Grade 4 rural teachers' pedagogical approaches of teaching geometric patterns in Sekhukhune East District, Limpopo Province, South Africa

by

Meta Nchabeleng

submitted in accordance with the requirements for

the degree of

MASTER OF EDUCATION

in the Department of Mathematics Education

&

at the

UNIVERSITY OF SOUTH AFRICA

SUPERVISOR: **Prof Hlamulo Wiseman Mbhiza**

February 2026

Table of Contents

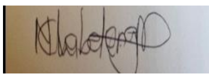
As the candidate's supervisor, I have approved this thesis for submission.

Supervisor: Prof Hlamulo Mbhiza

27/02/2026

DECLARATION

I declare that **Grade 4 rural teachers' discourses and approaches of teaching geometric patterns in Sekhukhune East District, Limpopo Province, South Africa** is my original unaided work and has not been submitted to another university before for assessment or degree purposes. All the consulted sources in this study have been duly acknowledged both in the text and on the end reference list. I completely understand that actions will be taken by the University of South Africa against me if evidence suggest that this is not my original work and that sources have not been acknowledged.



Meta Nchabeleng

27/02/2026

Date

ABSTRACT

This study explored the pedagogical approaches employed by Grade 4 teachers in the Sekhukhune East District when teaching geometric patterns, using Shulman's Pedagogical Reasoning and Action (PRA) framework as an analytical lens for understanding how teachers conceptualise, transform, and enact mathematical content in real classroom settings. Through a qualitative multiple-case study design involving five teachers, the research employed non-participatory and unstructured classroom observations, Video-Stimulated Recall Interviews (VSRIs) and Semi-structured Individual Interviews. One or two episodes per teacher were purposively selected to allow for rich, close-up analysis of the teachers' pedagogical approaches. These data were examined using a custom set of PRA-aligned recognition rules that captured teachers' actions in relation to content comprehension, transformation of representations, instructional discourse, formative assessment, reflection, and pedagogical growth.

The cross-case findings reveals that despite working in resource-constrained rural classrooms, all five teachers enacted pedagogical approaches that foregrounded the structural features of geometric patterns. Teachers consistently relied on concrete and contextual representations, such as matchsticks, coloured chalks, learners' bodies, and familiar artefacts to make abstract concepts visible and accessible. They emphasised structural awareness by intentionally directing learners to identify growth rules, examine repeating units, analyse colour or shape changes, and predict future terms. Lessons were characterised by high levels of learner participation through demonstrations, board work, and group-constructed patterns, with teachers using probing, reasoning-eliciting questions to drive conceptual engagement and support generalisation. Formative assessment was embedded throughout teaching processes as teachers responded to learner thinking in real time, used misconceptions as learning opportunities, and encouraged peer validation of correctness.

The study contributes methodologically by operationalising PRA through recognition rules that made teachers' reasoning visible in empirical classroom analysis, particularly in relation to stabilising structure and transforming content. The findings carry implications for practice, suggesting the need for purposeful use of representations, structured questioning, and multilingual mediation; for policy, highlighting the need for resource provision and PRA-aligned professional development; and for future research, calling for expanded studies on pattern pedagogy, language, and longitudinal development of algebraic thinking. The study offers a nuanced understanding of how rural Grade 4 teachers navigate the teaching of geometric patterns and shows that meaningful, cognitively rich mathematics teaching can be enacted even in challenging contexts.

Keywords: Pedagogical Reasoning and Action; mathematics; geometric patterns; Grade 4; rural education; early algebra

ABBREVIATIONS

ASSESS ADJ – Instructional Adjustment

ASSESS AFL – Formative Assessment Moves

ASSESS AOL – Summative Assessment Evidence

ASSESS FEED – Feedback

CAPS – Curriculum and Assessment Policy Statement

CK – Content Knowledge

COMP KNOW – Content Accuracy

COMP MIS – Identification of Misconceptions

COMP STRUCT – Structural Awareness

COMP VOC – Use of Spatial Vocabulary

DBE – Department of Basic Education

ICT – Information and Communications Technology

INSTEXPL – Explanatory Talk
INSTQUEST – Questioning for Reasoning
INSTEXEMP – Exemplification
INSTPART – Learner Participation
INSTORG – Lesson Organisation

LOTL – Language of Teaching and Learning

LPREC – Limpopo Provincial Research Ethics Committee

NEWCOMP-GROW – Evidence of Growth
NEWCOMP-SHIFT – Change in Teaching Approach
NEWCOMP-APP – Application of New Understanding

PRA – Pedagogical Reasoning and Action

REFL INSIGHT – Pedagogical Insight

REFL PLAN – Future oriented Reflection

REFLINA – Reflection in Action

REFLONA – Reflection on Action

TRANSREP – Representational Choices
TRANSCTXT – Contextualisation
TRANSCPA – Concrete-Pictorial-Abstract Progression.
TRANSSIMP – Productive Simplification

VSRIs – Video-Stimulated Recall Interviews

ACKNOWLEDGEMENTS

First and foremost, I would like to thank God for granting me the opportunity to be admitted into the master's degree, and for giving me the strength, wisdom and perseverance to complete this dissertation. Your love and faithfulness in my life amazes me every day.

I would also like to express my deepest and sincere gratitude to my supervisor, Professor Hlamulo Wiseman Mbhiza, your unwavering support, encouragements and guidance throughout this research study enabled me to persevere and complete my dissertation successfully. Your research expertise and dedication have had a profound impact on my studies and I am grateful for your patience. Your exceptional commitment to us as your students is truly inspiring and I am amazed by your selflessness and willingness to go above and beyond to ensure that we complete our studies successfully. Your humbleness has made a lasting impression on me and I appreciate how you have never made me feel incompetent in areas where I had no knowledge of but rather empowered me to reach my full potential and achieve my goals. You truly have a heart of gold and may God bless you and enlarge your territory.

To my parents, Pelwane Idah Nchabeleng and Matotomale (Kaizer) Nchabeleng, thank you for your prayers, unconditional love and the support you have given me throughout my academic journey. I would also like to send my deepest gratitude to my mother for instilling the love of Mathematics in me, when I was starting my undergraduate degree, my mother and I had a conversation about my majors as I was not sure on which subjects to choose but my mother boldly said, Mathematics has to be one of your majors and of course I took her advice and chose mathematics as one of my majors and ever since then I fell in love with the subject and I knew from that moment onwards that this is the subject that I will want to pursue further studies in and I am also grateful to her for always motivating me to further my studies and for encouraging me to delve in the fascinating world of mathematics. To my brother, Fortune Nchabeleng, thank you for supporting me during the course of this study as you made sure that my child is well taken care of while I was busy with my studies. To my child, Thatego Nchabeleng, your existence has brought such a profound impact in my life and motivated me to be the best version of myself. I thank God every day for bringing you in my life at the time that He did. Thank you for respecting my study times and for being patient with me when I would call you countless times to come and refill my water bottle while I was busy writing my dissertation.

I would also like to extend my heartfelt gratitude to the school principals, teachers and learners of the schools where I collected data, for allowing me into their schools and classrooms. I would also like to thank the Limpopo Provincial Research Ethics Committee and Office of the Premier for granting me the permission to conduct my research in public schools. It would not have been possible for me to conduct this research study without your permission. I am truly thankful. To the University of South Africa M&D Bursary, thank you for making it possible for me to pursue my studies without any financial burden and I will forever be grateful for your financial aid.

And lastly, I would like to thank myself for never allowing certain circumstances to derail me, for consistently pushing myself to work hard and achieve everything that I set out to accomplish. Thank you for recognising that nothing worth having comes easily.

DEDICATION

I dedicate this dissertation to my child, Thatego Nchabeleng. Thank you for being a blessing in my life and I will forever thank God for your existence. You are loved and please bear in mind that “Hard work beats talent when talent doesn’t work hard” -*Tim Nokte.*

List of Tables

List of Figures

Chapter 1: Understanding the Teaching of Geometric Patterns

1.1 Introduction and background of the study

Mathematics learning is fundamentally important as it lays the groundwork for learners' cognitive development and problem-solving abilities, influencing their academic trajectories and shaping their future opportunities (Mbhiza, 2017). Effective mathematics teaching is particularly vital in primary education, where young learners begin to form their understanding of core concepts that will be expanded upon in later schooling (Ibrokhimovich & Mirzaxolmatovna, 2022). Within this context, the teaching of geometric patterns assumes a critical role, as it not only introduces learners to essential mathematical concepts such as symmetry, sequences, and spatial reasoning but also fosters skills in logical thinking and creativity (Du Plessis, 2018). Research indicates that when geometric patterns are taught effectively, including through diverse pedagogical approaches such as cooperative learning and inclusive practices, learners demonstrate improved engagement and comprehension, thereby enhancing their overall mathematics achievement (Bature et al., 2016). Consequently, the incorporation of geometric patterns into the curriculum must be approached with pedagogical rigor, ensuring that teachers have the necessary resources and training to deliver these lessons effectively, especially in under-resourced rural contexts where teaching challenges are prevalent (Andersson et al., 2023; Dahal et al., 2022).

Geometric patterns, defined as sequences of shapes, lines, or objects characterised by repetition are vital in nurturing critical thinking and problem-solving skills among learners (Du Plessis, 2018). However, the pedagogical paradigms surrounding the teaching of such concepts in rural settings like Sekhukhune in the Limpopo Province of South Africa pose significant challenges, which highlight the necessity for rigorous research. Notably, there remains an evident scarcity of research focused specifically on the teaching of geometric patterns within the Intermediate Phase in rural classrooms (Mdodana-Zide, 2023; Nkambule, 2022). This study aims to explore teachers' discourses and pedagogical approaches employed by Grade 4 as they navigate the teaching geometric patterns in Sekhukhune East District.

The education system in South Africa continues to be fraught with challenges, notably in rural areas and schools where resource scarcity and insufficient teacher training prevail (DBE, 2020). The Sekhukhune East District exemplifies these issues, characterised by dilapidated infrastructure and a lack of educational materials, which severely hinders learner access to quality mathematics education. Alarmingly, research addressing mathematics education in South Africa has traditionally marginalised rural contexts (Mbhiza, 2021), necessitating studies like the current one to diversify and expand the research locale and landscape concerning the teaching of mathematics in these areas. A focus on rural mathematics teaching in the current study not only demonstrate the necessity for equitable research representation but also aims to illuminate the unique dynamics that influence rural mathematics education, facilitating the development of targeted interventions that could enhance teaching practices and learner outcomes (Mbhiza, 2024; Nhlumayo, 2024; Adelabu et al., 2022).

In addition to the above discussion, the Grade 4 level in South Africa serves as a crucial transitional phase in mathematics education, marking the shift from foundational learning to more complex and abstract mathematical concepts (Tshesane, 2014). This grade level is pivotal because it is typically where learners are introduced to more sophisticated mathematical ideas. Research has highlighted the importance of effective pedagogical approaches in this transition; for instance, the use of guided discovery learning has been shown to enhance learner achievement in mathematics by fostering active participation and deeper understanding of content (Said et al., 2019; Mabhoza and Olawale, 2024). Moreover, the Southern and Eastern African Consortium for Monitoring Education Quality (SACMEQ) assessments indicate that learners often struggle in these early years, suggesting that effective teaching in Grade 4 can significantly impact overall learner performance and attitude towards mathematics (Mawela & Mahlambi, 2021). Therefore, ensuring that Grade 4 teachers are equipped with the necessary pedagogical skills and content knowledge to teach geometric patterns effectively is critical; it does not only supports the immediate educational needs of learners but also addresses broader issues of educational equity and quality in mathematics teaching across South Africa (Mbhiza, 2024). It is the recognition of Grade 4 as a critical transitory year that motivated me to explore and understand how rural teachers in Sekhukhune District make mathematical concepts relating to the concept of geometric patterns available for learners through the discourses and approaches they employ during teaching. The following section presents the problem statement for the current study.

1.2 Problem statement

The teaching of geometric patterns is vital for fostering foundational mathematics skills in learners, yet insufficient research exists regarding its implementation in rural contexts, particularly in the Sekhukhune East District of Limpopo Province. This absence of empirical research within rural areas and schools limits our understanding of how Intermediate Phase teachers navigate the often-cited resource constraints and pedagogical challenges when teaching mathematics in those contexts (Nkambule, 2017; Mbhiza, 2024). While it is widely acknowledged that rural schools suffer from a lack of learning materials, inadequate infrastructure, and insufficient training for teachers, the specific strategies that teachers employ to overcome these barriers in the teaching of geometric patterns remain largely unexplored (DBE, 2020; Grissom et al., 2015). This gap in literature leads to a significant lack of empirical knowledge regarding how teachers in these environments teach and facilitate learners' understanding of mathematical concepts, with a particular focus on geometric patterns.

While there exists a consensus about the widespread inadequacy of teaching and learning resources in rural schools, it remains unclear how teachers effectively navigate these resource constraints when addressing topics such as geometric patterns (Ingersoll & Tran, 2023; Mabena et al., 2021; Mbhiza et al., 2024). The limited number of studies conducted on this subject, particularly in the Limpopo Province and in other rural South African schools, illustrate the urgency for a comprehensive exploration of pedagogical approaches in this context. The current study explored teachers' pedagogical approaches while teaching geometric patterns at Grade 4 level in rural Sekhukhune East District, to address the identified research gap.

Prior studies suggest that successful pedagogical approaches involve employing visual aids, connecting geometric data to real-life scenarios, and encouraging rule-based learning, which can enhance learners' understanding (Mamolo & Glynn-Adey, 2023). Nevertheless, without targeted research in these South African rural classrooms, it is unclear how effectively these strategies are implemented and how they influence learner engagement and comprehension. In addition, socio-economic disparities prevalent in rural settings further complicate educational outcomes, as they limit opportunities for meaningful engagement and diminish learners' confidence in mathematics (Mawela & Mahlambi, 2021).

Furthermore, numerous studies indicate that South African learners consistently underperform in mathematics, with those in rural areas particularly affected by this trend (Mosimege & Winnaaar, 2021; Taylor, 2021). The Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ) has reported that South African learners demonstrate low performance in mathematics compared to their peers from other countries, and this issue is exacerbated in rural contexts where resource constraints are pronounced (Mawela & Mahlambi, 2021). It can be said that the lack of adequate teaching materials and infrastructural support, combined with socio-economic challenges, further impedes these learners' ability to engage effectively with the mathematics curriculum. Moreover, the teaching methodologies employed in these rural settings are reported to often fail to accommodate the diverse learning needs of learners, which contributes to low engagement and understanding of essential mathematical concepts (Mabhoza & Olawale, 2024). Addressing these disparities requires focused research and intervention strategies aimed at understanding the unique challenges faced by rural teachers and learners in mathematics and developing effective pedagogical practices to improve learners' learning outcomes.

Moreover, despite an increasing recognition among mathematics education researchers worldwide that effective methodologies must include the provision of challenging tasks that foster high-level thinking among learners (Cai & Rott, 2024; Silver & Mesa, 2011), there is a notable absence of studies that specifically investigate how rural teachers engage learners with such tasks within the context of geometric patterns. This lack of research not only hampers our understanding of effective pedagogical strategies but also contributes to the perpetuation of educational inequities in rural settings (Mbhiza, 2021). As a result, this study aims to fill this research gap by examining the teaching practices employed by Grade 4 teachers in Sekhukhune East District when introducing geometric patterns, thus promoting a deeper understanding of how to create an effective learning environment that caters to the unique challenges faced by rural teachers and their learners.

1.3 Rationale of the study

This investigation into the teaching of geometric patterns within the context of Grade 4 mathematics in rural Sekhukhune East District, Limpopo Province, addresses a significant gap in the existing literature on mathematics education in South Africa. Despite the recognised importance of geometric patterns as a foundational component for developing critical thinking and problem-solving skills, there is a notable lack of empirical research focusing specifically on the pedagogical practices of rural teachers in teaching these concepts (Mbhiza, 2024). The unique challenges faced by teachers in rural settings, such as inadequate resources, insufficient infrastructure, and socio-economic barriers, have often been overlooked, limiting our understanding of how to create engaging and effective learning experiences in these environments.

Research highlights the importance of contextualised teaching that resonates with learners' lived experiences and cultural backgrounds, which can significantly impact their engagement and comprehension in mathematics (Maja, 2023). By examining how teachers navigate resource constraints and adapt their teaching methods, this study sought to illuminate effective practices that could not only enhance understanding of geometric patterns but also promote a supportive learning environment conducive to meaningful engagement with mathematical concepts.

This study is particularly timely as it aligns with calls for contextualised research methodologies that reflect the unique challenges of rural education (Sibuyi et al., 2024). Furthermore, as South Africa continues to grapple with educational disparities, the findings of this research will inform teacher training programs and resource allocation strategies tailored to the specific needs of rural teachers. By investigating the intersection of geometric patterns, pedagogy, and learner engagement, this study aspires to provide a valuable research knowledge for improving mathematics education in disadvantaged contexts, thereby contributing to broader discussions of educational equity and inclusivity across South Africa. In the following sub-sections, I specifically elaborate on the rationale for each key tenet underpinning the focus of the current study.

1.3.1. Emphasising Grade 4 Mathematics

The Intermediate Phase, particularly Grade 4, plays a pivotal role in establishing learners' mathematical comprehension and serves as a critical transition point for building skills that are essential for future academic success. Research underscores the importance of equipping learners in this stage with robust mathematical competencies, as they form the foundation not only for personal financial literacy but also for careers increasingly reliant on analytical abilities and data processing (Perrill, 2020; Mensah & Baidoo-Anu, 2022). Addressing the challenges present in mathematics instruction at this level holds significant potential for influencing learners' long-term success in mathematics. Conducting research focused on Grade 4 teachers is vital to bridge the existing research gap and identify effective teaching practices that can enhance mathematics education at this crucial level.

1.3.2. Concentration on Geometric Patterns

Geometric patterns are fundamental concepts that underpin the development of algebraic reasoning and problem-solving skills (Blanton et al., 2015; Pinto & Cañadas, 2021; Radford, 2018). However, studies have shown that learners frequently struggle to express their understanding of patterns in general, specifically geometric patterns, which can detrimentally affect their performance in subsequent educational phases (Sitabkhan & Platas, 2018). It is essential to investigate how teachers facilitate the learning of geometric patterns to address this gap and bolster the mathematical abilities of learners, ensuring they are adequately prepared for future mathematics learning.

1.3.3. Focusing on Teachers Rather than Learners

Although learner achievement remains critical, this study prioritised the experiences of teachers for several compelling reasons. Primarily, teachers are instrumental in crafting learning environments that foster the development of mathematical understanding (Mbhiza, 2021). Their expertise in both subject content and pedagogical techniques significantly shapes their capability to effectively relay complex mathematical ideas to learners (Herbert et al., 2015). Additionally, it is crucial to recognise the specific pedagogical approaches utilised by mathematics teachers, especially in rural settings, where it is often cited in education research that many teachers lack comprehensive training and content knowledge (Msimango et al., 2020; Nkambule, 2017). A closer examination of teachers' pedagogical approaches provides valuable insights necessary for addressing systemic issues and enhancing the quality of mathematics teaching; hence I decided to research with teachers in the current study.

By focusing on the key areas discussed above, this study makes a significant contribution to the field of mathematics education, particularly in rural contexts, enriching our understanding of effective practices and fostering equitable educational opportunities for all learners. In essence, this research not only sought to fill a critical gap in mathematics education literature but also to enhance the teaching and learning of geometric patterns in rural primary schools. It is currently assumed that addressing these dynamics will ultimately enrich the educational experiences of rural learners, preparing them for future academic challenges and enhancing their overall mathematical competence and confidence. The following section focuses on the purpose of the current study.

1.4 Purpose of the study

The primary purpose of this study is to explore the pedagogical approaches employed by teachers in rural Sekhukhune East District. This study aims to yield a comprehensive understanding of the unique pedagogical approaches that rural teachers employ while teaching geometric patterns, as well as the opportunities available for enhancing mathematics teaching in these contexts. The study contributes to the creation of context relevant teaching practices in the teaching of geometric patterns. That is, recognising the diverse cultural backgrounds of learners in Sekhukhune is important for fostering an inclusive and engaging learning environment. By examining the interplay between Grade 4 teachers’ pedagogical approaches and the context of rural education, this study sought to offer insights that can lead to the enhancement of pedagogical approaches for rural mathematics education.

Additionally, this study sought to identify and understand the challenges faced by rural teachers in Sekhukhune regarding the delivery of effective mathematics education, particularly in the context of geometric patterns. Through generating information on the pedagogical approaches teachers employed to overcome these challenges, the study sought to highlight effective practices that can enhance learner comprehension and application of geometric concepts. In line with this purpose, the following section presents the research objectives for the study.

1.5 Objectives of the study

The primary objective of this study is to investigate the teaching approaches of Grade 4 rural teachers in Sekhukhune East District, specifically in relation to the teaching of geometric patterns. The specific objectives are as follows:

a) To explore and understand the ways Grade 4 rural teachers in Sekhukhune East District introduce and develop concepts related to geometric patterns.
b) To examine the teachers' explanatory talk during lessons on geometric patterns.
c) To identify factors that shape rural teachers' pedagogical practices while teaching geometric patterns at Grade 4 level.

1.6 Research questions

To guide the current study in exploring the teaching approaches of Grade 4 rural teachers in Sekhukhune East District with respect to geometric patterns, I formulated the following research questions:

1.6.1 Main Research Question:

- The main research question is: What pedagogical approaches do Grade 4 teachers in Sekhukhune East District employ during the teaching of geometric patterns?

1.6.2 Sub-Research Questions:

To answer the main research question, the following sub-research questions were identified:

a) How do Grade 4 rural teachers in Sekhukhune East District conceptualise and develop the topic of geometric patterns in their teaching practices?
b) What are Grade 4 rural teachers' explanatory talk while teaching geometric patterns?
c) What factors influence the pedagogical approaches of rural teachers when teaching geometric patterns in Grade 4, and how do these factors shape their pedagogical choices?

1.7 Operational definitions of concepts in the study

Table 1. *Operational Definition of concepts*

Concept	Operational Definition
Rural	In this study, the term "rural" refers to environments characterised by a low population density and limited access to urban resources and infrastructure. This definition encompasses areas where the daily activities of individuals, including teaching and learning, are conducted largely independent of urban influences. As articulated by Mbhiza (2021), rural settings are marked by specific place-based challenges such as inadequate educational resources, poor infrastructure, and socioeconomic factors impacting residents' quality of life. Additionally, rurality is conceptualised as a distinct social and cultural context that shapes the lived experiences of its inhabitants, including teachers and learners within educational settings (Dube, 2020; Shikalepo, 2020). This operationalisation of the term emphasises the unique dynamics of rural education and the importance of localised approaches to address the needs and strengths of these communities and schools.
Pedagogical Approaches	In the context of this study, "pedagogical approaches" refer to the diverse methods and strategies that Grade 4 teachers employ to facilitate the teaching and learning of geometric patterns in mathematics. This term encompasses the teaching techniques, discourse patterns, and curricular frameworks through which teachers guide learners' understanding, engagement, and application of geometric concepts. Effective pedagogical approaches hinge on creating an interactive and responsive learning environment, wherein teachers actively engage learners in the learning process, adapt their teaching to meet diverse learner needs, and incorporate culturally relevant practices that resonate with the learners' backgrounds and experiences (Ruyembe, 2020; Cattaneo,

	2017). Furthermore, these approaches involve the application of innovative and context responsive methods and fostering critical thinking and problem-solving skills among learners. By examining the specific pedagogical approaches used by rural teachers, this study sought to illuminate how these teachers navigate the challenges of their teaching contexts, thereby enhancing the effectiveness of mathematics teaching in rural Sekhukhune East District.
Geometric Patterns	Within the South African Mathematics Education, "geometric patterns" are taught to help learners understand sequences of shapes, lines, and objects, which involves copying and extending simple patterns in the Foundation Phase to investigating and describing more complex pattern relationships in the Intermediate and Senior phases (Department of Basic Education, 2011). As enshrined in the CAPS (Curriculum and Assessment Policy Statement), Geometric patterns also encompass activities that promote understanding of symmetry, transformations (such as translations, rotations, and reflections), and number patterns that allow learners to develop algebraic thinking skills. This operational definition highlights the significance of both visual and conceptual components of geometric patterns, emphasising their role in enhancing critical reasoning and problem-solving abilities within the mathematics domain.
Grade 4	Grade 4 refers to the fourth level of primary education in the South African schooling system, which typically accommodates learners aged 9 to 10 years. This stage is crucial as it marks a transition from basic foundational skills acquired in earlier grades to more advanced competencies in various subject areas, including mathematics. The Grade 4 curriculum emphasises the development of critical thinking, problem-solving abilities, and conceptual understanding. Within the Curriculum and Assessment Policy Statement (CAPS), Grade 4 encompasses the teaching of geometric concepts that are essential for enhancing mathematical reasoning. At this level, learners are expected to engage with more complex mathematical ideas, fostering their ability to draw connections between different concepts and apply their knowledge in practical contexts. The educational focus is on not only mastering foundational skills but also preparing learners for the increased challenges they will face in higher grades (Department of Basic Education, 2011).

1.8 Division of chapters

Chapter 1: This chapter provides an overview of rural mathematics education, focusing specifically on geometric patterns. It details the background of the study, elaborating on rural mathematics education and clarifying the research context. Key components such as the problem statement, rationale for the study, purpose, research objectives, and research questions are presented. Additionally, this chapter includes an operational definition of the key concepts central to the study in Table 1.

Chapter 2: In this chapter, the literature review examines existing research on teachers' pedagogical approaches surrounding the teaching of geometric patterns in rural mathematics classrooms. It compares findings from previous studies on teaching approaches, rural mathematics education, and discusses the various challenges faced by teachers in these environments. The chapter also identifies research gaps that are addressed by the current study's focus.

Chapter 3: This chapter outlines the conceptual framework employed in the study. It emphasises the analysis of rural mathematics teachers' approaches related to teaching geometric patterns. The framework structures the data analysis and presentation, facilitating a systematic approach to the research processes.

Chapter 4: This chapter delineates the research methodology utilised to achieve the study's objectives. It covers the research paradigm, design, and approach, alongside details on the sampling strategy, participant selection, and the data analysis techniques employed in the study. I also discuss how ethical considerations were made in the current study as well as how I maximised the trustworthiness of the study.

Chapter 5: In this chapter, the presentation and analysis of data collected from semi-structured interviews, unstructured and non-participatory classroom observations, and Video-Stimulated Recall Interviews (VSRIs) is done. The focus is on five Grade 4 rural mathematics teachers and their conceptualisations and pedagogical approaches regarding geometric patterns.

Chapter 6: This chapter presents the cross-case analysis and discusses the findings derived from the data analysis.

Chapter 7: It provides recommendations for improving teaching practices and suggests directions for future research based on the insights gained from the study. Additionally, the limitations of the research are acknowledged in this chapter.

1.9. Chapter Summary

In this chapter, I introduced the study by providing a comprehensive overview of rural mathematics education, with a specific focus on the teaching of geometric patterns. The chapter outlines the background of the study, elucidating the significance of addressing mathematics education in rural contexts, particularly in the Sekhukhune East District. The chapter presents the problem statement, detailing the specific challenges that teachers face in effectively teaching learners on geometric patterns. Additionally, it clarifies the rationale behind the study, articulates the purpose, defines the research objectives, and formulates the research questions. Furthermore, the operational definitions of key concepts, such as "rural" and "pedagogical approaches," Grade 4 and Geometric Patterns are meticulously delineated to establish a clear delimitation for the study. The following chapter presents the literature review relating to the focus of the current study.

Chapter 2: Review of Literature on the Teaching of Patterns in Primary Schools

2.1. Introduction

This chapter provides a comprehensive review of literature relevant to the teaching of geometric patterns in Grade 4 within rural South African contexts, specifically relating to the region in focus, Sekhukhune East District in Limpopo. It begins by examining the evolution of pedagogical approaches in education, highlighting shifts from traditional, prescriptive models to adaptive frameworks that prioritise learner engagement and contextual responsiveness. The discussion then narrows to pedagogical approaches in mathematics, emphasising their role in shaping effective pedagogical approaches in resource-constrained rural environments. Following this, the chapter explores research on the teaching of patterns in primary schools, focusing on their significance in fostering algebraic thinking and mathematical reasoning. The review further addresses the role of explicit reasoning in the teaching of patterns, highlighting its importance for developing learners' ability to generalise and articulate mathematical relationships. Throughout, the chapter identifies gaps in existing research, particularly the scarcity of studies on Grade 4 pattern teaching in rural Limpopo and positions the current study as a response to the identified research gaps. In reviewing literature based on these interconnected themes, the chapter establishes a foundation for understanding the complexities of teaching geometric patterns and situates the study within broader discourses on educational equity and effective mathematics pedagogy.

2.2. Evolving Pedagogical Approaches in Education

The conceptualisation of pedagogical approaches in education has experienced a significant evolution, transitioning from rigid, prescriptive models to more dynamic, adaptive frameworks. Historically, pedagogy was rooted in behaviourist theories, such as those articulated by Skinner (1953), which focused on observable behaviours and reinforcement outcomes. The cognitive perspective, particularly Gagné's instructional design (Gagné, 2004), brought attention to mental processes and organised learning sequences, thereby laying the groundwork for more structured teaching approaches. Subsequently, the constructivist theories championed by Piaget (1950), Bruner (1961), and Vygotsky (1978) introduced notions of active knowledge construction and collaborative learning, which continue to be influential in contemporary education. However, the realities of teaching in rural schools present unique challenges that can hinder the effective implementation of these progressive pedagogical approaches. For instance, studies have shown that rural teachers often rely on traditional drilling methods due to a lack of resources and support, resulting in passive learning environments (Rasool, 2018; Muniandy & Yunus, 2022). This situation is compounded by the high turnover rates of teachers (Masinire, 2015), who may not have the opportunity to engage with innovative pedagogical strategies that promote critical thinking and active participation.

In addition, more recent pedagogical theories, including connectivism, have emerged to respond to the realities of learning in varied environments (Siemens, 2005). This shift demonstrates the importance of adaptable pedagogical strategies that cater to diverse learning contexts (Mbhiza, 2017). Du Plessis et al. argue that pedagogical effectiveness hinges on mobility and responsiveness rather than adhering to static models. This reconceptualisation emphasises approaches such as situated learning, service learning, and reflective practice, which aim to bridge the persistent gap between theory and practice, especially in teacher education (Barends, 2022).

Moreover, the integration of Information and Communication Technology (ICT) has been proposed as a critical factor in enhancing pedagogical effectiveness; however, its adoption in rural education globally remains limited, primarily due to infrastructural challenges and inadequate training for teachers (Shukla et al., 2016; Mwapwele et al., 2019; Hasin & Nasir, 2021; Shava, 2022). In the context of South Africa, the integration of ICT in education has garnered attention as a potential means to enhance the learning and teaching of mathematics. Even though technological enhancements in learning can significantly improve learner engagement and outcomes when accompanied by appropriate pedagogical frameworks, the successful implementation of such technologies in rural schools remains limited due to infrastructural challenges and insufficient teacher training. It can also be said that the limited research conducted within rural schools in South Africa has resulted in dearth of knowledge regarding how

ICT integration occurs in those contexts, especially considering the prevalence of resource constraints in those contexts. Notwithstanding this, the current discussion highlights the pressing need for teachers to effectively incorporate technology to elevate their pedagogical practices.

Moreover, the call for critical pedagogy in education, rooted in Freire's liberatory model, continues to influence contemporary thought by challenging power structures and advocating for social justice through education (Hunaepi et al., 2024). Although systematic reviews affirm its positive impacts on critical thinking and civic engagement, critiques point to conceptual ambiguities and inconsistencies between policy and practice (Brookings SPARKS Report, 2023). These critiques affirm the necessity for frameworks that are both globally informed and sensitive to local contexts. That is, the shift toward a more context-sensitive understanding of pedagogy is crucial, as articulated by Du Plessis et al. (2024). This perspective highlights the importance of developing pedagogical frameworks that prioritise adaptability and cultural relevance in rural classrooms. As noted by Mbhiza et al. (2024), understanding specific teaching practices employed by rural teachers in teaching geometric patterns remains under-researched, creating a gap in the literature that this study sought to address. By investigating the pedagogical approaches of Grade 4 teachers in Sekhukhune East District, the current study sought to contribute to a more comprehensive understanding of effective mathematics education in rural settings, ultimately aiming to enhance educational equity and improve learner outcomes in these often-overlooked contexts.

The evolving landscape of pedagogical approaches discussed in this section highlights a move toward fluid and adaptive frameworks that prioritise learner agency, integration of technology, and socio-cultural relevance. The current study fills gaps in the literature by specifically examining the pedagogical practices during geometric patterns lessons, thereby contributing to efforts to enhance the quality of mathematics education within rural South African schools and ensuring that pedagogical reforms are equitable and effective in meeting the needs of rural teachers and learners.

Table 2 presents the summary of the evolution of pedagogical approaches reflects a shift from traditional, prescriptive models toward flexible, context-sensitive frameworks that emphasise adaptability, learner engagement, and socio-cultural relevance. This progression highlights the growing importance of strategies that respond to diverse educational settings, particularly rural contexts where resource limitations and systemic challenges persist.

Table 2. *Key Points and Research Gaps in Pedagogical Approaches Research*

Theme	Key Points
Historical Foundations	- Behaviorism: Focus on observable behaviours and reinforcement (Skinner, 1953). - Cognitivism: Emphasis on mental processes and structured learning (Gagné, 2004). - Constructivism: Active knowledge construction and collaboration (Piaget, 1950; Bruner, 1961; Vygotsky, 1978).
Challenges in Rural Contexts	- Reliance on traditional drilling due to lack of resources (Rasool, 2018; Muniandy & Yunus, 2022). - High teacher turnover limits exposure to innovative strategies (Masinire, 2015).
Technology Integration	- ICT proposed to enhance pedagogy but adoption in rural schools is limited due to infrastructure and training gaps (Shukla et al., 2016; Mwapwele et al., 2019; Hasin & Nasir, 2021; Shava, 2022).
Critical Pedagogy	- Freire's model promotes social justice and critical thinking (Hunaepi et al., 2024). - Implementation challenges include conceptual ambiguity and policy-practice gaps (Brookings SPARKS Report, 2023).
Research Gaps	- Limited studies on ICT integration in rural South African schools. - Under-researched pedagogical practices in teaching geometric patterns.

2.3 Understanding Pedagogical Approaches in Mathematics

Pedagogical approaches in mathematics education have gained significant attention in recent literature, where they are recognised as pivotal in shaping the teaching and learning process as well as enabling learners' effective learning. Effective pedagogical approaches are critical for fostering learners' understanding of mathematical concepts, particularly in contexts characterised by diverse learner needs, such as rural environments (Mbhiza et al., 2024). Recent studies highlight the necessity for adaptive pedagogical approaches that cater to these unique settings (Vaughn et al., 2020; Palinussa et al., 2021). As noted by (Mbhiza, 2024), he emphasises the importance of exploring how rural mathematics teachers engage with and motivate learners in their classrooms, highlighting a gap in the literature related to pedagogical practices tailored to rural learners' contexts and experiences. While studies have been conducted focusing on the pedagogical approaches in mathematics within rural areas (Rasool, 2018; Sempe, 2024; Mbhiza et al., 2024), there is scarcity of research that has been conducted in Limpopo Province to explore and understand the pedagogical approaches teachers employ in the subject. The dearth of studies has resulted in the dearth of research knowledge regarding Grade 4 teachers' pedagogical approaches in teaching Geometric patterns. Accordingly, this study researched with Grade 4 rural mathematics teachers in Sekhukhune East District, to understand how they facilitate the learning of the topic.

Research indicates that traditional pedagogical approaches often fail to address the complexities of teaching mathematics in rural schools. Li et al. (2024) discusses broader educational inequities facing rural contexts and schools; however, they do not specifically focus on mathematics education or achievement gaps in the subject. Additionally, gaps in resources, both material and pedagogical further complicate the challenge faced by rural teachers. The current study's focus on teachers' pedagogical approaches responds to the call for more contextually relevant research in rural (mathematics) education (Masisire, 2015; Mbhiza, 2021; Nkambule, 2022). Through examining the pedagogical actions teachers made during the teaching of Geometric patterns, the study sought to provide insights into how rural teachers in the under-researched Limpopo region navigates the complexities and dynamics of teaching the subject within rural classrooms.

The existing literature highlights the need for a nuanced understanding of mathematics pedagogy that goes beyond mere content delivery, emphasising the need to fostering critical thinking and problem-solving skills in learners. In relation to this, Kanandjebo (2024) identifies the necessity of incorporating sustainable development principles into mathematics teaching, advocating for pedagogical approaches that connect mathematical concepts to real-world issues. This connection not only increases motivation among learners but also reinforces the relevance of mathematics in addressing contemporary challenges. Similarly, Mukavhi and Brijlall (2021) emphasise the importance of adapting teaching methods, particularly considering increased digitalised education during the COVID-19 pandemic, to better engage learners and reduce educational gaps caused by inequitable access to resources. This being the case, the lack of empirical research focusing on the teaching of mathematics within rural Limpopo Province has not offered insights into how teachers in those contexts connect concepts such as Geometric patterns to real-world issues, as well as how they adapt pedagogical approaches as advocated by Mukavhi and Brijlall (2021).

2.3.1. Mathematics Teaching within Rural South African Schools

The learning and teaching of mathematics in rural South African schools have garnered significant attention due to the persistent underperformance of learners in this critical subject area. Research indicates that rural schools face numerous challenges, including inadequate resources, high teacher turnover, and a lack of professional development opportunities for teachers (Masinire, 2015; Muremela et al., 2021; Mbhiza, 2021). These issues contribute to learning environments that are often unideal for effective mathematics teaching. Furthermore, the legacy of educational inequality in South Africa exacerbates these challenges, negatively impacting learners' mathematical literacy and overall academic performance (Muremela et al., 2021; Mbhiza, 2021).

The scarcity of qualified mathematics teachers in rural areas is particularly alarming, as studies have shown that a low number of graduates in mathematics and science education from teacher training institutions significantly contributes to this shortage (Muremela et al., 2021). Additionally, high teacher attrition rates disrupt the continuity of teaching and hinder learning experiences for rural learners. The lack of professional development further limits teachers' abilities to enhance their pedagogical skills, perpetuating a cycle of underperformance in mathematics (Muremela et al., 2021; Mbhiza et al., 2024). Advocating for the need to move away from the deficit understanding of rural schools and teachers, Sempe (2024) elaborates on effective teaching strategies employed by Grade 12 teachers in Free State Province when exploring calculus concepts, highlighting the importance of innovative methodologies in boosting learners' comprehension. Furthermore, Rasool (2018) highlight Foundation Phase teachers' teaching strategies in Mpumalanga Province, particularly in teaching number and geometric patterns, revealing a continued gap regarding how rural teachers can effectively convey complex mathematical ideas. As argued earlier, of concern for the current study is that the previous studies reviewed were in other provinces and not in Limpopo, and the level of schooling they focused on were either Foundation Phase or Further Education and Training (FET). The Intermediate Phase mathematics remains under-researched, particularly within the Limpopo Province. Hence, researching with Grade 4 mathematics teachers within the Sekhukhune region was one way of diversifying both the geographic locale and grade or phase focus.

In addition to the above discussion, while existing studies address broader issues in rural mathematics education, there remains a notable gap concerning specific pedagogical practices used by rural teachers to teach geometric patterns. The current literature primarily focuses on overarching challenges, leaving a void in understanding the significant approaches utilised by Grade 4 rural teachers in Sekhukhune East District. By investigating these practices, this study aims to raise awareness of the complexities inherent in teaching mathematics in rural environments and contribute to the discourse on educational equity in South Africa (Mbhiza, 2021; Mbhiza et al., 2024).

2.4. Research on the Teaching of Patterns in Primary Schools

The teaching of geometric patterns is a foundational aspect of mathematics education, particularly primary school level. A strong understanding of geometric concepts is essential for learners ' future success in mathematics, as it lays the groundwork for more complex mathematical reasoning and problem-solving skills (Wang et al., 2022). This emphasises the pedagogical importance of integrating pattern teaching into the mathematics curriculum to cultivate early algebraic reasoning and enhance overall mathematical competence. However, the specific pedagogical approaches employed by rural teachers to teach geometric patterns have not been extensively studied in the South African context as argued throughout this study. Research indicates that rural teachers often face unique challenges that can hinder their ability to effectively teach these concepts (Mbhiza et al., 2024).

Research indicates that learners in primary school typically develop their understanding of patterns through hands-on activities that involve identifying, creating, and describing patterns (Kaput, 2008; Mawela & Mahlambi, 2021). For instance, the effectiveness of manipulatives and visual aids in teaching patterns has been highlighted in recent literature, with specific methods such as the use of number lines, visual constructions, and interactive elements being effective at promoting understanding (Mukuka & Alex, 2024). However, the implementation of these pedagogical approaches is often hindered in under-resourced educational contexts, particularly in rural areas where access to materials and technology is limited (Mwapwele et al., 2019; Mbhiza et al., 2024). This is evidenced in studies that investigated the challenges faced by teachers in rural schools, where reliance on rote memorisation and traditional teaching methods is prevalent (Adelabu et al., 2022; Mdodana-Zide, 2023; Maharaj & Chauke, 2025; Nhlumayo, 2024).

Despite the recognised significance of teaching patterns in developing mathematical competence, research specifically targeting primary mathematics teachers' practices in rural South African contexts remains sparse. Existing literature primarily addresses the broader educational challenges without delving into the specific pedagogical approaches employed in teaching geometric patterns. This gap is crucial as it prevents the identification of effective pedagogical approaches tailored to the socio-cultural contexts of learners in rural communities. I argue that addressing these gaps requires targeted research that not only identifies effective pedagogical approaches but also considers the complexities of the rural educational landscape. Such an approach can ultimately foster educational equity and enhance mathematical outcomes for learners in primary schools. This discussion resonates closely with Venketsamy’s (2021) assertion that culturally relevant teaching practices can bridge the gap between learners' lived experiences and the formal curriculum, fostering a more inclusive learning environment. The following sub-section focuses on the review of literature focusing on the role of patterns and its mastery in fostering algebraic thinking and mathematical reasoning.

2.4.1. The role of patterns in fostering algebraic thinking

The teaching and learning of patterns in Grade 4 are foundational to developing mathematical thinking and serve as a crucial link to algebraic reasoning. Research emphasises the vital role patterns play in helping learners develop generalisation skills, structural awareness, and symbol sense competencies that are essential for grasping higher-order mathematical concepts (Rivera, 2013; Gould et al., 2023). In Grade 4 mathematics teaching, emphasis is placed on various types of patterns, including repeating, growing, and spatial patterns as stipulated in the Curriculum, Policy and Assessment Statement (CAPS) (DBE, 2011), which foster early algebraic thinking and enhance problem-solving abilities. This teaching is firmly rooted in constructivist principles, advocating for active engagement and exploration among learners. Rivera (2013) supports a graded theory of representation in pattern generalisation, emphasising the importance of moving from concrete to abstract forms of reasoning. Furthermore, Kaput's framework for algebraic reasoning positions pattern activities as essential for aiding learners in symbolising generalisations and engaging in syntactically guided reasoning (Kaput, 2008).

Recent studies have pointed to the effectiveness of manipulatives, visual models, and technology in teaching patterns. For instance, Furner (2024) argues that the integration of manipulatives and digital tools like GeoGebra not only enhances conceptual understanding but also alleviates math-related anxiety among learners. Differentiated instruction has also shown promise in improving mathematical literacy among Grade 4 learners, particularly when tailored to meet diverse learning needs and linked to learners' local contexts (Latumbo, 2024). Evidence suggests that successful teaching of patterns encompasses active learning, cooperative tasks, and problem-solving opportunities, allowing learners to identify regularities and articulate generalisations effectively (Bognar et al., 2025). Nevertheless, many teachers encounter difficulties in designing tasks that promote abstraction and generalisation, which can limit learners' transitions from arithmetic to algebraic thinking (Demonty et al., 2018). Thus, in the current study, it was interesting to observe how Grade 4 rural teachers designed and utilised tasks that promotes such abstraction ad generalisation.

Furthermore, research on early algebra highlights a strong association between pattern activities and the development of algebraic thinking. Systematic reviews have confirmed that engaging learners in pattern-based tasks fosters generalisation, functional thinking, and quantitative reasoning, all of which are crucial for Grade 4 learners as they begin to interpret relationships and express them symbolically (Sun et al., 2023; Ellis & Özgür, 2024). However, rural schools in South Africa face systemic barriers that severely hinder effective teaching of patterns, as studies indicate persistent resource shortages, inadequate teacher training, and infrastructural deficits that compel teachers to rely on rote methods instead of exploratory, inquiry-based approaches (Mdodana-Zide, 2023; Maharaj & Chauke, 2025). The limited integration of ICT further constrains opportunities for dynamic visualisation and interactive learning experiences, as emphasised by Shava (2022). These challenges highlight the urgent need for context-sensitive pedagogical models that leverage locally available resources while also promoting deeper conceptual understanding.

Despite the promising developments in pedagogical strategies for teaching patterns globally, there exists a significant gap in empirical research specifically focusing on Grade 4 pattern teaching in rural South African schools, especially regarding the teaching of geometric patterns. As argued throughout the current study, the existing literature often fails to explore how resource constraints shape pedagogical choices or how teachers adapt their teaching to local realities. This gap positions the current study as a critical contribution to understanding and improving mathematics education in under-resourced contexts. The reviewed literature affirms the pedagogical significance of patterns in fostering algebraic thinking and mathematical reasoning. While innovative strategies and technological tools augment learning in well-resourced settings, rural contexts necessitate adaptive, culturally relevant approaches (Maharaj & Chauke, 2025). Addressing these disparities requires targeted research and professional development initiatives that empower teachers to implement effective teaching approaches for patterns despite systemic challenges.

2.4.2. Promoting Competence with Shapes and their Relationships

The teaching and learning of geometric shapes and their relationships are essential components of mathematics education, particularly in the foundational years of schooling. Research establishes that competence with shapes is crucial for developing spatial reasoning and lays the groundwork for algebraic thinking (Mawela & Mahlambi, 2021). Early experiences with shapes also contribute to children's ability to visualise mathematical concepts, which is fundamental for higher-level mathematical proficiency (Mukuka & Alex, 2024). It is well-documented that children's natural affinity for recognising patterns and shapes forms a bridge toward more complex mathematical understanding (Dehaene, 1997; Du Plessis, 2018). However, as children progress through their educational journey, they often encounter challenges in mapping these concrete experiences to abstract representations, particularly when transitioning from arithmetic to geometric concepts.

Studies have shown that typically developing elementary learners exhibit both approximate number sense and an evolving understanding of shapes; yet this development is influenced by the quality and context of teaching they receive. For instance, Rivera (2010) investigated second-grade learners' pre-teaching competencies related to numerical pattern generalisation, demonstrating how foundational skills in recognising shapes can inform learners' progression toward more abstract mathematical concepts. This is further supported by research indicating that when children engage with shapes through hands-on activities, their ability to understand spatial relationships and develop geometric reasoning improves significantly (Kanandjebo, 2024).

However, in rural South African contexts, there are exacerbating factors that impede effective teaching in geometry and shapes. Challenges such as inadequate resources, limited teacher training, and lack of access to professional development hinder teachers' abilities to implement effective pedagogical practices that promote spatial reasoning and geometric competence. The reliance on rote learning methods in these environments as discussed in earlier sections fails to engage learners actively and limits opportunities for exploratory learning, which is essential for shaping their understanding of mathematical concepts (Bature & Atweh, 2016).

Despite the global advancements in geometric patterns pedagogy, there is a noticeable lack of empirical research focused specifically on how Grade 4 teachers in rural South Africa teach learners in geometric patterns concepts. The current study sought to address these gaps by investigating the specific pedagogical approaches employed by rural teachers, aiming to shed light on how these practices can enhance young learners' understanding of shapes and their relationships.

2.5. The Role of Explicit Reasoning in Teaching Patterns

The role of explicit reasoning in teaching patterns is critical as it facilitates learners' transitions from recognising superficial regularities to articulating generalised mathematical relationships. Research on early algebraic thinking indicates that learners often rely on recursive reasoning, predicting successive terms through addition or repetition; rather than developing explicit reasoning skills that involve formulating general rules to connect term positions with their corresponding values

(Beatty et al., 2018). In their study, Beatty et al. (2013) highlight the significance of explicit reasoning in fostering "perceptual agility," enabling learners to recognise patterns from multiple perspectives and express generalisations that extend beyond immediate observations (Lee, 1996; Beatty et al., 2018). This cognitive skill is paramount for functional thinking and serves as a bridge between arithmetic and algebraic reasoning in primary mathematics.

In the context of geometric patterns, explicit reasoning empowers learners to identify structural properties and articulate governing rules for growth, thereby laying a foundational understanding for algebraic representations (Rivera, 2013; Jackson & Stenger, 2024). However, research indicates that without deliberate scaffolding by teachers, learners may struggle to achieve this transition, often remaining engaged in rote learning without developing the capacity for higher-order reasoning (Mdodana-Zide, 2023; Maharaj & Chauke, 2025). Particularly in rural classrooms such as those found in Sekhukhune region, challenges such as limited access to manipulatives, technology, and professional development exacerbate reliance on repetitive drills, reinforcing recursive reasoning while seldom promoting explicit generalisation (Shava, 2022). Teachers often lack exposure to pedagogical strategies that emphasise structural thinking, such as utilising multiple representations; tables, graphs, and symbolic notation or encouraging learners to justify their reasoning (Sterner, 2024). Moreover, cultural and linguistic factors can significantly influence learners' abilities to interpret and articulate generalisations, emphasising the need for context-sensitive pedagogies that incorporate indigenous knowledge and locally relevant examples (Jojo, 2024; Gulaa & Jojo, 2024).

Recent systematic reviews advocate for explicit teaching approaches, including modelling, guided practice, and structured questioning, to enhance learners' abilities to generalise patterns effectively (Jackson & Stenger, 2024; Chen & Kalyuga, 2019). These strategies align with assertions made by Evans and Martin (2023), who suggests that breaking down complex reasoning tasks into manageable steps can alleviate cognitive overload and support mastery. In rural educational settings, explicit reasoning can be developed through cost-effective strategies, such as employing natural materials for pattern construction, integrating oral reasoning tasks, and utilising collaborative problem-solving to overcome resource limitations. In the context of the current study, explicit reasoning is not considered as simply a cognitive skill but a pedagogical necessity for effectively teaching geometric patterns in rural Grade 4. It enables learners to progress from rote pattern extension to functional thinking and algebraic generalisation (Beatty et al., 2018). The current study highlights the importance of designing interventions that prioritise explicit reasoning through structured, culturally responsive, and resource-adaptive strategies, thereby addressing systemic inequities and enhancing mathematical proficiency in rural classrooms.

2.6. Chapter Summary

This chapter critically reviews literature on mathematics pedagogy within rural South African schools, emphasising the evolution of teaching approaches, challenges in resource-constrained contexts, and the pedagogical significance of geometric patterns in Grade 4. It traces the shift from traditional, prescriptive models to adaptive, context-sensitive frameworks that prioritise learner engagement, technology integration, and socio-cultural relevance. The review highlights systemic barriers, such as inadequate resources, limited ICT adoption, and insufficient teacher training that hinder effective teaching in rural settings, while highlighting the role of patterns in fostering algebraic thinking and mathematical reasoning. It further explores the importance of explicit reasoning as a pedagogical strategy for enabling learners to generalise patterns and transition toward functional thinking. Despite global advancements, the literature reveals a critical gap in empirical research on Grade 4 pattern teaching in rural Limpopo, particularly in Sekhukhune East District, positioning the current study as a necessary contribution to developing context responsive and resource-adaptive pedagogical models aimed at promoting equity and improving learner outcomes in primary mathematics. Table 3 summarises the research gaps that the current study sought to address.

Table 3. *Identified Gaps in Literature in terms of Geographic Location*

Geographic Location	Existing Research Focus	Identified Research Gaps	Relevance to Current Study
Mpumalanga Province	Studies have explored pedagogical challenges and adaptive strategies in rural mathematics education in Mpumalanga Province (e.g., Rasool, 2018; Mbhiza et al., 2024).	Limited focus on specific mathematical topics such as *Geometric patterns* and how they are taught in rural classrooms.	Highlights the need for topic-specific pedagogical insights in rural settings.
Limpopo Province (General)	Sparse literature on mathematics pedagogy; some studies touch on rural education broadly (e.g., Mbhiza, 2021; Nkambule, 2022).	Lack of empirical studies on mathematics teaching practices, especially in intermediate phase (Grade 4).	Justifies the study's focus on Grade 4 mathematics pedagogy in Limpopo.
Sekhukhune (Limpopo)	As far as it could be determined, no known studies specifically addressing mathematics pedagogy in this district.	Absence of research on how teachers facilitate learning of *Geometric patterns* in rural classrooms.	Fills a critical gap by providing context-specific insights into pedagogical approaches.
Other Provinces (e.g., Eastern Cape, KwaZulu-Natal, Free State)	Some research exists on rural mathematics education and digital adaptation (Sempe, 2024).	These studies do not address Limpopo's unique socio-educational context or specific mathematical content.	Reinforces the need for geographically and contextually relevant research.

Table 3 illustrates that while rural mathematics education has been studied broadly, including some studies located in rural schools, there is a significant lack of research focused on Limpopo Province, particularly on Grade 4 teachers' pedagogical approaches to teaching Geometric patterns. Significantly, addressing this gap is crucial for generating context-specific insights that can inform effective teaching strategies in under-researched rural settings. The following chapter presents the theoretical frameworks espoused for the current study.

Chapter 3: Theoretical Framework – Employing Pedagogical Reasoning and Action

3.1. Introduction

This chapter sets out the theoretical and analytical framework that guides the study of Grade 4 rural teachers' pedagogical approaches to teaching geometric patterns in the Sekhukhune East District, Limpopo Province, South Africa. The chapter draws on Shulman's Pedagogical Reasoning and Action (PRA) to conceptualise how teachers understand, transform, plan, enact, assess, and reflect on teaching geometric patterns. I then operationalise each construct of PRA in this study, defining precise, recognition rules and data sources for analysis, to ensure that the observable statements that are made about teachers' pedagogical approaches during geometric patterns lessons are transparent, auditable and replicable. PRA was espoused for the current study because it offers a comprehensive analytic lens for examining how Grade 4 rural teachers teach geometric patterns in context.

3.2. Understanding the Conceptualisation of Pedagogical Reasoning and Action

Shulman (1987) positions PRA as a set of processes through which teachers operationalise their knowledge base, content knowledge, pedagogical knowledge, curriculum knowledge, knowledge of learners, knowledge of the educational contexts, and purposes, by shifting from an initial understanding of subject matter toward increasingly sophisticated and situationally responsive teaching action. Shulman developed the PRA framework to provide a model that explains how teachers transform their professional knowledge into effective teaching practice, emphasising comprehension, transformation, instruction, and reflection, arguing that these critical aspects of teaching had been largely overlooked in prior research and policy (Kong, 2025). Within the present study, PRA was espoused as the lens for examining how teachers interpret geometric pattern content, convert it into teachable forms appropriate for rural Grade 4 learners, and enact context-sensitive pedagogical approaches. As depicted in Figure 1, the PRA framework highlights the necessity of teachers possessing both deep content knowledge and pedagogical content knowledge (PCK), which Shulman (1987) describes as the blending of pedagogical and disciplinary knowledge required to represent educational ideas accessibly for diverse learners.

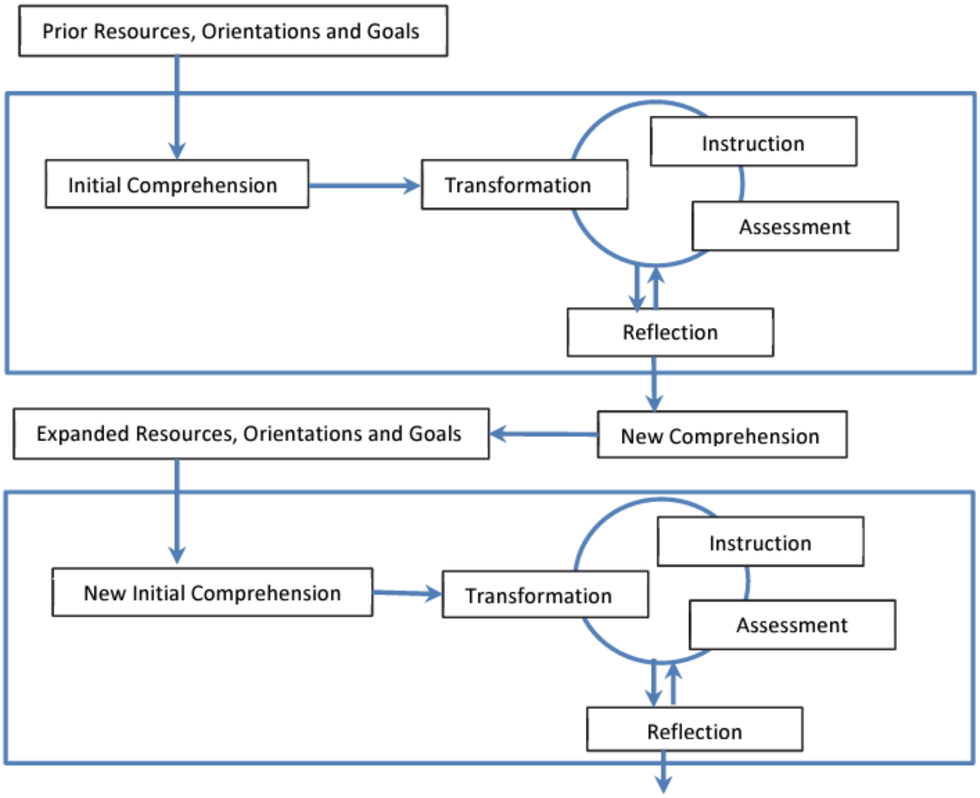


Figure 1. *Illustration of the Pedagogical Reasoning and Action Model (Adapted from Choy et al., 2024, p. 656).*

In rural schools like those located in Sekhukhune East, where resources, linguistic diversity, and learner support vary widely, PRA allowed me to foreground how teachers adapt their pedagogical choices to contextual realities. This aligns with Shulman's claim that teaching requires transforming one's understanding into representations, explanations, and actions that make learning possible for those "who do not yet understand" (Rusznyak, 2022, p. 11). In the following subsections, I discuss the different constructs of PRA, demonstrating their operationalisation within the context of the current study. It is important to note is that each construct of PRA is employed to explore and understand how Grade 4 rural teachers in Sekhukhune East taught the concept of geometric patterns.

3.2.1. Constructs of Pedagogical Reasoning and Action

As outlined in Figure 1, Shulman's (1987) PRA framework comprises a sequence of interconnected components, including comprehension of content, transformation of content, instruction, assessment, reflection, and the development of new comprehension. Taken together, these constructs describe how teachers mobilise their knowledge base to plan, represent, enact, assess, and improve their teaching practice to enable effective learning of the subject matter contents. Shulman view these interrelated pedagogical processes as the foundational elements of effective teaching. Accordingly, in this study, the PRA framework is further utilised as a tool for analysing teachers' reflective engagement with their own pedagogical practices, especially through Video-Stimulated Recall Interviews (VSRIs), which will be elaborated on in chapter 4. Shulman emphasises that pedagogical reasoning is never static; it involves ongoing reflection that feeds back into improved comprehension and re-design of pedagogical actions (Shulman, 1987; Mohamad et al., 2019).

3.2.1.1. Comprehension of Content

Shulman (1987) highlights that effective teaching is fundamentally rooted in a teacher's deep and coherent understanding of the subject matter, asserting that those who teach must first understand the content for themselves, a principle that shapes the conceptual foundation of this study's focus on Grade 4 rural teachers' pedagogical approaches to geometric patterns. This component of the PRA framework stresses that teachers' capacity to facilitate meaningful mathematical learning depends on the extent to which they themselves grasp the key concepts, structures, and relationships embedded in the curriculum (Nkambule, 2022; Mbhiza, 2023). In the context of geometric patterns, such understanding encompasses recognising and articulating features such as repeating and growing pattern structures, identifying units of repeat, interpreting relationships between successive elements, and understanding spatial properties such as symmetry, transformations, and tessellations (Mbhiza et al., 2024). A teacher's ability to unpack these ideas with precision and clarity is therefore essential for supporting learners' conceptual development, as established research consistently demonstrates the strong relationship between teachers' content knowledge and their teaching effectiveness (Ribeiro et al., 2020).

Within the rural context of Sekhukhune East, the importance of strong content comprehension is amplified by systemic and contextual constraints that shape teaching and learning (Magatsela, 2025). Rural teachers often work in environments characterised by limited access to manipulatives, visual aids, technological tools, and physical classroom materials, all of which are particularly valuable for teaching geometric patterns, where visualisation and hands-on exploration are central to conceptual understanding (Rasool, 2018). These constraints mean that the teacher's internalised knowledge becomes a primary resource for teaching, influencing how they generate examples, construct explanations, and design improvised teaching aids. Literature on rural education highlights how such contextual realities influence teachers' pedagogical decisions and their capacity to adapt mathematical content in ways that meet diverse learner needs (Masinire, 2020; Mbhiza, 2021; Nkambule, 2022). As a result, teachers with well-developed conceptual understanding are better positioned to create meaning-rich representations using limited resources, draw on local contexts such as cultural artefacts or environmental patterns, and guide learners toward seeing structure within familiar forms.

The study therefore interrogates how Grade 4 rural teachers' comprehension of geometric patterns informs their pedagogical practices, including their selection and sequencing of examples, use of analogies, incorporation of demonstrations, and development of learner-centred activities that make abstract spatial relationships accessible (Mpitso, 2025). Effective geometric-pattern teaching requires teachers not only to present examples but to strategically choose those that reveal underlying structure, draw attention to variations, and provoke learners to articulate general rules (Du Plessis, 2018). Teachers must also be able to anticipate common misconceptions, such as confusing a term with a unit of repeat, misidentifying symmetry lines, or mistaking a rotated figure for a different shape, which is only possible when the teacher possesses a secure grasp of the content's conceptual landscape (Shulman, 1987). This knowledge influences how they monitor learner thinking during teaching, respond to emerging misunderstandings, and scaffold learners toward more mathematically productive explanations (Strübe et al., 2014). The comprehension component of PRA provides an important lens for this study, enabling an exploration of how Grade 4 rural teachers' conceptual understanding of geometric patterns both informs and is shaped by their pedagogical choices, contextual constraints, and reflective practices. This sets the foundation for examining the next component of PRA: the transformation of content, where teachers convert this understanding into accessible and meaningful pedagogical representations for their learners, for learning geometric patterns in the current study.

3.2.1.2. Transformation of Content

The transformation of content within teaching frameworks, particularly in the context of Shulman's PRA framework, is pivotal for effective teaching in rural educational settings. This model emphasises how teachers translate their comprehension of mathematical concepts into forms that can be easily understood by learners during learning (Shulman, 1987). The focus of this study is specifically on the approaches utilised by rural Grade 4 teachers to convey geometric patterns in ways that align with their learners' experiences and educational requirements (Maudia et al., 2024; Marbun et al., 2022). Given the unique challenges presented by rural education, such as limited resources and diverse learner backgrounds, teachers are expected to develop creative and adaptive pedagogical approaches that facilitate effective mathematics learning (Sarif, 2025).

An essential aspect of content transformation involves not only simplifying complex mathematical ideas but also situating them within the learners' lived experiences (Wilmot & Goldschagg, 2023). For instance, rural teachers may incorporate local examples or culturally significant materials to illustrate mathematical concepts like geometric patterns. This contextualisation is crucial, as it can significantly enhance relatability and engagement among learners (Maudia et al., 2024). Existing research suggests that successful teaching necessitates a deep understanding of the learners' contexts and the capacity to relate mathematical ideas to their daily lives (Maudia et al., 2024; Mpitso, 2025). This practice strengthens the connection between mathematical concepts and learners' realities, thereby promoting deeper understanding and retention.

Furthermore, this study examines how teachers leverage various pedagogical strategies, such as modelling, guided practice, and hands-on activities, to facilitate the learning of geometric patterns in rural classrooms, where learners may benefit immensely from concrete representations of abstract concepts (Maudia et al., 2024; Er & Chongo, 2025). Thus, teachers' ability to transform content is closely linked to their pedagogical content knowledge (PCK), which encompasses the pedagogical methods and strategies that enable effective teaching of mathematical contents (Yeşildere & Akkoç, 2012). This study explored how the PCK of rural teachers influenced their capacity to engage learners and deliver meaningful teaching relating to the concept of geometric patterns. Specifically, teachers who possess a robust understanding of geometric patterns tend to employ effective strategies that foster learner engagement (Maudia et al., 2024; Hill & Uribe-Flórez, 2019).

3.2.1.3. Instruction

This study draws upon Shulman's (1986; 1987) concept of pedagogical content knowledge (PCK) to emphasise the importance of effective pedagogical approaches in fostering learner comprehension of mathematical concepts. Specifically, the investigation centers on the approaches employed by the participating Grade 4 teachers to facilitate understanding of geometric patterns, ensuring that learners learn and own knowledge and skills related to the concept. It was important for the current study to explore and gain insight into how teachers organised their classroom activities, what they said to the learners, as well as what they wrote on the board to bring the concept of geometric patterns into focus. Accordingly, I paid attention to teachers' explanations during the lessons, which involves the teacher's ability to communicate mathematical concepts clearly and effectively (Essien, 2021; Sempe, 2024). Effective explanations serve to clarify complex ideas, making them more accessible and fostering an inclusive learning atmosphere (Mbhiza, 2022). The analysis aims to uncover how teachers articulate geometric patterns and whether their explanations facilitate or impede learner understanding, thereby providing insights into the efficacy of their pedagogical approaches.

In conjunction with the explanations teachers provided during the observed lessons, exemplification is critical in teaching mathematical concepts, geometric patterns for Grade 4 level in the current study. According to Essien (2021), exemplification refers to the use of concrete examples and illustrations to elucidate mathematical ideas, to enable learners to have prototypes about the concept in focus. In this study, I paid attention to teachers' selection of specific examples and how these choices created classroom environments for learners to have conceptual grasp of geometric patterns. In addition, enabling learner participation is yet another vital aspect of effective pedagogy (Adler & Ronda, 2015). By fostering active engagement from learners, teachers facilitate an environment in which learners can collaboratively construct their understanding of geometric concepts (Mbhiza et al., 2024). This approach is grounded in constructivist pedagogies which advocate for exploration and inquiry as fundamental to the learning process. The current study evaluates how teachers encourage participation during lessons on geometric patterns and assesses how these practices facilitated opportunities for learners to learn the concepts.

3.2.1.4. Assessment Practices

Another component of PRA is assessment. According to Shulman (1987), assessing learners' understanding is a crucial component of effective teaching, particularly in the realm of mathematics education. In this study, I focused on how Grade 4 teachers in the Sekhukhune East District of Limpopo Province assess learners' comprehension of geometric patterns. Through employing both assessment of learning and assessment for learning techniques, teachers can gain valuable insights into the learners' grasp of the mathematical concepts and identify areas that require further support (Green, 2018). The concept of assessment of learning pertains to the summative evaluation methods used to determine if learners have reached specific learning outcomes, often through formal measures such as tests or standardised assessments which takes place at the end of the learning process (Alquraan & Al-Shaqsi. 2019). In contrast, assessment for learning focuses more on the formative aspects of assessment, emphasising continuous feedback that informs teaching and enhances learning processes (Veldhuis & Van den Heuvel-Panhuizen, 2020). Effective assessment practices are essential for providing clarity to both teachers and learners regarding the understanding of geometric patterns, as highlighted in the literature on assessment strategies (Maudia et al., 2024; Marbun et al., 2022; Sarif, 2025).

In the context of this study, I observed various assessment methods employed by rural Grade 4 teachers, including formative assessments, observational techniques, and informal assessments, such as questioning and classroom discussions. These strategies served not only to measure learners' understanding but also to enhance their learning experiences. For example, during observed lessons, when teachers identified specific areas where learners struggled with understanding geometric patterns, I explored how they adjusted their teaching strategies accordingly or lack thereof. This dynamic adjustment reflects the importance of a responsive approach to teaching, as it supports targeted support to help learners overcome their challenges with mathematical concepts.

Considering that teachers often serve as primary sources of knowledge within rural schools, assessment for learning becomes particularly significant. This approach allows teachers to better understand the varying levels of prior knowledge and engagement among learners, ultimately helping to create a more inclusive and effective educational environment (Yeşildere & Akkoç, 2012). Additionally, assessment techniques that are contextually relevant help to bridge the gap between the mathematical concepts and learners' lived experiences, increasing the likelihood of meaningful learning outcomes (Shulman, 1987).

3.2.1.5. Reflection on Teaching

Shulman (1987) argues that reflective practice is a vital component of effective teaching. Menegaz et al. (2018) reinforce this notion by asserting that reflection allows teachers to evaluate the success of their teaching strategies and make necessary adjustments to improve learner outcomes. Similarly, Purcell (2012) emphasises that reflection is an iterative process that informs teaching practices and enhances pedagogical reasoning. In this study, I explore how rural Grade 4 teachers reflect on their teaching of geometric patterns, considering both their successes and challenges within the classroom environment. Engaging in reflective practice enables teachers to critically analyse their pedagogical approaches, identify areas that require improvement, and adapt their methods to better meet the needs of their learners (Sun, 2022). Notably, I employed Video-Stimulated Recall Interviews (VSRI) as one of the data generation methods, allowing teachers to reflect on their observed lessons and providing me with deeper insights into their pedagogical reasoning. This is discussed in more detail in chapter 4.

Research indicates that reflective teaching can significantly enhance teachers' understanding of their teaching strategies, teaching challenges, and personal beliefs regarding teaching and learning (Shulman, 2011; Shulman, 2013). For example, seasoned teachers often utilise reflective journals or engage in discussions with colleagues to glean insights into their teaching practices, fostering improved pedagogical effectiveness and greater learner engagement (Ashraf & Zolfaghari, 2018).

Additionally, this study examines the various forms of reflection that participating teachers utilise, including reflection-in-action and reflection-on-action. Reflection-in-action occurs during the teaching process, enabling teachers to make real-time adjustments based on learners' responses and level of engagement (Mbhiza, 2019). In contrast, reflection-on-action takes place after the lesson, as seen in the Video-Stimulated Recall Interviews, which allows teachers to assess what strategies were effective and which were not (Redmond, 2015; Mbhiza, 2019). This emphasis on reflective practice highlights the importance of a continuous feedback loop in teaching, wherein teachers adapt their strategies in response to learner needs and classroom dynamics, to maximise opportunities for mathematics learning for the learners. The process of reflection is envisaged to enable teachers to reach new comprehension about their teaching, addressing the final component of Shulman's (1987) PRA.

3.2.1.6. Development of New Comprehension

The final aspect of Shulman's (1987) PRA emphasises the continuous growth of teachers' professional knowledge and pedagogical skills as they learn from their teaching experiences. In this study, this concept is utilised to investigate how rural Grade 4 teachers in Sekhukhune East deepen and refine their understanding of effective teaching related to geometric patterns over time, especially as they navigate the challenges of teaching in rural classrooms. Through reflective practices, particularly facilitated using VSRI, teachers may gain opportunities to critically assess the strengths and weaknesses of their pedagogical actions. This reflective process has been shown to support the identification of effective practices and highlight areas needing further enhancement (Ashraf & Zolfaghari, 2018; Shulman, 2013). Engaging in such reflection not only improves pedagogical decision-making but is also linked to enhanced learner outcomes (González, 2017). As teachers become more skilled at contextualising geometric pattern concepts in ways that resonate with their learners' lived experiences, they promote deeper understanding (Awofala, 2014; Lotz-Sistka, 2007). Shulman (1987) and Shanley et al. (2019) asserts that effective teaching is not a static set of skills; rather, it is an evolving and dynamic process, with teachers' knowledge continuously shaped and reshaped through their engagement with practical experiences. Thus, the component of new comprehension allowed me to assess whether and how the participating teachers reflected on their classroom practices to reach new understanding of their pedagogical practices and how they could better facilitate effective learning by adapting pedagogical practices that can enable learners' epistemological access. Table 4 details the operationalisation and recognition rules for the different components of PRA for the current study.

Table 4. *PRA Components with Recognition Rules, Codes, and Code Descriptions*

PRA Component	**Operationalisation of PRA Components**	**Recognition Rules & Code Descriptions**
1. Comprehension of Content	Evidence that the teacher accurately understands and explains geometric pattern concepts (e.g., unit of repeat, symmetry, transformations, tessellations); uses correct spatial vocabulary; identifies and addresses learner misconceptions; reveals conceptual coherence in explanations.	**COMPKNOW – Content Accuracy:** Instances teachers demonstrate correct and coherent understanding of geometric-pattern concepts as set out in the curriculum (e.g., repeating/growing patterns, symmetry). **COMPVOC – Use of Spatial Vocabulary:** When the teacher accurately employs mathematical language such as "unit of repeat," "pattern rules" etc. **COMPMIS – Identification of Misconceptions:** When the teacher identifies or addresses common learner misconceptions (e.g., confusing term and unit of repeat, misidentifying pattern rules). **COMPSTRUCT – Structural Awareness:** When the teacher explains or highlights underlying mathematical

		structure (e.g., identifying the pattern rule or structural generalisation).
2. Transformation of Content	Instances where the teacher converts geometric pattern content into accessible representations (manipulatives, drawings, local artefacts); simplifies abstract ideas without distorting them; contextualises tasks in learners' lived experiences; demonstrates movement from concrete → pictorial → abstract and vice versa.	**TRANSREP – Representational Choices:** When the teacher converts geometric pattern ideas into concrete, pictorial, or abstract representations (tiles, drawings, diagrams). **TRANSCTXT – Contextualisation:** Used when the teacher links geometric pattern ideas to familiar rural contexts (e.g., beadwork, brick patterns, traditional crafts etc.). **TRANSCPA – Concrete-Pictorial-Abstract Progression:** When the teacher deliberately moves learners from physical manipulation to drawing and then to reasoning/abstraction. **TRANSSIMP – Productive Simplification:** When the teacher simplifies complex concepts without distorting mathematical meaning.
3. Instruction (Teaching Approaches)	Observable teaching actions involving explanations, questioning, modelling, exemplification, and organisation of learning activities; use of examples and non-examples;	**INSTEXPL – Explanatory Talk:** When the teacher offers clear, logically sequenced explanations of geometric patterns. **INSTQUEST – Questioning for Reasoning:** When the teacher employs probing, focusing, or reasoning-eliciting questions to support understanding. **INSTEXEMP – Exemplification:** When the teacher selects examples and

	facilitation of learner participation; clarity and sequencing of instructional steps.	non-examples to illustrate pattern structure. **INSTPART – Learner Participation:** When the teacher prompts active engagement (e.g., learners constructing patterns, responding, demonstrating on board etc.). **INSTORG – Lesson Organisation:** Applied when the teacher structures the lesson coherently, transitions smoothly, and ensures instructional continuity and coherence.
4. Assessment (Assessment for/ of Learning)	Teacher's use of formative checks (questioning, probing, quick sketches, oral responses), summative tasks, and feedback to determine learner understanding; responsiveness to learner errors; evidence-based instructional adjustments.	**ASSESSAFL – Formative Assessment Moves:** When the teacher checks understanding during the lesson through questioning, observation, mini-tasks, or quick sketches. **ASSESSAOL – Summative Assessment Evidence:** When the teacher administers end-of-lesson/term tasks that evaluate mastery of geometric pattern content. **ASSESSFEED – Feedback:** When the teacher provides feedback that clarifies misconceptions or offers next steps. **ASSESSADJ – Instructional Adjustment:** When the teacher adapts instruction based on assessment evidence (e.g., reteaching, slowing down, adding examples).

5. Reflection on Teaching	Moments where the teacher reflects during teaching (reflection-in-action) or after teaching (reflection-on-action), especially in VSRIs; evaluates success of strategies; identifies limitations; considers alternative approaches; links reflection to future planning.	**REFL-INA – Reflection-in-Action:** When the teacher makes real-time adjustments during teaching in response to learner cues. **REFL-ONA – Reflection-on-Action:** When the teacher reflects after the lesson (especially during VSRIs) on what worked, what did not, and why. **REFL-INSIGHT – Pedagogical Insight:** When the teacher articulates new realisations about their teaching or learners' thinking. **REFL-PLAN – Future-oriented Reflection:** When reflection leads to concrete intentions for modifying future geometric pattern lessons.
6. Development of New Comprehension	Evidence that the teacher has gained new pedagogical or content insights after reflection; modifies future instruction; articulates improved understanding of geometric pattern teaching; demonstrates professional growth over time.	**NEWCOMP-GROW – Evidence of Growth:** When the teacher shows development in pedagogical reasoning or increased conceptual clarity about geometric patterns. **NEWCOMP-SHIFT – Change in Teaching Approach:** When the teacher demonstrates a shift in strategies or representations based on reflection or experience. **NEWCOMP-APP – Application of New Understanding:** When newly gained insights are visibly enacted in subsequent teaching episodes (lesson planning, resource choice, questioning).

3.6 Chapter Summary

This chapter presented the theoretical and analytical framework that guided my examination of Grade 4 rural teachers' pedagogical approaches to teaching geometric patterns in the Sekhukhune East District of Limpopo Province. Centred on Shulman's Pedagogical Reasoning and Action (PRA) framework, the chapter outlined how PRA provides a comprehensive lens for understanding the interconnected processes through which teachers comprehend, transform, plan, instruct, assess, reflect, and ultimately develop new comprehension in relation to their teaching practices. Each component of PRA was elaborated and operationalised for use in the current study, with clear recognition rules. By adapting PRA to the rural context and to the specific demands of teaching geometric patterns in Grade 4 mathematics, the chapter demonstrated the framework's relevance and explanatory power for capturing the complexity of teachers' pedagogical approaches. PRA establishes the conceptual tools necessary for analysing how teachers enact and refine their teaching of geometric patterns and sets the foundation for the methodological and analytical processes discussed in the subsequent chapters.

Chapter 4: Research Methodology

4.1. Introduction

This chapter outlines the methodology employed in this study, highlighting the usage of the qualitative approach through classroom observations, semi-structured recall interviews (VSRI) and semi-structured individual interviews. It provides a comprehensive account of my investigation into the pedagogical approaches of Grade 4 rural mathematics teachers during lessons on geometric patterns in the Sekhukhune East District. The primary goal of this chapter is to articulate the research methodology that underlies the investigation, demonstrating how the espoused methods enabled me to illuminate the teachers' pedagogical approaches in the teaching of geometric patterns. The chapter thoroughly details the research paradigm, approach, design, sampling techniques, sample, data generation methods, and analytical procedures employed to analyse, interpret, and present the data. Additionally, I discuss the measures I took to ensure the trustworthiness of the study, including aspects such as credibility, transferability, dependability, and confirmability. Ethical considerations that guided the research process, including ethical approvals, gatekeepers and access to the schools' requests, informed consent, confidentiality, anonymity, and the participants' right to withdraw, are also addressed. The chapter aims to offer a clear understanding of how the research was conducted and the rigor behind the methodology, ensuring the trustworthiness of the findings related to teachers' pedagogical practices in teaching Grade 4 geometric patterns.

4.2. Research Paradigm

A research paradigm refers to the overarching worldview or philosophical orientation that shapes how a researcher understands reality, determines what is worth investigating, and guides how knowledge should be generated and interpreted in a study (Okesina, 2020; Mbhiza, 2024). As Guba and Lincoln (1994) caution, researchers must be explicit about the paradigmatic assumptions that inform their work, because these assumptions influence every methodological choice that they make in a study, from the framing of the research questions to the interpretation of findings. In this study, I adopted the interpretivist paradigm as the most appropriate philosophical stance, considering the study's focus on exploring how Grade 4 rural teachers in the Sekhukhune East District teach geometric patterns.

The interpretivist paradigm is grounded in the belief that reality is not fixed or externally measurable; rather, it is socially constructed, shaped through people's interactions, meanings, and lived experiences (Mbhiza, 2024). It acknowledges that multiple realities exist and that individuals interpret their world in unique and context-dependent ways (Abou-Assali, 2014). This paradigm aligns closely with the aim of the current study, which seeks to gain an in-depth understanding of the pedagogical practices employed by rural teachers when teaching geometric patterns. Because I consider these practices to be deeply embedded in the social, cultural, linguistic, and resource-based realities of rural schooling, an interpretivist lens allows the researcher to capture the complexity and situatedness of teachers' experiences rather than reducing them to generalisable variables.

Ontologically, interpretivism assumes a relativist perspective, meaning that knowledge is subjective and shaped by the context in which it is produced (Mahadi & Husin, 2021). This is especially suitable for a multiple case study design, where the uniqueness of each teacher's context is central to understanding how geometric patterns are taught in diverse rural classrooms. I recognise that each teacher constructs meaning differently, shaped by their background, their learners, their resource environment, and their pedagogical histories. In line with this ontological position, the interpretive paradigm enables me to foreground these contextual variations rather than treat them as anomalies in the current study.

Epistemologically, interpretivism views knowledge as co-constructed between the researcher and participants through interactive dialogue, engagement, and reflection (Kumatongo & Muzata, 2021). This means that understanding emerges from the researcher's interpretation of teachers' narratives, classroom interactions, and reflections, rather than from detached observation. In this study, the interpretivist stance made it possible for me to engage comprehensively with rural teachers' explanations, rationales, and reflections about their teaching of geometric patterns, thereby yielding rich, contextualised insights into their pedagogical approaches while teaching geometric patterns. The use of semi-structured individual interviews, classroom observations and VSRIs aligns strongly with this paradigm, as these methods allowed teachers' voices, experiences, and interpretations to be foregrounded. In addition to this discussion, adopting the interpretivist paradigm provided the philosophical foundation necessary for exploring the nuanced, human-centred, and context-sensitive nature of rural teachers' pedagogical practices. It enabled me to capture not just what teachers did when teaching geometric patterns, but why they did it, how they make sense of their choices, and how their social and educational contexts shaped such pedagogical approaches. In line with the espoused interpretive paradigm, the following discusses the research design for the current study.

4.3. Research design

A research design can be understood as a blueprint or structured plan that guides the entire research process, ensuring that the methods employed and the procedures followed in a study logically connect to the research questions and addresses the objectives (Babbie & Mouton, 2008; Creswell, 2012). According to Bautista et al. (2017), a study's research design provides a systematic framework that helps the researcher determine what data to collect, how to collect it, and how analyse and interpret it so that the study yields credible, coherent, and meaningful insights about the subject under scrutiny. A robust research design also supports the development of valid theoretical interpretations by ensuring that data are collected authentically and analysed rigorously (Bautista et al., 2017). For this study, I employed a multiple case study research design, to enable an in-depth examination of how Grade 4 rural teachers in the Sekhukhune East District teach geometric patterns. A case study design is particularly suitable for research that seeks to understand phenomena within real-life contexts, especially when the boundaries between the phenomenon and its context are not clearly defined (Simons, 2009). This aligns well with the interpretivist paradigm underpinning the study, which prioritises understanding the subjective experiences, meanings, and practices of participants within their natural teaching environments in Sekhukhune East District.

Case studies can take several forms, including single instrumental, collective (multiple), and intrinsic case studies (Stake, 2005; Yin, 2003). A single instrumental case focuses on a specific issue using one case to illustrate it, while an intrinsic case is used when the case itself is of primary interest (Wheeler & Montgomery, 2009). In contrast, a collective (or multiple) case study involves examining several cases to illuminate broader patterns and deepen understanding of a phenomenon across contexts (Siljander, 2011). Utilising the multiple case study design in the current study allowed for a comparison of practices, reasoning processes, and contextual influences across cases, thereby generating a more nuanced and comprehensive understanding of the pedagogical approaches used to teach geometric patterns.

In addition to the above discussion, the multiple case study research design strengthened the study's analytical depth by highlighting the diversity of ways in which teachers engaged with geometric pattern content, despite facing similar contextual challenges. It also enabled the identification of both common patterns and unique variations in how rural teachers explained, represented, and assessed geometric patterns in their classrooms. As Mpitso (2025) suggest, multiple case studies enhance the robustness of findings by enabling cross-case analysis and triangulation of evidence. In the current study, this design was essential for capturing the complexities of rural pedagogical practice, including how contextual factors shaped pedagogical decisions and how teachers adapted their approaches to meet learners' needs during the lessons. Employing a multiple case study research design provided a systematic and contextually sensitive way for exploring the pedagogical approaches of Grade 4 rural teachers as they taught geometric patterns. It enabled the study to move beyond surface-level descriptions toward an in-depth, evidence-based understanding of teaching within rural South African mathematics classrooms. The following section presents the chosen research approach that I employed in the current study.

4.4. Research Approach

Research in the social sciences commonly employs one of three research approaches: qualitative, quantitative, or mixed-methods. The quantitative research approach draws on structured, standardised procedures that emphasise measurement, numerical data, and statistical analysis to explain patterns or test hypotheses (Creswell, 2014; Poth, 2016). While the quantitative approach can provide valuable large-scale trends, they are limited in their ability to capture the context-dependent, nuanced, and subjective experiences of participants (Anthony & Jack, 2009), experiences which I consider central to understanding teachers' pedagogical reasoning and pedagogical approaches in real classrooms. Mixed-methods research, which combines elements of both qualitative and quantitative approaches, may offer a broader view of a research problem; however, it can sometimes constrain the richness of qualitative insights by forcing inherently interpretive phenomena into quantifiable categories (Bleiker et al., 2019). Considering the aims of the present study, I espoused the qualitative approach as it offered the most appropriate methodological fit.

Qualitative research, as defined by Creswell and Creswell (2017), relies on participants' perspectives which are context-dependent, emphasises open-ended inquiry, and generates data primarily in the form of descriptive accounts, narratives, observations, and interactions. This approach is particularly valuable for studying teaching and learning because it allows the researcher to observe and interpret classroom practices as they naturally unfold, capturing not only what teachers do, but why they do it and how they make sense of their practices (Mbhiza, 2021). In the current study, qualitative inquiry aligned closely with the interpretivist paradigm, which holds that human behaviour must be understood within the social, cultural, and historical contexts in which it occurs, Sekhukhune East District in the current study (Lincoln & Guba, 2007).

Furthermore, qualitative research is guided by an interest in understanding the meanings teachers construct about their pedagogical actions (Hanson & Grimmer, 2007). This was critical for uncovering how Grade 4 rural teachers think about geometric patterns, how they interpret curriculum expectations, how they articulate mathematical ideas through explanatory talk, and how they negotiate constraints such as limited resources or learner misconceptions. Through utilising classroom observations, interviews, and reflective conversations, the study generated rich, thick descriptions of teachers' reasoning, discursive practices, and instructional choices; data that provide a textured picture of pedagogical work in rural mathematics classrooms related to the teaching of geometric patterns. This depth of understanding would not have been possible through quantitative methods, which are better suited to examining measurable outcomes rather than the processes, meanings, and contextual influences that shape teaching practices (Mbhiza, 2021). In contrast, qualitative research enabled a holistic and interpretively grounded examination of rural teachers' pedagogical approaches to geometric patterns, foregrounding the lived realities, experiences, and voices of the teachers themselves as they taught the topic.

According to Creswell and Poth (2016), qualitative research begins with the formulation of exploratory research questions that seek to understand how and why particular social or educational phenomena occur, rather than attempting to measure or quantify them. This aligns directly with the aims of the current study, which seeks not merely to document what teachers do when teaching geometric patterns, but to interpret how they make sense of their teaching, how they reason pedagogically, and how contextual conditions shape their instructional decisions. Creswell and Poth further emphasise the importance of grounding qualitative inquiry in relevant theoretical frameworks, as these frameworks serve as interpretive lenses through which data are understood and analysed. In this study, Shulman's PRA framework played this guiding role, helping to structure the investigation around the interconnected processes of comprehension, transformation, instruction, assessment, reflection, and the development of new understanding.

4.5. Introducing the Study Context

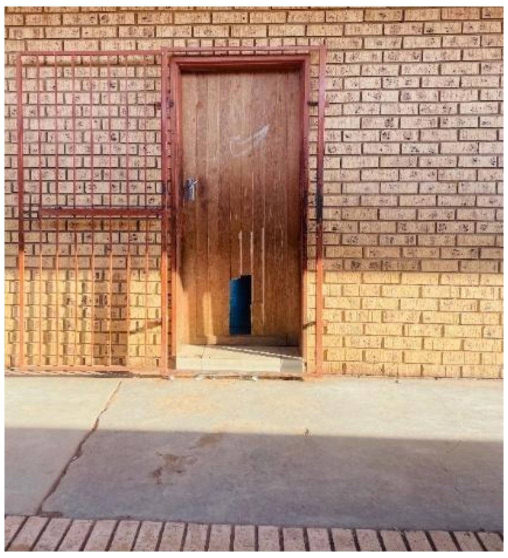
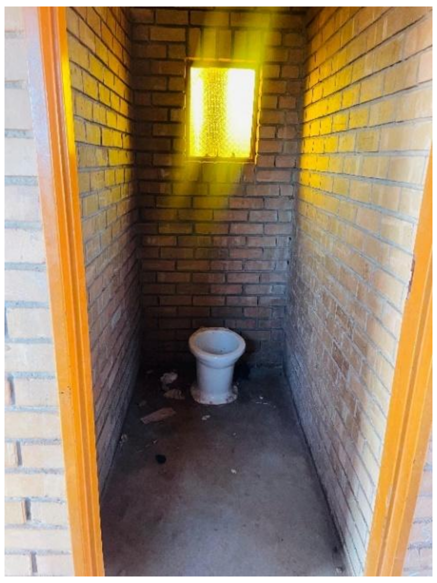
The current study was conducted in the Sekhukhune East District, which encompasses three circuits: Seotlong Circuit, Mohlaletse Circuit, and Lepellane Circuit. This region is situated in a deeply rural area within the Fetakgomo-Tubatse Local Municipality. Residents of this district face significant travel challenges, requiring them to journey over 100 kilometers to reach the nearest towns of Polokwane and Burgersfort. The community is predominantly composed of the Bapedi, also known as the Pedi people, who speak Sepedi as their primary language. The local population grapples with numerous substantial challenges, including inadequate infrastructure, poor road conditions, and a high poverty rate.

The region continues to struggle with issues related to service delivery, where residents sometimes endure months without access to water. Many parts of the area are characterised by untarred roads, including those that lead to various schools, which can often become unmanageable, particularly during adverse weather conditions such as rainy seasons. Consequently, the poor condition of these gravel roads (as illustrated in Image 1) hampers teachers' ability to arrive at their schools on time and negatively impacts learner school attendance. Within the Sekhukhune East

District, teachers encounter a range of difficulties in their classrooms and schools, including outdated and damaged infrastructure exemplified by old buildings (shown in Image 2), broken doors and windows (depicted in Images 3 and 4), and persistent water shortages, where taps remain non-functional for weeks at a time (as highlighted in Image 8). The lack of adequate hygiene (represented in Image 7) and poor sanitation (shown in Images 5 and 6) compound these issues, alongside overcrowded classrooms and limited teaching and learning resources. Moreover, the relationship between teachers and parents is often weak, with minimal support from families.

While global education systems, including many urban South African schools are advancing towards 4IR and 5IR learning environments through the integration of Artificial Intelligence (AI) and digital technologies, communities and schools in the Sekhukhune East District remain excluded from these developments due to persistent structural inequalities and widespread poverty. As a result, learners and teachers in this district continue to face severe limitations in accessing digital tools, infrastructure, and technological resources, reinforcing the digital divide and constraining opportunities for innovative, technology-enhanced teaching and learning. In essence, the challenges faced in the Sekhukhune East District reflect a complex landscape of educational difficulties exacerbated by social and infrastructural limitations that impact both teaching effectiveness and learner learning outcomes. It was my assumption at the conceptualisation of the current study that, researching within this context would provide in-depth insights into how teachers navigate teaching mathematics amidst the conditions described above.

Photograph 1: *Images of the Characteristics of Sekhukhune East District*

 **Image 1:** *Gravel road to school*	 **Image 2:** *Old damaged buildings*
 **Image 3:** *Classrooms with broken doors*	 **Image 4:** *Classrooms with broken windows*
 **Image 5:** *Poor sanitation*	 **Image 6:** *Toilets with no doors*
 **Image 7:** *Poor sanitation (non-functional water basin)*	 **Image 8:** *Poor infrastructures (Non-functional water tap)*

The contextual conditions in the Sekhukhune East District significantly influence the effectiveness of teaching and learning, as demonstrated by various studies highlighting the relationship between socio-economic challenges and educational outcomes. Ongoing infrastructural deficiencies, such as inadequate school facilities and poor road conditions, are barriers to effective teaching and learning (Mohangi et al., 2016; Edwards, 2018). The lack of reliable access to resources exacerbates these issues; educational success in rural areas is often contingent upon sufficient infrastructure and support systems (Mbhiza, 2021; Nkambule, 2022). Furthermore, socio-economic factors such as high poverty rates and insufficient parental involvement contribute to a challenging teaching environment, hindering teachers' capacity to engage learners and deliver quality education (Arasomwan & Daries, 2024; Madimabe & Omodan, 2021). This scenario reflects a broader pattern observed in rural schooling systems in South Africa, where resource scarcity limits opportunities for effective pedagogy and innovative teaching practices (Dikeledi & Wet, 2023; Masinire, 2015). Teachers in the Sekhukhune East District must navigate these multifaceted challenges, which shape their pedagogical approaches and learning outcomes, highlighting the importance of contextual awareness in educational research and practice. As communities strive to overcome these challenges, understanding the interplay between context and educational effectiveness becomes crucial for developing targeted strategies that support teachers and learners alike.

4.6. Sampling Technique and Study Sample

Sampling is a critical aspect of research, as it significantly influences the representativeness of the study population as well as the comprehensiveness of the data that a study generates (Wilhelm et al., 2017). McMillan and Schumacher (2010) describe sampling as the selection process of a representative group from a larger population. In this study, I employed a purposive sampling, which is a non-probability sampling technique particularly regarded to be advantageous for achieving an in-depth understanding of specific phenomena rather than for generalising findings to a broader population (Yin, 2005). From using purposive sampling, I was able to intentionally select participants who possess specific characteristics relevant to the research question (see Table 5), thereby ensuring that the data collected would be meaningful and significant. This approach is often favoured in qualitative research, where the focus is on gaining comprehensive insights into a phenomenon rather than making statistical inferences (McMillan & Schumacher, 2010; Ahmad & Wilkins, 2025). In the context of the current study, I selected five teachers from five different schools who were actively teaching mathematics in Grade 4. This sampling criteria detailed in Table 5 facilitated a deeper exploration of the pedagogical challenges and practices that these teachers experience in teaching geometric patterns within rural settings.

Table 5. *Teachers Selection Criteria*

Research Location	Sekhukhune East District, Apel Cluster, Limpopo Province
Subject	Mathematics
Grade	Grade 4
Teaching experience	At least three years teaching Grade 4
Number of Teachers	5

The rationale for employing purposive sampling in this study was to ensure that the selected participants possessed the specific characteristics needed to offer valuable insights into Grade 4 rural teachers' pedagogical approaches to teaching geometric patterns. As articulated by McMillan and Schumacher (2010), purposive sampling enables researchers to choose individuals based on relevant qualities, making it particularly effective for exploring specific phenomena in detail. In this study, focusing on a targeted group of five Grade 4 mathematics teachers enabled me to gain a deeper understanding of their pedagogical approaches related to teaching geometric patterns, allowing for a nuanced exploration of their methods and perspectives in the context of rural education. For the current study, I chose five teachers from different school sites as participants, forming multiple cases. Considering that this study employed a qualitative approach, the five Grade 4 teachers were sufficient to provide a clear understanding of Geometric Patterns in rural classrooms settings specifically in the Apel Cluster of Sekhukhune East District.

4.6.1. Challenges in Schools and Recruitment of Participants

The principals of all the schools where I conducted data collection were very accommodating and showed no objections to my research activities within their institutions. However, one of the most significant challenges faced during this study was the recruitment of participants. Many teachers expressed reluctance to be video recorded despite their willingness to allow observations of their teaching. To address their concerns, I revisited the ethical documents and consent forms to thoroughly explain the significance of the video recordings, emphasising that they would be utilised solely for research purposes and that confidentiality and anonymity would be upheld in all writings and presentations emanating from the study. I again also provided the Ethical Certificates issued by the Limpopo Provincial Research Ethics Committee (LPREC) and the University of South Africa, which further reassured the teachers. It was only after these explanations that they agreed to participate in the video recording process. It is important to note that, all the prospective participants who did not consent to be videotaped during their lessons were excluded from the study, because video recording the lessons plays a vital role in the current study. The first being that the recorded lessons allow me to rewatch the lessons and provide comprehensive analysis of their pedagogical approaches. Secondly, the recorded lessons are used for the purposes of video-stimulated recall interviews, for both the teachers and me to revisit the lessons and engage in reflections about specific critical incidences during teaching. Despite the challenges encountered during recruitment, I was ultimately successful in enlisting participants for the study. Figure 2 illustrates the final sampling approach and the sample utilised in this research. All participants' names are assigned pseudonyms.

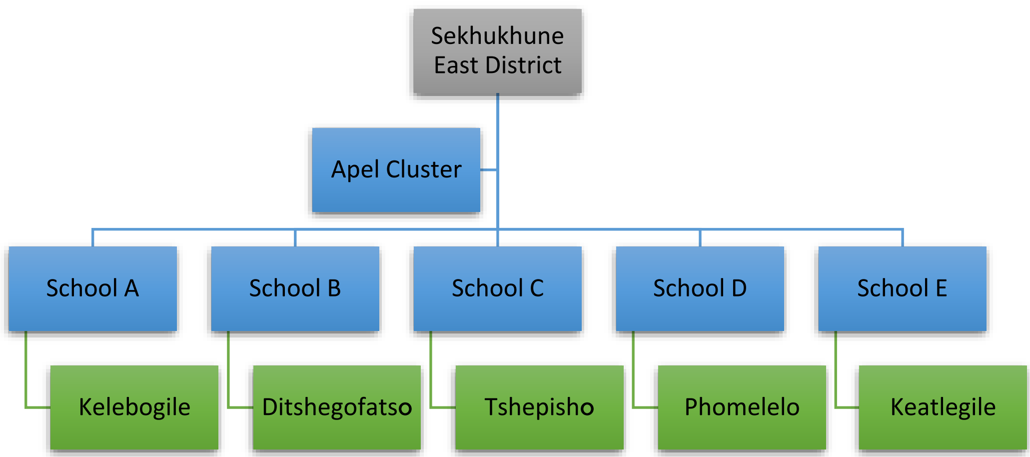


Figure 2: *The final sample that was used in the study.*

This comprehensive approach provided valuable insights into the pedagogical approaches employed by these teachers and enhanced my understanding of their pedagogical reasoning in rural contexts. Table 6 depicts the participants' biographical information.

Table 6. *Biographical Information of the Participants*

Pseudonym	***Gender***	***Age***	***Institution trained to be a teacher***	***Qualification***	***Number of years teaching***
1.Kelebogile	Female	30	University of Limpopo	Bachelor of Education (Intermediate and Senior phase)	5 years
2. Ditshegofatso	Male	56	Sekhukhune Collage of Education	Diploma in Junior Primary Teaching	25 years
3. Tshepisho	Female	55	Sekhukhune Collage of Education	Diploma & Advanced Certificates in Education	30 years
4. Phomelelo	Female	53	Sekhukhune Collage of Education	Diploma & Advanced Certificates in Education	18 years
5. Keatlegile	Female	47	University of South Africa	Bachelor of Education (Intermediate and senior phase)	12 years

The demographic profile of the five teachers participating in this study offers a diverse representation in terms of age, training backgrounds, and teaching experience, which contributes to a rich array of insights regarding the pedagogical approaches employed in the teaching of geometric patterns. The ages of the participants span from 30 to 56 years, encompassing both relatively novice teaching and seasoned practitioners, thereby allowing for a comparative analysis between early-career teachers and those with decades of experience in the educational system. The sample predominantly consists of female teachers, reflecting the national trend in South Africa where the teaching profession, particularly in foundation and intermediate phases, tends to be female dominated (Uka, 2025), while the inclusion of one male teacher provides a unique perspective within this largely feminised environment.

The participants' training backgrounds also vary, incorporating both university-based qualifications and college-based training, thereby highlighting the impact of differing educational pathways on their teaching styles. For instance, Kelebogile, the youngest participant, holds a B.Ed. from the University of Limpopo, which aligns with contemporary teacher education standards after the phase-out of colleges of education. In contrast, older participants, such as Ditshegofatso and Tshepisho, received their training from the former Sekhukhune College of Education, illustrating historical disparities in teacher preparation and the evolution of training practices over time. Some college-trained teachers later pursued Advanced Certificates in Education (ACE), demonstrating their commitment to professional growth in line with post-1994 education reforms (Msomi et al., 2025). The range of teaching experience among the participants, from 5 to 30 years, allows for an exploration of insights shaped by varying educational eras and policy environments, enriching the study's findings. The demographic diversity presented in Table 6 enhances the credibility and depth of the study by providing a nuanced understanding of the various challenges, pedagogical approaches, and professional identities of these teachers working within the specific context of teaching geometric patterns in their classrooms.

4.7 Research Methods

To gather comprehensive data for the proposed study, I employed three complementary qualitative research methods: semi-structured individual interviews, unstructured and non-participatory classroom observations, and Video-Stimulated Recall Interviews (VSRIs). The semi-structured interviews provided a platform to deeply explore the teachers' perspectives, beliefs, and decision-making processes concerning the teaching of geometric patterns in rural settings within the Sekhukhune East District. This method aligns well with similar case study designs used in prior qualitative research exploring teachers' teaching practices (Övez & Kıyıcı, 2018; Mpitso, 2025). Furthermore, semi-structured individual interviews are commonly utilised in qualitative studies to examine teachers' viewpoints and experiences, facilitating an in-depth understanding of their approaches to teaching geometric patterns in rural classrooms (Ndou, 2020).

In addition, I conducted unstructured, non-participatory classroom observations, which provided valuable insights into teachers' actual pedagogical approaches during lessons on geometric patterns. This observational method is particularly effective in research focused on inclusive education practices, where understanding teachers' behaviours and interactions with diverse learners is essential (Mpitso, 2025). Through the use pf unstructured observations, I was able to gain a greater understanding of Grade 4 teachers' pedagogical approaches of geometric patterns, allowing me to observe teaching practices without any direct involvement. Moreover, VSRIs were instrumental in enhancing my understanding of the teachers' decision-making processes and lesson implementation. During these interviews, teachers viewed recordings of their own lessons and reflected on their thoughts and actions, which provided a deeper layer of insight into their teaching practices (Mbhiza, 2019; Mbhiza, 2021). Collectively, semi-structured individual interviews, unstructured and non-participatory classroom observations, and VSRIs not only enriched the data gathered but also facilitated a comprehensive exploration of the complexities involved in teaching geometric patterns in rural classrooms. I provide the operationalisation of these three methods in the current study in what follows.

4.7.1 Unstructured, Non-Participatory Classroom Observations

To comprehensively understand the pedagogical approaches utilised by the participants in this study, direct observation proved to be an essential method, particularly within the qualitative research framework that aims to elucidate the patterns, structures, and dynamics of teacher interactions and behaviours (Guthrie, 2011). During lessons on geometric patterns, classroom observations documented the established rules of engagement for learners, the teacher-learner power dynamics, and the organisation of pedagogical activities and tasks (Guthrie, 2011). The insights gained from these observations illuminated the teachers' conceptualisations of teaching geometric patterns within their schools. Similar to the findings of Fetters and Rubinstein (2019, p. 555), observation allowed me to discern the characteristics that define "what counts as mathematical" and assess "what counts as effective" in teachers' pedagogical approaches during the teaching of geometric patterns.

As mentioned earlier, I employed an unstructured approach as recommended by Cohen et al. (2013), which enabled me to avoid premature judgments about the teachers' methodologies and focus instead on their pedagogical practices related to geometric patterns (Mbhiza, 2021). As Bell (2005) articulated, unstructured observations allow researchers to approach the data without preconceived definitions, enabling patterns to emerge organically (p. 185). Moreover, all classroom observations were conducted in a non-participatory manner (Cohen et al., 2011; Green & Thorogood, 2014), ensuring that my presence was unobtrusive, thereby allowing the teaching and learning processes to unfold naturally without my direct interference. This approach aligns well with qualitative research principles that advocate for studying phenomena in their natural contexts (Pratt, 2015). Each observation was videotaped, and summarised recordings were prepared (Appendix A), which facilitated review during the data organisation and analysis phase to ensure comprehensive capture of pertinent information relating to the teaching of geometric patterns. Table 7 outlines details regarding the observed lessons for each participating teacher, providing a structured overview of the educational contexts that were examined in this research. Furthermore, the observation approach I employed in the current study aligns with the need for detailed observational studies that can reveal the intricacies of teaching practices, thereby addressing the critical gap identified by researchers who argue for a deeper understanding of teachers' pedagogical decisions (Rozesahegyi, 2019; Mathayas, 2025).

Table 7. *Observed Lessons for each Teacher*

Teacher	***Number of lessons observed***	**Duration**	**Summary of lessons observed**
Kelebogile	3	45 minutes each	- Understanding geometric shapes - Identifying and creating geometric shapes - Recognising patterns
Ditshegofatso	3	45 minutes each	- Understanding geometric shapes - Identifying and creating geometric shapes - Recognising patterns
Tshepisho	3	45 minutes each	- Understanding geometric shapes - Identifying and creating geometric shapes - Recognising patterns
Phomelelo	2	45 minutes each	- Understanding geometric shapes - Identifying and creating geometric shapes
Keatlegile	3	45 minutes each	- Understanding geometric shapes - Identifying and creating geometric shapes - Recognising patterns

Table 7 illustrates that all five teachers were primarily engaged in lessons focused on foundational mathematical skills, specifically understanding geometric shapes, identifying and creating shapes, and recognising patterns, aligned to the Curriculum, Assessment and Policy Statement (CAPS) (Department of Basic Education, 2011). Most teachers (Kelebogile, Ditshegofatso, Tshepisho, and Keatlegile) were observed across three 45 minutes lessons each, providing a comprehensive view of their instructional approaches and consistency in practice, while Phomelelo was observed twice, still offering sufficient insight into her teaching of geometric patterns-related content. The uniformity in lesson duration and content demonstrates that the participating teachers operated within similar curricular pacing and planning structures.

4.7.2 Semi-Structured Individual Interviews

Semi-structured individual interviews are a qualitative research method that integrates elements of both structured and unstructured interviews, typically consisting of open-ended questions that allow for flexibility in the conversation (Maudia et al., 2024; Marbun et al., 2022). This format enables both the interviewer and interviewee to seek clarification and delve deeper into particular topics as necessary, promoting a more nuanced understanding of participants' experiences and perspectives relating to teaching Grade 4 geometric patterns (Sarif, 2025). In this study, I conducted face-to-face semi-structured interview with each of the five participating mathematics teachers, aiming to gain in-depth insights into their behaviours, experiences, and views regarding the teaching of geometric patterns (Appendix E). This approach aligns with previous research that indicates semi-structured interviews are particularly effective for exploratory studies, as they facilitate a rich exploration of complex phenomena (Wilmot & Goldschagg, 2023).

In contrast to semi-structured interviews, structured interviews employ a rigid format with predetermined questions and limited opportunities for follow-up, which can restrict the depth of responses and fail to capture the complexities of participants' experiences (Yeşildere & Akkoç, 2012). On the other hand, unstructured interviews offer a more conversational approach, allowing participants to guide the discussion; however, this can sometimes lead to inconsistencies and a lack of focus on specific research questions (Er & Chongo, 2025). The semi-structured format utilised in this study struck a balance between these two interview approaches, enabling me to navigate the discussion toward predetermined topics relevant to the research objectives while also remaining open to emergent themes that arose during the conversation. That is, the use of semi-structured interviews allowed me to tailor up subsequent questions in cases where teachers provided information that required further elaboration.

The decision to use face-to-face semi-structured interviews was informed by the need for direct engagement, allowing for the establishment of rapport and interpersonal trust, which are essential in qualitative research contexts (Hill & Uribe-Flórez, 2019). I assumed that this interpersonal connection could lead to more honest and reflective responses from participants, especially when discussing their teaching practices in a rural setting that may involve sensitive issues or challenges they face. Moreover, the face-to-face interaction facilitated an immediate follow-up on responses, enhancing the depth of the data collected and ensuring that nuances in teachers' experiences were fully explored. The combination of these advantages made semi-structured individual interviews an optimal choice for obtaining comprehensive insights into the teaching of Grade 4 geometric patterns, ultimately contributing to a richer understanding of pedagogical practices within rural classrooms (Mafakheri, 2021; Smart & Linder, 2017). Table 8 presents the time it took for each teacher's interview to conclude.

Table 8. *Duration of Each Teacher's Semi-Structured Individual Interview*

Teachers' pseudonyms	***The duration of the interview***
1. Kelebogile	30 minutes 5 seconds
2. Ditshegofatso	35 minutes 50 seconds
3. Tshepisho	28 minutes 47 seconds
4. Phomelelo	25 minutes 55 seconds
5. Keatlegile	29 minutes 16 seconds

The semi-structured individual interviews conducted in this study ranged in duration from approximately 26 to 36 minutes, providing a flexible framework for exploring participants' insights into teaching geometric patterns. Each interview time varied based on the depth of responses and the dialogue generated, highlighting the adaptability of the semi-structured format (Mbhiza, 2021). For instance, Kelebogile's interview took 30 minutes and 5 seconds, while Ditshegofatso's interview extended to 35 minutes and 50 seconds, indicating that some teachers had more extensive experiences or reflections to share. This variability highlights the benefit of semi-structured interviews in allowing participants to express their views comprehensively while providing room for probing questions that could elicit further elaboration on key themes.

4.7.3 Video-Stimulated Recall Interviews (VSRIs)

Video-Stimulated Recall Interviews (VSRIs) are a qualitative research method in which participants review video recordings of their own teaching practices and reflect on their decision-making during those moments (Mbhiza, 2019; Petersen & Nel, 2024). In an increasingly digital world, teachers and researchers alike are incorporating technology into their studies, leading to a growing interest in methods like VSRIs that facilitate self-reflection and analysis (Mbhiza, 2019). As highlighted by Mbhiza (2021) and Sempe (2024), the use of VSRIs has become more prevalent as it provides valuable insights into the decision-making processes of teachers regarding their teaching practices. The rationale for conducting VSRIs is the methodological promise that "cognitive processes can be investigated by inviting subjects to recall, when prompted by a video sequence, their concurrent thinking during that event" (Lyle, 2003, p. 861). One of the key advantages of VSRIs is that they afford participants the opportunity to observe the recorded lessons and reflect on past events, enabling them to better understand their teaching methods and make informed adjustments for improvement. In this study, the use of VSRI was not only aimed to uncover the underlying thought processes of teachers but also to provide them an opportunity to articulate and refine their understanding of effective teaching practices for geometric patterns.

In the context of the current study, I noticed that many teachers employed demonstration methods while teaching geometric patterns. This observation prompted me to engage the participants during the VSRI to discuss the significance of using demonstration in their teaching. By revisiting their recorded lessons, teachers could critically evaluate their pedagogical approaches, fostering a deeper awareness of effective teaching practices and areas for potential enhancement. The ensuing discussion revealed not only the rationale behind their pedagogical choices but also how such methodical approaches impacted learners' understanding of the concepts being taught. Consider the following VSRI exchange as an example of how this process was conducted:

1. ***Researcher:*** *I have noticed that you were using a demonstration method in all your lessons, like here where you were calling your learners to come forth and used them as examples. What is your reason for using the demonstration method?*
2. ***Keatlegile:*** *I always use the demonstration method in my lessons because it promotes an active learning, it also makes my learners to pay attention and participate in my lessons. It is important to use these kinds of demonstrations, the ones the learners are exposed to; remember in our area the children might not be exposed to some things, so using things in their surroundings to demonstrate is important.*
3. ***Researcher:*** *I see what you mean. Speaking of participation in class, I have noticed that your learners were using Sepedi to answer all your questions even though you were code switching. What could be the reason for that?*
4. ***Keatlegile:*** *Most of our learners in Grade 4 are still using Sepedi to speak and to write even on subjects like Mathematics but as we speak the Department of Education has already launched the usage of mother tongue language in all the South African schools in Grade 4.*

In this study, reflective conversations with teachers were facilitated by identifying critical incidents, which served as focal points for discussion (Appendix C). When I identified these incidents, I guided the teachers toward specific aspects of their teaching that warranted reflection. To further enrich these discussions, I provided open-ended questions designed to prompt meaningful dialogue regarding their pedagogical practices, as exemplified by the phrases "*I have noticed that … what is your reason* …" (line 1), "*I have noticed that … What could be the reason …*" (line 3). Teachers were also encouraged to identify their own critical incidents, allowing them to initiate conversations about elements of their lessons they felt needed further exploration or clarification regarding their teaching choices and actions as can be seen in line 5 in the following exchange in relation to image 9.

5. ***Kelebogile:*** *You see in this part of the lesson, I used coloured chalks to make the patterns. I like to use coloured chalks during my Geometric Patterns lessons because coloured chalks capture learners' attention, which makes them to participate in class*
6. ***Meta (Researcher):*** *It makes so much sense as I saw the increment of participation in your lesson*
7. ***Kelebogile:*** *Yes, coloured objects are very important in our Grade 4 class, because learners are also learning to create patterns of colours.*

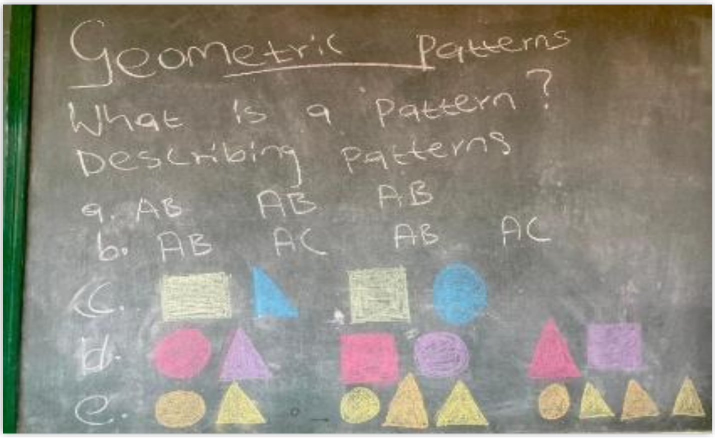


Image 9: *Geometric Patterns Activity*

This dual approach created an environment where both the researcher and the teachers engaged in a collaborative reflective process, fostering an atmosphere of shared inquiry and learning. In other words, the VSRIs in this study were semi-structured, allowing the flexibility in the reflective conversations. According to Mbhiza (2021), the semi-structured format of the VSRIs allow for open dialogue, wherein teachers can elaborate on their thoughts and provide context to their decisions beyond their initial teaching moment. This method of inquiry supports the concept of reflective practice, which has been shown to enhance teachers' ability to assess their performance and ultimately improve learner outcomes (Snell et al., 2018). Allowing teachers to reflect upon critical incidents enhanced their self-concept as teachers and promoted growth in their pedagogical approaches. In relation to the above discussion, Çukur (2022) posits that reflecting together helps teachers construct their own knowledge about effective pedagogical practices. Through this iterative reflective process, the study aimed to foster a deeper understanding of teaching practices among rural teachers, ultimately enriching their teaching effectiveness and enhancing student learning outcomes.

4.8 Data Organisation and Analysis

In qualitative research, the analysis of data often takes place within an interpretive framework, focusing on the significance and symbolic meanings derived from participants' experiences. According to Maree (2007), the goal is to thoroughly examine the qualitative data to draw out these meaningful interpretations. McMillan and Schumacher (2014) emphasise that understanding individuals' perceptions, attitudes, emotional responses, and experiences of specific phenomena is crucial for grasping their significance. Cohen et al. (2013) further elaborate that qualitative data analysis centers on interpreting participant perspectives to uncover themes, patterns, and categories, ensuring that the analysis reflects the participants' lived experiences accurately. For this study, Sepedi was the medium of instruction in the selected schools, which provided a conducive environment for effective communication during data collection. My strong proficiency in Sepedi enabled me to conduct all the necessary transcriptions and formal data analyses independently, ensuring that the data captured accurately represented the teachers' language and pedagogical approaches. This linguistic capability was crucial for comprehensively understanding the nuances of the teachers' instructional methods and their engagement with learners in discussions about geometric patterns.

As I could effectively interpret the nuances and terminologies used by the teachers in their lessons, it allowed me to create detailed and contextually relevant records of their teaching sessions. As I transcribed the recorded lessons, I took great care to ensure that both the spoken language and the associated non-verbal cues during both interviews and classroom observations were adequately captured, recognising their importance in understanding teachers' cognitive processes and content knowledge. This thorough engagement with the transcripts allowed for a more meaningful analysis of the qualitative data collected throughout the study. The attentiveness to detail in transcriptions also facilitated the identification of patterns, themes, and pedagogical strategies related to the teaching of geometric patterns, which were crucial for addressing the research questions. Additionally, my fluency in Sepedi helped bridge any potential gaps in communication, fostering authenticity in the data gathered. This approach aligns with the findings of Fournier and O'Neill (2020), who highlight the significance of language proficiency in qualitative research for accurately interpreting and analysing information that participants provide. Thus, my ability to navigate and understand the linguistic context of the study not only enhanced the reliability of the data but also enriched the overall depth of the analysis.

4.8.1. Analysis of the Classroom Observations

The classroom observations were transcribed verbatim and actions recorded, with images of board work attached to the transcripts (Appendix D). The observation transcripts were not merely records of spoken words; they also documented teachers' actions and their board work, all of which contributed significantly to the data analysis. The analyses aimed to interpret both verbal utterances and non-verbal actions, as these elements are crucial for understanding the teachers' cognitive processes and content knowledge in mathematics, geometric patterns for the current study. Following Cohen et al. (2007), this analysis is integral to interpreting and contextualising the study's findings in Chapter 5. In my analysis of the qualitative data collected from the classroom observations, I adopted content analysis as the primary analytical approach (McMillan & Schumacher, 2014). This method proved suitable for identifying and categorising themes and patterns within the data, facilitating a deeper understanding of teachers' experiences with teaching geometric patterns. This process involved coding the data and classifying it into differing themes, which provides a structured way to observe commonalities and differences in the teachers' pedagogical approaches (McMillan & Schumacher, 2014).

Specifically, during this process, I made margin notes to highlight significant elements of the lessons, focusing specifically on the pedagogical approaches employed through the horizontalisation of the transcripts. Horizontalisation allowed me to categorise the lessons according to Shulman's (1987) PRA framework discussed in chapter 3, following the recognition rules I established. As I classified the components of PRA, I considered different aspects, including the nature of teachers' explanations, their engagement with learners, assessments of learners' understanding, and feedback mechanisms as well as their reflection during the lessons. Furthermore, I documented the gestures and visual mediators utilised by the teachers to enhance learners' comprehension. This stage involved extracting both verbal and observable classroom behaviours, guided by the PRA framework, which enabled me to uncover the pedagogical approaches utilised during the observed lessons. To minimise irrelevant data, I focused my analysis on the information most relevant to the research objectives, effectively filtering out extraneous information which I put aside as 'dross' (Mbhiza, 2021). Based on the analysis of classroom observations, several key codes were identified that circulate around the components of pedagogical reasoning and action (PRA). These codes help to elucidate the specific practices that Grade 4 rural teachers employ in their approaches to teaching geometric patterns. That is, the analysis of classroom observations led to the development of initial codes related to the teaching geometric patterns in rural school contexts as represented in Table 9.

Table 9. *Initial Codes Emerging from the Analysis*

Identified Codes
• Concrete representations,
• Incremental reasoning,
• High learner participation,
• Formative assessment,
• Structured questioning, and
• Pattern generalisation as the core conceptual focal point.

These initial codes were subsequently refined into final themes after a thorough analysis of data collected from the semi-structured interviews and VSRIs, which were also examined in conjunction with the classroom observation data.

4.8.2. Analysis of the Semi-Structured Interviews and VSRIs

The analysis of the semi-structured interviews and VSRIs drew on the methodological principles of thematic analysis as outlined by Braun and Clarke (2006). Following their recommendation that researchers begin by immersing themselves thoroughly in the data, after the transcription of the interviews, I engaged in repeated readings of all interview transcripts to familiarise myself with the participants' narratives and identify salient features within their accounts. This stage provided the foundation for the subsequent coding and interpretation processes, allowing me to gain a nuanced understanding of the teachers' perspectives on their pedagogical practices relating to the teaching of geometric patterns.

In line with Braun and Clarke's (2006) second phase, I generated initial codes by systematically highlighting statements of significance within the transcripts. To support this analytic process, I employed the technique of horizontalisation. This ensured that the participants' utterances were examined without prematurely prioritising certain ideas over others, thereby allowing a wide range of potential codes to emerge. These codes were then organised into broader categories, which served as the preliminary thematic structures. At this stage, the interview-derived codes were compared with insights from the classroom observations, enabling the triangulation of data and enhancing the overall coherence of the developing thematic patterns (Mbhiza, 2021). As the analysis progressed, I moved into Braun and Clarke's (2006) theme-reviewing phase, refining and adjusting the thematic categories to ensure both internal consistency and fidelity to the data set. This iterative process helped consolidate themes that meaningfully captured the teachers' reasoning, the language in-use, and decision-making processes. The refinement of themes provided a basis for deeper interpretive analysis, which was supported through my integration of Critical Discourse Analysis (CDA), particularly as conceptualised by Fairclough (1992; Fairclough, 1995; 2013).

The use of Fairclough's CDA enriched the thematic analysis by foregrounding the relationship between language, practice, and broader socio-cultural structures. Drawing on Mbhiza's (2021) elaboration of mathematics discourse as a dynamic interplay between linguistic expression and social practice that shapes individuals' perceptions of reality, I examined how the themes identified in the teachers' narratives were embedded within larger ideological and institutional contexts. This discursive lens enabled me to explore not merely what teachers said, but how their language reflected and sometimes contested prevailing assumptions about mathematics teaching and learning, specifically the teaching of geometric patterns.

Fairclough's (1995) CDA framework comprises three dimensions: textual description, discursive interpretation, and socio-historical explanation. This comprehensive model offers a nuanced lens through which to analyse language and its socio-political implications, particularly in educational contexts. In the current study focusing on the teaching of geometric patterns in the Sekhukhune East District, these dimensions allowed for a deeper exploration of the information provided by the teachers, illuminating not just what was communicated, but also how their linguistic choices reflect and potentially challenge prevailing assumptions regarding mathematics education. The first dimension, textual description, involves a detailed examination of the teachers' spoken language and the visual materials they utilised during the semi-structured interviews. Through analysing the specific words and phrases the participating teachers utilised during the interviews, I aimed to grasp how their choice of language and their lived experiences framed the teaching of geometric patterns.

According to Fairclough (1995), the second dimension, discursive interpretation, allows for an exploration of how the teachers' language choices align with broader discourses in mathematics education. This aspect of critique focuses on identifying and understanding the implications of their language in relation to established educational norms and practices. The language used by teachers was not just a medium of communicating their pedagogical approaches; it also played a role in shaping the learners' understanding of geometric concepts and in reinforcing or contesting traditional power dynamics in the classroom. In examining these discursive elements, the study analysed how teachers' linguistic choices reflected their teaching philosophies and the potential impact of these philosophies on how they in turn presented geometric patterns lessons.

Finally, the socio-historical explanation calls for situating the discourse within its broader social and historical context (Faiclough, 1995; Mbhiza, 2021). This dimension facilitated an understanding of how various societal factors, such as educational policies, cultural attitudes toward mathematics, and historical trends in teaching practices as well as perspectives relating to what it means to teach and learn mathematics within rural classrooms. In this study, I paid attention to the statements teachers made about the teaching of mathematics within rural schools, highlighting both the challenges and successes, allowing for a comprehensive understanding of the challenges and implications faced by rural mathematics teachers. Through the integration of thematic analysis and CDA, the study produced a detailed account of how teachers articulate, justify, and enact their pedagogical choices in mathematics classrooms. The analysis of the semi-structured interviews, classroom observations, and Video-Stimulated Recall Interviews (VSRIs) led to the identification of key themes and sub-themes that capture the essential findings of the study, as presented in Table 10.

Table 10. *Themes and Sub-themes Themes*

Themes	Sub-themes
Strong Reliance on Concrete and Contextual Representations (TRANSREP, TRANSCTXT)	
Emphasis on Repetition and Structural Awareness in Pattern Growth (COMP-STRUCT)	
High Levels of Learner Participation Through Demonstrations and Board Work (INST-PART)	- *Movement from Teacher-Led Demonstrations to Learner-Constructed Patterns (TRANSCPA, INSTEXEMP)* - *Correction of Misconceptions Through Peer Engagement (COMPMIS)*
Use of Probing Questions to Drive Reasoning (INST-QUEST)	- *Formative Assessment Embedded Throughout Instruction (ASSESSAFL, ASSESSFEED)*

Through the analytical approach detailed above, the study reveals the critical role geometric patterns play in both educational contexts and everyday experiences, while also addressing the challenges faced by teachers regarding teaching resources. The findings demonstrate the need for effective pedagogical strategies and proper educational tools to enhance the teaching and learning of geometric concepts in rural classrooms.

4.9. Upholding Ethical Standards

Before commencing the research, I obtained ethical clearance from both the University of South Africa (Appendix A) and the Limpopo Provincial Research Ethics Committee (LPREC) (see Appendix B). This two-tiered process not only ensured compliance with ethical guidelines but also safeguarded the rights and welfare of all participating teachers involved in the study. It is crucial that educational research, particularly when involving video recordings of teaching practices, is conducted within a framework that priorities ethical considerations, ensuring that participants are treated with respect and integrity throughout the research process (Yesuf, 2024). To uphold the principles of anonymity, privacy, and confidentiality, I implemented a comprehensive strategy aimed at mitigating potential risks. Informed consent was obtained from the participants, learners, and their guardians prior to the study's initiation (Appendix F) (Villiger, 2025). Clear communication regarding the purpose of the research and how video recordings would be utilised was essential for ensuring that all participants understood their rights and the voluntary nature of their involvement, especially considering the challenges of those who did not want to be video recorded as discussed earlier.

I also took measures to anonymise participants and their respective schools by removing identifiable information and utilising pseudonyms in all reports, which helped create a secure environment that encouraged honest sharing of experiences. Additionally, secure data handling protocols were established, with all video recordings stored in encrypted formats and accessible only to the researcher and supervisor, thereby preserving the integrity of the data and protecting participants' privacy. The ethical rigor applied in this study enhances the reliability of the findings and contributes to the broader discourse on ethical practices in educational research.

4.10. Trustworthiness of the Study

As in any empirical research, it is imperative for me as the researcher to ensure that the data collected authentically represents the perspectives and experiences of the participants involved. Enhancing the trustworthiness of a qualitative study necessitates the implementation of several strategies designed to bolster its credibility, transferability, dependability, and confirmability (Anney, 2014). The following sections details how these strategies were employed in the current study, which specifically examines the teaching of geometric patterns by Grade 4 rural teachers in Sekhukhune East District.

4.10.1. Dependability

Dependability refers to the consistency of a study's findings over time and across different contexts, highlighting the necessity for a thorough and systematic approach to the research process (Anney, 2014). This ensures that the study can be replicated and that its results are reliable. To enhance dependability in the current research, I implemented several strategies including the use of VSRIs, regular audits of data collection and analysis procedures, and comprehensive documentation of the research process. Such meticulous measures demonstrate that the findings are not only credible but can also withstand critical scrutiny.

In this study, I emphasised the importance of maintaining clear communication and establishing regular check-ins with participants to address any uncertainties and gather additional insights regarding their experiences and pedagogical approaches. The VSRI sessions provided opportunities for teachers to use their own words to offer interpretive elaborations on their pedagogical approaches and were invaluable for clarifying interpretations of the findings. Furthermore, conducting audits of both data collection and analysis procedures ensured that any procedural modifications were documented and justified, promoting transparency throughout the research process. Additionally, thorough documentation of the research process allowed for an organised and systematic approach to data management. Each stage of the study was meticulously recorded as demonstrated in the appendices provided, which promotes trust in how the research was conducted and increases the study's overall dependability.

4.10.2. Transferability

Transferability refers to the degree to which the findings of a qualitative study can be applied or generalised to different contexts, settings, or populations (Anney, 2014). In this study, the focus was on capturing the richness and depth of the collected data rather than making sweeping generalisations. To enhance transferability, I provided detailed descriptions of the research context, the participants involved, and the methodologies utilised during data collection and analysis. By presenting a comprehensive account of the teachers' perspectives, interpretations, and experiences in teaching geometric patterns, I aimed to allow readers to evaluate the applicability of the findings to their own contexts. This approach ensures that those working in different educational environments can draw meaningful connections to their own situations, thereby enabling broader applications of the insights gained from this research. The in-depth information relating to the context of the study provided allows for a clearer understanding of how specific pedagogical practices might resonate across diverse educational landscapes.

4.10.3. Confirmability

Confirmability pertains to the extent to which the findings of a qualitative study can be corroborated by others, ensuring that the results genuinely reflect the perspectives of the participants rather than the biases of the researcher (Anney, 2014). In the context of this study, my supervisor played a vital role as an inquiry auditor, helping to verify the validity of the analyses, conclusions, and recommendations drawn from the data.

This oversight not only reinforced the integrity of the research findings but also ensured that the raw data was closely aligned with the interpretations made throughout the study, thereby enhancing the overall confirmability. To further strengthen confirmability, I adopted practices that promote transparency and rigor throughout the research process. Regular reviews of the data analysis, along with detailed documentation of the procedures used, allowed for an objective evaluation of the research findings. Through maintaining thorough records of my analytical processes and decisions, I facilitated an external audit trail that others could follow, thus confirming the authenticity of the conclusions drawn. This approach is particularly relevant in qualitative research, where understanding the context and the researcher's role is crucial for assessing the trustworthiness of the outcomes (Mpitso, 2025).

In addition, by fostering an environment of open feedback and dialogue with the teachers, I ensured that their voices were authentically represented in the findings. Implementing member checking further allowed participants to review and provide feedback on their representations in the research, thus enhancing the credibility of the results pertaining to their teaching of geometric patterns. This collaborative approach contributed to a richer understanding of the teachers' teaching of geometric patterns and further solidified the confirmability of the research.

4.10.4. Credibility

Credibility is defined as the extent to which the researcher's interpretations accurately reflect the data provided by participants (Korstjens & Moser, 2017). To ensure the credibility of this study, I employed several strategies, including member checking and triangulation. After transcribing the semi-structured interviews, I shared the transcripts with the participating teachers to gather their feedback. Following this, I held follow-up discussions with the teachers to ascertain whether my interpretations aligned with their intended meanings articulated during the interviews. This iterative process of feedback not only validated the findings but also strengthened the overall credibility of the research. Member checking allowed the participating teachers to confirm that their perspectives were accurately captured and represented in the analysis. Their involvement in reviewing the transcripts and engaging in dialogue regarding the interpretations fostered a collaborative environment in which teachers felt their insights were valued. This practice aligns with qualitative research principles, which emphasise the importance of participant input in ensuring that the findings resonate with their lived experiences and positionality (Mbhiza, 2019). By incorporating their feedback, I was able to refine the analyses and ensure that the conclusions drawn were rooted in the participants' realities.

In addition, triangulation was employed as a strategy to enhance the credibility of the study. Employing multiple sources of data, including classroom observations, semi-structured interviews, and VSRIs, I was able to cross-verify the findings and identify common themes across different data points. This multifaceted approach increased the robustness of the findings and provided a comprehensive understanding of the pedagogical practices of Grade 4 teachers concerning geometric patterns. Through the integration of member checking and triangulation, the study's credibility was significantly bolstered, ultimately contributing to the trustworthiness and richness of the qualitative data collected.

4.11. Chapter Summary

This chapter provided a detailed overview of the methodology employed in the study, focusing on the qualitative approach utilised to investigate the pedagogical practices of Grade 4 rural mathematics teachers in the Sekhukhune East District, specifically regarding geometric patterns. The chapter delineated the various methodological choices implemented in the research, including classroom observations, semi-structured interviews, and VSRIs. I described in detail the research paradigm, research design, research approach, sampling methods, and analytical procedures used to interpret and present the findings in the current study. The chapter addresses the strategies employed to ensure the trustworthiness of the study, emphasising the importance of credibility, transferability, dependability, and confirmability in qualitative research and how these strategies were employed in this study. Ethical considerations are also a focal point, detailing the steps taken to secure ethical approvals, obtain informed consent, ensure confidentiality, and uphold participants’ rights throughout the research process. The following chapter presents and analyses the data generated from classroom observations, semi-structured interviews and VSRIs.

Chapter 5: Data Presentation and Analysis

5.1 Introduction

The purpose of this chapter is to provide a detailed analysis of participants' pedagogical approaches while teaching Grade 4 geometric patterns in Sekhukune East District. As the research design for this study is a multiple case study, each teacher (i.e. Phomelelo, Ditshegofatso, Keatlegile, Kelebogile, and Tshepisho) represents a distinct case that contributes to understanding the broader context of teaching Grade 4 geometric patterns. According to Mbhiza (2021), episodes can be derived from various data collection methods, including classroom observations, semi-structured interviews, and Video-Stimulated Recall Interviews (VSRIs). In this chapter, I thoroughly discuss the pedagogical approaches observed during the lessons of the participating teachers[1]. I utilise data obtained from the semi-structured interviews and VSRIs to enhance the insights garnered from the classroom observations. While classroom observations served as the primary data source, semi-structured interviews were instrumental in providing biographical information about the teachers and their pedagogical experiences regarding geometric patterns. This in turn helps to illuminate factors that shape teachers' pedagogical choices and approaches during lessons on geometric patterns. The VSRIs provided further clarity into the teachers' behaviours and language during their lessons, enriching my understanding of their pedagogical approaches.

Furthermore, visual aids, such as photographs or excerpts from the classroom observations, are provided to illustrate the observed teaching and learning processes in action, allowing for a more comprehensive view of the pedagogical approaches under consideration. I begin the data presentation and analysis with the case of Phomelelo's teaching. In this chapter, I draw on Shulman's (1987) Pedagogical Reasoning and Action (PRA) framework, with particular emphasis on the recognition rules established in Chapter 3, to analyse and interpret the teachers' pedagogical approaches. This framework provides a structured lens through which to examine how

[1] Guided by the considered comprehensiveness of the selected episodes in answering the research questions, I draw from 2 lessons from the cases of Phomelelo, DItshegofatso and Keatlegile and 1 lesson each for Kelebogile and Tshepiso.

teachers understand content, transform it for instruction, and enact it within their specific classroom contexts.

5.2. The Case of Phomelelo's Teaching

Phomelelo, one of the five participating teachers in this study, brings extensive experience in teaching Grade 4 mathematics within a rural school context in the Sekhukhune East District. Her daily teaching realities are shaped by persistent challenges such as severely overcrowded classrooms, a shortage of teaching resources, and inadequate infrastructure. She related during the semi-structured interview that these conditions place significant pressure on teaching and learning processes and undermine efforts to deliver quality mathematics teaching. Reflecting on her experiences during our semi-structured interview, Phomelelo explained:

> *"In all honesty, teaching mathematics in rural schools often takes a toll on our careers and our well-being. We work under unfavourable conditions, one classroom can have more than seventy learners, and the shortage of teaching aids keeps increasing, forcing us to improvise. As a result, we resort to using cardboards, plastic bottles, and other available materials just so our learners can clearly understand the content. You know, when teaching geometric patterns, I have seen a video where they use sticky shapes to put on the board to show learners how patterns work, here I have to draw them"*

This statement highlights the emotional and professional strain placed on teachers, showing that the difficult working conditions negatively affect both their career satisfaction and overall well-being. Phomelelo's reference to overcrowded classrooms, sometimes exceeding seventy learners, demonstrates how learner–teacher ratios compromise effective teaching, classroom management, and individual learner support (see Meier & West, 2020). The chronic shortage of teaching aids reveals structural inequalities in resource allocation, which according to Phomelelo forces them to adopt improvised strategies using everyday materials such as cardboards and plastic bottles. This improvisation reflects the teacher's resilience and commitment, but it also illustrates the persistent lack of adequate educational support in rural contexts. To this case analysis, two episodes from Phomelelo's lessons were selected: one from the first observed lesson and another from the second. Figure 3 outlines the topics represented in the selected episodes.

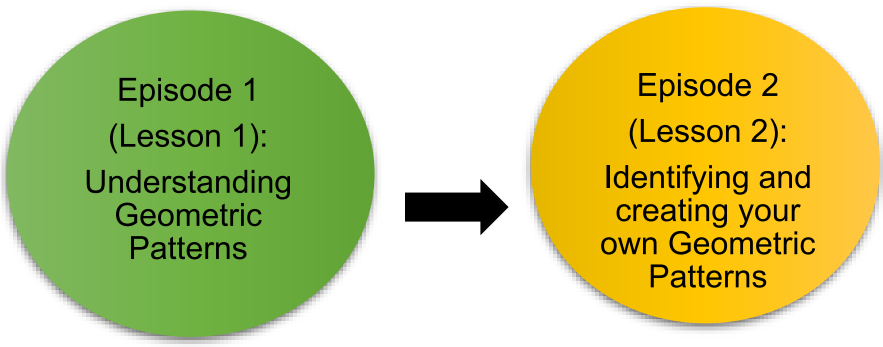


Figure 3: Phomelelo's selected episodes

5.2.1. Episode 1 (Lesson 1): Understanding Geometric Patterns

In a rural classroom accommodating more than seventy learners, Phomelelo began the lesson by introducing the concept of patterns. She initiated the discussion by asking learners to share their understanding of what patterns are. After receiving several responses, she clarified that a pattern refers to a repetitive sequence of numbers, shapes, or other mathematical objects. She further explained that patterns can be classified into numerical and geometric patterns, before proceeding to elaborate specifically on geometric patterns. The excerpt below captures the interaction that unfolded during this introductory phase of the lesson.

1. **Phomeleo:** Good morning class, we are going to talk about patterns today! So, is there anyone who can tell us what Patterns are?
2. **Learner 1:** "ke di paterone" (translated as it is patterns)
3. **Phomeleo:** he is saying, "di paterone, jwale le a di tseba di paterone?" (so do you all know patterns?)
4. **Learners:** yes
5. **Phomelelo:** okay. we have different types of patterns and our pattern today is Geometric patterns but we firstly have to know what patterns are as we cannot talk about Geometric patterns without knowing what patterns are, so a pattern is a repetitive sequence of colours, numbers, letters, shapes or objects. "Ke dilo tše dingwe le tše dingwe tša go te pusheletša, ekaba maletere, di nomoro goba di bopopego. Dilo tšeo re di bitša di pattern ka ge di tlabe di tsamaya ka tatelano ya go swana ebile ya go se fetoge" (it is anything that repeats itself, which it can be in the form of letters, numbers or objects so we call them patterns as it has the same sequence). And I said a pattern can be what? "Ke boletse dilo tse tharo" (I mentioned three things) so it can be what?
6. **Learner 2:** Colours, numbers, objects or shapes
7. **Phomelelo:** And a pattern with numbers is called a numeric pattern but today we are going to talk about geometric pattern. Geometric patterns need to be completed so

what do you do when you complete a pattern, for example, if I give you the pattern and I ask you to complete the pattern so what are you going to do?

8. **Learners:** (just stares at her and did not answer)
9. **Phomelelo:** You are going to repeat the very same shapes to complete the pattern. Let’s say I have a triangle and a circle in pattern 1 so in pattern 2 you are going to start with the triangle and then the circle. you can see that this pattern is in the same order. So, anyone with the question?
10. **Learners:** yes
11. **Phomelelo:** okay what are your questions?
12. **Learner 3:** can I add other different shapes in the pattern?
13. **Phomelelo:** no, if you can add the different shape that was not part of the pattern 1 and pattern 2 then your pattern would be wrong
14. **Learner 3:** okay I understand
15. **Phomelelo:** Okay so as we have agreed that patterns can be shapes or letters or colours and that when we write patterns it should be the same sequence and you don’t change anything it should be the same throughout the whole pattern. And after completing the pattern you should describe the pattern so what do you do when you describe the pattern? so you talk about the order of the shapes “ore botsa gore pattern ya rena ena le eng? ekaba enale di shape goba enale dinomoro goba enale di colour” (you tell us what our pattern consists of, is it shapes or numbers or colours) so in our pattern how will we describe it? who can try?
16. **Learner 4:** Our patterns have shapes
17. **Phomelelo:** He is saying our patterns has shapes, he is correct but what kind of shapes as you must name the kind of shapes that are in the pattern
18. **Learner 5:** the pattern has triangles and circles
19. **Phomelelo:** very correct! And when you complete it, you should add the same triangle and circles in the same order so if I have started with the triangle then you will also have to start with the triangle. Now let us look at the example that I have given (see image 1)

Image 1: The Example Phomelelo Provided

Table 11 presents an analysis of Phomelelo’s teaching in this introductory segment of the lesson using Shulman’s (1987) Pedagogical Reasoning and Action (PRA) framework. I integrate evidence from the above exchange with the corresponding codes as established in chapter 3, illustrating how Phomelelo’s pedagogical moves align with the key components of content comprehension, transformation, instruction, and assessment.

Table 11. *PRA Coding for Phomelelo's Turns 1 to 19*

Turns	Evidence from Exchange	PRA Component Codes
1	Introduces lesson by asking learners what patterns are.	INSTQUEST
2	Learner responds ("ke di paterone").	INSTPART
3	Teacher restates learner answer and checks collective understanding.	INSTEXPL; ASSESSAFL
4	Group response ("yes").	INSTPART
5	Defines patterns as "repetitive sequences" of colours, numbers, shapes, objects; uses Sepedi clarification; emphasises sequence and repetition.	COMPKNOW; COMPVOC; TRANSCTXT / TRANSSIMP; INSTEXPL
5	Asks learners to recall the three things that form patterns.	INSTQUEST; ASSESSAFL
6	Learner lists colours, numbers, objects/shapes.	INSTPART; ASSESSAFL
7	Explains numeric vs geometric patterns; asks how to complete a pattern.	COMPKNOW; INSTEXPL; INSTQUEST
8	Learners do not respond.	ASSESSAFL
9	Explains that pattern must be repeated in same order; uses triangle–circle as an example; links pattern rule to representation.	COMPSTRUCT; TRANSREP; INSTEXEMP; INSTEXPL
10–11	Learners say they have questions; teacher invites questions.	INSTPART; INSTQUEST
12	Learner asks if they can add different shapes.	ASSESSAFL; INSTPART
13	Teacher corrects the misconception (cannot add a new shape).	COMPMIS; ASSESSFEED; COMPSTRUCT
14	Learner confirms understanding.	INSTPART; ASSESSFEED
15	Recap types of patterns; explains pattern description; uses Sepedi to clarify; prompts description.	INSTEXPL; TRANSCTXT; COMPVOC; INSTQUEST
16	Learner says pattern has shapes.	INSTPART; ASSESSAFL
17	Teacher prompts learner to specify which shapes.	INSTQUEST; INSTEXPL
18	Learner identifies triangles and circles.	INSTPART; ASSESSAFL
19	Teacher affirms and reiterates rule of repeating shapes in same order using example.	INSTEXPL; COMPSTRUCT ; INSTEXEMP

Phomelelo’s pedagogical moves in this lesson reflect a strong alignment with Shulman’s (1987) PRA framework. Her clear explanation that patterns consist of sequences that repeat illustrates coherent content comprehension, supported by accurate mathematical vocabulary such as “sequence”, “same order,” and “repeats,” signalling the disciplined use of mathematical language essential for early geometric patterns teaching (COMPKNOW; COMPVOC). In relation to this, Riccomini et al. (2015, p. 238) assert that, “providing appropriate academic language support is important for all learners, especially in the mathematics classroom, where the ongoing development of explicit mathematical vocabulary is essential”. By identifying and correcting Learner 3’s misconception in line 12 about adding new shapes to a pattern, she reveals her ability to surface and address learner misunderstandings, an important dimension of pedagogical content knowledge noted by Shulman (1987) as central to effective teaching (COMPMIS; COMPSTRUCT).

Furthermore, it can be said that Phomelelo’s transitions between English and Sepedi to explain key ideas illustrates thoughtful transformation of content, as she adapts abstract mathematical structures into contextually meaningful representations for a multilingual rural classroom (TRANSCTXT; TRANSSIMP). Her use of a simple triangle–circle pattern to model repeating sequences further reflects deliberate representational choices that align with the concrete–pictorial–abstract trajectory widely regarded as effective in mathematics teaching (Leong et al., 2025) (TRANSREP; TRANSCPA). Phomelelo’s facilitation of learner responses and her coherent sequencing of teaching steps reflect effective lesson organisation (INSTPART; INSTORG). In addition, her continuous questioning and immediate correction of misconceptions exemplify assessment for learning, as she uses in-the-moment checks for understanding and provides responsive feedback to guide learners’ mathematical reasoning (ASSESSAFL; ASSESSFEED). The lesson continued with Phomelelo inviting learners to identify how the pattern progresses. Consider the following exchange.

20. **Phomelelo:** we will be now completing the pattern so which shapes will I draw next?
21. **Learners:** triangle and a circle
22. **Phomelelo:** Very good! (she drew the triangle and the circle (see image 2) on the chalkboard)
23. **Phomelelo:** now who can describe our pattern?
24. **Learner 6:** our pattern has triangles and circles

25. **Phomelelo:** correct (she wrote the description of the pattern on the chalkboard. See image 2)

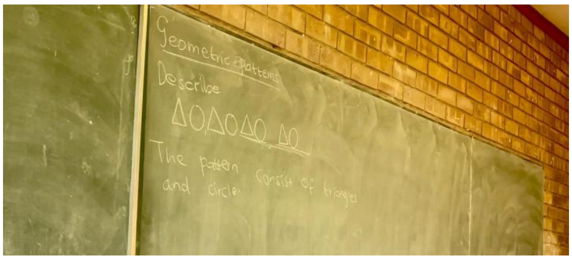


Image 2: description of the pattern provided by Phomelelo

26. **Phomelelo:** Now Let us do another example in our textbook. Let us look at page 129 there is an example there, but we are only going to talk about activity 1 and I will also write it on the chalkboard (see image 3). The first pattern we have '***AB AB'*** this is our pattern so what are we going to do when we complete our pattern? Are we going to add colours or objects or add letters?
27. **Learners:** We are going to add letters
28. **Phomelelo:** Which letters?
29. **Learners: *'AB AB AB'***
30. **Phomelelo:** Yes! '***AB AB AB ABC ABC'*** (see image 4)
31. **Learners:** No
32. **Phomelelo:** Why are you all saying No?
33. **Learners:** Because you have added ***'C'*** in your pattern
34. **Phomelelo:** that is very good, and yes, it is because I added ***'C'*** whereas my pattern started as '***AB AB'.*** So, as I have said that when you complete your pattern you should complete it in the same order you do not add or remove anything. Is that clear?
35. **Learners:** Yes
36. **Phomelelo:** so, the correct answer will be ***'AB AB AB AB'*** (see image 5). So if in our first pattern we used shapes and in our second pattern we used letters, is that correct?
37. **Learners:** no
38. **Phomelelo:** why is it not correct?
39. **Learner 7:** because you said if the pattern has shapes, then we will only be completing it by adding the same shapes and if the pattern has letters, then we will only complete it by writing the same letters
40. **Phomelelo:** that is very correct

Image 3: example from the textbook

Image 4: answer given by the teacher

Image 5: Patterns corrected by learners

The sequence from Turns 20–25 moves beyond naming shapes to articulating the structure of a repeating pattern: the class completed the pattern and then described it, which surfaced the idea of a unit of repeat and order (even though those exact labels are not used). The teacher's prompts ("*which shapes will I draw next?"; "who can describe our pattern?*") and the act of documenting the description publicly on the chalkboard can be interpreted to function as explanatory talk and worked exemplification (INSTEXPL; INSTEXEMP), while also checking current understanding (ASSESSAFL). According to Kundu and Bej (2020) in their examination of teaching practices, such prompts not only check for student understanding, but also facilitate a supportive environment for learners to articulate their thoughts and reasoning about mathematical concepts. The textbook item ("AB AB") in image 3 provides a clean representational case for repeating patterns. When Phomelelo wrote "AB AB AB ABC ABC" (Turn 30 and image 4), the learners' immediate rejection (Turn 31) and justification (Turn 33) created a diagnostic moment: learners recognised a violation of the unit by introducing "C". This is exemplary use of a nonexample to expose and correct a common misconception (COMPMIS) and to make the pattern rule explicit (COMPSTRUCT).

In addition to the above, the teacher's follow-up ("*complete it in the same order; you do not add or remove anything*") is high-value feedback (ASSESSFEED) that codifies the generalisation for future tasks. Across Turns 26–36, the teacher cycles through elicitation → learner response → public recording → challenge/nonexample → learner justification → teacher consolidation. This pattern exemplifies assessment for learning: questions are not merely rhetorical; they elicit reasons *("Why are you all saying No?"),* enabling learners to externalise their rule-based reasoning. The corrected completion "AB AB AB AB" (Turn 36) then becomes a worked benchmark the class can emulate, with explicit criteria for correctness. As researchers such as Smith et al. (2020) point out, enhancing learners' mathematical discourse by prompting them to articulate their understanding is crucial for developing mathematical literacy and reasoning. The lesson continued and the following exchange took place:

41. **Phomeleo:** so now let's continue with other examples (she drew a circle followed by a triangle, then a rectangle, circle, triangle and a rectangle) so here is my pattern, what will I draw next?
42. **Learner 8:** we will draw a rectangle, triangle and a circle
43. **Phomelelo:** is he correct?
44. **Learners:** no
45. **Phomelelo:** why is he not correct?
46. **Learner 9:** because he has started with a rectangle instead of starting with a circle then followed by a triangle just as you have done
47. **Phomelelo:** wow! That is very good, please clap hands for her
48. **Learners:** (claps hands)
49. **Phomelelo:** when you get home, make sure that you continue creating your own patterns using different shapes to see that you understand akere (okay).

In this exchange, Phomelelo continues to demonstrate coherent pedagogical reasoning through deliberate use of examples, questioning, and formative assessment aligned with Shulman's (1987) PRA framework. By drawing a repeating sequence of shapes: circle, triangle, rectangle, and asking learners what should come next, she employs clear representational choices (TRANSREP) that make the underlying structure visible while using the example to drive reasoning (INSTEXEMP). Her follow-up prompt ("what will I draw next?") functions as targeted questioning for reasoning (INSTQUEST), inviting learners to apply the established pattern rule. When Learner 8 provided an incorrect completion, Phomelelo leverages this as a formative assessment opportunity (ASSESSAFL), prompting peers to evaluate and justify the correctness of the response. Learner 9's explanation demonstrates strong structural awareness (COMPSTRUCT) by identifying the incorrect starting point and restating the expected order of shapes in alignment with the original pattern. Phomelelo's praise and encouragement to "clap hands" offer immediate feedback (ASSESSFEED) and reinforce positive mathematical reasoning, supporting a climate of active learner participation (INSTPART). Her closing instruction urging learners to create their own patterns at home extends explanatory guidance (INSTEXPL) and encourages independent reinforcement of the pattern concept. This segment illustrates effective integration of representation, reasoning, assessment, and learner engagement, showing that Phomelelo consistently transforms content and guides learners toward deeper structural understanding of repeating geometric patterns. Table 12 presents a detailed coding of the exchanges from Turns 20 to 40 using Shulman's (1987) PRA framework.

Table 12. *PRA Coding for Phomelelo's Turns 20 to 40*

Turns	Evidence from Exchange	PRA Component Codes
20–22	Teacher cues completion, elicits next shapes; affirms and models on the board.	NSTQUEST; INSTEXPL; INSTEXEMP; INSTPART
23–25	Prompts description; learner names shapes; teacher records a description on the board.	INSTQUEST; INSTPART; INSTEXPL; INSTORG; ASSESSAFL
26–27	Transitions to textbook example, scopes task (Activity 1), rewrites on board; elicits what to add for completion.	INSTORG; INSTEXPL; INSTEXEMP; INSTQUEST
28–29	Narrows question ("Which letters?"); learners produce correct extension "AB AB AB".	INSTQUEST; INSTPART; ASSESSAFL
30– 31–34	Teacher intentionally/accidentally writes "AB AB AB ABC ABC"; learners reject; teacher solicits reason; learners cite rule violation; teacher consolidates principle (keep the unit intact).	ASSESSAFL; ASSESSFEED; COMPMIS; COMPSTRUCT; INSTEXEMP (nonexample)
35–36	Confirms understanding and supplies corrected completion "AB AB AB AB".	ASSESSFEED; COMPKNOW
36–40	Cross-case check: shapes vs letters across tasks; learners reason back to rule; teacher validates.	COMPSTRUCT; INSTQUEST; INSTPART; INSTEXPL; ASSESSAFL
41	Teacher draws repeating pattern (circle → triangle → rectangle → circle → triangle → rectangle) and asks what comes next.	TRANSREP; INSTQUEST; INSTEXEMP
42	Learner 8 provides an incorrect continuation ("rectangle, triangle, circle").	ASSESSAFL; INSTPART
43	Teacher asks class to evaluate correctness.	INSTQUEST; ASSESSAFL
44	Learners collectively respond "no."	INSTPART; ASSESSAFL
45	Teacher asks learners to justify why it is incorrect.	INSTQUEST; ASSESSAFL
46	Learner 9 explains correct structural rule: pattern must start with circle then triangle.	COMPSTRUCT; COMPMIS; INSTPART
47	Teacher praises learner's explanation and encourages applause.	ASSESSFEED; INSTPART
49	Teacher encourages learners to create their own patterns at home to reinforce understanding.	INSTEXPL; INSTORG; ASSESSADJ

5.2.2. Episode 2 (Lesson 2): Identifying and creating their own patterns

The second lesson opened with Phomelelo inviting learners to recall the content taught in the previous session. By prompting learners to retrieve prior knowledge, she employed an instructional strategy widely recognised for strengthening learner engagement, supporting conceptual consolidation, and enhancing active participation in mathematics classrooms (Hundeland et al., 2020; Han et al., 2019). This approach enabled her to assess learners' retention while creating a meaningful bridge between the previous lesson on geometric patterns and the new learning objective focused on generating patterns independently. The following exchange captures how Phomelelo activated learners' prior understanding to prepare them for creating their own patterns.

1. **Phomelelo:** Good morning class before I could start with my topic today, do you guys still remember what we have learned yesterday?
2. **Learners:** yes
3. **Phomelelo:** so, who can tell us what is it that we learned yesterday?
4. **Learner 1:** we have learned about Geometric Patterns and how to describe them
5. **Phomelelo:** very good! Please clap your hands for her
6. **Learners:** (claps hands)
7. **Phomelelo:** now that we all know what Geometric Patterns are and even know how to describe them. Today we are going to learn on how to create our own patterns so is there anyone who can come to the chalkboard to try and draw their own patterns?
8. **Learner 2 and 3:** Yes!
9. **Phomelelo:** Please draw different patterns and not copy each other's patterns and remember when you create a pattern it should be the same pattern throughout

Two learners went to the chalkboard to create and draw their own geometric patterns as shown in image 6.

Image 6: Learners creating and drawing their own Geometric Patterns

An important feature of this episode is Phomelelo's deliberate decision to invite learners to the chalkboard to construct their own geometric patterns. This pedagogical move significantly elevated learner participation, shifting learners from passive recipients of information to active contributors in the mathematical activity (INSTPART). In asking learners to independently produce patterns, she created an environment in which learners must apply their existing understanding of repetition, sequence, and order, which are core structural elements of geometric patterns (Du Plessis, 2018). This aligns with Shulman's (1987) emphasis on engaging learners in tasks that reveal their thinking and allow the teacher to observe how they transform content through their own reasoning. Phomelelo's instruction that learners draw "*different patterns*" and "*not copy each other's patterns*" (Turn 9) reflects purposeful instructional organisation (INSTORG), ensuring that each learner's work becomes an authentic demonstration of understanding of the work covered the previous day rather than imitation. At the same time, she reminds them that a valid pattern "*should be the same throughout*," reinforcing the underlying structural rule of repetition, an instance of content comprehension (COMPSTRUCT) which was her main emphasis in the previous lesson. As learners constructed patterns publicly on the board, Phomelelo was able to observe and evaluate their ideas in real time, an example of formative assessment moves (ASSESSAFL), since each drawing offers insight into learners' grasp of sequence, repetition, and correctness.

During VSRI, I asked Phomelelo to reflect on why she started her lesson where she had ended the previous day and she said:

> *"Beginning where we ended wasn't just a recap; I wanted to assess that the learners understood what we covered the previous lesson, to ensure that they understand, remember I asked them to continue creating their own patterns, so checking that they did that, very important."*

This VSRI reflection demonstrates a high level of reflective teaching as described in Shulman's (1987) PRA framework. Her statement shows strong Reflection-in-Action (REFLINA) because she articulated that beginning the lesson where she previously ended was an intentional, real-time pedagogical decision and not a routine recap but a strategic pedagogical move. This signals that she adapted her teaching responsively rather than relying on fixed lesson scripts. She also displays Reflection-on-Action (REFLONA) by evaluating the effectiveness of this strategy after the lesson. She explicitly links her decision to the need to assess whether learners had retained and understood the concepts introduced earlier, showing her awareness of the role of continuity in conceptual development (Scott et al., 2011). This aligns with the recognition that formative assessment is central to monitoring learner understanding (ASSESSAFL).

The above statement further reveals Pedagogical Insight (REFLINSIGHT). Phomelelo recognises that revisiting previous content supports cognitive reinforcement and ensures that misconceptions do not persist. This demonstrates her understanding that the coherence of mathematical learning depends on revisiting and strengthening foundational ideas, especially in topics such as geometric patterns where the structure builds cumulatively (Du Plessis, 2018). Importantly, when she notes that she had asked learners to continue creating patterns at home and wanted to "*check that they did that*," she shows Future-oriented Reflection (REFLPLAN). This indicates that she views learning not as isolated events but as a developmental trajectory that requires follow-up, consolidation, and verification. Checking learners' self-created patterns also acts as a formative assessment strategy, enabling her to gather evidence of understanding, detect errors, and adjust instruction accordingly (ASSESSAFL;

ASSESSFEED). The reflective statement resonates with Phomelelo's utterance during the semi-structured interviewed:

> *"it is very important gore (that) I repeat some things for these learners, because most of them there is no one to monitor their homework at home; so, when I start a lesson, I always make sure that I check that what I taught they understand, before I start with something new. That is how I support their learning."*

Additionally, the introductory activity reflects an element of productive transformation of content (TRANSREP), as the learners' chalkboard drawings served as pictorial representations that help make abstract notions of pattern structure visible (Shulman, 1987). The public space of the chalkboard also allowed misconceptions to surface visibly, enabling the teacher to provide feedback (ASSESSFEED) as needed and to affirm correct reasoning. Encouraging learners to "*come to the chalkboard*" thus served dual purposes: it can be said that it promoted mathematical agency while giving the teacher rich opportunities to adapt teaching responsively. The following exchange continued the lesson:

10. **Phomelelo:** is the first pattern correct?
11. **Learners:** yes
12. **Phomelelo:** it is correct and what about the second pattern?
13. **Learners:** it is also correct
14. **Phomelelo:** Who can tell us how did they create their patterns
15. **Learner 4:** They drew a pattern of circles, triangles and squares
16. **Phomelelo:** it is indeed correct so who can describe the patterns that they have drawn?
17. **Learner 4:** our pattern has shapes
18. **Phomelelo:** our pattern has shapes but what kind of patterns?
19. **Learner 5:** our pattern has circle, triangles and squares
20. **Phomelelo:** is he correct?
21. **Learners:** yes

In this exchange, Phomelelo used a series of probing questions to check learners' understanding and evaluate the correctness of their patterns, demonstrating consistent formative assessment practice (ASSESSAFL) throughout the interaction. Her initial questions about the correctness of the first and second patterns function as reasoning-eliciting prompts (INSTQUEST), encouraging learners to evaluate mathematical work rather than passively accept it. As learners respond, their active engagement reflects effective facilitation of learner participation (INSTPART), a key pedagogical strategy for building mathematical sense-making (Basheti et al., 2019). When she asked learners to explain how the patterns were created, Phomelelo moved beyond correctness toward eliciting conceptual reasoning, using explanatory talk

(INSTEXPL) and probing for detail. Her follow-up question, "*what kind of shapes*?" demonstrates attention to mathematical vocabulary precision (COMPVOC), pushing learners to specify the structural components of their patterns rather than provide surface-level descriptions. Learners' responses reveal growing understanding of geometric pattern composition, and the teacher's confirmation of correctness served as feedback that reinforces accurate conceptual explanations (ASSESSFEED) (Shulman, 1987). This exchange shows how Phomelelo strategically combined questioning, clarification, and verification within the PRA framework to deepen learners' comprehension of pattern structure while maintaining high learner involvement. Consider the following exchange alongside image 7.

22. **Phomelelo:** and once again, what is our topic today?
23. **Learners:** Geometric patterns
24. **Phomelelo:** Good! And we now do know how to complete and describe them right?
25. **Learners:** yes
26. **Phomelelo:** okay now let's continue and look at activity 4 on page 130. So, how many triangles do we have in pattern 1?

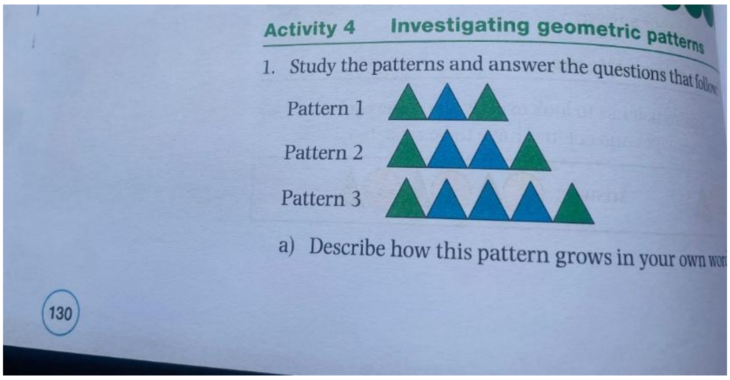


Image 7: Activity 4 on page 130 of the textbook

27. **Learners:** three triangles
28. **Phomelelo:** Good and how many green triangles are in pattern 1?
29. **Learner 6:** Two green triangles
30. **Phomelelo:** And how many blue triangles are in pattern 1?
31. **Learner 7:** It is only one blue triangle
32. **Phomeleo:** Now let's look at pattern 2, how many green triangles are there?
33. **Learner 8:** Two green triangles
34. **Phomelelo:** And how many blue triangles?
35. **Learner 9:** The blue triangles are two
36. **Phomelelo:** Then now let's look at pattern 3. How many triangles in total?
37. **Learner 10:** There are five triangles
38. **Phomelelo:** And how many triangles are green?
39. **Learner 11:** Two triangles
40. **Phomelelo:** And how many are blue?
41. **Learner 12:** Three triangles

42. **Phomelelo:** let us now imagine how pattern 4 would look like. So how many triangles would we have in pattern 4?
43. **Learner 13:** six triangles
44. **Phomelelo:** is it correct?
45. **Learners:** yes
46. **Phomelelo:** correct, please clap hands for her
47. **Learners:** (claps hands)
48. **Phomelelo:** so how many green triangles would pattern 4 have?
49. **Learner 15:** three green triangles
50. **Phomelelo:** is that correct?
51. **Learner 16:** no
52. **Phomelelo:** why is it not correct?
53. **Learner 16:** pattern 4 will still have two green triangles
54. **Phomelelo:** that is correct and how many triangles would be blue?
55. **Learner 17:** four blue triangles
56. **Phomelelo:** correct

In this segment, Phomelelo demonstrates a well-structured pedagogical approach characterised by systematic questioning, sustained formative assessment, and strong attention to pattern structure. By revisiting the topic and confirming learners' recall (Turns 22–25), she engaged in explanatory talk (INSTEXPL) and used formative checks (ASSESSAFL) to ensure conceptual continuity before introducing Activity 4 from the textbook. This can be interpreted as a deliberate representational choice (TRANSREP) that anchors learning in a clear visual context (Image 7). Her guided walkthrough of patterns 1–3, prompting learners to count total, green, and blue triangles, reflects purposeful instructional organisation (INSTORG) and continuous reasoning-eliciting questioning (INSTQUEST), enabling learners to observe how the pattern grows (from 3 to 4 to 5 triangles) and how specific attributes remain constant (green triangles consistently equal 2), demonstrating emerging structural awareness (COMPSTRUCT). When learners were asked to imagine Pattern 4 (Turn 42), she shifted from concrete to mental representation, extending the representational reasoning required to generalise the pattern rule. The incorrect suggestion of "*three green triangles*" (Turn 49) reveals a misconception (COMPMIS), which peers successfully corrected by referencing the invariant structure of the pattern; an impressive indicator of learner-driven reasoning (INSTPART) supported by teacher facilitation. Phomelelo's affirmative feedback (ASSESSFEED) reinforced accurate mathematical justification, while her consistent sequencing of prompts across all patterns enhances conceptual clarity. Table 13 presents the codes summarised codes for episode 2, using Shulman's (1987) PRA framework.

Table 13. *PRA Coding for Phomelelo's Episode 2*

Turns	Evidence from Exchange	PRA Component Codes
Turns 1–6	Teacher opens the lesson by asking what patterns are; learners respond, teacher defines patterns clearly using examples (letters, numbers, shapes) and supports understanding in Sepedi.	INSTQUEST; INSTPART; COMPKNOW; COMPVOC; TRANSCTXT/TRANSSIMP; ASSESSAFL
Turns 7–9	Teacher distinguishes numeric vs geometric patterns; explains how to complete patterns; uses triangle–circle sequence to model pattern structure.	INSTEXPL; INSTEXEMP; COMPSTRUCT; TRANSREP
Turns 10–14	Learners raise questions; misconception emerges about adding new shapes; teacher corrects misconception and reinforces rule.	INSTPART; COMPMIS; ASSESSFEED; ASSESSAFL
Turns 15–19	Teacher reviews how to describe patterns; prompts learners to specify shapes; learners identify triangles and circles; teacher affirms and re-explains pattern order.	INSTEXPL; INSTQUEST; INSTPART; COMPSTRUCT; COMPVOC
Turns 20–25	Teacher completes pattern on board; asks learners to describe pattern; records correct description.	INSTQUEST; INSTEXPL; INSTORG; TRANSREP; ASSESSAFL
Turns 26–34	Teacher introduces Activity 1 ("AB AB"); learners complete pattern; teacher writes incorrect nonexample; learners reject it and justify; teacher consolidates pattern rule.	INSTEXEMP – Nonexample; INSTQUEST; ASSESSAFL; COMPSTRUCT; COMPMIS; ASSESSFEED
Turns 35–40	Class confirms corrected pattern ("AB AB AB AB"); teacher checks cross-representation rule; learner explains consistency between shapes and letters.	INSTQUEST; COMPSTRUCT; INSTPART; ASSESSFEED
Turns 41–49	New repeating pattern (circle→triangle→rectangle); learner gives incorrect continuation; peers correct order; teacher praises; encourages pattern creation at home.	TRANSREP; INSTEXEMP; INSTQUEST; COMPSTRUCT; COMPMIS; INSTPART; ASSESSFEED; ASSESSADJ
Turns 50–56	Learners examine growth of green vs blue triangles in Pattern 4; misconception corrected by peers; teacher confirms.	ASSESSAFL; COMPSTRUCT; INSTPART; INSTQUEST; ASSESSFEED

The following section focuses on the case of Ditshegofatso's teaching of geometric patterns.

5.3 The Case of Ditshegofatso's Teaching

Unlike many rural schools that still rely on traditional chalkboards due to infrastructural limitations, Ditshegofatso's classroom presented a notable exception. Upon entering his Grade 4 mathematics class, I found that the school had transitioned to using whiteboards, offering a comparatively modern and flexible teaching environment. This distinctive classroom context provided an important backdrop against which to examine his pedagogical approach to teaching geometric patterns. For this case, two lesson episodes were selected for detailed analysis. Episode 1 (Lesson 1) focuses on *Understanding Geometric Patterns*, where Ditshegofatso introduced the concept of geometric patterns, guides learners in identifying repeating units, and supports them in describing the structure of given patterns. Episode 2 (Lesson 2) shifts towards *Identifying and Creating Own Geometric Patterns*, with an emphasis on learner-generated examples, pattern construction, and the application of pattern rules through independent reasoning. Together, these two episodes illuminate how Ditshegofatso made geometric patterns concepts available to the learners, moving from foundational understanding to creative application, as illustrated in Figure 4 below.

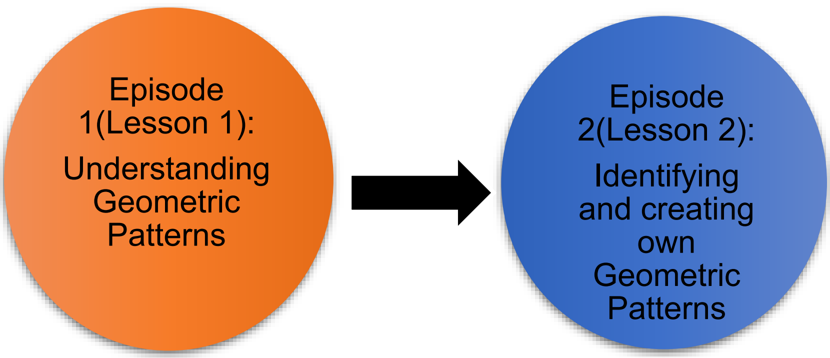


Figure 4: Ditshegofatso's selected episodes

5.3.1. Episode 1 (Lesson 1): Introduction to geometric patterns

Ditshegofatso opened his first lesson by introducing the concept of geometric patterns through familiar, real-life examples drawn from the learners' home and school environments. Through situating patterns within everyday practices, such as the layout of burglar windows and construction practices at home, he not only contextualised the content but also demonstrated strong content knowledge and an awareness of learners’ lived realities. Ditshegofatso verbally outlined the objectives of the lessons as follows: “*by the end of this lesson, everyone should be able to know what patterns are, they should also know what Geometric patterns are and how to draw and complete the geometric patterns given”.* After outlining the lesson objectives, he invited learners to articulate their existing understanding of patterns and to identify geometric patterns visible within their immediate surroundings. This approach provided a meaningful entry point into the lesson, grounding abstract mathematical ideas in concrete and observable contexts. The following exchange illustrates how he introduced and developed the concept of patterns in the classroom.

1. **Ditshegofatso:** Geometric patterns? But before we could know about geometric patterns let’s start with patterns because before you could know geometric patterns you should know patterns first. In our class here, we are having some patterns. At home there, mama and papa when they do something at home, they mostly use patterns. When they build their houses, they use patterns. So, what is a pattern?
2. **Learners:** (remained silent)
3. **Ditshegofatso:** anyone to try?
4. **Learner 1:** A pattern is something that shows and shows again
5. **Ditshegofatso:** That is the answer from her mind so anyone to try again?
6. **Learner 2:** patterns are things that repeat themselves
7. **Ditshegofatso:** that’s very good so let’s see our windows and look at the burglar windows, don’t we have patterns there
8. **Learners:** We do
9. **Ditshegofatso:** now can someone just stand up and show us the pattern on the window burglar windows
10. (One learner stood and pointed the patterns on the burglar window)
11. **Learner 3:** This is the burglar window I see a pattern here and I also see the same pattern and here (see image 8)
12. **Ditshegofatso:** Please clap hands for her. Do you all see the pattern on top
13. **Learners:** Yes
14. **Ditshegofatso:** Do you now see the same pattern in the middle
15. **Learners:** We do
16. **Ditshegofatso:** Are they not the same
17. **Learners:** They are!

Image 8: learner pointing a pattern of geometry on the burglar window

Ditshegofatso's teaching in through Turns 1 to 17 reflects a strong alignment with Shulman's (1987) PRA framework, particularly in his integration of content knowledge, contextualisation, and learner-centred pedagogical practices. His introduction of geometric patterns through familiar real-life examples (Turn 1) demonstrates deliberate transformation of content into meaningful, contextualised representations (TRANSCTXT), enabling learners to connect mathematical ideas to lived experiences. According to Tsiouri (2025, p. 30), "By appreciating the geometric beauty of their surroundings, students who engage in these activities also develop their link between mathematical concepts and useful applications". This grounding in everyday artefacts such as burglar windows also exemplifies effective representational choice (TRANSREP), as Ditshegofatso moved from abstract definitions to concrete, observable structures. According to Hundeland et al. (2020), combining verbal and physical illustrations highlights the importance of learning mathematics, connecting complex concepts to real-life experiences. This approach also assists learners in understanding the concept better as learners understand the concept better when they can relate mathematical concepts in their surroundings (BjÖrklund & Palmér, 2024). Throughout the above exchange, he employed probing questions (INSTQUEST) to elicit learners' ideas and diagnose their level of understanding (ASSESSAFL), particularly when learners initially remained silent (Turn 2) or provided partial reasoning (Turn 4–6). This supports formative assessment by allowing him to tailor his explanations in response to what learners reveal (Shulman, 1987). His follow-up prompts encouraged learners to articulate more precise definitions, supporting the development of content accuracy and vocabulary use (COMPKNOW; COMPVOC).

Furthermore, Ditshegofatso's encouragement for learners to physically point out patterns in the burglar windows (Turn 9–11 and image 8) fostered active participation (INSTPART) and cultivated structural awareness (COMPSTRUCT) as learners identified the repeating units within the visual pattern. His affirming feedback (ASSESSFEED) reinforced correct reasoning and bolstered learner confidence. The discussion of how the patterns repeat "*on top*" and "*in the middle*" (Turns 12–17) demonstrates intentional explanatory talk (INSTEXPL), guiding learners toward recognising sameness, repetition, and structure; all foundational aspects of geometric pattern understanding. Ditshegofatso's pedagogical approach in this segment of the lesson reveals thoughtful planning, effective use of context, and responsive teaching, all of which align with the PRA framework and support learners' conceptual engagement with geometric patterns. The lesson continued with the following interactive exchange:

18. **Ditshegofatso:** Now let's do our pattern in class; 5 boys come in front (Started arranging the boys). Now class can you see the pattern?
19. **Learners:** Yes
20. **Ditshegofatso:** Now how many boys are facing on the left-hand side
21. **Learners:** Three
22. **Ditshegofatso:** And how many boys are facing on the right-hand side
23. **Learners:** two
24. **Ditshegofatso:** didn't they form a pattern?
25. **Learners:** they did.

Image 9: Demonstration of learners on geometric patterns

In this segment, Ditshegofatso demonstrated highly interactive and conceptually grounded pedagogy by transforming an abstract mathematical idea into a physically embodied learning experience (INSTPART). Through arranging five boys at the front of the class to model alternating orientations (Turn 18), he used a powerful representational choice (TRANSREP) that aligns with the PRA requirement for making mathematical structures visible through concrete, familiar resources. One way of interpreting this is that the kinaesthetic demonstration heightened learner participation (INSTPART) and supports learners in noticing pattern regularities through observation rather than passive listening. The teacher's pedagogical approaches resonate with the findings by Edwards (2019) and Nemirovsky and Ferrara (2009) that kinaesthetic modelling enhances conceptual grounding in mathematics. His follow-up questions about how many boys face left versus right (Turns 20–23) function as formative assessment (ASSESSAFL) and reasoning-eliciting prompts (INSTQUEST), enabling him to gauge whether learners could identify the embedded structure within the arrangement. When he asked whether they recognised this as a pattern (Turn 24), he deliberately reinforced structural awareness (COMPSTRUCT), guiding learners to articulate that a pattern exists not merely because objects are arranged in a group, but because they follow a repeating rule. The class's affirmative responses indicate conceptual uptake, while his use of real learners as teaching tools contextualises the mathematics in a way that is culturally responsive and pedagogically effective.

26. **Ditshegofatso:** Now let me draw my own geometric pattern but before I forget, we have a 2-dimension patterns so today I am going to use triangles in our geometric pattern lesson so I will draw three triangles on the first pattern and on the second pattern I have drawn six triangles so how many triangles are we going to have in the third pattern? Anyone to come and draw the triangles (one learner went to the whiteboard to draw the triangles)
27. **Learner 4:** drew ten triangles
28. **Ditshegofatso:** Is he correct?
29. **Learners:** no
30. **Ditshegofatso:** so how many triangles are in pattern 3, please check our pattern correctly
31. **Learner 5:** (drew nine triangles)
32. **Ditshegofatso:** is she correct
33. **Learners:** yes
34. **Ditshegofatso:** clap hands for her
35. **Learners:** (clapped hands)

Image 10: Learner completing the geometric pattern on the whiteboard

In this segment of the episode, Ditshegofatso used a clear visual representations and learner-centred engagement to build understanding of geometric pattern growth. His decision to draw three triangles for Pattern 1 and six for Pattern 2 reflects a deliberate representational choice (TRANSREP) designed to make the multiplicative growth structure visible. By inviting a learner to draw Pattern 3 on the whiteboard (Turn 26), he actively promoted learner participation (INSTPART) and shifted ownership of mathematical reasoning to the class. When the learner incorrectly drew 10 triangles, Ditshegofatso used this as a formative assessment opportunity (ASSESSAFL), prompting the class to evaluate and justify correctness rather than simply supplying the answer himself. This aligns with the PRA emphasis on eliciting learner reasoning rather than procedural recall (Shulman, 1987). His follow-up prompt: “check our pattern correctly” in Turn 30, functions as guided feedback (ASSESSFEED), directing learners’ attention to the underlying pattern rule and encouraging self-correction. Learner 5’s revision to nine triangles demonstrates emerging structural awareness, which the teacher validates through affirming feedback and collective applause, reinforcing a supportive classroom climate.

Furthermore, Ditshegofatso’s pedagogical moves in the episode illustrate effective transformation of content, strategic questioning, and responsive assessment practices that strengthen learners' conceptual grasp of pattern generalisation. Table 14 summarises the coding of Ditshegofatso's teaching approaches using the PRA framework and the current study's recognition rules.

Table 14. *PRA Coding for Ditshegofatso's Episode 1*

Turns	Evidence From Exchange	PRA Component Codes
1	Introduces geometric patterns; explains that understanding patterns comes first; uses real-life examples (home building, burglar windows).	COMPKNOW; TRANSCTXT; INSTEXPL
2–3	Learners remain silent; teacher encourages attempts to respond.	ASSESSAFL; INSTQUEST; INSTPART
4–6	Learners attempt definitions; teacher refines answers and validates correct reasoning.	INSTPART; ASSESSAFL; COMPKNOW; ASSESSFEED
7–8	Teacher directs attention to burglar windows as concrete examples; learners confirm seeing patterns.	TRANSREP; TRANSCTXT; INSTEXPL
9–11	Learner identifies repeating structure in window; teacher praises and makes pattern visible to whole class.	INSTPART; COMPSTRUCT; ASSESSFEED
12–17	Teacher asks learners to observe repetition in top and middle sections of the window; learners confirm sameness.	INSTQUEST; INSTEXPL; COMPSTRUCT; INSTPART
18	Teacher begins class pattern activity using five boys; arranges them to model alternating directions.	TRANSREP; INSTPART; INSTEXPL; INSTQUEST
19	Learners acknowledge seeing the pattern.	INSTPART
20–23	Teacher asks how many boys face left/right; learners respond correctly.	ASSESSAFL; INSTQUEST; INSTPART
24–25	Teacher checks if arrangement forms a pattern; learners confirm.	COMPSTRUCT; INSTEXPL; ASSESSAFL
26	Teacher draws his own geometric pattern (triangles); introduces Pattern 1 (3 triangles) and Pattern 2 (6 triangles); asks learners to determine Pattern 3; invites a learner to draw it.	TRANSREP; INSTEXPL; INSTQUEST; INSTPART
27	Learner draws 10 triangles (incorrect).	ASSESSAFL
28–29	Teacher checks correctness; learners say "no."	INSTQUEST; INSTPART; ASSESSAFL
30	Teacher prompts class to re-examine pattern structure ("check our pattern correctly").	ASSESSFEED; INSTQUEST
31	Learner draws 9 triangles (correct).	ASSESSAFL; INSTPART
32–33	Teacher checks correctness; class affirms.	INSTQUEST; INSTPART; ASSESSAFL
34–36	Teacher praises learner and encourages applause; reinforces successful reasoning.	ASSESSFEED; INSTPART

5.3.2. Episode 2 (Lesson 2): Identifying and creating own geometric patterns

> *If there is one thing I try and do, is to allow these kids a space for them to do the mathematics, you know, to think and tell me what they see. For patterns, it is important that they see what makes patterns patterns, and they have to be given space to tell me their own mathematics – how they see things* **(Ditshegofatso, semi-structured interview)**

This statement was foundational in my observation of Ditshegofatso's teaching, as I was interested to explore how he puts this into practice. In the second lesson, Ditshegofatso continued building on the foundational concepts introduced in the previous session by guiding learners toward identifying and creating their own geometric patterns. To support this shift from recognition to construction, he brought matchsticks to class and explained that these would serve as concrete manipulatives for demonstrating and exploring geometric patterns. This hands-on approach set the stage for an interactive and practical engagement with the concept. The following exchange captures how the lesson unfolded. Consider the following exchange:

1. **Ditshegofatso:** so today, we will be talking about patterns using the things we use in our everyday lives, where we will be using our match sticks as our examples. (He took three match sticks) so how many match sticks am I holding?
2. **Learner 1:** three match sticks
3. **Ditshegofatso:** very good! (he now took six match sticks) so how many match sticks do I have now?
4. **Learner 2:** six match sticks
5. **Ditshegofatso:** (took nine match sticks out) so how many match sticks do I have now?
6. **Learner 3:** nine match sticks
7. **Ditshegofatso:** Good! So, who can describe our pattern?
8. **Learner 4:** our pattern is added by three match sticks
9. **Ditshegofatso:** that is very good! Now I am going to draw pattern 1 and pattern 2 (he drew pattern 1 and pattern 2) anyone who can come and complete pattern 3?
10. **Learner 5:** (raised his hand)
11. **Ditshegofatso:** please come
12. **Learner 5:** (went to the whiteboard and drew pattern 3 as shown in image 11).
13. **Ditshegofatso:** is he correct?
14. **Learners:** yes
15. **Ditshegofatso:** anyone who can tell me why everyone is saying, he is correct?
16. **Learner 7:** he is correct because our pattern is growing by adding two match sticks and that is what he did and our pattern 3 has seven match sticks, he then added another two match sticks in pattern 4, which gave us 9 match sticks
17. **Ditshegofatso:** that is very correct, please clap hands for her
18. **Learners:** (claps hands)

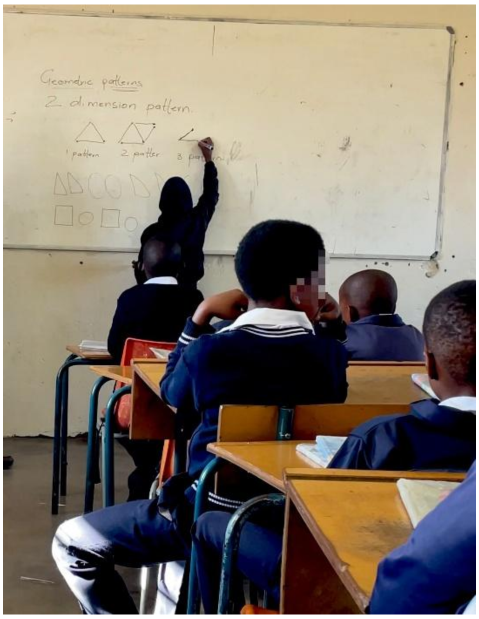


Image 11: learner drawing the match sticks to complete the geometric pattern

In this segment, Ditshegofatso demonstrated pedagogical reasoning that aligns closely with Shulman's (1987) PRA framework, particularly in the ways he transformed content and employed learner-centred teaching strategies. Through introducing the lesson through everyday objects, using matchsticks, the teacher drew on learners' lived experiences and grounds mathematical ideas in familiar, tangible resources (Akbaşlı, 2021). This according to Shulman (1987) reflects TRANSCTXT (Contextualisation) and TRANSREP (Representational Choices), as he converts an abstract number pattern into a concrete, manipulable form. Research supports this approach: using concrete materials enhances conceptual understanding and helps learners visualise mathematical structures (Carbonneau et al., 2013; Ball, Thames & Phelps, 2008). His sequential presentation of 3, 6, and 9 matchsticks reveals the underlying structure of a growing pattern, demonstrating COMPKNOW (Content Accuracy) and intentional COMPSTRUCT (Structural Awareness).

In addition, Ditshegofatso's consistent questioning, "*how many matchsticks am I holding?*", "*who can describe our pattern?",* and "*is he correct?*" illustrates purposeful INSTQUEST (Questioning for Reasoning). These questions are not merely procedural; they function as formative assessment moves (ASSESSAFL) to check understanding and uncover learners' reasoning, aligning with Black & Wiliam's (1998) emphasis on formative assessment as a driver of learning. When Learner 4 articulated that the pattern "*is added by three matchsticks*," the teacher recognised and validated correct reasoning, providing positive feedback (ASSESSFEED), which is known to reinforce mathematical confidence and accuracy (Hattie & Timperley, 2007). Inviting a learner to the whiteboard to complete Pattern 3 reflects strong INSTPART (Learner Participation) and positioned learners as active constructors of mathematical knowledge rather than passive recipients (Mpitso, 2025). This aligns with constructivist perspectives in mathematics education, which emphasise learner agency in making sense of mathematical structures (Piaget, 1970; Fosnot & Dolk, 2001). When the class agreed that the learner is correct and another learner justifies why ("*our pattern is growing by adding two matchsticks…*"), they demonstrate the ability to articulate generalisations, an indicator of developing structural reasoning. Ditshegofatso's prompt, "*why is he correct?",* elicits this deeper justification, demonstrating COMPSTRUCT and INSTQUEST, and aligning with literature that stresses the importance of mathematical explanation in developing conceptual understanding (Stylianides, 2007).

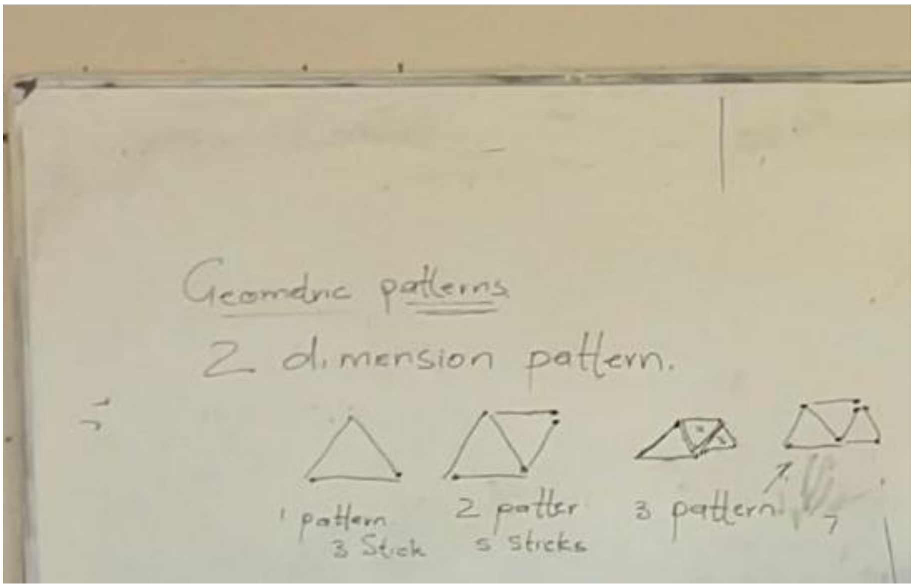


Image 12: Geometric patterns completed by a learner on the whiteboard

The lesson continued with the following interaction:

19. **Ditshegofatso:** how many match sticks makes four triangles?
20. **Learner 8:** nine match sticks
21. **Ditshegofatso:** how many match sticks would make five triangles?
22. **Learner 9:** eleven match sticks
23. **Disthegofatso:** how many match sticks would make six triangles?
24. **Learner 10:** thirteen match sticks
25. **Ditshegofatso:** so, do you all see how many match sticks do we keep adding?
26. **Learner 11:** yes

27. **Ditshegofatso:** how many?
28. **Learner 12:** two match sticks
29. **Ditshegofatso:** that is very good.

In this exchange, Ditshegofatso demonstrates effective mathematical pedagogy by guiding learners through a pattern-generalisation task involving matchsticks and triangles. His sequence of questions, "*how many matchsticks make four triangles?",* "*five triangles?",* "*six triangles?"*, reflects deliberate reasoning-eliciting questioning (INSTQUEST)**,** which aligns with the Shulman's (1987) PRA expectation that teachers prompt learners to articulate relationships rather than merely compute answers. Research shows that such questioning fosters conceptual understanding and supports mathematical generalisation (Boaler, 2016; Sfard, 2008). As learners correctly produced the counts (9, 11, 13), he used their responses as formative assessment evidence (ASSESSAFL)**,** monitoring whether they could recognise the additive pattern in the growing structure (Du Plessis, 2018).

When he asked, "*do you all see how many matchsticks we keep adding?*", Ditshegofatso steered learners toward recognising the structural regularity in the pattern, demonstrating COMPSTRUCT (Structural Awareness)**.** This aligns with research emphasising that identifying the invariant additive or multiplicative structure in growing patterns is central to early algebraic reasoning (Mulligan & Mitchelmore, 2013; Papic, 2015). The learner's response, "*two matchsticks*" signals a successful shift from noticing individual cases to identifying the growth rule. In affirming the response with "*that is very good*," the teacher provided positive, conceptual feedback (ASSESSFEED)**,** reinforcing the generalisation process, which literature shows to be crucial for building learners' mathematical confidence and accuracy (Hattie & Timperley, 2007). Furthermore, the use of matchsticks as manipulatives illustrates strong representational transformation (TRANSREP)**,** as he converts an abstract pattern relationship into a tangible, visual and countable form. Concrete materials have been shown to support pattern recognition and structural reasoning in primary mathematics (Sudihartinih & Purniati, 2020). His facilitation of repeated learner contributions also reflects effective learner participation (INSTPART)**,** an essential part of constructivist teaching approaches that promote active meaning-making (Voon et al., 2020). Collectively, this segment reveals how Ditshegofatso integrates representation, questioning, formative assessment, and feedback to scaffold learners' understanding of pattern growth, aligning tightly with Shulman's PRA framework and best practices in mathematics education.

Table 13 illustrates the number of matchsticks required to construct one to six connected triangles, providing a structured visual representation of how the geometric pattern grows.

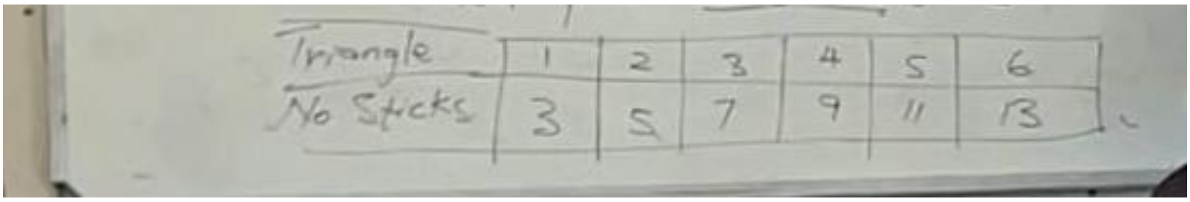


Image 13: geometric patterns in a table form

Ditshegofatso's use of this table reflects a deliberate representational choice (TRANSREP), as he transforms the abstract numerical relationship between triangle number and matchstick count into an organised, accessible format for learners. By directing learners' attention to the incremental increases shown across the row of matchstick values (3, 5, 7, 9, 11, 13), he supports the development of structural awareness (COMPSTRUCT), enabling them to recognise the constant growth rule underlying the pattern. During VSRI conversations relating to this segment of the lesson, Ditshegofatso reflected that:

> *When I introduced the table of values, my plan was to help learners see the relationship in a clearer and more organised way. When we were only working with the matchsticks physically and when we drew them on the board, some learners could recognise the pattern, but they struggled to describe how it was growing. So, putting the numbers into a table, I wanted to make the pattern more visible numerically and structured so that they could compare the number of triangles with the number of matchsticks side by side. The table helped them notice the way the pattern was growing, by two and that was why many of them could eventually explain that we keep adding two matchsticks each time.*

In view of this reflection, the table also functions as a tool for formative assessment (ASSESSAFL), allowing the teacher to check whether learners can deduce and articulate the rule governing the pattern. Furthermore, by prompting learners to interpret the table and justify the pattern's growth, Ditshegofatso engaged in reasoning-eliciting questioning (INSTQUEST), thus promoting deeper conceptual understanding of geometric pattern generalisation. The teacher concluded the lesson by encouraging learners to extend their exploration independently, instructing them to continue constructing additional triangles and to record the corresponding number of matchsticks in the table of values, thereby reinforcing their understanding of the underlying numerical relationship. Table 15 depicts the summary of the coding of the teacher's pedagogical approaches across the Turns 1 to 29.

Table 15. *PRA Coding for Ditshegofatso's Episode 2*

Turns	Evidence From Exchange	PRA Components Codes
1–3	Introduces lesson using everyday objects (matchsticks); shows 3, then 6 matchsticks; asks learners to count.	TRANSCTXT; TRANSREP; INSTEXPL; INSTQUEST; COMPKNOW
4–6	Learners correctly count matchsticks (6, then 9). Teacher affirms answers.	INSTPART; ASSESSAFL; ASSESSFEED
7–8	Teacher asks learners to describe the pattern; learner states pattern grows by adding 3 matchsticks; teacher validates.	INSTQUEST; COMPSTRUCT; INSTPART; ASSESSFEED
9–12	Teacher draws Pattern 1 and Pattern 2; invites learner to complete Pattern 3; learner constructs correct drawing.	TRANSREP; INSTPART; INSTEXPL; INSTORG
13–14	Teacher checks correctness; class confirms answer.	INSTQUEST; ASSESSAFL; INSTPART
15–17	Teacher asks why pattern is correct; learner explains that pattern is growing by adding 2 matchsticks; teacher praises.	COMPSTRUCT; INSTQUEST; ASSESSFEED; INSTPART
19–23	Teacher asks how many matchsticks make 4, 5, and 6 triangles; learners respond (9, 11, 13).	INSTQUEST; ASSESSAFL; INSTPART; COMPSTRUCT
25–29	Teacher asks what is being added each time; learner responds "two matchsticks"; teacher affirms.	INSTQUEST; COMPSTRUCT; ASSESSFEED; INSTPART

5.4. The Case of Keatlegile's Teaching

The case of Keatlegile's teaching focuses on two selected lesson episodes that illuminate his instructional approach to geometric patterns in the Grade 4 mathematics classroom. As depicted in diagram 5, Episode 1 (Lesson 1) centres on *Understanding Geometric Patterns*, where Keatlegile introduces learners to the foundational ideas of patterns, repetition, and structure. Episode 2 (Lesson 2) then progresses to *Identifying and Creating Geometric Patterns*, allowing learners to apply and extend their understanding through generating their own examples. Together, these episodes provide insight into how Keatlegile scaffolds learning across sequential lessons, moving from conceptual introduction to active construction and offer a lens through which his pedagogical reasoning and classroom practices can be analysed within the broader aims of this study.

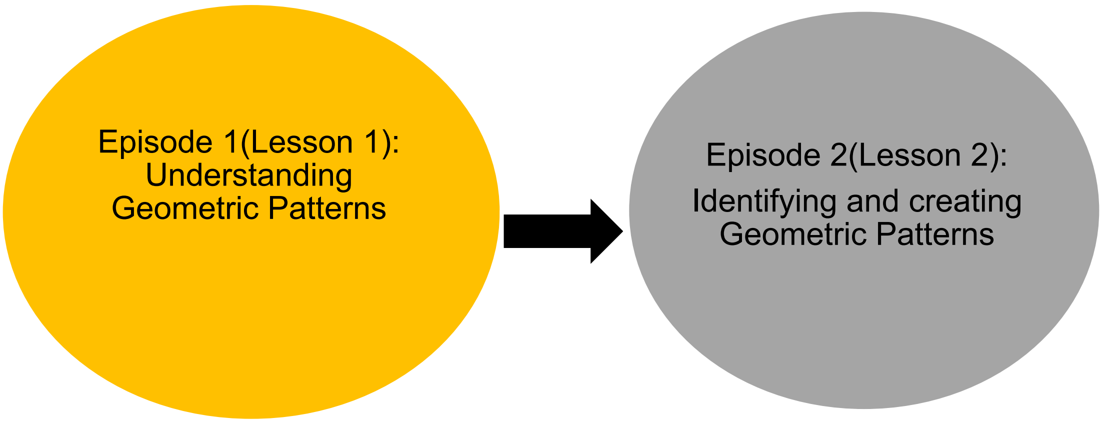


Figure 5: Keatlegile's selected episodes

5.4.1. Episode 1(Lesson 1): Understanding Geometric Patterns

Keatlegile introduced the lesson on geometric patterns by inviting a few learners to the front of the classroom (see Image 14), enabling those seated to count them aloud while pointing. This strategy aligns with Gardee and Brodie's (2015) observation that the act of pointing, especially in early mathematics learning, supports learners' cognitive engagement and strengthens their ability to connect concrete objects to emerging mathematical ideas, particularly for those who may struggle with abstract concepts. Beyond promoting active participation, Keatlegile also used the learners at the front as a concrete demonstration tool through which she could introduce and illustrate key ideas related to geometric patterns, thereby providing a meaningful and accessible starting point for the lesson. Consider the following exchange:

1. **Keatlegile:** Good morning my beautiful children. Our topic today is geometric patterns but before we can talk about it, I need learners to come in front. (Learners went in front). Now let's count them
2. (Learners counting the learners in front)
3. **Keatlegile:** So how many are they?
4. **Learners:** They are eight
5. **Keatlegile:** Good! now two learners turn to the right and two learners turn to the left, do like that until the last pair. Is this not a pattern?
6. **Learners:** It is
7. **Keatlegile:** Now again one learner raise your hands up, the next one put your hands down, the next one raise your hands up, the one next to him put your hands down, do like this until the last one. So, do you all see them?
8. **Learners:** Yes
9. **Keatlegile:** So is this not a pattern
10. **Learners:** It is
11. **Keatlegile:** So, do you all see how one can form a pattern?
12. **Keatlegile:** Yes

Image 14: learners in front of the class for demonstration purposes

13. **Keatlegile:** so how many learners came to the front?
14. **Learner 1:** eight (8) learners
15. **Keatlegile:** Good! Now I am going to arrange them in a geometric pattern to see the kind of a pattern that they will form (She demonstrated the geometric pattern using her learners as examples, see image 15).

Image 15: learners' demonstration on geometric patterns

In this opening segment, Keatlegile used embodied, highly participatory teaching strategies to introduce geometric patterns, aligning closely with Shulman's (1987) notion of transforming content in ways that make it accessible to learners. Calling learners to the front of the class and having the rest count them aloud, she employed representational choices (TRANSREP) that brought mathematical ideas into a concrete, observable form. This strategy also drew on contextualisation (TRANSCTXT), connecting abstract pattern concepts to learners' bodies and everyday sense-making processes; an approach supported by Gardee and Brodie (2015), who argue that pointing and physical engagement enhance young learners' capacity to grasp mathematical structure. Keatlegile's repeated use of actions, having learners turn left and right in alternating pairs, and later instructing learners to raise and lower their hands in a repeating sequence further demonstrates exemplification (INSTEXEMP), as she created multiple physical examples of repeating patterns through embodied demonstration. The observable action of using learners for demonstration resonates with Keatlegile's utterance during semi-structured interview. She stated that: "*one thing about teaching in this place is that you must be creative, you need to find things in the classroom to show the learners the concept, because we don't have resources at all*". Thus, it can be said that using learners for demonstration was one of her creative ways to make the concept available to learners.

Keatlegile's questions ("*Is this not a pattern?" (Turn 5, Turn 9), "Do you all see them?" (Turn 7)*) illustrate reasoning-eliciting questioning (INSTQUEST), prompting learners to articulate their understanding and recognise repetition and regularity within the arrangement. These questions function simultaneously as formative assessment moves (ASSESSAFL), allowing her to gauge whether learners can identify structural similarities across different embodied pattern configurations. When learners affirmed the presence of patterns, she leveraged these responses to deepen structural awareness (COMPSTRUCT) by reinforcing that patterns emerge through systematic repetition, a foundational idea in early algebraic reasoning (Mulligan & Mitchelmore, 2013; Papic, 2015).

Her pedagogical approach also reflects strong learner participation (INSTPART), as learners actively enacted and observed pattern behaviours rather than merely listening. This aligns with constructivist orientations in mathematics education, which emphasise engagement with manipulatives and embodied actions as essential for conceptual development (Mbhiza, 2021). Through arranging learners into geometric formations (Turn 15), Keatlegile transformed the classroom into a living representation of geometric patterns, showing the effectiveness of using learners' bodies as a concrete medium for exploring abstract mathematical ideas. This choice resonates with previous literature that emphasise kinaesthetic modelling to be effective in facilitating deeper conceptual understanding, especially in early grades where learners benefit from physical demonstrations (Nemirovsky & Ferrara, 2009).

16. **Keatlegile:** how many learners are bending and facing on the left-hand side?
17. **Learner 2:** 4 learners
18. **Keatlegile:** and how many learners are bending and facing on the right-hand side?
19. **Learner 3:** 4 learners
20. **Keatlegile:** is this not a pattern?
21. **Learners:** it is
22. **Keatlegile:** Good! Is there anyone who can make their own geometric patterns with the learners in front?
23. **Learner 4:** I can
24. **Keatlegile:** okay come and arrange your own geometric patterns
25. **Learner 4:** (asked two learners to face each other, the other two pair to face each other on the opposite direction, she did that with all the four pairs) this is my pattern
26. **Keatlegile:** (asking learners who are sitting down) did she formed a pattern?
27. **Learners:** yes
28. **Keatlegile:** Good! Please clap hands for her
29. **Learners:** (claps hands)
30. **Keatlegile:** thank you very much, please sit down
31. **Learners in front:** (went back to their seat)
32. **Keatlegile:** as I have already stated, we are going to explore geometric patterns, so who can tell me what they think geometric patterns are?
33. **Learner 5:** it is a pattern that has geometric
34. **Keatlegile:** mmmh! Mmmh! (giggling) is that correct?
35. **Learners:** yes
36. **Keatlegile:** (laughing) So Geometric patterns are patterns that consists of shapes that repeats over and over again (she drew triangle and a square) let us all look at the example that I have drawn. What do you see?
37. **Learner 6:** I see shapes
38. **Keatlegile:** correct! it is indeed shapes but what kind of shapes?
39. **Learner 7:** triangle and a square
40. **Keatlegile:** that is correct. So, if I want to complete my pattern, what would be my next shapes?
41. **Learner 8:** it will be another triangle followed by a square
42. **Keatlegile:** correct and if I am continuing with my pattern, the next pattern I will draw a square and triangle, right?
43. **Learners:** yes
44. **Keatlegile:** No, that is not correct. If I have started with a triangle shape then even on the next pattern, I have to start with a triangle. Is that very clear?
45. **Learners:** yes

This segment demonstrates Keatlegile's continued use of embodied, participatory teaching to support learners' emerging understanding of geometric patterns. Her questioning about the number of learners facing different directions (Turns 16–19) reflects strategic formative assessment (ASSESSAFL) and reasoning-eliciting questioning (INSTQUEST), enabling her to check learners' recognition of structure while guiding them toward the mathematical idea that patterns depend on repetition and regular arrangement. The use of learners' bodies to model alternating orientations shows purposeful representational transformation (TRANSREP), converting an abstract pattern rule into a concrete and highly visible demonstration, a practice that has been shown in previous research to strengthen learners' understanding of pattern regularities (Nemirovsky & Ferrara, 2009; Gardee & Brodie, 2015; Du Plessis, 2018). Her invitation for a learner to create her own geometric pattern using peers (Turns 22–25) illustrates learner participation (INSTPART) and positioned learners as co-constructors of mathematical knowledge, consistent with constructivist views that conceptual understanding deepens when students are empowered to generate and test their own ideas (Thompson, 2020). In asking the class to evaluate the constructed pattern (Turn 26–27), Keatlegile employed peer-supported formative assessment (ASSESSAFL) and reinforced structural awareness (COMPSTRUCT), as learners had to justify whether the arrangement qualifies as a pattern. Keatlegile's praise (Turn 28) functioned as positive feedback (ASSESSFEED).

The transition to formal representations (Turns 32–45) marks an important shift from concrete to abstract reasoning. When she asks learners to define geometric patterns and then introduces shapes on the board (triangles and squares), she engages in explanatory talk (INSTEXPL) and vocabulary development (COMPVOC), helping learners link earlier embodied experiences to disciplinary language. This movement aligns with the concrete–pictorial–abstract (CPA) progression (TRANSCPA), which facilitates conceptual transfer from manipulatives to symbolic or geometric representations (Akar et al., 2022). Keatlegile's challenge to the learner's incorrect idea that the next shape sequence could begin with a square (Turn 44) demonstrates effective identification and correction of misconceptions (COMPMIS). Her explanation that the pattern must start with the same shape each time reflects strong structural awareness (COMPSTRUCT), reinforcing to learners that pattern integrity relies on maintaining the order of the repeating unit. This pedagogical move is supported by research emphasising the importance of explicitly addressing misconceptions in early algebraic reasoning (Mulligan & Mitchelmore, 2013). Below is the exchange that took place during VSRI, in which I enquired on Keatlegile's use of learner-centred approach, particularly the use of learners' physical bodies for demonstrations:

1. **Meta (researcher):** I have noticed that you make use of the learner-centred approach in all your lessons, why is it important to you?
2. **Keatlegile:** I like using the learner-centred approach in all my lesson because I have seen so much improvements when using this approach, my learners are able to recall the concepts easily and also see so much improvement in the classroom participation
3. **Meta (researcher):** that is very true, learner-centred approach is very important, which now leads to my second question. I have also noticed how you like using learners to demonstrate patterns in your lessons so do you think this approach influence learners' understanding of geometric patterns?
4. **Keatlegile:** oh yes! I always use my learners when demonstrating geometric patterns as it is an effective way of making mathematical concepts to be understood better and not forgetting how it makes learning enjoyable
5. **Meta (researcher):** well, you are right as I have seen how all the learners wanted to come forward to be used as examples.

In this VSRI exchange, Keatlegile's reflections highlight a clear commitment to learner-centred pedagogy, demonstrating what Shulman describes as reflection-on-practice, where teachers justify pedagogical decisions based on observed improvements in learning. Her emphasis on enhanced recall and increased classroom participation aligns with research showing that learner-centred approaches promote deeper engagement and conceptual understanding, particularly in mathematics education where active involvement helps learners internalise structures and relationships (Adler & Ronda, 2015). Her explanation that using learners' bodies to demonstrate geometric patterns makes "*mathematical concepts… understood better*" reflects strong TRANSREP (transforming abstract ideas into concrete representations) and TRANSCTXT, both central to Shulman's (1987) PRA model. Furthermore, Keatlegile's recognition that this approach "*makes learning enjoyable*" reflects an awareness of the affective dimension of teaching, acknowledging that motivation and enjoyment contribute significantly to mathematical learning. Table 16 depicts the summary of Keatlegile's pedagogical approach in the current episode.

Table 16. *PRA Coding for Keatlegile's Episode 1*

Turns	Evidence From Exchange	PRA Components Codes
1–4	Teacher greets class, introduces topic, calls learners to the front, and asks the class to count them; learners count aloud and provide the total.	INSTORG (lesson setup), TRANSREP (learners as concrete representations), INSTQUEST, INSTEXPL, INSTPART, ASSESSAFL
5–10	Teacher alternates learners' positions (left/right; hands up/down) and asks whether these arrangements form patterns; learners repeatedly confirm.	TRANSREP (embodied modelling), INSTEXEMP (examples of patterns), INSTQUEST, COMPSTRUCT (recognising repeating structure), INSTPART, ASSESSAFL
11–15	Teacher checks conceptual understanding, asks how many learners came to the front, and arranges them into a geometric pattern for demonstration.	INSTQUEST, ASSESSAFL, INSTEXPL, TRANSREP, INSTORG, COMPSTRUCT
16–21	Teacher asks how many learners face left versus right; learners respond; teacher confirms whether this is a pattern; learners agree.	INSTQUEST, ASSESSAFL, INSTPART, COMPSTRUCT
22–29	Teacher invites a learner to create her own geometric pattern using peers; learner arranges pairs; peers evaluate correctness; teacher praises the learner and class claps.	INSTPART (learner agency), TRANSREP, INSTEXEMP, ASSESSAFL, ASSESSFEED
30–33	Learners return to seats; teacher transitions to conceptual discussion and elicits learners' definitions of geometric patterns; learner gives incomplete response.	INSTORG, INSTQUEST (eliciting prior knowledge), ASSESSAFL, INSTPART
34–36	Teacher probes correctness, clarifies definition ("patterns that consist of shapes that repeat"), draws triangle and square, and asks learners what they see.	COMPKNOW, COMPVOC, INSTEXPL, TRANSREP, TRANSCPA (concrete → pictorial), COMPMIS
37–39	Learners identify shapes ("triangle and square"); teacher confirms.	INSTPART, ASSESSAFL, INSTEXPL
40–43	Teacher asks how to complete the pattern; learner gives correct continuation; teacher presents a nonexample (incorrect sequence); class incorrectly agrees.	INSTQUEST, COMPSTRUCT, INSTEXEMP (non-example), ASSESSAFL
44–45	Teacher corrects misconception, explains that the pattern must start with the same shape, and learners confirm understanding.	COMPMIS (correction), COMPSTRUCT, ASSESSFEED, INSTEXPL, INSTPART

5.4.2. Episode 2 (Lesson 2): Identifying and Creating Geometric Patterns

Keatlegile began the second lesson by inviting learners to recall what they had learned in the previous session, an approach that plays a crucial role in reinforcing memory and activating prior knowledge before introducing new mathematical ideas. One learner responded by explaining that they had learned about geometric patterns and the repetition of shapes such as triangles and circles. As noted by Scott et al. (2011), eliciting learners' prior knowledge is an essential pedagogical practice in mathematics education because it enables teachers to build conceptually on what learners already understand, thereby creating a foundation for deeper and more connected learning. This approach aligns with the PRA framework, which emphasises the importance of linking new instructional content to what learners have previously encountered (Shulman, 1987). Following this recap, Keatlegile asked for a volunteer to come to the chalkboard and create their own geometric pattern, and one learner stepped forward to draw the pattern shown in Image 16. The exchange that unfolded is presented below.

1. **Keatlegile:** Good morning class. How are you?
2. **Learners:** Good morning mam. We are good and how are you?
3. **Keatlegile:** I am also good. So, is there anyone who still remember what we did yesterday?
4. **Learners:** yes
5. **Keatlegile:** who can tell us?
6. **Learner 1:** we learned about Geometric patterns
7. **Keatlegile:** what is it about Geometric patterns that we learned?
8. **Learner 2:** it has shapes that repeats themselves
9. **Keatlegile:** Good! So today we are going to continue learning about Geometric patterns using match sticks. If in pattern 1, I have 2 match sticks and in pattern 2, I have 4 match sticks then how many match sticks will pattern 3 have?
10. **Learner 3:** it will have 6 match sticks
11. **Keatlegile:** is it correct?
12. **Learners:** yes
13. **Keatlegile:** and how many match sticks will pattern 4 have?
14. **Learner 4:** 8 match sticks
15. **Keatlegile:** so how many match sticks are we adding on each pattern?
16. **Learner 5:** we are adding two match sticks on each pattern
17. **Keatlegile:** now who can come to the chalkboard and draw their own geometric patterns using the match sticks?
18. **Learners:** (remained silent)
19. **Keatlegile:** anyone to try?
20. **Learner 6:** (went to the chalkboard and created their own patterns)

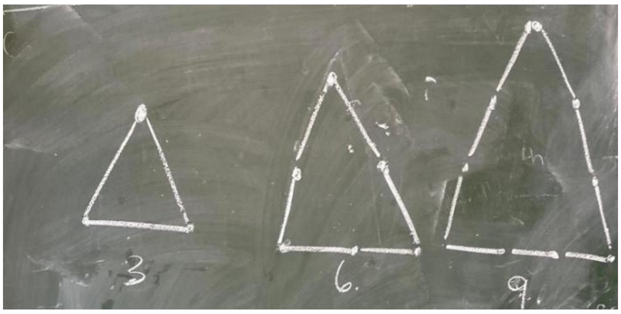


Image 16: a geometric pattern that was created by a learner

Keatlegile begun by asking learners whether they remember the previous lesson (Turns 3–5), which reflects INSTQUEST (Questioning for Reasoning) and ASSESSAFL (Formative Assessment Moves), as she checked what learners retained from the earlier lesson. Activating prior knowledge is a key pedagogical strategy in mathematics education because it enables learners to connect new content to existing conceptual frameworks (Scott et al., 2011; Mbhiza, 2021). When learners correctly describe geometric patterns as "*shapes that repeat*" (Turn 8), Keatlegile affirmed their reasoning, illustrating ASSESSFEED (Feedback) and COMPSTRUCT (Structural Awareness), as she guideed attention toward the repeating unit that defines geometric patterns. Her introduction of matchsticks to represent pattern growth (Turn 9) demonstrates TRANSREP (Representational Choices), where she transformed abstract content into a concrete, manipulable form; an approach shown to support understanding of pattern generalisation (Mulligan & Mitchelmore, 2013). She then asked learners to determine how many matchsticks appear in subsequent patterns (Turns 9–14), using a sequence of reasoning-eliciting questions (INSTQUEST) that encourage learners to recognise the additive structure of the pattern (Du Plessis, 2018). The learners' responses, correct identification of 6 and 8 matchsticks, provide opportunities for ASSESSAFL, enabling Keatlegile to verify that they are reasoning with the growth rule. Her follow-up question about how many matchsticks are added each time (Turn 15) reflects a shift from noticing individual cases to identifying a general rule, a hallmark of developing algebraic reasoning (Du Plessis, 2018). When learners respond that "*two matchsticks*" are added each time (Turn 16), she reinforced this insight, strengthening their structural understanding (COMPSTRUCT).

In addition to the above, when Keatlegile invited learners to construct their own geometric patterns on the chalkboard (Turns 17–20), she promoted INSTPART (Learner Participation) and offered learners agency in representing mathematical ideas; an approach consistent with Adler and Ronda's (2015) iteration that active learner involvement is a precondition for effective mathematics learning. Her encouragement for volunteers also reveals *reflection-in-action*, adjusting her teaching based on the quiet classroom (Turn 18) and prompting again for engagement. The lesson continued with the following exchange:

21. **Keatlegile:** is the pattern correct?
22. **Learners:** yes

23. **Keatlegile:** and how is the pattern growing?
24. **Learner 7:** it is growing by adding three match sticks
25. **Keatlegile:** that is very good! So, I will draw another geometric pattern and one of you will have to complete it. (she drew the geometric pattern) Who can come and complete the pattern?
26. **Learner 8:** I can try
27. **Keatlegile:** Good, please come
28. **Learner 8:** (went to the chalkboard and completed the pattern)
29. Keatlegile drew another geometric pattern on the chalkboard and asked her learners to come to the chalkboard to complete them (see image 17)

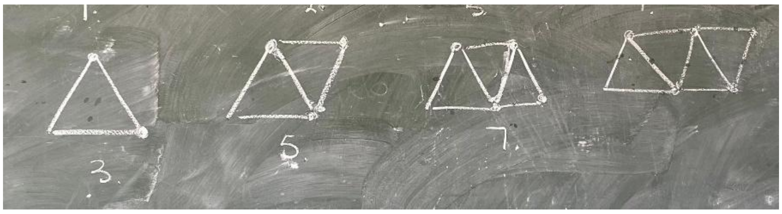


Image 17: geometric pattern that was completed by a learner on the chalkboard

30. **Keatlegile:** pattern 4 ye ae ngwadileng ke yona? (is the pattern 4 that he has written, correct?)
31. **Learners:** ee, ke yona (yes, it is correct)
32. **Keatlegile:** ke ka lebaka laeng lere ke yona? (why are you saying it is correct?)
33. **Learners:** ke ka lebaka la gore o okeditse ka mahlokwa a ma bedi go pattern 4 (it is because he added two match sticks on pattern 4)
34. **Keatlegile:** correct! Now let us imagine how many match sticks will pattern 6 have, so how many match sticks will we have in pattern 6?
35. **Learners:** (remained quiet)
36. **Keatlegile:** why are you all quiet?
37. **Learners:** re sa nagana karabo (we are still thinking about the answer)
38. **Keatlegile:** mmmhh (thinking) okay. I will wait for your answers then.
39. (after few minutes)
40. **Learner 9:** pattern 6 will have 13 match sticks
41. **Keatlegile: h**ow did you get your answer?
42. **Learner 9:** our pattern is being added by two match sticks so I wrote the pattern on a piece of paper, then continued to add two match sticks until I got to pattern 6
43. **Keatlegile:** that is very good. Did you all hear what he said?
44. **Learners:** yes
45. **Keatlegile:** Good! This is exactly how you should all do when you are given the pattern number and they ask you to write the number of match sticks in that pattern. I will give you another example. So, if I have three squares in pattern 1, six squares in pattern 2 and 9 squares on pattern 3, how many squares will I have in 7?
46. **Learner 9:** pattern 7 will have 21 squares
47. **Keatlegile:** that is very good but I cannot have one learner who is always answering my questions. What is happening with others?
48. **Learner 10,11 and 12:** (laughing) Ebe re sa nagana di Karabo (we were still thinking about answers)
49. **Keatlegile:** haha! Di Karabo tseo tsaka ka di nyaka go mehlala ye mengwe (I want those answers of mine on other examples).

Keatlegile's initial question: "*Is the pattern correct?"* (Turn 21), functions as INSTQUEST (Reasoning-Eliciting Questioning) and ASSESSAFL (Formative Assessment Move), prompting learners to evaluate work rather than simply accept it.

This supports the development of critical mathematical reasoning, as emphasised by Sfard (2008), who argue that formative questioning deepens understanding. When learners correctly identify that the pattern grows by adding three matchsticks (Turn 24), Keatlegile affirmed their reasoning (ASSESSFEED) and strengthens COMPSTRUCT (Structural Awareness) as learners articulated the underlying rule governing the pattern. Her next pedagogical move, drawing a new geometric pattern and inviting volunteers to complete it (Turns 25–28) reflects both TRANSREP (Representational Choices) and INSTPART (Learner Participation), positioning learners as active constructors of mathematical knowledge. When the class evaluated the correctness of the next pattern (Turns 30–33), Keatlegile again employed peer-supported formative assessment, reinforcing learners' capacity to justify correctness through reference to the growth rule. This kind of peer-mediated reasoning is well supported in the literature for strengthening mathematical argumentation skills (Franke et al., 2009; Adler & Ronda, 2015).

Keatlegile's shift to predictive reasoning, asking learners to imagine the number of matchsticks in Pattern 6 (Turn 34) represents a move into generalisation, a key developmental step in early algebra (Mulligan & Mitchelmore, 2013). The learners' silence (Turn 35) becomes diagnostic evidence (ASSESSAFL), prompting her to ask why they are quiet (Turn 36), a move that aligns with in-the-moment pedagogical adjustment, encouraging metacognition and reducing performance anxiety (Desoete & Craene, 2019). When a learner finally provided the correct answer (Turn 40) and explained their method (Turn 42), Keatlegile again reinforced structural reasoning through COMPSTRUCT, highlighting the learner's process of repeatedly adding two matchsticks. Her follow-up example involving squares (Turn 45) shows COMPKNOW (Content Knowledge) and INSTEXPL (Explanatory Talk) as she scaffolded learners toward recognising multiplicative and additive growth in patterns. When the same learner provided the answer again (Turn 46), Keatlegile humorously noted that she wants contributions from others, which serves two pedagogical purposes: promoting equitable INSTPART and modelling a collaborative classroom culture (Shulman, 1987; Adler & Ronda, 2015). Her remark on Turn 49, encouraging learners to give answers "*on other examples*", illustrates ASSESSFEED, guiding learners toward greater independence and engagement. In reflecting about language in-use in the previous exchange, Keatlegile reflected on the usage of both Sepedi and English during the lesson. Consider the following VSRI conversation:

1. **Meta:** I have noticed that in all of your lessons, like you did here, you were using Sepedi more than English, so why do you prefer to use Sepedi as your language of teaching and learning?
2. **Keatlegile:** most of our learners in rural schools often struggle to understand the English language especially in Grade 4 as they just come from the foundation phase where their language of teaching and learning is their home language so we often struggle with them when using English
3. **Meta:** but does that not disadvantage them when coming to writing assessments?
4. **Keatlegile:** it does, that is why I always explain to them each and every word in Sepedi, which is time consuming
5. **Meta:** do they improve if you do that?
6. **Keatlegile:** yes, most of them do
7. **Meta:** and does it not bother you that you have to now translate the whole question paper in Sepedi?
8. **keatlegile:** not really and besides now the Department of Education has introduced the usage of mother tongue in Grade 4 so our teaching resources consists of two languages which are Sepedi and English
9. **Meta:** including textbooks?
10. **Keatlegile:** no, at the moment it is only the lessons plans and question papers that we get from the district

In this VSRI exchange, Keatlegile demonstrates thoughtful pedagogical reasoning about her language choices, highlighting how the use of Sepedi supports learner comprehension in a rural Grade 4 context. Her reflection shows awareness of a key challenge in the transition from the foundation phase, where learners are taught primarily in their home language—to Grade 4, where English becomes increasingly dominant and often poses a barrier to understanding. In choosing to incorporate Sepedi, she enacted a form of contextualised instruction, ensuring that learners grasp mathematical concepts before encountering the additional cognitive load of English. Although she acknowledged that this approach is time-consuming and may create challenges during assessments, she mitigates this by explicitly teaching vocabulary in both languages, reflecting purposeful instructional adaptation aligned with pedagogical reasoning. Her mention of the Department of Education's introduction of mother-tongue based bi-lingual teaching in Grade 4 further indicates her awareness of the current policy intentions and shifts. The lesson concluded with Keatlegile assigning homework to the learners, as reflected in Turns 50 to 55 and illustrated in Image 18.

50. **Keatlegile:** I am going to give you home activity and I want you to ask your parents, sisters and brothers at home to help you, which I will check them tomorrow to see if

they indeed helped you. So let us open on page 134 on revision, number 2. Did you all open that page?

51. **Learners:** yes
52. **Keatlegile:** Good! So, they have given us pattern 1, pattern 2 and pattern 3. You are going to draw those patterns and answer the questions that follows. Is that clear?
53. **Learners:** yes
54. **Keatlegile:** is there any question regarding the home activity?
55. **Learners:** no

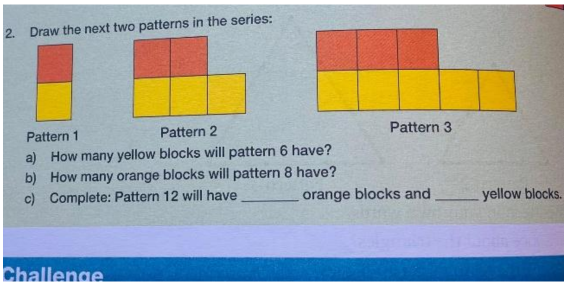


Image 18: home activity that was given to learners

The activity shown in Image 18 requires learners to analyse Patterns 1–3, draw subsequent stages, and determine the number of orange and yellow blocks in later positions (Patterns 6, 8, and 12). This homework consolidates the structural reasoning developed in class, where learners repeatedly identified how patterns grow by constant increments. It also reinforces the pattern-generalisation processes practised earlier, such as identifying repeating units, counting increases, and predicting future stages. In this way, the homework meaningfully links to the lesson's core mathematical processes of identifying growth rules, extending patterns, and justifying how quantities change from one stage to the next. In essence, the homework phase of the lesson reflects INSTORG (effective lesson closure through task assignment), TRANSREP (shifting pattern work from concrete class demonstrations to pictorial representations), and ASSESSAFL (formative assessment beyond the lesson to check understanding of pattern growth). Table 17 summarises the coding of the teacher's pedagogical approaches for episode 2, using Shulman's (1987) PRA framework, particularly the operationalised codes for the current study.

Table 17. *PRA Coding for Keatlegile's Episode 1*

Turns	Evidence From Exchange	PRA Components Codes
1–4	Teacher greets class, checks wellbeing, and asks learners to recall what was taught the previous day; learners correctly respond.	INSTQUEST (eliciting prior knowledge), ASSESSAFL (checking retention), INSTORG (lesson framing), INSTPART (learner responses)
5–9	Teacher probes what learners remember about geometric patterns; learners mention repeating shapes; teacher introduces matchstick pattern 1 → 2 → 3 (2, 4, 6 matchsticks).	INSTQUEST, COMPKNOW (content accuracy), TRANSREP (matchsticks as manipulatives), ASSESSAFL, COMPSTRUCT
10–12	Learners provide correct answers (6 matchsticks); teacher verifies correctness with class; learners affirm.	INSTPART, ASSESSAFL, ASSESSFEED
13–16	Teacher asks how many matchsticks in Pattern 4 (8 matchsticks) and asks learners to identify the growth rule ("adding two matchsticks").	INSTQUEST, COMPSTRUCT, ASSESSAFL, INSTPART
17–20	Teacher invites volunteers to draw their own geometric patterns on the chalkboard; initial silence; prompts again; a learner comes forward and constructs a pattern.	INSTPART, INSTORG, TRANSREP (student-generated representations), ASSESSAFL
21–24	Teacher asks if the pattern is correct; learners evaluate; teacher asks how it grows; learner explains "growing by adding three matchsticks."	INSTQUEST, ASSESSAFL, COMPSTRUCT, INSTPART
25–29	Teacher draws another geometric pattern and invites a learner to complete it; learner does so; teacher draws additional patterns and asks class to evaluate correctness.	TRANSREP, INSTEXEMP, INSTPART, ASSESSAFL
30–33	Teacher asks if Pattern 4 is correct; learners say yes; teacher asks why; learners justify rule ("added two matchsticks").	INSTQUEST, COMPSTRUCT, ASSESSAFL, INSTPART
34–38	Teacher asks learners to predict number of matchsticks in Pattern 6; learners initially silent; teacher probes why.	ASSESSAFL (diagnostic silence), INSTQUEST, INSTORG, INSTPART
39–42	After thinking time, learner states Pattern 6 has 13 matchsticks and explains method (adding by 2 each time). Teacher praises and highlights reasoning.	COMPSTRUCT, INSTPART, ASSESSFEED, TRANSREP
43–45	Teacher gives a new example (3, 6, 9 squares → Pattern 7 = 21 squares).	INSTEXPL, COMPKNOW, COMPSTRUCT, TRANSREP
46–49	Same learner answers again; teacher encourages broader participation; other learners explain they were thinking; teacher encourages contributions for future examples.	INSTPART, ASSESSFEED, ASSESSAFL, INSTORG
50–55	Teacher assigns homework (page 134): draw Patterns 1–3 and answer pattern-growth questions; checks for understanding of task; learners confirm.	INSTORG (lesson closure & homework setup), TRANSREP, ASSESSAFL (checking clarity), INSTEXPL

5.5 The Case of Kelebogile's Teaching

This case presents one selected episode from Kelebogile's Grade 4 mathematics teaching on understanding geometric patterns. Selecting a single, information-rich episode is consistent with a qualitative, case-study orientation that privileges depth over breadth, enabling a thick, contextualised description of pedagogical practice within a clearly bounded event (the lesson episode), rather than a thin overview across many lessons (Stake, 1995). The chosen episode was retained because it is analytically comprehensive: within a short span it captures Kelebogile's core pedagogical moves for pattern teaching (e.g., activating prior knowledge, introducing representations, eliciting and probing learner thinking, addressing misconceptions, and consolidating structure), allowing robust coding with the study's espoused theoretical framework and offering sufficient variation of evidence (teacher talk, learner responses, board work/artefacts) for credible interpretation. Methodologically, the episode aligns with the study's case-study design by bounding the case to a single, naturally occurring lesson. One carefully selected episode provided adequate analytic saturation for the study's aims and a coherent window into Kelebogile's pedagogical approaches in teaching Grade 4 geometric patterns. Figure 6 depicts the focus of the episode in focus, which is presented and analysed in the following sub-section.

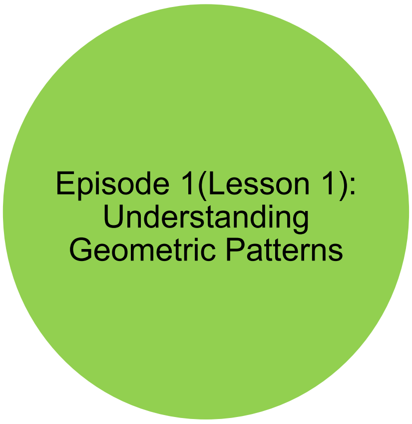


Figure 6: Kelebogile's selected episodes

5.5.1. Episode 1 (Lesson 1): Understanding Geometric Patterns

The following exchange captures the early phase of Kelebogile's lesson, where she introduces the concept of geometric patterns through familiar classroom objects and guided questioning. This episode illustrates how she scaffolds learners' understanding of pattern growth and structure, gradually transitioning from concrete materials (coloured chalks and drawings of soccer balls) to more abstract reasoning about how geometric patterns develop across successive stages.

1. **Kelebogile:** Good morning class. Is there anyone who can guess what our topic is for today?
2. **Learners:** yes/no
3. **Kelebogile:** okay before I can tell you, let us all pay attention and look what I am holding (holds coloured chalks in her hands) so, what is it that I'm holding on my hands?
4. **Learners:** you are holding chalks that have different colours
5. **Kelebogile:** Good! So, anyone who can guess what I will be teaching about today?
6. **Learner 1:** Drawings
7. **Kelebogile:** mmmh! Yes, we will be drawing but what will we be drawing?
8. **Learner 2:** shapes?
9. **Kelebogile:** correct! Shapes. So, our topic today is Geometric Patterns, which is the repetitive sequence of shapes and other mathematical objects (she drew two soccer balls on the chalkboard on pattern 1, she continued to draw pattern 2 where she drew four soccer balls) so how many soccer balls will I have on pattern 3?
10. **Learner 2:** six soccer balls
11. **Kelebogile:** correct. So how many soccer balls are we adding in this pattern?
12. **Learner 3:** we are adding two soccer balls
13. **Kelebogile:** very good! This is how geometric patterns works. We look at how a pattern is growing
14. Kelebogile took her colour chalks and drew pattern 1, where it had 3 circles that had two green colour and one yellow colour, she then continued to draw pattern 2, where it had four circles, with two green circles and two yellow circles.

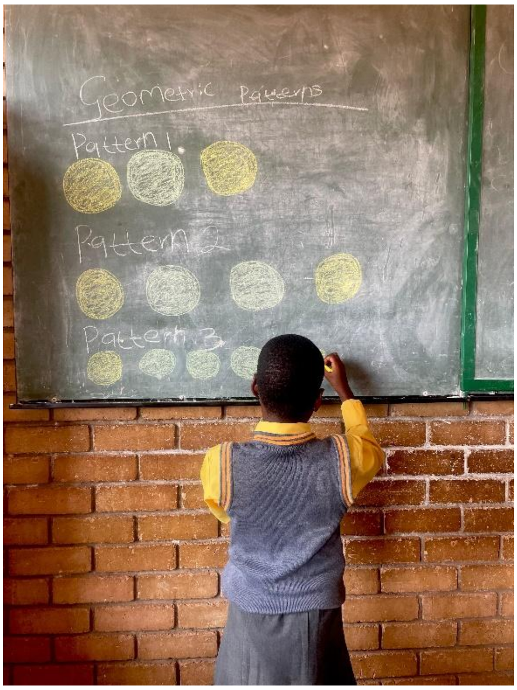


Image 19: a learner completing pattern 3 on the chalkboard

15. **Kelebogile:** I am going to do another example on the chalkboard and one of you is going to complete it right?

16. **Learners:** yes
17. **Kelebogile:** (draws pattern 1 with three circles that consists of two yellow circles and one green circle, she then proceeded to draw pattern 2 that has four circles, which consists of two yellow circles and two green circles)
18. So, who can tell me how is my patterns growing?
19. **Learner 4:** it is growing by adding one circle
20. **Kelebogile:** good but now I want someone who can describe our pattern, where you will talk about the colours
21. **Learner 5:** it is growing by one green circle and one yellow circle
22. **Kelebogile:** that is very good. Please clap hands for her
23. **Learners:** (claps hands).
24. She left a blank space on pattern 3 and asked one of her learners to complete it (see image 19). The following exchange took place:
25. **Kelebogile:** as we have stated that our patterns are growing by adding one green circle and one green, yellow, so who can come to complete pattern 3?
26. **Learner 6:** me, me, me!!!
27. (she went to the chalkboard and draw pattern 3)
28. **Kelebogile:** is it correct?
29. **Learners:** yes
30. **Kelebogile:** Good!

Kelebogile's pedagogical actions in this segment illustrate a deliberate effort to ground the concept of geometric patterns in concrete, familiar representations, consistent with TRANSREP (Representational Choices) and TRANSCTXT (Contextualisation) from the PRA framework (Shulman, 1987). By first drawing learners' attention to coloured chalks (Turns 3–4) and later introducing soccer-ball drawings to model pattern growth (Turn 9), it can be said that she transformed abstract mathematical ideas into accessible visual anchors, an approach well supported by research showing that concrete representations enhance young learners' ability to generalise patterns (Mulligan & Mitchelmore, 2013; Papic, 2015). Her use of repeated questioning ("*what will we be drawing?*", "*how many soccer balls will I have*?", "*how many are we adding?*") reflects INSTQUEST (Questioning for Reasoning) and ASSESSAFL (Formative Assessment Moves), as she continually checked learner understanding and guided them to articulate the structural rule governing the pattern.

When learners correctly identified that the pattern grows by adding two soccer balls (Turn 12), Kelebogile provideed ASSESSFEED (Positive Feedback), reinforcing conceptual accuracy and strengthening COMPSTRUCT (Structural Awareness); the recognition of stable numeric and visual relationships within a geometric pattern (Du Plessis, 2018). Her subsequent shift to colour-based patterns (Turns 14–18), where she varied the attributes of the circles (green/yellow), demonstrates further TRANSREP as she highlights that patterns can grow not only by quantity but also through repeating colour arrangements. This aligns with literature indicating that variation in multiple attributes supports flexible pattern awareness (Goldin, 2020).

In asking learners to describe the pattern "*using colours*" (Turn 20) and inviting them to complete Pattern 3 on the chalkboard as depicted on image 20 (Turns 25–29), she promoted INSTPART (Learner Participation) and created opportunities for mathematical communication. Learner participation is a key component of effective mathematics instruction because it supports internalisation of concepts through active involvement (Sfard, 2008; Adler & Ronda, 2015). The peer-evaluation phase, checking whether the drawn pattern is correct (Turn 28) again reflects ASSESSAFL, positioning learners as co-evaluators of mathematical work, consistent with formative assessment principles emphasised by Green (2023). This exchange demonstrates that Kelebogile integrated representational modelling, guided questioning, learner participation, and structure-focused reasoning when teaching geometric patterns.

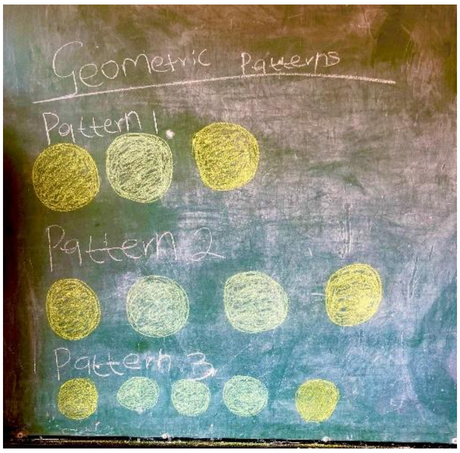


Image 20: completed Pattern 3 that was drawn by a learner

Kelebogile then invited the learners to observe the three generated patterns in image 20 and asked them to verbalise their observations as captured in the following exchange.

31. **Kelebogile:** now let us go through our patterns once again. So how many circles do we have in pattern 1?
32. **Learners:** 3 circles
33. **Kelebogile:** and how many yellow circles and how many green circles in pattern 1?
34. **Learners:** two yellow circles and one green circle
35. **Kelebogile:** Good. Now let us look at pattern 2? How many circles in pattern 2 and how many are green and how many are yellow?
36. **Learners:** there are four circles in pattern 2, which two are yellow and two are green
37. **Kelebogile:** Good. Now we look at pattern 3, how many circles are there and how many are yellow and how many are green?

38. **Learners:** in pattern 3, there are 5 circles, which two are yellow and three are green
39. **Kelebogile:** That is very good. Now let us imagine pattern 4, how many circles will we have in pattern 4 and how many will be yellow and how many will be green?
40. **Learner 7:** pattern 4 will have six circles
41. **Kelebogile:** that is very correct now how many circles will be yellow and how many will be green?
42. **Learners:** (remained silent)
43. **Kelebogile:** Mmmh! Anyone who can try? Even if your answer is not correct. It is okay, just try.
44. **Learner 8:** pattern 4 will have three yellow circles and three green circles
45. **Kelebogile:** is that correct?
46. **Learners:** (remained silent)
47. **Kelebogile:** well, it is not correct but anyone who is willing to try again?
48. **Learner 9:** pattern 4 will have two yellow circles on the first and last then four green circles in the middle
49. **Kelebogile:** Wow! That is very good. Please clap hands for her.
50. **Learners:** (claps hands)

In this segment, Kelebogile systematically guided learners through identifying how the pattern grows by repeatedly asking them to quantify circles and their colours across patterns 1 to 3, demonstrating INSTQUEST (reasoning-eliciting questioning) and continuous ASSESSAFL as she checks understanding at each stage. Her careful breakdown of total circles and colour distribution reflects strong COMPSTRUCT, helping learners attend to multiple dimensions of pattern growth. Her practice aligned with recent studies showing that decomposing pattern attributes enhances learners' structural reasoning (Papic & Mulligan, 2022; Warren, 2023). When she prompted learners to "*imagine pattern 4*" (Turn 39), she shifted them toward anticipatory thinking and generalisation, illustrating TRANSREP (moving from concrete board representations to mental representations) and encouraging early algebraic reasoning, which current research identifies as foundational for mathematical generalisation in the primary years (Blanton, 2024). During VSRI, Kelebogile reflected on her pedagogical intention for instructing the learners to imagine what pattern 4 would be, she said:

> *When I asked the learners to imagine Pattern 4, my intention was to help them move beyond what they could see on the board and start thinking about how the pattern grows from one stage to the next. I wanted them to realise that geometric patterns are not only about copying what is already drawn, but about understanding the rule that drives the pattern to continue. By imagining the next pattern, they are pushed to use the structure we discussed, like how many circles increase and how the colours change, and apply it independently. I also know that asking them to visualise the next step builds their confidence in reasoning without relying on me. That moment helps me see who understands the growth rule and who still needs support.*

The statement reflects Kelebogile’s intention to move learners from relying on visible examples toward independently applying the underlying growth rule of geometric patterns, demonstrating her focus on developing their reasoning and visualisation skills. When learners became silent (Turn 42), she responded with encouragement, “*even if your answer is not correct, just try*”, creating a supportive learning climate and modelling ASSESSFEED through supportive feedback rather than correction, which resonates with studies emphasising productive struggle and mistake-friendly participation (Hiebert & Lefevre, 2023; Russo & Hopkins, 2024). Her follow-up invitation for more attempts when the first answer is incorrect (Turns 46–48) illustrates purposeful COMPMIS (surfacing and correcting misconceptions) and maintained INSTPART, enabling deeper reasoning as the next learner successfully identified the correct composition of pattern 4. The praise, (“*please clap hands for her*”, Turn 50) reinforces correct structural insight through positive feedback (ASSESSFEED), which has been shown to strengthen learners’ confidence and engagement in mathematics (Söderström & Palm, 2024).Kelebogile’s teaching reflects a strong alignment with the PRA framework by blending structured questioning, formative assessment, representational reasoning, and error-tolerant participation to support Grade 4 learners’ understanding of geometric pattern structure. The lesson continued with the following exchange:

51. **Kelebogile:** as I have already given you examples, now we are going to write class activity on geometric patterns. I will draw pattern 1, 2 and 3 then you will be required to complete pattern 4 on your classwork books, but before I can proceed on giving you class activity, is there anyone who does not understand how geometric patterns are formed?
52. **Learners:** no
53. **Kelebogile:** so, do you all understand?
54. **Learners:** yes
55. **Kelebogile:** okay! Good!

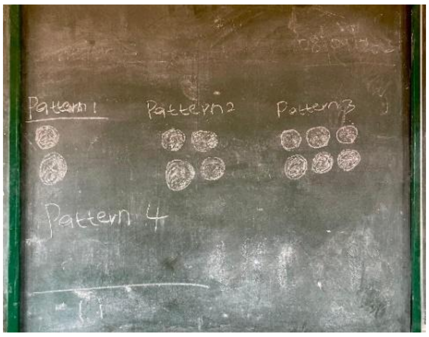


Image 21: class activity that was given to learners

56. **Kelebogile:** Is there anyone who is still writing the classwork activity?
57. **Learners:** no
58. **Kelebogile:** okay, now let us do corrections for our activity, so if I have two circles in pattern 1, then four circles in pattern 3. Who can tell us, how many circles did I add on pattern 2 and pattern 3?
59. **Learner 7:** you added two circles on each pattern
60. **Kelebogile:** correct and if I have added two circles on each pattern then how many circles will we have on pattern 4?
61. **Learner 8:** pattern 4 will have eight circles

Kelebogile demonstrates deliberate structuring of learning by transitioning from demonstration to independent classwork, reflecting INSTORG (Lesson Organisation) as she clearly outlines the expectations for Patterns 1–4 before learners begin the task (Shulman, 1987). Her question, "*is there anyone who does not understand how geometric patterns are formed?"* (Turn 51), served as an important ASSESSAFL (Formative Assessment Move), enabling her to gauge conceptual readiness before proceeding, which aligns with recent research emphasising formative checks as essential for strengthening mathematical understanding in primary learners (Ayalon & Wilkie, 2023; Castro-Superfine et al., 2024). Once learners confirmed understanding, she proceeded with confidence and later asked who was still writing (Turn 56), again engaging in diagnostic assessment to manage pacing and ensure no learner is left behind. During correction (Turns 58–61), her questions about how many circles were added across patterns represent INSTQUEST (Reasoning-Eliciting Questions) and reinforce COMPSTRUCT (Structural Awareness) by prompting learners to articulate the constant additive rule. In relation to this, previous studies highlight the importance of explicitly identifying pattern growth rates to build early algebraic reasoning (Blanton et al., 2022; Warren & Cooper, 2023). By affirming learners' correct reasoning ("correct"), she provides ASSESSFEED (Feedback), a pedagogical move shown to enhance motivation and accuracy in mathematics classrooms (Söderström & Palm, 2024). Of interest to note is that the lesson steps Kelebogile took in her lesson cohered with the information she provided during semi-structured interview:

> "*I show my learners the concept, let them do it and then ask them to explain their thinking to me Once this is done and I see that they understand, I give them homework to make sure that they continue learning when they get home. This makes it easy to continue teaching the following day, because the things they learned the previous lesson, they still remember*."

It was interesting to observe her description of her teaching approach was translated into action during classroom observation. The lesson culminated in Kelebogile assigning the homework to the learners as reflected in the following statement alongside image 22.

62. **Kelebogile:** that is very correct. Now let us open on page 133, which will be our home activity. There is pattern 1, pattern 2 and pattern 3 so what is it that you are going to do? You are going to draw those patterns then continue to draw pattern 4 and pattern 5 then we will mark it tomorrow.

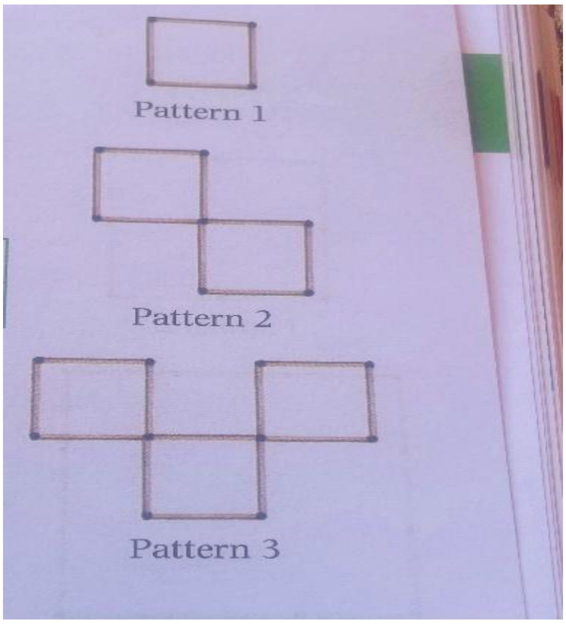


Image 22: home activity that was given to learners

The lesson concluded with Kelebogile assigning a homework activity that required learners to extend the geometric patterns from Patterns 1–3 to 4 and 5, reinforcing the structural reasoning developed during classroom teaching. This homework task served as a purposeful continuation of the mathematical processes emphasised throughout the lesson: identifying the pattern rule, recognising growth across stages, and applying the rule independently. Through directing learners to complete later stages of the pattern on their own, she provided an opportunity for consolidation and transfer of learning beyond the immediate classroom context, which is a key pedagogical consideration for strengthening retention and deepening understanding (Mbhiza, 2021). Specifically, the activity, as shown in Image 22 aligns closely with the conceptual focus of the lesson, ensuring continuity between classroom learning and independent practice, while reinforcing learners' ability to generalise pattern rules and communicate their understanding through drawings. Table 18 provides a summary of the coding of Kelebogile's pedagogical approaches throughout the episode.

Table 18. *PRA Coding for Kelebogile's Episode 1*

Turns	Evidence From Exchange	PRA Components Codes
1–4	Teacher activates prior knowledge, and prompts guesses about the lesson topic. Learners identified geometric patterns.	INSTORG, INSTQUEST, ASSESSAFL, INSTPART
5–9	Teacher uses coloured chalks and questioning to lead learners toward identifying shapes; introduces geometric patterns using soccer-ball patterns for Pattern 1 and Pattern 2; asks how many in Pattern 3.	TRANSREP (use of concrete objects & drawings), TRANSCTXT (connecting to familiar objects), INSTEXPL (explanation), INSTQUEST (reasoning questions), COMPKNOW (content accuracy), COMPSTRUCT
10–13	Learners correctly identify quantity in Pattern 3; teacher verifies and reinforces rule that pattern grows by adding 2 soccer balls.	INSTPART, ASSESSAFL, ASSESSFEED (positive feedback), COMPSTRUCT
14–18	Teacher shifts to colour-based patterns (yellow/green circles); draws Pattern 1 & 2; asks learners how the pattern is growing. Learners identify increases in circles and colour combinations.	TRANSREP (pictorial representation), TRANSCPA (moving from concrete → pictorial), INSTQUEST, COMPSTRUCT, INSTEXPL, INSTPART
19–23	Learners describe pattern using colours; teacher praises and invites class applause.	INSTPART, ASSESSFEED, ASSESSAFL, COMPVOC
24–30	Teacher leaves a blank space for Pattern 3 and invites a learner to complete it; learner completes; class confirms correctness; teacher affirms understanding.	INSTPART, TRANSREP, INSTQUEST, ASSESSAFL, INSTEXEMP, ASSESSFEED
31–38	Teacher systematically revisits Patterns 1–3, asking for total circles and colour distribution; learners respond accurately.	INSTQUEST, ASSESSAFL, INSTEXPL, COMPSTRUCT, TRANSREP
39–43	Teacher asks learners to *imagine* Pattern 4; learners initially silent; teacher encourages risk-taking and reasoning attempts.	TRANSREP, INSTQUEST, ASSESSAFL, ASSESSFEED, INSTPART
44–50	Learners provide two attempts at Pattern 4 colour-distribution; teacher evaluates, corrects misconception, and praises correct explanation.	COMPMIS, COMPSTRUCT, ASSESSFEED, INSTPART, INSTEXPL
51–55	Teacher introduces class activity.	INSTORG, INSTQUEST, ASSESSAFL, INSTEXPL
56–61	Teacher checks who is still writing; leads corrections; asks learners to determine growth rule and predict Pattern 4 total circles; learners respond with correct rule ("add two each pattern").	ASSESSAFL, INSTQUEST, COMPSTRUCT, INSTPART, ASSESSFEED
62	Teacher assigns homework to extend patterns to Patterns 4 and 5; signals continuity by stating work will be marked next day.	INSTORG, TRANSREP, ASSESSAFL

5.6 The Case of Tshepisho's Teaching

Like the rationale provided in Kelebogile's case, a single episode was chosen for the case of Tshepisho's teaching, because it offered sufficient analytic depth and richness, providing clear evidence of the teacher's pedagogical approaches as aligned with the aims of this study. In qualitative research particularly within a case study design, such purposeful selection of an information-rich episode enables thick description, close analysis, and meaningful interpretation of how a teacher mediates mathematical content in context (Mpitso, 2025). During semi-structured interview, Tshepisho remarked:

> *The challenges are mainly around resources, there is always a shortage of teaching aids, manipulatives, and sometimes even basic materials like charts. But I've learned to improvise. I use whatever is available: matchsticks, stones, bottle tops, old cardboard, even learners' bodies sometimes. Another challenge is large class sizes, which makes it difficult to give every child individual attention. To manage that, I group learners so that they can support each other, and I walk around a lot during lessons to check who is struggling. The rural context forces you to be creative all the time.*

The selected episode was especially comprehensive in illustrating Tshepisho's strategies for eliciting learner thinking, guiding pattern identification, and supporting the creation of geometric patterns, thereby offering a coherent and complete window into her pedagogical approaches.

Figure 7: Tshepisho's selected episode

5.6.1. Episode 1 (Lesson 2): Identifying and Creating Own Geometric Patterns

This episode captures Tshepisho's approach to introducing geometric patterns by anchoring the lesson in textbook examples and prompting learners to think about how patterns grow either by number or by size. This episode was selected because it offers a clear illustration of how Tshepisho transitions from concept introduction to guided reasoning and finally to learner-generated completion of patterns, aligning closely with the study's interest in understanding teachers' pedagogical approaches to geometric patterns at Grade 4 level. The lesson started with the following exchange:

1. **Tshepisho:** Good morning class. our topic today is Geometric Patterns, which is on page 129. Can we all open page 129
2. **Learners:** okay (learners opened page 129)
3. **Tshepisho:** this what we will be learning about today. We will be learning how patterns grow by numbers or by size. Is there anyone who can think of a pattern that grows bigger or a pattern that grows by adding shapes?
4. **Learner 1:** you mean when the pattern is added by more shapes?
5. **Tshepisho:** yes, now let us look at the examples (showed a pattern of triangles growing in size) so is there anyone who can describe the pattern?
6. **Learner 2:** the shapes of the triangles are growing bigger and bigger
7. **Tshepisho:** correct! The triangles are growing bigger by size. Now I am going to draw the triangles and one of you will come to complete the pattern on the chalkboard. (she drew the pattern) so who can come and complete it?
8. **Learners:** (remained silent)
9. **Tshepisho:** anyone please come and try
10. **Learner 3:** me!
11. **Tshepisho:** please come
12. **Learner 4:** (went to the chalkboard and completed the pattern)
13. **Tshepisho:** is it correct?
14. **Learners:** yes
15. **Tshepisho:** it is indeed correct

Tshepisho demonstrated well-structured pedagogical moves aligned with the study's espoused theoretical framework. She started the lesson with clear orientation and shared text navigation (Turns 1–2), reflecting INSTORG (Lesson Organisation) and INSTEXPL (Explanatory Talk) as she established the lesson focus. Her prompt asking learners to think of patterns that "*grow by numbers or size*" (Turn 3) illustrates INSTQUEST (Reasoning-Eliciting Questions), encouraging conceptual engagement and activating prior knowledge. This approach is supported by recent research highlighting that eliciting learner thinking early in a lesson supports deeper mathematical reasoning (Ayalon & Wilkie, 2023). When a learner attempted to articulate the idea of patterns growing by adding shapes (Turn 4), Tshepisho affirmed and extended their thinking, an example of ASSESSFEED (Feedback) and

ASSESSAFL (Formative Assessment Move), consistent with findings that responsive feedback promotes conceptual accuracy and learner confidence (Söderström & Palm, 2024). Relating to Turns 1 to 15, Tshepisho reflected during the VSRI conversation, she said:

> *In today's lesson, my goal was to introduce learners to the idea that patterns can grow either by number or by size. When I began the lesson by asking the question, my intention was to check what they know. I wanted to see how they understood growth in patterns before giving them any formal explanation. Learner 1's response showed me that at least some learners already associated pattern growth with the addition of elements. That helped me confirm that starting with a question placed the learners in the right thinking. When I displayed the example of triangles increasing in size and asked if anyone could describe the pattern, Learner 2 responded that the triangles were "growing bigger and bigger." This confirmed that visual examples helped them interpret growth not only in number but also in size, which was an important thing for the lesson. At this point, I felt the class was ready to engage with the pattern more actively.*

Showing a visual pattern of triangles that grow in size (Turn 5, image 23) and asking learners to describe it, Tshepisho effectively used TRANSREP (Representational Choices) to make structure visible, particularly by transitioning from verbal description to visual representation. When learners correctly described the pattern as "*growing bigger and bigger*," Tshepisho validated this and explicitly labeled the growth mechanism, reinforcing COMPSTRUCT (Structural Awareness). Wilkie and Clarke (2016) emphasise that the use of visual growth patterns strengthens learners' structural awareness and early algebraic reasoning. Her invitation for learners to complete the pattern on the chalkboard (Turns 7–12) illustrates INSTPART (Learner Participation), positioning learners as active constructors of knowledge, a pedagogical move aligned with contemporary views that active engagement supports mathematical sense-making (Russo & Hopkins, 2024). The silence when Tshepisho first asked for a volunteer (Turn 8) became a moment of ASSESSAFL, and her gentle prompt "*anyone please come and try*" (Turn 9) created a welcoming classroom environment for learners to participate actively in learning, which Hiebert and Lefevre (2023) illustrate as vital for encouraging risk-taking in mathematics learning. When the learner completed the pattern and the class confirmed correctness (Turns 13–15), Tshepisho's affirmation served as ASSESSFEED, reinforcing accurate application of the pattern rule.

Image 23: Geometric patterns with different sizes

Tshepisho asked her learners if they know different types of shapes and they answered yes. The following exchange took place:

16. **Tshepisho:** let us continue, so in geometric patterns, we use different shapes that forms a pattern, what I mean is that you have to draw and repeat the shapes. As I have given you examples, is there anyone who can come to the chalkboard to draw and create their own geometric patterns that grows by size
17. **Learners:** (remained quiet and seated)
18. **Tshepisho:** Anyone who can try, please?
19. **Learner 5:** I can come and try
20. **Tshepisho:** please come
21. **Learner 5:** (went to the chalkboard and created her own geometric patterns that grew by size, she drew three different triangles that had three different sizes)
22. **Tshepisho:** is it correct?
23. **Learners:** yes
24. **Tshepisho:** Do you all see that she has created the triangles that are the same in shape and also the same in sizes throughout her pattern?
25. **Learners:** yes

In this segment of the lesson, Tshepisho deepened conceptual engagement by explaining that geometric patterns require drawing and repeating shapes (Turn 16), a move reflecting INSTEXPL (Explanatory Talk) and strengthening learners' conceptual clarity about pattern formation. Her prompt inviting learners to create their own patterns on the chalkboard evoked INSTQUEST (Reasoning-Eliciting Questions) and encouraged INSTPART (Learner Participation), though the initial silence (Turn 17) served as ASSESSAFL (Diagnostic Evidence), signalling hesitancy or uncertainty. Her gentle encouragement *("Anyone who can try?"* Turn 18) illustrates ASSESSFEED (Supportive Feedback), aligning with Russo and Hopkins' (2024) emphases about the importance of reducing performance pressure to promote mathematical risk-taking. When Learner 5 volunteered and constructed a size-growing triangle pattern on the board (Turn 21), Tshepisho's pedagogical choice to use learner-generated representations exemplifies TRANSREP (Representational Choices), shifting authority to learners and supporting deeper ownership of mathematical reasoning. This pedagogical approach is supported by recent findings that learner-constructed representations enhance conceptual flexibility in pattern generalisation (Blanton et al., 2022).

In addition to the above, Tshepisho's follow-up question (*"Is it correct?"* Turn 22) invited peer evaluation, demonstrating ASSESSAFL and cultivating a collaborative verification environment shown to strengthen meta-reasoning (Söderström & Palm, 2024). When she drew attention to both the sameness in shape and the systematic growth in size (Turn 24), Tshepisho explicitly reinforced COMPSTRUCT (Structural Awareness), thereby helping learners recognise the invariant and changing features of a pattern, a core element of early algebraic thinking (Wilkie & Clarke, 2016). This combination of clear explanation, gentle prompting, learner participation, and structured reasoning reflects a pedagogical approach that is responsive, inclusive, and aligned with current best practices for supporting learners' understanding of geometric pattern growth (Du Plessis, 2018). The lesson continued and culminated with the following exchange:

26. **Tshepisho:** (asking the learner that drew the pattern) so can you tell us how you made your pattern?
27. **Learner 5:** the pattern that I drew has circle, triangle and square, circle, triangle and square
28. **Tshepisho:** that is good, so anyone who can tell us how will the pattern continue, anyone?
29. **Learner 6:** the pattern will continue by adding the circle, triangle and square
30. **Tshepisho:** that is very good!

Image 24: geometric pattern that was created and drawn by a learner

The following exchange took place:

31. **Tshepisho:** now I want you to pair in groups and each group should create their own geometric patterns that is not similar to any group

32. **Learners:** (started grouping themselves and started creating their own geometric patterns)
33. **Tshepisho:** any group that is done can choose one person to come and draw their group's geometric pattern on the chalkboard
34. **Learners:** (first group chose one person to go and draw their group's geometric pattern and all the groups did the same)

In this segment, Tshepisho continued to scaffold learners' engagement with geometric patterns by first asking the learner who constructed the pattern to explain her reasoning (Turn 26), a pedagogical approach that reflects INSTQUEST (Reasoning-Eliciting Questioning) and ASSESSAFL (Formative Assessment) because it allowed the teacher to make the learner's thinking visible. Previous studies highlight that prompting learners to verbalise their mathematical processes strengthens metacognition and conceptual clarity (Ayalon & Wilkie, 2023; Hsu & Silver, 2024). When the learner described the repeating sequence "*circle, triangle, square*" (Turn 27), Tshepisho affirmed this and extended it to the whole class by asking how the pattern continues (Turn 28). This illustrates COMPSTRUCT (Structural Awareness) and INSTEXPL, as she directed learners to generalise the underlying growth rule, foregrounding pattern structure instead of isolated cases.

Once another learner identified the continuation ("*adding the circle, triangle and square*" – Turn 29), Tshepisho's positive reinforcement (ASSESSFEED) further strengthened learner confidence, aligning with recent findings that affirming feedback supports mathematical engagement and reduces anxiety (Söderström & Palm, 2024). The shift in Turn 31 to group work; asking groups to create geometric patterns not similar to any other group demonstrates INSTPART (Learner Participation) and TRANSREP (Learner-Generated Representations). Group-based pattern construction encourages collaborative mathematical reasoning, a practice shown to support flexible thinking and concept co-construction among primary learners (Russo & Hopkins, 2024; Blanton et al., 2022). One way of interpreting this is that by requiring unique patterns, Tshepisho also promoted productive variation, a technique Papic and Mulligan (2022) identify as crucial for deepening structural reasoning and helping learners distinguish invariant from changing features in patterns. Finally, her instruction that each group select a representative to draw its pattern on the chalkboard (Turn 33) reflects INSTORG (Lesson Organisation) and reinforced public mathematical communication. Table 19 presents the summary of Tshepisho's pedagogical approaches, using the PRA recognition rules.

Table 19. *PRA Coding for Tshepisho's Episode 1*

Turns	Evidence From Exchange	PRA Components Codes
1–2	Directs class to the textbook (page 129), and ensures all are on the correct page.	INSTORG, INSTEXPL, INSTPART
3–4	Teacher explains that the lesson focuses on patterns growing by number or size and invites learners to think of such patterns; learner attempts interpreting the idea.	INSTQUEST, ASSESSAFL, INSTEXPL, INSTPART
5–7	Teacher uses a visual example (triangles growing in size), prompts learners to describe the pattern; learner identifies size growth; teacher affirms; teacher draws triangles and invites completion.	TRANSREP, COMPSTRUCT, INSTQUEST, ASSESSFEED, INSTEXPL
8–12	Silence when asked to complete the pattern; teacher encourages participation; learner volunteers and completes the pattern on the chalkboard.	ASSESSAFL, INSTPART, ASSESSFEED, TRANSREP
13–15	Teacher checks correctness of the completed pattern; class confirms; teacher validates.	ASSESSAFL, INSTPART, ASSESSFEED
16–18	Teacher reinforces that geometric patterns involve repeating shapes; invites a learner to draw their own pattern; initial silence; prompts again.	INSTEXPL, INSTQUEST, ASSESSAFL, INSTPART
19–25	Learner draws a size-growing triangle pattern; teacher asks if correct; class agrees; teacher highlights sameness of shape and systematic size growth.	TRANSREP, COMPSTRUCT, INSTPART, ASSESSFEED, INSTEXPL
26–30	Teacher asks the learner to explain how she made the pattern; learner describes repeating sequence (circle-triangle-square); teacher asks how the pattern continues; learner responds with correct repeating unit.	INSTQUEST, ASSESSAFL, COMPSTRUCT, INSTPART, ASSESSFEED
31–34	Teacher assigns group activity: each group must create a unique geometric pattern; learners organise into groups and begin constructing; groups select representatives to present on the chalkboard.	INSTORG, INSTPART, TRANSREP, INSTEXEMP, ASSESSAFL

Chapter 6: Cross-Case Analysis, Findings and Discussions

6.1. Introduction

This chapter presents a cross-case analysis of the five participating teachers, drawing together the detailed pedagogical evidence presented in Chapter 5 to answer the study's predetermined research questions. Building on the turn-by-turn analyses, PRA coding and interpretive insights discussed previously, this chapter synthesises how Grade 4 teachers in the Sekhukhune East District conceptualise, explain, and enact the teaching of geometric patterns. In addressing the main research question: *What pedagogical approaches do Grade 4 teachers in Sekhukhune East District employ during the teaching of geometric patterns?* the chapter integrates findings across cases to illuminate commonalities, variations, and contextual influences on practice. The findings presented in this chapter therefore moves beyond individual cases toward a holistic understanding of the pedagogical landscape, culminating in consolidated findings and overall conclusions that reflect the broader implications of the study.

6.2. Cross-case patterns in Teachers' Pedagogical Approaches

Across the five cases in this study, clear cross-case patterns emerged in how teachers supported learners' comprehension of geometric patterns by foregrounding structural features: units of repeat, invariants, and growth rules, as the organising ideas of the lesson. Rather than treating patterns as sequences to be copied, the teachers consistently pressed learners to articulate *why* a continuation was valid, using questioning, embodied demonstrations, concrete materials, and public inscriptions to make the governing structure visible and debatable. These pedagogical actions illustrate a shared orientation toward conceptual transparency: surfacing the rule, representing it in multiple forms, and warranting it through learner-generated justification. In doing so, the teachers' approaches closely reflect the COMPSTRUCT domain of PRA and align with Shulman's (1987) assertion that managing the central ideas of content, its structure, representations, and justifications is fundamental to pedagogical reasoning. The sub-section that follows elaborates how this structural emphasis manifested across cases and how it shaped learners' opportunities to notice, describe, and generalise geometric pattern relationships.

6.2.1. Comprehension & Structure of Content (COMPKNOW; COMPSTRUCT)

Across the five cases, teachers consistently framed geometric patterns around repetition, order, and growth rules, pressing learners to articulate "*what comes next*" *and why*, a through-line visible in their questioning, public recording of ideas, and peer-justified corrections (COMPSTRUCT; INSTQUEST/ASSESSAFL). For example, Phomelelo repeatedly consolidated the unit of repeat by staging non-examples (e.g., inserting "C" into "AB AB"), then orchestrating learner rebuttals that made the governing rule explicit, an approach that surfaced structure and normalised rule-based justification. Keatlegile and Kelebogile likewise pushed learners to name the invariant; "*add two matchsticks*," "*add two circles*," or maintain colour composition, thereby tying visible variation to an underlying stable relation; these moves anchored correctness in *structure*, not mere pattern copying (Du Plessis, 2018). Ditshegofatso extended this structural emphasis through embodied enactments (alternating orientations of learners) and then tabular representations that made constant differences salient and invited functional descriptions across stages, an explicit bridge from noticing to generalising. Tshepisho broadened the conception of growth from *number* to size, maintaining "*sameness of shape*" while highlighting systematic increases in magnitude, thus widening learners' sense of what counts as a growth rule beyond additive accumulation. In relation to the teachers' pedagogical actions, Rizos and Gkrekas (2024, p. 5) iterates that "when teaching students to recognize patterns, hands-on activities can make the mathematical terms and sequences easier to be grasped". Collectively, the participating teachers' pedagogical moves resonates closely with the COMPSTRUCT component of PRA and Shulman's (1987) call to manage the *ideas* of the lesson; what counts as the rule, how it is represented, and how it is warranted in mathematical discourse.

In addition to the above discussion, recent studies on early algebraic thinking suggest that such attention to structural invariants is foundational but insufficient unless learners are supported to express generalities beyond specific cases (e.g., moving from "+2 each time" to "for any pattern number n, …"). In the current study, Ditshegofatso's table-of-values work is exemplary: tables can re-present repeated addition as a function-like mapping and make constant differences visible, a representational shift repeatedly associated with progression from *arithmetic noticing* to *generalised/symbolic reasoning*. Likewise, the Concrete–Pictorial–Abstract (CPA) trajectories evident across cases, embodied/concrete modelling (learners, matchsticks), pictorial inscriptions (board drawings), and increasingly abstract descriptions ("*add 2 each time*") mirror contemporary guidance that CPA is most powerful when teachers explicitly connect stages and cycle flexibly among them, not treat them as linear rungs (Chang et al., 2017). The frequent use of manipulatives and embodiment (e.g., matchsticks, learner formations) accords with recent reviews showing potential gains when concrete activity is tightly coupled to bridging talk and representational linking; conversely, effects are weaker when materials are used without conceptual connections; an important caution for practice (Van Dijk & Rietveld, 2021).

Furthermore, the teachers' routines of elicitation → peer evaluation → consolidation mapped onto formative assessment principles known to yield positive (albeit variable) effects when enacted regularly and tied to content-specific intentions (e.g., "*state and justify the growth rule*"). Their handling of productive struggle, treating silence as thinking time, re-voicing prompts, and celebrating justified corrections aligns with current syntheses that advocate *supported* challenge: design tasks rich enough to demand explanation, then scaffold so that struggle remains *productive* rather than discouraging. The centring of structure (unit of repeat, invariant features), engineering representation shifts (including Ditshegofatso's strategic use of tables), and leveraging formative dialogue, the cases collectively instantiate the very pathways that recent research links to robust early algebraic generalisation (Du Plessis & Prinsloo, 2025), while also signalling a next step: making nth-term reasoning explicit to consolidate learners' movement from patterned cases to general statements. The following section provides a cross-case synthesis of the ways participants transformed Grade 4 geometric patterns contents during teaching.

6.2.2. Transformation of Content (TRANSREP; TRANSCTXT; TRANSCPA)

All teachers transformed abstract pattern structures into accessible representations by orchestrating Concrete→Pictorial→Abstract (CPA) progressions that were tightly coupled to talk about the underlying growth rule. For example, in *Phomelelo, Episode 1*, the lesson moved from a concrete/pictorial triangle–circle pattern drawn on the board to increasingly explicit verbalisation of the repeat unit and its lawful extension (*Turns 9, 20–25, 30–36*), a representational shift that anchored learners' justifications when a deliberate non-example ("AB AB AB ABC ABC") was introduced and then corrected to "AB AB AB AB." In *Keatlegile, Episode 1*, the class began with embodied demonstrations (learners as the pattern, alternating left/right and hands up/down) before moving to pictorial shape sequences on the board and abstract statements about order preservation (*Turns 5–10, 30–45*).

In *Ditshegofatso, Episode 1*, an embodied kinesthetic arrangement of five boys facing alternating directions was re-represented as drawings and then, crucially, in a table of values in *Episode 2* to make constant differences salient and invite general rules (*Episode 1 Turns 18–25; Episode 2 image/table segment and Turns 19–29*). Likewise, *Kelebogile, Episode 1* wove familiar artefacts (coloured chalks, soccer balls) into colour-counting patterns that transitioned to pictorial inscriptions and abstract descriptions of "*add one yellow and one green each time*" (*Turns 14–23, 31–38*). *Tshepisho, Episode 1* leveraged textbook visuals of triangles growing by size to set a pictorial baseline, then transferred authority to learners' board constructions and peer evaluation, culminating in abstracted statements distinguishing sameness of shape from systematic increase in size (*Turns 5–15, 19–25*). According to Shulman's (1987) conception of pedagogical reasoning and transformation, the teachers' pedagogical sequences exemplify how teachers convert disciplinary knowledge into *pedagogically powerful representations* by strategically moving from concrete experiences to pictorial depictions and finally to abstract generalisations.

6.2.3 Instructional Interaction (INSTQUEST; INSTEXPL; INSTPART; INSTEXEMP)

In the current study, the teachers used reasoning-eliciting questioning to activate prior knowledge, press for descriptions, and justify continuations, cycling through elicitation → public recording → peer evaluation → consolidation in ways that closely mirror Shulman's PRA loop of transformation → instruction → evaluation (Shulman, 1987). For example, in *Phomelelo, Episode 1*, the teacher's prompts ("*which shapes will I draw next?", "why are you all saying no?*") led learners to articulate and defend the unit of repeat, with the non-example "AB AB AB ABC ABC" publicly recorded, rejected by peers, and then consolidated as "AB AB AB AB" (*Turns 20–36, 30–36*). This sequence exemplifies how *worked example ↔ non-example* pairing and peer justification stabilise rule statements rather than mere extensions (Barbieri et al., 2023). Similarly, *Keatlegile, Episode 1* opened with embodied questioning ("Is *this not a pattern?*") as learners alternated left/right or hands up/down) before shifting to board-based sequences where correctness and order preservation were interrogated collectively (*Turns 5–10; 30–45*). The move from bodily arrangements to shared inscriptions sustained participation; learners first *saw* the regularity, then *said* and *checked* it together.

In *Ditshegofatso, Episode 1*, questioning was yoked to embodied formations ("*how many boys face left/right?*") and, in *Episode 2*, to tables of values that were constructed and debated in public space; learners not only supplied the next case but were prompted to name the constant difference ("+2") and justify it across rows (*Episode 1, Turns 18–25; Episode 2, Turns 19–29*). In addition, in *Kelebogile, Episode 1*, the teacher's prompts ("*how many yellow/green in Pattern 1–3?*"; "*imagine Pattern 4*") compelled learners to track attribute-based growth and then test candidate continuations through whole-class appraisal (*Turns 31–43; 44–50*). *Tshepisho, Episode 1* combined textbook visuals and board completions with questions that pressed learners to distinguish sameness of shape from systematic increase in size, with peers publicly validating or challenging proposed continuations (*Turns 5–15; 19–25*).

The embodied and contextualised transformations that preceded or accompanied questioning (e.g., learners physically forming alternating patterns; pointing out burglar-window symmetry; manipulating matchsticks) align with recent classroom-based research (da Ponte et al., 2023) showing that kinaesthetic enactments and familiar artefacts help learners perceive regularity and invariants; provided teacher to the bridge to inscriptions and rules, as occurred in these lessons through public board work and tables. In tandem, umbrella and scoping reviews on manipulatives and hands-on learning caution that such tools are maximally effective when embedded in guided discourse that surfaces structure, precisely the function served by the elicitation→recording→peer-evaluation→consolidation cycles across these episodes. The questioning-centred orchestration documented in *Phomelelo (Episode 1, Turns 20–36)*, *Keatlegile (Episode 1, Turns 5–10; 30–45)*, *Ditshegofatso*

(Episode 1, Turns 18–25; Episode 2, Turns 19–29), *Kelebogile (Episode 1, Turns 31–50)*, and *Tshepisho (Episode 1, Turns 5–15; 19–25)* reflects a reproducible PRA-aligned cycle in which teacher prompts elicit conjectures, public records preserve and focalise ideas, peers evaluate continuations and reasons, and the teacher consolidates the generalisation (Shulman, 1987). The next section focuses on the participating teachers' pedagogical approaches linked to assessment for learning and feedback.

6.2.4. Assessment for Learning & Feedback (ASSESSAFL; ASSESSFEED)

A distinctive feature across all five classrooms was the continuous, in-the-moment use of formative assessment, enacted through questioning, peer evaluation, and responsive feedback that shaped the flow of instruction rather than merely checking correctness. In *Phomelelo, Episode 1*, for instance, the teacher repeatedly posed verification questions *("Is it correct?")* followed by justification prompts *("Why?"),* using learners' responses to diagnose understanding of the repeat unit when the non-example "AB AB AB ABC ABC" appeared, and subsequently guiding the class to articulate why "AB AB AB AB" was the lawful continuation (*Turns 20–36*). Such content-specific questioning aligns with Moskos' et al. (2025) findings that formative assessment has stronger effects when embedded regularly and tied directly to the conceptual target, in this case, the growth rule rather than being used episodically or generically.

Similarly, in *Keatlegile, Episode 1*, formative checks were woven through the embodied activity; questions such as *"Is this not a pattern?"* prompted learners to evaluate the correctness of alternating bodily arrangements, and later, board-based completions elicited peer judgement and teacher probing (*Turns 5–10; 30–45*). In *Episode 2*, the teacher treated learner silence around predicting Pattern 6 as meaningful diagnostic evidence, re-prompting learners and normalising re-attempts (*Turns 34–38; 39–42*). Sortwell et al. (2024) emphasise that such adaptive responses, allowing time, prompting re-attempts, and validating partial reasoning—enhance equity and benefit lower-performing learners when feedback is specific and supportive. In *Ditshegofatso's Episode 1*, questioning embedded within an embodied formation *("How many boys face left/right?")* functioned as immediate checks for understanding (*Turns 18–25*). By *Episode 2*, the construction of a table of values became a public tool for formative assessment, enabling learners to identify, articulate, and justify the constant difference (+2) across patterns (*Turns 19–29*). These moves map onto evidence that public recording of thinking (tables, worked examples) and whole-class reasoning discussions are particularly effective in supporting early algebraic generalisation, especially when combined with regular prompts that draw attention to structural invariants.

In *Kelebogile, Episode 1*, the teacher systematically asked learners to state and justify the number of colours in patterns 1–3, and later, to evaluate proposed continuations of Pattern 4 (*Turns 31–43; 44–50*). During corrections (*Turns 58–61*), prompts like *"How many did I add?"* required learners to express the growth rule precisely. Meta-analytical work underscores that formative assessment is most effective when feedback has a high degree of task specificity, guiding learners toward the mathematical idea itself—here, the invariant increases in elements—rather than offering general praise or correction (Maskos et al., 2025). A similar pattern appeared in *Tshepisho, Episode 1*, where initial hesitation after being asked to complete a size-growing pattern was interpreted not as disengagement but as diagnostic information, followed by supportive re-prompting that invited a volunteer attempt and peer validation (*Turns 5–15; 19–25*). Sortwell et al. (2024) emphasise that formative assessment contributes most reliably to learning when teachers treat mistakes, hesitations, and partial answers as resources for instruction, guiding learners to refine and articulate their reasoning—precisely the stance evident in Tshepisho's facilitation.

The formative assessment moves enacted across *Phomelelo (Episode 1, Turns 20–36)*, *Keatlegile (Episode 1, Turns 5–10; 30–45; Episode 2, Turns 34–42)*, *Ditshegofatso (Episode 1, Turns 18–25; Episode 2, Turns 19–29)*, *Kelebogile (Episode 1, Turns 31–50; 58–61)*, and *Tshepisho (Episode 1, Turns 5–15; 19–25)* reflect a high-leverage cycle of elicitation, evaluation, justification, and consolidation. This cycle aligns with recent literature showing that frequent, content-specific, feedback-rich formative assessment, particularly when combined with public reasoning and opportunities for re-attempts, is among the most powerful drivers of conceptual learning in mathematics, especially in early algebraic domains such as pattern generalisation (Sortwell et al., 2024).

6.2.5. Reflection (REFLINA; REFLONA; REFL PLAN)

A salient thread across the cases is the pervasive presence of teacher reflection, visible *in the moment* and *after the fact*, and functioning similarly as Shulman theorised: a mediating bridge that feeds back into new comprehension and forward planning in the PRA cycle. In reflection-in-action, teachers adjusted while teaching, re-prompting after silence, code-switching for clarity, and varying representations. For example, *Keatlegile, Episode 2* treated learners' initial quietness around Pattern 6 as diagnostic silence, then re-prompted and legitimised re-attempts (*Turns 34–42*); *Tshepisho, Episode 1* similarly responded to hesitation by issuing a gentle second invitation before moving to peer evaluation (*Turns 5–15; 19–25*); and *Phomelelo, Episode 1* deliberately deployed a non-example ("AB AB AB ABC ABC"), then pivoted on learners' rejection to consolidate the lawful continuation, thereby *steering* discourse in real time (*Turns 30–36*). These moves exemplify Shulman's claim that skilled teachers manage ideas as they unfold, continuously transforming and tailoring content to learners' responses (Shulman, 1987).

Equally visible was reflection-on-action, where teachers articulated choices that structured subsequent lessons: starting where a prior lesson ended, using tables to clarify growth, and setting homework to check transfer. For instance, *Phomelelo, Episode 2* intentionally reopened with retrieval of the previous day's work and then assigned consolidation tasks; *Ditshegofatso, Episode 2* reported (in VSRI) that moving from concrete matchsticks to a table of values was a planned decision to make the constant difference visible and to support learners in verbalising the general rule (*Episode 2, Turns 19–29*; table segment); and *Kelebogile, Episode 1* closed with classwork corrections and homework designed to extend patterning beyond the lesson (*Turns 51–62*). Such after-lesson reasoning exemplifies Shulman's PRA feedback loop in which reflective review *re-feeds comprehension* and *re-shapes future action* (Shulman, 1987).

These reflective practices are not merely anecdotal; they resonate with recent elaborations of the teacher-knowledge literature that continue to position PRA as a productive lens for analysing live pedagogical decision-making. Contemporary accounts emphasise the centrality of *representation selection*, *adaptation to learner response*, and *iterative planning* as signatures of expert practice, exactly what is seen when teachers cycle from embodied enactments to pictorial inscriptions to tables/rules and then plan subsequent lessons to press generalisation. In Shulman's (1987) formulation, reflection is the indispensable bridge between instruction and new comprehension; the case material here shows that bridge operating repeatedly: *Ditshegofatso's* shift to tables after embodied work, *Keatlegile's* decision to reopen with prior content and then set forward-leaning tasks, and *Tshepisho's* movement from textbook visuals to learner-authored board work, each illustrates reflective *redirection* of the next pedagogical step in order to make the growth rule more explicit and public (Jo & Huh, 2022; Shulman, 1987).

Concretely, episode-specific traces of reflection can be read alongside the PRA categories. In *Phomelelo (Episode 1, Turns 20–36)*, the non-example was not a misstep, but a reflective probe used to assess and then refine learners' understanding of the repeat unit, evaluation feeding back into transformation. In *Keatlegile (Episode 1, Turns 5–10; 30–45; Episode 2, Turns 34–42)*, noticing hesitation triggered adaptive pacing and a return to representation and questioning that put structure back into view. In *Ditshegofatso (Episode 1, Turns 18–25; Episode 2, Turns 19–29)*, planned re-representation through a table of values functioned as a reflective design move to catalyse generalisation. In *Kelebogile (Episode 1, Turns 31–50; 51–62)*, post-task corrections and homework reflected on learners' partial explanations and targeted what to press next (the explicit rule). And in *Tshepisho (Episode 1, Turns 5–15; 19–25)*, a reflective stance toward silence and peer evaluation allowed the teacher to tune prompts and transfer authority to learners while keeping the invariant (sameness of shape) and the variant (size increase) in focus. Taken together, these cycles instantiate Shulman's (1987) PRA in action: reflection continuously re-configures representation, questioning, and tasking so that new comprehension becomes the platform for next-step planning.

6.3. Findings and Discussions

Drawing on the detailed individual case analyses presented in Chapter 5 and the cross-case synthesis that followed in section 6.2., this section presents the key findings that illuminate how Grade 4 teachers in the Sekhukhune East District mediate the teaching of geometric patterns. Across classrooms, a coherent constellation of pedagogical practices emerged, reflected in both the shared patterns of pedagogical reasoning and the nuanced variations shaped by context, learner needs, and teacher decisions. These findings are organised into four interrelated themes that capture the dominant pedagogical tendencies evident in the data. The first theme, *Strong Reliance on Concrete and Contextual Representations (TRANSREP, TRANSCTXT)*, reflects teachers' sustained use of physical artefacts, embodied demonstrations, familiar objects, and locally meaningful examples to ground abstract pattern structures in learners' immediate experiences. This reliance on concrete and contextual supports was evident across all episodes and reflects a deliberate effort to transform mathematical ideas into accessible forms.

The second theme, *Emphasis on Repetition and Structural Awareness in Pattern Growth (COMPSTRUCT)*, highlights how teachers foregrounded the unit of repeat, growth rules, and invariance across representations. Whether through verbal prompts, non-examples, or table-based reasoning, teachers consistently oriented learners toward identifying and articulating the structural features that define geometric patterns. The third theme, *High Levels of Learner Participation Through Demonstrations and Board Work (INSTPART)*, captures the interactive nature of pattern-teaching episodes. Here, two sub-themes emerged. The first sub-theme, *Movement from Teacher-Led Demonstrations to Learner-Constructed Patterns (TRANSCPA, INSTEXEMP)*, showing how teachers gradually shifted representational authority to learners through board work, embodied modelling, and pattern completion tasks. The second sub-theme, *Correction of Misconceptions Through Peer Engagement (COMPMIS)*, reflecting the prominent role of peer evaluation, collective correction of non-examples, and collaborative explanation in refining learners' structural understanding.

The fourth theme, *Use of Probing Questions to Drive Reasoning (INSTQUEST)*, reveals teachers' strong reliance on questioning to elicit explanations, activate prior knowledge, and push learners toward justification and generalisation. Embedded within this practice is the sub-theme *Formative Assessment Embedded Throughout Instruction (ASSESSAFL, ASSESSFEED)*, as teachers continuously evaluated learner thinking through cold calls, "why" questions, strategic pauses, peer checks, and feedback moves that shaped the direction of instruction moment by moment. Taken together, these themes illustrate a pedagogical landscape characterised by multimodal representation, structurally focused reasoning, interactive learning, and richly formative discourse.

Table 20. *Themes and Sub-themes for the Study*

Themes	Sub-themes
Strong Reliance on Concrete and Contextual Representations (TRANSREP, TRANSCTXT)	
Emphasis on Repetition and Structural Awareness in Pattern Growth (COMPSTRUCT)	
High Levels of Learner Participation Through Demonstrations and Board Work (INSTPART)	- *Movement from Teacher-Led Demonstrations to Learner-Constructed Patterns (TRANSCPA, INSTEXEMP)* - *Correction of Misconceptions Through Peer Engagement (COMPMIS)*
Use of Probing Questions to Drive Reasoning (INSTQUEST)	- *Formative Assessment Embedded Throughout Instruction (ASSESSAFL, ASSESSFEED)*

The sections that follow elaborate each theme in turn, drawing on episode-specific evidence to demonstrate how these pedagogical approaches manifested across the participating teachers' classrooms.

6.3.1. Strong Reliance on Concrete and Contextual Representations (TRANSREP, TRANSCTXT)

A dominant theme across all five teachers' episodes is their consistent reliance on concrete, contextual, and embodied representations to introduce and develop learners' understanding of geometric patterns. This approach aligns closely with the TRANSREP (Representational Choices) and TRANSCTXT (Contextualisation) components of Shulman's (1987) PRA framework, which emphasise the teacher's role in transforming abstract mathematical ideas into familiar and meaningful forms that learners can interpret and manipulate. Across episodes, teachers drew repeatedly on learners' lived realities, classroom artefacts, or physical bodies. For example, Phomelelo (Turns 5–9) used everyday classroom materials, cardboard shapes and familiar patterns such as alternating circles and triangles to concretise the repeating unit in geometric patterns. In another instance, Ditshegofatso (Turns 18–25) used learners' bodies to physically model an alternating directional pattern, asking them to face left and right in sequence. This move not only exhibited TRANSREP by converting an abstract idea into a kinaesthetic representation but also leveraged TRANSCTXT, drawing directly from the children's physical presence to build embodied understanding. Similarly, Keatlegile (Turns 5–10, 16–25) used coloured chalks and the physical arrangement of learners standing, bending, and raising hands to demonstrate repetition and variation, practices that made abstract pattern structure perceptually available and grounded in learners' immediate environment.

The reliance on household-level contexts is also illustrated in Kelebogile's case, where she framed geometric patterns using soccer balls (Turns 9–14) and later colour-based circle patterns to help learners notice visual and quantitative growth. Tshepisho (Turns 5–7) likewise showed growing triangles and then invited learners to complete patterns based on size enlargement, using shapes familiar from everyday objects, thereby smoothing the transition from concrete visuals to more conceptual reasoning. This cross-case pattern of using familiar materials mirrors the findings of recent mathematics education research. Recent studies (Sen & Guler, 2022; Mulligan, 2025) emphasise that young learners develop structural reasoning more effectively when exposed to concrete, varied representations that explicitly highlight repeating attributes. Similarly, Vassar (2017) show that concrete and embodied modelling reduces cognitive load and supports learners' transition into early algebraic thinking by helping them visualise invariant features across growing patterns. These findings resonate with the teachers' choice to use chalks, matchsticks, learners' bodies, and pictorial shapes as accessible starting points for exploring geometric structure.

Moreover, the teachers' use of contextual examples aligns with the PRA framework's expectation that teachers should actively transform content to fit learners' cognitive, linguistic, and cultural backgrounds. For instance, Phomelelo's bilingual explanations in Sepedi (Episode 1, Turns 5 and 15) not only provided linguistic access but situated pattern reasoning within the familiar language practices of rural learners, strengthening both comprehension and engagement. This aligns with recent multilingual education research (Essien, 2021) which highlights that home-language mediation facilitates access to mathematical meaning, especially for learners transitioning from the Foundation Phase. The usefulness of concrete representations was also evident in how frequently teachers transitioned from teacher-produced examples to learner-constructed ones. For instance, Tshepisho (Turns 21–25, 31–34) and Keatlegile (Turns 22–29) encouraged learners to create their own patterns on the chalkboard or using their peers. These moves reflect TRANSREP in the sense that learners shift from consuming representations to producing them, a transition that research now frames as essential to developing agency and internalising mathematical structures (Blanton, 2024; Russo & Hopkins, 2023). The teachers' collective reliance on concrete and contextual representations reveals a shared pedagogical understanding grounded in the belief that abstract pattern structure becomes intelligible when it is made visible, physical, and relatable. The following section focuses on the second theme that emerged from the analysis of the data.

6.3.2. Emphasis on Repetition and Structural Awareness in Pattern Growth (COMPSTRUCT)

The second cross-case theme emerging from all five teachers' lessons is their continuous emphasis on structural awareness, the recognition of how a pattern grows, repeats, or changes according to a consistent rule. This aligns with the COMPSTRUCT code in Shulman's (1987) PRA framework, which in the context of the current study refers to teachers helping learners notice the *underlying structure* of geometric patterns ideas rather than surface features. Across the episodes, teachers deliberately foregrounded pattern growth through counting, colour differentiation, size changes, and identification of repeating sequences, which is consistent with previous research emphasising that structural awareness is central to early algebraic reasoning (Sen & Guler, 2022).

Phomelelo, for instance, repeatedly drew learners' attention to the *order* and *repetition* of shapes. In Turns 9 and 13 of her lesson, she explained that if a pattern begins with a triangle followed by a circle, then every subsequent repeat must maintain this order: "*you don't add or remove anything; it should be the same sequence*." This aligns with COMPSTRUCT as she made explicit the idea of the *repeating unit*, prompting learners to justify why an incorrect pattern ("AB AB AB ABC ABC") violated the pattern rule.

This form of structural talk is highlighted in recent literature as crucial for helping learners shift from perceptual observations to rule-based generalisations (Blanton, 2024).

Ditshegofatso similarly emphasised growth structure when working with matchstick and learner-body patterns. In Episode 2 (Turns 19–29), he asked learners repeatedly, *"How many matchsticks are we adding?" "How many will Pattern 6 have?"* His insistence that learners articulate why a pattern grows by *two matchsticks* each time reflects COMPSTRUCT, focusing learners' thinking on the invariant growth rate rather than isolated quantities. When a learner explained that she continued adding two matchsticks *"until I got to pattern 6,"* he validated the explanation, reinforcing structurally grounded reasoning. Relating to this, Ayalon and Wilkie (2023) affirm that encouraging learners to articulate growth rules improves their ability to model and generalise patterns.

In Keatlegile's selected episode, structural awareness emerged prominently through colour-based patterns. In Turns 31–45, she guided learners to count how the number of yellow and green circles changed from Pattern 1 to Pattern 3. When she asked, *"How many circles will Pattern 4 have, and how many will be yellow or green?"* she pushed learners beyond counting toward analysing the *internal composition* of each pattern stage. When learners proposed incorrect distributions, she used COMPSTRUCT and COMPMIS to correct misconceptions, emphasising that *"pattern 4 will still have two yellow circles and four green circles in the middle."* This echoes recent studies showing that analysing multi-attribute patterns (number, colour, orientation) strengthens learners' capacity to detect varying and invariant aspects of structure (Thompson & Carlson, 2017). Kelebogile, likewise, foregrounded pattern growth by asking learners how many soccer balls were being added (Turns 11–13) and later how colour combinations changed across patterns. Her utterance, *"This is how geometric patterns work; we look at how a pattern is growing,"* demonstrates explicit structuring of COMPSTRUCT. She then shifted to patterns growing by one green and one yellow circle (Turns 19–23), helping learners understand that the structure of the pattern lies not only in the number of shapes but also in their attribute-based arrangement. Rivera (2018) highlight that drawing attention to both quantitative and qualitative structure deepens learners' generalisation skills.

Furthermore, in Tshepisho's lesson, structural awareness was reinforced through patterns growing by *size*, an element often overlooked in early pattern teaching. In Turns 5–7, she highlighted that the triangles *"grow bigger and bigger,"* encouraging learners to describe the pattern using structural language rather than shape recognition alone. Later (Turns 28–30), she asked learners how a pattern of repeated shapes (circle-triangle-square) would continue, prompting them to identify the repeating unit. Her focus on size- and sequence-based growth aligns with Mason (2018) emphasis that structural awareness is fostered when learners examine multiple pattern attributes across consecutive terms. Across all five teachers, then, the emphasis on repetition and structural awareness was not incidental, it formed a core pedagogical approach. Teachers guided learners through: counting elements in patterns (e.g., number of circles, matchsticks, triangles), examining colour distribution (yellow vs green), identifying repeating units (circle-triangle-square, triangle-circle), and predicting future terms *("How many in Pattern 4 or 6?").*

This prevalent emphasis on structure over appearance aligns with the PRA expectation that effective teaching requires making the essence of content visible (Shulman, 1987). The collective focus on structural awareness across the five teachers demonstrates strong pedagogical coherence: each teacher consistently supported learners in recognising what *drives* a geometric pattern (the rule) rather than merely describing what it *looks like*. It can be said that such pedagogical moves are foundational to learners' emerging algebraic understanding, providing the conceptual building blocks for generalisation, reasoning, and pattern modelling in the intermediate grades. The next theme focuses on the levels of learner participation that teachers enabled during the lessons.

6.3.3. High Levels of Learner Participation Through Demonstrations and Board Work (INSTPART)

The third prominent theme across all five teachers' teaching episodes is their strong and deliberate enablement of learner participation, operationalised through demonstrations, board work, group pattern construction, and peer validation. This theme aligns directly with INSTPART in the operationalised recognition rules of Shulman's (1987) PRA framework, which recognises learner engagement as central to pedagogical reasoning and classroom decision-making. Participation was not incidental, rather it was purposefully embedded as a core mechanism for enabling Grade 4 learners to make sense of geometric patterns through *action*, *observation*, and *collective reasoning*. Across cases, teachers invited learners to the board to complete or generate patterns. This aligns with recent findings by Russo and Hopkins (2024), who emphasise that open classroom participation enhances learners' mathematical agency and supports metacognitive clarity. Similarly, Adler and Ronda (2015) assert that to enable effective mathematics learning, teachers should prioritise fostering active learner participation during the lessons.

Taken together, the teachers' pedagogical approaches across the selected episodes reveal a deeply embedded belief that mathematical understanding develops through participation. The participants used participation not only for engagement but as a pedagogical tool: to expose learners' thinking (diagnostic), to strengthen conceptual clarity (explanatory), to verify correctness (evaluative), and to build confidence and agency (affective). This theme powerfully illustrates the PRA model's principle that effective teaching involves active learner involvement as a core pedagogical resource. Through positioning learners as demonstrators, evaluators, and creators of patterns, all five teachers cultivated classrooms where geometric pattern reasoning was co-constructed, negotiated, and made visible through participation, mirroring contemporary mathematics-education research that emphasises learning-through-doing as fundamental to developing early algebraic reasoning (Adler & Ronda, 2015; Mbhiza, 2021; Sfard, 2008). The following sub-theme demonstrates the ways the teachers enabled learners' active participation in learning.

6.3.3.1 Movement from Teacher-Led Demonstrations to Learner-Constructed Patterns (TRANSCPA, INSTEXEMP)

Across all five teachers' lessons, a clear pedagogical progression emerged whereby teachers began by modelling geometric patterns themselves and then gradually shifted responsibility to learners, who were asked to complete, extend, or generate their own patterns. This progression reflects the Concrete–Pictorial–Abstract (CPA) movement captured in the TRANSCPA code of the PRA framework and the deliberate use of exemplification (INSTEXEMP) to scaffold learning. The CPA progression is pedagogically significant because it allows learners to first *observe* correct mathematical structure, then *manipulate* representations, and finally *infer or construct* patterns independently (Blanton, 2024). This pedagogical shift was evident in Phomelelo's teaching in episode 1, where she first demonstrated a repeating unit (triangle → circle) on the board (Turns 7–9) and later invited learners to extend the pattern (Turns 20–25). When a learner incorrectly proposed adding different shapes, she used the example as a *non-example* (INSTEXEMP), prompting peers to articulate what makes a pattern valid (Turn 33). This movement toward learner-constructed justification aligns with TRANSCPA, because learners transitioned from viewing teacher examples to judging and creating patterns themselves.

Ditshegofatso similarly began with teacher-generated examples using matchsticks, showing three, six, and nine matchsticks (Episode 2, Turns 1–6) before calling learners to the board to draw Pattern 3 (Turns 26–27). When the first learner mis-counted, he used the incorrect attempt as a non-example (INSTEXEMP), encouraging the class to "*check our pattern correctly*" (Turn 30). Eventually, a learner produced nine triangles, demonstrating a movement into the pictorial and abstract stages of the CPA trajectory by reasoning from the growth rule rather than copying. Recent studies (Salingay & Tan, 2024) confirm that such transitions reinforce mathematical autonomy and help learners internalise structural relationships rather than merely reproduce surface features.

Keatlegile demonstrated this sub-theme even more explicitly by using learners' bodies to model patterns (Turns 16–25), then shifting to student-generated pattern constructions. After arranging learners to show alternating directions as a demonstration, she invited a learner to create her own pattern using peers (Turn 22). The class evaluated whether the pattern was correct (Turn 26), illustrating INSTEXEMP and peer-supported structural reasoning. Later, when she transitioned to drawing patterns on the board (Turns 36–45), she again demonstrated first and then asked learners to complete or describe the next shapes, reflecting the CPA movement from embodied to pictorial to conceptual. This aligns with findings by Russo and Hopkins (2024), who argue that learner-constructed examples strengthen generalisation because they require learners to apply rules independently.

Kelebogile also followed this structure in Episode 1. She introduced geometric patterns using soccer balls (Teacher demonstration: Turns 9–12), then added colour-based patterns and asked learners how they grew (Turns 14–18) and eventually left an incomplete pattern for a learner to finish (Turn 24). The class evaluated correctness (Turn 29), illustrating a transition into INSTEXEMP where both correct and incorrect examples provide opportunities for learners to articulate the growth rule. This reflective co-construction is supported by Hsu and Silver (2024), showing that pattern-completion tasks stimulate learners' structural generalisation when accompanied by teacher scaffolding.

In Tshepisho's episode, the transition is particularly clear. After demonstrating triangles growing in size (Turns 5–7), she invited learners to complete the next triangle (Turn 7). When learners hesitated, she encouraged them ("*Anyone please come and try*", Turn 9), lowering the affective barrier to participation. Once the learner completed the pattern (Turn 12), she checked correctness with peers (Turns 13–15) before moving into learner-constructed patterns (Turns 16–25), where learners created their own repeating sequences. Ayalon and Wilkie (2023) affirm that such shifts from demonstration to independent construction build learners' confidence and support their transition from imitating to generating mathematical structure, which is fundamental for algebraic readiness.

Across cases, therefore, the movement from teacher demonstrations to learner-constructed patterns served crucial purposes:

- It scaffolded learners' pattern-generalisation abilities by gradually reducing dependence on the teacher.
- It allowed misconceptions to surface naturally, providing opportunities for corrective feedback through nonexamples (INSTEXEMP).
- It offered learners agency, shifting them from passive observers to active creators of patterns.

It is important to note that this pedagogical strategy is particularly powerful in Grade 4, where learners are transitioning from arithmetic to early algebraic thinking (Mateboge, 2024).

6.3.3.2 Correction of Misconceptions Through Peer Engagement (COMPMIS)

The second sub-theme representing *INSTPART* is the strategic use of peer engagement to surface and correct misconceptions, aligning with the COMPMIS element of the PRA framework. Rather than correcting errors unilaterally, teachers consistently invited learners to evaluate and rectify one another's thinking. This pedagogical approach positioned misconceptions not as failures but as opportunities for collective reasoning, a practice increasingly recommended in contemporary mathematics education research (Ayalon & Wilkie, 2023). Across the observed lessons, misconceptions often emerged when learners misidentified growth rules, reversed sequences, or introduced non-pattern elements. For example, in Phomelelo's lesson, when she deliberately wrote "AB AB AB ABC ABC" on the chalkboard (Episode 1, Turn 30), learners immediately rejected the pattern, "*No, because you added C*" (Turn 33). Here, peer explanation, rather than teacher correction, was central to refining structural understanding. This aligns with the PRA notion that correction of misconceptions becomes more meaningful when supported by peer-mediated reasoning and dialogic clarification (Shulman, 1987).

Similarly, in Ditshegofatso's Episode 2, when a learner incorrectly drew 10 triangles for Pattern 3 (Turn 27), he did not correct the misconception directly. Instead, he prompted the class: "*Is he correct?"* (Turn 28). Learners responded "*No*" and provided justification after reevaluating the structure. This moment reflects COMPMIS supported by INSTQUEST and ASSESSAFL, where the teacher strategically facilitated peer-led correction. Du Plessis (2018) show that when learners articulate *why* a pattern is incorrect, they strengthen their structural awareness and minimise procedural guessing.

In addirion, in Keatlegile's colour-pattern episode, she asked, "*How many yellow and how many green circles in Pattern 4?*" (Turns 40–43). When one learner proposed an incorrect distribution (Turn 44), the class remained silent, showing uncertainty until another learner corrected the misconception: "*Pattern 4 will have two yellow circles and four green circles*" (Turn 48). The teacher used this as a teaching moment, praising the explanation, thus reinforcing conceptual precision through peer-generated clarity. Kelebogile also relied on peer engagement for misconception correction. After a learner attempted Pattern 3, she asked the class, "Is it correct?" (Turn 29). Learners' affirmative or corrective responses allowed her to use peer validation to consolidate structural reasoning. In Tshepisho's classroom, misconceptions concerning repeating units and pattern continuation were similarly corrected through peers. When learners were asked how the pattern would continue (Turn 28), the class collaboratively reasoned about the repeating sequence "*circle, triangle, square*." Later, when the teacher requested unique group-generated patterns (Turns 31–34), peer feedback naturally emerged as groups compared and justified the correctness of each other's patterns; an extended form of peer-mediated misconception correction.

The episodes presented in the current study demonstrate a shared pedagogical strategy: teachers used learners' incorrect and/or incomplete responses as low-stakes opportunities for peers to reason aloud, justify their thinking, and refine structural understanding. This aligns with the PRA framework, which emphasises that addressing misconceptions is not merely error-correction but a meaning-making process grounded in learners' own explanations (Shulman, 1987). Papic and Mulligan (2022) contend that collective reasoning around misconceptions significantly enhances early algebraic thinking and pattern generalisation. The use of peer engagement as a tool for misconception correction was a consistent and effective pedagogical move across all teachers. It positioned learners as mathematical thinkers capable of evaluating, correcting, and justifying pattern structure, deepening both conceptual understanding and classroom discourse. The next section focuses on the final theme that emerged from the analysis of the data.

6.3.4. Use of Probing Questions to Drive Reasoning (INSTQUEST)

Another unifying pedagogical feature I identified across all five teachers' episodes was the deliberate and consistent use of probing questions to stimulate learners' mathematical reasoning relating to the geometric patterns' topic at Grade 4 level. These questions, such as *"How is the pattern growing?", "How many will be next?", "Is this correct?", "What do you see?", "Why are you saying that?"*; functioned as central tools for eliciting learner thinking, surfacing misconceptions, and guiding learners toward structural generalisation. This pattern aligns directly with INSTQUEST in the espoused theoretical framework, which emphasises the teacher's role in asking questions that deepen conceptual engagement rather than solicit rote or superficial responses (Shulman, 1987). Across episodes, teachers used questioning to shift learners from noticing surface features to identifying pattern structure, prompting learners not simply to repeat shapes but to articulate the underlying rules for the patterns they worked with. The questions they asked were essential in moving learners from concrete descriptions to structural explanations, an approach supported by Blanton (2024), showing that reasoning-eliciting questions foster early algebraic thinking by directing learners' attention to regularity and relationships. I elaborate on this finding in the discussion of the sub-theme in what follows.

6.3.4.1. Formative Assessment Embedded Throughout Instruction (ASSESSAFL, ASSESSFEED)

Another sub-theme that emerged across the five teachers' pedagogical approaches is the continuous embedding of formative assessment practices throughout the teaching of geometric patterns. This resonates strongly with the ASSESSAFL and ASSESSFEED components of the PRA framework, which emphasise real-time monitoring of learners' understanding, using learner responses to adapt teaching, and providing immediate feedback that supports conceptual development. Across cases, formative assessment was not an isolated event; it was ingrained in questioning, demonstration, peer interaction, and pattern-completion tasks. The teachers used diagnostic questioning to check understanding at strategic points. For example, Phomelelo asked learners, *"So what do you do when you complete a pattern?"* (Episode 1, Turn 7) and later checked whether her intentionally incorrect pattern ("AB AB AB ABC ABC") violated the structure (Turns 31–33). Her open prompts enabled learners to reveal their thinking, reasoning and communication about mathematical ideas, and her follow-up, "*it is because I added C*" served as ASSESSFEED, reinforcing correct structural awareness. Such real-time diagnostic questioning is supported by findings from Ayalon and Wilkie (2023), who demonstrate that formative assessment deepens learners' structural generalisation in pattern tasks.

Ditshegofatso similarly embedded formative checks in his matchstick-counting lesson. After a learner incorrectly drew 10 triangles in Pattern 3 (Episode 2, Turn 27), he asked the class, *"Is he correct?"* (Turn 28). This invited peer validation (a key element of ASSESSAFL) and helped surface misunderstandings about the pattern's additive growth. When another learner corrected the error by drawing 9 triangles (Turn 31), his response, "*is she correct?",* continued the cycle of assessment through peer reasoning especially without leading the learners to the correct answer. In relation to this, Castro-Superfine et al. (2024) emphasises that peer-based formative assessment strengthens learners' confidence in articulating mathematical rules and enhances conceptual coherence.

In Keatlegile's colour-pattern lesson, formative assessment appeared through repeated checks such as: *"How many circles in pattern 1?"*, *"How many are yellow and green?"* (Turns 31–38) and later *"Is this not a pattern?"* (Turn 20). These checks confirmed not only counting accuracy but also understanding of attribute-based structure, allowing her to assess whether learners recognised consistent colour distribution. When learners proposed incorrect numbers for Pattern 4 (Turn 44), her response, "*It is not correct but anyone willing to try?"* illustrated ASSESSFEED that was both corrective and encouraging. Such error-tolerant formative assessment practices are increasingly recognised as essential to sustaining learner participation and productive struggle in mathematics (Russo & Hopkins, 2024).

Kelebogile's formative assessment moves were particularly explicit. After drawing Patterns 1–2 and leaving Pattern 3 blank, she asked the class, *"Is it correct?"* (Turn 29). Their confirmation acted as peer validation, while her follow-up questions about how many soccer balls were added (Turns 11–12) assessed learners' understanding of the additive growth rule. Furthermore, her statement *"This is how geometric patterns work"*, served as ASSESSFEED, clarifying conceptual expectations. Moreover, in Tshepisho's episode, formative assessment was used to encourage reasoning and reduce fear of failure. When learners were initially silent in response to her prompt to complete the pattern (Turn 8), she asked, *"Anyone please come and try?"* (Turn 9), signalling a supportive assessment culture. Later, after a learner generated a pattern, she asked, *"Is it correct?"* (Turn 22), again drawing on peer evaluation, decentring herself from the provision of answers, but allowing learners to engage in exploration of the behaviour of the presented patterns (see Sfard, 2008). Her comments, *"Do you all see that she created triangles that are the same in shape and same in size?"* (Turn 24) again served as ASSESSFEED, guiding learners to focus on the structural criteria of correctness. It was interesting to observe lessons in which formative assessment was utilised in the manner that is non-threatening, participatory, and focused on reasoning, particularly in developing pattern generalisation skills.

6.4. Chapter Summary

This chapter presented a comprehensive cross-case analysis of the five participating teachers, synthesising the detailed turn-by-turn pedagogical evidence, drawing from the PRA framework, and interpretive insights outlined in Chapter 5 to address the study's central research question on the pedagogical approaches employed by Grade 4 teachers in the Sekhukhune East District when teaching geometric patterns. In drawing together the individual cases, the chapter illuminated the shared strategies, distinctive practices, and contextual influences shaping how teachers conceptualised, explained, and enacted the teaching of geometric patterns, thus moving the discussion beyond isolated episodes toward a holistic understanding of the broader pedagogical landscape. The consolidated findings illustrate key thematic patterns across cases and provide the foundation for the study's overall conclusions. The next chapter brings the study to its close, presenting these conclusions in full and offering recommendations for practice, policy, and future research.

Chapter 7: Conclusions and Recommendations

7.1. Introduction

This final chapter synthesises the study's core insights and translates them into actionable implications by first answering the predetermined research questions that framed the current study. It revisits the main research question. *What pedagogical approaches do Grade 4 teachers in the Sekhukhune East District employ during the teaching of geometric patterns?* and the sub-questions concerning (a) how teachers conceptualise and develop the topic in practice, (b) the nature and function of their explanatory talk during teaching, and (c) the contextual factors that shape pedagogical choices in rural classrooms. Drawing on the turn-by-turn analyses presented in chapter 5, PRA-based codings, and cross-case synthesis in chapter 6, the first section distils how teachers transform content (e.g., concrete–pictorial–abstract moves), orchestrate questioning and formative assessment, and navigate contextual constraints to support Grade 4 learners' understanding of geometric patterns. Building on these answers, the subsequent section offers Recommendations in three strands: Recommendations for Practice (classroom-level guidance to strengthen representation, questioning, and assessment for learning), Recommendations for Future Research (directions for extending the evidence base on pattern pedagogy, rural mathematics education contexts, and teacher learning), and Recommendations for Policy. The chapter then acknowledges the study's Limitations to clarify scope and transferability, before closing with a concise Study Conclusion that synthesises the study's contribution to knowledge and practice in the teaching of Grade 4 geometric patterns within the South African context.

7.2. Answering Predetermined Research Questions

Synthesising across the five cases in this study, the participating Grade 4 teachers in the Sekhukhune East District typically enacted a coherent repertoire of pedagogical approaches that make geometric patterns visible, participatory, and structurally explicit**.** With respect to the main research question: *What pedagogical approaches do Grade 4 teachers employ when teaching geometric patterns?* the cross-case evidence shows consistent use of (i) representational transformation to ground ideas in manipulatives, pictorials, and familiar artefacts (TRANSREP, TRANSCTXT), (ii) explanatory talk to define, model, and re-voice key ideas in accessible language (INSTEXPL, COMPVOC), (iii) reasoning-eliciting questioning to orient learners toward pattern rules rather than surface features (INSTQUEST), and (iv) embedded formative assessment to probe understanding, surface misconceptions, and provide immediate guidance (ASSESSAFL, ASSESSFEED). These moves are routinely orchestrated within a concrete–pictorial–abstract progression (TRANSCPA), with teachers first demonstrating or co-constructing an example (often including a purposeful non-example) and then shifting responsibility to learners to complete, extend, or generate patterns (INSTEXEMP, INSTPART). Collectively, the pedagogical spine is one of guided inquiry toward structural generalisation, with teachers deliberately positioning learners to articulate "what changes, what stays the same, and why", the core of geometric pattern thinking.

In addition to the above, addressing Sub-Research Question (a) *How do Grade 4 rural teachers in Sekhukhune East District conceptualise and develop the topic of geometric patterns in their teaching practices?* teachers framed patterns as systems governed by rules that can manifest through quantity (additive growth), attribute (colour/shape), or magnitude (size). Lesson development commonly opened with contextual anchors (matchsticks, classroom fixtures, coloured chalks, learners' bodies) to reduce abstraction and establish a shared reference point (TRANSCTXT), then pivoted to worked examples that foregrounded the unit of repeat and order (INSTEXEMP, COMPSTRUCT). Progression was intentionally incremental: teachers moved from identifying and counting elements in given patterns, to describing the repeating unit and its order, and finally to predicting unseen terms. Throughout, teachers emphasised rule-talk ("What are we adding?"; "How is it growing?"), which consolidates conceptual coherence and supports transfer as tasks shift from shapes to letters, colours, or size sequences.

For Sub-Research Question (b**)** *What are Grade 4 rural teachers' explanatory talk while teaching geometric patterns?* explanations were clear, brief, and structurally oriented, frequently supported by bilingual re-voicing to mediate key vocabulary (INSTEXPL, COMPVOC). Teachers habitually define a pattern in terms of repetition and order and then immediately instantiate the definition through a board-based or embodied example, tightening the link between language and representation. Explanatory talk also served diagnostic purposes: teachers paraphrased learners' contributions, contrasted correct and incorrect continuations, and highlighted why an insertion or re-ordering breaks the rule (COMPMIS, COMPSTRUCT). This style of teacher classroom talks positioned definitions, examples, and justifications as a single explanatory chain: *name it, show it, explain why it works (or doesn't)*, which appears to be effective for maintaining class-wide focus on structure rather than appearance.

Responding to Sub-Research Question (c) *What factors influence the pedagogical approaches of rural teachers when teaching geometric patterns in Grade 4, and how do these factors shape their pedagogical choices?* three influences recur. First, resource realities in rural schools encourage low-cost, high-impact representations (learners' bodies, everyday objects, textbook figures), reinforcing a hands-on orientation (TRANSREP, TRANSCTXT). Second, linguistic diversity motivates strategic bilingual mediation, where teachers toggle between English and the Sepedi home language to secure conceptual access while gradually building mathematical disciplinary vocabulary (INSTEXPL, COMPVOC). Third, class size and time constraints make formative routines indispensable: quick whole-class checks, peer validation of pattern continuations, and concise feedback loops enable teachers to monitor understanding and adjust pacing at scale (ASSESSAFL, ASSESSFEED, INSTORG). It is important to note that these contextual factors do not merely constrain practice; they shape a pragmatic pedagogy that privileges visible structure, collective participation, and economy of explanation.

Moreover, the Pedagogical Reasoning and Actions lens clarifies that effective lessons hinge on transforming content into accessible forms, eliciting and steering reasoning toward general rules, and using assessment in the flow of teaching to consolidate understanding. The result is a recognisable pathway from teacher-led modelling to learner-constructed generalisations, underwritten by purposeful examples/non-examples, learners explain their thought process, and continual emphasis on *what repeats, in what order, and by how much*. Thus, the findings reveal that, despite varied school conditions, teachers converge on a structurally focused, participatory, and assessment-rich approach to teaching Grade 4 geometric patterns.

7.3. Recommendations

The findings of this study highlight several pedagogical strengths across the five participating teachers, as well as areas in which support, refinement, and further development are necessary for enhancing the teaching and learning of geometric patterns in Grade 4 classrooms. These recommendations are therefore organised to speak to three interconnected domains: practice**,** future research, and policy. Each set of recommendations draws directly from the cross-case analysis and the PRA-informed interpretations presented earlier, offering targeted and contextually grounded guidance. While these recommendations stem from a rural South African context, many hold broader relevance for mathematics education in similar schooling environments characterised by resource constraints, and diverse learner needs.

7.3.1. Recommendations for Practice

At the classroom level, the study recommends that teachers continue to strengthen the use of concrete and contextual representations, but with more intentional progression toward abstract structural reasoning. While the teachers demonstrated strong competence in using manipulatives, colours, textbook diagrams, and embodied demonstrations, greater emphasis on explicitly linking these representations to generalisable rules may deepen learners' algebraic readiness. Teachers should also further develop strategic questioning routines that scaffold learners into producing verbal and written justifications for pattern extensions, as probing questions were shown to be central to conceptual understanding. Additionally, teachers would benefit from systematically incorporating peer explanation protocols, since learners across cases showed improvement in engagement and mathematical talk when allowed to evaluate one another's pattern completions and reasoning. Finally, the study recommends that teachers strengthen practices that support mathematical sense-making, including the deliberate use of home language to build conceptual access while gradually anchoring vocabulary in English to support assessment demands.

7.3.2. Recommendations for Future Research

Future research should build on this study by exploring longitudinal developments in learners' pattern generalisation skills as they transition from Grade 4 into upper primary, where algebraic reasoning becomes more explicit. Given that this study focused on classroom-level pedagogical approaches, further research should examine how teachers' underlying mathematical knowledge, professional development experiences, and beliefs about mathematics influence their implementation of geometric pattern teaching. Another important area for investigation is the role of language in geometric pattern learning: researchers could analyse how different forms of code-switching, translanguaging, or bilingual scaffolding impact learners' ability to articulate pattern rules. Additionally, because this study was limited to one district, future research could undertake comparative case studies across regions or provinces to examine whether the pedagogical patterns identified here hold across broader contexts. Finally, research should further examine how the CPA approach and learner-generated representations influence learners' long-term conceptual understanding in pattern-rich topics beyond Grade 4.

In addition, future research should build on the analytic power of the recognition rules by examining how these classroom moves, such as COMPSTRUCT-based stabilisation of structure, TRANSREP/TRANSCTXT/TRANSCPA decisions, INSTQUEST-driven discourse orchestration, and ASSESSAFL/ASSESSFEED-based responsiveness function across different mathematical topics, grade levels, and school contexts. Because the recognition rules operationalise Shulman's PRA model into observable, codable pedagogical actions, future studies could investigate whether specific combinations of these moves (e.g., the sequencing of representational transformation followed by peer-mediated misconception correction) are reliably associated with deeper learner generalisation and early algebraic reasoning. Comparative or longitudinal designs would allow researchers to trace how teachers' PRA enactment evolves over time and whether professional development targeted at structure recognition and representational fluency shifts teachers' real-time decision-making. Additionally, future research could explore how these recognition rules might be adapted as tools for teacher self-reflection, mentoring, or coaching, examining their usefulness not only for analysis but also for professional growth.

7.3.3. Recommendations for Policy

At the policy level, the findings highlight the need for targeted support for mathematics teaching in rural contexts, particularly where resource constraints shape pedagogical choices. Policy should ensure that teachers receive structured professional development aligned with frameworks such as the PRA model, emphasising representational fluency, formative assessment, and pedagogical reasoning. Additionally, the Department of Basic Education could expand its recent efforts to provide bilingual learning and assessment materials, as teachers frequently relied on their home language to ensure learner access to mathematical concepts. Policymakers should also consider resourcing schools with low-cost manipulatives and visual support materials to strengthen the effectiveness of concrete–pictorial–abstract teaching approaches. Finally, the study recommends that curriculum policy more explicitly articulate the developmental progression of geometric and algebraic patterning, ensuring that teachers have clear guidance on expectations at Grade 4 and how these link to later curriculum phases.

7.4. Limitations of the Study

Although this study generated rich insights into Grade 4 teachers' pedagogical approaches to geometric patterns, several limitations must be acknowledged when interpreting the findings. First, the study was conducted with only five teachers within a single rural district, which, while appropriate for an in-depth qualitative case study, limits the generalisability of the findings to broader South African contexts or more diverse school environments. The study's reliance on a single or limited number of episodes per teacher, particularly in cases where only one episode offered sufficient analytic depth also means that the full range of each teacher's pedagogical repertoire may not have been captured. Additionally, classroom observations were influenced by the natural constraints of the school day, including large class sizes, resource limitations, and the presence of the researcher, which may have subtly shaped teacher or learner behaviour (observer effect). The use of semi-structured interviews and VSRI reflections provided valuable insights into teachers' reasoning, but these self-reported explanations may not fully capture the complexity of them in-the-moment decision-making.

Second, while the PRA-aligned recognition rules offered a powerful analytic tool for detailing pedagogical processes, their application was limited by the quality and granularity of the available classroom data. For example, nuances of teacher tone, gesture, or micro-level learner interactions may not have been fully captured in the video footage or transcripts, constraining the depth of representational or discourse-based coding. The study also focused specifically on geometric patterns, meaning that findings may not extend neatly to other mathematical topics that require different forms of reasoning or representation. Furthermore, although the recognition rules provided clarity and precision, they may benefit from additional refinement in future research to ensure their applicability across varied teaching contexts, content strands, and teaching styles. Despite these limitations, the study lays a strong foundation for understanding how rural Grade 4 teachers navigate the teaching of geometric patterns, while highlighting the contextual and methodological boundaries within which the findings should be interpreted.

7.5. Study Conclusion

This study set out to explore the pedagogical approaches employed by Grade 4 teachers in the Sekhukhune East District when teaching geometric patterns, using Shulman's (1987) Pedagogical Reasoning and Action (PRA) framework as both a theoretical and analytic lens. Through detailed turn-by-turn analysis, recognition-rule coding, and cross-case synthesis, the study reveals that teachers consistently draw on concrete representations, richly contextualised examples, probing questions, and embedded formative assessment to support learners' development of structural awareness. The findings demonstrate that despite working in resource-constrained rural classrooms, the participating teachers enacted pedagogical practices that were conceptually coherent, learner-centred, and aligned with the foundational principles of early algebraic reasoning. Their collective practice reflects a pragmatic yet robust enactment of the PRA cycle, characterised by transformation of content into accessible forms, orchestration of meaningful mathematical discourse, continuous monitoring of understanding, and learner-centred participation that builds toward generalisation.

Beyond identifying shared patterns of practice, the study contributes theoretically by refining PRA for empirical use through the development and application of recognition rules, which made teachers' pedagogical reasoning visible at a granular level. The study therefore not only documents how teachers teach geometric patterns but also demonstrates how pedagogical reasoning unfolds in action in rural South African classrooms. In doing so, it adds meaningful insight into the mediating role of context, language, and resources in shaping pedagogical decision-making. Ultimately, the study demonstrates the importance of supporting teacher expertise in representational fluency, formative assessment, and learner-centred facilitation, areas that are vital for strengthening early algebraic thinking and improving mathematics teaching and learning more broadly.

References

Abou-Assali, M. (2014). The link between teacher professional development and student achievement: A critical view. *International Journal of Bilingual & Multilingual Teachers of English*, *2*(01).

Adelabu, F. M., & Alex, J. (2024). Mathematical Knowledge for Teaching in the Senior Phase: Entry-Level Teacher Candidates' Subject Content Knowledge. *Futurity Education*, *4(3), 269-290*.

Adelabu, F. M., Marange, I. Y., & Alex, J. (2022). GeoGebra Software to Teach and Learn Circle Geometry: Academic Achievement of Grade 11 Students. *Mathematics Teaching Research Journal*, *14*(3), 2-16.

Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. *African Journal of Research in Mathematics, Science and Technology Education*, *19(3), 237-254*. https://journals.co.za/doi/abs/10.1080/10288457.2015.1089677

Ahmad, M., & Wilkins, S. (2025). Purposive sampling in qualitative research: A framework for the entire journey. *Quality & Quantity*, *59(2), 1461-1479*.

Andersson, A., Foyn, T., Simensen, A. M., & Wagner, D. (2023). Storylines in voices of frustration: Implications for mathematics teacher education in changing times. *Education Sciences*, *13(8), 816*.

Anney, V. N. (2014). Ensuring the quality of the findings of qualitative research: Looking at trustworthiness criteria. *Journal of emerging trends in educational research and policy studies (jeteraps)*, *5(2), 272-281.*

Anthony, S., & Jack, S. (2009). Qualitative case study methodology in nursing research: an integrative review. *Journal of advanced nursing*, *65(6), 1171-1181.*

Arasomwan, D. A., & Daries, G. (2024). Music-based pedagogies to teach conversational skills to early childhood care and education's learners: The educators' constraints and the proposed solutions. *E-Journal of Humanities Arts and Social Sciences, 5(7), 1219-1237.*

Babbie, E. & Mouton, J. (2008). *The practice of social research*. Cape Town: Oxford University Press Southern Africa.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special?.

Barends, Z. (2022). Pedagogical choices to integrate theory and practice: Conceptualisation and insights for literacy teacher education. *Reading & Writing-Journal of the Reading Association of South Africa*, *13(1), 333*.

Bature, B. (2016). The role of information and communication technology as a tool for effective teaching and learning of mathematics. *J. Appl. Comput. Math*, *1(5), 333*.

Bature, I. J., & Atweh, B. (2016). Achieving quality mathematics classroom instruction through productive pedagogies. *International Journal of Educational Methodology*, *2(1), 1-18.*

Beatty, R. (2018). Connecting Algonquin loomwork and western mathematics in a grade 6 math class. In *Toward equity and social justice in mathematics education* (*105-125*). Cham: Springer International Publishing.

Beatty, Ruth, Day-Mauro, Mary & Morris, Kate (2013). Young students' explorations of growing patterns: Developing early functional thinking and awareness of structure. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

BJÖRKLUND, C., & PALMÉR, H. (2024). Toddlers learning the meaning of counting words as increase in discerned aspects of number. *Mediating mathematics*, 61.

Blanton, M. (2022). The Role of Learning Progressions in" Democratizing" Students' Access to Algebra. *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J. S. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for research in Mathematics Education*, *46(1), 39-87.*

Blanton, P. (2024). Algebraic Reasoning and Modeling: Meaningful Curriculum. In *Third Symposium Proceedings. New Ways of Teaching and Learning: Aemilia Hotel, Bologna, Italy. August 6-10, 2024* (*Vol. 13, p. 44*). WTM-Verlag Münster.

Bleiker, J., Morgan-Trimmer, S., Knapp, K., & Hopkins, S. (2019). Navigating the maze: Qualitative research methodologies and their philosophical foundations. *Radiography*, *25*, S4-S8.

Boaler, J. (2016). Designing mathematics classes to promote equity and engagement. *The Journal of Mathematical Behavior*, *41,172-178.*

Bognar, B., Mužar Horvat, S., & Jukić Matić, L. (2025). Characteristics of effective elementary mathematics instruction: A scoping review of experimental studies. *Education sciences*, *15(1), 76.*

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative research in psychology*, *3(2), 77-101*.

Brookings Institution. (2023). Strengthening Pedagogical Approaches for Relevant Knowledge and Skills (Report No.2023-01). https://www.brookings.edu/research/strengthening-pedagogical-approaches-for-relevant-knowledge-and-skills

Brunner, K. (1961). A schema for the supply theory of money. International Economic Review, *2(1), 79-109*.

Cai, J. & Rott, B. (2024). On understanding mathematical problem-posing processes. *ZDM - Mathematics Education, 56(1), 61–71*. https://doi.org/10.1007/s11858-023-01536-w

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of educational psychology*, *105(2), 380.*

Castro Superfine, A. (2024). Moving forward while looking back. *Journal of Mathematics Teacher Education, 27, 137–138.*

Chang, S. H., Lee, N. H., & Koay, P. L. (2017). Teaching and learning with concrete-pictorial-abstract sequence: A proposed model. *The Mathematics Educator*. https://repository.nie.edu.sg/bitstreams/1bc521cc-ef36-4ad8-b994-4cab3ee7d37b/download

Chauke, T., & Maharaj, N. (2025). Teachers' Perceptions on Addressing Rural School Challenges for Quality Teaching and Learning.

Chen, J. J., Bredesen, R., Cooper, S. L., Du, K., Garda, A. E., Janchenko, L., ... & Croke, J. (2023). Radiation Oncology Providers' Practices and Preferences for Delivering Patient Education on Radiation Therapy: A Mixed Methods Study. *International Journal of Radiation Oncology, Biology, Physics*, *116(3), e3-e4.*

Chen, O., Retnowati, E., & Kalyuga, S. (2019). Effects of worked examples on step performance in solving complex problems. *Educational Psychology*, *39(2), 188-202.*

Cheung, K. S., Chan, K. C. A., Cheung, A., Chan, P. K., Luk, M. H., Chiu, K. Y., & Fu, H. (2025). Current trends of unicompartmental knee arthroplasty (UKA): choosing between robotic-assisted and conventional surgeries and timing of procedures. *Arthroplasty*, *7(1), 6.*

Cohen, E., Hoz, R., & Kaplan, H. (2013). The practicum in preservice teacher education: A review of empirical studies. *Teaching education*, *24(4), 345-380.*

Creswell J.W. (2012). Educational research: Planning, conducting, and evaluating quantitative and qualitative research (4th ed). Boston, MA: Pearson.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage publications.

Creswell, J. W., & Poth, C. N. (2016). *Qualitative inquiry and research design: Choosing among five approaches*. Sage publications.

Çukur, H. S. (2022). Dealing with Critical Incidents: Experiences of Turkish Novice EFL Teachers. *LLT Journal: A Journal on Language and Language Teaching*, *25(2), 691-708.*

da Ponte, J. P., Martins, M., Quaresma, M., Gomes, P., & Rodrigues, B. (2025). Needed skills and knowledge and challenges of the mathematics lesson study facilitator: a scoping systematic literature review. *International Journal for Lesson & Learning Studies, 14(4), 415-430.*

Dehaene S (1997) The number sense. New York: Oxford University Press.

Department of Basic Education. (2020). Curriculum and Assessment Policy Statement (CAPS) Document.

Desoete, A., & De Craene, B. (2019). Metacognition and mathematics education: An overview. *Zdm*, *51(4), 565-575*.

Du Plessis, J., & Prinsloo, L. (2025). Mapping the landscape of algebraic thinking and pattern generalisation in early childhood and primary education: A SCOPUS-BASED REVIEW. In *ICERI2025 Proceedings* (pp. 2192-2200). IATED.

Du Plessis, J., (2018). Early algebra: Repeating pattern and structural thinking at foundation phase. South African Journal of Childhood Education, 8(2), pp.1-11. https://www.scielo.org.za/pdf/sajce/v8n2/03.pdf

Dube, B. (2020). Rural online learning in the context of COVID 19 in South Africa: Evoking an inclusive education approach. REMIE: Multidisciplinary Journal of Educational Research, *10(2),135-157.* https://dialnet.unirioja.es/descarga/articulo/7606310.pdf

Eck, C. J., & Edwards, M. C. (2019). Teacher shortage in school-based, agricultural education (SBAE): A historical review. *Journal of Agricultural Education*, *60(4), 223-239.*

Edwards Jr, D. B., & Higa, S. (2018). The global education policy of school-based management in conflict-affected contexts: Current reach, prominent rationales, and future research. *Policy Futures in Education*, *16(3), 306-320*.

Elayeh, E. R., Hammad, E. A., Tubeileh, R. H., & Basheti, I. A. (2019). Use of secret simulated patient followed by workshop based education to assess and improve inhaler counseling in community pharmacy in Jordan. *Pharmacy Practice (Granada)*, *17*(4).

Ellis, A. B., & Özgür, Z. (2024). Trends, insights, and developments in research on the teaching and learning of algebra. *ZDM–Mathematics Education*, *56(2), 199-210.*

Essien, A. A. (2021). Rethinking exemplification in mathematics teacher education multilingual classrooms. *Exploring new ways to connect*, *39*.

Evans, P., & Martin, A. J. (2023). Load reduction instruction: Multilevel effects for motivation, engagement, and achievement in mathematics. *Educational Psychology*, *43(10), 1125-1143.*

Fairclough, N. (1992). Discourse and text: Linguistic and intertextual analysis within discourse analysis. *Discourse & society*, *3(2), 193-217.*

Fairclough, N. (1995). *Media discourse* (9-14). London: Edward Arnold.

Fairclough, N. (2013). *Critical discourse analysis: The critical study of language*. Routledge.

Fetters, M.D., & Rubinstein, E.B. (2019). The 3 Cs of Content, Context, and Concepts: A Practical Approach to Recording Unstructured Field Observations. Annals of https://doi.org/10.1370/afm.2453

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction*. Heinemann, 88 Post Road West, PO Box 5007, Westport, CT 06881.

Fournier, G., O'Neill, S., & Moreno Sala, M. T. (2021). Strategic approaches to learning sight-singing at college level: An investigation using Q Methodology. *Psychology of Music*, *49(6), 1429-1448.*

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60(4), 380-392.*

Furner, J. M. (2024). Blazing New Pathways for Digital Engagement in Mathematics Using GeoGebra, Math Manipulatives, and Picture Books in a STEM World to Nurture Confident Young People in Mathematics. *FDLA Journal*, *8(1), 11.*

Gagné, F., 2004. Transforming gifts into talents: The DMGT as developmental theory. High ability studies, *15(2), 119-147.*

Gardee, A., & Brodie, K. (2015). A teacher's engagement with learner errors in her Grade 9 mathematics classroom. *Pythagoras, 36(2), 293*. https://doi.org/10.4102/pythagoras.v36i2.293

Green J, Thorogood N (2014) Qualitative methods for health research. Introducing qualitative methods, 3rd ed. Sage Publications, Los Angeles

Green, A. (2018). Assessment of learning and assessment for learning. *The TESOL encyclopedia of English language teaching*, 1-6. https://onlinelibrary.wiley.com/doi/abs/10.1002/9781118784235.eelt0352

Grissom, J. A., Kern, E. C., & Rodriguez, L. A. (2015). The "representative bureaucracy" in education: Educator workforce diversity, policy outputs, and outcomes for disadvantaged students. *Educational Researcher*, *44*(3), 185-192.

Gula, Z., & Jojo, Z. (2024). Harnessing Indigenous Knowledge for Teaching of Mathematics for Sustainable Development in Rural Situated Primary Schools. *African Journal of Research in Mathematics, Science and Technology Education*, *28(3), 404-421.*

Guthrie, J. T., & Barber, A. T. (2011). Best practices in motivating students to read. *Best practices in literacy instruction*, 177-198.

Han, S., & Xu, X. (2019). How far has the state 'stepped back': An exploratory study of the changing governance of higher education in China (1978–2018). *Higher education*, *78(5), 931-946*.

Hanson, D., & Grimmer, M. (2007). The mix of qualitative and quantitative research in major marketing journals, 1993-2002. *European journal of marketing*, *41(1-2), 58-70.*

Hasin, I., & Nasir, M. K. M. (2021). The Effectiveness of the Use of Information and Communication Technology (ICT) in Rural Secondary Schools in Malaysia. Journal of Education and e-learning Research, *8(1), 59-64.*

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of educational research*, *77(1), 81-112*.

Herbert, S., Vale, C., Bragg, L. A., Loong, E., & Widjaja, W. (2015). A framework for primary teachers' perceptions of mathematical reasoning. International Journal of Educational Research, *74, 26-37.*

Hood Cattaneo, K. (2017). Telling active learning pedagogies apart: From theory to practice. *Journal of new approaches in educational research, 6(2), 144-152.*

Hunaepi, H., and Suharta, I. (2024). Transforming education in Indonesia: The impact and challenges of the Merdeka belajar curriculum. Path of science, *10(6), 5026-5039.* https://doi.org/10.22178/pos.105-31

Hundeland, P. S., Carlsen, M., & Erfjord, I. (2020). Qualities of mathematical discourses in kindergartens. *ZDM*, *52(4), 691-702.*

Ibrokhimovich, F. J., & Mirzaxolmatovna, X. Z. (2022). The most important role of mathematics in primary school. *Galaxy international interdisciplinary research journal*, *10(3), 652-655.*

Ingersoll, R. M., & Tran, H. (2023). Teacher shortages and turnover in rural schools in the US: An organizational analysis. *Educational Administration Quarterly*, *59(2), 396-431*.

Iqbal, M., Bhattacharjee, A., Sarif, Z., Das, D., & Das, A. (2025, August). Mental Health Monitoring in Higher Education Using Machine Learning and Computer Vision. In *2025 6th International Conference on Big Data Analytics and Practices (IBDAP)* (pp. 70-74). IEEE.

Jackson, J. L., & Stenger, C. L. (2024). Methods of Explicitly Teaching Generalization in the Mathematics Classroom and Indicators of Success: A Systematic Review. *International Journal of Education in Mathematics, Science and Technology*, *12*(4), 1109-1126.

Jo, I., & Huh, S. (2022). Pedagogical Reasoning and Action (PR&A): A Framework for Research on Professional Knowledge of Geography Teaching. In *Geography Teacher Education and Professionalization* (pp. 291-304). Cham: Springer International Publishing.

Jojo, Z. (2024). Culturally responsive pedagogy for the promotion of understanding Mathematics: The case of rural situated primary schools in Post-apartheid South Africa. *Interdisciplinary Journal of Sociality Studies*, *4*(s1), 1-15.

Kanandjebo, L. N. (2024). Mathematics as a driver for sustainable development: Pre-service teachers' perspectives on awareness, challenges, and implications for teacher training. *Interdisciplinary Journal of Education Research*, *6*, 1-17.

Kawasaki (eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education (PME)* (Vol. 4, p. 81-88). Belo Horizante, Brazil: PME. https://doi.org/10.1007/978-94-007-0014-

Kong, S. H. (2025). Music education training for kindergarten teachers: a workshop integrating Shulman's pedagogical content knowledge and pedagogical reasoning and action model. *Journal of Early Childhood Teacher Education*, *46*(1), 85-102. https://www.tandfonline.com/doi/abs/10.1080/10901027.2024.2410399

Kumatongo, B., & Muzata, K. K. (2021). Research paradigms and designs with their application in education. *Journal of Lexicography and Terminology (Online ISSN 2664-0899. Print ISSN 2517-9306).*, *5*(1), 16-32.

Latumbo, I. L. (2024). Differentiated Instruction in Teaching Fourth Grade Mathematics: Basis for an Instructional Module. *Psychology and Education: A Multidisciplinary Journal*, *17*(6), 1-1.

Lee, V. E. (1996). Understanding High School Restructuring Effects on the Equitable Distribution of Learning in Mathematics and Science. Revised.

Li, R., Cao, Y., Tang, H., & Kaiser, G. (2024). Teachers' scaffolding behavior and visual perception during cooperative learning. International Journal of Science and Mathematics Education, *22, 333–352*. https://doi.org/10.1007/s10763-023-10379-6

Lyle, J. (2003). Stimulated recall: A report on its use in naturalistic research. *British Educational Research Journal*, *29(6), 861–878.* https://doi.org/10.1080/0141192032000137349

Mabena, N., Mokgosi, P. N., & Ramapela, S. S. (2021). Factors contributing to poor learner performance in mathematics: A case of selected schools in Mpumalanga province, South Africa. *Problems of Education in the 21st Century*, *79(3), 451.*

Mabhoza, Z., & Olawale, B. E. (2024). Chronicling the Experiences of Mathematics Learners and Teachers on the Usage of Guided Discovery Learning (GDL) in Enhancing Learners' Academic Performance. Research in Social Sciences and Technology, 9(1), 141–155. https://doi.org/10.46303/ressat.2024.8

Madimabe, M. P., & Omodan, B. I. (2021). Investigating the effects of e-learning as a method of curriculum dissemination for rural TVET college students.

Mafakheri, M. (2021). Investigating The Relationship Between Parenting Styles And Depression And Anxiety And Educational Performance, By Distance Learning In Primary School During Coronavirus Crisis. *Journal of Emergency Health Care*, *10(4), 94-101.*

Magatsela, K. B. (2025). Understanding the Factors Behind Low Matric Performance in Limpopo's Government Schools: Insights From the Sekhukhune Region. *Journal of Education and Science of Teaching*, *1*(2), 1-11. https://journals.co.za/doi/abs/10.10520/ejc-joest_v1_n2_a1

Mahadi, F., & Husin, M. R. (2021). Case study approach: outlook on learning strategy and style among malaysian rural pupils. *Int. J. Acad. Res. Progress. Educ. Dev*, *10*, 35-48.

Maharaj, N., & Chauke, T. A. (2025). Teachers' Challenges in the Rural Schools in South Africa. *International Journal*, *16*, e2025235.

Maharjan, M., Dahal, N., & Pant, B. P. (2022). ICTs into mathematical instructions for meaningful teaching and learning. *Advances in Mobile Learning Educational Research, 2(2), 341-350.*

Maja, M. M. (2023). Teachers' perceptions of integrating technology in rural primary schools to enhance the teaching of English first additional language. *Journal of Curriculum Studies Research*, *5(1), 95-112.*

Mamolo, A., & Glynn-Adey, P. (2023). Tangible connections within the mathematical horizon: Exploring the Dihedral Calculator. *ZDM–Mathematics Education*, *55(4), 793-805.*

Marbun, Y. M. R. (2022). Yanty Maria Rosmauli Marbun 1 , Rianita Simamora 2 Dosen Pendidikan Matematika FKIP Universitas HKBP Nommensen Pematangsiantar Email :VII(1).

Maree, J. G. (2007). *Shaping the story*. Van Schaik Publishers.
Masinire, A. (2015). Recruiting and retaining teachers in rural schools in South Africa: Insights from a rural teaching experience programme. *Australian and International Journal of Rural Education*, *25(1), 1-13.*
Mason, J. (2018). Structuring structural awareness: a commentary on Chapter 13. In *Building the foundation: Whole numbers in the primary grades: The 23rd ICMI Study* (pp. 325-340). Cham: Springer International Publishing. https://link.springer.com/chapter/10.1007/978-3-319-63555-2_14
Maudia, N., Awodeyi, A. F., & Mohammed, A. S. (2024). Enhancing pedagogical content knowledge in mathematics teachers through collaborative professional development. *Interval: Indonesian Journal of Mathematical Education*, *2(1), 36-49.*
Mawela, A. S., & Mahlambi, S. B. (2021). Exploring Teachers' Views on Code-Switching as a Communicative Technique to Enhance the Teaching of Mathematics in Grade 4. *International Journal of Educational Methodology, 7(4), 637-648.*
Mbhiza, H. (2021). Rural Teachers` Teaching of Algebraic Functions Through a commognitive Lens. Interdisciplinary Journal of Rural and Community Studies*, 3(1) 10-20* https://pubs.ufs.ac.za/index.php/ijrcs/article/download/582/514
Mbhiza, H. (2021). Rural Teachers` Teaching of Algebraic Functions Through a commognitive Lens. Interdisciplinary Journal of Rural and Community Studies, *3(1) 10-20* https://pubs.ufs.ac.za/index.php/ijrcs/article/download/582/514
Mbhiza, H. W. (2017). A critical exploring of Grade 10 rural learners` experiences and attitude towards learning mathematics in Acornhoek classrooms. Mpumalanga province (Doctoral dissertation) University of the Witwatersrand, South Africa.
Mbhiza, H. W. (2024). Behind the Love and Stories: Rural Learners' Reasons and Motivations for Learning Mathematics. Interdisciplinary Journal of Sociality Studies, 4, 1-12. https://pubs.ufs.ac.za/index.php/ijss/article/view/1076/821
Mbhiza, H. W., & Sempe, M. (2024). Mathematics discourse in instruction during functions lessons in grade 10 rural classrooms. *Interdisciplinary Journal of Rural and Community Studies*, 6, 1-20.
Mbhiza, H. W., & Zondo, A. S. (2025). Teachers' Perceptions of their Learners, Teaching and Rurality in Mathematics Teaching. *Indonesian Journal of Education and Social Sciences*, *4(2), 321-333.*
McMillan JH & Schumacher S 2014. *Research in education: Evidence-based inquiry* (7th ed). Harlow, England: Pearson Education Limited.
McMillan, J. H., & Schumacher, S. (2010). Research in Education: Evidence-Based Inquiry, MyEducationLab Series. *Pearson*.
Mdodana-Zide, L., & Mafugu, T. (2023). An interventive collaborative scaffolded approach with a writing center on ESL students' academic writing. *Journal of Culture and Values in Education, 6(2), 34-50.*

Meier, C., & West, J. (2020). Overcrowded classrooms–the Achilles heel of South African education?. *South African Journal of Childhood Education*, *10(1), 1-10.*

Mensah, E., & Baidoo-Anu, D. (2022). Towards quality and equitable education in South Africa: Unpacking the relationship between teacher factors, students socioeconomic background and mathematics achievements. Research in Mathematics*, 9(1), 2088645.* https://www.tandfonline.com/doi/pdf/10.1080/27684830.2022.2088645

Mogorosi, D. E., & de Wet, A. (2023). Teachers'roles in realising human rights in challenging contexts.

Mohangi K. Krog S. Stephens O. Nel N. 2016 "Contextual Challenges in early literacy teaching and learning in Grade R rural schools in South Africa" **Per Linguam** 32 1 DOI: https://doi.org/10.5785/32-1-646

Moreno-Armella, L., Hegedus, S. J., & Kaput, J. J. (2008). From static to dynamic mathematics: Historical and representational perspectives. *Educational Studies in Mathematics*, *68(2), 99-111.*

Moser, A., & Korstjens, I. (2017). Series: Practical guidance to qualitative research. Part 1: Introduction. *European journal of general practice*, *23(1), 271-273.*

Mouza, C., Ottenbreit-Leftwich, A., Karlin, M., & Mathayas, N. (2025). Implications of the TPACK framework for developing computationally literate preservice teachers. In *Handbook of Technological Pedagogical Content Knowledge (TPACK) for Educators* (*227-250*). Routledge.

Mpitso, V. B. (2025). *Grade 2 Rural Teachers' Pedagogical Reasoning and Actions During Lessons on Number and Geometric Patterns in Acornhoek, Mpumalanga Province, South Africa* (Master's thesis, University of South Africa (South Africa)).

Msimango, N., Fonseca, K., & Petersen, N. (2020). Mentoring and preparing primary school mathematics teachers. Perspectives in Education, *38(2), 272-284.* https://journals.ufs.ac.za/index.php/pie/article/download/4383/3941

Msomi, X. A., Mabusela, M. S., Mkhize, M. V., & Suknunan, S. (2025). Teacher perceptions of skills development for curriculum change: training and empowerment of accounting teachers for effective Accounting curriculum delivery.

Mukavhi, L., Brijlall, D., & Abraham, J. (2021). Digital instructional and pedagogical gaps in mathematics education during COVID-19: APOS theory based laboratory methodology in higher education. *Universal Journal of Educational Research; Vol. 9, Issue 6*.

Mukuka, A., & Alex, J. K. (2024). Student Teachers' Knowledge of School-level Geometry: Implications for Teaching and Learning. *European Journal of Educational Research*, *13*(3).

Mulligan, J. (2022). Pathways to Early Mathematical Thinking in Kindergarten. *Special Issues in Early Childhood Mathematics Education Research: Learning, Teaching and Thinking*, *5, 155.*

Mulligan, J. T. (2025). The Role of Spatial Reasoning in Growing and Spatial-Repeating Patterns in First and Second Graders' Structural Development of Mathematics. *Education Sciences*, *15(11), 1479.*

Mulligan, J. T., & Mitchelmore, M. C. (2013). Early awareness of mathematical pattern and structure. In *Reconceptualizing early mathematics learning* (pp. 29-45). Dordrecht: Springer Netherlands.

Mulligan, J. T., & Mitchelmore, M. C. (2013). Early awareness of mathematical pattern and structure. In *Reconceptualizing early mathematics learning* (pp. 29-45). Dordrecht: Springer Netherlands.

Muremela, G., Kutame, A., Kapueja, I., & Adigun, O. T. (2021). Retaining scarce skills teachers in a South African rural community: An exploration of associated issues. African Identities, *21(3), 1–17.* https://doi.org/10.1080/14725843.2021.1965864.

Mwapwele, S. D., Marais, M., Dlamini, S., & Van Biljon, J. (2019). Teachers' ICT adoption in South African rural schools: a study of technology readiness and implications for the South Africa connect broadband policy. *The African Journal of Information and Communication*, *24, 1-21.*

Ndou, N. (2020). Educators' views on the provision of quality education at selected public schools in Limpopo Province. *Africa Education Review*, *17(4), 3-16*.

Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics*, *70(2), 159-174*.

Nhlumayo, B. S. (2024). Rural Primary School Principal's Leadership Strategies for ICT Integration. *Research in Social Sciences and Technology*, *9(1), 171-184*.

Nkambule, T. (2022). Exploring working conditions in selected rural schools: teachers' experiences. South African Journal of https://www.ajol.info/index.php/saje/article/view/229366/216538

Nkambule, T. C. (2017). Student teachers' perceptions of a Wits rural teaching experience project: What to learn and improve. South African Journal of Higher Education, *31(1), 191-206*. https://www.journals.ac.za/sajhe/article/view/901/763

Okesina, M. (2020). A critical review of the relationship between paradigm, methodology, design and method in research. IORS Journal Handbook of qualitative research of Research & Method in Education, 101): *57-68*.

Övez, F. T. D., & Kiyici, O. D. (2018). 6th Grade Students' Views about Mathematical Teaching Based on Technology Integration. *World Journal of Education*, *8*(5), 160-171.

Palinussa, A. L., Molle, J. S., & Gaspersz, M. (2021). Realistic mathematics education: Mathematical reasoning and communication skills in rural contexts. *International Journal of Evaluation and Research in Education*, *10(2), 522–534*. https://doi.org/10.11591/ijere.v10i2.20640

Papic, M. (2015). An early mathematical patterning assessment: Identifying young Australian indigenous children's patterning skills. *Mathematics Education Research Journal*, *27(4), 519-534.*

Perrill, E. (2020). The Arch Meets the Line: Geometries of Innovation and Conveyance. *de arte*, 55(2), 23-45. http://libres.uncg.edu/ir/uncg/f/E_Perril_Arch_2020.pdf

Petersen, J. A., & Nel, B. P. (2024). The Use of VSR as an In-house Professional Development Tool for Joint Reflection in a Primary School in the Western-Cape, South Africa. *African Journal of Research in Mathematics, Science and Technology Education*, *28(1), 109-119.*

Piaget, J. (1950). Introduction to Genetic Epistemology. Volume II: Physical Thought. Paris: Presses Universitaires de France.

Piaget, J. (1970). Intelligence and biological adaptation. *The Processes of Adaptation*, *1(1), 69-84.*

Pinto, E., & Cañadas, M. C. (2021). Generalizations of third and fifth graders within a functional approach to early algebra. Mathematics Education Research Journal, *33(1), 113-134.*

Pratt, M. G. (2015). Crafting and selecting research questions and contexts in qualitative research. In *Handbook of qualitative organizational research* (177-185). Routledge.

Rasool, Z. (2018). Emotions in community research. In Re-imagining Contested Communities (115-122). Policy Press.

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. Reading & Writing Quarterly, *31(3), 235 252*. https://doi.org/10.1080/10573569.2015.1030995

Rivera, F. D. (2010). Visual Roots of Mathematical Cognitive Activity. In *Toward a Visually-Oriented School Mathematics Curriculum: Research, Theory, Practice, and Issues* (59-97). Dordrecht: Springer Netherlands.

Rivera, F. D. (2018). Pattern generalization processing of elementary students: Cognitive factors affecting the development of exact mathematical structures. *Eurasia Journal of Mathematics, Science and Technology Education*, *14*(9), em1586. https://www.ejmste.com/article/pattern-generalization-processing-of-elementary-students-cognitive-factors-affecting-the-5536 \

Rivera, N. (2013). *Cooperative learning in a community college setting: Developmental coursework in mathematics* (Doctoral dissertation, Arizona State University).

Rizos, I., & Gkrekas, N. (2024). Pattern recognition among primary school students: The relationship with mathematical problem-solving. *Contemporary Mathematics and Science Education*, *5(2),* ep24010. https://www.conmaths.com/article/pattern-recognition-among-primary-school-students-the-relationship-with-mathematical-problem-solving-14689

Rozsahegyi, T. (2019). Observations. In M. Lambert (Ed.), Practical research methods in Education: An early researcher's critical guide (23-34). Routledge.

Russo, J., & Hopkins, S. (2023). Not so Simple Addition: Comparing Student Performance and Teacher Perceptions of Retrieval. *International Journal of Science and Mathematics Education*, *21(8), 2279-2301.*

Rusznyak, L. (2022). Teacher choices in action: An emergent pedagogical response and intervention. In *Pedagogical Responsiveness in Complex Contexts: Issues of Transformation, Inclusion and Equity* (111-131). Cham: Springer International Publishing. https://link.springer.com/chapter/10.1007/978-3-031-12718-2_7

Ruyembe, C. E. M. (2020). Approach to Pedagogy and Scenarios Poor People Face in the Pursuit of Basic and Higher. *Pedagogy in Basic and Higher Education: Current Developments and Challenges*, 43.

Rycroft-Smith, L., & Gould, T. (2021). The number line: A flexible and useful model. Cambridge Mathematics: Espresso, *35.* Retrieved from https://www.cambridgemaths.org/Images/espresso_35_the_number_line.pdf

Said, R., Sudiyanto, S., & Usodo, B. (2019). The Use of Guided Discovery Learning Model In Mathematics Subject. Proceedings of the 1st Seminar and Workshop on Research Design, for Education, Social Science, Arts, and Humanities, SEWORD FRESSH 2019, April 27 2019, Surakarta, Central Java, Indonesia. https://doi.org/10.4108/eai.27-4-2019.2286897

Salingay, N., & Tan, D. (2018). Concrete-pictorial-abstract approach on students' attitude and performance in mathematics. *International Journal of Scientific & Technology Research*, *7(5), 90-111.* https://www.academia.edu/download/57067805/Concrete-pictorial-abstract-Approach-On-Students-Attitude-And-Performance-In-Mathematics.pdf

Scott, P., Mortimer, E., & Ametller, J. (2011). Pedagogical link-making: a fundamental aspect of teaching and learning scientific conceptual knowledge. *Studies in Science education*, *47(1), 3-36.*

Sempe, M. (2024). *Grade 12 Rural Mathematics Teachers' Discourses During Calculus Lessons: A Case of Effective Teachers in Qwa-Qwa, Free State Province, South Africa* (Master's thesis, University of South Africa (South Africa)).

Sen, C., & Guler, G. (2022). Reasoning skills of children aged between 4 and 6 years in repeating pattern tasks. *International journal of mathematical education in science and technology*, *53(7), 1894-1915.* https://www.tandfonline.com/doi/abs/10.1080/0020739X.2022.2070085

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge university press.

Shava, E. (2022). Reinforcing the role of ICT in enhancing teaching and learning post-COVID-19 in tertiary institutions in South Africa. *Journal of Culture and Values in Education*, *5(1), 78-91.*

Shikalepo, E. E. (2020). Challenges facing teaching at rural schools: A review of related literature. *International Journal of Research and Innovation in Social Science*, *4(5), 211-218.*

Shukla, D. (2016). Student's Perceived Level and Teachers' Teaching Strategies of Higher-Order Thinking Skills, a Study on Higher Educational Institutions in Thailand. Ongkharak: Department of Business Administration, St. Theresa International College, Thailand.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard educational review*, *57(1), 1-23*. https://meridian.allenpress.com/her/article-abstract/57/1/1/31319

Sibuyi, N. A., Segooa, M. Y., Molise, H. V., Modiba, N. S., & Mafumo, T. (2024). Investigation into the Challenges Experienced by School Management Teams (SMTs) Post-Pandemic in Rural High Schools in the Mopani-West Education District. *Research in Social Sciences and Technology*, *9(2), 109-132*.

Siemens, G. (2005). Connectivism: Learning as network-creation. *ASTD Learning News*, *10(1), 1-28.*

Siljander, P. (2011). What are we looking for?—Pro critical realism in text interpretation. *Journal of Philosophy of Education*, *45(3), 493-510*.

Silver, E. A., & Mesa, V. (2011). Highly accomplished teachers of mathematics and effective instructional practice: Probing the intersection. *Expertise in mathematics instruction: An international perspective*, *63-84.*

Simons, H. (2009). Case study research in practice.

Sitabkhan, Y. A., & Platas, L. (2018). Early mathematics counts. RTI Press. 68-76. 185 Sofianingsih, E., Kartono, K., & Ahmadi, F. (2023). Analysis of students' knowledge of class ii about the use of the visual aids, traditional abacus. International Jo0urnal of Research and Review, *10(8), 859-868.* https://doi.org/10.52403/ijrr.202308110

Skinner, B.F. (1953). Some contributions of an experimental analysis of behaviour to psychology as a whole. American Psychologist, *8, 69-78*

Smart, J., & Linder, S. M. (2017). Development, validation, and implementation of the elementary mathematics motivation inventory (EMMI): examining motivational constructs in elementary mathematics. *Fields Mathematics Education Journal*, *3(1), 2.*

Snell, J., & Lefstein, A. (2018). "Low ability," participation, and identity in dialogic pedagogy. *American Educational Research Journal, 55(1), 40-78*.

Söderström, S., & Palm, T. (2024). Feedback in mathematics education research: a systematic literature review. *Research in Mathematics Education*, 1-22. https://www.tandfonline.com/doi/abs/10.1080/14794802.2024.2401488

Sortwell, A., Trimble, K., Ferraz, R., Geelan, D. R., Hine, G., Ramirez-Campillo, R., ... & Gkintoni, E. (2024). *A Systematic Review of Meta-Analyses on the Impact of Formative Assessment on K-12 Students' Learning: toward Sustainable Quality Education. Sustainability 2024, 16 (17), 7826*.

Stake, R. E. (2005). Qualitative case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), Handbook of qualitative research (3rd ed., 443-466). Thousand Oaks, CA: Sage.

Stake, R., & Visse, M. (1995). *Case study research*. Cham: Springer.

Strübe, M., Tröger, H., Tepner, O., & Sumfleth, E. (2014). Development of a Pedagogical Content Knowledge test of chemistry language and models. *Educación química*, *25*(3), *380-390.* https://www.sciencedirect.com/science/article/pii/S0187893X14705531

Stylianides, A. L. (2007). Proof and proving in school mathematics. *Journal for research in Mathematics Education*, *38(3), 289-321.*

Sudihartinih, E., Purniati, T., & Rohayati, A. (2020, July). A Rasch Model Analysis of Mathematical Understanding of Prospective Mathematics Teachers in The Conic Concept. In *MSCEIS 2019: Proceedings of the 7th Mathematics, Science, and Computer Science Education International Seminar, MSCEIS 2019, 12 October 2019, Bandung, West Java, Indonesia* (p. 1). European Alliance for Innovation.

Sun, L., Kangas, M., Ruokamo, H., & Siklander, S. (2023). A systematic literature review of teacher scaffolding in game-based learning in primary education. *Educational Research Review*, *40*

Tshesane, H. M. (2014). Exploring Grade 4 learners' use of models and strategies for solving addition and subtraction problems. University of the Witwatersrand, Johannesburg (South Africa).

Tsiouri, E. (2025). Teaching geometry and painting: A path to integrating art and mathematics. *European Journal of Education Studies*, *12*(4).

Van Dijk, L., & Rietveld, E. (2021). Situated anticipation. *Synthese*, *198(1), 349-371.* https://link.springer.com/article/10.1007/s11229-018-02013-8

Vassar, A. (2017). *The use of manipulative materials in early place value instruction: a cognitive load perspective* (Doctoral dissertation, UNSW Sydney). https://unsworks.unsw.edu.au/bitstreams/c90ab024-2c71-43d1-8137-26e34eb04960/download

Venketsamy, R., Smart, L., & Hu, Z. (2021). Creating and leading a learning environment in diverse Foundation Phase classrooms in a South African school. *Journal for the Education of Gifted Young Scientists*, *9(4), 359-376.*

Villiger, D. (2025). Informed consent under ignorance. *The American Journal of Bioethics*, *25*(7), 126-138.

Voon, X. P., Wong, S. L., Wong, L. H., Khambari, M. N. M., & Syed-Abdullah, S. I. S. (2023). Developing pre-service teachers' computational thinking through experiential learning: hybridisation of plugged and unplugged approaches. *Research and Practice in Technology Enhanced Learning*, *18, 006*. https://doi.org/10.58459/rptel.2023.18006

Vygotsky, L. (1978). Interaction between learning and development. Readings on the development of children, *23(3), 34-41*. https://ia.eferrit.com/ea/a6589cd862231ed3.pdf

Wang, L., Zeng, J., Ran, X., Cui, Z., & Zhou, X. (2022). Different cognitive mechanisms for process-open and process-constrained problem solving. *ZDM–Mathematics Education*, *54(3), 529-541*.

Warren, E. A. (2023). Addressing deficit constructs through critical counter-narratives and transformative coaching. *Middle School Journal*, *54(1), 14-22.*

Wheeler, D. L., & Montgomery, D. (2009). Community college students' views on learning mathematics in terms of their epistemological beliefs: a Q method study. *Educational studies in mathematics*, *72(3), 289-306.*

Wilhelm, A. G., Munter, C., & Jackson, K. (2017). Examining relations between teachers' explanations of sources of students' difficulty in mathematics and students' opportunities to learn. *The Elementary School Journal*, *117(3), 345-370.*

Wilkie, K. J., & Clarke, D. M. (2016). Developing students' functional thinking in algebra through different visualisations of a growing pattern's structure. *Mathematics Education Research Journal*, *28(2), 223-243.* https://link.springer.com/article/10.1007/s13394-015-0146-y

Wilkie, K. J., Ayalon, M., & Kanj, S. Z. (2023). Exploring ways to engage disaffected mathematics students through formative assessment processes with rich tasks. *Teaching and Teacher Education*, *132, 104256.*

Wilkie, K. J., Ayalon, M., & Kanj, S. Z. (2023). Exploring ways to engage disaffected mathematics students through formative assessment processes with rich tasks. *Teaching and Teacher Education*, *132, 104256*.

Wilmot, D., & Goldschagg, P. (2023). Exploring the inner workings of the Southern African Geography Teachers' Association Google Group. *Journal of Geography Education in Africa*, *6, 79-103.*

Xu, C., Di Lonardo Burr, S., & LeFevre, J. A. (2023). The hierarchical relations among mathematical competencies: From fundamental numeracy to complex mathematical skills. *Canadian Journal of Experimental Psychology/Revue canadienne de psychologie expérimentale*, *77(4), 284.*

Yeşildere İmre, S., & Akkoç, H. (2012). Investigating the development of prospective mathematics teachers' pedagogical content knowledge of generalising number patterns through school practicum. *Journal of Mathematics Teacher Education*, *15(3), 207-226.*

Yesuf, S. A. (2024). Ethical clearance paper as a bottleneck. *Pan African Medical Journal*, *47*(1).

Yin, R. K. (2003). Case study research: Design and methods (3rd ed.). Thousand Oaks, CA: Sage.

Yin, R. K. (Ed.). (2005). *Introducing the world of education: A case study reader*. Sage.

Zheng, S. X., & Leong, K. E. (2025). Metacognition's mediating effect on undergraduate achievement goals and mathematical modelling competency. *International Journal of Instruction*, *18(2), 455-472.*

Appendices

Appendix A: Unisa Research Ethics Approval

UNISA university of south africa

UNISA COLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 08 January 2025

Decision: Ethics Approval form

Ref: **2024/11/15/000000382/07/RB**
Name: **Ms Meta Nchabeleng**
Student No.: **57423652**

Dear **Ms Meta Nchabeleng**

Researcher(s): Name: **Ms Meta Nchabeleng**
E-mail address: 57423652@mylife.unisa.ac.za
Telephone: **0715276066**

Supervisor: Name: **Prof Hlamulo Mbhiza**
E-mail address: mbhizhw@unisa.ac.za
Telephone: **0769019192**

Title of research: *Grade 4 Rural Mathematics Teachers' Discourses of Teaching Geometric Patterns in Sekhukhune, Limpopo Province, South Africa.*

Qualification: MEd (Mathematics Education)

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above-mentioned research. Ethics approval is granted for the period **2025/01/08** to **2028/01/08.**

The ***low risk level*** *application was reviewed by the Ethics Review Committee on* ***15 November 2024*** *in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

Appendix B: Limpopo Department of Education Ethics Approval

CONFIDENTIAL

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

OFFICE OF THE PREMIER

Office of the Premier

Research and Development Directorate

Private Bag X9483, Polokwane, 0700, South Africa

Tel: (015) 230 9910, Email: mokobij@premier.limpopo.gov.za

LIMPOPO PROVINCIAL RESEARCH ETHICS

COMMITTEE CLEARANCE CERTIFICATE

REVIEW DATE: 16 APRIL 2025

PROJECT NUMBER: LPREC/103/2025: PG

SUBJECT: GRADE 4 RURAL MATHEMATICS TEACHERS' DISCOURSES OF TEACHING GEOMETRIC PATTERNS: A CASE IN SEKHUKHUNE DISTRICT.

RESEARCHER: M NCHABELENG

Chairperson: Prof I Swarts

ISwarts

Chairperson: Limpopo Provincial Research Ethics Committee

The Limpopo Provincial Research Ethics Committee (LPREC) is registered with National Health Research Council (NHREC) Registration Number **REC-111513-038.**

Note:

i. **This study is categorized as a Low Risk Level in accordance with risk level descriptors as enshrined in LPREC Standard Operating Procedures (SOPs)**

ii. **Should there be any amendment to the approved research proposal; the researcher(s) must re-submit the proposal to the ethics committee for review prior data collection.**

iii. **The researcher(s) must provide annual reporting to the committee as well as the relevant department and also provide the department with the final report/thesis.**

iv. **The researchers will be required to make presentations of the study findings and recommendations at the Provincial Research Conference/Departmental Research Day.**

v. **The ethical clearance certificate is valid for 12 months. Should the need to extend the period for data collection arise then the researcher should renew the certificate through LPREC secretariat. PLEASE QUOTE THE PROJECT NUMBER IN ALL ENQUIRIES.**

Appendix C: Video-Stimulated Recall Interview Transcript

Meta (Researcher)	**Keatlegile**
I am with Miss Keatlegile and we are going through the videos that were recorded during her lessons. So Miss Keatlegile, I have observed that when you were starting your lessons, you were always introducing the topic of your lessons, what is it important to you to always introduce the lesson's topic?	I always make sure that I introduce my lesson's topic everyday to my learners because it helps learners to understand what to expect and also helps them to prepare themselves mentally
Okay. It makes so much sense and I understand you perfectly well. I have also seen that you always made sure that it is the learners that come to the chalkboard to complete the patterns themselves and not you so how important it is for learners to draw those diagrams for themselves and not you drawing those diagrams for them?	Yes, I always ask my learners to complete the geometric diagrams on their own because it makes them to develop critical thinking and analytical skills and it also makes them not to forget what they wrote on the chalkboard
Okay. Understandable! Another thing that I found so interesting in your lessons when teaching geometric patterns is when you were using coloured chalks, care to share what motivated you to use them?	I like to use coloured chalks in my geometric patterns lessons because the coloured chalks capture my learners' attention, which then makes them to participate in my class
You are very right as I have seen increment participation in your lessons	Yeap! Coloured chalks are very important in geometric patterns especially in Grade 4
Here, I can see how you were emphasising to your learners the importance of being able to describe the patterns so how important it is for learners to know how to describe the patterns?	I always emphasise the importance of knowing how to describe the patterns because it assists me as the teacher to know if my learners understand the topic, it also encourages them to reflect on their own understanding and also helps them to develop mathematics vocabulary

Appendix D: Classroom Observation Transcript

Phomeleo: Good morning class, we are going to talk about patterns today! So, is there anyone who can tell us what Patterns are?
Learner 1: "ke di paterone" (translated as it is patterns)
Phomeleo: he is saying, "di paterone, jwale le a di tseba di paterone?" (so do you all know patterns?)
Learners: yes
Phomelelo: okay. we have different types of patterns and our pattern today is Geometric patterns but we firstly have to know what patterns are as we cannot talk about Geometric patterns without knowing what patterns are, so a pattern is a repetitive sequence of colours, numbers, letters, shapes or objects. "Ke dilo tše dingwe le tše dingwe tša go te pusheletša, ekaba maletere, di nomoro goba di bopopego. Dilo tšeo re di bitša di pattern ka ge di tlabe di tsamaya ka tatelano ya go swana ebile ya go se fetoge" (it is anything that repeats itself, which it can be in the form of letters, numbers or objects so we call them patterns as it has the same sequence). And I said a pattern can be what? "Ke boletse dilo tse tharo" (I mentioned three things) so it can be what?
Learner 2: Colours, numbers, objects or shapes
Phomelelo: And a pattern with numbers is called a numeric pattern but today we are going to talk about geometric pattern. Geometric patterns need to be completed so what do you do when you complete a pattern, for example, if I give you the pattern and I ask you to complete the pattern so what are you going to do?
Learners: (just stares at her and did not answer)
Phomelelo: You are going to repeat the very same shapes to complete the pattern. Let's say I have a triangle and a circle in pattern 1 so in pattern 2 you are going to start with the triangle and then the circle. you can see that this pattern is in the same order. So, anyone with the question?
Learners: yes
Phomelelo: okay what are your questions?
Learner 3: can I add other different shapes in the pattern?
Phomelelo: no, if you can add the different shape that was not part of the pattern 1 and pattern 2 then your pattern would be wrong
Learner 3: okay I understand
Phomelelo: Okay so as we have agreed that patterns can be shapes or letters or colours and that when we write patterns it should be the same sequence and you don't change anything it should be the same throughout the whole pattern. And after completing the pattern you should describe the pattern so what do you do when you describe the pattern? so you talk about the order of the shapes "ore botsa gore pattern ya rena ena le eng? ekaba enale di shape goba enale dinomoro goba enale di colour" (you tell us what our pattern consists of, is it shapes or numbers or colours) so in our pattern how will we describe it? who can try?
Learner 4: Our patterns have shapes
Phomelelo: He is saying our patterns has shapes, he is correct but what kind of shapes as you must name the kind of shapes that are in the pattern
Learner 5: the pattern has triangles and circles
Phomelelo: very correct! And when you complete it, you should add the same triangle and circles in the same order so if I have started with the triangle then you

will also have to start with the triangle. Now let us look at the example that I have given (see image…)

Phomelelo: we will be now completing the pattern so which shapes will I draw next?
Learners: triangle and a circle
Phomelelo: Very good! (she drew the triangle and the circle (see image..) on the chalkboard)

Image..: completed pattern
Phomelelo: now who can describe our pattern?
Learner 6: our pattern has triangles and circles
Phomelelo: correct (she wrote the description of the pattern on the chalkboard. See image…)

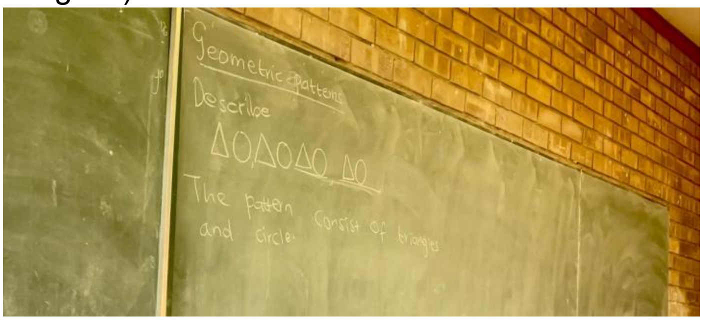


Image..: description of the pattern
Phomelelo: Now Let us do another example in our textbook. Let us look at page 129 there is an example there, but we are only going to talk about activity 1 and I will also write it on the chalkboard (see image…). The first pattern we have ‘***AB AB’*** this is our pattern so what are we going to do when we complete our pattern? Are we going to add colours or objects or add letters?
Learners: We are going to add letters
Phomelelo: Which letters?
Learners: ‘*AB AB AB’*

Phomelelo: Yes! '***AB AB AB ABC ABC'*** (see image…)
Learners: No
Phomelelo: Why are you all saying No?
Learners: Because you have added ***'C'*** in your pattern
Phomelelo: that is very good, and yes, it is because I added ***'C'*** whereas my pattern started as '***AB AB'.*** So, as I have said that when you complete your pattern you should complete it in the same order you do not add or remove anything. Is that clear?
Learners: Yes
Phomelelo: so, the correct answer will be ***'AB AB AB AB'*** (see image..). So if in our first pattern we used shapes and in our second pattern we used letters, is that correct?
Learners: no
Phomelelo: why is it not correct?
Learner 7: because you said if the pattern has shapes, then we will only be completing it by adding the same shapes and if the pattern has letters, then we will only complete it by writing the same letters
Phomelelo: that is very correct

Image..: example from the textbook

Image..: answer given by the teacher

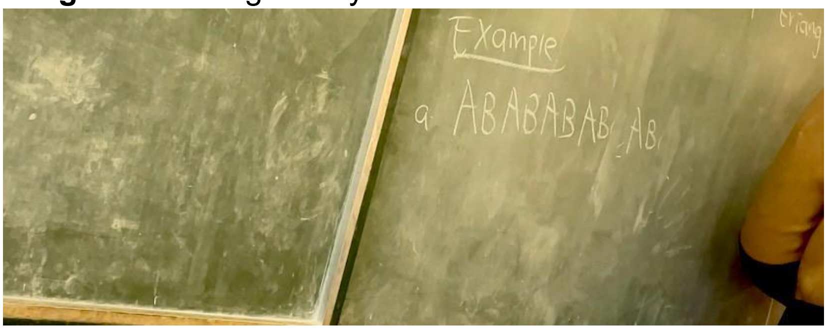


Image..: answer that was corrected by learners

The lesson continued and the following exchange took place:
Phomeleo: so now let’s continue with other examples (she drew a circle followed by a rectangle in pattern 1 and in pattern 2, she drew the very same shapes) so here are my pattern 1 and pattern 2. What will I draw in pattern 3?
Learner 8: we will draw a rectangle and a circle
Phomelelo: is he correct?
Learners: no
Phomelelo: why is he not correct?
Learner 9: because he has started with a rectangle instead of starting with a circle then followed by a rectangle just as you have done in pattern 1 and pattern 2
Phomelelo: wow! That is very good, please clap hands for her
Learners: (claps hands)

Appendix E: Semi-Structure Interview Transcript

Meta (Researcher): *To begin, can you tell me about your teaching experience and your background in mathematics? How has that shaped the way you teach today?*
Tshepisho: *My experience in teaching mathematics has honestly been wonderful. Over the years, I've learned that mathematics requires you to be flexible, because no two classes are ever the same. Each learner brings their own strengths and challenges. My background in mathematics has really helped me gain confidence. I understand the concepts well, so when I explain to the learners, I do so knowing that I can handle whatever questions they bring. That confidence also allows me to try out different methods and adapt my teaching whenever I see that something isn't working. I've grown to realise that teaching mathematics is about patience, creativity, and constant learning.*

Meta (Researcher): *How long have you been teaching mathematics in Grade 4?*
Tshepisho: *I have been teaching Grade 4 mathematics for 30 years now. It's a long time, and I've seen many curriculum changes, different classes, and different challenges, but I still enjoy it. Every year brings something new.*

Meta (Researcher): *What kinds of challenges have you faced when teaching mathematics in rural classrooms, and how do you usually overcome them?*
Tshepisho: *The challenges are mainly around resources, there is always a shortage of teaching aids, manipulatives, and sometimes even basic materials like charts. But I've learned to improvise. I use whatever is available: matchsticks, stones, bottle tops, old cardboard, even learners' bodies sometimes. Another challenge is large class sizes, which makes it difficult to give every child individual attention. To manage that, I group learners so that they can support each other, and I walk around a lot during lessons to check who is struggling. The rural context forces you to be creative all the time.*

Meta (Researcher): *How do you normally introduce the topic of geometric patterns to yourGrade 4 class?*
Tshepisho: *I always start with something they can see. For example, the burglar bars on the windows, the tiles on the floor, the pattern on someone's jersey—anything that is in front of them. I've found that when they see a real example, they quickly understand that patterns are all around us. From there, I introduce the idea of repetition or growth, depending on the type of pattern I want them to learn.*

Meta (Researcher): *What teaching methods and resources do you rely on for teaching geometric patterns?*
Tshepisho: *I like using diagrams because they give learners a clear picture. Real-life designs also work well, I sometimes bring items from home or ask learners to point out patterns they know. Matchsticks are one of my favourite resources because they are simple, cheap, and learners can manipulate them to build their own patterns. I find that when they build patterns by themselves, they remember the concept much better.*

Meta (Researcher): *How do you assess whether learners understand geometric patterns?*

Tshepisho: *I mostly give class activities during the lesson, short tasks where they have to extend a pattern or create a new one. I walk around and check their work. I also give home activities to reinforce what we learned, and the next day I go over them with the class. Sometimes I ask learners to explain their thinking aloud because that helps me see whether they truly understand or are just copying.*

Meta (Researcher): *How do you engage your learners and promote discussion in your mathematics lessons?*
Tshepisho: *Asking questions is my main method. I ask many "why" and "how" questions. I also call learners to the chalkboard to complete or draw patterns. When they draw something, I ask the class to discuss whether it is correct and why. This builds confidence and gets everyone involved.*

Meta (Researcher): *Do you think culture plays a role in learners' understanding of geometric patterns?*
Tshepisho: *Yes, definitely. Our learners grow up seeing patterns in their traditional clothing, crafts, beadwork, and even in the way houses are decorated. Different cultures use different shapes and colours, so when I teach patterns, I sometimes refer to those examples. It helps them connect mathematical patterns to patterns they already know.*

Meta (Researcher): *How do you accommodate learners who struggle with the language of teaching and learning?*
Tshepisho: *I use pictures and diagrams because pictures don't need translation. I also allow learners to code-switch. If they understand the concept better in Sepedi, I let them explain it that way, and then I help them find the correct English mathematical terms. This helps a lot, especially in rural contexts.*

Meta (Researcher): *Do you have support systems for learners who struggle with geometric patterns?*
Tshepisho: *Yes. I usually give them extra activities so that I can see exactly where they are getting stuck. Sometimes I work with them in small groups. I also pair them with learners who are stronger in the topic. Peer support is very effective because learners learn well from each other.*

Meta (Researcher): *Do you attend mathematics workshops to improve your teaching?*
Tshepisho: *Yes, I try to attend every workshop that the district offers. New methods and examples help a lot, especially in areas like patterns where learners need clear and simple explanations.*

Meta (Researcher): *Do you have any ideas to improve the teaching of geometric patterns in rural classrooms?*
Tshepisho: *Yes. I think we should promote the use of recycled materials such as cardboard, bottle tops, and old paper. These can be turned into shapes and patterns. Learners enjoy making their own materials, and it makes the concepts more concrete. Also, teachers in rural areas should be supported with simple teaching kits that include basic shapes, colour cards, and manipulatives*.

Appendix F: Lesson Plan

MATHEMATICS LESSON PLAN
GRADE 4
TERM 3: July – September

PROVINCE:	Limpopo
DISTRICT:	SEKHUKHUNE EAST
SCHOOL:	School B
TEACHER'S NAME:	Ditshegofatso
DATE:	28/07/2025
DURATION:	1 Hour

1. **TOPIC: NUMERIC AND GEOMETRIC PATTERNS:** Geometric patterns(**Lesson 4**)

2. **CONCEPTS & SKILLS TO BE ACHIEVED:**

By the end of the lesson, learners should know and be able to :

- Investigate and extend geometric patterns looking for relationships between numbers, including patterns:
  - limited to sequences involving a constant ratio.
  - of learners' own creation
  - represented in tables
- describe and justify the general rules for observed relationships in own words

Approved

DEPARTMENT OF EDUCATION

28 -07- 2025

Khumalo MR

P.O. BOX 708, LEFALANE, 0741

LIMPOPO PROVINCE

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Page 26 Grade 4 Term 3 and 4 Mathematics Les

Grade: 4 Subject Mathematics

Topic: Exploring geometric patterns Date 04/08/2025.

Subject: Learners should be able to identify and create geometric pattens

Activities

- Explain pattens using shapes, colours and positions
- learners create their own patterns using shapes
- learners draw and describe their patterns to the rest of the class.

Assessment:

Activity 4 (page 130. Vobolous Textbook)
class Activity.

1 Study the patterns and answer the questions that follows.

pattern 1 △▲△

patten 2 △▲▲△

pattern 3 △▲▲▲△

a) Describe how this pattern grows in your own words

b) Draw patterns 4 and 5

Approved: 04/08/2025