: Mosimege et al. (2017, p. 12)

For example, drawing from my own experience as a mathematics teacher and researcher, out of the 182 Grade 10 learners in my school, only 11 are studying pure mathematics. Additionally, only nine out of 151 Grade 11 students are pursuing mathematics and unfortunately, just eight out of 148 Grade 12 candidates who sat for the NSC (National Senior Certificate) were enrolled in pure mathematics. This is because learners lose interest in Mathematics as a subject and it might be because they are unable to understand mathematical concepts such as algebraic expressions (Christou et al., 2022).

In the research conducted by Baybayon and Lapinid (2024), learners' answers in algebraic expressions and equations, showed inadequate prerequisite skills and concepts which notably influence performance in mathematics. Baybayon and Lapinid (2024) further highlighted that some errors indicate that learners need to improve their test-taking skills and should thoroughly review their solutions before submitting their work to avoid careless mistakes. The preceding discussions suggest that the understanding of algebra, particularly the concept of algebraic expressions is a major problem both in and outside of South Africa and it is a pressing concern that requires attention.

Social settings or environments can act as both enablers and obstacles in understanding algebraic expressions, as with other mathematical concepts (Hausberger, 2020). In terms of an obstacle, the environment could create negative perceptions about the concept of algebraic expressions and mathematics as a subject.

Research by Gezahegn (2007), which examined many school stakeholders in Ethiopia, discovered that learners' perceptions of mathematics were a contributing factor in their academic failure. According to the researcher, most stakeholders in the study thought mathematics was a challenging subject. Most of the stakeholders (being children) claimed to have inherited that adverse perception from their families. Therefore, if learners are given such a bad impression, it is challenging to persuade them that they can succeed in mathematics, and algebraic expressions in particular.

To maximise the success of learners' understanding of algebra and algebraic expressions, it is crucial to understand the challenges of algebraic materials and find ways to enhance learning with these challenges (Lee et al., 2020). Given this context, the study investigated Grade 9 learners' comprehension of algebraic expressions.

1.2 Problem Statement

Algebraic expressions, as a section in algebra, form one of the most critical topics in the school mathematics curriculum (Essuman et al., 2024). Generally, algebraic applications are in most cases found in all mathematics and Science fields and covers a third of secondary school mathematics curriculums (Daud & Ayub, 2019). The concept of Algebraic expressions is considered one of the most pivotal topics in high school which makes it very important for learners to be more proficient in solving algebraic problems if overall achievement of students in national examinations is to be attained (Daud & Ayub, 2019). Notwithstanding all the relevance highlighted in Section 1.1, topics related to algebraic expressions remain under-researched and require further attention (Po urnara et al., 2016; Daud & Ayub, 2019). Mastering these topics has the potential to significantly improve mathematics pass rates for learners (Jupri et al., 2015).

TIMSS (2019) report revealed that, the mathematics grades for learners in England dropped from an average of 528 in 2015 to 519 because new Grade 9 learners had not learned algebraic expressions, which had degraded their complete acquisition of concepts and it had in turn led to them achieving lower marks after committing major mistakes while solving critical problems (Mullis et al.,2020).

The TIMSS 2023 report also reveals that out of the 44 countries, only 12 countries representing 27%, improved their average score in Algebra between the 2019 and 2023 TIMSS assessment. Thirty-two (32) out of the 44 countries (73%) registered a drop in their average score in Algebra including the top five best performing countries as shown in Table 1.1 and Table 1.2. For instance, Singapore obtained an average score of 619 for Algebra in the TIMSS 2019, but 605 in TIMSS 2023. Chinese Tapei dropped from 618 to 612, Korea dropped from 609 to 595, Japan dropped from 602 to 593 and Hong Kong dropped from 584 to 577 (See Table 1.1 & 1.2).

Another critical issue with regards to learning and understanding algebraic expressions amongst learners is incorrect procedures and this is crucial because it inhibits accurate solutions (Edo & Tasik, 2022). Many learners (both lower and higher levels) lack basic understanding of algebraic expressions and commit several mistakes when solving problems. They (learners) only commit to memory a limited number of formulas, algorithms and facts without understanding the underlying algebraic concept (Edo & Tasik, 2022).

Pramesti and Retnawati (2019) revealed that Algebra is a crucial subject that begins in middle school, and it assists in understanding the notions of the other areas. However, learners have several difficulties when studying mathematics in the early stages. For this reason, Pramesti and Retnawati (2019) investigated individuals' challenges with mathematics, specifically algebraic expressions during the first year. The research employed a descriptive method using a phenomenological technique. The research included 65 middle school pupils from Indonesia. Finally, the study findings suggested that students struggled with grasping the concept and operations in algebra. As a result, instructors' strategies for teaching algebra content must be devised to reduce pupils' challenges in understanding algebra.

Additionally, according to Ojaleye and Awofala (2018), algebra is a critical area in the field of mathematics where learners struggle and do not do well. This has necessitated the need to undertake a study on how learners understand algebraic expressions.

In the struggle to master algebraic expressions, South Africa has been no exception. Most learners in South Africa usually struggle to understand mathematics in general, because they (learners) lack basic numerical skills (Mtshali, 2019). For instance, the achievement in both general Mathematics and Algebra by South African learners according to TIMSS, 2016, was below average (Mosimege et al., 2017).

Again, the Grade 9 learners’ performance in ANA (Annual National Assessment) which operated between 2012–2014 saw a pattern of dismal performance in Algebra and Mathematics in general because the national averages for Grade 9 were 12.7%, 13.9% and 10.8% respectively over the three years (Schollar, 2022).

As shown in Figure 1.1 below, a declining performance especially in grade 9 continually tormented the educational system of South Africa.

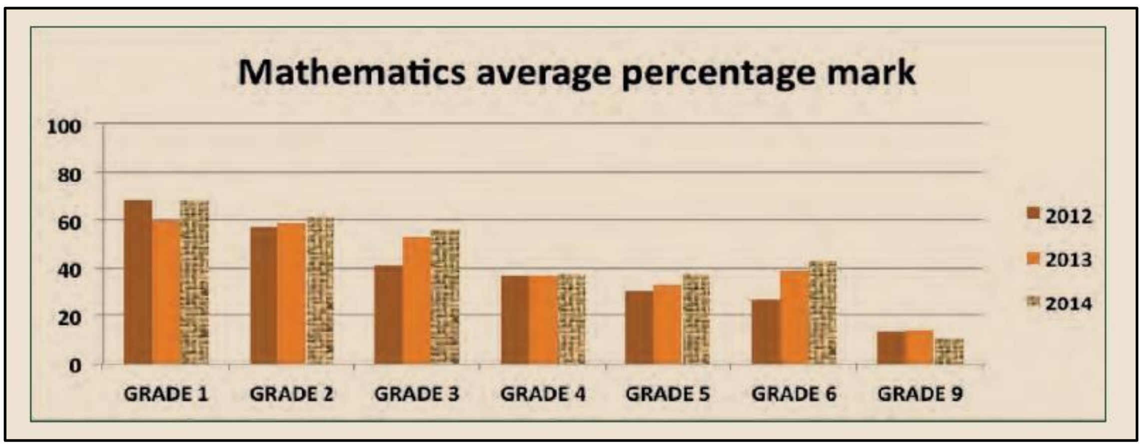


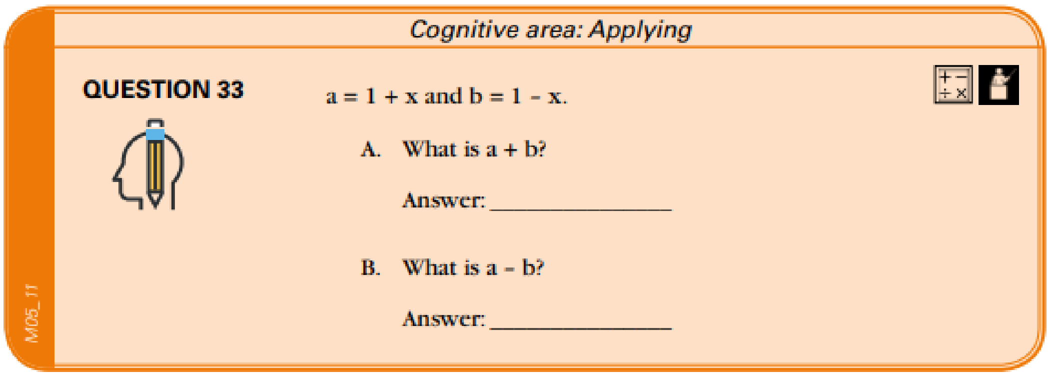
Figure 1.1

Average marks for mathematics in ANA from 2012–2014.

Source: Department of Education [DBE] (2015 p. 41)

This consequently suggests that there is a huge Mathematics problem in the South African Education system (Pournara, 2020). According to Akhtar et al. (2020), more challenges are observed when learners fail to interpret the word-problems into mathematical statements (algebraic expressions). The frequently cited reason was learners' lack of the necessary mathematical patterns and cognitive skills (Pournara et al., 2016; Daud & Ayub, 2019; Akhtar et al., 2020).

As shown in the extract from the 2015 TIMSS Diagnostic report (Figure 1.2), South African learners performed way below the international average in terms of providing correct answers for algebraic expression items.


Cognitive area: Applying

QUESTION 33

$a = 1 + x$ and $b = 1 - x$.

A. What is $a + b$?

Answer: ______________

B. What is $a - b$?

Answer: ______________

M05_11

Learner performance

Question 33A	Correct	Incorrect	Omitted
South Africa	15,1	80,6	3,2
International average (n=38)	37,3	47,0	14,6

Question 33B	Correct	Incorrect	Omitted
South Africa	1,4	93,4	4,0
International average (n=38)	13,9	68,2	16,8

Figure 1.2

An Algebraic Expression Question and its Analysis in TIMSS 2015.

Source: Mosimege et al. (2017, p. 46)

It is clear from Figure 1.2 that Algebra as a topic is a serious challenge amongst South African learners in the 9th grade. According to the report, majority of the learners struggled to comprehend the algebraic questions to the extent that some of them re-stated the questions but in words rather than algebraic terms with one of the reasons being linked to language.

According to TIMSS, 2019 report, although South Africa showed a slight improvement and progress in the standard of mathematics, it (South Africa) was among the five lowest-performing countries in average mathematics achievement (DBE, 2020). Breaking down the TIMSS 2019 report by province, learners in South Africa's Northern Cape Province recorded the lowest achievement improvement (+11) points in mathematics (DBE, 2020). This result made the Northern Cape one of the six provinces with a mathematics achievement score lower than the National Score of 398.

Many authors have linked the inability of learners to proceed to STEM studies and the increased rate of failure in mathematics to the inability of learners to master the subject of mathematics, particularly, algebraic expressions (Tunc & Bagceci, 2021).

Furthermore, Ying et al. (2020), emphasised that most learners fail algebraic expressions due to recognition difficulties, where leaners fail to differentiate between algebraic expression types and hence, the need for learners to be helped when learning the topic. This implies that unless an exploratory study is done, South African learners, particularly in the Northern Cape province, shall continue to underperform in mathematics due to the continued inability of learners to completely understand and master mathematics that relates to algebraic expressions. Another reason that motivated me to conduct this study is that algebraic expressions extend across learners' mathematics education from elementary through high school (Essuman et al., 2024).

Learners not only continually study algebraic expressions across grades, but their conceptual understanding of the topic also evolves from basic algebraic concepts to complex concepts starting from Grade 7 and extending to the upper secondary level (Grade 12), in conjunction with their advancing mathematics competency (DBE, 2014). For instance, learners start Algebra in Grade 7 and are required to be able to recognise and distinguish between constants and variables in an expression, equation or a formula. When they reach the 12^{th} grade, they (learners) are expected to be able to factorise third-degree polynomials including examples which require the factor theorem (DBE, 2014).

1.3 Research Questions

1.3.1 Main Research Question

The primary research question guiding this study is:

How do Grade 9 learners understand algebraic expressions?

1.3.2 Secondary Questions

The study seeks to address the following questions:

1. What strategies do Grade 9 learners use to solve algebraic expression problems?
2. How do Grade 9 learners make connections when solving problems involving algebraic expressions?
3. What conceptual gaps manifest when learners solve algebraic expression problems?
4. How can the conceptual gaps be minimised?

1.4 Aims and Objectives of the Study

The aim of this study was to investigate and describe how Grade 9 learners understood the concept of algebraic expressions. This aim was achieved through the following objectives:

- Describe the strategies Grade 9 learners use to solve algebraic expression problems.
- Explain how learners make connections when solving problems involving algebraic expressions.
- Identify the conceptual gaps that manifest when learners solve problems involving algebraic expressions.
- Suggest ways to minimise these conceptual gaps.

1.5 Purpose Statement

This study investigated how Grade 9 learners understood and solved algebraic expression problems. It explored the conceptual gaps and challenges learners in Grade 9 experience when solving algebraic expression problems and how these challenges could be minimised. The study was designed to better understand how learners connect, apply and use learned concepts when solving problems in algebraic expressions. The outcome of this research offers practical evidence which could help learners to use different learning techniques to solve algebraic expression problems.

1.6 Rationale and Significance of the Study

The significance of algebra is highly acknowledged in our world today (Wasserman, 2018). The advancement of science, engineering and technology relies heavily on the contributions of algebra to the shared understanding of society and the world (Juraev & Bozorov, 2024). Therefore, focusing on the teaching and learning of algebra is crucial and very important in our schools to help foster 21st-century competence including engineering and technology amongst the emerging graduates.

Algebra offers learners the ability to work with abstract concepts and to put those concepts into practical situations (Alam & Mohanty, 2024). However, algebra constitutes one of the major branches of high school mathematics where learners frequently experience difficulties (Rahayu et al., 2024).

To help high school learners overcome algebraic difficulties, it is essential to provide guidance and opportunities for them from an early stage, such as primary school.

This study will therefore help in enhancing policies relating to algebraic expressions to determine whether it is safe to introduce them in the earlier years of learning, and to see whether this will bridge the conceptual gaps and reduce the challenges learners face when they are in their high school years of learning.

Information gathered from this study will assist teachers and school management, particularly heads of mathematics departments in providing support on techniques to use for effective teaching of algebraic concepts.

The study will also assist curriculum designers in identifying the challenges both teachers and learners face with regards to algebraic expressions enabling them to put measures in place to help learners perform better.

Moreover, this research work will establish a strong foundation for future study of this kind which is needed to achieve progress and address a variety of learner concerns in understanding algebraic expressions.

This study has the aptitude to improve the quality of mathematics education for learners in Grade 9 and above which would be beneficial to the following sectors of the society:

Learners: By learning from their mistakes, the study can inspire learners to develop a greater interest in mathematics-related courses and disciplines.

Teachers: The study could support teachers in designing more effective teaching strategies and approaches for algebra instruction.

Education Department: The valuable information from this study can help in the development of learning materials such as textbooks, workbooks and online resources, benefiting policy makers and stakeholders in the education sector.

1.7 Theoretical Framework

A theoretical framework is a set of interconnected ideas or models that provide direction for a research study or business venture (Stahl, 2023). Put differently, the theoretical framework is a constructed structure that upholds a theoretical investigation.

Varpio et al. (2020) similarly define a theoretical framework as a cohesive and interrelated collection of concepts and presumptions constructed by a researcher based on one or more theories to support research. This suggests that it (the theoretical framework) forms the basis of a study in theoretical or conceptual terms. In other words, a theoretical framework is a design that assists a researcher to conduct a study using an existing idea or theory. The current study adopted Richard Skemp's (1976) theory of understanding, which distinguishes between two main types of understanding: instrumental and relational.

Skemp describes instrumental understanding as the type of understanding that enables learners to use mathematical rules or procedures to perform tasks without knowing or understanding why the rules work. Relational understanding, on the other hand, occurs when learners understand how and why the mathematical rule or procedure works (Skemp, 1976).

Skemp's theory was used for this study due to its emphasis on the concept of understanding around which this study revolves.

1.8 Research Paradigm

A research paradigm is very important in a study as it guides scientific findings through their underlying assumptions and helps minimise bias in research methods (Yong et al., 2021). Positivism and interpretivism are the two main common paradigms used in philosophy for scientific research and analysis (Junjie & Yingxn, 2022). This study adopted the interpretivist paradigm because, it best fits qualitative research design (Turin et al., 2024; Junjie & Yingxn, 2022). Interpretation of this study was conducted subjectively based on participants' results, their explanations and how they made sense of their experiences through dialogue without any omission (Kodisang, 2022).

1.9 Research Design and Methodology

1.9.1 Research Design

The study employed a qualitative case study research design. Case study research involves an in-depth and thorough analysis of a particular event, scenario or organisation within a designated time and place (Hancock et al., 2021; Schoch, 2020). A case study was employed to enable me, as the researcher, to collect various types of data, which in this context included a test and interviews. These data collection techniques also allowed me to pose insightful and thorough questions to extract information on issues related to algebraic expressions.

1.9.2 Research Approach

This study used a qualitative research approach. This approach prioritises the collection of data on events that occur naturally (Muzari et al., 2022; McMillan & Schumacher, 2010). This research approach is thus deemed appropriate for the current study as I directly tested the learners and personally interviewed them on how and why they came up with their answers.

1.10 Population and Sampling methods

The study's target population comprised the entire Grade 9 learners of Circuit eight (8) in the Frances Baard District of the Northern Cape Province. There were 1085 Grade 9 learners in the circuit.

The sample of this study comprised 40 learners who were purposively selected from a single school in Circuit eight (8) from the Frances Baard district.

Sampling purposively is the deliberate selection of research participants or materials based on their ability to contribute particular qualities that are needed and important for a theme, concept or phenomenon of interest (Creswell & Poth 2017).

1.11. Data Collection Instruments

The two main data collection instruments used for this study were test and a face-to-face semi-structured interview. A set of structured test items (Appendix A) on algebraic expressions was given to participants (learners) to answer, followed by a semi- structured interview (Appendix B) with open-ended questions with selected learners.

Twelve (12) learners were subsequently chosen to be interviewed out of the 40 participants who wrote the test. These 12 learners were selected based on their solutions to evaluate their understanding of algebraic expressions and explore their answers.

1.12. Analysis and Interpretation of Data

The qualitative data collected were analysed using conceptual thematic analysis (Braun & Clarke, 2016). The process includes becoming acquainted with the data; code generation; organising code into themes, defining & labeling themes and producing a report that presents sufficient evidence from these themes (Campbell et al., 2021). The themes that emerged from this study include: the common strategies learners employ when solving algebraic expression problems, the connections learners create when solving algebraic expressions and the conceptual gaps that manifest in learners' solutions.

1.13. Credibility

Credibility can be defined briefly as the confidence placed in the truth of a research study (Dahlgren, 2022). To ensure the credibility of this research, I relied on peer debriefing. This procedure involves seeking assistance and guidance from an expert or professional (Katz et al., 2022), which in this case was my research supervisor. Secondly, I had a prolonged engagement and persistently observed the participants to establish whether their responses were genuine, reliable, and consistent (Amin et al., 2020).

1.14 Research Ethics

When conducting research interviews, researchers are expected to examine the ethical issues that arise when using any research instrument on research subjects. First, I submitted application form to the Ethics Review Committee of UNISA College of Education (CEDU) and permission was granted (Appendices C1 & C2). Furthermore, a formal request was made to the Northern Cape Department of Education to obtain permission (Appendix D) for conducting the study in the chosen school. The research adhered to ethical standards to minimise the potential for unexpected harm, safeguard the confidentiality of the interviewees' information, mitigate the danger of exploitation, and effectively inform the respondents about the study's purpose. Additional ethical considerations I followed include:

1.14.1 Receiving Approval to Conduct the Study

I obtained an approval letter from the Provincial Education Office and consent from the selected school (Appendix E) to get permission for participating in the research.

1.14.2 Informed Consent

I ensured that I provided consent documents (Appendix F) to all participants, together with an invitation letter to participate in this research.

1.14.3 Confidentiality and Anonymity

I used alphanumeric codes to identify all the forty (40) learners who participated in the study. To protect personal and institutional information, I committed to protecting the primary information obtained and to publishing it only after explicit permission from participants or the school.

1.14.4 Safety of Participants

I made a commitment to ensure that participants would be safeguarded against any kind of emotional or physical harm for taking part in the study. I achieved this objective by obtaining permission from the relevant authority to conduct the research with the participants.

1.15 Limitations and Delimitations

1.15.1 Limitations

‘Research limitations’ refers to the constraints that affect the design of a study and have an impact on the overall research (Ross & Bibler Zaidi, 2019). In other words, limitations are the weaknesses in the study that could impact the research findings. The limitations of this study are as follows:

Firstly, the 40 participants used was notably small as compared to the entire Grade 9 population in the Frances Baard district. The main reason for selecting such a small sample was due to lack of resources and time constraints.

Secondly, I was a full-time Mathematics teacher employed by the Northern Cape Department of Education and conducted the research in addition to my basic teaching duties. This created time constraints with respect to the research scope and as a result the study was limited to 40 learners.

Having pointed out these limitations, it was assumed that learners possessed certain basic background knowledge regarding algebraic expressions. These assumptions included the fact that they (learners) have mastery of addition, subtraction, division and multiplication of both negative and positive whole numbers. Secondly, they were assumed to be knowledgeable about variables and the fact that they (variables) are symbols used in expressions to represent numbers and that, two different variables cannot be added or subtracted. It was also assumed that, learners understood and can apply the basic concept of the distributive property. Moreover, it was my assumption that learners could add and subtract simple fractions with different denominators with ease.

1.15.2 Delimitations

'Delimitations' refers to the specific choices a researcher makes that need to be explicitly stated in a research study (Akanle et al., 2020). In other words, they (delimitations) outline the bounds the researcher establishes for the research or the choices made by the researcher to narrow the study's focus and scope (Chasokela & Sibanda, 2025).

The two concepts, understanding and algebra, are broad and extensive and have their challenges. The study therefore focused on learners' understanding of algebraic expressions and the conceptual gaps associated with it. The study was done in only one selected school in the Northern Cape. This was due to both limited time and resources.

Secondly, it was to allow for an extensive exploration of the research problem within a manageable scope. Also, the school authorities readily provided consent and were willing to provide the necessary support, making it possible to conduct the research. Moreover, the selected school was in close proximity to me as the researcher.

1.16 How my Interest in the Research Study Developed

Previous studies highlighted that learners perform poorly in the domain of algebra (Mtshali, 2019; Akhtar et al., 2020; Ojaleye & Awofala, 2018). However, no study has been conducted previously to understand algebraic expressions, the strategies learners use, the conceptual gaps and challenges they experience, and the strategies adopted to overcome such challenges, particularly in the Northern Cape. This revealed a research gap that needed to be explored and sparked my interest. I also discovered that several authors (Baidoo, 2017; Powell et al., 2019) argue that learning of algebraic expressions is mainly done in the advanced stages. I, however, believe that these stages are too late for most learners to benefit from the topic because teaching and learning of algebraic expressions start in Grade 7 (DBE, 2012). I believe that if algebra was to be introduced earlier than Grade 7, then there might be a possibility for improvement in high school, especially in Mathematics and related subjects such as Science. In this regard, I was interested and eager to find out if earlier introduction of the concept would help improve student performance in Grade 9, and this triggered the research interest even more.

1.17 Definition of Key Terms

- **Understanding** of a concept is the ability to describe, explain and use the concept in various ways across different situations. It (understanding) involves the ability to know more than isolated facts (Ncube & Luneta, 2025).

- **Algebraic Expression** is a combination of numeric and symbolic notation with a variety of variables and operations like subtraction, addition, division and exponentiation (Stemele & Jina Asvat, 2024 ; Chaika, 2019). An example of an algebraic expression is : $8x + 3y - 9$.

- **Mathematical Connection** is the link between different mathematical concepts, ideas, or representations which enable learners to establish relationships between problems, particularly in terms of mathematical language (García-García, 2024 ; Kenedi et al., 2019). It is also the ability of learners to acknowledge how other components or topics of mathematics are interconnected and build upon one another, forming a cohesive whole.

- **Misconception** is an incorrect idea, preconception or an inaccurate viewpoint based on inadequate reasoning or comprehension (Kurudirek et al., Karim, 2025 ; Abakah & Brijlall, 2024). An example of a misconception in algebraic expressions is $3x + 5 = 8x$.

1.18 Chapter Outline

This study comprises five (5) chapters outlined as follow:

- **Chapter 1: Introduction**

The first chapter outlines a brief description of the study background and justify the importance of the research. Also included in chapter one are: the problem statement, the research questions, the aims and objectives, the purpose statement, rational and significant of the study, limitations and delimitations, how my interest in the research study developed, definition of key terms and chapter outline.

Chapter 2: Review of Literature

The second chapter highlights previous studies conducted by different scholars related to learning and understanding of algebraic expressions. It focuses on studies related to learners' in-depth understanding of algebra and algebraic expressions. These include: the context of educational practice in South Africa, algebra and its significance in mathematics, the notion of algebraic expressions, general learner performance in algebraic expressions, challenges learners experience when solving algebraic expressions, the concept of understanding, learners' understanding of solving algebraic expression problems, the concept of mathematical connections, and strategies used by learners in solving algebraic expressions. The chapter also discusses the theoretical framework (Skemp's theory of understanding) of the study and the conceptual gaps associated with learning algebraic expressions.

- **Chapter 3: Methodology**

Chapter three outlines how the research was conducted and fully explains the research paradigm, the research approach, the research design, the population and sampling method used, the instruments used for collecting data and the data analysis procedure. The chapter also discusses the credibility, trustworthiness and the ethical considerations undertaken by the research study.

- **Chapter 4: Analysis and Interpretation of data**

The fourth chapter presents the analysis and interpretation of data from a test and interviews using various suitable statistical tools and techniques. The content of chapter four also includes the demographics of the school, presentation of data and the various themes that were identified during the analysis of the data, namely: common strategies employed by learners in solving algebraic expression problems, connections created in the process of solving algebraic expressions and conceptual gaps identified in the learners' solutions.

- **Chapter 5: Summary and Conclusion**

Chapter five consolidates all findings of the research and presents the significant results depicted after conducting this study. Also included in this chapter are the reflection on the utility of the theoretical framework, recommendations based on the findings of the study and reflexivity. The chapter concludes the study with a discussion of important suggestions for future research and the limitations associated with this research.

1.19 Conclusion

This chapter set the tone for the study by giving the statement of the problem, the aim and objectives, the research questions and the importance of the study, among others. It therefore briefly showed what I intended to do. The next chapter reviews literature on algebraic expressions and the theoretical framework used for this study.

2. CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

The previous chapter presented the introduction to the research study and outlined the rationale, aims and objectives. This chapter aims to provide a review of the literature on the understanding of algebraic expressions amongst grade 9 learners. In other words, the chapter reviews how scholars understand the structure, elements and principles that govern algebraic expressions and how such knowledge could be applied effectively to solve problems.

Literature review can generally be defined as the systematic method of collecting, discussing, synthesising and analysing published information or previous research in a specific subject area (Snyder, 2019; Linnenluecke et al., 2020). The primary purpose of revealing literature is to gain a deeper understanding of existing research that is relevant to a specific concept or field of study (Snyder, 2019; Western Sydney University Library, 2024). It also serves as the basis for developing a new framework, theory or model (Rowley & Slack, 2004; Snyder, 2019).

There are several compelling reasons why it is essential for learners to develop a deep understanding of mathematics. First, mathematics provides a powerful means of promoting logical reasoning, developing intellectual rigor and cognitive discipline (Lin, 2023). Moreover, deep comprehension of mathematical concepts plays an important role in understanding concepts or contents that are covered in several academic disciplines, including most of the sciences like life, agriculture, physical and social (Park et al., 2021).

This chapter begins by situating the curriculum. In other words, it places or contextualises the curriculum within specific settings or environments. Secondly, it elaborates on the content related to algebraic expressions as addressed in educational systems beyond South Africa. Moreover, it provides an overview of the concept of understanding, while the last section offers a detailed description of algebraic expressions concept and its significance.

The subsequent sections highlight the mathematical connections used when solving algebraic expression problems, the various strategies learners use in solving algebraic expressions, the difficulties learners face as well as the potential solutions to improve learners' understanding and mastery of algebraic expressions. The chapter furthermore recounts the theoretical framework that situates the study within the broader academic discourse (Lim & Koay, 2024).

Skemp's theory of understanding was the guiding framework for this research. It provides a structure for teachers to guide students in moving from merely performing procedures to understanding and connecting the concepts behind algebraic expressions, leading to a deeper and more comprehensive grasp of the algebraic concept. After conducting a comprehensive literature review, significant gaps were identified. The study further illustrates how addressing these research gaps can facilitate the attainment of the aims and objectives of the current investigation. The next section considers a variety of factors that influence how education is offered, experienced and perceived by learners, teachers, and policymakers in South Africa.

2.2 The Context of Educational Practice in South Africa

Access to high-quality education is recognised as a fundamental human right by the South African Constitution and is considered a prerequisite for achieving national development goals (Arends et al., 2017). Mathematics education is highly valued in all education systems worldwide (Compayan, & Dollete, 2019). This is because the successful application of mathematical knowledge plays a significant role in various facets of human life.

Mathematics is an essential subject for advancing human knowledge to higher levels. It serves as a tool to cultivate individuals' ability to think logically and critically (Rossouw & Steenkamp, 2025). In South African high schools, Mathematics is a five-year mandatory subject (as pure Mathematics or Mathematics literacy), from Grade 8 to Grade 12 (Machisi, 2023; DBE, 2011). Learners for instance study the same mathematical concepts in the GET (General Education and Training) band particularly in senior phase (Grades

7, 8 & 9). They only differ in their level of complexity. When they get to Grade 10, learners are expected to choose one of the three (3) mathematical components together with the other six subjects. These three components include mathematics, technical mathematics and mathematical literacy (Machisi, 2023; DBE, 2011).

To acquire a national senior certificate (NCS) in Grade 12, Mathematics is one of the four mandatory subjects that learners must have together with one home language, one first additional language and Life Orientation (DBE, 2011).

Mathematics is allocated 4.5 hours of teaching time in the high school curriculum (DBE, 2011, 2011b, 2014). That is undoubtedly amongst the highest teaching times in the entire school curriculum.

As part of mathematics, the topic ‘Algebraic expressions’ is a key component of Algebra, a fundamental aspect of mathematics which concentrates on the manipulation of mathematical symbols to solve expressions, equations and represent relationships. These expressions include mathematical operations, variables and constants, serving as foundational tools for more advanced topics such as equations, functions, and polynomials (Musi, 2022; Essuman et al., 2024).

2.3 Algebra and its Significance in Mathematics

2.3.1 The Concept of Algebra

Algebra can generally be defined as the branch of mathematics that deals with operations, symbols, mathematical structure and numerical relationships (Juraev & Bozorov, 2024; Silverman, 2022). It (algebra) can also be described as the section of mathematics that identifies and generalises patterns (Doğru, 2025). This involves recognising a pattern, articulating it briefly in words, and using symbols to represent it in general terms.

In summary, algebra can be explained as the component of mathematics that operates with symbols and numbers as well as the rules that govern their manipulations. These symbols are called variables and are usually denoted by alphabetical characters.

2.3.2 Importance of Algebra

Algebra remains a crucial aspect of mathematics that is taught in schools to enhance and cultivate mathematical understanding. It (algebra) serves as a bridge between basic arithmetic and advanced mathematics education, encompassing both elementary and college-level concepts and beyond (Compayan & Dollete, 2019; Adu et al., 2015). This suggests that if learners experience difficulties in acquiring algebraic skills during their high school education, their ability to comprehend advanced mathematical concepts in the future is jeopardised.

Algebra provides learners with the required mathematical abilities to excel in different disciplines like Physics, Chemistry, Computer Science, Engineering, Economics, Biology and Data Science (Messer, 2021). Although the concept of algebra is looked at from different perspectives, the general consensus is that it prepares learners for mathematical thinking in every aspect of mathematics and in real life (Aktürk, 2023; Bilbao et al., 2023; Van de Walle et al., 2014). The next section defines *algebraic expressions*, the central aspect of algebra, by clarifying its meaning, context, and scope.

2.4. The Notion of Algebraic Expressions

An algebraic expression, according to Gyampoh et al. (2020), is a mathematical representation made up of variables, numbers and mathematical operations. Similarly, other researchers (Faujia et al., 2024; Kieran, 2018a) define it as a concept under algebra made up of terms which can be variables and constants using different operations. A term in this context refers to a product of factors which can be numerical or in the form of variables (Cox, 2020; Trager, 2022). As a component of an algebraic expression, a coefficient relates to a numerical factor in a term. (Gallian, 2021; Trager, 2022). Variables in algebraic expressions are placeholders which are used to represent unknown quantities. They (the variables) are usually denoted by a lower-case letter such as x, y or a (Kilhamn et al., 2022). A constant in an expression is a value or number that never changes in the expression. It is constantly the same (Kieran, 2018b).

For example, in the algebraic expression: $-4y + 11$, y is the variable, -4 is the coefficient, 11 is the constant while $-4y$ and 11 are called terms. It is also acknowledged that terms that have similar algebraic factors are referred to as like terms, whilst terms with different algebraic terms are unlike terms (Toth, 2021; Makonye & Khanyile, 2015). Algebraic expressions have several representations which are monomial, binomial, trinomial and polynomial expressions (Toth, 2021).

An expression with one term is known as a monomial whereas an expression with two unlike terms is known as a binomial. A trinomial is an expression with three unlike terms. A polynomial on the other hand is an expression in which the exponent is a whole number or a non-zero integer (Courtois & Bard, 2007; Chaika, 2020).

Examples of each type of expression are reflected in the examples below:

- Example of monomial expression: $4xy$; $5x(2y)$; $7x^2$
- Example of binomial expressions: $2x^2 + 6$; $8xy + 3$; $5(x + y) - 3x$
- Example of trinomial expression: $6xy + 14v - 24$; $-3x - 9y + 2$
- Example of polynomial expressions: $ax^3 + by - ax^2 + ca$; $4x^2 + 2x + 6y - 3$.

Also, important to note is that algebraic expressions comprise an integer constant, a variable and the algebraic operation which includes addition, subtraction, multiplication division and an exponent that is a rational number, as reflected in the example: $2x^2 - 6x + 5$.

To ensure that this research remains pertinent to educators and policymakers, it is essential to locate the curriculum, thereby facilitating informed recommendations for improvement.

2.5 Locating Curriculum

Locating curriculum means understanding where something takes place, how it is influenced and experienced by those involved, like policy makers, schools and other communities. The importance of algebraic expressions makes it mandatory to be studied in almost every curriculum worldwide (Edo & Tasik, 2022). In Singapore for instance, an algebraic expression is formally introduced in Grade 6 of primary schools, with special emphasis placed on developing algebraic concepts and algebraic manipulation skills (Ng, 2004). According to Ng (2004), in Singapore, teaching goals that relate to algebra are specified in the curriculum and the teacher's guide and paramount among these goals is to simplify algebraic expressions in one variable that involves addition and subtraction.

Algebra in Ghana is introduced when learners reach junior high level and is continued throughout high school, colleges and then in universities (Adu et al., 2015). Algebra topics that are studied in the Ghanaian mathematics curriculum at the junior high level include algebraic expressions (simplification and factorisation), linear equations, relations, mapping and functions (Adu et al., 2015).

In their study, Fey & Smith (2017) report that in the United States of America (USA), conventional elementary mathematics programmes include almost twenty-four months promoting learners' skills related to the symbolic operations necessary for resolving algebraic problems and creating similar algebraic equations.

Similarly, in investigating the reasons behind the low algebraic achievements of learners in Sweden, Bråting (2019) examined how algebraic understanding is presented in the curriculum used by Swedish schools and the textbooks employed. The authors note that the analytical approach they used was based on the categorisation of "major concepts," which have been shown to be effective in enhancing learners' algebraic knowledge at the elementary school level. The findings suggest that operational reasoning, notations, statements and equations are quite well depicted in both the course and the teaching manuals. Nevertheless, generalised mathematics is underdeveloped in both the curricula and the teaching materials (Bråting et al., 2019).

All the scenarios highlighted, underscore the importance of including algebraic expressions as a topic in the curriculum across the globe and the need to prioritise it.

Curriculum and Assessment Policy Statements (CAPS) is the document that guides curriculum development, teaching, learning and assessment (DBE, 2012) in South Africa. This educational policy document is issued by South Africa's Department of Basic Education (DBE). It also provides a framework for teachers to plan and deliver effective teaching and learning experiences, ensuring consistency and quality across the education system. This document (CAPS) outlines the values, skills, knowledge, attitudes or information which the learners are expected to acquire in each grade and subject (DBE, 2012). Learners in South Africa are initially taught simple Algebra when they are 12-13 years old, at the beginning of Grade 7 (DBE, 2012; Heidari & Asghary, 2019). This phase of learners' educational career is referred to as senior phase comprising Grades 7-9.

The algebra learned in Grades 7,8 and 9 serves as a basis for more advanced algebraic comprehension acquired in Grades 10, 11 and 12, culminating in a highly complex and competitive examination following Grade 12 known as the NSC (National Senior Certificate) or "matric exam". The 9th grade year for a South African student is one of the most important years of their lives since it marks the transition from the General Education and Training (GET) band to the Further Education and Training (FET) band stage (Karimi, 2020).

Furthermore, upon completion of Grade 9, learners will be able to leave the disciplinary school system with a GET credential. They have the option of entering the labour field or further their studies. Because 9th grade is such a vital year for a learner, this research study concentrates on 9th-grade learners (Ehrlich, 2017). The focus on exit level for senior phase helps in evaluating how well learners are prepared for the FET (Grades 10-12) phase.

The curriculum specifications for Grades 7-9 illustrate that algebraic expressions grow from basic to complex concepts. For instance, in Grade 7, learners concentrate on identifying and interpreting relationships and rules represented symbolically. They focus on how to recognise constants and variables in formulae or equations.

Grade 8 learners are expected to use cumulative, associative and distributive laws for rational numbers and laws of exponents to add and subtract like terms in algebraic expressions, multiply and divide integers and monomials, binomials and trinomials. Grade 8 curriculum also focuses on simplifying expressions that involve addition, multiplication and division. They (grade 8 learners) are expected to calculate the squares, square roots cubes and cube roots of monomials (expression with one term).

In Grade 9, the algebraic manipulations done in Grade 8 are extended to incorporate how integers and monomials are multiplied by a polynomial, how polynomials are divided by integers or monomials, and calculate the product of two binomials as well as the square of a binomial. Moreover, Grade 9 learners are expected to factorise expressions that involve common factors, difference of two squares and trinomials (DBE, 2011). This increase in the level of complexity from one grade to the next shows that learners' comprehension of algebraic expressions must be improved at all times (Heidari & Asghary, 2019).

Table 2.1 presents an extract from the CAPS document to show prior knowledge needed from Grade 8 and what is to be taught as new knowledge in Grade 9 according to the South African curriculum.

Table 2.1

Connection between Prior Knowledge from Grade 8 and New Concept to be Taught in Grade 9

	Key concepts	

PRIOR KNOWLEDGE		**Revise the following done in Grade 8:** -- recognise and identify conventions for writing algebraic expressions -- identify and classify like and unlike terms in algebraic expressions -- recognise and identify coefficients and exponents in algebraic expressions
NEW CONCEPTS	**Algebraic language**	Recognise and differentiate between monomials, binomials and trinomials
Revision	**Expand and simplify algebraic expressions**	**Revision** add and subtract like terms in algebraic expressions multiply integers and monomials by divide the following by integers or monomials simplify algebraic expressions involving the above operations Determine the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms Determine the numerical value of algebraic expressions by substitution

	Expand and simplify algebraic expressions	Extend the above algebraic manipulations to include: -- Multiply integers and monomials by polynomials -- Divide polynomials by integers or monomials -- The product of two binomials -- The square of a binomial
	Factorise algebraic expressions	Factorise algebraic expressions that involve: -- common factors -- difference of two squares -- trinomials of the form: ♦♦ $x^2 + bx + c$ ♦♦ $ax^2 + bx + c$, where *a* is a common factor. Simplify algebraic expressions that involve the above factorisation processes. • Simplify algebraic fractions using factorisation.

Source: DBE (2012, pp. 23-24)

Figure 2.1 also shows how the concept of algebraic expression in the South African curriculum becomes a transition process where learners learn from simple to complex and how new concepts are built on prior knowledge.

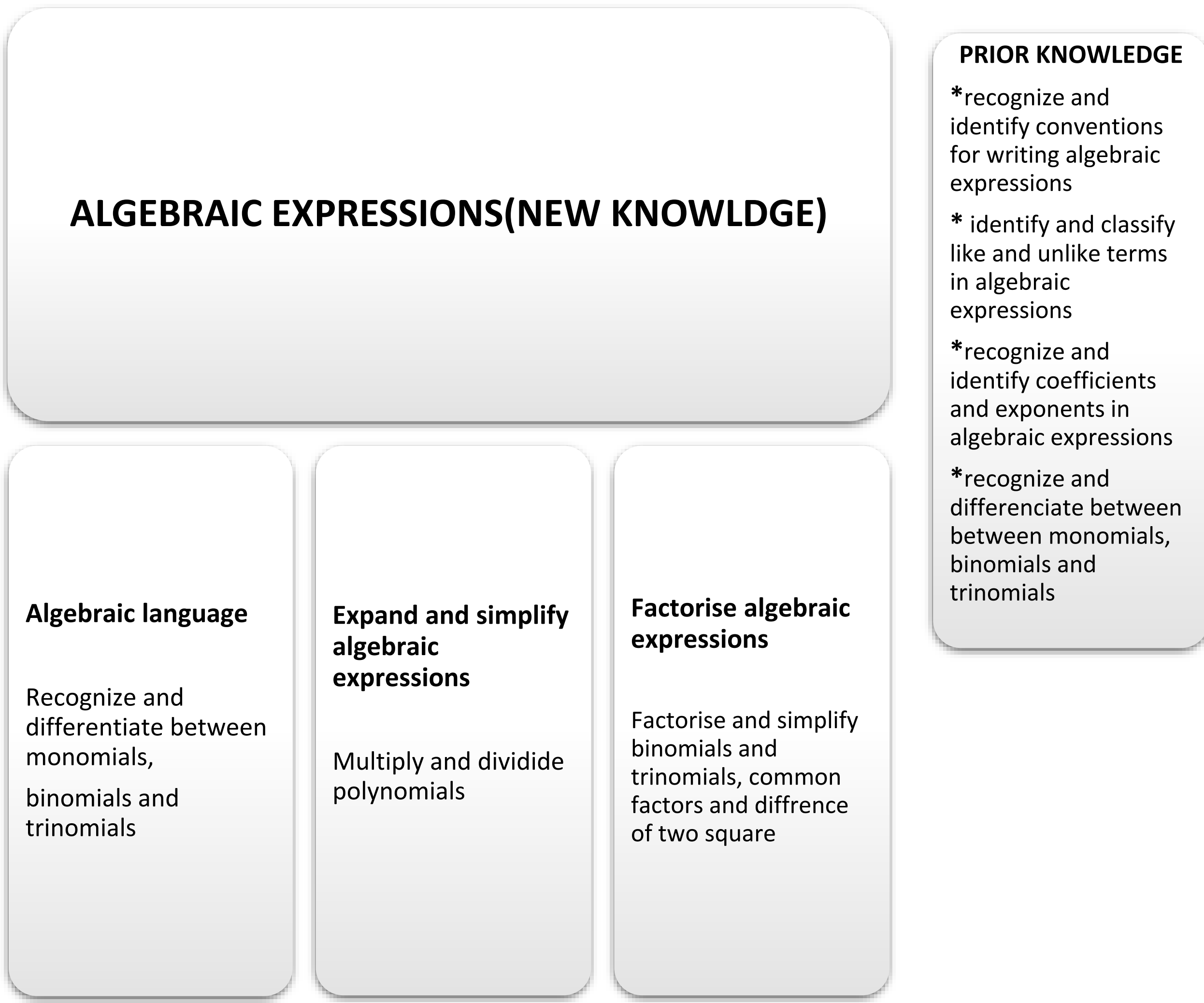


Figure 2.1

Transition Process of Algebraic Expression in South African Curriculum

Source: DBE (2012, p. 23–24)

2.6 General Learner Performance in Algebraic Expressions

Powell (2019) identified that learners often face proficiency problems when dealing with algebraic expressions in the GET phase and at college level as well. The research was conducted to assess the algebraic skills among 362 college students. The participants were also exposed to measures of fact fluency in mathematics, rational numbers, spelling accuracy, reading fluency, and mathematics anxiety. The findings consequently revealed that instruction in rational numbers is imperative to enhance the algebraic skills among the students.

The Annual National Assessment 2014 diagnostic report by DBE (2015) described the low Mathematics performance of learners graphically as shown in Figure 2.2.

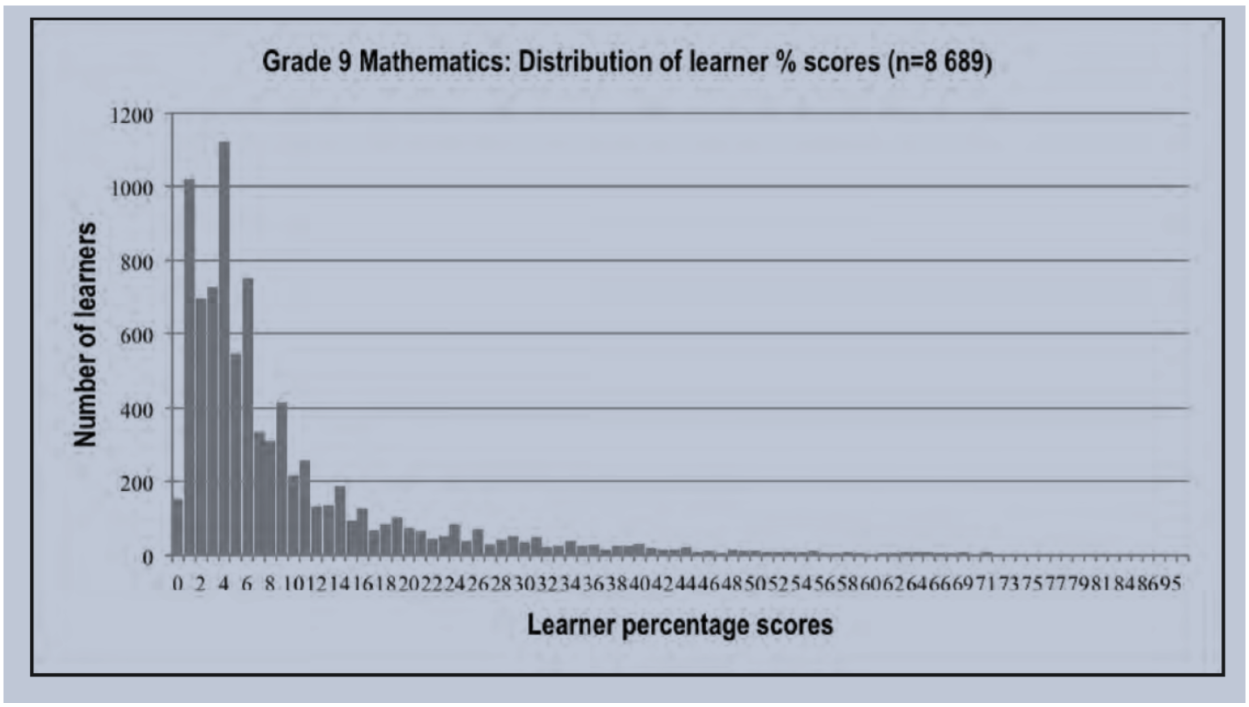


Figure 2.2

Distribution of Learner Mathematics Percentage Score

Source: (DBE, 2015, p. 53)

It can be observed from the figure that, the score most frequently attained by learners was 4%. Moreover, the distribution of learners' scores in Mathematics progressively shifted towards the lower end of the scale (DBE, 2015).

According to the report, paramount among the specific areas of weakness included algebraic expressions, specifically simplification, factorisation and product of binomials for which learners require urgent support and remediation (DBE, 2015). The performance in algebraic calculations according to the DBE (2015) report also shows that fundamental aspects of Algebra such as terminology and basic operations (addition/subtraction and multiplication/division) were not adequately understood by learners.

For instance, in Figure 2.3, learners were to simplify the algebraic fractions $\frac{x^2-4x}{x^2-2x-8}$

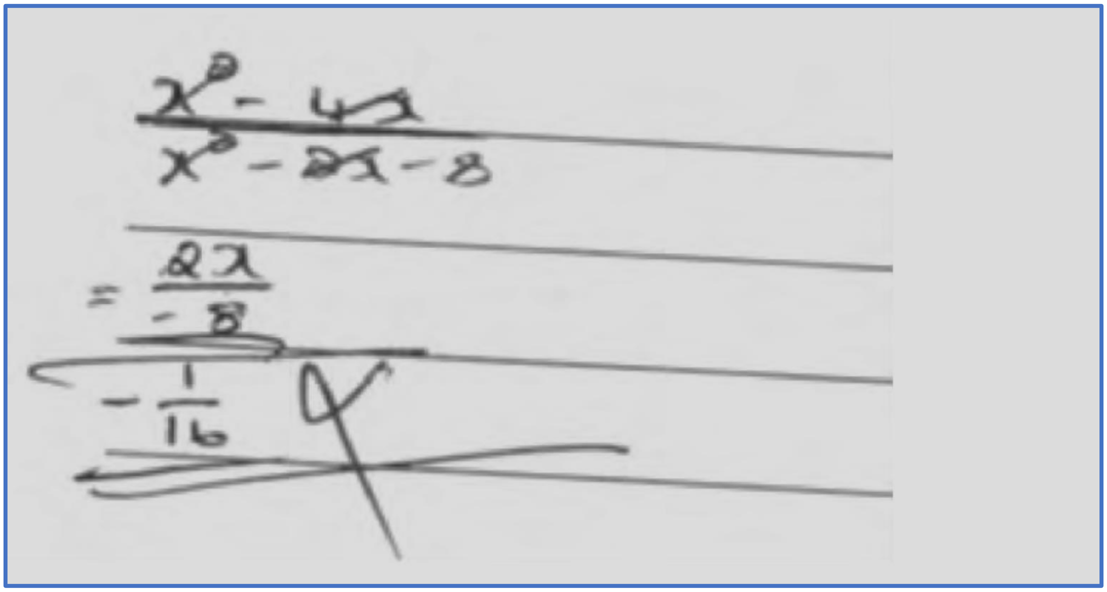


Figure 2.3

Learner's Solution to a Simplification Question

Source: DBE (2015, p. 58).

The primary skill learners needed to utilise in the above question was factorisation of the numerator by taking out common factors and by factorising the trinomial in the denominator. As shown in Figure 2.3, the learner applied an incorrect mathematical procedure by dividing x^2 in the numerator by x^2 in the denominator as well as dividing $-4x$ by $-2x$. From the error displayed, it can be concluded that the learner lacked the understanding of the difference between a term in an expression and its factors.

The next example required learners to factorise the difference of two squares. However, a significant percentage of learners did not seem to understand the concept of factors, as shown in Figure 2.4 (DBE, 2015).

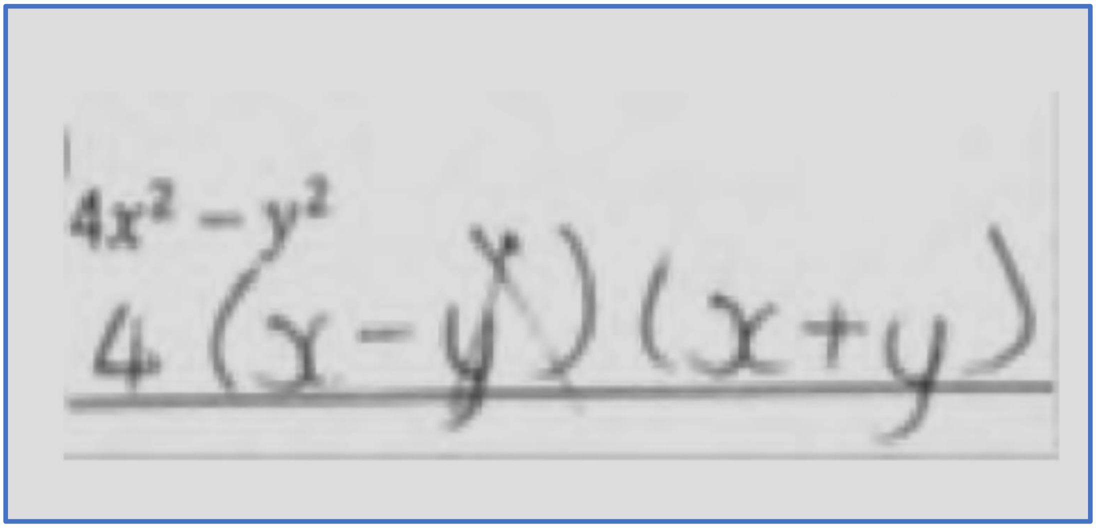


Figure 2.4

Learner's Solution to Factorisation Question

Source: DBE (2015. p. 59).

According to the report, the common error among learners was that they wrongly took out 4 as a common factor and did not realise that 4 is a square number (DBE, 2015). Furthermore, the leaner did not notice that this is the difference of two squares.

The Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ) highlights the understanding of analysis which should be inculcated in learners to obtain higher results in their academic performance (DBE, 2017). According to the SACMEQ (IV) report, there was a notable progression in learners' mathematics achievement at the GET level (DBE, 2017). As highlighted in the study, learners scored 552 points in Mathematics. The result reflected a 57-point rise in Mathematics compared to the previous research results. South Africa, according to DBE (2017), had the highest improvement rates in reading and mathematics among the 13 nations that participated in the study. However, there is more room for improvement. According to the study, there was a considerable decrease in the percentage of non-numerate learners at the GET level, demonstrating that acquiring the foundational skills of numeracy is a crucial goal that demands special focus in our classrooms (DBE, 2017). The study further asserts that more improvement is possible if learners are exposed to mathematical questions with higher cognitive demands.

Pournara (2020) asserted that learners' achievement in mathematics is low in South African education institutions which have few resources. Aside from their exceptional Grade 12 scores, very little is understood about the performance of learners in high-performing, well-facilitated secondary institutions. Pournara (2020) encouraged teachers in Grades 8 and 9 to give particular attention to all four basic operations which are addition subtraction, multiplication, and division of integers and algebraic expressions. The outcome of the research also mentioned that although learners in Quintile 5 scored better in the examinations, the learner achievement trends in the examinations were identical between learners from both the highest quintile institutions and those from lower quintile institutions, such as 1 and 2. The report further indicates that the capacity of Quintile 5 learners to accurately calculate linear equations is significantly influenced by their difficulty in reducing two algebraic variables to a single term, especially when solving problems involving subtraction.

It should be noted that the highest quintile institutions (4 & 5) are schools that serve the least poor communities whereas lower quintile schools (1, 2 & 3) are schools serving the poorer communities which are declared no-fee schools (Chanee, 2020; Akombelwa, 2018).

Figure 2.5 illustrates the response given by a learner on an algebraic expressions question 33(a) and 33(b) during the TIMSS examination (Mosimege, et al., 2017. p. 47). According to the TIMSS report, only 37.3% of learners answered 33(a) correctly whereas only 13.9% of learners were able to answer 33(b) correctly (see Figure 2.5). Incorrect answers for these particular questions counted 47.0% and 68.2%, respectively.

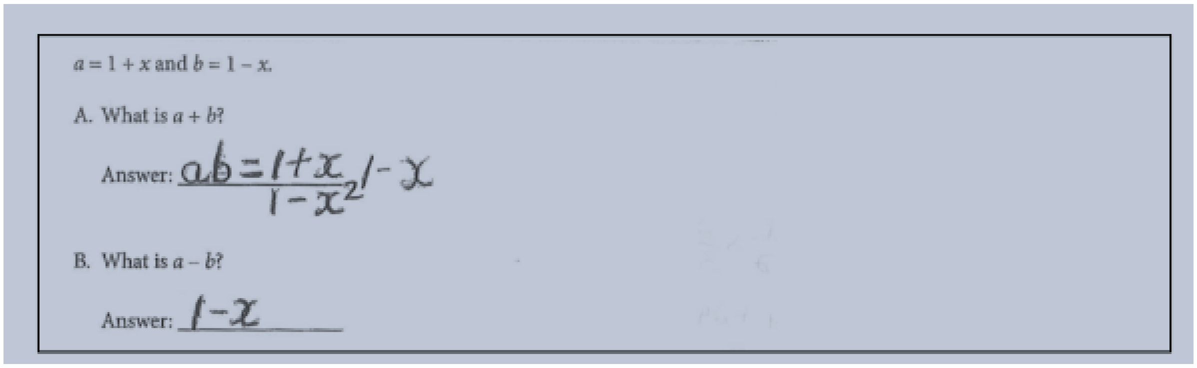


Figure 2.5

Learner's Response to Algebra Question 33(a) and 33(b) of TIMSS 2015

Source: Mosimege et al. (2017, p. 47)

From Figure 2.5, it can be seen that the learner had a challenge with substitution and distribution law. Considering these findings, I deemed it necessary to investigate the understanding difficulties of learners with regards to algebraic expressions.

2.7 Challenges Learners Experience when Solving Algebraic Expression Problems

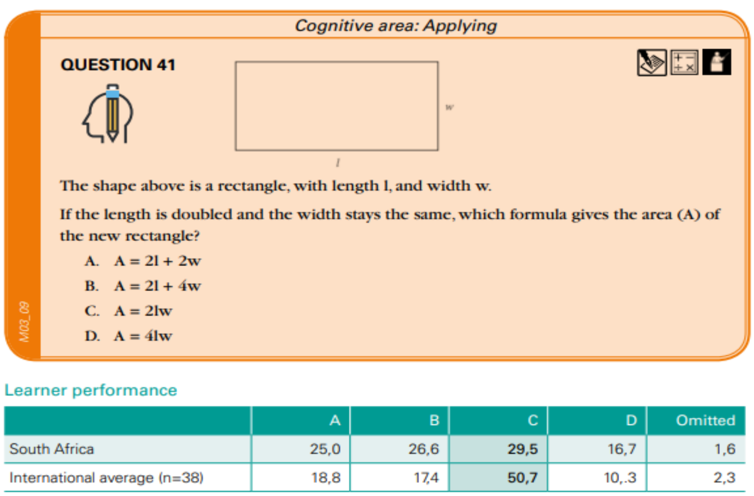
The next section discusses the various forms of challenges learners face when dealing with algebraic expression problems as follows:

2.7.1 Misinterpretation of Algebraic Concept and Language

The challenges faced by learners are more obvious when solving a problem that expects them to translate a word problem into a mathematical statement (Ying et al., 2020; Kenney & Ntow, 2024).

A study conducted by Moleko (2018) suggested that learners should have the ability to read and understand word problems before they can successfully solve them. Nonetheless, several researchers (Obina et al., 2025; Rachmawati et al., 2023) realised that learners encounter difficulties not understanding mathematics word problems when they read, and this makes it even more challenging for them to represent these word problems algebraically.

In agreement with this, Musi (2022) affirms that learners who find it difficult to read and understand the language of 'learning and teaching' face challenges that make it even more difficult for them to correctly solve a given word algebraic expression problem successfully. For example, learners may represent a word problem, such as three less than a number as $3 - x$ simply because the expression 'less than' follows three in the word representation (Musi, 2022). In other words, the learner might not understand that the expression represents subtracting a certain number from 3. This challenge further suggests that connecting abstract symbols and procedures (like equations, expressions or operations) to meaningful interpretations or practical use is key. Additionally, Mosimege et al. (2017) reveal how South African learners struggle with algebraic terms and language as shown in Figure 2.6 from the 2015 TIMSS items diagnostic report (Question 41, p. 54).



	A	B	C	D	Omitted
South Africa	25,0	26,6	**29,5**	16,7	1,6
International average (n=38)	18,8	17,4	**50,7**	10,.3	2,3

Figure 2.6

Percentage Distribution of how Learners Answered Question 41 of the TIMSS report

Source: Mosimege et al. (2017, p. 54)

As shown in Figure 2.6, only 29.5% learners had the answer correct as compared to the international average of 50.7%. According to the report, South African learners battled with understanding the conceptual difference between the terms “perimeter” and “area” which might have been worsened by using unfamiliar language or term such as “width” instead of the familiar word “breadth” (Mosimege et al., 2017). In this problem, even though the dimensions are given as variables, learners still need to understand and apply the formula for calculating area. The report emphasised the importance of exposing learners to different mathematical terms with the same or similar meanings during instruction. This can be done through practical and drilling exercises and should also place emphasis on the understanding of formulas.

Moleko and Mosimege (2021) similarly reveal that some of the major factors affecting learners’ ability to solve algebraic problems include clarity of concepts, text comprehension and terminology, mathematical language use, vocabulary and the structure of the word problems. For instance, when writing an algebraic expression representing “a product of two consecutive numbers”, learners write xy instead of “$x(x+1)$ or $(x-1)x$” and the reason behind this is because x and y are consecutive in alphabetical order. However, that does not make them the consecutive numbers (Musi 2022).

Another challenge in algebra, as highlighted by scholars such as Joseph (2020) and Mudaly and Narriadoo (2023), is the issue of vocabulary and algebraic language. Learners’ inability to understand algebraic terms such as constant, coefficient, variable, degree, expand, simplify, factorise and the like also influence their poor performance in algebraic expressions (Samuel et al., 2016; Ying et al., 2020). Thus, this suggests the importance of being sensitive to issues related to mathematical language and learners’ increasing ability to express themselves using it. It is crucial that learners understand and master mathematical language to perform well in this subject (Joseph, 2020). Because learners continue to struggle with the terminology used in mathematics, teachers are encouraged to devote time to creating strategies to address the vocabulary problem. This is because most teachers do not make the effort to become proficient in mathematical language themselves (Inganah et al., 2023).

Similarly, Baidoo (2017) conducted research to examine the performance of Grade 10 learners in algebraic fractions and analysed the mistakes they generally make while simplifying these fractions. The study found that most of the participants did not understand the meaning of the term ‘simplify’, stating that their knowledge of essential algebraic simplification and expressions was weak.

2.7.2 Inability to Analyse and Manipulate Algebraic Expressions

A different challenge identified with regards to solving algebraic expression problems is the learners’ inability to unravel the order in which algebraic expressions must be understood and manipulated with the order of natural language. Musi (2022) referred to it as “the parsing obstacle.” For instance, when given the expression $\mathbf{8 - 3y}$, learners may read from left to right as $\mathbf{8 - 3}$ giving **5** and consider the full expression to be equivalent to $5y$. When given $\mathbf{y + 6}$, learners may read it as y and 6, and interpret this as $6y$ (Musi, 2022). This suggests that learners should engage in activities that focus more on understanding like and unlike terms. The inability of learners to successfully manipulate algebraic expressions also manifests when dealing with expressions that involve the negative sign (Booth et al., (2017). According to Yasseen et al. (2020), individuals who don’t fully or accurately understand the minus sign are more likely to rely on improper methods when solving algebraic expression problems. Because negative is abstract, it is particularly challenging for learners transitioning from arithmetic to algebraic reasoning. Most learners typically associate the negative sign exclusively with the basic process of subtracting (Yasseen et al., 2020).

2.7.3 Lack of Structural Comprehension

In the study by Wang (2015), 29 papers from the ERIC (Education Resources Information Center) library were organised into a taxonomy derived from the research journals which were grouped into five classifications: algebra material, cognition barrier, teaching concerns, learning concerns, and transitional information. The study's findings revealed that learners lacked a structural comprehension of algebraic expressions. For instance, they divided algebraic formulas into different component parts. However, more frequently than not, learners' operations of simplification appeared to be extremely arbitrary. For example, $4(6x - 3y) + 5x$ was reduced as $4(6x - 3y + 5x)$ and at times it was often misinterpreted as $4(6x + 5x) - 3y$. The findings from Wang (2015) also indicated that learners are often challenged and confused about brackets when solving algebraic expression problems.

Das (2020) conducted a study aimed at determining the error computation in mathematics problem-solving involving brackets and identifying misconceptions about bracket usage in arithmetic. The study revealed that learners are mostly unaware that brackets can be used as symbols for grouping terms and also as a multiplier operator in additive situations. Moreover, most students do not understand expressions that involve brackets and often struggle to solve them from left to right (Das, 2020). The study finally indicated that if a bracket is not used properly, the value of the mathematical result is completely changed (Das, 2020).

2.7.4 Inadequate Knowledge of the Concept of Numbers

According to Reeder (2017), Algebra serves as a foundational concept in high school mathematics that facilitates learners' advancement into STEM careers. The study further contends that learners encounter numerous problems related to algebra, which are rooted in a complex array of issues. According to Reeder (2017), the notable challenge experienced while learning algebra was an inadequate understanding of rational numbers. This challenge adversely affects the competence, efficiency and eventually the achievement of learners in algebra. Therefore, acknowledging and understanding the challenges encountered by learners necessitates attentive support from adequately trained teachers possessing both pedagogical and content expertise, which ultimately influences learners' success in algebraic concepts (Osei & Agyei, 2024a; Oppong et al., 2024).

Similarly, Samuel et al. (2016) conducted research to investigate the difficulties learners encounter when working with algebraic equations. The difficulties were examined using three criteria: manipulation of algebraic expression, formulation of algebraic equations and translation of word problems into algebraic symbols and variables. According to the study, most learners were unable to solve algebraic equation problems because they lacked the fundamental knowledge. Secondly, formulating algebraic equation from a given situation was a real challenge for learners because they lacked adequate mastery of algebraic language (Samuel et al., 2016).

2.7.5 Lack of Understanding in Algebraic Simplification

Most learners usually misinterpret the term "simplify" as to end up with a single term (Makonye & Stepwell, 2016; Pournara et. al, 2016; Moru & Mathunya, 2022). For instance, the expression $5a + 4b$ is often simplified as $9ab$. This usually happens because most learners think terms can be added or subtracted regardless of different variables.

When viewed as a mathematical notation, algebra requires learners to be utterly fluent in the language. This implies that understanding algebra involves the capacity to read, write, and manipulate integers as well as symbolic analogies in formulae, statements and algorithms (Mphuthi & Machaba, 2018). Furthermore, knowledge of algebra is linked to the ability to fully understand algebraic patterns, equations and formulae, which is sometimes alluded to as symbolic awareness (Mphuthi & Machaba, 2018).

In simplifying algebraic expressions, learners at times are not able to recognise the like terms in an expression especially when there is an exchange in the order of the variables, for instance, ab and ba (Moru & Mathunya, 2022). This occurs when learners develop a pattern for identifying like terms based on how variables are presented in an expression. It usually creates a challenge for such learners to restructure the pattern to recognise the like terms with interchangeable positions of the variables (Moru & Mathunya, 2022). It also indicates that when learners are solving algebraic expression problems, they mostly do not deeply understand the concept of commutative property of multiplication (Moru &

Mathunya, 2022). This is because mathematically, “$a \times b$" and "$b \times a$" both give the same result because $ab = ba$ irrespective of the relative positions of the variables.

2.7.6 Failure to Identify and Connect Algebra in Everyday Lives

Learners’ inability to relate algebraic learning to real-life situations is another challenge that often hinders their understanding of algebraic expressions (Azumah, 2023). Research was conducted by Utami et al. (2017) to examine the challenges learners in junior high school experience when solving algebraic conceptual problems requiring a skills level of creative thinking. The study discovered that although some learners successfully performed the fundamental operations of the concept examined, they were unable to recognise similar scenarios in their daily surroundings. Consequently, those learners found it challenging to perform the required analysis needed at the highest level.

2.7.7 Lack of Understanding of Negative Numbers and Terms

According to Sarımanoğlu (2019), when solving algebraic expression problems, learners often disregard the negative sign particularly during the manipulation of the expression. Additionally, they (the learners) do not realise that brackets can be used for both grouping and signal multiplication. For instance (20 – 7) = 13 and – (20 – 7) = -13 (Booth et al., 2017).

Pournara et al. (2016) also contend, in similar research, that learners often neglect to pay close attention to mathematical operations (both negative and positive) when solving algebraic expression problems. The research also disclosed that learners recognised the negative sign as an operation that represents subtraction only. This means that when learners are solving algebraic problems, they do not always take into account the signs that classify the terms as positive or negative but only consider the signs between the terms.

2.8 The Concept of Understanding

The inability of learners to grasp basic mathematical concepts is a significant problem often encountered in mathematics as a subject. Lack of learners' ability to grasp principles is a key issue that is frequently encountered in arithmetic (Teguh et al., 2021). As a result, it is imperative to unpack the meaning of *understanding*. This is because learners are expected to learn with understanding. Understanding is pivotal in this study. Alfiyansah et al., (2020) explained understanding as a dissociative construct in which a person may use notions to symbolise a separate or physical thing, including a person, scenario, or knowledge.

Al-Mutawah et al. (2019) asserted that the key indicator of understanding is the ability of learners to identify, label and create instances of concepts, apply principles and integrate related concepts. Understanding is described as possessing the necessary skills and expertise in connection to a range of evidence to enable intelligent behaviour (Collins & Evans, 2019). All these definitions seem to suggest that understanding is much more than a mere collection of facts, information and analytics.

One trademark of mathematical comprehension or knowledge is a learner's foundation for understanding why a particular arithmetic solution is correct or why a quantitative principle makes sense (Arifin & Herman, 2018). In other words, a learner's ability to explain why a certain mathematical answer is valid or why a mathematical rule makes sense is one of the hallmarks of mathematical comprehension.

Arifin and Herman (2018), discovered in their study that in order to handle more challenging issues and expand their intelligence in the actual world, learners must first comprehend the principles of mathematics. In other words, learners would need to know the basic concepts of mathematics to equip them to understand complex concepts.

Understanding the concept of algebraic expressions means that a learner can interpret, exemplify and classify expressions (Dwijayanti, 2023). With regards to interpretation, a learner who understands the concept of algebraic expression has the ability to change a sentence (word problem) into a mathematical expression. For example: John puts books in 2 large boxes and 3 small boxes. This can be represented as $2x + 3y$ as an expression. Understanding algebraic expressions by exemplifying involves the ability of a learner to provide examples of the various types of algebraic expressions. For instance, a learner will be able to state examples of monomials (expressions with one term), binomials (expressions with two terms), and trinomials (expressions with three terms). Lastly, understanding algebraic expressions means a learner can group or separate algebraic expressions from other expressions with reasons (Dwijayanti, 2023). Nevertheless, learners require the related comprehension of algebraic expressions that utilise abstract concepts including alphabetical characters to hold places for unknown numbers, to hold any value or to represent unknowns. This understanding has been found to be beneficial not only in developing mathematical skills, but also in solving real-world business issues, economic history, pharmaceuticals, scientific knowledge, technology, and all other facets of everyday life (Makonye & Stepwell, 2016).

The next section discusses learners' understanding of algebraic expressions, including their prior knowledge and reasoning. This is significant in relation to the study's objectives and will provide insight into how learners construct their understanding of the concept.

2.9 Learners' Understanding of Solving Algebraic Expression Problems

Several studies (Verschaffel, 2024; Newton et al., 2020) maintain that a critical learning outcome in solving algebraic problems is the acquisition and flexible application of multiple strategies across various circumstances. Edo and Tasik (2022) asserted that conceptual understanding of problem features is one of the most crucial types of prior knowledge needed for solving algebraic problems. These include variables, terms, and negative signs.

Furthermore, Goldin (2020) noted that learners employ representations to support their understanding of mathematics by transforming abstract notions such as algebraic concepts into tangible ideas through the application of logical reasoning. Mathematical representations can be illustrated in two forms: visual and non-visual. Visual representation of mathematics includes tables, charts, and diagrams, whereas non-visual representation encompasses mathematical equations and models (Ratumanan et al., 2022; Goldin, 2020). This implies that mathematical concepts, such as algebraic expressions, should be represented in their simplest form to help learners understand and use them effectively. This is because representation functions as a medium or tool for conveying mathematics concepts in relation to the process of solving mathematical problems (Mainali, 2021).

2.10 The Concept of Mathematical Connection

Understanding, on the other hand, could be defined from the perspective of how learners make connections when learning or solving mathematical problems. This understanding refers to a learner's ability to grasp not only isolated facts or procedures but also the relationships among mathematical concepts. In relation to connection, understanding involves recognising and integrating these relationships to form a coherent and meaningful structure of knowledge (Rodríguez-Nieto & Alsina, 2022; Darling-Hammond et al., 2024). Mathematical connection has been defined by many scholars based on their respective fields of study. Pambudi et al. (2020) for instance, defined mathematical connection as the ability of learners to associate their existing knowledge with the situation they face. Mathematical connection according to Kenedi et al. (2019), is part of a network of interconnected knowledge made of critical concepts to help understand and establish relationships between mathematical ideas, concepts, procedures and the real world. These authors argue that mathematical connection capability manifests when a learner can link one concept to another and should be developed as early as the elementary age. Hasbi (2024) describes mathematical connection as the ability to identify, utilise and understand mathematical concepts, establish links between ideas to form a comprehensive relationship and apply mathematics to non-mathematical situations.

Mathematical connection can be classified into two broad categories: internal and external mathematical connection (Siregar & Surya, 2017; García-García, 2024). Other researchers (Gamboa et al., 2021) use the term 'intra' and 'extra' mathematical connections. Internal or intra-mathematical connections include connections between mathematics topics or concepts while external or extra- mathematical connection involves the connection with other subjects and everyday life (Siregar & Surya, 2017; Gamboa *et al.*, 2021; García-García, 2024).

In summary, mathematical connection can be defined as the relation or linkage between mathematical concepts, between mathematics and other fields of study as well as the relationship with everyday life. It can also be defined in the context of this study as the ability of learners to relate or connect algebraic expressions to prior knowledge, other subjects and everyday life (Siregar & Surya, 2017; Kenedi et al., 2019; Pambudi et al. 2020; Jung et al., 2024). For example, the ability to correctly simplify the expression $(4x + 5)(2x - 6)$ requires the application of concepts such as exponents, distributive property and integers to successfully expand the expression, group the terms and do the calculations to complete the simplification.

The following sections discuss further issues related to mathematical connections, drawing on relevant research and theoretical perspectives to highlight both their importance and learners' poor ability to make such connections.

2.10.1 Importance of Mathematical Connections

The ability to connect mathematically is one of the most essential skills a learner can have (Arifin, 2024; NCTM, 2014). This is because good mathematical connection ability helps learners accurately solve mathematical problems (Altay et al., 2017; Pambudi et al., 2020). It is therefore not surprising that more emphasis has been placed on mathematical connections in the national curricula in numerous countries including the United States of America (NCTM, 2014), Australia (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2018) and Singapore (Ministry of Education [MOE], 2018).

The ability of learners to recognise the relationship between mathematical concepts and connect them easily enables them to understand mathematics itself and opens up opportunities for these learners to develop mathematical skills, ultimately giving them greater mathematical power (Kenedi et al., 2019). According to Arifin (2024), learners usually change their perception about mathematics after developing a mathematical connections ability. They then see mathematics as an integrated whole, rather than an isolated body of knowledge where concepts are divided.

Siregar and Surya (2017) also emphasised the importance of learners possessing mathematical connection ability, asserting that it enhances learners' cognitive skills, helps them understand and remember mathematical concepts, and apply them in daily life. Additionally, García-García and Dolores-Flores (2021) argue that mathematical connection skills are highly needed by learners, especially when solving problems that require relationships between mathematical concepts and other concepts in mathematics, as well as between mathematics and other disciplines or in real-life situations. García-García (2024) supports this statement by revealing that when learners establish strong connections between mathematical concepts, they are well equipped to transfer their knowledge to new situations, apply mathematical principles in real-world contexts, and solve more challenging problems.

According to García-García (2024), there is a profound relationship between mathematical connections and mathematical understanding because mathematical connections serve as the framework on which mathematical understanding is built. Consistent with this assertion, researchers (García-García & Dolores-Flores, 2021; Mason et al. 2010) also affirmed that making connections goes beyond procedural knowledge and improves critical thinking, conceptual understanding and problem-solving.

2.10.2 Low Ability in Mathematical Connection amongst Learners

Despite all the above-mentioned importance, mathematical connection ability among learners remains poor in almost every part of the world (OECD, 2019; Saputri & Zulkardi, 2020). A study conducted on geometry in Indonesia for instance, revealed that learners continue to struggle with making connections between two-dimensional and three-dimensional concepts (Hasbi, 2024).

Saminanto and Kartono (2015, as cited in Pambudi et al., 2020) reported that learners' mathematical connection skills in solving daily life problems reached an average score of 2%. Siregar & Surya (2017, as cited in Pambudi et al., 2020) asserted that the mathematical capability of junior high learners to associate ideas in mathematics and the ability to associate mathematics with everyday life problems reached an average of 51.11% and 17.78%, respectively. In the study by Kenedi et al. (2019) where learners were given a task that required them to recognise and utilise the relationship between mathematical concepts, the study revealed that out of the 120 sampled learners, 21 (17.5%) got a score of 3 out of 5, 74 learners (61.7%) scored 2, 21 learners (17.5%) scored 1 while four learners (3.3%) scored zero(0).

Kenedi et al. (2019) recount how learners performed dismally when given mathematics problems that required a connection between mathematics and science. The study revealed that no learner scored higher than 2 out of 5 points. 23 learners (19.2%) scored 2 points. 15 learners (12.5%) scored 1 out of 5 points and 82 (68.3%) of the learners scored zero (0). The study in its conclusion revealed that the ability of learners to connect mathematical materials into another fundamental science as a coherent unity was very weak.

The poor performance of learners in making mathematical connections with everyday life was no exception. Kenedi et al. (2019) for instance, stated the outcome of a task given to learners with regards to connecting mathematical concepts in everyday life as follows: 34 learners representing 28.34% scored 2 out of 5 points; 25% of the learners scored 1 out of 5 points and as many as 56 learners representing 46.67%, scored zero (0).

It should be noted that weak mathematical connection ability in learners have dire consequences for their overall mathematics performance. According to Kartikasari and Widjajanti (2017), weak mathematical connection ability causes lack of meaning and interest for learners to learn mathematics and this has a detrimental effect on both the learning process and outcome of mathematical achievement.

All the mathematical connection problems discussed above indicate that learners struggle to make mathematical connections independently. This statement aligns with Stillman et al. (2020)'s perspective which asserts that learners are not inherently capable of connecting mathematical concepts.

2.10.3 Indicators of Mathematics Connection

Mathematical connection ability of learners in solving problems is measured by three main indicators (Kenedi et al., 2019; Arifin, 2024; Siregar & Surya, 2017).

The first indicator according to Kenedi et al. (2019) and Arifin (2024) is the ability of learners to recognise and understand the relationship between the various concepts in mathematics. The ability to connect mathematically requires learners to be able to identify and use the connection between the ideas in learning mathematics. In the context of this study, mathematical connection ability requires learners to grasp and recognise algebraic expression problems and relate those problems to some known mathematical concepts. In the connection process, learners should be able to understand how newly acquired information is directed to information that has been received earlier. This is because understanding previous mathematical concepts enables learners to learn and master new concepts. For instance, a learner with a mathematical connection ability should be able to relate the concept of perimeter and area in simplifying algebraic expressions.

The second mathematical connection indicator is the ability of learners to understand how the ideas and concepts in mathematics and other disciplines interconnect to produce a coherent unity (Arifin, 2024; Kenedi et al., 2019; Siregar & Surya, 2017). Mathematical connection ability requires learners to be able to recognise the connection between mathematics and other subjects like the sciences.

Mathematics serves as the fundamental language of science and many other disciplines, and vice versa (Demirkuş, 2023). This means that learning mathematics is related to other learning. Therefore, the ability to connect one mathematical concept with a concept beyond mathematics (other subjects) is a proof of strong mathematical connection ability.

For example, the terms ‘mono’ and ‘bi’ are root words in science that represent one and two respectively (Malini et al., 2020), and therefore, a monocot plant refers to flowering plants with seeds that contain only one embryonic leaf (Fu, 2024). In the same vein, ‘mono’ in the world of algebra also represents a single or one, and thus the term monomial represents an algebraic expression with a single or only one term, while binomial represents an expression with two terms (Bolondi et al., 2020; Theba et al., 2024).

The third indicator with regards to mathematical connection is the ability to recognise and apply mathematics in everyday life (Kenedi et al., 2019; Arifin, 2024; Siregar & Surya, 2017). Mathematics is closely related to daily life (Gaurifa & Harefa, 2024) and thus, mathematical concepts not only link to other disciplines but also to real-life situations (Verschaffel et al., 2020).

As one of the unique disciplines, mathematics helps improve thinking ability and most importantly, contributes to daily problem solving and the same in the world of work (Dare et al., 2021). In other words, mathematics connection involves learners’ ability to create association between mathematics and real-life experience. However, some studies have shown that learners' ability to connect mathematical knowledge to real-life situations is very low (Kenedi et al., 2019; Siregar & Surya, 2017).

Mathematical connection in relation to this study involves learners' ability to relate and apply mathematical concepts or principles to practical situations they encounter in everyday life (Banerjee, 2021). These include the ability to relate and solve algebraic tasks that involve word problems. For example, the total measurement and cost of materials needed to fence a rectangular piece of garden, as well as the number of litres and cost of fuel required to travel a specific distance. In this problem for instance, learners are expected to multiply the cost per litre of fuel by the total distance travelled.

2.11 Strategies used by Learners in Solving Algebraic Expressions

The term *strategy* is defined as a plan that comprises a series of activities designed to achieve a specific educational goal (Romiszowski, 2024; Hasbi, 2024).

Yahya and Shahrill (2015) also explain a strategy as a process that involves careful and efficient use of cognitive skills for a particular context or task where individuals engage in activating plans and integrating new content into meaningful knowledge structures. A strategy in the context of this study refers to a plan, an approach or method used to solve an expression. These plans, approaches or methods include specific procedures or steps such as grouping like terms, distributing brackets, factorising or simplifying an expression to arrive at a solution.

Several strategies are employed by learners when solving algebraic expression problems (Hibi & Assadi, 2022; Yahya & Shahrill, 2015). Unfortunately, these thinking strategies by learners are mostly associated with challenges, mistakes and errors. Paramount among these strategies by learners include bracket decoding, changing operations, reduction strategy, trial and error and finding the greatest common denominator, to mention a few.

2.11.1 Bracket Decoding

Bracket decoding is one of the most common strategies normally used by learners when solving algebraic expressions. It (bracket decoding) simply refers to the act or process of solving a mathematical expression which involves brackets by performing the operation within the bracket by first following the order of operations. (Hibi & Assadi, 2022; Osei & Agyei, 2024b).

According to Hibi and Assadi (2022), most learners in Grade 9 struggle with simplifying expressions that involve brackets. This assertion was made as most participants in their study simplified the expression $(a + 5)3$ as $3a + 5$. According to the study, learners during the interview indicated that the distribution is done by multiplying what is outside the bracket which is 3 by the first term a, and then add 3a to the second term in the bracket which is 5. The study concluded that most learners were unable to decode the bracket correctly and moreover, their understanding of the distribution of multiplication was limited and imprecise.

2.11.2 Trial and Error.

Another area that showcased learners' strategies when solving algebraic expression problems is when they use factorisation. According to a study by Yahya and Shahrill (2015), most learners used trial and error when factorising both algebraic and quadratic expressions. The study further revealed that a high percentage of learners could not explain the term 'factorisation' during an interview. Moreover, most of them were unable to relate factorisation to distributive law. The learners attempted to group common terms in brackets but were unable to successfully factorise the given expressions. For instance, learners factorised the expression $mv - mu$ as $m^2(v - u)$ as shown in the Figure 2.7 below.

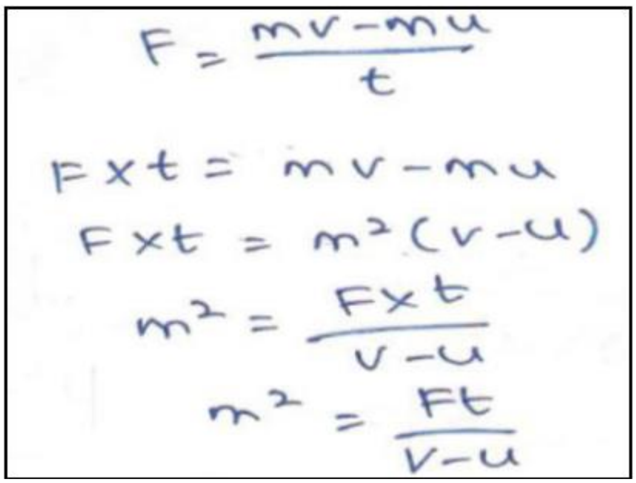


Figure 2.7

A Learner's Solution on Solving Factorisation Question

Source: Yahya & Shahrill (2015 p. 4)

According to the study, learners committed this error because they thought the two $'m'$ s in the question meant that they could use the law of exponents to simplify. Learners were also misled by the rule of cross multiplication. It was also revealed in the study that the few learners who exhibited some conceptual understanding of factorisation only reflected rote learning and lacked relational understanding of algebraic expressions concept.

2.11.3 Reduction

Reduction in algebraic expressions is the act of simplifying an expression to its most basic form and as concise as possible using mathematical rules (Kieran, 2018b; Hibi & Assadi, 2022). This strategy is usually used by learners when simplifying algebraic expressions. However, learners believe that reduction by simplifying involves abbreviating similar expressions without considering the mathematical operations (Strand, 2021; Hibi & Assadi, 2022). In other words, they (learners) believe reduction means to reduce an expression to the smallest number of terms or the complexity of the expression as much as possible. For instance, Hibi and Assadi (2022) revealed in their study that when learners were asked to simplify the expression $\frac{-8+2b}{2}$, they did so by eliminating the number 2 from both the numerator and denominator to get $-8+b$. This was due to the incorrect application of mathematical rule of reducing terms and similar expressions by cancelling the common factors between the numerator and denominator (Hibi & Assadi, 2022; Musi, 2022).

2.11.4 Finding the Greatest Common Denominator

Finding the Greatest Common Denominator (GCD) is a common strategy that learners use when solving algebraic expressions involving fractions (Mbeki & Mbekwa, 2022). This strategy involves factoring both the numerator and denominator of the fraction. Secondly identifying the highest common factor and lastly, simplifying by dividing both the numerator and denominator by that greatest common factor. However, several researchers (Hibi & Assadi, 2022; Baidoo et al., 2020; Mbeki & Mbekwa, 2022) believe that learners have more difficulty finding the GCD when solving algebraic fractions. For instance, instead of learners following the right procedures like the one discussed above, they simply subtract the numerator and denominator from each other and conclude that they (the numerator and denominator) are separate numbers (Hibi & Assadi, 2022).

2.12 Theoretical Framework

2.12.1 Introduction

The previous section provided a critical review of the literature and an analysis of the relevant theory on some essential concepts related to understanding algebraic expressions. This section reviews the theoretical underpinnings associated with the understanding of algebraic expressions. The section discusses Richard Skemp's theory which serves as a guiding framework for this study.

2.12.2 Skemp's Theory of Understanding

The theory of Richard Skemp serves as the guide for this study. According to Skemp (1976) which serves as the main or fundamental source of the theory, understanding a concept entails assimilation into suitable schema, which involves forming a link between widely accepted ideas, facts and procedures. The theory of Skemp asserts that a deepened understanding of a specific concept ultimately enables its application in real life.

From Skemp's point of view, it is usually common and evident to make errors when applying a technique in practice without possessing a thorough understanding of it. This theory reinforces the idea that understanding involves the ability to construct a conceptual structure (also known as schema) to link or connect what is already known with new learning. In other words, mathematics is very broad and involves an extensive hierarchy and thus learners cannot form any particular concept until they have formed all the subsidiary ones upon which it depends.

Skemp (1976) distinctly differentiated between two types of understanding: 'instrumental' and 'relational'. The next section discusses these two types of understanding, their characteristics and indicators.

2.12.2.1 Instrumental understanding

Skemp (1976) briefly explained instrumental understanding as the ability to apply rules without reason. In the context of this study, it refers to the ability to use a memorised procedure or rule to solve an algebraic problem without understanding why the procedure or rule works. The instrumental understanding approach prioritises learners' rule-base strategies that focus on the process of completing tasks (Herheim, 2023). It thus implies that instrumental understanding requires memorisation of formulae, rules or procedures. Moreover, mental structures (schemas) formed by instrumental understanding are difficult to change. For instance, learners who instrumentally understand the concept of average only know the rule or procedure for calculating the basic average of given numbers (Noraini, 2009).

The basis of instrumental approach is characterised by a strong focus on the teacher demonstrating what must be done while learners provide rapid responses. It involves the methodical adherence to rules and procedures over time without apparent reasons or justification (Skemp, 1976; Mellin-Olsen, 1984; Herheim, 2023). These authors looked at the approach from the perspective that it is a type of understanding characterised by advantages such as immediate and more evident rewards (right answers), easily understandable and offers solutions quicker than relational mathematics. Herheim (2023) described it (instrumental understanding) as learning by doing without reflection.

2.12.2.2 Relational understanding

According to Skemp (1976), relational understanding involves the construction of a conceptual framework from which an individual or a learner can generate unlimited strategies from start to finish. It (relational understanding) requires recognising patterns, understanding structures and connecting previous understanding to new concepts (Herheim, 2023). For instance, a learner with relational understanding should be able to identify the various features or characteristics that make an expression monomial, binomial or a trinomial.

Contrary to instrumental, relational understanding is an inter-related understanding that transcends beyond memorisation. The emphasis is on recognising mathematical connections and relationships, understanding the rationale behind rules and procedure, how the rule works and when it makes sense to use them. It also involves understanding the interconnections between mathematical structures and concepts (Herheim, 2023).

This type of understanding suggests that learners' ability to know what should be done, why it should be done and when it should be applied is a clear indication of relational understanding. It entails putting in effort to understand the concept and technique, as well as connecting the new concept with a pre-existing one. In the context of this study, relational comprehension occurs when learners are competent in solving algebraic expression problems by accurately applying the guidelines with the correct processing operations and explaining the reasoning behind their solutions.

A learner with proficient relational understanding, can solve problematic issues with little guidance (Noraini, 2009; Skemp, 2006).

Moreover, the learner can derive specific rules or procedures from a broader mathematical relationship. In other words, such a learner understands the what, the how, and also *the why*.

Skemp (1976) highlighted four (4) important features of relational understanding in mathematics. These include the fact that relational understanding is more flexible and adaptable to new tasks, it is easier to recall, it is more effective as a goal, and lastly, it is easier to structure and extend. Instrumental understanding on the other hand is more short-term, rigid, and context dependent (Herheim, 2023). For instance, learners with instrumental understanding cannot link prior or existing knowledge to a new mathematical concept. To get more insight into Skemp's theory, the study now explores the characteristics of the two types of understanding.

2.12.2.3 Characteristics between relational and instrumental understanding

Herheim (2023), which extends Skemp's theory of understanding in the current context, lists five (5) distinctive but interrelated characteristics between instrumental and relational understanding as shown in Table 2.2 below.

Table 2.2

Characteristics of Skemp's Theory

	INSTRUMENTAL UNDERSTANDING	**RELATIONAL UNDERSTANDING**
1	Learners treat new ideas as disconnected facts and unrelated to previous ideas	Learners relate new ideas to previous ones, and integrate them into conceptual systems
2	Learners memorise information and carry out rules/procedures without focusing on how or why	Learners look for patterns and underlying features to express how and why
3	Learners practise an exercise discourse, accept explanations as given without questioning	Learners examine the logic of explanations critically
4	Learners express explanations rigidly. e.g.: rely on visual prototypes	Leaners express explanations flexibly

5	Learners do not reflect on the processes of developing an understanding	Learners reflect on the processes of developing an understanding

Source: Herheim (2023, p. 393)

According to Herheim (2023), the first characteristic with regards to learners demonstrating instrumental understanding is the fact that they (the learners) treat new ideas as disconnected facts which do not relate to previous ideas. Learners exhibiting Instrumental understanding find it difficult to link previous knowledge to newly constructed mathematical concepts. Learners with instrumental understanding can apply algebraic expression concepts such as adding like terms when coefficients are whole numbers. However, they are not able to relate it to algebraic fractions. Learners with instrumental understanding at this level have the capability to solve algebraic problems correctly. However, they are unable to use mathematical procedures correctly in solving the problem (Permata & Prabawanto, 2019).

Relational understanding, on the other hand, requires learners to relate concepts and integrate them into conceptual systems (Herheim, 2023). According to Sahin et al. (2015), relational understanding of the context of derivatives, for instance, requires that learners understand and make sense of certain connections among derivatives which include the slope of the tangent, the concept of limit and the rate of change. In the context of this study, a relational understanding of the concept of algebraic expression demands making sense of concepts including variables, coefficients, as well as simplification, factorisation and algebraic language in word problems.

Secondly, according to Herheim (2023), learners with instrumental understanding memorise information and execute rules and procedures without focusing on how these procedures and rules are used or why they exist. Although instrumental understanding requires learners to comprehend mathematical concepts by following rules, most learners fail to adhere to the rules of writing in mathematics (Permata & Prabawanto, 2019).

Moreover, learners misrepresent mathematical symbols or signs when solving algebraic problems. For instance, they misrepresent absolute value symbols for brackets and at times, misinterpret brackets for addition especially when expanding an expression. This is particularly significant, as it is through symbols that, a person is able to explain and communicate an idea (Skemp, 1987; Permata & Prabawanto, 2019). Additionally, those who approach mathematical concepts with an instrumental perspective can solve mathematical problems in accordance with a demonstrated procedure without undergoing any manipulation process (Permata & Prabawanto, 2019). For instance, they (the learners) are able to write mathematical expressions or results in the simplest form.

Relational understanding, on the other hand, equips learners to always search for patterns and traits to demonstrate (the how) and express the reason (the why) to justify their answers (Herheim, 2023). This implies that this approach does not only require learners to know related concepts of algebraic expression, but also to be able to explain the role of these concepts in terms of their meaning, why and how they are related to algebraic expressions (Sahin et al., 2015). For instance, learners who understand the concept of algebraic expression relationally can explain how and why the expression $2ab + ba$ is equal to $3ab$.

The third characteristic of instrumental understanding according to Herheim (2023) is the fact that learners accept every answer and explanation without question or doubt. This happens because learners who understand mathematical concepts instrumentally, never find any point of connection with anything they have already learned (Star, 2020).

On the other hand, learners exhibiting relational understanding practise dialogic discourse (Herheim, 2023). This means that learners at this stage can develop their own ideas and share with others, being other learners or teachers. They also have the ability to follow logical reasoning and are able to retrieve organised and relevant knowledge (Star, 2020; Herheim, 2023). It can also be realised when learners feel comfortable and can explain mathematical concepts in their own words and recognise them in different contexts (Pujakusuma & Pramuditya, 2023).

The fourth characteristic exhibited by learners with instrumental understanding is the fact that they often express explanation in a more rigid manner. Rigid explanation relies and depends on rules and is not adaptive to context. The rigidity of an instrumental understanding amongst learners is intense to such an extent that if expressions are not exactly the same, algebraically, they do not recognise them as similar. For instance, learners with instrumental understanding rely on prototype thinking which is a situation where learners have difficulties in making sense of an expression that differs from their prior experience due to an absence of pattern recognition and the inability to connect the concept to their existing knowledge (Herheim, 2023). According to Herheim (2023), learners with relational understanding, however, express explanation flexibly. For learners to gain such a flexible and deep understanding of the concept of algebraic expressions, they should study beyond just the normal, a standard or basic expression. Learners with relational understanding have the ability to solve complex algebraic expression problems rather than simple and standard ones (Herheim, 2023; Watson & Shipman, 2008).

Lastly, a learner with an instrumental understanding according to Herheim (2023), is unable to reflect on the process of building and deepening comprehension. When given a task, learners with this type of understanding only scan through the information without the edge to process and understand it (Abdullah, 2009). Inappropriate application of concept is also one of the characteristics exhibited by leaners with instrumental understanding. Learners with instrumental understanding often apply remembered concepts and procedures blindly with the hope that they will produce the correct answer (Abdullah, 2009; Herheim, 2023, Chang, 2019). However, learners with relational understanding have the ability to reflect on the process of developing an understanding and transform knowledge across tasks and contexts when solving mathematical problems (Abdullah, 2009; Herheim, 2023).

Other researchers (Rachmawati et al., 2021; Maclellan, 2014) who equally strengthened the conceptual foundation of the Skemp's theory of understanding in current context also provided distinguishing features between learners who exhibit instrumental and relational understanding based on their abilities. For instance, Rachmawati et al. (2021) mentioned six (6) characteristics of students with relational understanding.

Firstly, leaners with relational understanding can explain a concept and the reason to justify that concept. Secondly, such learners reflect before they act. Thirdly, they strive to find solutions to the given mathematical problem. Also, they can adapt to any task using prior knowledge. Moreover, such learners strive to understand any mathematical concept they come across. Lastly, learners with relational understanding enjoy doing mathematics problems for their own sake (Rachmawati et al., 2021). On the other hand, although learners with instrumental understanding can provide direct answers to certain mathematical questions, they are unable to explain the reason behind mathematical concepts. They are sometimes unable to make progress when solving mathematical problems. Learners with instrumental understanding mostly memorise formulas and procedures and lastly, they cannot find mathematics fun (Maclellan, 2014; Rachmawati et al., 2021). The next section explains the various pointers or indicators of Skemp's theory.

2.12.3 Indicators of Skemp's Relational Understanding

Several researchers (Rachmawati et al., 2021; Kuncorowati et al., 2017; Herheim, 2023) highlighted the main indicators that can be extracted from Skemp's theory of relational understanding.

The first indicator is the ability to classify objects based on needs that form a concept (Rachmawati et al., 2021; Kuncorowati et al., 2017). In the context of this study, leaners with relational understanding have the ability to classify given terms based on their known characteristics and patterns. For instance, a learner with relational understanding will be able to classify terms into monomial, binomial, trinomial, etc.

The second indicator focuses on the ability of a learner to apply concepts algorithmically (Rachmawati et al., 2021; Kuncorowati et al., 2017). In the context of this study, it involves the ability of a learner to logically apply an algebraic procedure correctly. For instance, in an expression such as $(x + y)(2x + 4)$, learners are expected to first expand by applying the distributive property, group the like terms together and simplify further.

According to Rachmawati et al. (2021), the third indicator of relational understanding is learners' ability to make connections between various mathematical concepts. Learners who demonstrate relational understanding have the ability to connect and merge various mathematical ideas when solving problems that involve algebraic expressions. They achieve this by linking new ideas to previous learned concepts (Herheim, 2023). For example, to calculate the perimeter or area of a rectangle given the dimensions $4x + 6 \text{ and } 7x + 4$ as the length and width respectively, learners with relational understanding are expected to link and incorporate concepts from other content areas such as 2-D shapes (rectangle to be precise) and exponents (DBE, 2012).

The fourth indicator of relational understanding according to Skemp, refers to learner's ability to build the foundational knowledge needed for understanding specific mathematical concepts (Rachmawati et al., 2021; Kuncorowati et al., 2017; Skemp, 1976; Skemp, 2006). Certain mathematical problems require specific prior knowledge or conditions to be effectively solved. For instance, factorising a polynomial such as $x^2 + 6x + 5$ requires learners to specifically identify the factors of the constant that when added will be equal to the coefficient of x, which is 6.

The last indicator in Skemp's relational understanding is the ability of learners to articulate explanations with logical reasoning and flexibility (Herheim, 2023; Rachmawati et al., 2021; Skemp, 1976). A learner with relational understanding has the ability to provide an explanation with good reasons, vividly and logically. For instance, the learner will be able to provide a tangible as justification for their approach to every algebraic problem.

Table 2.3 summarises the pointers or indicators of Skemp's relational understanding as discussed in section 2.12.3 and serves as a model for this study.

Table 2.3

Indicators of Skemp's Theory

Pointers / Indicators	**Description (expectations from learners)**	**Examples**
1. Classify using patterns and features.	Able to accurately classify expressions based on patterns and correct features.	State if an algebraic expression is a monomial, binomial or trinomial
2. Apply algebraic concepts algorithmically	Able to logically and correctly apply mathematical procedures to solve algebraic problem	Solving an expression such as $(3x + y)(2x + 4)$ By first expanding, identifying common terms and then simplify
3. Interconnectedness of concepts	Able to relate and connect prior knowledge to new mathematical concepts and integrate various mathematical concepts	Finding the area or perimeter of a rectangle with both length and width given an expression form.
4.Developed requirement for concept	Able to know or identify the best and necessary procedure to use for a	To factorise a trinomial such as $x^2 + 5x + 6$

	specific mathematics problem.	
5. Express or provide explanations flexibly	Able to freely express themselves given reasons and explanations to defend their solutions to algebraic problems.	Explain with good reason why the expression $4(x+6)$ is a monomial.

Source: Adapted from Herheim (2023)

2.13 Research Gap

There has been evidence from studies that suggest that learning about algebraic expressions can be done at the advanced stages of learning (Baidoo, 2017; Powell et al., 2019). However, I believe that these stages are too late for most learners to benefit from the topic because algebraic expressions concept starts as early as Grade 7 (DBE, 2012). It is imperative to discover problems early rather than to concentrate on the later stages which are of little or no benefit to the learners. It is for this reason that this research focuses on Grade 9 which tends to be a critical grade in a child's educational journey. If learners are well-versed with this aspect of algebra, they will eventually be able to progress to higher grades without difficulty in the future. Learners will be able to grasp concepts of different topics in Mathematics and other integrated subjects such as Science.

Again, as discussed in the first section of this chapter, learners still hold algebraic misconceptions. They are still adding variables with constants, add unlike terms, and factorise expressions wrongly. This means that we need to explore more on how best we can help learners understand the topic.

Moreover, in spite the numerous studies (Mtshali, 2019; Akhtar et al., 2020; Ojaleye & Awofala, 2018) highlighting learners misunderstanding and dismal performance in the domain of algebra, little or none of such study, per my knowledge, has been conducted that aims to comprehend the perspective of learners in grade 9 belonging to selected schools in the Northern Cape province specifically on how they (learners) understand algebraic expressions, the strategies they use and how they relate algebraic expressions to other mathematical concepts. This clearly reveals a crucial gap including the lack of evidence on how learners in this setting (Northern cape) interpret and make sense of algebraic expressions as well as the misconceptions and challenges they may have.

Thus, the current study seeks to bridge this gap by concentrating specifically on the understanding of algebraic expressions amongst Grade 9 learners in the Northern Cape province. This will contribute to the wider field of mathematics education by providing evidence that can enhance teaching and learning strategies as well as curriculum design.

2.14 Conclusion.

This chapter aimed at analysing the literature to provide a clear picture of what it means to understand algebraic expressions. The chapter begins by locating curricula which expands and specify concepts and skills covered under algebraic expressions, subsequently providing a description of the concept of understanding and algebraic expressions as well as their importance. It goes on further to discuss the theoretical framework adopted for the study to offer a lens through which the data and findings on the understanding of algebraic expressions can be analysed and interpreted.

The chapter contributed in addressing the research question “what strategies do grade 9 learners use to solve algebraic expression problems” by discussing some common strategies learners use when solving algebraic expressions including: bracket decoding, reduction, trial and error and FGCD (finding greatest common denominator). This chapter also provided vital information on the research question “how do grade 9 learners make connections when solving problems involving algebraic expressions” by outlining more details on the concept of mathematical connections and its indicators.

Moreover, the chapter played a critical role in addressing the research question “what conceptual gaps manifest when learners solve algebraic expression problems” by discussing about seven (7) challenges learners experience when solving algebraic expression problems which in effect are conceptual gaps including: misinterpretation of algebraic concept and language, lack of structural comprehension and inability to analyse and manipulate algebraic expressions to mention a few. The research methodologies and approaches used in this study are discussed in the next section.

3. CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter describes the journey to fulfilling the research objectives and to answering the research questions. The chapter also details the methodology used for collecting, analysing and interpreting data for this study. Highlighted in this chapter is the research paradigm adopted, including sampling strategy, the research instruments used, the credibility and the ethical issues that emerged when conducting this research. The study aimed to explore Grade 9 learners' understanding of algebraic expressions.

The chapter begins by first, presenting the synopsis of the data collection and analysis methodologies as shown in Figure 3.1.

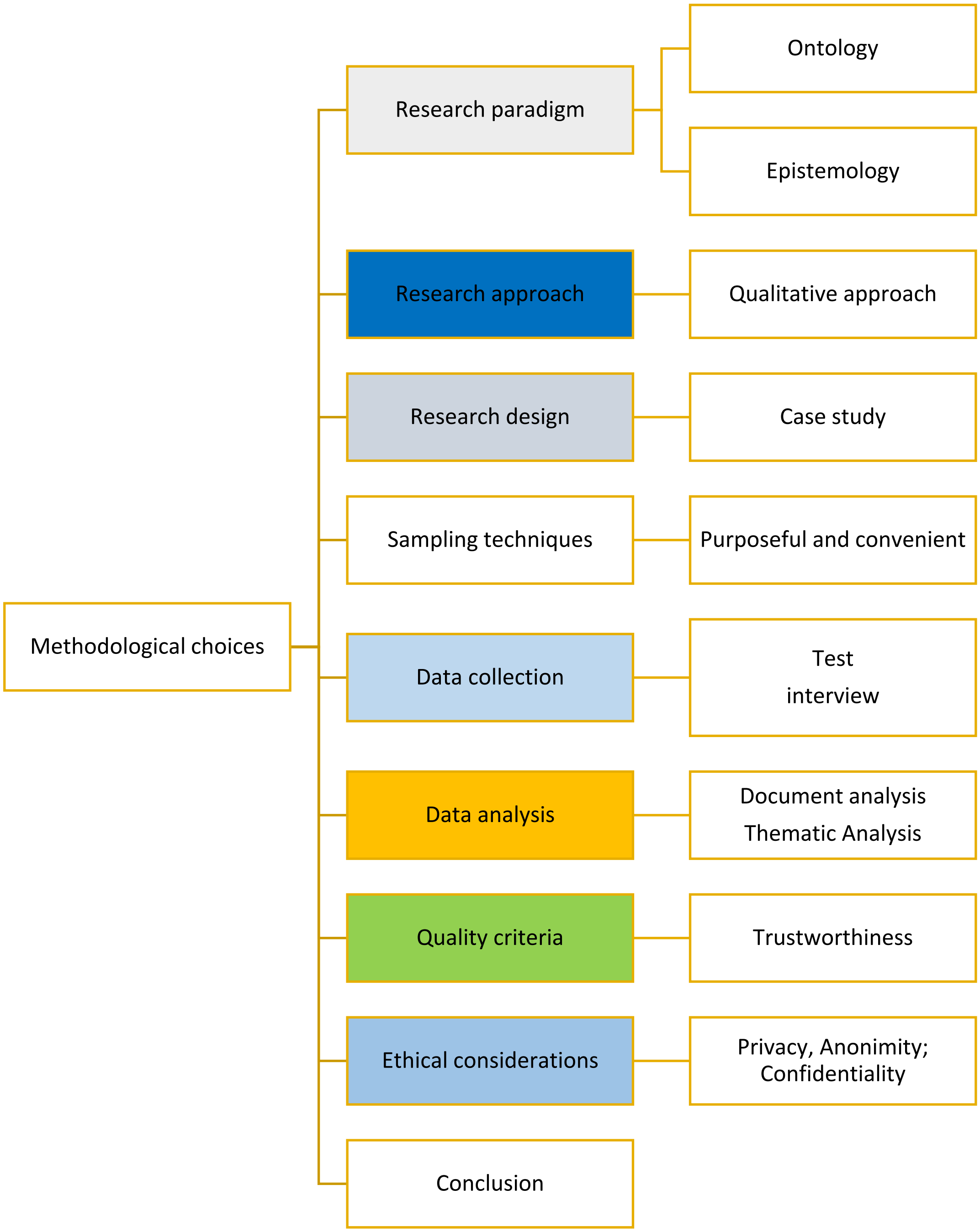


Figure 3.1

An Overview of Chapter 3

Figure 3.1 shows the choices I made regarding how the research should be conducted. Such choices were informed by the objectives of the study and how they impacted the design, procedures for collecting data and the result analysis. Sampling criteria were used to ensure that every learner in the population had equal chance of being selected (Rahman et al., 2022). Most importantly, the welfare of the participants was considered including the credibility (Auspurg & Brüderl, 2024) of the research. The next section presents my worldview and how it provided perspectives on conducting and evaluating research.

3.2 Research Paradigm

Determining the guiding research philosophy was crucial for selecting a suitable research strategy for this study. Research philosophy can be explained as the opinions or views about how empirical data can be collected, reviewed, interpreted and used (Saunders et al., 2015; Saliya, 2023). It also entails the development of knowledge through a well-defined system of beliefs and assumptions, and the assumptions can either be rooted in human knowledge (epistemology), the nature of reality (ontology), or the influence, beliefs and values of the researcher which is known as axiological assumptions (Saunders et al., 2015; Al-Ababneh, 2020; Moroi, 2021). This in turn determines the research strategy to be taken by the researcher and the research methodology to be adopted as well (Moroi, 2021).

The research philosophy chosen dictates the direction of the research when addressing the particular challenge of interest (Cazeaux, 2017**;** Khatri, 2020; Saliya, 2023). The current research was exploratory, and data were analysed qualitatively using figures to represent the thoughts of the research participants. For this reason, the interpretivist paradigm was adopted.

3.2.1 Interpretivist Research Paradigm

An Interpretivist is of the view that learners' understanding is influenced by how individual participants explain their own situations (Kodisang, 2022). In other words, an interpretivist believes that a situation is best analysed based on the participant's experience. Gillani (2021) further explains that in the context of research, since research participants' worldviews personify the truth and meaning of a phenomenon, the truth and reality cannot be separated from them. Put differently, without the qualitative voices of research participants, the actual truth about a phenomenon cannot be deemed complete. These voices cannot be numerically captured or objectively expressed. For these reasons, this study adopted an interpretivist approach to qualitatively understand the exact and true nature of how Grade 9 learners perceived algebraic expressions.

Choosing the interpretivist paradigm for this study was also partly guided by the fact that it incorporates diverse perspectives from individuals and groups in the analysis (Pervin & Mokhtar, 2022). This is because interpretivism presumes that realities are infinite, constructed socially and comprehensive (Moroi, 2021; Creswell & Poth, 2018). It can therefore be concluded that the interpretivist paradigm allows for multiple viewpoints which are useful in enabling the acquisition of insightful and extensive information about the understanding of algebraic expressions in Grade 9.

According to Moroi (2021), the interpretivist paradigm is grounded in three specific assumptions: ontological, epistemological, and methodological. Moroi (2021) explains that while ontology is about reality, epistemology is about how we learn about that reality, and methodology is about the assumptions researchers have when conducting their studies. The subsequent paragraphs explain how the three assumptions are confined to interpretivism and apply to this study.

3.2.2 Ontological Assumptions:

Several scholars (Creswell & Poth, 2018; Moroi, 2021; Khatri, 2020) summarise the definition of *ontology* as the study of reality or existence. In a research study, researchers' presumptions about what makes sense are the focus of ontology. An in-depth understanding of ontology is essential for comprehending how researchers interpret their data in research (Khatri, 2020). The ontological beliefs of a researcher influence how they formulate research questions, recognise their significance and approach data analysis (Khatri, 2020; Moroi, 2021). This study investigated learners' understanding of algebraic expressions. Interviews with the selected learners played a significant role in gaining a sense of reality. This study focused on what it means to understand algebraic expressions. This approach also guided my investigation into how this topic is perceived and understood by learners in different contexts.

3.2.3 Epistemological Assumptions

Khatri, (2020), describes epistemology as the research paradigm that deals with how knowledge is gained from different sources. Similarly, Al-Ababneh (2020) contends that epistemology deals with the nature of knowledge, its possibility, scope and general basis. In a research study, epistemology describes how we come to know something as well as how we know the truth or reality (Khatri, 2020). In simple words, epistemology is concerned with the nature and the scope of knowledge (Moroi, 2021; Ugwu et al., 2021). In this study, I focused on how knowledge about algebraic expressions is acquired, justified and validated by learners. I was also concerned about the nature of the knowledge acquired and how learners apply such knowledge.

3.2.4 Methodological Assumptions

Moroi (2021) explains methodological assumptions as the philosophies that guide data gathering and determine methods in research. This implies that methodological assumptions in a paradigm simply include participants (learners), the instruments used to gather data, and measures for analysing data through which knowledge is gained about the research problem (Khatri, 2020). In other words, the choice of methodology drives and informs the research processes (Kodisang, 2022). In this study, learners' opinions were presented without any omission, and most importantly, I concentrated on what was valuable in the research process.

Qualitative research approach is the methodological choice relevant and used for this current study. The methodological assumptions shaped the design of the study and guided decisions about the data collection process, research data collection instruments and analysis techniques. The assumption in this study was that learners learn through constructive processes (Hite et al., 2024) and that is why I used a test and interviews to understand their thought processes. To be more direct, the current research followed processes to understand how Grade 9 learners comprehend algebraic expressions. In qualitative research, the concepts of credibility, transferability, dependability, and confirmability are the criteria used to assess or evaluate the research (Moroi, 2021). These criteria are further explained in the subsequent sections.

3.3 Research Approach: Empirical Framework

The next section discusses issues related to the research approach used for this study, starting from the research setting. It further discusses the issues related to the population and sampling techniques used. This section also includes an in-depth discussion of the research instrument used for data collection and analysis. The last two sub-sections detail the credibility and ethical concerns of the study.

3.3.1 Research Settings

A research setting is described as the natural environment in which participants are located, where a qualitative researcher conducts research (Creswell, 2003; Lofland et al., 2022). The study was conducted at a public non-fee-paying high school (Grade 8-12) in the Frances Baard district, Northern Cape. The high school has an enrolment of over 1200 learners. Most of these learners get to school on foot while others use public transport funded by the Northern Cape Department of Education (NCDOE). I chose the school because of its easy access and, most importantly, the availability and willingness of the participants, as well as the fact that all the Grade 9 learners are studying mathematics (purposeful sampling).

3.3.2 Qualitative Research Approach

The research approach used for this current study was qualitative. The justification for using a qualitative research approach is thus to provide learners (Grade 9) with an opportunity to explain their understanding of algebraic expressions. Secondly, the qualitative approach is most suitable as data are collected directly from learners in a natural setting (Lofland et al., 2022). Also, the focus of a qualitative research method is the collection of data on naturally occurring events (McMillan & Schumacher, 2010).

Moreover, qualitative research deals more with issues on social phenomena (Usman et al., 2025).

Creswell (2008) describes the qualitative research approach as a tool used by people (individuals and groups) to explore and understand human and social problems. Brace (2018) describes qualitative research as a method of study that presents, analyses and explains text and representations of the raw data collected by a researcher. Qualitative research is subjective in nature (Creswell, 2008; Dehalwar & Sharma, 2024). This is because qualitative research allows the researcher to subjectively collect data, interpret, and analyse findings comprehensively from themes (Kodisang, 2022).

The study adopted qualitative research as the most suitable approach also because it allowed participants (learners) to express themselves as to how they understand algebraic expressions and what they see as the challenge in understanding the concept. This is because qualitative research is participatory in nature which offers significant opportunities for meaningful participant engagement and gives value to what individual participants can communicate by sharing their experience (Stutterheim & Ratcliffe, 2021).

Moreover, the qualitative research approach perfectly harmonised with the research problem, which aimed to investigate not only the understanding of algebraic expressions among grade 9 learners but also, more significantly, how they (learners) interpret algebraic expression concepts and the reason behind the strategies they use when solving algebraic problems. Since the research problem is inherently explanatory, it necessitated an approach that encapsulates learners' experiences, how they reason and make meaning, as opposed to limiting their responses to numerical evaluation or measures. The thematic analysis of the participants' responses, explanations and interactions allowed me to trace patterns in the way they think as well as the misconceptions they had in algebraic expressions.

3.4 Research Design

Kumar (2018) explains research design as a plan of action that a researcher uses to answer research questions and address research objectives in a more valid and accurate manner. It is also a framework by the researcher that reflects the method used in collecting and analysing data (Leavy, 2022). McCombes (2022) further explains research design as the technique a researcher uses to answer research questions using empirical data. This study aimed to explore the understanding of algebraic expressions amongst Grade 9 learners. For this reason, I adopted a case study qualitative research design.

A case study is an extensive investigation of a specific research problem which is carried out examining variables like events, people, choices and other social structures (Muzari et al., 2022). It can further be defined as a practical investigation that examines a phenomenon from a real-life standpoint (Muzari et al., 2022).

Notable features of a research case study approach include the fact that it provides a detailed encounter with the research participants (Hancock et al., 2021). Secondly, a case study possesses openness to multiple sources of data (Urbinati et al., 2020). Thirdly, it employs a limited number of research participants (Schoch, 2020). Additionally, a case study approach can offer practical insights because it enables a thorough investigation using various tools. It was also employed in this study to help identify how learners solve and apply the rules or principles in algebraic expressions. The case study design moreover enabled me to obtain a full grasp of how the participants understand and reason when solving algebraic expressions including word problems (Hancock et al., 2021). The features mentioned above are pivotal to the current study, as I directly engaged with the participants in a school setting.

Interviewing the participants directly also helped in adding voice to facts and figures that show the extent to which learners struggle to understand the concept of algebraic expressions, which justifies the overall rate of students' failure in the province, the district, and at the school in question. Exploration is guided by Skemp's theory of understanding (relational), and data were collected through a learner’s test (worksheet) and one-on-one interviews.

3.5 Research Population and Sampling

3.5.1 Target Population

Casteel and Bridier (2021), briefly define target population as the precise individuals, units, objects or events that will be considered in a research study. It (the target population) consists of every potential case that a study is intended to be based on (Hair et al., (2016). The target population of this study consisted of all Grade 9 learners in Circuit 8 of the Frances Baard District, Northern Cape Province. There were 1085 Grade 9 learners in the circuit. However, conducting research of this size was not feasible for several reasons. Paramount among them was the impact of financial and human resource constraints. Because of such setbacks, there was a need to sample participants. I chose Grade 9 for this study because it is a transitional period where learners move from the GET to the FET phase. Most elementary mathematics concepts are done in the GET phase, while learners are exposed to more advanced mathematical concepts in the FET phase, making it important to study how they navigate this shift (Turner et al., 2020). Moreover, it (Grade 9) is the grade in which learners formalise and extend their understanding and application of topics like algebraic expressions and other advanced mathematical concepts (Lika, 2021).

3.5.2 Sample Size

A sample is a group of participants from whom data are collected (McMillan and Schumacher, 2010). This implies that a sample represents a portion of the total population, which typically represents the entire group under study. Casteel and Bridier (2021) similarly described sampling as the process of selecting representative cases from within the target population for a research study. The sample of this study was forty (40) learners from a single school in the Frances Baard district. The school was conveniently chosen due to factors such as its close proximity, research consent from the school, financial considerations, and suitability of the school for the research.

I made sure every participant was in Grade 9 and attended the selected school. Golzar et al. (2022) briefly explain convenience sampling as the process of selecting that is suitable for and accessible to a study.

3.5.3 Sampling Technique

Sampling can be grouped into two main kinds (Saunders et al., 2015). These are probability and non-probability sampling. In probability sampling, each participant has an equal chance to contribute to primary evidence (Pace, 2021), whereas with non-probability sampling, members do not have an equal opportunity to contribute to primary evidence (Rahman, 2023).

Non-probability purposive sampling was adopted for this study. I purposively sampled learners across different performance levels (intellectual or academic abilities) to identify patterns and insights into how various groups learn, process information, and overcome challenges.

A sample of 40 learners was selected from 203 Grade 9 learners in the selected school for this study. The 40 selected learners were later numbered L1 to L40. Details of the sampling process are explained in Section 4.2 of this study.

3.6. Purposive sampling

Purposive sampling, also known as judgmental sampling, involves the intentional or conscious selection of participants based on their personal characteristics to help clarify or explain a specific theme, concept, or phenomenon (Etikan et al., 2016; Campbell et al., 2020). In other words, it (purposive sampling) is a non-random sampling technique that requires the researcher to determine what information is necessary and then look for participants who are able and willing to provide the information through knowledge, experience or characteristics (Etikan et al., 2016; Shamsudin et al., 2024).

The participants in this study were purposefully sampled as indicated earlier. Forty (40) learners were sampled to write the algebraic expression test and out of the 40 learners, 12 were purposefully selected for an interview. The selection of these individuals (interviewees) was based on their responses to the algebraic expression questions. They were purposively selected for interviews to obtain deeper insights into their reasoning processes, clarify their test responses, and to learn more about the cognitive and affective factors that may have impacted their performance.

3.7 Research Instrument and Data Collection Techniques

The current study employed two research data collection instruments: a test and interviews.

3.7.1 Algebraic Expression Test

This data collection instrument consisted of test items based on learners' understanding of algebraic expressions (See Appendix A). Twenty (20) algebraic expression questions were administered to all the 40 sampled learners. The test had a duration of 80 minutes (1 hour, 20 minutes). All items (questions) for the test were adapted from a Grade 9 textbook, learners' workbook as well as national and international assessments (TIMSS and ANA). Questions were framed in a manner that made the strategies used evident.

3.7.2 Description of the Test Items

The test was divided into five sections. Section 1 which represented Question 1, consisted of seven (7) questions. These questions mostly tested the ability of learners to recall some basic algebraic expressions. Also, it was aimed at testing learners' ability to classify algebraic expressions in their respective types such as monomials, binomials and trinomials. Moreover, it was intended to test learners' ability to simplify algebraic expressions (DBE, 2012).

The second section of the test (Question 2) consisted of three sub-questions. These questions aimed at testing learners' algebraic manipulation skills. The section tested learners' ability to multiply (expand) integers and monomials, binomials and trinomials using the right mathematical procedure (DBE, 2012).

Question 3 consisted of four sub-questions. The purpose of the questions was to assess learners' ability to factorise algebraic expressions. Each question represented a different type of factorisation as captured in the Grade 9 syllabus or the annual teaching plan (DBE, 2012).

Section 4 (Question 4) consisted of only two questions. This section intended to test learners' substitution skills, their ability to apply exponential laws and simplify fractions.

Section 5 was a single question aimed at testing learners' conceptual understanding of algebraic fraction, negative integers as well as like and unlike terms.

The last section (Question 6) consisted of three (3) word problems. Section 6 aimed to assess learners' ability to interpret and translate word problems into algebraic language and solve them.

3.7.3 Semi-structured Interviews

McMillan (2012) defines an interview as a two-party dialogue between an interviewer and participants where the interviewer asks questions to find out the opinions, beliefs, views, ideas, and behaviour of a participant. The current study used a face-to-face semi-structured interview. The reason for adopting face-to-face was as follows: First, I could note gestures by the respondents. Secondly, I could notice the willingness to participate or not with regards to the participant, as well as incidents where the respondent would not be comfortable with some of the questions. Lastly, I could note questions where respondents may be hesitant and incidents where respondents may not be telling the truth.

The reason for adopting semi-structured interviews was also to facilitate a healthy dialogue with the participants. Moreover, semi-structured interviews provided an opportunity for me to ask follow-up questions to help better understand the feedback of participants.

Most importantly, the use of semi-structured interviews allowed me to explore the participants' thoughts and confirm what I had noticed or discovered during the test. Twelve (12) learners were selected for the interviews. Learners were selected based on their test answers. All twelve learner participants were interviewed using the same questions, structure and strategy. The interview took place the day after the test was written. This was to help learners vividly remember how they came up with their answers and the reason behind their solutions when asked in the interview as it was still fresh in their minds. The questions in the interview were asked using simple and more understandable language and were very brief. Questions from the interview guide (see Appendix B) were what was used for the interview.

Each interview lasted between 30-45 minutes. Interviews were recorded (audiotaped) and later transcribed.

3.7.4 Pilot Study

A pilot study, including both tests and interviews, was conducted prior to the commencement of actual data collection. The primary purpose of the pilot study was to assess the feasibility of the data collection instruments. It was to validate both the test items and interview questions as data collection instruments. The pilot study was carried out with grade 9 learners in one of the high schools with a similar socio-economic background to the participating school in the Namakwa district in the Northern Cape. The main research setting, and the pilot study field were far apart. To preserve the integrity of the research, the pilot school and the participating school were not close to each other and that prevented any exchange of information.

Thirty- four (34) grade 9 learners from the same class participated in the test and five (5) learners were interviewed. The pilot study did not feature all the research processes involved in the main study. The learner’s responses from the pilot study helped me reformulate some of the test questions to avoid ambiguity and unnecessary repetition of questions from the interview guide. The pilot study also helped me to estimate the time and necessary resources needed to complete the main study. Moreover, it helped me to anticipate any diversion by the sampled participants during the data collection process in the study.

3.8. Data Analysis Procedure (Method)

According to Flick (2013), data analysis involves the classification, interpretation and organisation of materials to give meaning to what they represent. This process also includes defining the situation, notifying patterns, categories, themes and regularities (Khoa et al., 2023). Pertaining to this study, data analysis involved the activities performed in converting data to significant information which could be used in finding answers to the research questions. In this study, data from the interviews were analysed using thematic analysis. Thematic analysis is the technique or process used in identifying themes and patterns within collected data (Dawadi, 2020; Braun & Clarke, 2022). The process of thematic analysis is summarised by Braun and Clarke (2006) in six (6) phases. They are familiarisation, coding, searching for themes, reviewing themes, defining and naming themes and writing them up. The analysis of the data from the interview was achieved by following all six aforementioned phases, including the construction of themes and transcribing the recorded interview (McMullin, 2023), among others.

With regards to the test, the scripts of the 40 participants were analysed using content analysis. Content analysis refers to an in-depth and well-organised observation of the content of a particular material to identify themes or patterns within it (Anwar et al., 2024). Content analysis was used to help account for the nature of understanding, challenges and conceptual gaps identified in the test.

3.9 Credibility and Trustworthiness

For the findings of this research to be trusted, it had to be of good quality. The current study used a qualitative research design, and the criteria for evaluating a qualitative research study includes credibility, dependability, confirmability and transferability (Ahmed, 2024). These components that depict the quality of research are discussed in the next sub-sections.

3.9.1 Credibility

McMillan and Schumacher (2010) defined credibility as the degree to which the outcome of research is judged to be real, precise, exact and suitable. In other words, credibility determines whether the research results are genuine and can be trusted. Similarly, Gagani (2019), explained credibility as the confidence placed in the truth of the research.

To ensure the credibility of this research, I relied on peer debriefing. This procedure involves seeking assistance and guidance from an expert or professional (Anney, 2014). In this instance, the expert or professional was my supervisor. The guidance, support and feedback from my supervisor were outstanding and pivotal in this study as it helped improve the quality of this research. Secondly, I had a prolonged engagement with the participants and persistently observed them to establish whether their responses were genuine, reliable, and consistent. I also reflected with participants to ensure that I correctly understood their intentions, making the study credible (Ahmed, 2024). Credibility was additionally strengthened through member checking (Subrahmanyam, 2025; Vella, 2024). I did this by sharing a summary of the initial themes with a few of the participants who affirmed through their feedback that the interpretations precisely represented their response and experiences. Lastly, I applied more than one method of data collection which were testing and interviewing as stated earlier.

3.9.2 Triangulation

Triangulation is the use of different techniques or methods, data and observations in research to investigate one phenomenon (Donkoh & Mensah, 2023). It is a research strategy used to improve the validity and credibility of the research findings (Rankweteke, 2020). The current study employed multiple data collection tools (tests and interviews) to validate the collected data. The written test revealed the depth of understanding of algebraic expressions amongst learners, whereas the interviews provided insight through the participants' own narrations. This means the participants' responses were not just taken based on the written tests but tested in a face-to-face real-life situation.

3.9.3 Dependability

Anney (2014) defined dependability as the consistency of research outcomes over a period. According to Ahmed (2024), a study is dependable when it has a complete and transparent audit trail that details the research path from data collection to interpretation. To ensure the dependability of this study, the methods and procedures used for data collection were described in detail. These methods included a test and interviews. Moreover, participants were briefed about the findings and recommendations.

3.9.4 Confirmability

Confirmability is the extent to which participants influence the outcome of research rather than the researcher's interest (Kassa et al., 2024). In other words, for a study to be confirmable, the data and interpretation of the results must not be from the researcher's imagination but must be derived from the data. Confirmability was strengthened in the current study through the use of open-ended questions during the interviews. Participants were given considerable opportunities to express their views on the given test items, as well as how and why they (the learners) answered the test questions in the manner they did. Confirmability was also ensured in the current study, as I ensured that all responses from participants were included in the findings.

3.9.5 Transferability

Transferability is the degree to which a study could be applied or generalised to different contexts comparable to the situation, circumstances and context wherein the study was conducted (Anney, 2014; Muzari et al., 2022). Transferability in the context of this current research is the degree to which the analysis, strategies, and interview protocol would produce results that can be applied to similar research (Kaae & Traulsen, 2020). To ensure transferability of findings to different settings, an in-depth interpretation of the collected data should be given for different researchers to transfer it to other contexts (Alborough & Hansen 2023).

Other scholars can adopt or adapt methodological processes from this study because I provided rich descriptions of the research context (Stalmeijer, Brown & O'Brien, 2024). Moreover, participants' responses from the interview were reported as quotations to avoid misinterpretation or distortion of meaning. Afterwards, all such quotations were properly analysed. Equally important, the current study was conducted in such a way that the methodologies used could be transferred from the Frances Baard District to other districts, provinces and countries. According to Dlamini (2017) and Kodisang (2022), transferability is made easier when a study describes in detail how participants were purposely selected. This makes it possible for the reader to compare their samples to that of a study. Moreover, the indicators of Skemp's theory, which was used as the framework of this study is applicable to other mathematical concepts and subjects.

The external validity of this study refers to the extent to which the findings can be transferred or generalised beyond the actual research setting (Drisko, 2025). Though this study was conducted with learners from a single high school and among grade 9 learners, the findings may have relevance for other classrooms and grade levels where learners have comparable background knowledge and are taught under similar conditions. For instance, if this study identified an effective strategy to enhance learner understanding of algebraic expressions, such a strategy could be rationally adapted for learners in other districts, schools, or learners in similar grade levels that use the same curriculum.

To sum up this concept in the context of this study, transferability was supported through the provision of rich, detailed descriptions of the population and participants, including the criteria and process used for their selection. Furthermore, the study enhanced transferability by thoroughly explaining the rationale and procedures for purposively selecting specific participants for interviews. These detailed accounts enable readers to assess the applicability of the findings to other contexts with similar characteristics.

3.10 Ethical Issues

Taylor et al. (2015) assert that acceptance criteria must be followed to ensure that the research is ethically valid. Ethical considerations form a very important part of any research study. It lays a strong foundation in terms of establishing a good rapport between the interviewer and the interviewee (Alderson & Morrow, 2020). The next section discusses how ethical principles were adhered to in this study.

3.10.1 Obtaining Informed Consent

A researcher is required to get consent to collect data when a study involves human participants (Millum & Bromwich, 2021). Before data were collected for this study, participants were approached. Afterwards, a consent form (Appendix F) was sent to learners and their parents (or guardians) respectively. Consent letters were sent to parents because minors were involved. Formal letters were also sent to the provincial Head of Department (Appendix D) and the school principal (Appendix E) to seek permission to conduct the research in the chosen district, and the school, respectively. All letters informed the respective recipients about the purpose and nature of the study. Most importantly, permission was obtained from the UNISA College of Education Ethics Review Committee for ethical issues. The certificates (Appendix C1 and C2) are attached for ease of reference. Last, a letter of appreciation was sent to all participants (learners) and their parents or guardians.

3.10.2 Voluntary Participation and Right to Withdraw

According to Dahal (2024), research participants have the right to voluntarily withdraw from a study. All learners in this study were informed that participation was voluntary and had no impact or connection with their education in any way while at school. Moreover, participants were informed that they had the right to withdraw at any point in time without any risk or harm. No participant was compensated for taking part in the research.

3.10.3 Ensuring Confidentiality and Anonymity

Confidentiality is a state where a researcher is aware of a participants' identity but takes the appropriate precautions to keep that information hidden from others (Williams et al., 2024). Anonymity, on the other hand, simply refers to a situation in which a research participant is never known or identified by anyone in the study (Kang & Hwang, 2021). To enforce confidentiality and anonymity, I ensured that the data collected were secured and completely inaccessible by third parties. All participants' personal information was kept confidential. Moreover, I ensured none of the participants were subjected to any form of emotional, physical or mental distress. Also, all information gained from participants were used solely for academic purposes and not commercial purposes. Participants were informed not to provide any identification information or their names when writing the test. Participants were coded with pseudonyms to protect their identities. Learners' scripts were coded and marked anonymously to avoid research bias as well. They (the participants) were referred to as L1, L2 and so on.

The data would be stored in a lockable cabinet for five (5) years and would be password-protected. I applied the UNISA Research Policy, which stipulates that data must be stored or saved for a period of five years after the research is completed.

3.11 Summary

This chapter gave an insight into how data were collected and the rationale behind it. The chapter also reviewed the research paradigm and justified the choice of research method used. Also, the chapter discussed the population, the sampling procedure and the instruments used for collecting data. Moreover, it discussed the research settings, data analysis procedure, issues of trustworthiness and ethical issues. All the processes and procedures that were followed as described in this chapter were to ensure that quality and first-hand data were obtained to help find the right answers to all the stated research questions of the study. The next chapter discusses the findings and interpretation of the study.

4. CHAPTER 4 PRESENTATION OF DATA

4.1 Introduction

This chapter provides a systematic presentation of the findings. Drawing on the analysis of primary data, the findings are categorised under three main themes, namely, common strategies employed by learners in solving algebraic problems, connections created in the process of solving algebraic expressions, and conceptual gaps identified. The themes surfaced during the data analysis and coding process (Salmona & Kaczynski, 2024) and were framed within the context of Skemp's theory. In other words, Skemp's theory was used for interpreting arguments in this chapter. The complex data analysis of this study was structured using Braun and Clarke (2016)'s model.

It is essential to note that the primary objective of this study was to investigate how Grade 9 learners comprehend algebraic expressions. The following objectives provided a framework for achieving the aim of the study.

- Explore and describe the strategies used by learners in solving problems involving algebraic expressions. In other words, the different methods and approaches learners use in solving algebraic expression problems.

- Explain and reflect on how learners make connections to deepen their understanding of algebraic expressions. This refers to strategies used by learners to build a deeper understanding of how various concepts and techniques are interrelated and work together to simplify, manipulate and solve problems involving variables and constants.

- Identify and highlight the conceptual gaps that manifest when learners solve algebraic expressions.

- Suggest possible ways on how the conceptual gaps in Grade 9 learners' understanding of algebraic expressions be minimised.

The findings in this study are presented in accordance with the critical discoveries and participants' responses to the test questions and interviews. To preserve original context and ensure proper attribution, responses from participants are quoted verbatim. The data collection techniques used were aimed at answering the following research question: ***How do Grade 9 learners understand algebraic expressions?*** Subsequently, a group of learners was purposively chosen and interviewed to elicit their perspectives and the rationale behind their responses. These learners were probed to explain how and why they arrived at specific answers on their tests. The school's context and demographics are presented in the next section. This information helped me as a researcher to propose interventions or strategies relevant to address specific educational gaps.

4.2 Demographics of the School

Data were collected from only one school in the Frances Baard District in the Northern Cape Province. The language used for instruction at the school is English. A total of 40 learners (16 boys and 24 girls) were sampled for this study as mentioned in the previous chapters. The school had five (5) classrooms for Grade 9 with an average of 40 learners per class. There were 203 Grade 9 learners with two (2) different teachers in the school.

The five classes were distributed amongst the two teachers as follows: Three classes (9A, 9B & 9C) were assigned to Teacher 1 with a class total of 40, 41 and 41 respectively. Teacher 2 was assigned with two classes (9D & 9E) with a class size of 40 and 41 respectively. The teachers provided me with a list of each class, along with their academic records, for sampling purposes.

Participants for the interviews and written test were selected using a non-probability purposive sampling technique. I purposively sampled learners across different performance levels (intellectual or academic abilities) to identify patterns and insights into how various groups learn, process information, and overcome challenges. The aim was to ensure that the findings are not skewed towards one group. From each class, I sampled eight (above average, average and below average performance) learners making a total of 40 learners who participated in the study.

The performance of the school in terms of Grade 9 Mathematics had seen a significant improvement for the past three years. The average percentage of Grade 9 Mathematics was 47.4% in 2022, 49.6% in 2023 and 50.8% for 2024. Despite incremental improvements in performance over the past three years, algebraic expressions continued to pose a significant challenge (Stemele & Jina Asvat, 2024). However, with a proven record of achievement, it was easier for me to identify the factors that led to the sustained success. It is also important to note that understanding demographics enabled me to suggest interventions that are appropriate for the learner group. A thorough analysis of the data collected in the study is presented in the next section, along with a detailed overview of the framework outlining the themes that emerged.

4.3 Presentation of Data

Figure 4.1 presents a framework for presenting the findings in this chapter. To put differently, the identified patterns from the data are systematically classified into themes and categories and subsequently presented as findings.

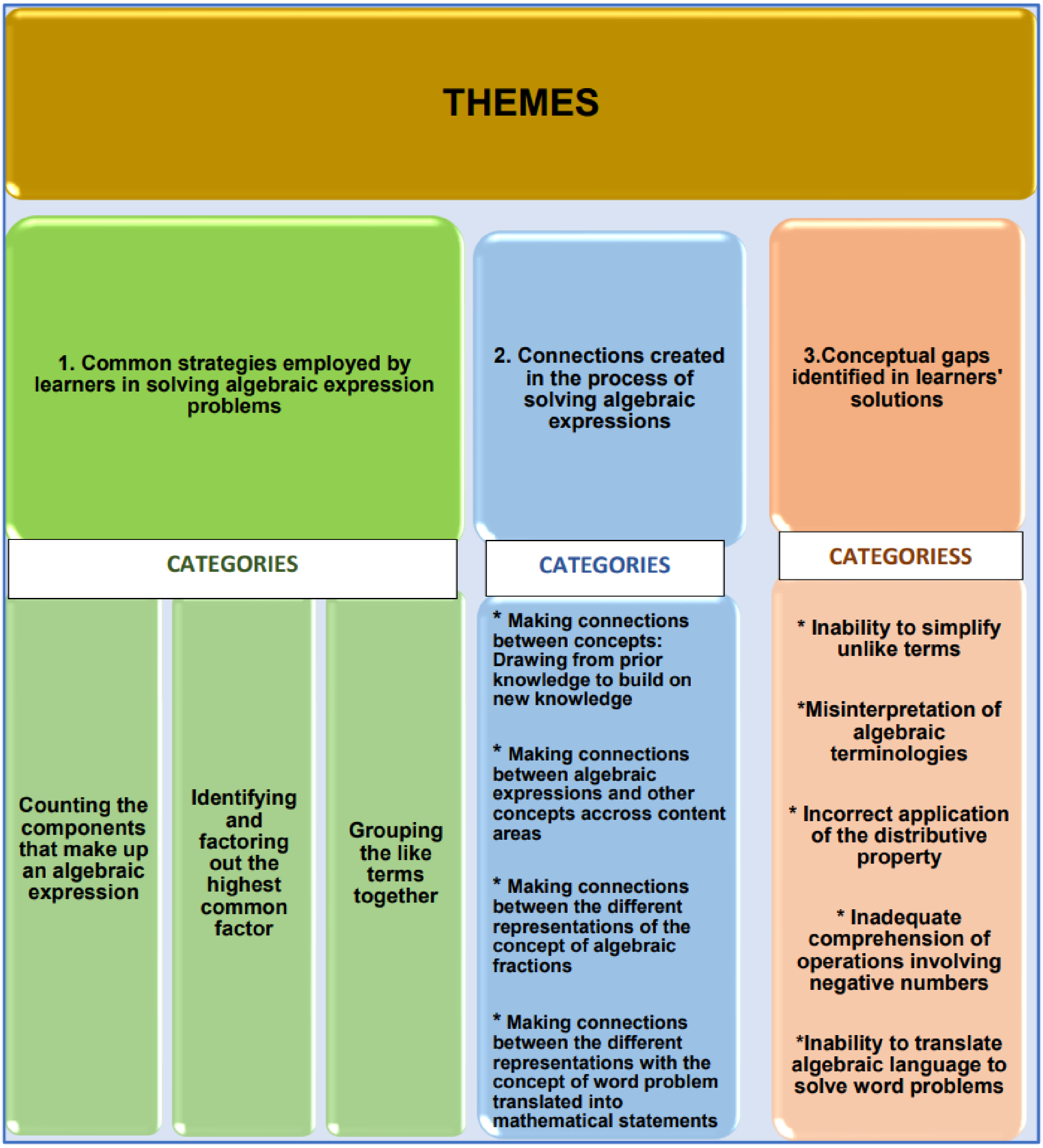


Figure 4.1

Three themes identified in learners' responses

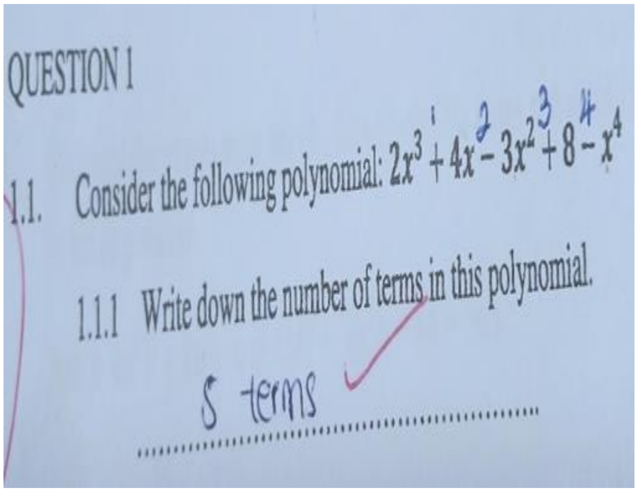
As depicted in Figure 4.1, the findings obtained from the written test and interviews are systematically organised into themes and categories. The first theme identified in the analysis was ***the common strategies employed by learners while solving algebraic expression problems***. The term *strategies* refers to the methods and techniques learners use when solving the questions. The second theme that emerged was the ***mathematical connections learners made in understanding algebraic expressions***. The last theme manifested was *the* ***conceptual gaps learners exhibited while solving algebraic expression problems***. These themes are discussed in the next sections, starting with the common strategies learners use when solving algebraic expression problems.

4.4 Theme 1: Common Strategies Employed by Learners in Solving Algebraic Expression Problems

The theme in this context refers to the typical or frequently used approaches, methods or techniques by learners when attempting to understand and solve problems. These strategies can vary based on the nature of the problem. For instance, logical or conceptual (Resnick & Stieff, 2024) and the learner's individual preferences or cognitive processes. In the context of problem-solving, these strategies include:

4.4.1 Counting the Components that make up an Algebraic Expression

This refers to instances where learners demonstrated the easiest and most convenient way to determine the correct number of terms in an expression. For instance, learners were required in Question 1.1.1, to write the number of terms in the given expression $2x^3 + 4x - 3x^2 - x^4$. The strategy used by learner **L16** was to count the number of basic operations in the given expression and add 1. **L8** on the other hand counted the number of terms sequentially using the degree of x. Figures 4.2 and 4.3 illustrate these observations.



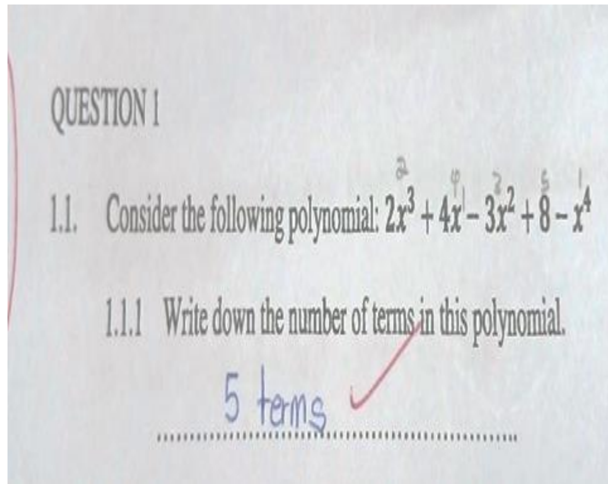


Figure 4.2: *Learner L16's response* **Figure:4.3**:*Learner L8's response*

Figures 4.2 and 4.3 reveal that the learners understood the question and they used different ways of determining the number of terms. What is evident from the two solutions is that learners understood that a constant is also a term with the variable x to exponent zero (x^0). 57% of the learners got this question wrong. From the 43% of learners who got it right, 34 % used L8's strategy and 9% used L16's strategy. It appears that L16 counted the number of basic operations and added 1 to obtain the answer, whereas L8 counted the number of terms. To find out more about their reasons behind using this strategy, two learners were interviewed. The excerpt below explains what transpired between the researcher and the learners. Note that in the excerpt, **'R'** represents the researcher and 'L' represents the learner.

Excerpt 1: Learner L16

R: *Kindly turn to the first page of your test paper. Question 1.1.*

L16: *Yes, I am seeing it.*

R: *Please, go through the question.*

L16: *Ok, I have finish.*

R*:* *You got it right. Congratulations!*

L16*:* *Thank you, sir.*

R*:* *Can you please explain to me how you got your answer?*

L16*:* *You see, my teacher Mr …… said, plus and minus put terms together. So, you see in the question, the first plus (+) joins two (2) terms, the first minus (-) joins two terms, the second plus (+) also joins two and the last minus (-) joins 2 terms. So, in all, we have 4 signs (2 plus and 2 minus) joining 5 terms together.*

R*:* *I have seen some numbers written on top of each sign. Where from that? It's that what helped you in getting the right answer?*

L16*:* *Yes sir.*

R*:* *How?*

L16*:* *You see, 1st sign joins two terms, 2nd signs join three terms, 3rd signs will be able to join four terms and then 4th signs join five terms. Or you can write 1, 2, 3, 4 on top of the plus (+) and minus (-), then added 1 to get 5 terms.*

R: *So, if I get you right, you mean you count the number of signs and then add 1 to get the total number of terms?*

L16: *Yes, yes.*

R: *Oh, I see.*

The dialogue below (Excerpt 2) is a description of how Learner 8 came up with a solution.

Excerpt 2: Learner L8

R*:* *Let's check your answer in Question 1.1.*

L8*:* *Ok.*

R: *You got it right. Congratulations!*

L8: *Thank you.*

R: *Can you explain to me how you got your answer?*

L8: *I always put numbers on the terms and count them to be sure.*

R: *Is that the strategy you use to answer questions like this?*

L8: *Yes, if you look at my test paper, I have written 1, 2, 3, 4 and 5 on top of all the numbers. (He meant terms). The last number is 5 so it has five terms.*

R: *Ok, I see.*

The two excerpts above (excerpts 1&2) clearly suggest that learners can recognise the number of terms in an expression by indicating counting numbers on each term or operations (starting from 1) and then count all to arrive at the total number of terms in an algebraic expression.

Undoubtedly, the strategies used by the learners were effective and can be applied to similar problems. On the other hand, the responses provided by learners who answered incorrectly were as follows:

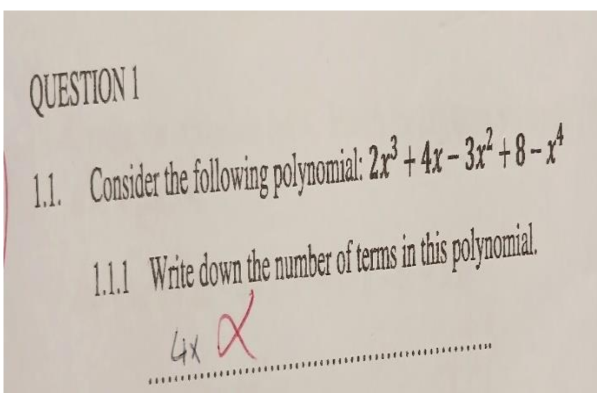


Figure 4.4 *Learner L36’s response*

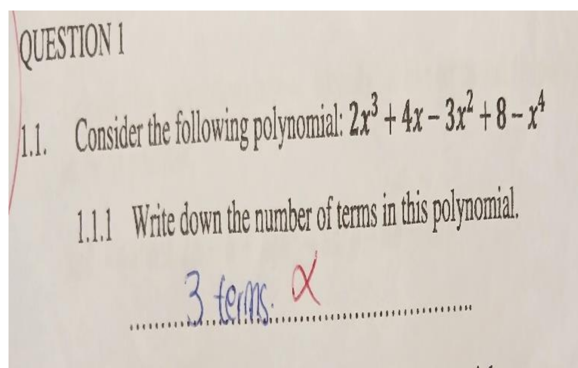


Figure 4.5 *Learner L6’s response*

It can be observed from the extracts that these learners appear to have no idea about what the number of terms in an expression means or how to determine such numbers.

4.4.2 Identifying and Factoring out the Highest Common Factor

Another strategy that emerged from the learners' test was the one where learners visibly showed an identified factor when solving some algebraic expression problems. This strategy was used mostly in factorisation questions that asked learners to identify a common factor. For Question 3.2, for instance, learners were asked to factorise the expression $25x + 15x^3 - 10x^5$. The purpose of the question was to assess learners' ability to break down the expression into its components or factors.

It was observed in most learners' solutions that after identifying the correct factor of the terms, they divided each term by the identified factor within the brackets. Figure 4.6 illustrates this observation from one of the learners.

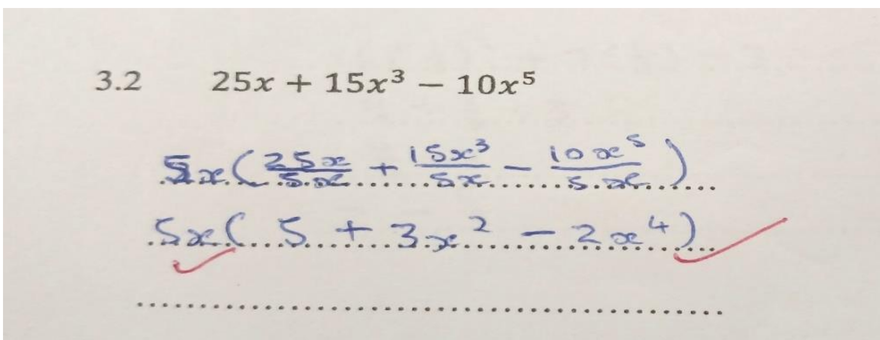


Figure 4.6

Learner L34's response

Figure 4.6 reveals how well the learner understands the concept of highest common factor (HCF) by factoring the right factor out. It also shows how well the learner understands the concept of factorisation by visibly dividing the identified factor by each term to get the right dividend. In explaining his strategy and reason in an interview, Learner L34 responded as follows:

Excerpt 3: Learner L34

R: Let's quickly check your answer for Question 3.2 on page 3.

L34: Ok.

R: You got it right. Good job for that. Can you take me through how you got your answer?

L34: I first checked a number that can divide 25, 15 and 10 with no remainder and that number is 5. I also realised that I can also include x *to the 5 because it is also everywhere. So, the common terms to bring out is* $5x$*. So, I brought it out.*

R: Ok, I see that. Now, how did you get the remaining part of your answer? I can see some divisions in the bracket. Can you explain to me how you did it?

L34: Yes, so I divided all the terms, each of them by $5x$

R: Ok. I see. Why did you do it that way if I may ask?

L34: I divided them on the paper because I wanted to be sure of the answers and did not want any mistake. Our teacher taught us this way too.

It appears from the interview that L34 and the other 11% of learners who got the question right have a conceptual understanding of factorisation. In other words, the learner has a grasp of the underlying principles and ideas behind breaking down a number or an algebraic expression into its factors, rather than just memorising steps or algorithms. Eighty-nine (89%) of the learners, on the other hand, got Question 3.2 and most of the factorisation questions wrong. Figures 4.7; 4.8; 4.9 & 4.10 illustrate how most learners lacked understanding of the concept of factors and HCF in solving questions of this nature.

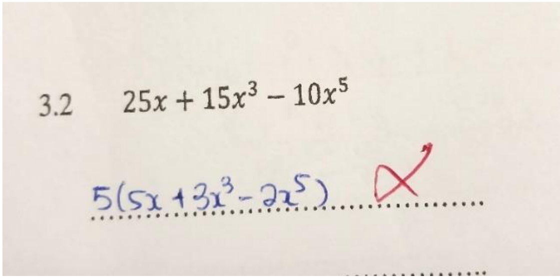


Figure 4.7: *Learner 37's response*

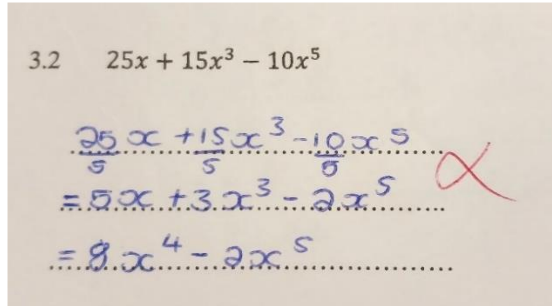


Figure 4.8: *Learner 18's response*

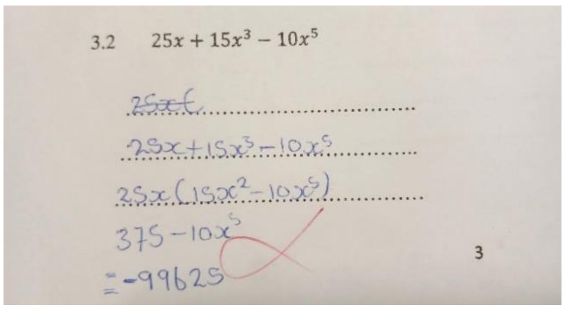


Figure 4.9: *Learner 17's response*

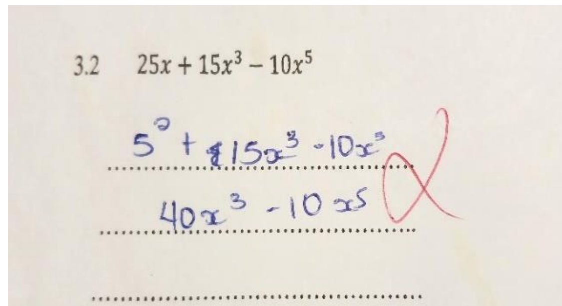


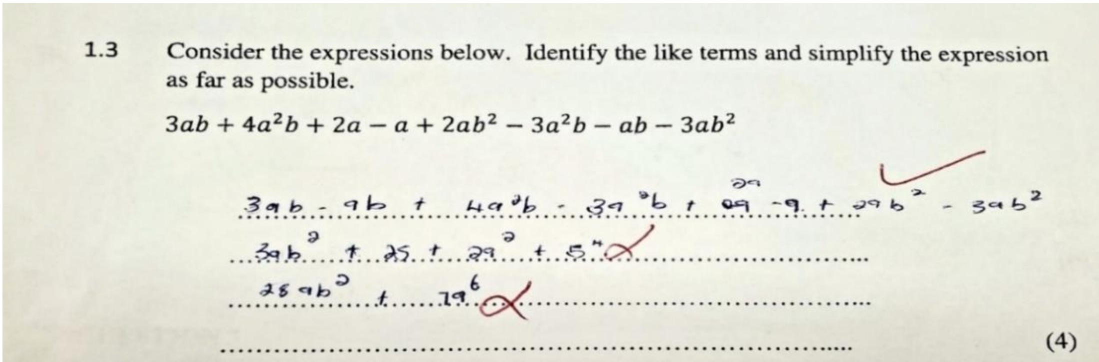
Figure 4.10: *Learner 5's response*

L37 and L18 seem to have an idea about the lowest common factor, however, it looks like they experience a challenge when it comes to identifying or recognising a variable as part of a factor as well as what it means to factorise an expression. L17 and L5 appear not to know how to factorise the given trinomials.

4.4.3 Grouping the Like Terms together

Grouping the like terms was another strategy that learners employed in the test specificallpay with simplification of expressions. An example was Question 1.3 which required learners to simplify: $3ab + 4a^2b + 2a - a + 2ab^2 - 3a^2b - ab - 3ab^2$. The question was intended to examine how learners identify like terms and simplify completely. After the researcher had analysed the learners' responses, it was observed that the majority (68%) of them were able to secure the first point out of four by correctly grouping all like terms together. However, none of these learners was able to correctly simplify the expression after the grouping.

Figure 4.11 illustrates an example of how Question1.3 was solved.



1.3 Consider the expressions below. Identify the like terms and simplify the expression as far as possible.

$3ab + 4a^2b + 2a - a + 2ab^2 - 3a^2b - ab - 3ab^2$

$3ab - ab + 4a^2b - 3a^2b + 2a - a + 2ab^2 - 3ab^2$

$3ab^2 + 25 + 2a^2 + 5^4$

$28ab^2 + 7a^6$

(4)

Figure 4.11*: Learner L3's response*

As shown in Figure 4.11, the learner was able to categorise terms into their respective groups, although this was done without introducing the brackets. However, after grouping the like terms correctly, the learner did not know what else to do. Nothing below the first step conformed to mathematical reasoning. Learner L3 further explained the reason behind the answer in an interview as follows:

Excerpt 4: Learner L3

R: *Open to Page two (2) of your paper and look at Question 1.3. Gradually go through the question and also your answer.*

L3: *I have finished.*

R: *In your first step, you were able to identify and group all the like terms to their respective partners if I may put it that way. How did you come out with your answers in the 3rd step, I mean your last answer?*

L3: *The question also said simplify so I added* $3ab^2$ *and* 25 *to get* $28ab^2$ *then* $2a^2$ *and* 5^4 *to get* $7a^6$

R: *Okay, let's look at* $3ab^2$ *and* 25 *first. Are they like terms?*

L3: No.

R: Are $2a^2$ *and* 5^4 *also like terms?*

L3: No, they are not.

R: Then, why did you add them if I may ask?

L3: I did that to get a simple or short answer because I wanted to simplify.

Learner L3 seems to have a different meaning of the term ‘simplify’. According to the learner, the question appeared to suggest that the terms were to be *reduced to the shortest number of terms* possible, if not to a single term. In this instance, Learner L3 may have predicted the behaviour of an algebraic expression to be the same as an arithmetic expression.

4.5 Theme 2: Connections Created in the Process of Solving Algebraic Expressions

The second theme identified, was the mathematical connections learners made when solving algebraic problems. This refers to the recognition of underlying patterns and structures within algebraic contexts. In respect to this study, making mathematical connections pertains to the ability of learners to relate concepts, methods and ideas across different areas of mathematics (García-García, 2024). Most importantly, mathematical connection involves the ability of learners to recognise patterns, make use of prior knowledge and master how different mathematical concepts are interrelated (Duman & Aydoğan Yenmez, 2024; Jung et al., 2024; Hasbi, 2024; Arifin, 2024). This also includes application of concepts in various contexts.

4.5.1 Making Connections Between Concepts: Drawing from Prior Knowledge to Build on the New Knowledge

Acquiring a new concept requires establishing connections with previously learned concepts or those that have been internalised as prior knowledge. This process involves integrating new information with existing cognitive structures, facilitating deeper understanding and retention. Some items in the test were purposefully designed to uncover how learners utilise mathematical connections in solving algebraic expression problems. Question 2.2 for example expected learners to simplify the expression $(2x^3 - 1)\ (2x^3 + 1)$. The question aimed to assess how learners apply and relate the concepts of the distributive property and exponents, which are the prior knowledge needed when simplifying an algebraic expression question of this type (DBE, 2012). Ninety-eight per cent (98%) of learners could solve this question or part of it to obtain a single point out of the three. Figures 4.12 and 4.13 affirm and show how learners who experienced challenges solved the problem.

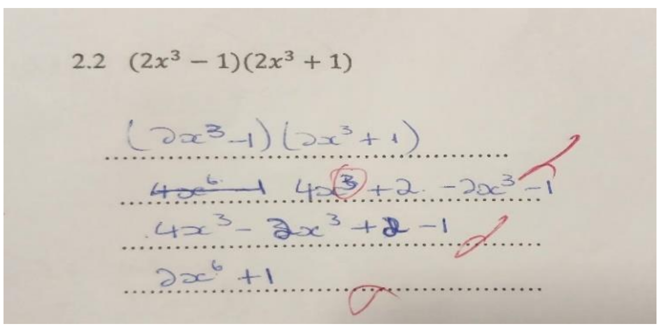


Figure 4.12 *: Learner 14's response*

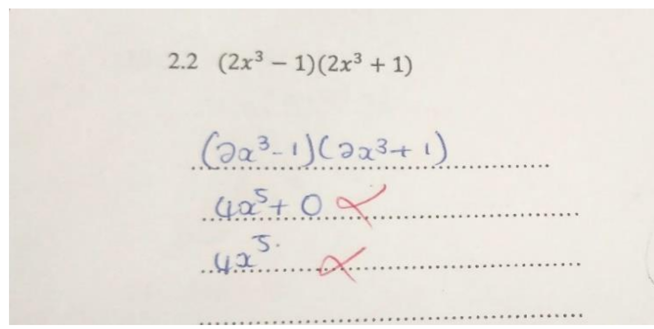


Figure 4.13: *Learner 23's response*

As revealed in both Figures (4.12 & 4.13), it seems learners have difficulty in distributing terms in brackets and applying the exponential laws. **L14** for instance appears to understand the distributive property but slipped in writing the second term. However, the learner appears to have difficulty applying the first law of exponents, specifically in not adding the exponents after multiplying the first terms in each bracket. **L23**, on the other hand, seems to have a challenge in applying the distributive property and exponential laws. The solution lacks mathematical coherence or fails to align with established mathematical principles. L14 provided further insight regarding the rationale behind her answers.

Excerpt 5: Learner L14

R: Please open your paper on Page 3. Question 2.2

L14: Ok. I am on 2.2

R: Good. Let's look at your second step. Can you take me through how you ended up getting your answer?

*L14: Ok. First, I multiplied $\mathbf{2x^3}$ by $\mathbf{2x^3}$ and I got $\mathbf{4x^3}$, I multiplied $\mathbf{2x^3}$ by $\mathbf{1}$ to get the $\mathbf{2}$. Eish! I made a mistake there, I was writing $\mathbf{2x^3}$ not **2.** Then I multiplied $-\mathbf{1}$ by $\mathbf{2x^3}$ to get $-\mathbf{2x^3}$ and last, I multiplied $-\mathbf{1}$ by **1** to get $-\mathbf{1}$*

*R: Ok. If I may ask, how did you get $\mathbf{4x^3}$ as the first term**?***

*L14: It's from $\mathbf{2x^3} \times \mathbf{2x^3}$**.** Two times two equals **4** and then $\mathbf{x^3}$ is the same so you attach to the **4** to get $\mathbf{4x^3}$.*

R: So, from there, you simplified to get your answer, right?

L14: Yes.

It can be deduced from the conversation that the learner (L14) recalled the concept of the distributive property and executed it. However, in so doing, the learner could not recall the concept of exponents and how it works. Hence, the learner got $\mathbf{4x^3}$ as the answer of $\mathbf{2x^3} \times \mathbf{2x^3}$**.** In summary, the learner could not connect all the necessary prior knowledge needed to solve the given algebraic problem.

Figure 4.14 illustrates the response of one of the few learners (2%) who answered Question 2.2 correctly.

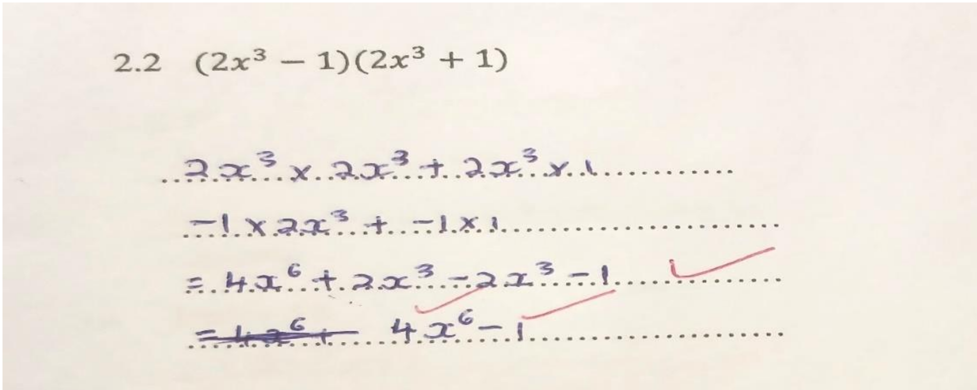


Figure 4.14*: Learner 20's response*

Figure 4.14 reveals that L20 can apply and integrate prior knowledge (the concept of exponents), which contributed to getting the correct answer. It is also revealed that **L20** can apply and connect the prior knowledge which led to successfully solving the problem. L20 was later engaged in an interview to give more details on how the problem was solved.

Excerpt 6: Learner L20

R: *Can we check your answer on Question 2.2?*

L20: *Ok Sir*

R: *Can you quickly take me through how you got your answer?*

L20: *First, I multiplied what is inside the front bracket to everything also in the back.*

R: *The front and back bracket, do you mean the first and second bracket?*

L20: *Yes, I multiplied everything in the first bracket to the second bracket and I take it one by one. So, you can see I first multiplied* $\mathbf{2x^3}$ *by* $\mathbf{2x^3}$ *and also* $\mathbf{2x^3}\ by\ \mathbf{1}$ *and after that I took* $\mathbf{-1}$ *and multiplied it by* $\mathbf{2x^3}$ *and also* $\mathbf{1}$

R: *Oh good. If I may ask, how did you get the* $\mathbf{4x^6}$ *in your answer?*

*L20: I multiplied $2x^3$ in the first bracket to $2x^3$ in the second bracket to get that. Because from what we did last week, if I multiply then I should add the exponents which are **3** and **3** to give **6**.*

It can be inferred from the conversation that the learner has prior knowledge of both the distributive property and the application of laws of exponents. The learner successfully connected and applied prior concepts (distributive property and laws of exponents) to solve the current problem.

4.5.2 Making connections between algebraic expressions and other concepts across content areas

Mathematical connection, as indicated earlier in this chapter, involves the ability of learners to understand how different mathematics concepts are interrelated. Question 4.1, for example, asked learners to calculate a given expression where the variables of the expression must be substituted with numbers. The purpose of the question was for learners to realise that while learning algebraic expressions, they can make connections with the concepts of exponents and integers. Contrary to expectation, 37 out of the 40 learners, representing approximately 92.5% of the learners, received zero points for their incorrect answers. Only 7.5% were able to correctly substitute the given values. However, they could not move beyond that first step. It appears learners experience some challenges in simplifying integers.

Figures 4.15 & 4.16 show some of the solutions provided by learners.

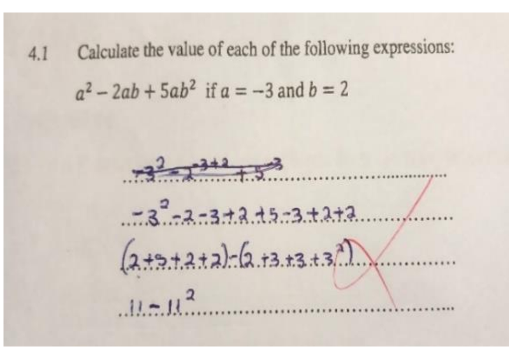


Figure 4.15: *Learner 13's response*

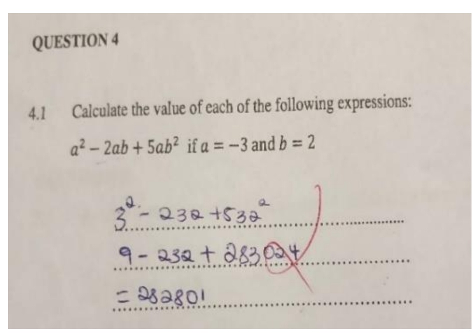


Figure 4.16: *Learner 40's response*

It is obvious from these figures that most learners have problems in substituting numbers in an expression, particularly in the use of basic operations (multiplication, addition and subtraction) on exponents and integers. Both excerpts suggest learners did not understand what they were doing, and this was how the majority (92.5%) of learners presented their work as indicated earlier. Few learners (7.5%) who obtained 1 out of 3 points seemed to have been able to substitute values correctly in the expression as displayed in Figure 4.17. Nevertheless, they appear to demonstrate a limited conceptual understanding of both exponents and integers and therefore cannot proceed further.

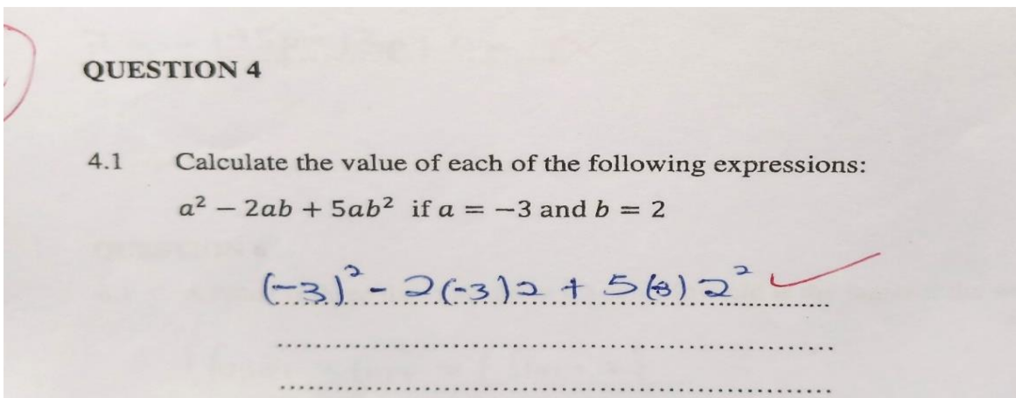


Figure 4.17: *Learner 11's response*

In explaining how the answer was obtained during an interview, L11 responded as follows.

Excerpt 7: Learner L11

R: Lets quickly turn to Page 4 of your paper. Looking at Question 4.1.

L11: Ok.

R: I can see you started very well but you did not complete your answer.

L11: Yes.

R: Anyway, can you take me through what you did in your first step that made you get 1 point?

L11: In the question, a is -3 and b is 2. So, I just look at the question, where there is a , I put -3 and where there is b, I put 2. If a or b has numbers on top, then I do the same.

R: Oh, that's great! I can see you did very well in your substitution. But what happened after that? You didn't continue. Why?

L11: Yes, I was very confused about what to do next. I forgot about how the exponent is done and also the negative numbers. I did not want to do and get them wrong.

R: Oh ok. I see.

The solution described in Excerpt 7 demonstrates proficiency in performing substitution accurately. However, the learner faced challenges in manipulating exponents and integers which impeded his ability to complete the task. Additionally, a fear of making mistakes seemed to discourage the learner from proceeding with the solution of the problem. Similarly, the problem of substitution in solving algebraic expression problems also surfaced in Question 4.2 where almost none of the learners could successfully simplify (add and subtract) common fractions after substitution. Question 4.2 required learners to determine the value of the expression $6a^2 + 2a - 3$ if $a = \frac{1}{3}$.

The extracts below illustrate how most learners became stuck immediately after substitution.

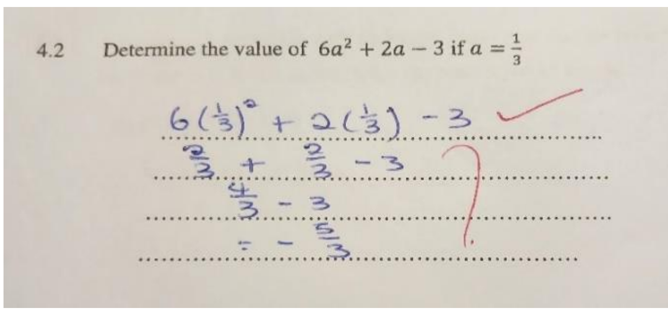


Figure 4.18*: Learner 9's response*

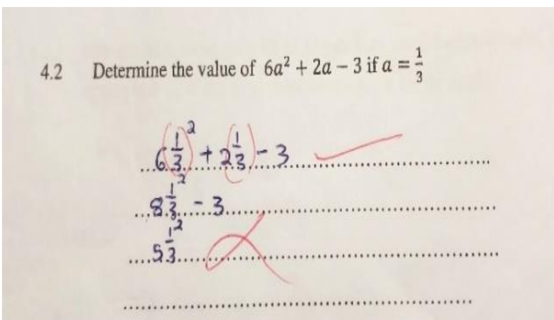


Figure 4.19*: Learner 39's response*

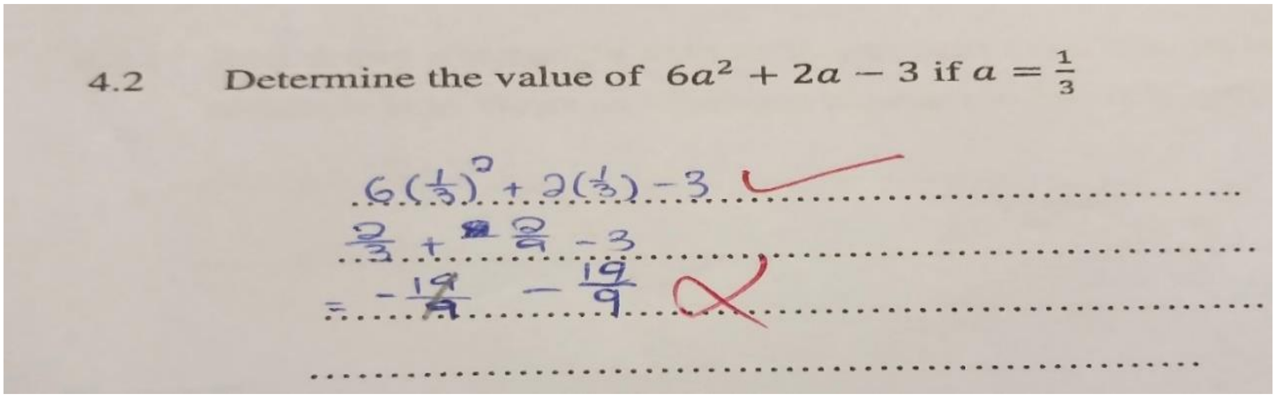


Figure 4.20: *Learner 21's response*

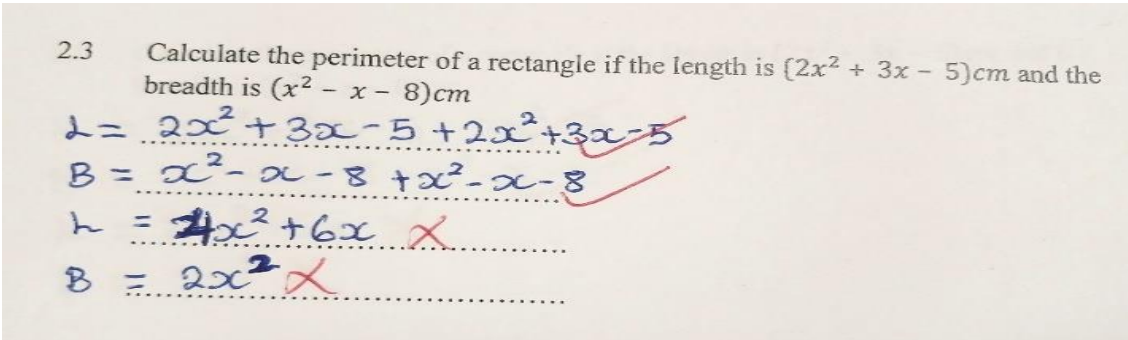
It can be seen from these three figures that after the substitution, learners could hardly find the square of a fraction. It appears that manipulating fractions also presented a significant challenge.

Establishing connections between the concept of algebraic expressions and measurement also has the potential to enhance learning. I therefore aimed to emphasise the importance of making such connections, a fundamental aspect inherent in the nature of mathematics, by investigating how these connections manifest in the context of the relationships between various concepts. Question 2.3 was designed to make learners realise that algebraic expressions can be deconstructed and understood within any given context. The main purpose of the question was to firstly, assess learners' conceptual understanding of the concept of perimeter and find out the different ways they can prove this when the dimensions of the rectangle are provided as expressions.

Question 2.3 was phrased as follows: *Calculate the perimeter of a rectangle given the length and breadth as* $(2x^2 + 3x - 5)$ and $(x^2 - x - 8)$ *respectively*.

One approach to answering the question involves doubling each dimension and calculating the sum of the doubled dimensions. Most learners added the length and breadth twice which is not a wrong approach. However, they forgot to further add the two sides (length and breadth) together as the total perimeter of the rectangle. Others also decided to add the two sides (length and breadth) and forgot to multiply their answer by

2 to get the total perimeter. Figure 4.21 illustrates the typical response of most learners to the question.


2.3 Calculate the perimeter of a rectangle if the length is $(2x^2 + 3x - 5)cm$ and the breadth is $(x^2 - x - 8)cm$

$L = 2x^2 + 3x - 5 + 2x^2 + 3x - 5$

$B = x^2 - x - 8 + x^2 - x - 8$

$L = 4x^2 + 6x$

$B = 2x^2$

Figure 4.21 : *Learner L33's response*

According to Figure 4.21, although Learner **L33** is aware that the perimeter of a rectangle is given by the formula 2L + 2B, the learner struggled to correctly sum the two expressions. It appears that the learner encountered difficulty when adding negative constants, possibly overlooking one of the negative signs or making a careless mistake. Upon examining the sum of the expressions for 2B, it became clear to the researcher that a similar reasoning error was made in the calculation. This suggests that the learner may disregard any term involving a negative number, indicating a potential misconception. Another observed challenge is that the learner did not indicate the units of measurement. This also suggests a gap in understanding the fundamental principles of measurement which serve as an essential part of communicating results in scientific and practical contexts. In explaining the solution, L33 responded as follows:

Excerpt 8: Learner L33

R: Let's look at Question 2.3. Can you explain to me how you got your answer?

L33: I know perimeter is putting all length and breadth all together.

R: Yes. Alright, go on I am listening

L33: So, I put the length and the other length together and breadth and breadth together

R: *But I can see that after adding the 2 lengths and 2 breadths together, they remained separated and I thought you said perimeter means put all of them (sizes) together.*

L33: *Yes, I should put the L and B together, but I forgot to do it. It's like, I didn't know how to put them together again.*

R: *Ok. I get it.*

From this engagement, it appears that L33 has a challenge in adding terms, particularly those that are negative integers.

Figure 4.22 displays how learner L38 solved the problem.

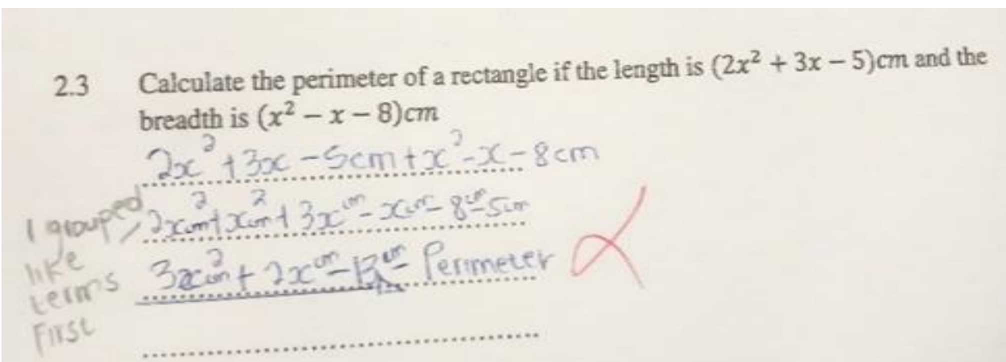


Figure 4.22 *: Learner 38's response*

From the solution for L38 (Figure 4.22), it is very obvious that the learner does not know the formula for calculating the perimeter of a rectangle. This means that the answer will not be correct immediately. Learner L38 is among the 95% of learners who did not know the formula nor how to calculate the perimeter of a rectangle when the sides are presented as expressions. This may be due to their limited exposure to applying algebraic expressions within the context of measurement. When probed on how the question was answered, the conversation unfolded as follows:

Excerpt 9: Learner L38

R: Can we go through your Question 2.3? That's page 3.

L38: Ok, am there.

R: Kindly explain to me the strategy (method) you were using here.

L38: I first added the length and breadth together.

R: What is your understanding of perimeter in the first place?

L38: To put all the length and all the breadth together.

R: So, is that what you think you did?

L38: You see, I was supposed to add it again or I can times it by 2 but I missed that.

R: You mean you forgot to do that?

L38: Yes sir, I forgot.

R: Alright.

From this response, it appears that learner L38 has a basic understanding of how to calculate the perimeter. However, the learner was not sure. The explanations suggest that although the correct answer was not obtained, the learner understood the definition of perimeter. The primary challenge appears to be its application in various contexts.

4.5.3 Making Connections between Different Representations with the Concept of Algebraic Fractions

Question 5 aimed to figure out how participants could completely break down an algebraic fraction to its simplest form. Most learners performed well on this question. However, there are few who could not get the correct answer. They began by breaking down the fraction into smaller fractions, splitting and dividing each term of the numerator by their respective denominators. Figures 4.23 and 4.24 show some of the responses from learners.

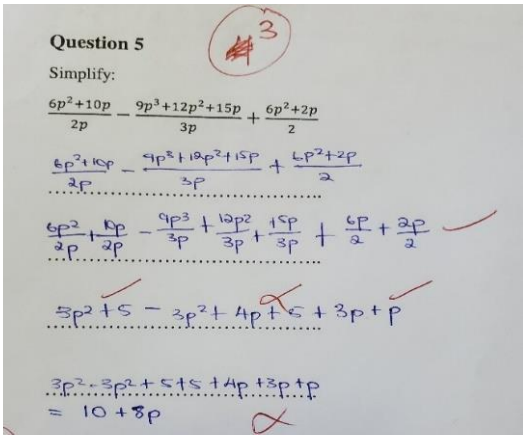


Figure 4.23: *Learner 28's response*

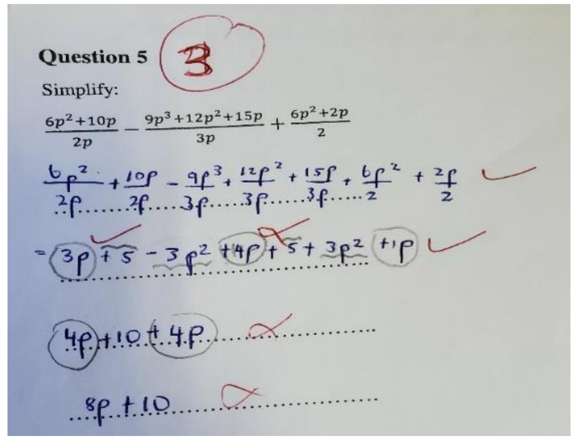


Figure 4.24: *Leaner 12'response*

Thirty-eight per cent (38%) of the learners missed the last but one step while 62% got it completely wrong. It is evident from Figures 4.23 and 4.24 that both learners understood how to break the algebraic fractions down into simpler components or parts that are easier to work with. However, learners seem to experience a little difficulty by not recognising the fact that the negative sign in front of the second fraction affects all terms in the numerator. Hence, their final simplified answers were wrong. To explain how the question was solved, Learner L28 explained as follows:

Excerpt 10: Learner L28

R: *Can we turn to Question 5? It's on Page 5.*

L28: *Ok. I am on Page 5.*

R: *Good. I can see you got 3 out of the total marks. That's good. well done.*

L28: *Thank you.*

R: *Can you take me through the method or strategy you used to help you almost get the full marks?*

L28: What I did was that I break the top terms (numerator) and divided each by the down (bottom) or denominator. For instance, the first one I broke $\frac{6p^2+10p}{2p}$ *into* $\frac{6p^2}{2p} + \frac{10p}{2p}$.

R: What was your reason for using such method if I may ask?

L28: Well, it makes it easy than the other one that my teacher also said you find the L.C.M of the down number(denominator) first. That was long and confusing.

From the explanation above, the learner used the said strategy (splitting numerator and dividing by the same denominator) for various reasons, including the fact that it is the simplest and the only strategy they know. Almost none of the learners could get the final answer right, because of their inability to distribute the negative sign in the second term.

As stated earlier, 62% of the learners got Question 5 wrong with zero points. Responses of some of these wrong answers are displayed in the Figures 4.25, 4.26, 4.27 and 4.28.

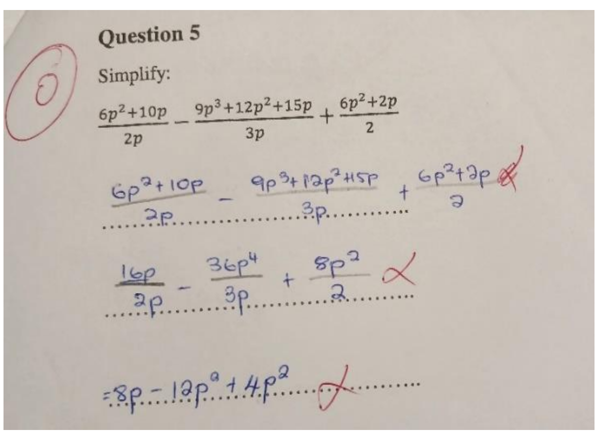


Figure 4.25*: Learner 24's response*

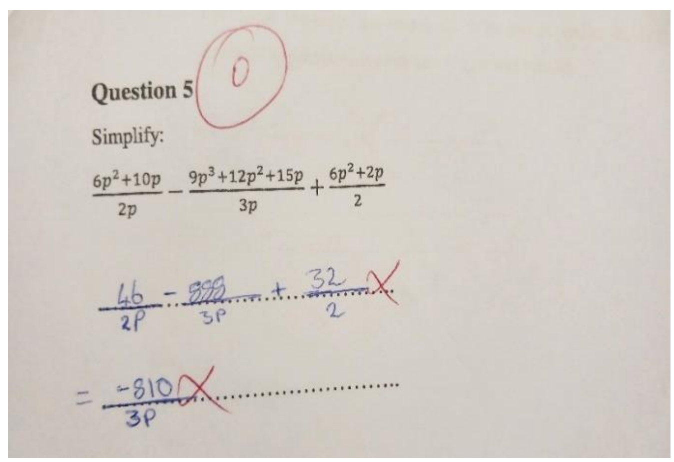


Figure 4.26*: Learner 15's response*

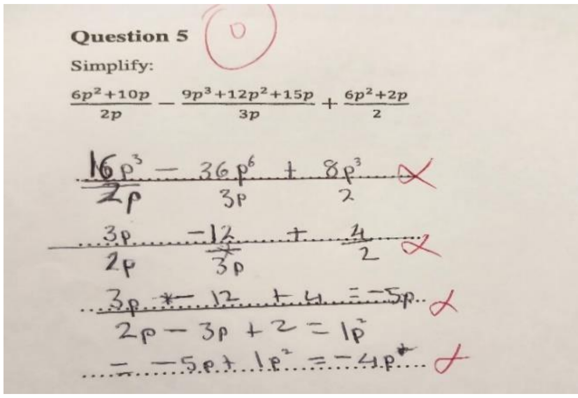


Figure 4.27*: Learner 30's response*

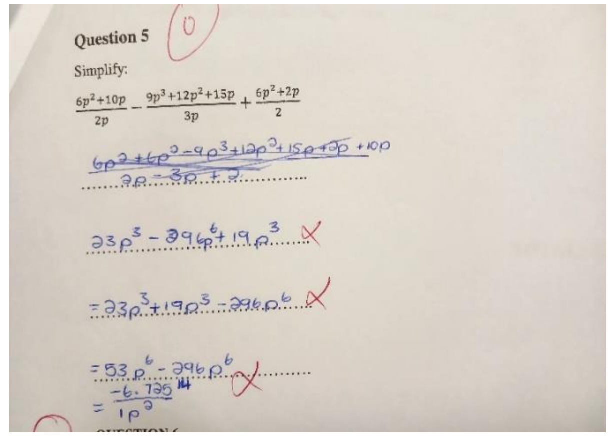
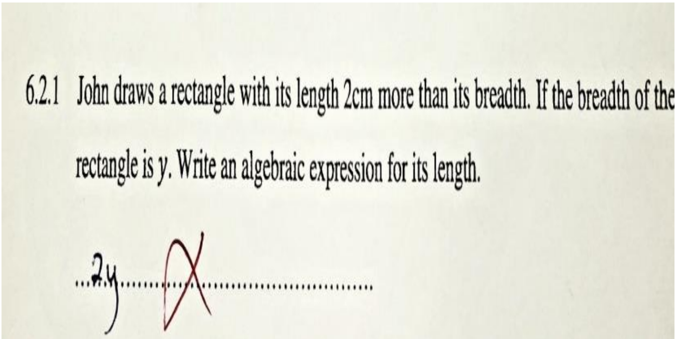


Figure 4.28*: Learner 31's response*

These learners experienced some challenges in decomposing the fractions. Looking at these responses, L24 and L30 added the unlike terms (the numerators). L15 and L31 on the other hand seem not to have any idea of how to solve such problems, as the approaches they had made no mathematical sense.

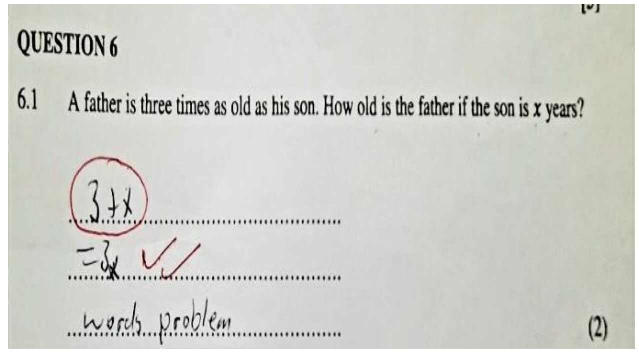
4.5.4 Making Connections Between the Different Representations with the Concept of Word Problems Translated into Mathematical Statements

The outcome of the test also indicated that most learners were unable to translate word problems into mathematical statements. The result suggested that they (learners) are struggling to understand how to convert the information provided in the problem into a form that can be solved using mathematical operations. This typically involves identifying key variables, operations, and relationships described in the problem, and then expressing them as an equation or mathematical expression. The inability to do this may indicate gaps in comprehension, difficulty in interpreting the language of the problem, or challenges in applying mathematics concepts in a real-life situation. Figures 4.29 and 4.30 illustrate this observation.



6.2.1 John draws a rectangle with its length 2cm more than its breadth. If the breadth of the rectangle is y. Write an algebraic expression for its length.

2y



QUESTION 6

6.1 A father is three times as old as his son. How old is the father if the son is x years?

3+x

=3x

words problem

(2)

Figure 4.29*: Learner L1' response*

Figure 4.30: *Learner L25's response*

Learner L1 seems to have misunderstood the concept of "more than". Instead of interpreting it as addition, the learner understood it as multiplication. That is why the learner wrote "y" instead of "2 + y". If we examine Learner L25, the solution reveals the opposite. The learner seemed to interpret the term "times" as addition and believed that the number 3 and the variable x can be added together. It appears that learners lack both mathematical knowledge and reasoning and logical thinking to figure out how to approach the problem and what steps to take to solve it.

When delved into how they came out with their answers, L1 explained as follows:

Excerpt 11: Learner L1

R: Let's look at Question 6.1 on page 5

L1: Ok

R: Please can you explain to me how and why you came out with your answer?

L1: In the question, the length is more than the breadth by 2. And it says the breadth is y. So, 2 times the y will give the breadth.

R: I want to be clear here. what does it mean by the statement "more than" in the question?

L1: It means, multiply by 2. So, "2 more than" means multiply by 2. That is why I got

$2y$ because it is 2 times y.

R: Ok. I see.

From the interview, it appears the learner has a problem or challenge in interpreting the algebraic language "more than". It may be this reason that L1 interpreted the statement more as multiplication instead of addition.

Learner L25 also explained how the answer was obtained as follows:

Excerpt 12: Learner L25

R: Let's go to Page 5 and look at Question 6.1

L25: Ok

R: Can you please explain to me how you got $3 + x$ in your first step?

L25: The father is 3 times older than the child in the question. So, I add 3 to the father age to get the child age.

R: Ok, I see. So how did you end up getting $3x$?

L25: Oh, I put them together to get the $3x$. That is 3 and x.

Learner L25 seems not to understand the statement "3 times" as multiplying by 3. The learner recognised it as addition. Although the learner got $3x$ as the final answer, the explanations say otherwise. This means the learner had little or no idea what was being done which was translating the given statements to algebraic expressions.

4.6 Theme 3: Conceptual Gaps Identified in the Learners' Solutions

The third aspect of this study's analysis was the conceptual gaps that manifested in learners' solutions when solving algebraic expression problems. After thorough analysis of the data, five (5) common gaps were identified. These conceptual gaps resulted from the challenges, misunderstandings and errors manifested by learners when solving the problems. In other words, these gaps were revealed from the first two themes of this study which are the strategies used and connections made by learners when solving algebraic expression problems. Figure 4.31 displays the identified gaps.

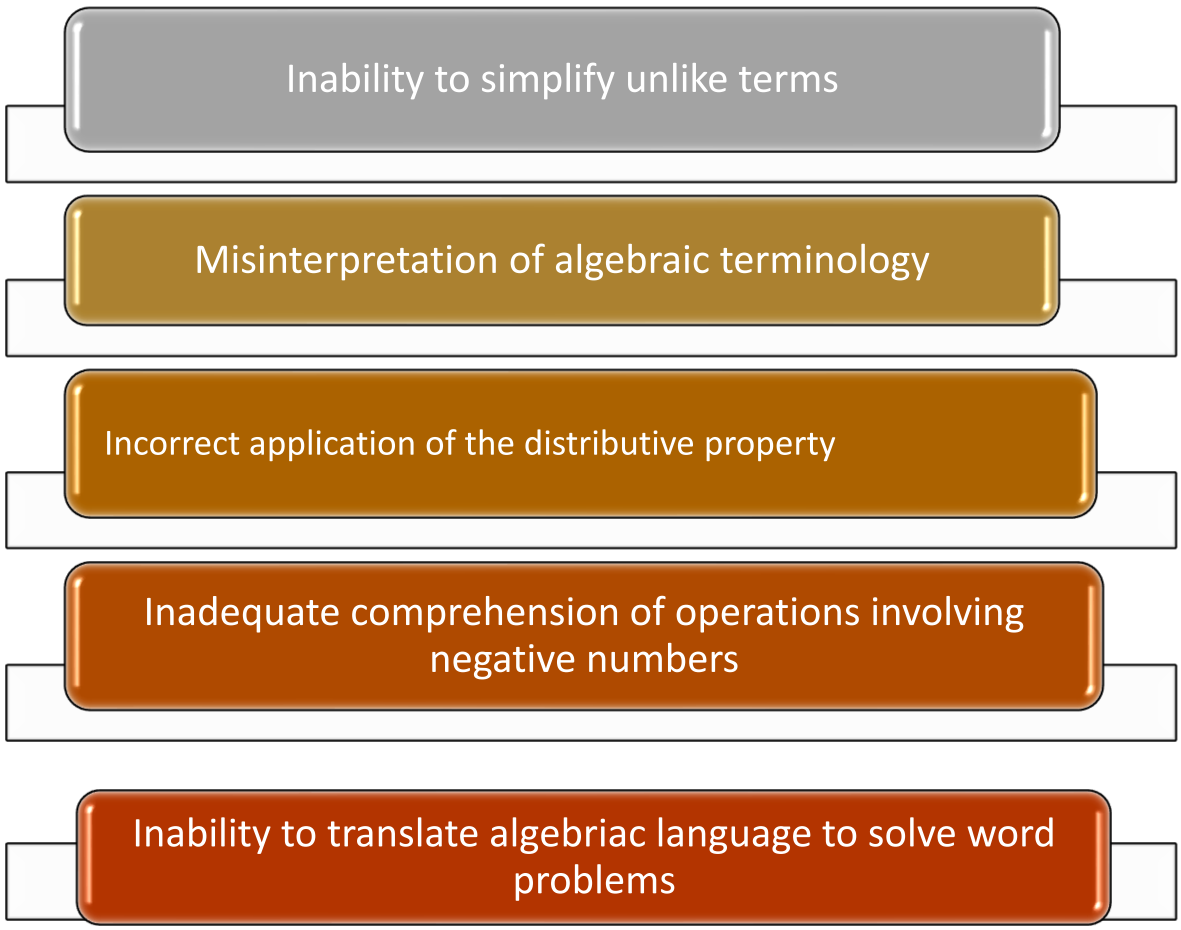


Figure 4.31

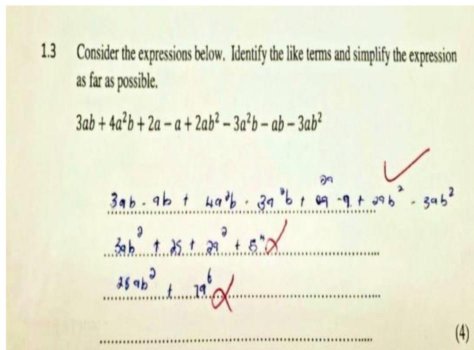
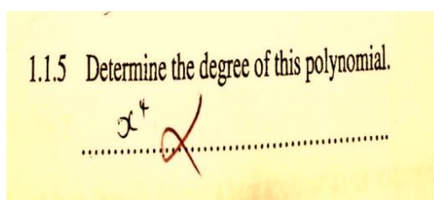
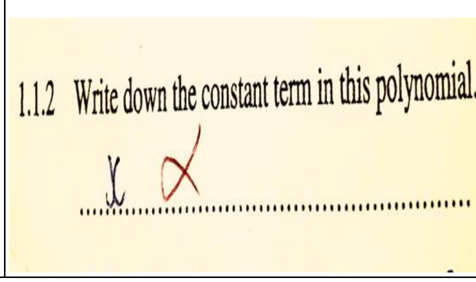
Conceptual Gaps Identified in Learners' Solutions

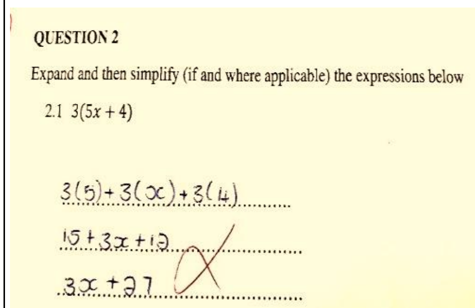
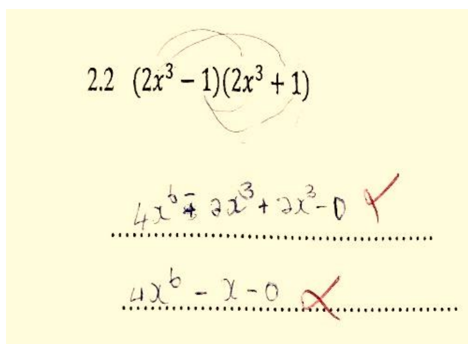
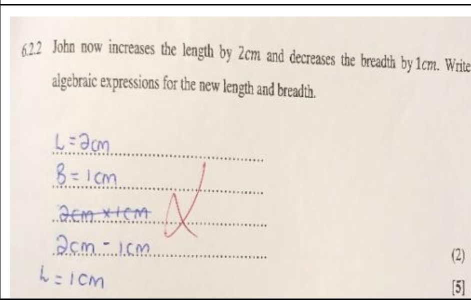
4.6.1 Interpretations of the Conceptual Gaps

The identified gaps were briefly interpreted for easy understanding and summarised in Table 4.1. The table indicates the conceptual gap, a brief description of the identified gap, a sample of learner's work revealing the challenge and the implications.

Table 4.1

Interpretation of the Identified Conceptual Gaps

CONCEPTUAL GAP	DESCRIPTION	SAMPLE LEARNER'S WORK	IMPLICATIONS
Inability to simplify unlike terms	This category consisted of all learners' challenge with regard to combining algebraic terms.		As shown in the sampled work, the learner was able to put terms in their respective groups. However, after grouping the like terms correctly, the learner does not seem to know what else to do. As shown in the solution, nothing below the first step made mathematical sense.
Misinterpretation of algebraic terminologies	This category consisted of all challenges relating to algebraic expression terminologies or vocabularies used in algebraic expression questions	 	From the first solution, the learner seems to understand the algebraic term 'degree' to be the term with the highest exponent hence wrongly wrote x^4 as the degree of the expression. Similarly, in second solution, the learner seems to confuse the

			algebraic term ‘constant’ with ‘variable’
Incorrect application of the distributive property	The category consisted of learners’ challenge with regard to product of algebraic terms and the distributive law.	 QUESTION 2 Expand and then simplify (if and where applicable) the expressions below 2.1 $3(5x+4)$ 3(5)+3(x)+3(4) 15+3x+12 3x+27	The learner seems to understand brackets to be multiplication and the fact that the outside multiplier needs to be multiplied by every term inside the bracket. However, inseparable terms were unnecessarily separated and makes the entire expansion wrong.
Inadequate comprehension of operations involving negative numbers.	This category consisted of the challenges in respect to operations and direction sign.	 2.2 $(2x^3-1)(2x^3+1)$ $4x^6 \mp 2x^3 + 2x^3 - 0$ $4x^6 - x - 0$	From the solution, although the learner seems to understand the FOIL (First, Outside, Inner, Last) method concept. However, there is a challenge regarding multiplying negative numbers by a positive number (-1×1) as well as adding a negative term to a positive one $(-2x^2 + 2x^2)$.
Inability to translate algebraic language to solve word problems	This category consisted of challenges pertaining to word problems	 6.2.2 John now increases the length by $2cm$ and decreases the breadth by $1cm$. Write algebraic expressions for the new length and breadth. L=2cm B=1cm ~~2cm×1cm~~ 2cm-1cm L=1cm (2) [5]	The learner seems not to understand the statements “increase by and “decrease by” and therefore cannot translate

			the problem to an expression.

4.6.2 Coding the Conceptual Gaps

The five conceptual gaps were later coded using numbers. This was done to identify and project the frequency and dominance of such gaps among learners across all questions. It also helped in presenting the conceptual gaps graphically for easy understanding. Table 4.2 displays the coded numbers used to represent each conceptual gap.

Table 4.2

Code Description

CONCEPTUAL GAP	**CODE**
Inability to simplify unlike terms	2
Misinterpretation of algebraic terminologies	4
Incorrect application of the distributive property	6
Inadequate comprehension of operations involving negative numbers	8
Inability to translate algebraic language to solve word problems	10

Using the codes mentioned above (Table 4.2), I recorded the results of all 40 learners and analysed the conceptual gaps for each question per learner. This analysis helped in identifying the frequencies at which each challenge or error was committed by learners. The frequencies were also recorded, converted into percentage and displayed graphically. Before that, a general description of the test items per section with its corresponding dominant conceptual gap is briefly discussed.

4.7 General Description of Test Items per Section

Although most learners were able to answer the first section of the test item (**Question 1)** which consisted of seven questions (1.1.1 to 1.2), it was also dominated by **Code 4** which is represented by **"Insufficient knowledge of algebraic terminologies"** with the highest in question 1.2 where 29 learners (72.5%) got their answers wrong.

Question 2 (2.1- 2.3) expected learners to primarily expand and simplify given expressions. About eighty-one per cent (81%) of learners' answers were marred with challenges and errors, where **"incorrect application of the distributive property"** representing **Code 6** was dominant. Most learners could not expand the expression to remove brackets, as the distributive property rule was wrongly applied. Other learners wrongfully added the unlike terms within the brackets instead of multiplying by the pre-multiplier or expanding using the FOIL (First, Outside, Inner, Last) method. Most learners also regarded the bracket sign as addition rather than multiplication and added terms.

Question 3 consisted of four questions (3.1, 3.2, 3.3 & 3.4) and required learners to factorise the given expressions. Solutions from learners were dominated by **Code 2 (Inability to simplify unlike terms).** This challenge constituted more than sixty per cent (60%) of the overall conceptual gaps represented by Code 2. Surprisingly, learners did not understand the term "factorise" and consequently did not know what to do. They ended up wrongfully adding unlike terms including those in brackets.

Question 4 consisted of only two questions (4.1 & 4.2). It (question 4) had most of the learners substituting the unknown by numeric values. More than ninety per cent (90%) of the solutions learners provided were flawed by **Code 8 (Inadequate comprehension of operations involving negative numbers).** Most learners substituted the given values into the expressions without attaching exponents and the correct sign as given in the question. Most learners also struggled to simplify (multiply, add and subtract) integers, especially those with negative signs.

Question 5, which was a single question, was dominated by both **Codes 2 and 8** (**Inability to simplify unlike terms** and **Inadequate comprehension of operations involving negative numbers**). Besides the fact that most learners could not divide a monomial by a polynomial, they were inappropriately adding terms that could not be added, and moreover struggled with the negative sign.

Question 6 (6.1, 6.2.1 & 6.2.2) expected learners to exhibit their word problem skills. Learners performed dismally in these test items due to their inability to interpret and translate the given word problems to expressions. This item was obviously dominated by **Code 10 (Inability to translate algebraic language to solve word problems).** More than seventy-eight per cent (78%) of learners could not translate the algebraic statements such as "more than" and "increase by" as addition and "decrease by" as subtraction. Question 6.2.2 for instance was a follow-up question of question 6.2.1. Surprisingly, none of the learners was able to properly translate the question and solve it correctly.

Table 4.3 displays the frequency at which learners exhibited each conceptual gap which was then converted to percentages and presented graphically.

Table 4.3

Percentages of the Conceptual Gap

CONCEPTUAL GAP	**CODE**	**Percentage (%)**
Inability to simplify unlike terms	2	32%
Misinterpretation of algebraic terminologies	4	28%
Incorrect application of the distributive property	6	11%
Inadequate comprehension of operations involving negative numbers	8	16%
Inability to translate algebraic language to solve word problems	10	13%

As indicated in table 4.3, 32**%** of the identified conceptual gaps were in the category of learners "Inability to simplify unlike terms" whereas **28%** of the answer codes were in the category of "misinterpretation of algebraic terminologies". "Inadequate comprehension of operations involving negative numbers" also contributed **16%**. "Inability to translate algebraic language to solve word problems" constituted **13%** while "Incorrect application of the distributive property" accounted for **11%** of the total conceptual gaps.

Generally, learners had **17%** correct answers. These correct answers were borne out of learners' ability to use the correct mathematical approaches or strategies in solving the algebraic expression questions. It also includes the learners' ability to efficiently connect algebraic concepts to solve the problems. Also, **10%** of the questions were unattended to. No effort was made to answer those question. The remaining **73%** of the given answers had some level of challenges, misunderstanding or errors and these were classified as conceptual gaps and illustrated in table 4.3. The overall contribution of individual identified gaps was further illustrated graphically as shown in Figure 4.32.

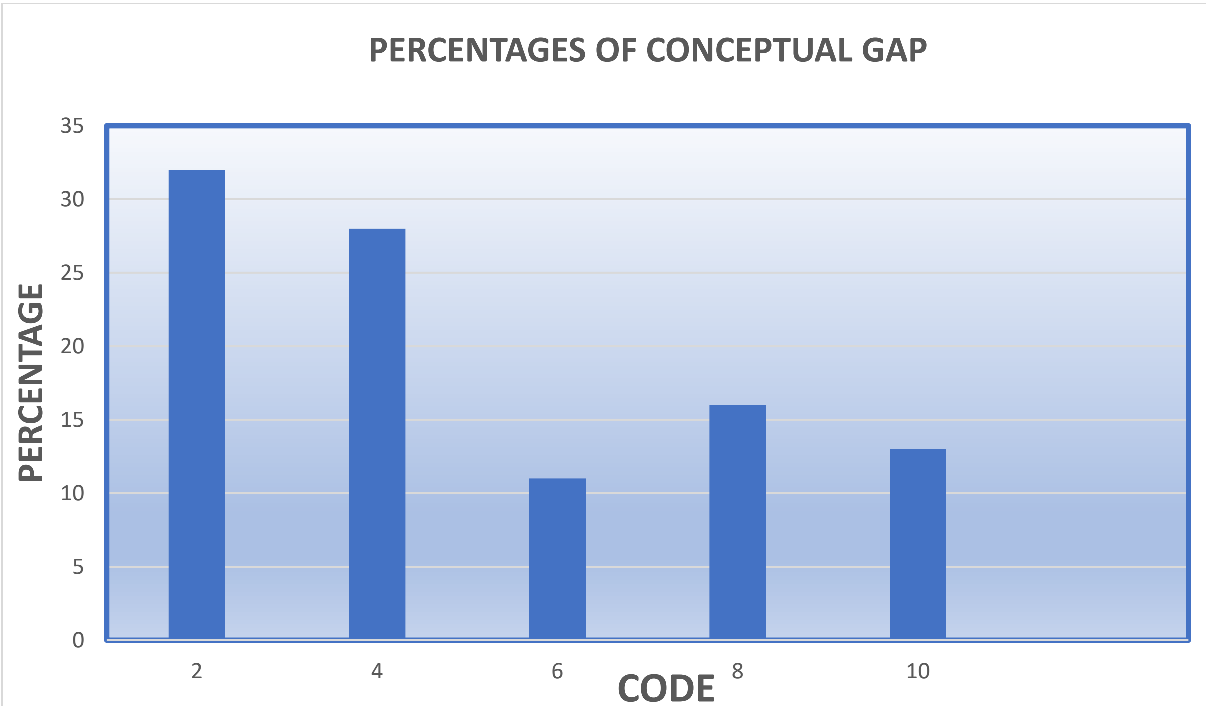


Figure 4.32

Graphical Representation of the Conceptual Gaps

Evidently, the predominant conceptual gap or challenge was the learner's inability to simplify unlike terms, which is denoted by Code 2. This was followed by code 4 (Insufficient knowledge of algebraic terminology). Next was Code 8 (inadequate comprehension of operations involving negative numbers). After that was learners' inability to translate algebraic language to solve word problems (code 10) with the least dominant challenge being the incorrect application of the distributive property represented by Code 6.

4.8 Conclusion

This chapter presented, interpreted and discussed the outcome of the test items and interviews from selected learners. The discussion began by outlining the demographics of the selected school in this study. This was followed by the analysis and interpretation of the results. The interpretation of the results was based on three major themes that emerged from the learners' solutions to the written test. The first theme is the common strategies learners employ when solving algebraic expression problems. These strategies include how learners counted the components that made up a given expression. It also includes how learners identified and factored out the highest common factor of an expression, and lastly, the grouping of like terms together. The second theme that emerged from the learners' results are the mathematical connections created by learners in the process of solving algebraic expressions. These include connections between concepts by drawing from prior knowledge, connections between algebraic expressions and other concepts within the content area numbers, operations and relationships, connections across content areas and lastly, connections between different representations of the concept of algebraic fraction. The third and final theme from the learners' results is the conceptual gaps that manifested when learners solved algebraic problems. These conceptual gaps include learners' inability to simplify unlike terms, misinterpretations of algebraic terminologies, incorrect application of the distributive property, inadequate comprehension of operations involving negative numbers and lastly, their inability to translate algebraic language to solve word problems.

It is therefore worth noting that the three themes outlined and discussed in this chapter directly contributed in addressing majority of the study's research questions including: "what strategies do Grade 9 learners use to solve algebraic expression problems", "how do Grade 9 learners make connections when solving problems involving algebraic expressions" and " What conceptual gaps manifest when learners solve algebraic expression problems"?

The next chapter provides a conclusion of the discussion and suggests possible strategies and recommendations to teachers and all other interested stakeholders on how the conceptual gaps in Grade 9 algebraic expressions could be minimised.

5. CHAPTER 5 : SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

This is the final chapter of the research that summarises the findings, offer recommendations, and suggests areas for further study, focusing on the results from the test and interviews of the participants (Dale & Volpe, 2016). The study findings stem from the themes identified through data analysis, providing direct responses to the research questions. The data are also presented to support the analysis of Grade 9 learners' understanding of algebraic expressions and justify the reasoning behind their approaches. This chapter provides answers to the main research question: How do Grade 9 learners understand algebraic expressions, with reference to the following secondary questions:

- What strategies do Grade 9 learners use to solve algebraic expression problems?
- How do Grade 9 learners make connections when solving problems involving algebraic expressions?
- What conceptual gaps manifest when learners solve algebraic expressions?
- How can the conceptual gaps be minimised?

5.2 Summary of Findings in Terms of Research Questions

The study explored how Grade 9 learners understand and solve problems in algebraic expressions. It specifically focused on how learners use strategies and make conceptual connections when solving problems involving algebraic expressions. Additionally, it identified gaps in learners’ understanding and proposed strategies for addressing and minimising these gaps. The subsequent section provides responses to the research questions that guided this study.

5.3.1 Research Question 1: What Strategies do Grade 9 Learners Use to Solve Algebraic Expression Problems?

This question highlights and describes the various strategies learners used when solving algebraic expressions. The study identified three (3) main strategies, ranging from how learners determine the components of an expression to how it is simplified. The strategies emerged after analysing the test results and the interviews. Among the main strategies observed are: (a) Counting the components that make up an algebraic expression. (b) Identifying and factoring out the highest common factor and (c) Grouping the like terms together. A detailed analysis and presentation of the aforementioned strategies follow:

5.3.1.1 Counting the components that make up an algebraic expression

The findings revealed that learners employed two different approaches to identify the number of terms in an expression. One approach used by 22 of the participants representing 54%, was that learners counted the number of basic operations and added 1 (see Section 4.4.1, Figure 4.2, Excerpt 1). The second approach, used by 9% of the learners, involved counting the actual terms instead of the operational signs. To be accurate, learners wrote counting numbers on top of each term starting from one as explained by learner L8 in Chapter 4 (see Figure 4.3). However, 37% of the learners could not use either of the two mentioned approaches nor any other in determining the number of terms in the given expressions, meaning that, a significant number of the learners lacked the knowledge of how the number of terms in an expression is determined. The finding also indicates that these strategies used by the majority of the learners were the specific method or procedure recommended by their teacher as stated by learner L16 (see Excerpt 1). This is in line with Herheim (2023) who indicated that teachers typically provide students with predetermined strategies or methods intended for use in problem-solving contexts. This is what we call over-reliance on teacher-directed instruction (Estaji & Jafari, 2022). To illustrate this, consider that learner L16 was unable to provide a tangible explanation when asked about the reason for using their approach. Some of the statements or answers learner L16 gave during the interview were:

"*You see, my teacher, Mr …… said ………..*"

"Mr ………...told us to always to do it this way"

This finding also suggests that most learners demonstrated an instrumental understanding (Skemp, 1976). These learners mastered mathematical procedures or rules without justification (Skemp, 1976; Herheim, 2023; Noraini, 2009).

5.3.1.2 Identifying and factoring out the highest common factor

The second strategy identified was that when factorising an expression, learners first identify the correct factor and then visibly divide each term in the bracket by the identified factor. Five (5) learners, representing 12% of the participants, were able to factorise the given expressions using the same strategy. However, most of the participants including Learner L34 (see Excerpt 3, Section 4.4.2), could not use this strategy to completely answer questions that involved factorisation of either binomials or trinomials correctly. This finding is in line with various scholars (Hibi & Assadi, 2022; Baidoo et al., 2020; Mbeki & Mbekwa, 2022) who affirm that learners usually have more difficulty identifying the greatest common denominator when solving algebraic problems that involve fractions and do not follow the right procedure to get their answers right (see Section 2.10.5). Moreover, the inability of most learners (88%) to identify the correct factor and solve the problems indicates that they lack relational understanding (Skemp, 1976). This is because as discussed in Section 2.14.3, one of the indicators (4th) of Skemp's relational understanding is the ability of learners to develop the necessary requirements for certain mathematical concepts (Rachmawati et al., 2021; Skemp, 1976; Skemp, 2006) which in this instance was the identification of the correct factor.

5.3.1.3 Grouping like terms together

The grouping strategy was used by learners when simplifying expressions that involved both like and unlike terms. The finding revealed that when simplifying algebraic expression questions, most learners demonstrated grouping skills and completed the first step by putting the terms in their respective groups. However, learners were unable to extend their understanding beyond the grouping of terms to fully simplify the expressions. The reason for learners' inability to go beyond grouping terms was their lack of conceptual understanding of simplifying algebraic expressions, as expressed by Learner L3 in the interview in section 4.4.3 (see Excerpt 4). This finding agrees with Moru and Mathunya's (2022) observation, which confirms that learners' inability to simplify expressions is mostly due to a misunderstanding of algebraic expression concepts, and this usually leads to what is known as commutative-like *term error*.

In summary, the findings revealed that learners demonstrated an understanding of algebraic expressions by utilising certain common strategies, including counting terms in an expression, identifying and factoring out the highest common factor, and grouping like terms. However, most of the learners could not use the strategies throughout to obtain the correct answers and also could not justify the strategies used. The reasons behind this, which align with the findings of other scholars, include learners' misunderstanding of the algebraic expression concept (Moru & Mathunya, 2022) and a lack of relational understanding (Skemp, 1976). I agree with the assertion of both scholars in that, a real understanding of a mathematical strategy used by the learners should have enabled most them to get the correct answers to the given algebraic expression problems.

I conclude this section and directly respond to the second research question: How do Grade 9 learners make connections when solving problems involving algebraic expressions?

5.3.2 Research Question 2: How do Grade 9 Learners Make Connections when Solving Problems Involving Algebraic Expressions?

In making connections, learners demonstrated an understanding of algebraic expressions by relating prior knowledge to new concepts, linking algebraic expressions and other concepts across content areas, linking different representations of the concept of algebraic fractions and linking different representations with the concept of word problems. These four (4) mathematical connections abilities exhibited by learners are briefly discussed as follows:

5.3.2.1 Making connections between concepts by drawing prior knowledge

The data revealed how learners intended to make mathematical connections between concepts by integrating known concepts (prior knowledge) with new knowledge when solving algebraic expression problems. The finding also revealed that a handful of the learners were able to fully connect, integrate or build on prior knowledge to solve new problems or concepts. Although few learners were able to demonstrate this connection skill, the majority (98%) could not exhibit this connection proficiency, as discussed in Section 4.5.1. This is because most learners could not recall and apply more than one prior concept to solve an algebraic expression problem as required (Faulkner et al., 2020). This finding is consistent with what Herheim (2023) regards as the first common trait about learners with instrumental understanding, which indicates that they treat new ideas as disconnected facts that do not relate to previous ideas (Cui et al., 2023). In other words, they struggle to connect their prior knowledge to newly constructed mathematical concepts (see Section 2.12.2.3 and Table 2.2). It also reveals and confirms that the majority of the participants do not have relational understanding in algebraic expression but rather instrumental understanding (Skemp, 1976).

5.3.2.2 Making connections between algebraic expressions and other concepts across content areas

The findings also showed how learners tried to relate algebraic expressions and other mathematical concepts, such as integers and exponents. This mathematical connection ability was exhibited by learners predominantly in Question 4. Nonetheless, only few of the learners (7.5%) managed to get the first step (substitution) right but could not go further to provide the correct solution. They (the learners) gave a common reason for their inability as “being confused” and “not knowing what to do next” as pointed out by learner L11 (see Excerpt 6). The remaining majority (92.5%) on the other hand showed no sign of this category of mathematical connection ability. This may be due to a lack of problem-solving strategies (Urban & Urban, 2025) or an inability to understand whether the problem requires multiple steps or concepts to be integrated (Li et al., 2025). Another finding from the test revealed that as many as 95% of the learners were unable to calculate the perimeter of a rectangle. It means learners cannot see how different concepts fit into a broader framework of mathematical knowledge. They could not recall the formula to be used. This understanding agrees with Luecke (2025) who indicated that sometimes concepts are taught in isolation. The remaining 5% who could recall and apply the formula had issues in finalising their answers and had difficulties dealing with negative terms, disregarding any term involving a negative number. They also did not indicate the units of measurement with their answers, which signals a lack of understanding of basic principles of measurement. As a researcher, I view this as lack of essential context and clarity.

These findings reveal that the majority of the learners lacked relational understanding (Skemp, 1976). According to Rachmawati et al. (2021), one key indicator of relational understanding is the ability to correlate other mathematical concepts. Such learners have the ability to relate, connect and integrate other mathematical concepts when solving algebraic expression problems (see Section 2.12.3).

5.3.2.3 Making connections between different representations of the concepts of algebraic fractions.

This category of mathematical connection was applied by learners when solving problems that involved the simplification of algebraic fractions. While 25 of the learners (62%) did not have an idea of how the given algebraic fraction could be simplified and got zero points, the remaining 38% were able to simplify algebraic fractions in the test to some extent by breaking down each term into smaller fractions and further divide the terms of the numerator by their respective denominators (see Section 4.5.3). However, none of the 38% of learners was able to completely solve the given problem successfully to the end. This challenge of not fully solving the problem is evident from the perspective of lacking sufficient information (Sidenvall et al., 2024). It might also mean that learners struggled with the correct approach to solve the problem. In other words, they used inappropriate operations (Musi, 2022). They could not recognise the effect or impact of a negative sign in front of a term and could not distribute it. The learners did not realise that the introduction of a negative sign changes the magnitude of a number. This finding coincides with Ncube (2016)'s assertion that learners usually familiarise themselves with the division of algebraic fractions without properly considering the condition that comes with the common factor in the denominator being cancelled. Baidoo (2017) also revealed that learners' inability to simplify expressions involving fractions usually occurs when their knowledge of essential algebraic simplification and expressions is weak (see Section 2.11.4).

5.3.2.4 Making connections between the different representations with the concept of translating word problems into mathematical statements

Findings from the test showed that learners faced difficulties in making mathematical connections in algebraic word problems (see Section 4.5.4). These challenges included difficulties in interpreting the language used in the word problem items (Question 6.1, 6.2.1 & 6.2.2) and the inability to apply mathematical concepts in real-life scenarios. These findings resonate with the study conducted by Samuel et al. (2016), which concluded that learners' inability to solve algebraic equations and word problems is due to a lack of mastery of mathematical language.

In summary, although learners demonstrated mathematical connection abilities to some extent, the way they make connections could not strengthen their understanding of algebraic expressions (Musi, 2022; Baidoo, 2017). I express a similar view because the majority of the learners in the end struggled to connect previous knowledge to new concepts. They also struggled to simplify the algebraic fractions and experienced difficulties in translating and solving word problems.

The next section directly responds to the third research question: What conceptual gaps manifest when learners solve algebraic expressions?

5.3.3 Research Question 3: What Conceptual Gaps Manifest when Learners Solve Algebraic Expressions?

The main findings of this research question included the inability of learners to simplify unlike terms, learners misinterpreting algebraic terminologies, incorrect application of the distribution property, inadequate comprehension of operations involving negative numbers and inability to translate algebraic language to solve word problems. These identified conceptual gaps are briefly elaborated as follows:

5.3.3.1 Inability to simplify unlike terms

This conceptual gap (inability to simplify unlike terms) refers to the needless addition or subtraction of unlike terms (Stemele & Jina Asvat, 2024; Moru & Mathunya, 2022; Rafiepour et al., 2023; Pournara et al., 2016). The study revealed that some learners (32%) demonstrated a basic understanding of the concept of like terms by being able to group them. Nevertheless, most learners added terms of different variables (unlike terms). Some learners also added the coefficients and constant terms together. For instance, as mentioned in section 4.4.3, the 68% of learners who secured the first point in Question 1.3 by grouping, ended up adding and subtracting terms of different variables. As a result, none of these learners was able to present anything mathematically meaningful beyond the first step and provided incorrect answers. This extends the idea of Stemele and Jina Asvat (2024), who indicated that a weak foundation in addition and subtraction of integers is one major cause of learners' struggle in addition and subtraction of like terms.

Another dominant expression that exposed this conceptual gap (learners' inability to simplify unlike terms) was Question 2.1 which required learners to simplify the expression $\mathbf{3(5x+4)}$. Although most learners initially answered the question incorrectly, about 90% of those who succeeded by accurately expanding the expression to get $\mathbf{15x+12}$, went on to give $\mathbf{27x}$ as their final answer. Moreover, Question 3.1 expected learners to factorise the expression "$\mathbf{12x^2+4x^3}$". However, about 70 of learners wrongly joined the two terms to get "$\mathbf{16x^5}$" instead of factorising. In line with this finding, Musi (2022, p. 44) who refers to this conceptual gap as "parsing obstacle" (see section 2.7.2) asserted that the reason behind this challenge is that learners at times feel uneasy to leave a final answer in a form of expression with more than one term. In other words, learners feel comfortable combining their answers to a single term, regardless of the different variables.

5.3.3.2 Misinterpretation of algebraic terms

Misinterpretation of algebraic terms was the second conceptual gap identified after analysing the learners' results. The outcome of the test revealed that few learners showed an absolute understanding of the interpretation of algebraic terms. The greater part of the learners either did not know anything about the terms used in the test or they used them interchangeably. These terms include *"constant", "coefficient", "descending"* and *"degree"* to mention a few. For instance, when Question 1.1.2 asked learners to write the constant term of the expression $2x^3 + 4x - 3x^2 + 8 - x^4$, the most dominant answer (72%) given by learners was $'x'$. Most learners were completely clueless about the term *"constant"* in the world of algebra. The term "*degree"* of an expression or polynomial was also misconstrued as the term with the highest exponent by most of them (the learners). This finding is in consistent with Mosimege et. al (2016), who assert that South African learners generally struggle with algebraic terms and language when solving algebraic expression problems (see Section 2.7.1). Again, as documented in Section 2.7.5, scholars such as Makonye and Stepwell (2016) and Pournara et al. (2016) have revealed that one of the most common reasons learners perform poorly in simplifying expressions is that they misinterpret the term "simplify" as settling on a single term at the end.

5.3.3.3 Incorrect application of the distribution property

Findings from the test revealed that although some learners demonstrated understanding and proficiency in applying the distributive law, most learners struggled to properly operate the brackets. As a result, learners generally performed dismally in all questions that involved brackets which required the application of the distributive law. For instance, more than 75% of the learners did not know the precise term in the bracket to be multiplied by the pre-multiplier in questions such as $\mathbf{3(5x+4)}$. As some learners were multiplying the pre-multiplier only by the first term (inside the bracket) and leaving the other terms, others perceived the bracket as an addition symbol, rather than indicating multiplication, and added all terms together (both inside and outside of the brackets). Other learners also separated terms that could not be separated algebraically when multiplying to get $15 + 3x + 12$ as their answer.

Multiplying two binomials inside brackets such as $(2x^3 - 1)(2x^3 + 1)$ in Question 2.2 was no different. More than 90% of the learners had no idea how to start or where to begin when solving this given problem. This finding indicates that learners lacked the basic skill of operating with brackets such as expansion and the use of the FOIL method. The few learners who successfully distributed the questions but could not complete it also lacked Skemp's (1976) relational understanding. This finding aligns with Mbewe (2013), who states that learners often misapply procedures in other contexts when solving algebraic expression problems, including the application of the distributive property. Hibi and Assadi (2022) also confirm that simplifying an expression that involves brackets and its operations is a major challenge for most learners (see Section 2.11.1).

5.3.3.4 Inadequate comprehension of operations involving negative numbers

Although some learners demonstrated a level of understanding when dealing with terms and numbers involving the negative sign, working with negative terms was found to be a major issue after analysing the learners' results. This conceptual gap was mainly seen when learners were dealing with terms with negative signs whether adding, subtracting, multiplying or dividing. The finding was that most learners demonstrated a neglect of the minus sign. More than 87% of the test items that involved negative terms or numbers were simplified or solved incorrectly. It was evident that learners struggled with both operational and directional signs. This finding is in line with what Aydin-Güç and Aygün (2021) termed as 'the error of neglecting the minus'.

The finding also concurs with the study by Yasseen et al. (2020) which affirmed that while calculating algebraic expressions, learners who have an erroneous or partial knowledge of the minus sign are much more likely to refer to improper techniques (see section 2.7.2).

5.3.3.5 Inability to translate algebraic language to solve word problems.

The data from the learners' results also indicated that solving a word problem is a major challenge for learners in algebraic expressions. The finding was that most learners had a real challenge translating algebraic language into expressions. For instance, 78% of participants could not interpret the algebraic statements such as *"more than", "decrease", "increase", "times"* and other algebraic terms and language used when solving the word problem questions in the test items. There were instances where no learner was able to correctly translate a given word problem into an expression and solve it. Based on the learners' explanations during the interviews (see Excerpts 11 & 12 from Learners L1 and L25), it is clear that translating algebraic statements or language presents a significant conceptual gap in working with algebraic expressions among the learners. This finding is in line with that of Moleko and Mosimege (2021) which assert that text compression, terminology, mathematical language use and the problem structure are among the factors that inhibit learner understanding and the ability to solve word problems accurately. The finding also concurs with those of Dwijayanti and Nugroho (2020), which affirm that lack of understanding of basic concepts of algebraic expressions is a major factor in this conceptual gap.

In summary, five (5) common conceptual gaps were identified when learners solved the algebraic problems. These are learners' inability to simplify unlike terms, misinterpretation of algebraic terms, incorrect application of the distributive property, inadequate comprehension of operations involving negative numbers and lastly, inability to translate algebraic language to solve word problems.

The study strived to validate these claims by building on the work of Mudaly and Narriadoo (2023), Mosimege et al. (2017) and Makonye and Stepwell (2016) which emphasise that South African learners in general usually struggle with algebraic terms and language when solving algebraic expression problems. The findings of the study about learners' inability to simplify unlike terms coincide with the findings of Stemele and

Asvat (2024) and Rafiepour et al. (2023) who revealed that learners usually make errors and struggle to simplify expressions that involve different variables.

The findings on learners' incorrect application of the distributive property also add to what Hibi and Assadi (2022) and Stemele and Asvat (2024) revealed that, learners usually struggle and show less proficiency when solving algebraic problems that involve multiplication of terms and brackets. In respect of learners' inability to translate algebraic language and solve word problems, I express similar sentiment as Kennye and Ntow (2024), Musi (2022) and Myers et al. (2023). All these scholars acknowledge that learners usually misunderstand and struggle with algebraic language translation and perform dismally in solving such problems as revealed in this study.

Concerning learners' inadequate comprehension of operations involving negative numbers, I also concur with scholars like Yasseen et al. (2020), Aydin-Güç and Aygün (2021) and Naidoo (2023) who all affirm that when solving algebraic problems, learners usually struggle and are unable to solve problems especially when terms involve negative coefficients.

The next section provides the responses to the fourth and final research question: How can the conceptual gaps be minimised?

5.3.4 Research Question 4: How can the Conceptual Gaps be Minimised?

This question aims to identify efficient strategies that can be used to minimise the conceptual gaps, with the objective of strengthening learners' understanding. The various strategies suggested in this study are discussed next.

5.3.4.1 Improving learner understanding in simplifying algebraic expressions of like and unlike terms

The discussions on learners understanding of algebraic expressions suggest that using physical representation through diagrams or pictures was considered to be a useful and effective tool in helping learners understand like and unlike terms. According to Yıldız and

Akyüz (2019), teachers are said to have successfully applied multiple forms of representation and effectively engaged learners in the classroom if learners can visualise an algebraic expression in different forms. The study's findings also suggest using real-life scenarios in teaching algebraic expressions to boost learners' understanding of simplifying algebraic expressions. In support of these findings, Norman (2018) reveals that before using symbols to manipulate expressions, teachers must first use concrete objects to enhance learners' understanding of algebraic expression manipulation. I concur and declare the same sentiment as Musi (2022) who maintains that the use of concrete objects such as chocolates in a box with different colours is one of the best ways to teach the concept of like and unlike terms.

5.3.4.2 Employing different approaches to address algebraic terminologies and strengthen its understanding

The outcome of this study suggests that incorporating real-life illustrations can facilitate the teaching of mathematical content in a meaningful way. This, as a result, will show an alternative method of teaching algebraic expressions that provides learners the possibility to relate concepts to situations in real life. Through this, learners can understand and talk about algebraic terms like monomials, binomials, constant, coefficient, degree and the like. This aligns with Johnson (2016), who asserts that it is essential for teachers to use physical representations, as they serve as instruments for communicating and thinking mathematically. I therefore agree with Joseph (2020), who confirms that prior to teaching a new topic, learners must first be exposed to the mathematical language instruction to help improve learner understanding in the algebraic concept, its language and overall performance.

5.3.4.3 Improving learners’ ability to translate algebraic language to solve word problems.

With respect to improving learners’ ability to translate algebraic language to solve word problems, the results suggest that frequent application of practical and relevant scenarios helps learners to understand questions well and produce correct answers (Moleko & Mosimege, 2021).

The findings also reveal that when learners understand what is expected of them in a word problem, they will find it easy to solve the problem and translate it into a simple expression (Zong & Krishnamachari, 2023). It is therefore important for learners to be expressly exposed to algebraic expressions terms and language so that they (the learners) can establish and develop the understanding of word problems and the ability to translate and solve them effortlessly.

I align myself with Demirkus (2023), who recommends that when teaching algebraic word problem concepts, the problem should be broken down into smaller parts that can be understood easily. I believe this strategy will enable learners to easily understand the details of the problem and the requirements to solve it successfully.

5.3.4.4 Addressing misapplication of distributive property and operations involving negative numbers.

According to Star et al., (2015), one effective approach to promote understanding is to teach learners algebraic expressions through the facilitated whole class discussion strategy that demonstrates how different approaches are used for the same problem.

Additionally, allowing learners to compare and assess diverse methods to simplify expressions by inspiring them to connect the structures in problem and answering techniques to figure out the connection between similar and different problems, approaches and solutions can help in minimising misapplication of algebraic expression rules such as the misapplication of the distributive property and sign error (Tabach & Friedlander, 2008). I agree with the fine strategy suggested by Musi (2022) that in addressing misapplication of distributive property, learners should first, be taught the accurate and appropriate way of applying the rules and subsequently be given related problems with wrong or incorrect application of the rules to check if they (the learners) can recognise the misapplication and then asked to rectify the application of rules.

I conclude this section and respond next to the main research question: How do Grade 9 learners understand algebraic expressions?

As per the findings of this study, learners demonstrated their understanding of algebraic expressions by first using certain strategies in answering algebraic expression questions (Durkin et al., 2023). These strategies include the counting of the components that make up an algebraic expression, identifying and factoring out the highest common factor and the grouping of like terms together.

Learners also demonstrated understanding of algebraic expressions through mathematical connections (Sari et al., 2020). Learners exhibited these mathematical connections in four different ways: Connection between concepts by drawing on prior knowledge; connection between algebraic expressions and other concepts across content areas; connections between different representations with the concept of algebraic fractions and lastly, connections between different representations with the concept of word problem translation into mathematical statements (Sari et al., 2020; Jung et al., 2024).

Learners also displayed some conceptual gaps in the cost of demonstrating understanding in algebraic expressions (Edo & Tasik, 2022). These gaps included inability to simplify unlike terms, incorrect application of the distributive property, inadequate comprehension of operations involving negative numbers and inability to translate algebraic language to solve word problems. However, these conceptual gaps can be minimised through various strategies as identified in this study including: improving learner understanding in simplifying algebraic expressions' like and unlike terms (Musi, 2022 ; Ncube & Luneta, 2025); employing different approaches to address algebraic terminologies and strengthening its understanding (Kusmaryono, et al., 2024); improving learner's ability to translate algebraic language to solve word problems (Zong & Krishnamachari, 2023); and addressing misapplication of distributive property and operations involving negative numbers (Musi, 2022).

5.4 Reflecting on the Utility of the Theoretical Framework.

This research was carried out within the context of the understanding of algebraic expressions. The selected perspective was rooted in Richard Skemp's theory of understanding, providing specifically valuable in analysing the understanding of algebraic expressions in Grade 9 learners. The theory also contributed to a deeper understanding of learners' mathematical connections. The interpretation, findings and recommendations presented in this study were guided by Skemp's theory. For instance, one of the characteristics of Skemp's relational understanding is the ability of learners to relate new ideas to previous knowledge and integrate them into conceptual systems (Herheim, 2023). The indicators of Skemp's theory, for instance, informed the investigation as follows:

5.4.1 Interconnectedness of Concepts, Prior Knowledge and Experience

The test questions used in this study were developed such that they required the use of previous knowledge and experience to solve new problems. In other words, the questions required learners to relate and connect prior knowledge to new mathematical concepts and integrate various mathematical concepts in solving algebraic expression problems. For instance, in Question 2.3, learners needed the knowledge of 2-D shapes which influences how they would interpret, integrate and be able to calculate the perimeter of a given rectangle with both length and width given in a form of expression. They (the learners) also needed the knowledge of both exponents and integers which influences how they would expand or distribute an expression that involves brackets like Questions 2.1 and 2.2 of the test items. This is about using the prior knowledge and experience to build on new knowledge (Hattan et al., 2024).

5.4.2 Classifying using Patterns and Features

Some of the questions were also designed such that they expected learners to accurately classify algebraic expressions based on patterns and specific features (Herheim, 2023). Learners needed to be able to exhibit relational understanding abilities in other to be able to recognise, classify and differentiate between monomials, binomials, trinomials and whether an expression is a polynomial or not, using their specific characteristics. This is about demonstrating understanding of algebraic expressions based on patterns and features which is one of the indicators of Skemp's theory of understanding (Herheim, 2023).

In summary, to relate this study to Skemp's theory of understanding, learners who were able to answer algebraic expression questions but were not able to explain or justify the reason behind their procedures and solutions illustrates instrumental understanding. Inversely, the few learners who correctly answered the questions, applied all mathematical principles flexibly and explained the reasons behind their answers demonstrates relational understanding.

Therefore, the findings of this study do not only strengthen previous research studies but also highlights the fact that, even within a group of grade 9 learners from a single school in the Northern Cape, there exists difference or variations of understanding. Hence, this study confirms and support Skemp's distinction between instrumental and relational understanding and further emphasise the need to promote relational understanding in algebraic instructions in our classrooms.

5.5 Limitations of the Study

The study is liable to some limitations that needs to be recognised when interpreting the above-mentioned findings. Firstly, the forty (40) sampled participants used for this study was relatively small as compared to the entire Grade 9 population in the district and it is difficult to claim to have captured the overall performance in algebraic expressions. The small sample size limits the extent to which the outcome or result can be generalised to a wider population. Future research can therefore build upon the findings of this study using larger and more diverse samples across multiple districts or provinces to help enhance the transferability of the study findings and conclusions.

The study was carried out in one out of five districts and only one school in the Northern Cape province. The focus on a single district and school also narrows the geographical scope which suggest that, the study findings basically reflects the experiences of the learners in this specific setting.

The study centered on only one topic (algebraic expressions) and one grade (Grade 9) in the entire GET phase curriculum. The Skemp theory of understanding could also be tried in other subjects, topics and grades.

Moreover, a longer time would have given learners the opportunity to retake the test, use different approaches to answer the questions and then reflect on their learning. This would have enriched the data.

5.6 Recommendations Based on the Findings of the Study

The purpose of this study was to investigate how Grade 9 learners understand algebraic expressions, identify some conceptual gaps if any, and recommend some strategies to help minimise the gaps.

The study findings revealed that learners lack the fundamental concepts or pre-requisite knowledge and relational understanding in algebraic expressions. Moreover, it was revealed that the mathematical connection ability of learners was very low. Although the study focused primarily on learners. It is important to note that teaching and learning go hand in hand and as such, teachers cannot be left out. The recommendations of the study based on the findings are as follows:

5.6.1 Provide Learners with Questions Where They Need to Analyse Errors

The study recommends that teachers should pay special attention, monitor, document and carefully examine learners' misconceptions and errors when teaching algebraic expressions. In other words, they (teachers) should document the common mistakes and errors in algebraic expressions and use this record as a teaching resource to make sure these mistakes are not repeated. This can be achieved when learners are exposed to algebraic expression questions where they need to analyse errors.

5.6.2 Professional Development

The study recommends teachers to engage in continuous professional training which should include subject-specific content like mathematics, to provide them (teachers) with the relevant strategies to effectively teach mathematical concepts including algebraic expressions to learners. Professional learner communities (PLC) at district, cluster and school level should also be established with activities which give teachers the opportunity to work together and share the best teaching practices to help deal with the problems they experience in their Grade 9 classrooms.

5.6.3 Expose Learners to Practical and Real-Life Scenarios

The study recommends that teachers familiarise themselves with Skemp's theory of understanding and other problem-solving models or best teaching practices that promote critical thinking, self-discovery and relational understanding in learners to enable them to know the how and why of their solutions. When teaching algebraic expressions, learners should therefore be exposed to appropriate, practical and real life-scenarios to make the content more simple, realistic, engaging and understandable.

5.6.4 Constant Practice

The study further recommends that learners should be consistent and exert more effort particularly in practising different exercises on simplification of algebraic unlike terms, expansion or distribution of brackets and word problems. This is because constant practice can lead them (learners) to excellence.

5.6.5 Enhance Learners' Mathematical Connection Abilities

The study, moreover, recommend teachers to spend time to plan their lessons and arrange their programs and topics in such a manner that learners would understand, recognise and appreciate the mathematical connections between topics and the various parts of the mathematics syllabus including algebraic expressions.

5.6.6 Use Concrete Representation

With regards to classroom practice, the study recommends teachers to make good use of concrete representations like visual models, maths aids and practical examples to help learners when simplifying algebraic unlike terms. This practice makes abstract algebraic concepts more simple and understandable. Moreover, learners should be exposed to the concept of algebraic expressions in a gradual and progressive manner from simple to more complex task to help establish a strong conceptual foundation among learners.

5.7 Suggestions for Future Research

Considering the fact that the research was conducted in a single school and a rural community, I recommend that the same research be carried out in more schools including those in urban areas to explore common learner understanding and conceptual gaps in algebraic expressions for various learner populations in Grade 9. In other words, the study as stated earlier recommends the need to broaden the scope of inquiry to a larger and more diverse sample across districts and provinces to help enhance the transferability of the study findings.

This study explored the understanding of algebraic expressions with only Grade 9 learners. Extended research that can track learners from Grade 7 to 9 in the exploration of learner understanding of algebraic expressions would be worthwhile to endorse and support this and related research.

Moreover, it is recommended that a research be conducted to explore the teacher’s role in learner understanding, strategies used and conceptual gaps in algebraic expressions in Grade 9.

5.8 Reflexivity

Conducting this study was a unique and valuable learning journey. It made me understand the cyclic nature of research. The understanding of the nature of research made me realise that all phases of research is equitably important and interconnected (Nyanjom, 2018). I also came to the realisation that the analysis of the acquired data is not static nor linear (Salmona & Kaczynski, 2024). I therefore needed to keep putting together the data after several attempts through in-depth and careful reading so as to make sense of it. This was also possible through the outstanding and unwavering support from my supervisor due to a healthy supervisor-student relationship. This research journey moreover taught me that acquiring knowledge is a continuous process and as such, I must keep on learning. The process involved in conducting this research improved my research skills and enlightened me on aspects like ethical issues, the needs and safety of participants as well as the ability to be attentive and carefully listen to learners' interpretation on how they solve mathematical problems.

I will therefore entreat other research students to keep on reading, because knowledge is dynamic and for that reason, new findings and discoveries are consistently being made.

5.9 Conclusion

The findings of this study were presented in this last chapter. Among the critical issues deduced from the findings was the fact that there is a lot of work that needs to be done to improve the understanding of algebraic expressions among South African learners in grade 9.

The chapter also addressed all the study's research questions by providing answers to all the four stated research questions as well as the main research question "how do grade 9 learners understand algebraic expression" based on the study findings.

The chapter moreover provided recommendations based on the findings of the study to teachers, learners, policymakers and other stakeholders. These recommendations included the need for teachers to continuously develop themselves, provide learners with questions where they need to analyse errors, expose learners to practical and real-life scenarios and the fact that learners should practise constantly to grow towards perfection.

The limitations of this study were also specified in this chapter. These include the sample size and the geographical location of the school used.

Lastly, I reflected on my personal convictions, beliefs and experiences throughout the research journey, examining how these may have influenced both the process and the outcomes.

In conclusion, this research underscores the critical importance of developing learners' understanding of algebraic expressions, rather than relying solely on procedural fluency.

By shedding light on the common misconceptions and cognitive gaps learners face, the study contributes valuable insight into how mathematics instruction can be more effectively structured and implemented.

This study provides new enlightenment into how grade 9 learners understand algebraic expressions, addressing the deficit of research on this area in less explored settings like the Northern Cape province. The findings also highlight the importance of teacher education programs that train future teachers to identify and address learners' misconceptions about algebraic expressions by providing teachers with effective instructional strategies.

With regards to curriculum policy, the findings suggest the need for a more intentional sequencing of algebraic concepts. For instance, learners should be exposed to basic reasoning skills before abstract symbolic manipulations.

Ultimately, fostering a deep, connected understanding of algebraic expressions lays the foundation for success in more advanced mathematical thinking and problem-solving.

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APPENDICES

APPENDIX A: **TEST QUESTIONS AND MEMORANDUM**

ALGEBRAIC EXPRESSIONS TEST

LEARNER NUMBER: ....

INSTRUCTIONS

1. Answer all questions.
2. Show all working where necessary in the spaces provided.
3. The use of an electronic calculator is allowed.

QUESTION 1

1.1. Consider the following polynomial: $\mathbf{2x^3 + 4x - 3x^2 + 8 - x^4}$

1.1.1 Write down the number of terms in this polynomial.

………………………………………………….. (1)

1.1.2 Write down the constant term in this polynomial.

………………………………………………… (1)

1.1.3 Write down the co efficient of the term $-3x^2$

……………………………………………….. (1)

1.1.4 Rewrite the polynomial, arranging it in descending order of x

……………………………………………………………………. (1)

1.1.5 Determine the degree of this polynomial.

………………………………………….. (1)

1.2 Complete the table below

Algebraic Expression	Number of terms	Type of Polynomial	Constant term
$7x^3 - 8 + 2x^2$			
$\frac{x}{4} + 3(x^2 - x^4) + 10$			
$\frac{x^2+3}{2} - 9x$			

(6)

1.3 Consider the expressions below. Identify the like terms and simplify the expression as far as possible.

$3ab + 4a^2b + 2a - a + 2ab^2 - 3a^2b - ab - 3ab^2$

…………………………………………………………………

…………………………………………………………………...

…………………………………………………………………...

……………………………………………………………….....   (4)

[15]

QUESTION 2

Expand and then simplify (if and where applicable) the expressions below

2.1 $3(5x + 4)$

…………………………………………

……………………………………………

……………………………………………

……………………………………………   (2)

2.2 $(2x^3 - 1)(2x^3 + 1)$

……………………………………….

………………………………………….

………………………………………….

…………………………………………. (3)

2.3 Calculate the perimeter of a rectangle if the length is $(2x^2 + 3x - 5)cm$ and the breadth is $(x^2 - x - 8)cm$

…………………………………….

…………………………………….

……………………………………..

……………………………………..

……………………………………. (4)

[9]

QUESTION 3

Factorize the following expressions completely

3.1 $12x^2 + 4x^3$

………………………………..

………………………………....

……………………………….. (2)

3.2 $25x + 15x^3 - 10x^5$

…………………………………….

……………………………………..

……………………………………… (2)

3.3 $x^2 + 6x + 5$

………………………………………

………………………………………..

……………………………………… (2)

3.4 $2m^2 - 18$

………………………………………

………………………………………..

……………………………………… (3)

[9]

QUESTION 4

4.1 Calculate the value of each of the following expressions:

$a^2 - 2ab + 5ab^2$ if $a = -3$ and $b = 2$

……………………………………….............

…………………………………………………

………………………………………………… (3)

………………………………………………...

4.2 Determine the value of $6a^2 + 2a - 3$ if $a = \frac{1}{3}$

……………………………………………………..

……………………………………………………

…………………………………………………….

………………………………………………… (4)

[7]

Question 5

Simplify:

$\frac{6p^2+10p}{2p} - \frac{9p^3+12p^2+15p}{3p} + \frac{6p^2+2p}{2}$

……………………………………….

………………………………………...

………………………………………..

…………………………………. ……. (5)

[5]

QUESTION 6

6.1 A father is three times as old as his son. How old is the father if the son is x years?

…………………………………………….

…………………………………………….

……………………………………………. (2)

6.2.1 John draws a rectangle with its length 2cm more than its breadth. If the breadth of the rectangle is y. Write an algebraic expression for its length.

…………………………………………….

…………………………………………..... (1)

6.2.2 John now increases the length by $2cm$ and decreases the breadth by $1cm$. Write algebraic expressions for the new length and breadth.

……………………………………………

……………………………………………

…………………………………………….

……………………………………………. (2)

[5]

TOTAL: 50

MEMORANDUM

QUESTION 1

1.1.1 5 ✓

1.1.2 8 ✓

1.1.3 -3

1.1.4 $-x^4 + 2x^3 - 3x^2 + 4x + 8$ ✓

1.1.5 4 ✓ (5)

1.2

Algebraic Expression	Number of terms	Type of Polynomial	Constant term
$7x^3 - 8 + 2x$	**3** ✓	**Trinomial** ✓	**-8** ✓
$\frac{x}{4} + 3(x^2 - x^4)+10$	**3** ✓	**Trinomial**	**10**
$\frac{x^2+3}{2} - 9x$	**2**	**Binomial** ✓	**None** ✓

(6)

1.3 $3ab + 4a^2b + 2a - a + 2ab^2 - 3a^2b - ab - 3ab^2$

$= 3ab - ab + 4a^2b - 3a^2b + 2a - a + 2ab^2 - 3ab^2$

$= 2ab + a^2b + a - ab^2$✓✓✓✓ (4)

QUESTION 2

2.1 $3(5x + 4)$

$15x + 12$✓✓ (2)

2.2 $(2x^3 - 1)(2x^3 + 1)$

$4x^6 - 2x^3 + 2x^3 - 1$✓ (3)

$4x^6 - 1$✓✓

2.3 Perimeter = $2l + 2b$✓

$= 2(2x^2 + 3x - 5) + 2(x^2 - x - 8)$ ✓

$= 4x^2 + 6x - 10 + 2x^2 - 2x - 16$✓

$= (6x^2 + 4x - 26)cm$✓ (4)

QUESTION 3

3.1 $12x^2 + 4x^3$

$4x^2(3 + x)$ ✓✓ (2)

3.2 $25x + 15x^3 - 10x^5$

$5x(5 + 3x^2 - 2x^4)$ ✓✓ (2)

3.3 $x^2 + 6x + 5$

$(x + 1)(x + 5)$ ✓✓ (2)

3.4 $2m^2 - 18$

$2(m^2 - 9)$ ✓

$2(m + 3)(m - 3)$ ✓ ✓

(3)

QUESTION 4

4.1

$a^2 - 2ab + 5ab^2$

$a = -3$ and $b = 2$

$= (-3)^2 - 2(-3)(2) + 5(-3)(2)^2$✓

$= 9 + 12 - 60$✓

$= -39$✓ (3)

4.2 $6a^2 + 2a - 3$

$a = \frac{1}{3}$

$= 6\left(\frac{1}{3}\right)^2 + 2(\frac{1}{3}) - 3$✓

$= 6\left(\frac{1}{9}\right) + \frac{2}{3} - 3$ ✓

$= \frac{6}{9} + \frac{2}{3} - \frac{3}{1}$✓

$= \frac{6+6-27}{9}$

$= \frac{-15}{9}$

$= -1\frac{2}{3}$✓ (4)

QUESTION 5

5. $\frac{6p^2+10p}{2p} - \frac{9p^3+12p^2+15p}{3p} + \frac{6p^2+2p}{2}$

$= 3p + 5 - (3p^2 + 4p + 5) + (3p^2 + p)$ ✓✓✓

$= 3p + 5 - 3p^2 - 4p - 5 + 3p^2 + p$✓

$= 0$✓ (5)

QUESTION 6

6.1 $3 \times x$ ✓

$= 3x$ ✓ (2)

6.2.1 $y + 2\ cm$ ✓ (1)

6.2.2 Length $= y + 4\ cm$✓ (1)

Breadth $= y - 1\ cm$ ✓ (1)

TOTAL: 50

APPENDIX B: **INTERVIEW GUIDE**

INTERVIEW GUIDE

Process	Interview question
1. Look for question	Please let's look at question…. on page….
2. Read	Please read the question.
3. Explanation	Explain to me how you came up with this solution.
4. Consolidation	Why did you use this strategy?
5. Conflict	Ask conflicting questions if there is conflict in Solving

APPENDIX C1: ETHICAL CLEARANCE CERTIFICATE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2022/06/08

Dear Mr E Kwankye

Decision: Ethics Approval from 2022/06/08 to 2025/06/08

Ref: **2022/06/08/55094589/34/AM**
Name: Mr E Kwankye
Student No.:55094589

Researcher(s): Name: Mr E Kwankye
E-mail address: Kwankyeemmanuel@gmail.com
Telephone: 0719688717

Supervisor(s): Name: Dr SM Kodisang
E-mail address: kodissm@unisa.ac.za
Telephone: +27124293964

Title of research:

Grade 9 learners' understanding of Algebraic Expression. A case of a selected school in the Frances Baard District in the Northern Cape.

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2022/06/08 to 2025/06/08.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2022/06/08 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2025/06/08**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2022/06/08/55094589/34/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof Mpine Makoe
ACTING EXECUTIVE DEAN
qakisme@unisa.ac.za

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150

APPENDIX C2: ETHICAL CLEARANCE CERTIFICATE

UNISA university of south africa

UNISA COLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 08 January 2025

Ref: **2024/11/15/000000431/08/RB**
Name: **MR EMMANUEL KWANKYE**
Staff No.: **55094589**

Decision: Ethics Approval form

Dear **MR EMMANUEL KWANKYE**

Researcher(s): Name: **MR EMMANUEL KWANKYE**
E-mail address: 55094589@mylife.unisa.ac.za
Telephone: **0179688717**

Supervisor: Name: **Dr SM Kodisang**
E-mail address: kodissm@unisa.ac.za
Telephone: **012 429 3964**

Title of research: ***Grade 9 learners' understanding of Algebraic Expression. A case of a selected school in the Frances Baard District in the Northern Cape.***

Qualification: MEd

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above-mentioned research. Ethics approval is granted for the period **2025/01/08** to **2028/01/08**.

*The **low risk level** application was reviewed by the Ethics Review Committee on **15 November 2024** in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. The study's risk level is Category 2- Low risk: Human participants are involved and there is foreseeable risk of inconvenience. Non-vulnerable adult participants and non-sensitive information is involved.
9. No field work activities may continue after the expiry date **2028/01/08.** Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2025/01/08/00000431/07/RB** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

R. Bellonyai

Prof RB Monyai
Acting Head: CEDU Research
monyarb@unisa.ac.za

Prof Mpine Makoe
Executive Dean: CEDU
qakisme@unisa.ac.za

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX D: APPROVAL LETTER (PROVINCE)

IK Nkoane Education House
156 Barkly Road
Homestead
Kimberley 8301

Private Bag X5029
Kimberley 8300
Republic of South Africa

Tel: 053 839 6500
Fax: 053 839 6580/1
Web:ncdoe.ncpg.gov.za

DEPARTMENT OF EDUCATION

Enquiries: J N Horne
Contact No.: (053) 839 6536
Reference: L2.10.2.4.3
Date: 16 November 2022

16 Heuning Street
Jan Kempdorp
8550

Dear Mr E Kwankye

APPROVAL TO CONDUCT RESEARCH

The aforesaid matter with, Ref: L2.10.2.4.3, bears reference.

This letter serves to indicate that approval is granted to conduct research for the dissertation titled: **"Grade 9 learners' Understanding of Algebraic Expression; A case of a selected high schools in the Frances Baard District in the Northern Cape.**

The onus rests with you as the researcher to arrange appropriate and relevant time schedules with the sampled learners in order to conduct the research. A copy of this approval letter must be presented to the school (Principal and SGB) and the District Director as proof that permission for the research has been granted.

The following conditions must be strictly applied during the conduct of your research in the Northern Cape Department of Education (NCDoE). Approval may be withdrawn should any of the conditions listed below be flouted.

Criteria for approval	Comment
Value of the proposed research	The research topic covers a policy position of the Basic Education Sector and thus has value. The study aims to outline and substantiate how Grade 9 learners solve problems involving algebraic expressions.
Policy and Strategic Alignment	The research is aligned to policy and to the Medium Term Strategic Framework (MTSF). The outcome of the research serves to assist in teacher development which falls within the ambit of the priorities of the Basic Education Sector.
Potential benefits to the NCDoE	The NCDoE would benefit from the research as empirical evidence would be available that is derived from the Northern Cape experiences.

2030 NDP

APPENDIX E: PERMISSION AND APPROVAL LETTER (SCHOOL)

UNISA university of south africa

P.O.Box 1124
Jan kempdorp
8550

1st August, 2022.

The principal
Tlhwahalang High School
P. O Box 438
Jan Kempdorp
8550
Email: khathuamos97@gmail.com

Dear Sir,

PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

I, Emmanuel kwankye, am doing research under supervision of Dr S M Kodisang, a lecturer in the Department of Mathematics Education towards a M Ed at the University of South Africa. We are requesting you to permit us in a study entitled: Grade 9 learners' understanding of Algebraic Expression. A case at a High school in the Frances Baard District. The main aim of the study is to outline and substantiate how grade 9 learners solve problems involving algebraic expressions. The study will also explore on the connection between learners existing knowledge and their understanding of algebraic expression. Also, it will explore how learners make connections between concepts used in algebraic expressions.
We will be testing learners by administering test(worksheet) to 40 sampled grade 9 learners and later conduct a face-to-face unstructured interview with some of them.

The benefits of the study are that, it shall help in making policy changes relating to algebraic expressions to determine whether it is safe to introduce algebraic expressions in the earlier years of learning, and to see whether this will reduce the challenges faced by students when they are in their high school years of learning. Moreover, information gathered after conducting the study shall assist teachers and school management, particularly Head of Mathematics Departments in recognising the areas to focus upon with regards to algebraic expression. Thirdly, it will be useful for school administrators and teachers who desire to improve their learners' performance and eliminate the misconceptions of algebraic expressions.

If you would like to be informed of the final research findings, kindly contact Kwankye Emmanuel on 0719688717 or email: kwankyeemmanuel@gmail.com. Should you have any other concerns about the research, you may contact Dr SM Kodisang on 012 429 3964 or email: kodissm@unisa.ac.za.

Yours faithfully,

Emmanuel Kwankye

UNISA university of south africa

TLHWAHALANG HIGH SCHOOL

Tel: 073 842 0014
Tel/Fax: 073 842 0014
Email: tlhwahalanghighschool@hotmail.com
Khathuamos97@gmail.mail

Principal: K.A. Muvhango

Cell: 064 608 2448

P.O Box 438
Jan Kempdorp
Code: 8550

R. Nkomo Street
Val span
Jan Kempdorp
850

Dear Mr. Emmanuel Kwankye

Request for permission to conduct research in the school

Your letter dated 1st August 2022 refers

Your request for permission to conduct study/research at Tlhwahalang High School is hereby permitted.

The school will be happy if you would provide with a copy of your findings. The execution of your study project should not interfere with the normal teaching and learning at school.

We wish you success in your studies.

Regards.

Mr. K.A Muvhango (Principal)

DEPARTMENT OF EDUCATION
TLHWAHALANG HIGH SCHOOL
P.O. Box 438, JAN KEMPDORP 8550
2022 -10- 27
Principal/Deputy:
Tel/Fax: 053 456 1107
NORTHERN CAPE PROVINCE

APPENDIX F: LETTERS OF CONSENT

CONSENT LETTER TO PARTICIPANTS

Dear Sir/Madam,

I am a student at the University of South Africa (UNISA) pursuing a Master's Degree in Education. I am conducting research titled: Grade 9 Learners' Understanding of Algebraic Expressions: A Case of a Selected High School in Frances Baard District in the Northern Cape.

This is part the requirements of my degree programme.

I am therefore, asking you to take few minutes of your time to complete the attached test. Participation is voluntary. You are free to withdraw from participation should it deem right for you. You are assured that all your answers will be treated in confidential manner and will not in any way be identified with you as an individual.

This research is conducted under the supervision of Dr SM Kodisang. Any questions regarding this research can be directed to me or my supervisor through the following details:

Dr SM Kodisang (Supervisor)
Department of Mathematics Education
Tel: 012 429 3964 //0824665968
Email: kodissm@unisa.ac.za.

Emmanuel Kwankye (Researcher/Student)
P. O. Box 1124
Tel: 0719688717
Email: kwankyeemmanuel@gmail.com

Yours Sincerely

CONSENT/ASSENT TO PARTICIPATE IN THIS STUDY (Return slip)

I, ___________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions and am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

LETTER REQUESTING CONSENT FOR PARENTS TO PARTICIPATE IN A RESEARCH PROJECT

Dear Parent

Your son/daughter is invited to participate in a study entitled Grade 9 Learners' Understanding of Algebraic Expressions: A Case of a Selected High School in Frances Baard District in the Northern Cape.

I am undertaking this study as part of my master's research at the University of South Africa.

The purpose of the study is to outline how grade 9 learners understand algebraic expression and to determine whether it is safe to introduce algebraic expressions in the earlier years of learning, and to see whether this will reduce the challenges faced by students when they are in their high school years of learning.

It will also assist teachers and school management at Tlhwahalang High School in recognising the areas to focus upon with regards to algebraic expression.
Moreover, the study will be useful for school administrators and teachers who desire to improve their learners' performance and eliminate the misconceptions of algebraic expressions.

I am asking permission to include your child in this study because he/she is in grade 9 at Tlhwahalang High School.

I expect to have 39 other children participating in the study.

If you allow your child to participate, I shall request him/her to:

Complete a test: He/she will write a test with a duration of 60minutes (1 hour) maximum. The test will be a sample of algebraic expression questions. The test will be written in the school.
Take part in an interview: He /she will be interviewed face-to-face based on the test written in school at an appointed time. Interviews will be conducted formally (structured), questions will be asked verbally and these questions will be focused, clear and encourage open-ended responses.

Permission will be asked if videos or audio recordings will be done during the test and interview.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and will only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. Your child will receive no direct benefit from participating in the study; however, the possible benefits to education are that, the findings will serve as a

APPENDIX G: LANGUAGE EDITING LETTER

Letter of Proof of Editing
Emmanuel Kwankye
Ref. S02511

Sandra Smook
Mobile: +27 82 255 2884
sandra.smook@icloud.com
Plot 427, Kopjes Nedersetting, 9540

7 June 2025

TO WHOM IT MAY CONCERN

I, Sandra Smook, declare that I language edited the master's dissertation by Emmanuel Kwankye (Student Number: 55094589): '*Grade 9 Learners' Understanding of Algebraic Expressions: A Case of a Selected High School in Frances Baard District in the Northern Cape*', in Mathematics Education, to be submitted to the University of South Africa.

I considered English grammar, including spelling, punctuation and idiom, lay-out, formatting and referencing, according to the APA style and the UNISA guideline (supplied by the author) and in line with the given brief from the author.

The editing was done electronically, using MS Word *Track Changes*, to enable the author to accept or reject the suggested changes, thus retaining authorial discretion and the right to assert authorship.

I declare that I am qualified to do such editing as graduate of the University of Pretoria (UP) (BBibl, English, Library and Information Science), and UNISA (Honours), Master of Information Technology, (UP), Post graduate certificate in Editing, (UP), as well as being a member of the South African Translators Institute (SATI) and an Associate member and Accredited Text Editor of the Professional Editors' Guild.

Yours faithfully

7.June.2025

Sandra Smook

Sandra Smook, Editor. BBibl(UP) Majors in English and Information Science, Hons(UNISA), M.IT (UP),
Post graduate Certificate in Editing (UP), (cum laude)
linkedin/sandra-smook

APPENDIX H: TURNITIN REPORT

PAPER NAME	AUTHOR
COMPLETED DISSERTATION.docx	**EMMANUEL KWANKYE**

WORD COUNT	CHARACTER COUNT
44467 Words	**254100 Characters**
PAGE COUNT	FILE SIZE
191 Pages	**5.6MB**
SUBMISSION DATE	REPORT DATE
Jul 30, 2025 3:45 AM EDT	**Jul 30, 2025 4:11 AM EDT**

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