EFFECTS OF HEURISTIC-BASED PROBLEM-SOLVING INSTRUCTIONAL APPROACH FOR LEARNING ALGEBRA WITH CONCEPTUAL UNDERSTANDING IN GRADE 8

by

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DECLARATION

In accordance with the rules and regulations of the University of South Africa, I declare that this thesis, except where explicit reference is made to the contribution of others, is the result of my own original work. It has not been previously submitted to any other institution of higher education.

November 2023

Solomon Ketema Gebrie   Date

DEDICATION

This thesis is dedicated to my late wife Azalech Gedefa Rise

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ABSTRACT

In Ethiopia, the content domain in mathematics at elementary school level, covering grades 1-8, consists of four thematic areas. These are numbers, geometry, algebra, probability and data handling. However, learners have difficulty with algebra and solving algebraic problems when compared to other mathematical topics and problems. This study aimed to enhance the development of middle school learners' skills in algebra when responding to mathematics Model-Eliciting Activities (MEAs) drawn from a real-life context. The Models and Modelling Perspective (MMP) and the APOS (Action, Process, Object, Schema) theory were two frames that supported teaching activities in the experimental group. Mainly, MMP informed the design of an effective model-eliciting activity. The latter was used as a guide in designing a heuristic-based instructional sequence conducted in this study explaining the mental structure of learners and the method through which they attain the conception of algebra. To analyse the impact of the heuristic-based problem-solving instruction, 205 learners in four different middle schools in the North Wollo district (in Ethiopia) were selected by means of convenience sampling. The study used both qualitative and quantitative research methods.

The main instruments in this study were, 1) a classroom observation schedule; 2) model-eliciting activities; and, 3) an achievement test. The first and second instruments are qualitative components. The first was used to document teachers' teaching processes in participating classrooms during lesson observations. The second instrument was used to, 1) serve as a medium of interaction between the researcher and learners during the implementation of the heuristic-based problem-solving instruction; 2) assess learners' background knowledge on algebraic variables in their effort to solve the designed MEA; and, 3) assess the learning opportunities created by the designed MEA. The third instrument was the quantitative component used to measure the comparative effects of the heuristic-based instructional method.

The findings from the classroom observations revealed that participating schools employed comparable conventional teaching methods. The implementation of the problem-solving approach of instruction gave insights into how a heuristic-based problem-solving approach of instruction can be developed and used in grade 8 algebra lessons, and the factors that could influence conceptual development of learners in algebra. The results from the qualitative data also revealed that, 1) learners did not build relevant background knowledge in their previous schooling on algebraic variables that enabled them to understand and interpret the situations of the designed MEAs; and, 2) engaging learners in MEAs supports the goals of Science Technology Engineering and Mathematics (STEM) education. The findings from the quantitative component supported the initial hypothesis that participation in the heuristic-based problem-solving instructional method results in improved test scores in algebra.

Keywords

Problem-solving;

Model–eliciting activity

Heuristic-based problem-solving instruction

Models and modelling perspective

APOS theory

Genetic decomposition

Algebra lessons

Mathematics learning

STEM education

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ACRONYMS

ANCOVA:	Analysis of Covariance
APOS:	Action, Process, Object and Schema
COVID-19:	Coronavirus Disease of 2019
CTA:	Conventional Teaching Approach
EEDR:	Ethiopian Education Development Roadmap
HBPSI:	Heuristic–Based Problem-Solving Instruction
MEA:	Model-Eliciting Activity
MMP:	Models and Modelling Perspective
MoE:	Ministry of Education
NCTM:	National Council of Teachers of Mathematics
NEAEA:	National Educational Assessment and Examinations Agency
NLA:	National learning Assessment
SPSS:	Statistical Package for the Social Sciences
STEM:	Science Technology Engineering and Mathematics
TIMMS:	Trends in International Mathematics and Science Study

CHAPTER ONE

THEORETICAL BACKGROUND AND ORENTATION OF THE STUDY

1.1 INTRODUCTION

Teaching mathematics for understanding has caught the attention of mathematics education researchers for the last four decades (Schoenfeld, 2013). Educators in mathematics education have been constantly emphasising mathematical understanding as the most desirable instructional goal (Baker et al., 2022). However, despite four decades of research, several studies have shown that learners' low level of understanding of mathematical concepts and their problem-solving performance remains the main concern worldwide (Cai, 2016; Johansson & Strietholt, 2019). This evidence from the literature shows that a major research focus in mathematics education remains targeted on how mathematics instruction should be designed and taught in a way that supports learners to develop desirable and profound understanding of concepts and procedures (Jackson & Nieman, 2017; Li & Schoenfeld, 2019).

In Ethiopia, the content domains in mathematics at elementary school level, covering grades 1–8, include four thematic areas. These are numbers, algebra, geometry, data handling and chance or probability (Section 2.4.6). It is the researcher's view that algebra is one of the most important areas of school mathematics. In the same vein, researchers in other parts of the world highlight that success in algebra is required in the twenty-first century (e.g., Jupri & Drijvers, 2016; Kieran, Pang, Schifter & Ng, 2016). However, learners have difficulty with algebra and solving algebraic problems when compared to other mathematical topics and problems (Banerjee & Subramaniam, 2012; Jupri & Drijvers, 2016; Jupri, Drijvers & van den Heuvel-Panhuizen, 2014; Kieran et al., 2016). These authors also concluded that many of the difficulties in learning algebra come from learners' low level of understanding of basic concepts, especially that of variable and algebraic expression (Banerjee & Subramaniam, 2012). In line with this view, Lopez-Diaz and Pena (2021) concluded that most difficulties in learning mathematics in later grade levels were predominantly related to learners' low level of understanding of basic concepts such as numbers, formulas and algebraic variables in earlier and middle school.

Most of the evidence from the literature shows that researchers continually look for alternative instructional methods that support the development of learners' understanding of mathematical concepts (e.g., Cai, 2016; Kieran et al., 2016; Li & Schoenfeld, 2019; Rittle-Johnson, Schneider & Star, 2015). The National Council of Teachers of Mathematics (NCTM) recommends that learners should assess concepts in mathematics and construct new knowledge through problem-solving (NCTM, 2004). There is also widespread agreement among educators on the positive effect of teaching mathematics through problem-solving (e.g., English & Sriraman, 2010; French, 2002; McNeil, Uttal, Jarvin & Sternberg, 2009). They maintain that the problem-solving approach of teaching mathematics fosters the understanding of concepts. According to Lesh and Doerr (2003), the process of creating relationships and making connections between basic or fundamental concepts in mathematics by solving problems is one of the most important and fundamental aspects of problem-solving activities. However, Cai (2003) remarked that, "while teaching through problem-solving starts with problems, only worthwhile problems give learners the opportunity both to solidify and extend what they know and to stimulate their learning" (p. 247). Cai (2003) further noted that in order to teach mathematics through problem-solving, much more effort on the part of the teacher and learners must take place. During a classroom mathematics instruction, teachers need to consider issues such as learners' current knowledge of basic concepts, the way in which learners should be helped, selection of problems, and organisation of classroom discourse, among others.

Recent reform efforts and instructional practices in mathematics education have widely propagated the benefits of teaching mathematics through problem-solving by using model-eliciting activities (MEAs) as a class of problems (for examples, see, Aguilar, 2021; Arnon et al., 2014; Deniz & Kurt, 2022; Oferi-Kusi, 2017; see also, Sections 1.6.5, 2.2.3.1 & 2.5.1; Appendix A). According to Aguilar (2021), "MEAs are open-ended client-driven problem-solving activities in which learners working in small groups are encouraged to develop and generate useful solutions or models for a client" (p. 56). An MEA is open-ended as learners in groups may suggest various acceptable or useful solutions to the MEA (Chamberlin & Moon, 2005). It is also client-driven because the solutions are required to be reported to the client, where learners provide a detailed explanation of their model-solution.

One of the MEAs that were utilized in this study or research was "Garden Activity", designed by the researcher as a learner-centered activity (Appendix A). It could be seen as a school principal-driven MEA. In this MEA a school principal wanted to know the number of vegetable beds learners in grades 5 and 8 may be able to cultivate. Also, the principal would like to have a model or rule that enables her to find the number of vegetable beds that could apply for learners in grades 6 and 7. Therefore, the solution to this activity was required to be reported to the principal in a letter-format so that it helped her to consider the number of watering cans needed for garden use. The MEA was open-ended as learners in groups were asked to suggest various solutions to the given task (Section 4.5.3 & Appendix A). This is because the principal did not decide the exact number of learners assigned to a single vegetable bed. The MEA approach is also different from conventional word problems because it requires learners to evaluate the validity of the various solutions produced by the different groups in a whole class setting, and to improve, share and revise solutions produced by each group to find the optimal (satisfactory) solution (see, Lesh & Harel, 2003; see also, Section 2.2.3.1).

To this end, learners worked in a group of four or five on the MEA, consisting of a Garden Activity. The researcher supported the group through explanations to avoid language barriers in the process of understanding the MEA and facilitated the use of their strategies and approaches in order to develop concepts in algebra. Since the problem statement of the MEA did not explicitly refer to concepts of algebraic variables, learners had to interpret the MEA for themselves before any solution model could be developed. Then learners identified which variables they thought were relevant and which of them were not and then formulated, and refined a rule or model using the relevant variables. Learners in the group then developed a mathematical model that enabled them to find a solution. Finally, with the appropriate guidance of the researcher, learners tested and revised the developed model(s) in a whole class setting until they had determined what they believed to be an optimal solution to the Garden Activity and then shared the model that they developed. A group might take 90 - 120 minutes to complete the Garden Activity (Section 4.5.3 & Appendix A).

In this study, the phrase Heuristic-Based Problem-Solving Instruction (HBPSI) has been used to refer to the researcher's intervention or teaching method that largely embraced elements of MEAs designed by the researcher to be implemented in grade 8 mathematics classrooms. Hence, in this study the word heuristic has been conceptualised as referring to self-employed learners' problem-solving strategies or methods that they use to solve learner-centred real-life problems or MEAs (see, Sections 2.1, 2.1.1 & 2.1.2; Appendix A). Different authors have referred to this problem-solving approach as 'mathematical problem-solving heuristic' (Matousek, Dobrovsky & Kudela, 2022; Tan, 2018; TLS, Nurashari & Lie, 2021). Vogel, Kollar, Fischer, Reiss and Ufer (2022) and Dhlamini (2012) have emphasised that heuristic-worked examples and approaches provide a powerful instructional scaffolding mechanism to favourably support the acquisition of mathematical problem-solving skills. The heuristic approach to mathematics problem-solving encourages problem solvers to shift from relying overly on more conventionally prescribed problem-solving modalities, and rather move towards generating self-regulated problem-solving methodologies (Section 4.5.3).

The theoretical framework of this study that guided the HBPSI drew on two theories, namely, the Action, Process, Object, Schema (APOS) theory and the Models and Modelling Perspective (MMP) (Section 2.5.1). Each MEA used in this study was designed by the researcher based on the MMP to serve as a medium of instruction aimed at developing the conceptual understanding of learners in algebra at grade 8 level (Section 2.5). The instructional sequence that this study used was largely informed by the APOS theory (Section 2.5). It was believed that integrating these two theories could provide the researcher with valuable insight in terms of developing learners' meaningful understanding of concepts in the teaching and learning of algebra at grade 8 level.

Given the experimental segment of this study, the HBPSI was implemented by the researcher in the experimental group alongside the Conventional Teaching Approaches (CTA) that was implemented by incumbent teachers in control schools (Section 4.5.2). This kind of a research design gave the researcher the room to impose statistical data analysis procedures to measure and compare the effectiveness of the HBPSI and CTA on the mathematical problem-solving performance of grade 8 learners in the topic of algebra (Sections 4.5.1 & 4.7.2). In the experimental group, observed successes in learners' problem-solving processes were construed as evidence of development of learners' problem-solving skills as a result of integrating and implementing MEAs in the teaching and learning of algebra at grade 8 level (Section 4.6.5). The knowledge and skills gained through the designed instructional sequence for the intervention led the researcher to explain whether there was a significant performance difference in the post-test between learners in the experimental and control groups (Section 4.7.2.2). As a result, this performance difference was considered as evidence of the positive effects of the teaching treatment in comparison to the conventional approach of teaching employed by teachers in the control group (Section 4.7.3).

In addition to the previously mentioned purpose of the study, the analysis and explanation on the data collected also included: Firstly, the extent to which grade 8 learners achieved success in utilising their knowledge of algebraic variables in engaging to solve a mathematical model-eliciting activity. Here the difficulties groups of learners encountered were analyzed by identifying part of the communicative events in the problem-solving process of the designed MEA (Garden Activity) at the action conception stage (Section 4.5.3.1). These were the events where learners attempted to understand and interpret the designed MEA and apply algebraic concepts in the MEA problem-solving processes. To this end, the study used the suggestion made by Ursini and Trigueros (2001) on the knowledge required by learners to understand algebraic variables as unknown quantities. Secondly, the data collected showed the extent to which a mathematical modelling activity creates an opportunity for learning when grade 8 learners fully engage to solve the task. In this regard, the researcher assessed the learning opportunities created by the designed MEA with the aim of identifying from learners' conversational statements, manifested in the transcripts, and their modelling activities indicated on the Garden Activity sheet that possibly exhibited or elicited attributes of a Science, Techology, Engineering and Mathematics (STEM)-educated learner. The data constituted the recordings of communicative events in which group members fully participated and engaged to solve and produce a satisfactory solution to the MEA (Section 4.7). To this end, suggested attributes of STEM-educated learners, as also summarised by Morrison (2006) and Magiera (2013), were used as a tool to analyse and explain the relevant data. Thirdly, the data collected showed how the HBPSI could be implemented by mathematics teachers in the middle school (Section 5.4). The intervention used in this study was expected to produce positive results in terms of learners' knowledge development in learning.

Participants in this study consisted of grade 8 learners in the Amhara[1] region, which is in the north Wollo Province, in Ethiopia (Sections 3.2 & 3.3). The study focused on grade 8 mathematics learners because learning with understanding at the middle school constitutes a foundation for a higher level of understanding and improves learners' performance during the rest of their schooling (NCTM, 2000). Furthermore, the analyses of the data from grade 8 learner surveys in this region have revealed that learners have low achievement in mathematics (Bati, 2020; Ministry of Education [MoE], 2018). To collect data, this study used both qualitative and quantitative methods (Section 3.2). The qualitative research design had three components. The first was classroom observation. It was used to see whether any of the participating classes in the experimental classrooms had an advantage over the participating classes in the control classrooms, with regard to teaching and learning of mathematics (Sections 4.3.1 & 4.4). The second component of the qualitative method was used to explore the conceptual difficulties and understandings of algebraic variables that grade 8 learners have in their attempt to solve the MEA after these concepts would have been learned in their previous schooling from grades 5-8 (Section 4.6). Finally, the third component of the qualitative method was to identify the learning opportunities created by the MEA from the learners' conversational statements that possibly exhibit the attributes of a STEM-educated learner (Section 4.6). In the quantitative analysis, a

1. There are nine regional states in Ethiopia, which are based on ethnic territoriality. Amhara region is one of these regions.

pre- and post-test quasi-experimental design using a *non-equivalent control group design* was employed to measure and compare the effectiveness of HBPSI and CTA on learners' achievements in algebra (Sections 4.7.1, 4.7.2 & 4.7.3).

The participants in this study were 205 learners drawn from four schools. Four grade 8 classrooms; one in each school was selected through convenience sampling procedures. Learners (n = 100) in two classrooms were in the intervention condition and learners (n = 105) in two classrooms were in the control condition (Sections 3.3.1 & 3.3.2). Four grade 8 mathematics teachers in the participating classrooms also participated in the study. In addition, two departmental heads, one in the experimental and the other in the control schools, participated in the study to observe the teaching methods employed by teacher participants prior to the intervention. The four schools, four classrooms, four participant teachers and two department heads were all selected through convenience sampling procedures.

1.2 RATIONALE FOR THE STUDY

According to Deniz and Kurt (2022), an instructional model that uses MEAs in mathematics teaching is a prerequisite for successful and meaningful learning of the content of the formal curriculum and it has been recognised as a useful and innovative approach (see, also, Aguilar, 2021; Baker et al., 2022). In addition, Abassian, Safi, Bush and Bostic (2020) distinguish MEAs from other modelling approaches in mathematics education because of the emphasis on the learning of concepts in mathematics through constructing a model of a mathematical process. Maiorca and Stohlmann (2016) support the conclusion made by Abassian et al. (2020) by stating that MEAs are learner-centred activities in which learners use mathematical modelling to solve complex and real-life problems. Other authors have highlighted the need for an instructional model that uses MEAs in mathematics teaching to equip learners to solve real-life problems, introduce new concepts (English, 2016; Oferi-Kusi, 2017) and facilitate learners' application of mathematical knowledge by applying the learned mathematical concepts (i.e., to make mathematics practical).

The models and modelling perspective, also called model-eliciting perspective (Kaiser & Sriraman, 2006), adapt the goal of problem-solving to encompass the use of mathematical concepts and develop a deeper meaning for those concepts (Lesh & Doerr, 2003). There are recent empirical data showing that an instructional model that use MEAs based on APOS theory improves learners' performance at all levels of schooling (for examples, see, Arnawa, Yerizon, Nita, & Putra, 2019; Borji, Alamolhodaei & Radmehr, 2018; Kamid, Huda, Rohati, Sufri & Iriani, 2021; Oferi-Kusi, 2017). In the context of Ethiopia, there is limited research in the teaching and learning of mathematics based on APOS theory. Two particular studies of importance were conducted by Sebsibe and Feza (2019) and Gemechu, Michael and Atnafu (2020) using college learners as participants. These studies report on learners' difficulties that they demonstrate in understanding the limit of functions (Sebsibe & Feza, 2019) and learners' understanding on domain and range of several variables (Gemechu, Michael & Atnafu, 2020). At the time of conducting this study, it seemed that there was no study in Ethiopia that had been conducted to implement an instructional model using APOS theory.

The government of Ethiopia, through the declaration of the Ministry of Education [MoE] (1994), has called for a problem-solving approach to teaching, aiming to improve problem-solving capacity of the society and individuals for the country's socio-economic development. Furthermore, problem-solving is considered as one of the general objectives in the Ethiopian Education Policy. However, evidence from literature in the context of Ethiopia (e.g., Buishaw & Ayalew, 2013; Dawit, 2006; Diribsa, 2006; Gulfo & Obsa, 2015; Metassebia & Demmiss, 2002; Tesfamicael & Lundeby, 2019) has revealed that mathematics textbooks and the conventional teaching approach emphasise rote learning and memorisation and are less supportive to the development of learners' problem-solving skills. As a result, routine skill practice is the common approach and is overused by teachers and learners in the classroom during mathematics instruction in Ethiopian schools. In the context of this study, a task or problem that can be solved comfortably by familiar facts and procedures (no matter how difficult) is an exercise or a routine mechanical algorithmic task. Ketema (2021) conducted a qualitative study that examined grade 8 learners' understanding of algebraic concepts, namely, variable constructs and sub-constructs, after these concepts were taught using the conventional teaching approach from grades 5-8. The study found that participating learners were not able to identify unknown quantities or variables, which are possibly embedded in open-ended real-life problem situations (model-eliciting activity) in their effort to understand and interpret the designed MEA and apply algebraic concepts in the MEA problem-solving processes (Ketema, 2021). Ketema (2021) concluded that this might be the result of the conventional teaching approach largely employed in learners' previous schooling experiences.

A recent comparative study conducted by Tesfamicael and Lundeby (2019) has shown that more than 90% of the tasks in the topics of algebra, focusing on relations and functions, in the Ethiopian 9^{th} grade mathematics textbook are dominated by exercises that require lower-level cognitive demands (i.e., memorisation and procedures) with no real-life context. These authors suggested that relations and functions are mathematical concepts which are suitable to be represented with different real-life contexts in Ethiopian textbooks. It is against the above background that this study is being promoted. In this study, MEAs were used in order to help learners use the mathematics similar to the mathematics that is needed to solve problems arising in everyday life.

Given this background, this study could highlight the classroom changes from the conventional methods of teaching to instructional approaches that enhance learners' understanding of algebraic concepts and their performance. Conventional methods of teaching mathematics have limited learners' productive engagement in mathematical classroom discourse. On the contrary, using MEAs as a vehicle, the learner-centered approach based on MMP and APOS theory views learners as active participants in their own learning. This study highlighted how heuristic-based intervention could be developed and implemented to teach mathematical concepts. This study could also serve as a catalyst for further investigation on the effectiveness of the MEAs for other grade levels in other domains of mathematics in ways that support the goals of STEM education in mathematics classrooms.

1.3 THE PROBLEM STATEMENT

The desire to improve the learning outcomes in schools is central to many education reforms in both developing and developed countries. The current Ethiopian education system relies on the MoE (1994) education policy that emphasises on problem-solving ability. To further substantiate the education policy, extensive efforts were started under the General Education Quality Improvement Programme (GEQIP) in 2008; GEQIP was established with the aim of improving teaching and learning conditions of primary education, consisting of grades 1-8. However, despite the tremendous progress in expanding access to primary education, poor education quality is a major persisting challenge in Ethiopia (Le Nestour, Moscoviz & Sandefur, 2021). According to Eigbiremolen (2017), efforts to improve learning outcomes in Ethiopia are seemingly silent in reflecting the academic skills that learners were able to acquire during their years in primary school. As mentioned in Section 1.1, despite the importance that should be accorded to algebra, little attention, if any, has been given to the topic of algebra, and less has been reported in the literature in the context of Ethiopia about learners' understandings and the difficulties they experience with algebraic variables. Mainly, this study was conducted to shed light on the role of MEAs as a vehicle for meaningful problem-solving of algebra in grade 8 mathematics classrooms.

With regard to the achievement of learners in primary school, almost all national learning assessment (NLA) processes conducted in Ethiopia for the last two decades have shown that learners' achievement profiles, particularly in mathematics have declined dramatically (Bati, 2020). Specific to mathematics, the trends in the grade 8 achievement across the five NLAs in the years 2000, 2004, 2008, 2012, and 2016 are alarming. The average mathematics scores across the five consecutive NLAs were far less than the expected 50% minimum threshold that had been set by MoE (Bati, 2020) (see, also, Table 1.1). Although there is no data similar to the previous five NLAs after 2016, the researcher believed that the situation in grade 8 learners' achievement during the era of COVID-19 was even worse than the previous NLA's results. A prominent Ethiopian educator at the peak of COVID-19 already expressed concerns that there were pre-COVID-19 inequalities in access to quality education between learners in different locations in Ethiopia (e.g., urban and rural) and different parental economic status (higher and lower economic status) (Tiruneh, 2020). The prominent educator further pointed out that there are signs suggesting that the Corona virus could have a lasting impact on access to quality education and on learners' achievement in Ethiopia (Tiruneh, 2020).

Table 1.1: Grade 8 learners' mathematics achievement at the national level as a percentage

Academic years	2000	2004	2008	2012	2016	2020[2]
Mean scores	38.2	40.9	34.1	25.5	35.2	-

Source: *National Educational Assessment and Examinations Agency [NEAEA] (2016, cited in, Bati, 2020)*

Relative to educational norms that are set internationally, for example, the Trends in International Mathematics and Science Study (TIMSS), about 50% of the Ethiopian learners at the age of 12 fail to reach the low achievement benchmark for learners aged 10 years internationally (Singh, 2014). Bati (2020) noted that the mean achievement score of learners in Ethiopia was found to be significantly lower than the international mean score, like other African countries that participated in the International Large-Scale Assessments (ILSAs).

With regard to curricular concerns which does not provide time to implement instructional approach such as HBPSI, the new educational reform in Ethiopia under the education development roadmap (2018-2030) suggested that "in later phase of the primary education the curriculum should introduce higher order thinking skills through teaching science, technology, engineering and mathematics (STEM) subjects appropriate for the level" (MoE, 2018, p. 18). As a result, the new curriculum included presentation in group, reading assignment, project and problem-based learning and modelling of real-life asks to be used in mathematics classrooms. The new educational reform mitigates the above concern (shortage of time to cover contents) by replacing the crowded curriculum with the new curriculum which could provide an opportunity for teachers to apply MEA-guided intervention in mathematics classrooms in all levels.

2. In 2020 the six NLAs were not conducted for reasons that could possibly be attributable to be COVID-19 pandemic.

With this background, the researcher was able to identify the need to address issues of poor quality (i.e., emphasis on rote skills and memorisation) as well as low achievement in mathematics. One possible way to improve grade 8 learners' problem-solving skills and their performance was an initiative to design an innovative instructional methodology. In this regard, the researcher crafted HBPSI that aimed to deepen learners' understanding of mathematical concepts and enhance performance in algebra. Subsequently, the influence of this intervention on learners' mathematical performance in grade 8 would be measured. To this end, the following were premised as the aim and objectives of the study.

1.4 THE AIM AND OBJECTIVES OF THE STUDY

The aim of this study was to investigate the effect of heuristic-based instruction on the problem-solving performance of grade 8 learners. In order to achieve this aim, the following objectives were outlined:

1.4.1 To highlight the aspects of two theories, the models and modelling perspective (MMP) and the action, process, object and schema (APOS), that possibly influence learners' mathematical problem-solving performance;

1.4.2 To integrate the theories of MMP and APOS to develop the Heuristic-Based Problem-Solving Instruction to teach algebra in grade 8;

1.4.3 To use MMP as an instructional tool to design the model-eliciting-activities (MEAs) to enhance, if possible, the effectiveness of Heuristic-Based Problem-Solving Instruction when teaching the topic of algebra in grade 8;

1.4.4 To identify the difficulties that learners possibly encounter in relation to conceptions of variables in their attempt to solve the designed MEA;

1.4.5 To explain learners' success after they completed the designed MEA in relation to the attributes of a STEM-educated learner;

1.4.6 To compare the influence of heuristic-based problem-solving instruction and Conventional Teaching Approach (CTA) in teaching grade 8 learners problem-solving skills using the MEAs in the topic of algebra; and,

1.4.7 To statistically measure the comparable effects of Heuristic-Based Problem-Solving Instruction and CTA on learners' mathematical problem-solving performance in the topic of algebra in grade 8.

1.5 RESEARCH QUESTIONS

The study posed the following research questions:

1.5.1 What is the design feature of the Heuristic–Based Problem-Solving Instruction (HBPSI) to enhance meaningful learning and accelerated problem-solving performance of grade 8 learners in the topic of algebra?

1.5.2 How can the Heuristic–Based Problem-Solving Instruction be implemented in grade 8 algebra lessons to facilitate learning?

1.5.3 To what extent do grade 8 learners achieve success in utilising their knowledge of algebraic variables to engage with solving a mathematical model-eliciting activity?

1.5.4 To what extent does a mathematical modelling activity create an opportunity for learning when grade 8 learners fully engage to solve the task?

1.5.5 What is the comparative effect of the Heuristic–Based Problem-Solving Instruction (HBPSI) and the Convectional Teaching Approach (CTA) on the mathematical problem-solving performance of grade 8 learners in the topic of algebra?

1.6 KEY TERMS AND THEIR DEFINITIONS

The key terms and their operational definitions used in the study are explained in this section.

1.6.1 Problem

Krulik and Rudnick (1988) defined a mathematical problem as "a situation . . . that requires resolution and for which the individual sees no apparent or obvious means or path to obtain the solution" (p. 3). Schoenfeld (1992) defined a mathematical problem as, (a) a task that arouses learners' interest and engagement to get a resolution; and, (b) a task in which there is no obvious way or solution path. Based on these definitions, a problem in the study is learners' real-life task with no immediate solution and it serves as a medium to impart important mathematical concepts embedded in it.

1.6.2 Problem-solving

In this study solving of problems has been conceptualised as mental activities that require learners to reflect and engage in the processes of determining an answer for the MEA (ask questions, select variables and negotiate mathematical meaning, create or modify useful mathematical models, reflect on the solution process and think about the content of the problem) on the basis of previous experience and knowledge.

1.6.3 Learning

Puustinen and Pulkkinen (2001) described learning as, "the ways in which individuals regulate their own cognitive processes" (p. 269) in problem-solving. Learning in the context of this study is described as the ability of a group of learners to develop algebraic concepts imbedded in the MEA in the process of developing a model that enables a client to solve the problem and other similar problems.

1.6.4 Conventional Teaching Approach (CTA)

Conventional Teaching Approach (CTA) is referred to as the teaching techniques that teachers dominantly used in the control group of this study. In these teaching approaches teachers appeared to rely largely on the textbook examples and solution procedures. In CTA, learners appeared to rely largely on memorising countless formulas and teachers often looked for a single answer using predetermined mathematical algorithms. Furthermore, the majority of learners are passive receivers of ready-made knowledge of mathematics. In short, instruction using this approach can be characterised as teacher dominated, with no active involvement of learners, and no collaborating groups; learners are passive rather than active receivers of knowledge, with no inclusion of real-life context in the teaching process. It was the researcher's view that CTA was devoid of developing and enhancing problem-solving skills in learners.

1.6.5 Model-Eliciting Activity (MEA)

Maiorca and Stohlmann (2016) have stated that MEAs are learner-centred activities in which learners use mathematical modelling to solve complex, real-life problems. In solving MEAs, learners identify which variables they think are relevant and which of them are not and then formulate and refine a rule or model using the relevant variables. Learners in the group then develop a mathematical model that enables them to find a solution. With appropriate guidance, learners test and revise the developed model(s) in a whole class setting until they have determined what they believe to be an optimal solution to a MEA and then share the model that they developed. In this study, to complete an MEA, a group may take 90 - 120 minutes.

1.6.6 Heuristic–Based Problem-Solving Instruction (HBPSI)

Based on learners' day-to-day activities and their understanding of the MEA, the researcher described algebra concepts in grade 8 by encouraging learners to model a solution of the MEA based on the preliminary genetic decomposition (Section 2.4.1). The researcher supported learners through explanation to avoid language barriers in the process of understanding the MEA and facilitated group discussion (ask one another questions and evaluate one another's ideas and reasoning to make sense of the MEA, and share solution models in a whole class setting) to use their strategies and approaches in order to develop the associated algebra concepts in groups.

1.7 CONSTRAINTS AND DELIMITATIONS

It is anticipated that the research design used in the study may raise the issue associated with pre-test challenges that may have impacted on the notion of external validity (see, Section 3.2). Since this research was conducted in school classes, where non-randomised intact or already existing classes were used, assigning learners to a control group and experimental group using random techniques would not have been possible. However, the study used the pre-test session to verify the equivalent educational status of participating schools. Despite the sample in this study representing an appropriate size in relation to the target population, results cannot be extended beyond schools in north Wollo province where the study was conducted. Finally, the intervention in this study was conducted to deal with learners` performance as a result of participating in the HBPSI.

1.8 STRUCTURE OF THE THESIS

Chapter One of this thesis provides the introduction and a theoretical overview of the study. The chapter goes on to present the rationale and the problem statement, aim and objectives, and the research questions of the study. The first section of this chapter reflects on the HBPSI as a teaching tool to enhance learners' acquisition of problem-solving skills, problems and problem-solving processes, as well as problem-solving in the real-world context. The second section provides mathematical modeling, MEAs, collaborative learning through mathematical modeling; and finally, the theoretical framework of the study guiding the teaching treatment is explained.

In the first and second sections of *Chapter Two*, the discussion of the HBPSI together with the discussion of the integration of MEA for the teaching and learning of algebra is presented. The importance and challenges in conducting this innovative teaching approach in different levels of schooling from elementary to higher level is discussed.

The discussion on the importance and challenges in conducting this innovative teaching approach in different levels of schooling from elementary to higher level is presented. An overview of algebra is also given. Lastly, the theoretical framework guiding the HBPSI based on the APOS theory and the models and modelling perspective is presented.

Chapter Three provides a discussion on the research design, research methodology and sampling procedures employed in this study for the purpose of data collection to examine the effect of the intervention for the experimental group in comparison with learners in the control group. A comprehensive description of data collection tools is provided. The data collection tools were lesson observations, achievement test, heuristic-based problem-solving instruction and learners' conversational statements. Finally, the process of data collection and analysis are discussed.

Chapter Four discusses the data analysis and reports on the quantitative and qualitative findings. Firstly, the classroom teaching methods employed in the experimental and control groups are discussed. The discussion on how HBPSI was implemented is described. Then the difficulties learners encountered related to conceptions of variables were analysed by identifying learners' conversational statements dealing with difficulties. Thereafter substantial quantitative evidence based on the pre-test and post-test is presented and the way learners develop understanding of the concept in relation to their participation in the teaching treatment in the experimental group is discussed. Then statistical significance or non-significance on the effect of the intervention based on scores of learners in the test at two different stages is reported.

Chapter Five provides a summary of the study, discussing the research results in relation to the research questions and the theories employed in the study. The study conclusions and recommendations are then presented.

1.9 SUMMARY

The Heuristic-Based Problem-Solving Instruction (HBPSI) has been identified in this study as a viable and effective pedagogical method of improving learners' understanding of concepts in mathematics. Against this backdrop the study investigated the viability of the HBPSI in the learning of algebra in grade 8 in the context of Ethiopia. This study also investigated how one uses model-eliciting activity as an instructional tool and how to convert such knowledge into viable pedagogical practices in the context of Ethiopia. In the light of this, the introduction and background of the study, justification of the study, the problem statement, aim and objectives, research questions, and the structure of the study were briefly discussed.

CHAPTER TWO

PROBLEM-SOLVING AND THE LEARNING OF ALGEBRA

2.1 INTRODUCTION

The purpose of this chapter is to provide a review of relevant and related literature. Firstly, the notions of heuristics and problem-solving are reviewed. This review is followed by a wide-spreading review of literature in mathematical modeling, model-eliciting activities and the learning of algebra. Then this is followed by a discussion on the theoretical framework of the study.

2.2 HEURISTICS

Heuristic, as an adjective, is defined as “serving to discover” (Polya, 1973. p.112; Todd & Gigerenzer, 2000, p. 738) and in the case of problem-solving, discover and invent solutions to problems or discover and invent procedures to solve problems. Heuristics in relation to problem-solving are defined as experientially derived cognitive rules of thumb that serve as guides in problem-solving processes. Different explanations from the literature were offered by authors to the term heuristics: Heuristic methods, heuristic strategies, or simply heuristics. Heuristic strategies are the techniques one could use to solve mathematical tasks (Schoenfeld, 1988). Schoenfeld added that the heuristic approach as a method of teaching through discovery encourages mathematical thought communication among learners. For others, 1) heuristics are rules of thumb for making sense or progress on difficult problems (Polya, 1973); 2) heuristics are general suggestions on strategies that are designed to help when we solve problems (Bruner, 1960); and, 3) heuristics are methods or strategies which offer important clues that aid learners to understand a problem in a better way (Polya, 1973). Recent studies have also described heuristic skills as strategies that help learners to interpret and analyse problems (for examples, see, Gordon, 2021; Vogel et al., 2022).

Heuristic strategies identified in earlier literature include considering extreme cases, finding a pattern, modeling, and logical reasoning (Engel, 1998; Muis, 2004). In a recent study, Gordon (2021) noted that some of the standard heuristic strategies and procedures include “tinkering, describing, taking things apart, reasoning by analogy, trial and error, etc.” (p. 393). Gordon (2021) suggested that helping learners “think in terms of making the problem simpler via heuristic strategies such as tinkering, describing, taking things apart, reasoning by analogy, trial and error, etc. could well provide a fruitful avenue in their problem-solving efforts and mathematics reading” (p. 402).

2.2.1 The use of heuristics as a teaching tool

Generally, heuristic strategies have been identified as a crucial element for problem-solving (Polya, 1973; Schoenfeld, 1985). According to Schoenfeld, “heuristic strategies have become nearly synonymous with mathematical problem-solving” (Schoenfeld, 1985, p. 23). Higgins (1971) has highlighted the following four characteristics of the heuristic teaching technique:

i) Approaches content through problems;

ii) Reflects on problem-solving techniques in the logical construction of instructional procedures;

iii) Demands flexibility for uncertainty and alternative approaches; and

iv) Seeks to maximize learners’ actions and participation in the process of teaching.

Heuristic teaching typically includes one or more heuristic strategies or problem-solving techniques (Stone, 1983) and “aims to lead learners through well-chosen questions to discover facts, information, relationships and principles for themselves” (Butler & Wren, 1960, p. 167). This is supported by empirical data from Higgins’s (1997) study (Section 2.1.2). In recent study, Vogel et al. (2022) emphasised that heuristic-worked examples and approaches provide a powerful scaffolding mechanism to favourably support the acquisition of mathematical problem-solving skills. Gordon (2021) claimed that if an instructional model incorporated heuristic strategies in the teaching of mathematics on a regular basis, “learners would have the opportunity to gain greater insight into how the presented mathematics came to be, and in so doing become more educated and confident practitioners in the art and science of mathematical investigation" (p. 394). Gordon argued that "including heuristic strategies such as tinkering, describing, taking things apart, reasoning by analogy, trial and error, etc. will enable learners to make headway when no particular problem-solving technique is apparent" (p. 402). Gravemeijer (1999) also pointed out that heuristic teaching starts from everyday experiences in meaningful contexts and an understanding of the basic concepts are developed by connecting to these experiences through the use of mathematisation. Based on the suggestions of Gordon (2021), Gravemeijer (1999), Higgins (1971) and Butler and Wren (1960), the implementation of the heuristic-based teaching instruction in this study was conducted.

2.2.2 Empirical studies

Based on using heuristic strategies as teaching tools, a recent state of empirical research findings have shown that learners made use of invented heuristic strategies in solving mathematical tasks (Ketema, 2021; Oferi-Kusi, 2017; Vogel et al., 2022). In a study by Oferi-Kusi (2017), a learner-centred approach that employed modelling as a heuristic strategy and conventional teaching pedagogy was compared in an experimental study with 6^{th} grade learners to determine if there was a performance difference between groups of experimental conditions. Oferi-Kusi showed that there was a significant difference in learners' achievement in favour of learners in the intervention group compared to learners' scores in the conventional group. Ketema (2021) conducted a qualitative study that examined grade 8 learners' understandings of algebraic concepts (variable constructs and sub-constructs) after these concepts were taught using the conventional method of teaching from grades 5–8. The study indicated that participating learners, both in the experimental and control groups, were not able to identify unknown quantities (variables) in the open-ended real-life problem designed by the researcher at the beginning of the intervention. Ketema (2021) concluded that this difficulty may be related to the conventional teaching approaches that mainly focus on the teaching of procedures.

After the intervention (Ketema, 2021), there was a significant difference in learners' achievement in favour of learners in the intervention group who taught by using modelling as a heuristic strategy compared to learners' scores in the control group who taught using the conventional approach. In a study by Jonsson, Norqvist, Liljekvist and Lithner (2014), a learner-centred approach that employed reasoning as a heuristic strategy and conventional teaching pedagogy that employed a procedure-based teaching approach was compared in an experimental study with secondary school learners to determine if there was a performance difference between groups of experimental conditions (Jonsson et al., 2014). They showed that there was a significant difference in learners' achievement in favour of learners in the intervention group compared to learners' scores in the conventional group. Vogel et al. (2022) have found that teaching that employed worked examples and approaches as a heuristic strategy provide a powerful scaffolding mechanism to favourably support the acquisition of mathematical problem-solving skills compared with a conventional approach.

Empirical studies in the early years also reported similar research findings (for examples, see, Derwinger, Neely & Bäckman, 2005; Higgins, 1997). Higgins (1997) found that middle school learners trained in problem-solving techniques had a more positive attitude toward mathematics, and were more persistent in seeking solutions than were learners in more conventional classroom. In a study by Derwinger et al. (2005), elderly that were encouraged to create and practice their own mnemonic training (i.e., finding a pattern as a heuristic strategy) and those provided with a mnemonic strategy were compared. They showed that there was a significant performance difference in the recall task eight month later in favour of those encouraged to create and practise their own mnemonic training compared to those provided with a mnemonic strategy.

2.3 PROBLEM-SOLVING

Scholars in mathematics education developed the models and modelling perspective (MMP) and consider it as an alternate way to view mathematics teaching and learning. In the case of MMP, problem-solving is approached using MEAs. In MMP, problem-solving as an instructional strategy and is approached differently when compared with instructional proceedings in a conventional mathematics classroom (Lesh etal., 2000). In conventional mathematics classrooms, application problems are considered to be more complex and a subset of problem-solving. In the conventional approach, mathematical skills are first learned with no real-life context in the abstract sense. Then learners are taught general strategies of problem-solving and then they might work problems, if time allows them to do so, in a real-life context.

However, there is a shift in the way in which real-life problems are solved in the models and modelling perspective. In this perspective learners are given real-life problems which are worthwhile and meaningful. Then learners are required to develop a deep understanding of concepts in mathematics. In addition, solving problems in a real-life context in this perspective is less difficult compared to solving abstract problems with no real-life context. Based on this argument, problem-solving in the conventional mathematics classrooms is considered as a subset of these worthwhile and meaningful problems (i.e., MEAs) (Lesh et al.,2000, cited in, Maiorca, 2016) as shown in Figure 2.1.

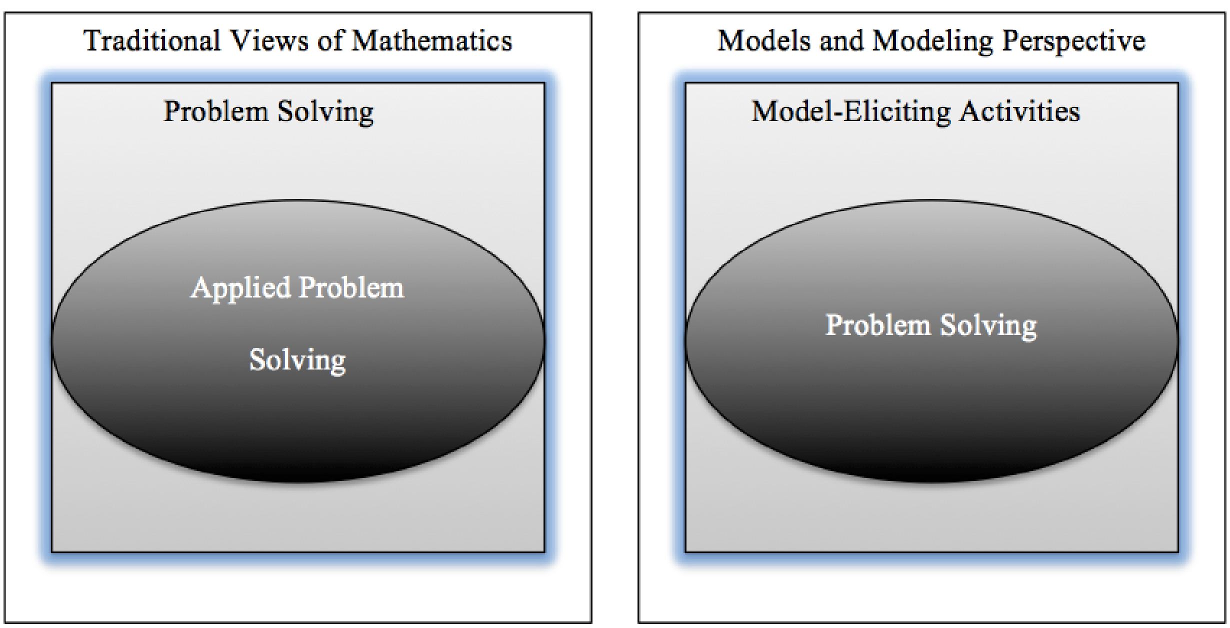


Figure 2.1: Conventional versus models and modelling perspective view of problem-solving

In MEAs, learners make use of the mathematics similar to the mathematics that is needed to solve problems arising in everyday life (Chamberlain, Payne & Kettler, 2019; Lesh et al., 2000; Maiorca, 2016). MEA places learners at the centre of their learning experiences, promoting them to discover, give meaning to their own learning of concepts embedded in the problem, and connect those understandings to other concepts (Chamberlain et al., 2019).

2.3.1 Issues on Teaching Mathematics through Problem-Solving

It is our day-to-day observation that young children, even before they realise what they are doing, are naturally curious, have a desire to make sense of, and invent ways of dealing with their world without interference from others. They are involved especially with quantities and different relationships involving quantities. Authors in earlier times suggested that instructional practices that promote invented strategies to solve problems, turn to help learners to extend their understanding (see, Kamii, 1989; Cai, 2003). Cai (2003) argued that teaching-learning using invented strategy in problem-solving is an approach to find a workable solution to a problem that exists in real life. From this perspective, it can be concluded that learners are capable of inventing problem-solving strategies or mathematical procedures.

Cai (2003) claimed that if learners are capable of inventing strategies out of problem situations (though inefficient) to solve the problem, then they are more likely to demonstrate promising features for understanding the concept in that specific domain. After reviewing earlier empirical studies, Cai (2003) concluded that learners in the elementary and middle schools are capable of inventing their own strategies to solve problems. However, Cai raised important questions such as: “How do learners learn to use invented strategies in the first place before any instruction takes place?” Kamii (1989) argued that children come to school at a younger age with intuitive ways of thinking about and doing mathematics. Cai (2003) further raised other important questions such as: What does teaching through problem-solving look like? How are appropriate mathematical problems selected? How is classroom discourse organized to appropriately guide learners engaging in the mathematical problem? This study considered the issues raised by Cai (2003) in the implementation of heuristic teaching instruction.

2.3.2 Problem-Solving Processes in Mathematics

Krulik and Rudnick (1988) defined a mathematical problem as "a situation . . . that requires resolution and for which the individual sees no apparent or obvious means or path to obtaining the solution" (p. 3). Schoenfeld (1992) defined a mathematical problem by stating that for any learner a mathematical problem is a task 1) in which the learner is interested and engaged and for which she wishes to obtain a resolution, and 2) for which the learner does not have a readily accessible mathematical means by which to achieve that resolution. A mathematical problem can be considered as an effective starting point of problem-based approach that entails thinking. Law (2008) contended that thinking takes place when a learner confronts a complex problem with a belief in himself/herself to solve the task. According to Lesh et al. (2000), the process of creating relationships and making connections between basic (fundamental) concepts in mathematics by solving problems is one of the most important aspects of problem-solving activities. He maintained that learning mathematics through problem- solving facilitates the development of learners' ability to solve applied problems successfully. Young was the first who explicated a method of problem-solving which is similar to recent heuristic methods. Young (1924) claimed that the execution of the action in solving mathematical problems comprised four phases, none of which on its own was the best:

1. *Grasping the problem* – Identifying the known and required quantities successfully.
2. *Work plan* - Deciding the steps required to arrive at the solution. If necessary, the first plan can be improved through time. But the problem solver should make an intelligent plan.
3. *Carrying out of the plan* – Test the plan in order to determine whether the plan will not lead to correct answer (in which case he tries another), or the result is attained.
4. *Testing the results* - Check the solution with the data of the first phase, and make certain that one has really done what he set out to do. (1924, p. 208, cited in, Higgins, 1997).

Polya's (1957) four well used and documented phases of the problem-solving process are, (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and, (4) looking back

(p. xvi-xvii). These four phases shape the basis of the learning of solving mathematics problems apparent in school mathematics curricula and their use in schools in many countries today. Even though George Polya is called the "father of mathematical problem-solving," the connection between Young's and Polya's four steps is clear. That is, the similarities far outweigh their differences.

The five-phase word problem-solving method developed by Krulik and Rudnick (1980) consists of, (1) reading; (2) exploring; (3) selecting a strategy; (4) solving the problem; and, (5) reviewing and extending. In the first phase the problem solver needs to see what the problem is asking for, or needs to restate the problem in his/her language to promote easy understanding. The second phase is essentially a crucial step in which the learner identifies what the problem is and symbolises it in such a way that it is easy to understand. The learner ask himself, "What is the problem like?" He/she is connecting the previous knowledge with the current and looking for patterns or is attempting to determine the concept or principle embedded in the problem. The learner might sketch a diagram or provide a description of the situation. Based on the result that could have been obtained in steps one and two, one draws a conclusion or makes a hypothesis about the way to solve the problem in the third phase. Once the method has been selected, the learner applies it to the problem in the fourth phase. In the final phase the learner verifies his or her answer and looks for variations in the method of solving the problem.

The problem-solving process developed by Mayer (1985) consists of, 1) problem translation; 2) problem integration; 3) solution planning; and, 4) solution execution. Problem translation requires the learner to read and understand the problem. Problem integration takes place when the learner creates a visual representation of the problem using schematic information. Then the learner hypothesises or makes a plan in order to solve the task and estimates a reasonable answer during the solution planning stage. Finally, solution execution occurs when the learner computes or does the arithmetic to obtain the final answer. The problem-solving process developed by Blum and Leiss (2007) consists of, (1) understanding; (2) structuring; (3) mathematising; (4) working mathematically; (5) interpreting; and, (6) validating. In the first stage the learner forms an initial idea about what is required as a final result of the problem. In the second stage the learner creates an idealised view of the problem. In the third stage the learner performs mathematical analysis, that is, the learner performs manipulations. In the fourth stage the learner may re-contextualize the mathematical result, that is, the learner explains the solutions based on the contextual situation of the task. In the last stage the learner verifies results against constraints by explaining the repeatability of the solution.

There are also other problem-solving processes developed by prominent scholars in the field of mathematics. It is important to note that each stage in each of these models involves a different combination of mathematical skills and different cognitive abilities. The problem- solving process developed by Suharnan (2005) comprises, (1) identification of the problem; (2) defining the problem through the thinking process about the problem and selecting the solution path; (3) exploring the possible solution and doing verification from several perspectives; (4) strategy implementation; and, (5) revision and evaluation of the result in relation to the strategy employed. According to Krulick and Rudnick (1996), problem-solving in mathematics encompasses five steps, namely, (1) reading and thinking; (2) analysing and planning; (3) organising strategy; (4) getting the answer; and, (5) confirming the answer. A three-phase problem-solving process consists of, (1) reading and understanding the problem; (2) organizing strategy and solving problem; and, (3) confirming of the answer and process.

According to Montague (2002), the cognitive processes associated with problem-solving encompass seven steps, namely, (1) comprehend text in the problem; (2) translate text to the language of mathematics; (3) identify the relationship in the problem situation; 4) devise a plan to solve the problem; (5) predict the solution; (6) regulate solution path as it is performed; and, (7) detect and correct mistakes in solving the problem. Nevertheless, there are insignificant gaps in the strategies that could be used during the solving of non-routine tasks. The demands that almost all problem-solving models place on the problem solver can be summarised as follows: elucidate the purpose to be attained in solving a given task at hand, outline a plan of action; perform the plan and revisit if needed; evaluate the processes after completion and verify the appropriateness of the result achieved (Ellerton & Clements, 1991). In this regard, Lester (2013) has summarised the demands on the problem solver as follows: translating and interpreting; planning, selecting strategies; identifying sub-goals; creating appropriate representations, and verifying that a solution has been found.

2.3.2.1 Overcoming problem-solving barriers through heuristic method of teaching

Lee and Heyworth (2000) identified four problem complexity factors that affect the cognitive difficulty in a mathematics problem; namely,

1. Perceived number of difficult steps; this reflected whether learners would encounter any difficulties in the problem-solving process. Difficult steps in the process were assumed to be those at which learners usually made non-trivial errors.
2. Number of steps required in solving the problem; this factor was used in relation to the number of steps that an expert would require to finish a problem.
3. Number of operations in the problem expression; determined by simply counting the number of operations (i.e., addition, subtraction, multiplication, division, and exponentiation) in the problem; and
4. Learners' degree of familiarity with the problem; familiar problems were considered as easy and unfamiliar problems were complex.

Researchers have identified factors that determine the level of difficulty of a mathematical task. For instance, according to Suharnan (2005), there are four factors that can influence the level of difficulty of a problem, namely, (1) problem understanding; (2) mental representation; (3) the coverage of problem; and, (4) problem imbalance. The findings conducted by Fasasi (2015) on the effect of heuristic strategies in the mathematics learning and teaching process corroborated the views that the heuristic teaching approach is an effective method of instruction to overcome learners' difficulties, as it is aimed at utilising the active, creative and constructive tendencies of the learner. Researchers have extensively investigated the use of heuristic strategies in the process of mathematics problem-solving (for examples, see, Adeleke, 2007; Brehovsky, Eisenmann, Ondrusova, Pribyl & Novotna, 2013; Camera, 2016; English & Kirshner, 2015; Ezeugo & Agwagah, 2000; Fasasi, 2015; Kolawole, 2007; Obodo, 1997; Odili, 2006). Furthermore, researchers have pointed out that the heuristic teaching approach benefitted learners in overcoming problem-solving barriers as they acquired a real understanding and clear notion of the concept (*Ibid*). This study hypothesised that contextual real-life problems that are presented in learners' daily life-experience can help them to develop their problem-solving skills.

2.3.2.2 Real-world problem-solving

The word 'realistic', as defined in the Oxford English Dictionary, means, representing things in a way that is accurate and true to life. Beswick (2012) has described the use of terms such as *authentic*, *real life* and *situated* as implying, "different degrees of distinctiveness from problem presentations that rely entirely on mathematical symbols" (p. 368). The issue of problems that encourage the development of learners' skills in applying and connecting classroom mathematics instruction to real world mathematics has been receiving much attention by mathematics education scholars and various reform curricula in many nations since the 1990s (Ball, 1995; Beswick, 2005; Leonard & Austin, 2012). Theoretically, tasks are pivotal to learners' learning because "they convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p.24). Mathematical tasks can shape the way learners think about mathematics and, thus, affect their learning. They can serve to hinder or improve interpretation of learners about the concept in the domain of mathematics (Henningsen & Stein, 1997). The NCTM (2000) emphasises the significance of using valuable tasks to intellectually engage and challenge learners. It states that regardless of the context, valuable tasks should have characteristics such as intriguing; with a level of challenge that invites speculation and hard work.

Hammer (1997) has advocated the use of real-world contextual tasks for good reasons, indicating that, (1) the realness of such tasks results in a positive return in building aware individuals who can contribute to society as a whole; and (2) such tasks help to organise concepts in specific mathematics domains in an appropriate manner, deepen understanding of mathematics, and learners learn by comprehending. Furthermore, the use of realistic contexts of tasks in mathematics creates an opportunity for easier cooperation among peers, is easier with different level of competence, creates an opportunity to restructure their older schema and provide an opportunity to learners in developing effective strategies that are suitable for solving a contextual real-life task (Trafton & Andrews, 2002). Trafton and Andrews (2002) further stated that tasks with real contexts are the basis for progressive mathematisation and through mathematising, learners could develop context-specific solution strategies from realistic situations. Trafton and Andrews (2002) have described a task that is grounded in well-designed real-life contexts as having one key feature: its role to stimulate and engage learners in mathematics learning.

Notwithstanding the growing importance for using real-life problems in mathematics education, they should be conducted with caution. On the one hand, applying problems that have a restricted and non-natural context is still the centre of mathematics instruction in many classrooms (Stylianides & Stylianides, 2007). That is, the nature of tasks in textbooks and their implementation in the classrooms is superficial; and such emphasis revolves around the process of "memorising" the mathematics concept. On the other hand, we need to make sure that the motivational features in a cognitively high-level real-life task should not overshadow the mathematics involved in the task (Stylianides & Stylianides, 2007). Watkins, Carnell and Lodge (2007) highlighted that learning is an active process in which the learner relates new information and experience to their real-life experience and may accommodate and assimilate new ideas. Through assimilation and accommodation, learners may adapt to the environment. Accordingly, Piaget (1970) defined intelligence in terms of effective interaction between assimilation and accommodation. In terms of problem-solving, assimilation would help the learner to absorb a mathematical concept and then filters and interprets it in terms of their existing knowledge. In accommodation, the learner uses this understanding in order to solve new problems.

To this end, one of the principal activities in the teaching-learning of mathematics is selection of tasks or problem situations that learners can assimilate in order to solve the task (Simon & Tzur, 2004). According to Piaget (1970), "to know is to assimilate reality. … knowing an object does not mean copying it - it means acting upon it" (p. 15). Learners therefore need to manipulate (e.g. with concrete materials). Baroody and Ginsburg (1990, p. 55) have corroborated Piaget's view by arguing that a learner does not merely imitate new concepts; instead, he/she assimilate the concept. This means that, the learner applies pre-existing knowledge to organise the new concept and react accordingly. As far as theoretical and empirical studies are concerned, research on the teaching-learning of mathematics investigates the practice of activating learners' real-life experiences (McNeil et al., 2009). Theoretically, real-life context-based mathematics instruction places learners at the centre of their learning experiences, promoting them to discover, give meaning to their own learning of concepts, and connect those understandings to other concepts (National Science Teacher Association, 2002). By considering learners' real-life knowledge around them as the starting point to introduce concepts, context-based instruction gives learners tasks with a real-world context to reach a level that allows higher-order thinking skills.

Based on situated cognition, learning is viewed as being situative, which means knowledge acquisition is tied to a specific context and situation (Boaler, 1993). Boaler (1993) went further to explain that mathematics instruction needs to be embedded in real-life contexts for various reasons. Among others, such instruction helps learners to see mathematical concepts in relation to their daily life experiences which can be used as a motivational tool towards mathematical problem-solving. Therefore, mathematics instruction in schools must be situated in real-world tasks familiar to learners to help them to transfer their knowledge to similar situations. From the cognitive theory perspective, real-life context-based mathematics instruction that serves as a catalyst for employing strategies that are efficient should facilitate performance and reduce working memory load (Sweller, 1988). In the same vein, Dhlamini (2012) pointed out that real-life contexts, if integrated appropriately, could provide an opportunity for teachers to connect mathematics instruction to learners' experiences. A learner may lose interest if the teaching learning process does not help them to comprehend the concept during instruction.

Empirically, several researchers have shown that solving problems in a real-world mathematical setting improves learners learning (e.g., improves mathematical reasoning). Boaler (1993) contended that using authentic real-life situations (e.g., world-wide, local, and individual examples) which learners are familiar with that they can analyse and interpret is thought to bring mathematics as a tool to understand reality. Sternberg and Grigorenko (2002) pointed out that, designing tasks in an authentic contextual setting and exposing learners to these problems not only help them understand the problem and arrive at a correct solution but also raise their motivation and interest in mathematics. The findings of the study conducted by Cheng (2013) confirmed that children were highly motivated to solve a problem that was familiar to their daily lives, for instance, a restaurant problem (Cheng, 2013), unlike the standard practice exercises or word problems drawn from the textbook. At the same time, Dhlamini (2012) indicated that designing a problem-solving instruction in a real-life context helps to improve the problem-solving skills of learners.

In Zevenbergen, Sullivan and Mousley's (2002) study the researchers, based on their direct experiences; reflect on the role of contexts in mathematics. These researchers have suggested that context-based mathematics instruction can enhance learners learning. The findings of the study conducted by Edwards and Edwards (2017) showed that the learners who are assigned to teachers participating in a professional development programme, which included working on tasks in authentic contexts, outperformed learners taught by teachers in the control group who did not participate in developmental programmes. Learning mathematics through solving real-world problems and immersion in rich and focused mathematics content helps teachers in their teaching and contributes in positive ways to learner schooling. In a research study by Ben-Chaim, Fey, Fitzgerald, Benedetto and Miller (1998), grade 7 learners who were encouraged to construct their own conceptual and procedural knowledge of proportionality through collaborative problem-solving activity performed better than learners with more conventional teacher-directed instructional experiences.

In recent studies, Bonotto (2011) pointed out that the process of bringing lived experiences of learners into mathematics instruction gives learners the opportunity to make connections to improve mathematical knowledge and deepen their understanding. This idea corroborates

Dindyal's (2009) proposition that exploring real-life mathematical problems helps to make mathematics more meaningful to learners and improves their understanding in other mathematics related deeper skills.

Other inconsistent and contradictory research findings documented the poor performance of learners in word problems set within a real context when compared to learners who performed the same problems in symbolic form (Baranes, Perry & Stigler, 1989). Other research studies have raised doubts over the effectiveness of context in solving word problems when drawing on learners' real-world knowledge (Verschaffel, Greer & De Corte, 2000). The results of these studies indicated that learners generally did not perform well (Verschaffel, Greer & De Corte, 2000). Sethole (2004) argued that focusing on procedural knowledge in mathematics instruction and the foregrounding of context may result in a loss of focus on the conceptual development of mathematical knowledge and make mathematical concepts invisible. Given the reports of these multiple studies on the use of real-life contexts for teaching mathematics, it is not easy to provide a general and conclusive answer on its effects. As a result, researchers have begun to look closely at how situations can be arranged in order to support learners to describe from their experience in the world around them when solving mathematical problems.

Various research studies have suggested concrete objects as a useful teaching tool. Several research studies have been conducted on the use of concrete objects in line with the goal of assisting learners to better understand mathematical concepts (Björklund, 2013; Burns, 2011; Driscoll, 1983; Englert, Raphael & Anderson, 1992; Freer, 2006; Moyer & Jones, 2004; Swan & Marshall, 2010). For instance, Golafshani (2013) stated that the fact that there are learners with different prior knowledge and with different needs in the same mathematics classroom means that teachers should focus on different strategies that are effective for these various groups in order to satisfy the various needs. Therefore, educators should consider interventions in mathematics instruction in order to help learners achieve higher academic success. McNeil et al. (2009), for instance, suggested that exposing learners to different learning materials such as real objects that reinforce real situations as interpreted in the contextual task activate learners' real-world knowledge when they solve word problems. McNeil et al. (2009) further explained that teachers can use actual money as a clue to help learners if a problem describes dividing money among people. McNeil et al. (2009) pointed out that the use of concrete objects as learning materials provide an opportunity for a learner to work to the best of his/ her ability by activating their real-world knowledge. From this it is possible to posit that one can use the actual chalk box as a learning material if situation of a problem describes finding the area of each face of a chalk box. In this way it is believed that a learner who is exposed to a chalk box and asked to explain about the number of faces of the box is expected to have better understanding of faces of any rectangular solid than those who have not been exposed to any related concrete object.

Several research studies on the teaching-learning of mathematics have shown that using concrete objects with elementary, middle, and secondary school learners resulted in an improved learning outcome (for examples, see, Balka, 1993; Bjorklund, 2013; Boggan, Harper & Whitmire, 2010; Bullmaster, 2013; Burns, 2011; Cope, 2015; Driscoll ,1983; Freer, 2006; Gauthier, Tarr & Bubb, 2010; Kontas, 2016; Laski, Jor'dan, Daou, & Murray, 2015; Liggett, 2017; Raphael & Wahlstrom ,1989). Therefore, there are good reasons why learners improve their performance when mathematical problems are presented with concrete objects rather than when they are presented without concrete objects (Hiebert & Wearne, 1993). McNeil et al. (2009) corroborated this view by explaining that, to approach a mathematical modelling problem using concrete objects assists in evoking the real world knowledge of learners and improving their performance. Contradictory research findings have documented the effect of concrete materials in assisting the performance of learners in solving mathematical tasks. A considerable number of researchers maintained that concrete objects that are highly realistic have little effect on improving the learning of mathematics. (Boakes, 2009; Enki, 2014; McClung, 1998; White & Dauksas, 2012).

Given the above dichotomous viewpoints, it is difficult to conclude that the teaching of mathematics with concrete objects will result in improving learning (Liggett, 2017). One promising approach in researcher literature involved mathematical problems in a real-world context where learners "forge a connection between whatever we were talking about in class and what went on in the lives of the individual members" (Tompkins, 1990, p. 658), and developed a deep understanding of the content (see, also, Springer, Stanne & Donovan, 1999). Learners thus share a sense of purpose and orientation, and an apparent set of roles (Reagan, Fox & Bleich, 1994) and become part of a community of people where vast real-life experiences of learners is the starting point for learning mathematics in the classroom. A key role for teachers lies in helping learners prepare for and embrace a change in such a teaching-learning approach, to tap into the engagement concept and spark some of the interest and motivation to learn in a difficult task, for an example, non-routine mathematics problem (Samson, 2015; Springer et al., 1999; Tompkins, 1990). By making use of modelling activities, mathematical modelling was used as a process-oriented problem-solving method in this study. Learners were required to talk and discuss in groups about real-life modelling activities by imagining the concrete objects involved in the activities.

2.3.3 Mathematical modelling

Pollak (1979) was among the first who described the mathematical modelling process in a way that could be used as a tool to promote mathematics instruction. Since then several scholars have made a great effort to analyse the process of mathematical modelling (for examples, see, Blomhoj & Jensen, 2006; Blum & Leiss, 2007; Blum & Ferri, 2009; Doerr & English, 2003). Although researchers in the mathematics education community share the idea that modelling in mathematics involves tasks in a real-life context (Kaur & Dindyal, 2009), however, they have offered different definitions for it (e.g., Chinnappan & Thomas, 2003; Daher & Shahbari, 2015; Fox & Surtees, 2010; Lingefjard, 2005). More recently, Daher and Shahbari (2015) have described mathematical modelling as the process of representing real-world situations using mathematics as a way to understand and solve a specified problem. Fox and Surtees (2010) explained that modelling requires the transition of mathematical knowledge acquired through learning to real-world mathematics embedded in a real-life problem situation. As a result, a learner attempting to solve a modelling activity should examine the relation between school mathematics and real-world mathematics using his/her developed model. Chinnappan and Thomas (2003) interpreted mathematical modelling as something that helps to motivate, develop, and illustrate the relevance of a particular mathematical content.

Mathematics instruction through models in mathematics presents learners with problems that describe real-life situations that require critical thinking skills. According to English (2009), mathematical modelling serves as a tool to take the learner beyond the conventional form of problem-solving taking place in today's classrooms. In contrast to solving a problem in the usual or conventional approach of teaching, modelling problems have the important mathematical constructs and relationships embedded within the problem context and learners elicit these as they work through the problem (English, 2009). As more precisely stated by Blum and colleagues, the purpose of mathematical modelling is to elevate the gap between reasoning about a situation in real-life and reasoning in the teaching learning of mathematics in the classroom (Blum, Galbraith, Henn, & Niss, 2007). The authors explained that mathematical modelling has been used as a fundamental tool to develop learners' competencies. As a result, they stressed the need for the integration of mathematical modelling into the mathematics curricula. By recognising the importance of mathematical modeling, the NCTM (2000) has stated that, "one of the most powerful uses of mathematics is the mathematical modelling of phenomena. Learners at all levels should have an opportunity to model a wide variety of phenomena mathematically in ways that are appropriate to their level" (p. 39). The importance of the modelling process in solving problems is to provide a link between school mathematics and real-world mathematics outside the school (Levy, 2015).

Whilst it is acknowledged that mathematical modelling is an essential part in every mathematics classroom, its implementation is not simple and easy, even for the most experienced teacher (Galbraith & Stillman, 2006). Mathematics instruction, through a modelling approach, requires learners to engage with a cognitively demanding task. Effective implementation of mathematics instruction through modelling requires teachers to take a considerable amount of time and preparation for appropriate scaffolding to avoid barriers encountered by learners (Galbraith & Stillman, 2006). During the modelling process in the classroom, learners in small groups engage in various activities that involve reporting and explaining the activities that provide them the opportunity of developing their communication skills (English & Walters, 2005). In addition, as a result of the modelling process, learning occurs in a social and individual context where learners learn by working together, which allows them to verbalise their thinking. Moreover, it also provides learners the opportunity for peer assessment, that is, assessing each other's work and as well as their own, and this form of learning takes place in group tasks. English and a colleague have affirmed that modelling tasks engage primary and middle school young learners in constructive dialogue and debate with their peers as they share ideas, question one another's claims, justify and refute arguments, and resolve conflicts (English & Walters, 2005).

Given the importance of modelling competencies in today's world, educators in the mathematics education community have used mathematical modelling as a vehicle for teaching mathematics (Lester & Cai, 2016), constructing new knowledge through non-routine mathematical tasks, motivating learners in order to capture their interest towards mathematics (Stanic & Kilpatrick, 1988), and transferring knowledge to other situations (Kaiser & Schwarz, 2010). Modelling is increasingly recognised as a powerful tool to promote the learning of mathematics to elementary and middle school learners. It helps learners understand various concepts and helps them understand the applicability of their learning in their day-to-day activities (Erchul, Grissom, & Getty, 2008). Specifically, mathematical modelling has been taken as a fundamental tool for mathematics instruction at the junior and elementary school level. It is a tool to develop learners' competencies in mathematics. Researchers have emphasised integrating modelling in mathematics curricula (Blum et al., 2007). According to Pollak (1979), at the earlier grade levels mathematical modelling is used to restore meaningful learning that increases learners' confidence and motivation towards mathematics (see, a.so, Burkhardt, 2006). Instructional programmes at all levels of schooling should enable learners to, "use representations to model and interpret physical, social, and mathematical phenomena" (NCTM, 2000, p. 67). Therefore, it is reasonable to argue for the teaching and learning of mathematical concepts through a modelling approach that is firmly grounded on solving mathematical tasks that are realistic and meaningful to learners' lives.

2.3.3.1 Modeling-eliciting activities (MEAs)

A team of educators in the 1970s developed MEAs aiming to mimic problems in a real-world context in science fields such as engineering and others (Lesh et al., 2000). The four components of an MEA, namely, an article, readiness questions, data, and problem-solving task, encourages learners to solve the MEA through a sequence of the following steps: (a) Learners start with an opening article; (b) Learners respond to MEA to assist them become familiar with the world around them and the situations of the MEA; (c) Learners work in small groups in solving the MEA; (d) Learners present their work in groups to the whole class; and (e) Learners in their groups revise and reflect on their model (Stohlmann, 2017). According to Stohlmann (2017), with regard to MEAs learners are confronted with a mathematical problem in real-life situations familiar to learners where the path to the solution is not immediately obvious. In attempting to produce a model consisting of conceptual structures, learners should go through a cycle of expressing, testing, and refining or revising their current ways of thinking in order to solve the given problem.

A model-eliciting activity is, "a problem-solving activity constructed using specific principles of instructional design in which learners make sense of meaningful situations and invent, extend, and refine their own mathematical constructs" (Kaiser & Sriraman, 2006, p. 306). MEA is a complex problem involving real situations. In order to deal with an MEA, learners should be involved in cognitive activities that elicit various responses such as comprehending and understanding the MEA, interpreting the situation of the MEA, developing strategies to develop a model that enables them to solve the MEA so that they develop understanding of the concepts in mathematics that are embedded in the MEA and deepen the understanding of concepts in mathematics (Lesh & Doerr, 2003). Learners in a small group have various shared responsibilities and interact to construct a model (English & Walters, 2005). Then learners are required to test and iteratively revise the model with the given data in the MEA (Lesh & Harel, 2003).

In the last four decades researchers in the mathematics education community have emphasised the role and significance of modelling activities in learners' attempts to solve problems and their attitude towards mathematics, problem-solving and reasoning, as well as communication

(Ercikan, McCreith & Lapointe, 2005). Researchers identified a particular benefit, among others, to learners during the process of solving modelling tasks: learners see the need for mathematisation (i.e., representation, quantification, analysis, categorisation, construction, and organisation) and the need to use two or more mathematical concepts as a whole (Lesh & Doerr, 2003; Lesh & Zawojewski, 2007). As long as learners engage in multiple modelling cycles that involve analysing information in the situation of the given MEA, testing the correctness of the solution of the MEA, refining and exploring concepts, and revising their current ways of thinking gradually increases, they expand or rebuild their existing knowledge and understanding of mathematical concepts (English, 2006). In the modelling process of solving problems, the teacher guides learners in a holistic approach that allows a focus on developing a conceptual understanding of learners in mathematics and their communication skills. Prominent researchers have pointed out that to gain insight into mathematics instruction through solving problems that are applicable in other related disciplines, learning through mathematical modelling is an acceptable approach in many countries around the world (English & Sriraman, 2010). As a result, learners' competencies improve to solve problems in a real-life context and develop their understanding of concepts in mathematics (English & Sriraman, 2010).

According to Chamberlin and Coxbill (2012), MEAs may help to, (1) analyse how the thinking process of learners takes place mathematically; (2) provide various strategies to learners in order to solve tasks in mathematics; (3) identify cognitive and meta-cognitive resources of learners; and, (4) identify and encourage gifted learners to develop their creativity. Specifically, one of the fundamental purposes of modelling is to reveal the way of thinking that is at play during problem-solving, especially the way learners mathematise the situation. Therefore, MEAs are taken as a useful tool that is helpful for assessment and instruction (Lesh et al., 2000). With regard to the assessment and cognitive aspect, MEAs, which are designed for a small group, provide at least two opportunities for learners' social development. Firstly, during the modelling process, learners share the responsibility for the construction of the mathematical model. Secondly, learners argue and explain their position to their peers, which in turn gives them the opportunities to develop their argumentation and communication skills (English, 2003).

2.3.3.2 Collaborative learning through mathematical modelling

From a theoretical perspective, when learners work with peers to solve MEAs, each learner may control his/ her problem execution through explaining his/ her way of thinking to group members, or an idea given by other group members (Lesh et al., 2000). From the point of view of methodology, it may also be easy to evaluate the way learners mathematise the situation during modelling with group members (Lesh et al., 2000). Therefore, as mentioned in Section 2.3.4.1, MEAs can be considered as a tool for assessment and instruction for learners (Lesh et al., 2000). Empirically learners comprehend ideas better if they share and communicate in a group during the learning process (Roschelle & Teasley, 1995). Several researches have shown that when learners in a group engage in mathematical modeling, their communication, problem-solving skills and achievement improved. In this regard, Roschelle and Teasley (1995) have defined collaboration as, “a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain a shared conception of the problem” (p. 235). Communication of learners in relation to their potential helps to add value to the shared understanding and solution of the problem (Roschelle & Teasley, 1995). In line with this, Goos, Galbraith and Renshaw (2002), pointed out that collaboration amongst groups of learners occur if learners explore their ways of thinking along with other group member’s ideas to construct a shared detailed response and understanding of MEA. The authors have further explained that the mathematical model proposed by learners to the given MEA interpreted in mathematical symbols to peers in the group as well as learners within the classroom.

Learning through mathematical modelling and collaborative learning present an ideal instructional model to promote communication and teamwork. Therefore, such an instructional approach makes the learning of mathematics meaningful and may help learners to develop productive learning skills in other related subjects. Also, this instructional approach provides learners with an opportunity to give meaning to their learning and promote useful discussion and dialogue among peers. MEAs allow learners to explore collaboratively. Many studies have shown that a key feature of MEAs is collaborative problem-solving in small groups as they all allow learners to explore real-life problems (see, Webb, 2008). When learners engage in MEAs, they are empowered to take responsibility for shared learning and each member becomes an active participant. MEAs also encourage peer participation and communication amongst peers which in turn provides learners with an opportunity to experience peer assessment, that is, to assess each other's work and their own work. Communication is a fundamental component in the processes of solving a modelling activity; as it contributes to a learner's social and mathematical development.

2.3.3.3 Challenges in implementing mathematical modelling with middle school learners

Although several research studies have concluded that mathematical modelling and MEAs have shown positive results with middle school learners, their difficulties are also well documented. One problem is scarcity of MEAs. Researchers have pointed out that there is a need for more MEAs in mathematics curricula at all levels of schooling to promote better mathematics instruction and deep understanding of the content. In addition, several research studies have pointed out that learners' lack of necessary knowledge in mathematics could cause a blockage in solving MEA (Galbraith & Stillman, 2006; Ng, 2011; Stillman, Brown & Galbraith, 2010), such as being unable to make assumptions which are relevant and understanding the situation of the MEA. Despite such blockages and learners' difficulties in modeling, the value of learning mathematics through modelling is recognised in literature. The real challenge on the part of the teachers or learners depends on whether or not both parties are committed to the success of this new way of learning. Learners as well as the teachers should embrace mathematics instruction through modelling (Brown, 1991).

2.4 STEM AND MATHEMATICS EDUCATION

Mathematics is a key discipline that is directly or indirectly related to different areas of science and engineering, which are at the heart of technological and scientific developments (Eş, Özdemir & Kaplan, 2019). The countries that have succeeded in producing knowledge and technology have recognised and acknowledged the importance of education, especially in the fields of mathematics and science (Yamak, Bulut & Dundar, 2014). In this regard, acquiring mathematical competencies is a condition for being able to open doors to a productive future, so these competencies need to be supported and provided with opportunities to gain a deep understanding of mathematics (NCTM, 2000). Recently, reform efforts and instructional practices in mathematics education in many countries of the world, including Ethiopia, have widely propagated the need for STEM learning in elementary and middle school mathematics classrooms. Educators and policymakers in each of these countries have “embraced” the slogan of STEM (Bybee, 2010; Ministry of Education [MoE], 2020). For example, the Ministry of Education (MoE) in Ethiopia has suggested that STEM learning should begin during early childhood when children are curious and creative and should continue through college (MoE, 2020).

2.4.1 Goals of STEM Education

Morrison (2006) and Magiera (2013) have suggested the following attributes of a STEM-educated learner that support the goals of STEM education.

2.4.1.1 Develops problem solvers

Learners should ask relevant questions as they investigate, collect, organise, and draw conclusions from the given information and apply their understanding to new situations (Magiera, 2013; Morrison, 2006). In line with this attribute of an educated STEM learner, it is clear from relevant literature that an MEA offers many benefits to a group of learners such as criticising each other, discussing and reaching the most relevant result, and applying their knowledge to similar but different situations (Zawojewski, Lesh, & English, 2003). In this regard, it can be concluded that MEAs have the potential to support the goals of STEM learning in mathematics classrooms.

2.4.1.2 Develops innovators

Learners apply concepts and principles of science, mathematics and technology to the engineering design process (Magiera, 2013). In the same way, an MEA offers benefits to learners such as use of mathematical language and tools, use of mathematical skills, and understanding and applying mathematical and non-mathematical concepts in solving similar real-life problems that are integrated with other disciplines such as engineering and science (Swan, Turner, Yoon & Muller, 2007).

2.4.1.3 Develops inventors

Learners can recognise the need to optimise problem solutions and work through the cycles of designing, testing, re-designing, and implementing solutions. (Magiera, 2013). Within MEAs, learners must go through several cycles of modelling to interpret and improve their products in ways that go beyond just providing an answer to offering unique solutions to a problem (Lesh & Doerr, 2003).

2.4.1.4 Develops self-motivation and self-reliance

Learners set their own agendas and gain self-confidence as they work within specified constraints (Magiera, 2013). Previous and recent research findings have maintained that learners who were taught by integrating MEAs in mathematics classrooms were able to examine all aspects of the problem, employ more than one strategy before they research a solution, strive to reach an optimal solution, were also happy to learn, and their self-confidence increased (Magiera, 2013; Topsakal, Yalcin & Cakir, 2022).

2.4.1.5 Develops logical thinkers

Learners can support their innovations and inventions with rational and logical use of science, mathematics, and engineering design processes (Magiera, 2013). In line with this, Capobianco (2011) recommended problem-based learning, that is, the MEA-guided learning in mathematics classrooms to explicitly help learners develop problem-solving abilities, teamwork, and use of science, mathematics, and technology.

2.4.1.6 Develops technological literacy

Learners understand the nature of technology and can apply technology appropriately. According to Magiera (2013), MEAs provide a context for learning in which learners can develop technological skills. This is because MEAs require learners to think about methods of gathering, organising, analysing, and displaying information. In conclusion, Magiera (2013) have suggested the following three reasons to demonstrate how MEAs can support the goals of STEM learning. Firstly, the context of solving MEAs allows learners to integrate their knowledge of concepts, which are found both inside and outside mathematics. Hence learners' problem-solving process of the MEA revealed how they are interpreting a mathematical situation through a purposeful documentation trail that promotes testing, refining, and extending their ways of thinking (Diefes-Dux, Moore, Zawojewski, Imbrie & Follman, 2004). Secondly, MEAs provide the context for learning new concepts and skills because the problems often require informal explorations and discovering concepts that have yet to be formally introduced. And finally, MEAs support the development of problem-solving abilities, dispositions, and expertise needed for analytical thinking.

2.5 OVERVIEW OF SCHOOL ALGEBRA

2.5.1 Definition

As a highly valued branch of mathematics, algebra deals with relationships and structures in mathematics and operations within those structures (Kieran, 1992). Researchers have been taking the view on the importance of algebra in developing learners' skills for their insightful explanation of various situations in the world around them (e.g., French, 2002). In this case, algebra can be conceived as an important milestone in developing mathematical skills of learners. Algebra involves various cognitive activities such as constructions and justifications of formulas that represent certain patterns, the construction of relationships and functions and functional notations, construction of algebraic expressions and equations and the construction of mathematical knowledge all of which are applicable in the real world. Ball and Bass (2003) put forth the following descriptions of school algebra that are firmly rooted in a proficiency for algebra.

- The skill to engage with algebraic relations in a flexible and meaningful way to deal with strategies that are important to represent situations, to manipulate them algebraically, and to solve the equations they represent to find a solution;
- A structural understanding of the basic operations of arithmetic and of the notational representations of numbers and mathematical operations to develop mathematical descriptions ,for example, number concepts, notation of fractions, exponents;
- A robust understanding of a function such as notion of the idea of its graphical and analytical representations; with repertoire of basic function, for example, linear, quadratic polynomials; exponential, trigonometric function; and using a function to examine the effect of one variable on other variable; and,
- Knowing how to identify and name significant variables to model quantitative contexts, recognizing patterns, and using symbols, formulas, and functions to represent those contexts. (Ball & Bass, 2003, pp. 44-45).

2.5.2 Conceptions of algebra

A number of different characterisations of algebra can be found in the mathematics education literature. For example, Usiskin (1988) described four conceptions of algebra: generalised arithmetic, the set of procedures used for solving certain problems, the study of relationships among quantities, and the study of structures. This can be viewed as a conceptual transition from arithmetic to algebraic thinking that leads learners to develop their mathematical knowledge. According to Demana and Leitzel (1988), performing necessary computations on numbers first and then exposing learners to real-life problems makes it achievable for these learners to understand algebra basic concepts. Kaput (1995) identified five aspects of algebra: generalisation and formalisation; syntactically guided manipulations; the study of structure; the study of functions, relations, and joint variation; and a modelling language. According to Katz and Barton (2007), there are four stages in terms of conceptual development of algebra. The first stage is geometric nature in which algebra concepts are geometric. The static equation-solving is the second stage where its emphasis is on searching numbers that satisfy specified conditions. The dynamic function is the third stage where motion is the underlying theme. Final stage is the abstract stage in which structure as the main concern.

In school algebra the four organising themes, as described by NCTM (1988), are: functions and relations, modeling, structure, and language and representation. The second category, modeling, was the focus of this study where the learners form the concept of algebra by interpreting it in phenomena in the real world familiar to them. Mathematical modelling as described by NCTM (2000), is “identifying and selecting relevant features of a real-world situation, representing those features symbolically, analysing and reasoning about the model and the characteristics of the situation, and considering the accuracy and limitations of the model.” (p. 303). The NCTM standards’ for school mathematics call for more contextual real world MEAs around learners’ contexts in order to help them develop deep understanding of the concept (NCTM, 2000).

In response to difficulties in learning algebra, Kaur, Carter and Yeap (2008) and Sinclair (2008) claimed that the modelling approach is a powerful tool to improve problem-solving skills among middle school learners. According to Ferrucci, Kaur, Carter and Yeap (2008), the modelling approach emphasises pictorial representation to analyse and represent quantity relationships in algebra problems, such as linear relationships. They pointed out that this approach helps learners to visualise abstract mathematical relationships in the form of a model in which learners can gain a deeper understanding of the concepts and skills to manipulate symbols as variables. They have further recognised the modelling approach as a critical algebraic problem-solving competence. According to these scholars, there are two phases in modelling approach instruction; the first involves examining relationships among variables in the situation. In the second phase, with a series of mathematical transformations, a model expressed in terms of symbolic expressions, graphs or tables is proposed (Ferrucci et al., 2008). In the education system of some countries (e.g., Mexico), algebra is learned through the modelling and problem-solving approach.

2.5.3 Conceptions of algebraic variables as unknown quantities

According to Leinhardt, Zaslavsky, and Stein (1990), variables such as time, length (e.g., distance, height), speed, temperature, weight, age, and number of people are considered contextualised variables (Leinhardt, Zaslavsky & Stein, 1990). The focus of this study was the number of people as a contextualised variable (Bestgen, 1980) that can be involved in the situation of a problem. According to Ursini and Trigueros, to understand a variable as an unknown quantity, learners should be able to 1) recognise and identify the presence of something unknown in the context of a problem that can be determined by considering the restriction of that situation; 2) symbolise quantities involved in the situation of the problem; 3) interpret symbols (letters) that appear in equations as the representatives of specific numbers; 4) substitute symbols in equations with numbers to create relations that are true numerical statements; and 5) determine unknown quantities that appear in equations by performing suitable algebraic and arithmetic operations (Ursini & Trigueros, 2001). This study also used the suggestion made by Ursini and Trigueros (2001) on the knowledge required by learners to understand variables as unknown quantities

2.5.4 Algebra and school mathematics

Since the mid-nineteenth century, secondary school algebra has been established as 1) a conceptual foundation of learners' mathematics education, 2) a gatekeeper for later (mathematics) learning, 3) an instructional language in mathematics and other related subjects, 4) a language to construct abstractions and make generalisations on number concepts, and 5) a prerequisite to effective learning of mathematical concepts in higher grades and transition to higher level learning (Usiskin, 1988). In the last couple of decades, researchers' work has emphasised the role of algebra at all levels of schooling (NCTM, 2000; Usiskin, 1988). NCTM proposed the teaching of algebra, as one of three focal areas, at all levels ranging from kindergarten to the end of secondary school (NCTM, 2000).

2.5.5 Misconceptions in Learners' Understanding of some Basic Concepts in Algebra

Variables – Several research findings about misconceptions of variables among learners are well documented (e.g., Asquith, Stephens, Knuth & Alibali, 2007). Misconceptions learners have include: viewing variables as labels (e.g., Usiskin, 1988); learners have a tendency to perceive two variables which are different as representing different values in equation (e.g., Ball, Thames & Phelps, 2008); and the inability to understand variables as varying quantities rather than a missing value (e.g., Usiskin, 1988).

Equality - Learners' misconceptions about the equal sign begins in early elementary schooling. In arithmetic, misunderstanding of learners about equally is the belief that it means “and the answer is” (Ball et al., 2008), instead of understanding it as its usage as a balance and the total value on both sides must be the same.

Algebraic Expressions – In the learning of algebra learners 1) struggle with algebra because they do not have a clear understanding of the variable, and 2) struggle by holding the view that a solution is always a number, not an expression (Kilpatrick, Swafford & Findell, 2001).

Algebraic Equations - Possible errors attributed to learners’ misconceptions in equation solving are incorrectly combining like terms, incorrect inverse operation, and incorrect use of property of distribution (Kilpatrick et al., 2001).

Research findings have been reported abroad around learners’ understanding of algebraic notation and learners’ understanding of literal terms and expressions (e.g., Kieran, 2016), and on systematic mistakes that learners make in simplifying algebraic expressions (e.g., Banerjee & Subramaniam, 2012). Banerjee and Subramaniam (2012) have revealed that the learning of algebra for many learners is difficult. Many of the difficulties in learning algebra come from learners’ poor understanding of two important concepts, namely the variable and algebraic expression. Successful learning of algebra is important as the findings of MacGregor (2004) disclosed that most difficulties in learning mathematics in later grades were predominantly related to learners’ poor understanding of basic algebraic concepts (e.g., algebraic variables) in earlier grades. Following MacGregor’s argument, it can be said that learners’ poor performance in mathematics in later grades can be influenced by their knowledge of algebra. This argument is particularly evident as algebra and all high-level mathematics courses are built based on the concept of the variable.

2.5.6 Algebra in grades 5-8 in Ethiopian mathematics curriculum

The content domains in mathematics at elementary school level (grades 1–8) in Ethiopia include four thematic areas. These are numbers, algebra, geometry, data handling and chance (probability). Specific to algebra, learners are introduced to the use of letters to represent numbers and to find the value of a variable or an expression by substituting letters for numbers in grades 5 and 6. Learners learn to compare expressions to determine whether they are equivalent or not in grades 7 and 8. They also learn to identify variables in real-life situations with real-life word problems and solve using linear equations in Grades 7 and 8 with the aim of developing knowledge in various specific domains in mathematics to solve problems encountered in their day-to-day life activities (MoE, 2018). Therefore, learners in grade 8 in Ethiopia have encountered all constructs and sub-constructs of algebraic variables in algebra set for upper elementary levels (grades 5–8). As one of the four content domains, the researcher in this study regarded algebra as one of the most important areas of school mathematics. But despite its importance, little attention (if any) has been given and less is reported in the literature in the context of Ethiopia about learners' understandings and their difficulties with algebraic variables.

2.6 THEORETICAL FRAMEWORK

The models and modelling perspective as well as the APOS theory are two main theories that guided and supported the implementation of the teaching treatment that was conducted in the experimental group for this study. Hence, the former was used in designing an effective modelling activity (MEA). The latter was used as a guide to map out the instructional sequence that was conducted in this study, also explaining the mental structure of learners and the method through which they tended to attain algebra conception.

2.6.1 Models and modelling Perspectives

There are several different interpretations of modelling in mathematics instruction that encompass a variety of pragmatics and theoretical perspectives. They are realistic, contextual, educational, didactic, conceptual, socio-critical, epistemological and cognitive perspectives (Kaiser & Sriraman, 2006). For example, a cognitive modelling perspective is used to analyse cognitive processes and emphasises modelling as a mental process such as abstraction or generalisation (Kaiser & Sriraman, 2006). The models and modelling perspective is aligned with the contextual aspect that has educational and psychological goals (concept introduction and development).

Lesh and Zawojewski (2007) claimed that, "the development of problem-solving abilities is highly interdependent and far more socially constructed and contextually situated than traditional theories have supposed" (p. 779). The models and modelling approach to solve an MEA emphasizes learners' group interaction as a way for them to adopt deep understanding in mathematics learning as they engage in revising, differentiating, and improving their thinking through communication within the group (Lesh & Zawojewski, 2007). Educators who advocate problem-solving through the models and modelling approach view learning not only as a cognitive process where learners use their knowledge of mathematics to arrive at the solution of a task but also as a meta-cognitive process such as attitudes and beliefs which are relevant for learning.

Following the work of several researchers in the field of business, engineering and science, researchers in mathematics education also created the foundation for developing effective sequences for instruction (guidelines) dealing with significant concepts in various mathematical domains. Thus beginning with many researchers, Lesh et al. (2000) developed and further refined the guidelines to design MEAs. The six design principles used as guidelines are the models and modelling perspective (Lesh et al., 2000). The models and modelling approach encourages the use of MEAs. Any MEA promotes six fundamental principles, namely the a) reality principle, b) model construction principle, c) self-evaluation principle, d) construct documentation principle, f) model-generalisation principle, and g) simple prototype principle (Chamberlin & Moon, 2005).

2.6.1.1 Reality principle

The reality principle entails real world mathematical problems that are familiar to the learners (Lesh et al., 2000). It also entails the application of previous knowledge which enables learners to extend their ability in using appropriate strategies that enable them to arrive at the correct solution (Lesh et al., 2000). Researchers have pointed out that problems in a real world situation familiar to the learner provide an opportunity for learners to make use of their creativity in their attempt to solve such problems (e.g., Chamberlin & Moon, 2005).

2.6.1.2 Model-construction principle

This principle ensures that the solution to the MEAs requires the construction of various cognitive activities that elicit various responses such as explanation, description, and justification for the situation that is mathematically significant (Lesh et al., 2000). Such products externalise how the learners interpret the situation and also reveal the type of mathematical quantities, relationships, and patterns that they need to take into account.

2.6.1.3 Self-evaluation principle

The self-evaluation principle ensures the inclusion of criteria which learners can recognise and apply to test and revise models by themselves (Lesh et al., 2000). The criteria that the MEA contain must provide an opportunity for learners to revise the cognitive activities that learners pursue in the MEA by themselves. The MEA should be in such a way that it provides an opportunity for learners to evaluate their responses and those of others and to assess the adequacy of the responses.

2.6.1.4 Model-documentation principle

The model-documentation principle ensures the inclusion of criteria which learners themselves can recognise to create some form of documents of clear explanation about the way of thinking about the process of solving the MEA (Lesh et al., 2000). The MEA presented in this study required learners to prepare a written document for the researcher while working on the activity to demonstrate their own thinking of the problem situation.

2.6.1.5 Model-generalization principle

The model-generalisation principle requires learners to generalise the mathematical model in such a way that another person at the same level of competency could apply it to a similar situation (Lesh et al., 2000). For example, the goal often is to develop a conceptual tool that goes beyond being useful for some specific purpose in a given situation - and that also is sharable (with others) and re-useable (in other similar situations).

2.6.1.6 Simple prototype principle

The problem entails developing a simple structure that enables learners to produce a reasonable solution (Lesh *et al*., 2000). The MEAs in this study were designed in relation to the six principles to serve as a medium of instruction aimed at developing learners' understanding of concepts in algebra at grade 8 level.

2.6.1.7 A Models and modelling perspective lens of model eliciting activities

Lesh and Doerr, (2003) have mentioned that, "models are conceptual systems (consisting of elements, relations, operations and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviours of other system(s) perhaps so that the other system can be manipulated or predicted intelligently" (p. 10). MEAs are designed in order to provide an opportunity for learners to elicit various cognitive activities such as creating, applying and adapting scientific and mathematical models to interpret, explain, and predict the behaviour of real-world systems.

2.6.2 APOS theory

The APOS theory is an outgrowth or the product of Piaget's theory of reflective mathematical abstraction to describe logical thinking development in learners (see, Piaget, 1973). APOS is an acronym for action, process, object, schema (Dubinsky & McDonald, 2001). The APOS theorists were among the first scholars to pay serious attention to the extension and expansion of the theory, to how learners come to understand concepts, and to think carefully about how this should inform pedagogic interventions. Beginning with these researchers in the 1980s, the APOS theory is progressed at a more advanced level by Dubinsky (1991). Since then, it has shown that it has found its place and homage in mathematics education research and is applied to various domains in mathematics. Although several researchers have used the APOS theory to apply Piaget's and Dubinsky's ideas in relation to learning to post-secondary mathematics, it has also been applied to elementary school and high school mathematics. Researchers suggest that the theory can be applied at all levels of schooling to different domains in mathematics (Arnon et al., 2014).

The APOS theory characterises learners' mathematical understanding in a particular concept as their inclination to react to situations in mathematics which are problematic by reflecting on the solution of the task in the context of the real world and the construction or reconstruction of actions, processes and objects, organising them into schemas to deal with this situation (Dubinsky, & McDonald, 2001). Dubinsky and McDonald explained that APOS theory in the understanding of various concepts in mathematics instruction assists in understanding the process of learning by providing clarification in various learners' cognitive activities during problem-solving and construction of knowledge in mathematics (Dubinsky & McDonald, 2001). This study was conducted with the belief that grade 8 learner participants in the experimental group could possibly develop the mental structures and mechanisms as a result of their participation in an intervention instruction design, based on the APOS theory (Dubinsky, 2000). The description of each conception, which is the action, process, object and schema, provided in the next paragraphs are consistent with those provided by Maharaj (2010).

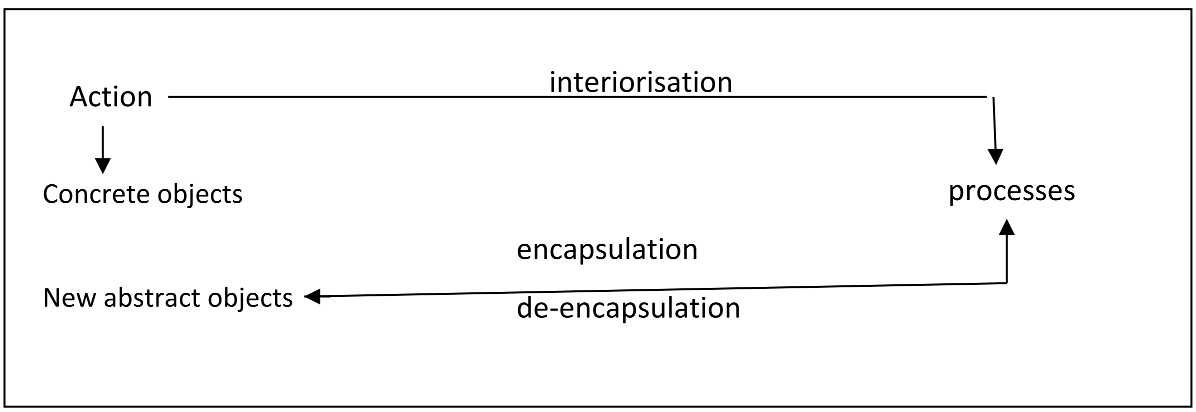
Action: An action conception is the transformation of a concept perceived by the learner as essentially external. It is external in the sense that the concept is conceived as an externally ordered command or externally directed transformation to do a specific operation. A learner with an action conception can perform the transformation based on learned detailed steps. Each step of the transformation needs to be performed explicitly, but not skipped, and instructed by external guidance on how to perform the operation. For this study, learners seemed to develop an action conception when they were able to form a concept that enabled them clarify the goal of the MEA and substitute the domain values to get the range values.

Process*:* When actions are repeated and interiorised, the actions collectively become a *process* (Breidenbach, Dubinsky, Hawks & Nichols, 1992). That is, an internal representation of the same action is constructed in a learner's mind, but not necessarily with extra stimuli. That is when the learner is able to imagine applying actions to a process and predicts its result without having to explicitly perform all steps or do the action then the process conception is transformed into an object conception. For example, learners seem to develop a process conception when they can at least apply reverse operation without being able to reliably perform the actions. Then, an action might be internalised to form process when an action is repeated and the learner has the opportunity to reflect on his/her action. At the process level, learners perform the same sort of transformations but only their mind.

Object: A learner is said to possess an object conception when he/she is able to view a process as consisting of a collection of single elements. We say that the encapsulation of processes into an object occurred (DeVries & Arnon, 2004) when learners apply rules in a specified sequence that goes beyond the ability to use them for calculation. For example, according to Baker, Cooley and Trigueros (2000), the object conception of function is a condition for effective understanding of function transformations.

Schema**:** According to Dubinsky (2000), the three aspects mentioned above and other schemas organised in a structured way are necessary to have complete knowledge of a concept. A schema as outlined by APOS theory for a specific mathematical concept is "coherent in the sense that it gives, explicitly or implicitly, means of determining which phenomena are in the scope of the schema and which are not" (Dubinsky & McDonald, 2001, p. 277). Therefore the coherence of a schema is what allows a learner to decide if it can be used in new and unfamiliar mathematical situations. However, this understanding is difficult to measure since it exists in the mind of the individual. What can be determined is the distinction between the proposed genetic decomposition and the subject's progress along the genetic decomposition pathway. Therefore a mathematical schema is considered as a collection of "action, process and object conceptions, and other previously constructed schemas, which are synthesized to form mathematical structures utilized in problem situations" (Trigueros & Martinez-Planell, 2010, p. 5; see, also, Figure 2.1).

Figure 2.2: APOS theory schema (based on Arnon et al., 2014)



2.6.2.1 Genetic decomposition

Genetic decomposition (Arnon et al., 2014) is a hypothetical model that involves the following: the theoretical analysis of the action conception that the learner performs explicitly; the process conception in which the learner imagines taking an action; object conception, which is where the learner sees a concept as a single element; and schema conception, in which the learner applies the previous conceptions. The basis for the construction of a genetic decomposition of a concept starts with a hypothesis based on previous experiences accumulated in teaching the concept, the researcher's current knowledge of APOS theory and the concept. Until it is tested experimentally, a genetic decomposition is a hypothesis and is referred to as preliminary (Arnon et al., 2014).

2.7. CONCEPTUAL FRAMEWORK

Polya's (1957) four well used and documented phases of the modelling process are, (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and, (4) looking back. In the first stage of Polya's model, the learner should fully understand the given problem and its ultimate goal. In the second stage the learner should devise a plan or recall a similar solved problem. In the third stage, the learner should validate each stage of the plan and modify any element of the strategy if it doesn't generate a shift from the initial plan to the desired goal. In the final stage the learner should check the solution and improve it if necessary.

Although, even today, many countries in the world included the influential works of Polya within their educational systems, one of the more outspoken critics of Polya's work is that it "did not provide the amount of detail that would enable people who were not already familiar with the strategies to be able to implement them" Schoenfeld (1992, p. 353), which according to English & Sriraman (2010, p. 264-265) the problem-solving stages of Polya are mostly "just names for large categories of processes rather than being well-defined processes in themselves".

Blum and Leiss (2007), defined mathematical modelling as a serial process constructed through several stages: (1) understanding; (2) structuring; (3) mathematising; (4) working mathematically; (5) interpreting; and, (6) validating. In the first stage the learner forms an initial idea about what is required as a final result of the problem. In the second stage the learner creates an idealised view of the problem. In the third stage the learner performs mathematical analysis, that is, the learner performs manipulations. In the fourth stage the learner may re-contextualize the mathematical result, that is, the learner explains the solutions based on the contextual situation of the task. In the last two stages the learner interprets the mathematical solution in real-world terms, and verifies results against constraints by explaining the repeatability of the solution.

Some researchers (e.g., Borromeo Ferri, 2018; Kohen & Gharra-Badran, 2022) comment on Blum and Leiss's (2007) modelling cycle by stating that, when viewed from a didactical view point, i.e., when the modelling cycle is considered as a tool with an aim to promote the understanding of mathematics by teachers and learners, then it should include clearly arranged steps illustrating the transitions between reality and mathematics.

There is one widely recognized approach in literature in the understanding of various concepts in mathematics instruction that assists in understanding the process of learning by providing clarification in various learners' cognitive activities during problem-solving and construction of knowledge in mathematics. This promising approach which underpins the theoretical framework of this study is the application of APOS theory in conjunction with models-and-modeling perspective (MMP) which relies on model-eliciting activity (MEA) as the central instructional resource.

2.8 SUMMARY

Mathematics problem-solving methods that have been cultivated in the conventional classroom setting need to change in order to help learners fully engage in solving real-life problems and develop the necessary mental construction. Solving problems in mathematics classrooms is no longer memorising facts and formulas to solve only textbook problems. In fact, the aim of school mathematics is to enable learners to approach, formulate, and refine problems beyond those they have studied, allowing them to organize and consolidate their mathematical thinking (NCTM, 2000). According to Greer et al. (2002), the models and modelling perspective (MMP), also called model-eliciting perspective (Kaiser & Sriraman, 2006), offers a way of introducing learners early to describe, construct, and explain phenomena, that is modelling (Lesh & Doerr, 2003). Abassian et al. (2020) distinguish MEAs from other modelling approaches in mathematics education because of the emphasis on the learning of concepts in mathematics through constructing a model of a mathematical process. There are recent empirical data which showed that an instructional model that use MEAs based on APOS theory improves learners performance at all levels of schooling (see, Arnawa et al., 2019; Borji et al., 2018; Kamid et al., 2021; Oferi-Kusi, 2017).

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 INTRODUCTION

It is often said that the design process of good research involves multiple stages of decision-making regarding the most appropriate design of scientific investigation. According to McMillan and Schumacher (2010), such decision-making processes involve, sample selection, instrumentation, data collection and the methods of data analysis that help the researcher to accurately respond to the research questions being investigated and the outcome of interest. The decisions with regard to the research methodology of this study were largely geared toward establishing the effects of the heuristic based problem-solving instruction (Section 1.4).

3.2 RESEARCH DESIGN FOR THE PROPOSED STUDY

According to Creswell (2013), a mixed-methods research approach involves integrating and collecting both the qualitative and quantitative data. The most important assumption in integrating the two methods lies in its power to provide an understanding of a research problem, and subsequent research questions, in a more complete way (see, Sections 1.3 & 1.5). This is because the combination of qualitative and quantitative approaches may overcome the limitations of single-method research. Therefore, one can accept the perspective that the mixed-methods research is an approach that possibly combine and integrate the strengths of both qualitative and quantitative approach compared to the strength of a single-method research. Another explanation that may describe this strength is that several challenges might have arisen if the qualitative and quantitative approaches tend to become disintegrated. According to prominent scholars in the field of mathematics education, there is still a need for data on the ways in which modelling practices influence learning, suggesting that mixed-method designs that seek to describe and explain any potential benefits of modelling tasks are assumed to be appropriate (Arnon et al., 2014; Cai, 2016; Tan, 2018).

To collect data, this study used both qualitative and quantitative methods. This was made possible since the study utilized some aspects of a Mixed Methods Research (MMR), consisting of a pre- and post-test quasi-experimental design, *non-equivalent control group design*, to collect quantitative data. To collect qualitative data, class observation of mathematics lessons of all four mathematics teachers in the participating schools was used before the implementation of HBPSI. Furthermore, analysis of learners' conversational statements of the experimental group during their discussion to solve the designed MEAs was used to collect qualitative data.

The non-equivalent control group design was a fitting design since this study would use intact or already existing classrooms in grade 8 (Section 3.3). It must be mentioned that it was anticipated that the use of this research design would probably jeopardize prospects of implementing rigorous random sampling procedures. Consequently, inability to optimally utilize randomized sampling procedures could possibly result in assembling or selecting two participating groups, experimental and control that would be deemed non-equivalent, in terms of scholastic performance and other variables of interest for this study. Hence, this study could be classified as a quasi-experimental research process. Quasi-experiments are experiments in which participants' assignment to experimental and control group is not achieved through random means, which is a natural expectation in quantitative research (Creswell, 2013). It is for this reason that the research design used in this study is a non-equivalent control group design, acknowledging that some aspects of true randomization have been compromised.

The use of a non-equivalent control group design helped the study to statistically measure and compare the effectiveness of the heuristic-based problem-solving instruction (HBPSI) and conventional teaching approach (CTA) on learners' achievements in algebra in grade 8 (Section 1.4). Both groups take a pre- and post-test. Only the experimental group receives the treatment (Creswell, 2013). Ofori-Kusi (2017) employed a pre- and post-test quasi-experimental design using a non-equivalent control group design in a study to investigate the effects of the heuristic approach algebra problem-solving instruction on the performance of learners. The gain in scores in the post-test in the Ofori-Kusi study was attributed to the implementation of the heuristic problem-solving instruction for the experimental group. Similarly, Dwiyogo, Kuswandi, Setyosari and Sudarman (2016) employed a pre- and post-test quasi-experimental design using a non-equivalent control group design to investigate the effects of learning strategy and cognitive style toward the learning of mathematical problem-solving. They argued that in homogeneous classrooms such design was appropriate.

The qualitative research design had three components of data collection. The first component involved classroom observations (Section 3.4). Classroom observation carried out in this study was basically aimed at ascertaining whether any of the participating classes in the experimental classrooms would possibly have a noticeable similarities or differences with the participating classes in the control classrooms with regard to learning and teaching of mathematics at grade 8. (see, Section 3.3). The second component of the qualitative research approach explored the possible conceptual difficulties, if any, and the possible understanding of algebraic variables that grade 8 learners would possibly manifest while attempting to solve the MEA after these concepts had been learned in their previous schooling from grades 5–8. This was done by analyzing learners' conversational statements during their discussion to solve the MEA. Finally, the third component of the qualitative research approach was meant to identify the learning opportunities created by the MEA from the learners' conversational statements that possibly exhibited the attributes of a STEM educated learner.

3.3 POPULATION OF THE STUDY AND THE SAMPLE

In the following sections, the population of the study and its sample are described.

3.3.1 Description of the population

The population in this study consisted of grade 8 mathematics learners drawn from 16 government schools that are located in eight towns in Northern Wollo province of Ethiopia. Prior to the commencement of this study, the researcher had noticed[3] that the government schools in these towns had similar features in terms of infrastructure, learner enrolment, and teacher-

3. North Wollo is the home region of the researcher, in Ethiopia.

learner ratio. This may be because resources for government schools' expansion and development are allocated by the central government. It is believed that every province in a particular region has had its equal share. Furthermore, the Ministry of Education in Ethiopia, through the regional state, recruits and assign teachers to every province of the region. In personal communication with the head of the province's education department the researcher ascertained that the previous year's distribution of new teachers across places where 16 participating schools were located was relatively equal. The researcher also observed that learners' results in the grade 8 regional examination around the areas where 16 schools were located had followed relatively similar patterns in previous years. In addition, in this province there is only one language of instruction[4] since this particular province is part of Amhara region which is the second largest region of Ethiopia and the language spoken in this region is the Amharic language, which is the national language of the country.

3.3.2 The sample of the study

The sample of the study consisted of grade 8 learners in mathematics classrooms (n = 205) from four elementary and junior secondary schools, two in Woldia and two in Kobo towns in the North Wollo province (see, also, Section 3.3.1). Four grade 8 classrooms, one in each school, were selected through convenience sampling. These were schools that the researcher had relations with, and as such the consent to conduct this research was granted. Moreover, the schools confirmed their availability for this research (see, also, Section 3.3.3). Learners (n = 100) in experimental schools in Woldia were in the intervention condition and learners (n = 105) in Kobo, which formed the control group, in the control condition. Four grade 8 mathematics teachers in participating classrooms took part in the study. Also, two department heads, one in the experimental schools and the other in the control schools participated in the study to observe the teaching methods employed by teacher participants prior to the intervention. The number of schools in the experimental and control groups was decided intentionally to reduce, if not eliminate all, the effects of inherent differences among the schools (Section 3.3.1). To ensure

4. In many parts of Ethiopia there may be more than one language of instruction in elementary schools within a short distance.

that this would be achievable, 205 participants in four schools were selected. Also learner-classroom ratio, textbook-learner ratio, teacher-learner ratio are almost similar in these schools.

3.3.3 Sampling techniques

The four schools, four classrooms, four participant teachers and two department heads were selected through convenience sampling, which was a suitable method to obtain schools that participated in this study (see, also, Section 3.3.2). A convenience sampling method is inconsequential in terms of generalisation of results. However, according to Gay, Mills and Airasian (2011), it is a non-randomisation approach of sampling employed to find, "whoever happens to be available at the time" (p. 140). Therefore participant schools involved in this study were chosen on the basis of their convenience and availability. There are many strengths associated with convenience sampling method; (a) it is less costly; (b) it is less time-consuming; (c) it is effective to administer easily; (d) it may substantially increase learners' participation; and, (e) it has a low participant attrition rate.

It must be mentioned that the drawbacks of this convenience sampling method are also well documented. One of its drawbacks, among others, is regarding the size and representativeness of the sample. Research studies depending on a convenience sampling method generate a limited generalisation of results. Even though every effort has been made to identify similarities, this does not mean that there are no inherent differences among the participating schools in the issue, for instance, teachers' preparation and motivation to teach a particular topic in grade 8 mathematics. Given the experimental segment of this study, the HBPSI was implemented by the researcher in the experimental group alongside the CTA that was implemented by incumbent teachers in control schools. This kind of a research design and its implementation gave the researcher the ability to impose statistical procedures on the study data in order to measure and subsequently compare the effectiveness of the HBPSI and CTA on the mathematical problem-solving performance of grade 8 learners in the topic of algebra.

The experimental and control schools were separated by a distance of 50km. This was done with the intention of preventing contamination. Contamination can occur when learners in the control group interact with learners in the experimental group or share study tools given to the experimental group (Shea, Arnold & Mann, 2004). Contamination for this study is could occur when the control group was exposed to the HBPSI that was intended for the experimental group.

3.4 CLASSROOM OBSERVATION IN PARTICIPANTING SCHOOLS

The teaching-learning experiences of teachers and learners and their interactions can be studied best by focusing on their activities through observation in a natural way in the mathematics classrooms (Gay et al., 2011). In this study the researcher and two department heads conducted observations in the participant classrooms.

3.4.1 Administering classroom observation

In this study, classroom observation before the implementation of the HBPSI was conducted by the researcher and two department heads in each of the four grade 8 classrooms in the control and experimental schools. It was aimed at identifying grade 8 teachers' teaching methods used in the four participant classrooms and comparing the method in relation to the HBPSI intended only for learners in the teaching intervention group (see Section 4.5). The study also evaluated whether or not there were any notable similarities or differences in the quality of learning achieved in the participating classrooms. The researcher adopted and used classroom observation schedule developed by Kotoka (2012). According to Kotoka (2012), classroom observation is an instrument to collect first-hand information in their natural setting, based on the lessons that the teacher brings to present in the classroom. The two department heads used the standardised lesson observation tool to assess teachers' performance in the teaching-learning process. It is standardised because lesson evaluation checklist used to evaluate teachers is prepared in a national level by curriculum experts. The data collected by the department heads during classroom visit was used for triangulation purposes (see, Section 3.8.1.1).

3.4.1.1 Classroom visits by the researcher

Each grade 8 mathematics teacher in participating classrooms was observed twice by the researcher. The classroom observation tool developed by Kotoka (2012) was adopted and used in this study to document classroom activities and the type of instruction offered by each teacher. In addition, the researcher used field notes to collect information. In these field notes the researcher documented the actions in the visited classrooms by taking into account issues such as, (1) the relationships of activities presented by the teacher, the mathematics concept taught and the corresponding lesson plan on these activities; (2) the effectiveness of teacher-learners and learner-learner communications; (3) learners' participation; (4) the authenticity of real-life problems included in the lesson of grade 8 teachers; and (5) ascertaining whether any of the participating classes in experimental schools would have an advantage over the participating classes, or not, in the control schools with regard to teaching and learning of mathematics. The role of the researcher was just limited to an observer of the discussions during lessons. However, the teachers were requested to provide the copies of their lesson preparations to the researcher and the heads before the commencement of the observation incidents.

3.4.1.2 Classroom visits by the heads of department

Each grade 8 mathematics teacher in the participating classroom was also observed once by each of the two heads. The standardised lesson observation tool to assess teachers' performance in the teaching-learning process was used by the department heads (see, Appendix D). In fact, each department head, in the context of Ethiopia, has the mandate to evaluate the learning process and teaching methods of teachers in their department in relation to the intention of the mathematics syllabus in both the first and second semesters of the academic year.

3.5 DESIGNING THE HEURISTIC-BASED PROBLEM-SOLVING INSTRUCTION

Conventional Teaching Approach (CTA) is referred to as teacher-focused teaching techniques used by participating teachers in the experimental and control schools of this study. Teachers in CTA give less emphasis to the development of learners' abilities in solving real life problems. Learners in CTA merely practice computation skills by recalling facts and formulas and imitating a solution procedure illustrated in their textbooks. As a result, learners In CTA appeared to rely overly on memorising formulas. Unlike CTA, HBPSI implemented by the researcher which was intended for the experimental group in this study allowed learners to reconstruct their previous knowledge to a deeper and more meaning full understanding of algebraic expressions. But participant teachers in the control group continued to use CTA for the control group. It was the researcher's view that CTA was devoid of developing and enhancing problem-solving skills in learners.

In this study the model-eliciting activities (MEAs), which are called authentic real-life problems, were used to assess the development of the learner in the learning of the topic of algebra. The MEA in this study was designed to serve as the medium of instruction aimed at developing the conceptual understanding of learners in algebra at grade 8 levels. The intervention informed the pedagogical approach that would possibly promote active participation of grade 8 learners in their algebra learning. Two theories were used as a framework in the intervention, namely, the models and modelling perspectives (MMP) and APOS (action, process, object, and schema) theory. The former used in designing an effective MEA. The latter was used as a guide in designing the instructional sequence used in the study explaining the mental structure of learners and the method through which they attain algebra conception.

The intervention involved the design of MEAs in learners' real-life context. Each MEA for the intervention was designed in such a way that the development of learners' understanding of concepts was hierarchical. This means that the problem-solving process of the designed MEA had hierarchical components in which the development of learners' understanding of concepts in algebra was guided. Components of all the six principles of the MMP were taken into account in designing each MEA (see, Section 2.6.1).

3.6 IMPLEMENTATION OF THE HEURISTIC-BASED PROBLEM-SOLVING INSTRUCTION

In the following sections, implementation of the HBPSI is described.

3.6.1 Implementers of the Heuristic-Based Problem-Solving Instruction

In this study the researcher implemented the HBPSI in the experimental schools. There were reasons the researcher implemented the HBPSI. Firstly, the HBPSI was completely new for grade 8 teachers in participating schools. As a result, there was a challenge in training these teachers on how to implement the HBPSI. Secondly, the grade 8 teachers in the experimental schools were overloaded with up to 30 periods per week in which case one period was 40 minutes. Hence, it would have been difficult for them to implement the HBPSI. Furthermore, the researcher wanted to work with learners to observe and guide their problem-solving processes on the designed MEAs, and this would be in line with the research questions 3 and 4 (Section 1.5). The researcher also wanted to ensure that the HBPSI in the experimental schools was implemented as intended by the study and as per its design, and that there was uniformity in implementing the aspects of HBPSI across all experimental classrooms.

3.6.2 Re-arrangement of learners for the heuristic approach to algebra problem-solving instruction

The 100 learner participants in the experimental group participated in the HBPSI in two different schools, and there were 50 learners from each school. Learners in the experimental group in each school were divided into 10 groups of five members. The groups were formed by the participants' mathematics teachers to ensure that the groups were heterogeneous in terms of the learner's mathematical achievement. The assumption was that such an arrangement could possibly contribute in facilitating the implementation of the HBPSI.

3.6.2.1 Rationale for a group setting intervention

Several researchers have pointed out that MEAs are better solved in a group setting consisting of three to five learners (for examples, see, Chamberlin & Moon, 2005; English & Walters, 2005). Based on APOS theory working in groups makes a difference to learners' confidence, engagement, and understanding, which in turn supports the premise of APOS based instruction (Arnon et al., 2014). Sfard (2010) suggested that, if you want your learners to think deeply about what they are learning in the classroom, they will also need to learn to communicate their mathematical ideas. In this study it was anticipated that such an instructional arrangement could encourage all members of a group to contribute to the production of more ideas through discussions, and that this approach could possibly create a sense of mutual support within a group.

3.6.3 Instruction and learning

In the following sections, implementation of the HBPSI model lesson is described.

3.6.3.1 Preparing for the intervention lesson

One week before the commencement of the intervention during the pilot study, the researcher completed all the MEAs. After that the researcher tested the MEAs with a friend/colleague. This was important as it helped to get insight into the way the researcher reflected on the real-life context of the MEA, which could in turn influence the teaching episode during intervention. When the researcher felt satisfied with the method of teaching, the intervention for the pilot study was initiated. After each lesson, the researcher thought about the way the activities of learning happened in that particular session. To this end, the researcher raised questions such as: *How did the teaching actions go with the class*? *Did all the learners in the respective groups participate*? *What questions were used to probe learners' understanding*? *How and when effectively intervene*? *What points need to be reinforced more*? Such questions helped the researcher to reflect on the teaching practice in the previous lesson and develop a more learner-focused teaching environment during in subsequent sessions. It also helped the researcher to engage learners in terms of finding mathematics interesting and enjoyable.

3.6.3.2 Genetic Decomposition

The MEAs in this study were used as the medium of instruction aimed at developing learners' understanding of concepts in algebra at grade 8 level. Based on APOS theory, the preliminary genetic decomposition of the task, which describes the learners' mental construction in learning algebra was made as follows:

1) **Action stage***:* Learners in a group were assisted by the researcher and their peers to avoid language barriers and to reflect on their understanding of the MEA until the researcher made sure that misunderstanding in dealing with the task was avoided. Then learners identified the variables that they thought would be relevant and those that they deemed not to be relevant. Then learners were guided to formulate a relationship or rule in the MEA and test their rule using a set of elements (input values) to give output values.

2) **Process stage:** Learners were required to repeat and reflect on these actions several times to strengthen their understanding. Then learners were guided to perform inverse operation by manipulating the rule in order to predict input value for a given output value.

3) **Object stage:** Learners were guided to notice similar problems in relation to the MEA and develop skills, which goes far beyond the ability to use the model (rule) developed from the given MEA to obtain the solution(s) of the MEA. Learners were also guided to manipulate the model they had already developed in order to help them modify and apply it to solve other MEAs in different situations.

4) **Schema:** Learners were given activities that enabled them to reflect in an interconnected pattern on the various stages of conceptions explained above, namely actions, processes and object conceptions in a structured manner. This was aimed at helping learners construct a complete knowledge of mathematical concepts embedded in the MEA. Using prompts the researcher in each stage intervened in the learners' modelling process to encourage them when necessary by respecting their ideas in the process.

The researcher incorporated more activities for a different situation that stimulated learners' real-life experiences from the grade 8 learners' mathematics textbook. This was with the hope that the designed MEAs could possibly make learners to recognise mathematics in their real-life, that is, to make mathematics practical. These activities were given to learners as homework activities to be done individually and in groups. Learners were expected to reflect on the prerequisite mathematical concepts involved in each activity and the various stages of its preliminary genetic decomposition at home and discuss this in each group in the following teaching session in order to bring common answers agreeable to all learners in the whole class.

3.6.4 Evidence for the positive effects of the teaching treatment on learners' performance after engaging in the intervention

When learners began to tackle more problems that they had not encountered before, and were now able to solve them correctly, this could be documented as evidence of the development of learners' problem-solving skills.

3.7 MEASURING EFFECTS OF THE HEURISTIC-BASED PROBLEM-SOLVING INSTRUCTION ON LEARNERS' ACHIEVEMENT

One of the purposes of this study was to determine the effects of HBPSI on grade 8 learners' achievement in algebra. The null and alternative hypotheses for the proposed study were expressed based on the problem statement of the study as follows.

3.7.1 Null hypothesis:

There is no statistically significant difference between the heuristic-based problem-solving instruction and conventional teaching approach in terms of grade 8 learners` achievement in the algebra test.

3.7.2 Alternative hypothesis:-

There is a statistically significant difference between the heuristic-based problem-solving instruction and conventional teaching approach in terms of grade 8 learners` achievement in the algebra test.

3.8 DATA COLLECTION INSTRUMENTS

The main instruments that were used in this study for data collection were, (1) classroom observation schedule before the implementation of the HBPSI (see, Section 3.4; see, also,

Appendix C; Appendix D); (2) the achievement test (Appendix B); and, (3) the model-eliciting activities or MEAs (Appendix A). The first two instruments helped the researcher to investigate and further ascertain the possible effect, if any, of the HBPSI in the learning of algebra for the experimental group compared to the control group where conventional teaching methods were implemented (Section 3.5). The MEAs in this study were used to serve as a medium of the HBPSI. These MEAs were aimed at developing learners' understanding of concepts in algebra at grade 8 level for the experimental groups through the preliminary genetic decomposition.

3.8.1 Purposes of data collection instruments

In the following sections, purposes of the HBPSI are described.

3.8.1.1 Pre-intervention classroom observation

Classroom observation carried out in this study before the intervention was basically aimed at ascertaining whether any of the participating classes in the experimental classrooms would possibly have a noticeable similarities or differences with the participating classes in the control classrooms with regard to learning and teaching of mathematics. The data collected by the department heads during classroom visit was used for triangulation purposes with the researcher's classroom observation gathered on the observation schedule. This was done to ensure that findings from the classroom observation were not overly dependent on a single source of data. It also helped the researcher to compare these classrooms experiences in relation to the HBPSI.

3.8.1.2 Achievement test

One of the dependent variables the researcher wanted to measure in the study was grade 8 learners' performance in algebra in the experimental and control classrooms. The status of learners' problem-solving performance in algebra was determined in terms of the achievement test scores. The same test (achievement test) was administered at pre- and post-stages of the experiment in both the control and experimental groups. To facilitate group discussion and to effectively implement the intervention learners were arranged in the form of heterogeneous groups with the help of participant teachers. Tests have been widely used to collect data (for examples, see, Dhlamini, 2012; Dhlamini & Mogari, 2012; Ofori-Kusi, 2017).

3.8.1.3 Model-eliciting activities

The main purpose of implementing the intervention was to explore the comparative effectiveness of the HBPSI and CTA on grade 8 learners' achievements in algebra. The MEAs in this study were used to serve as a medium of instruction aimed at developing learners' understanding of concepts in algebra at grade 8 level through the preliminary genetic decomposition. One of the designed MEAs (Garden Activity) was used to assess learners' background knowledge on algebraic variables after these concepts were taught using the conventional method of teaching from grades 5–8 (see, Appendix A). In this study it was expected that, as a result of learning the various algebra concepts set for primary school from grades 5–8, learners would have acquired some level of required background knowledge on the concepts of variables. Learners could be judged on their success in utilising their knowledge of algebraic variables in their effort to understand and interpret the designed MEA and apply algebraic concepts in the MEA problem-solving processes.

Also, the success of learners in solving the Garden Activity was possibly measured or ascertained in relation to the attributes of a STEM-educated learner. In this study it was also expected that learners could benefit from the learning opportunities created by the Garden Activity. It is expected that, with the help of the guidance of the researcher, these learning opportunities may enable learners to use the mathematics similar to the mathematics that is required to solve real-life problems upon which learners' success in developing higher-order thinking skills can be judged in relation to the attributes of a STEM-educated learner. In MEA, the mathematics that is required to solve real-life problems provides learners an opportunity to mathematise or interpret real-world situations (Lesh et al., 2000), thus developing higher-order thinking skills (Stohlmann, Maiorca, & Olson, 2014). MEAs have been identified as the typical medium not only to effectively foster deep understanding and critical thinking in learners (Lesh & Sriraman, 2005) but also to develop a more comprehensive understanding of algebra. The intervention informed the pedagogical approach that promotes active participation of grade 8 learners in their algebra learning.

3.8.1.4 Group worksheets

The purposes of the worksheet in this study was to familiarise learners with the HBPSI, thus creating opportunities to learn, and to increase time for learner engagement in problem-solving, that is, perseverance.

3.8.2 Development of data collection instruments

In the following sections, the development of data collection tools is discussed.

3.8.2.1 Pre-intervention classroom observation

In this study, a classroom observation tool developed by Kotoka (2012) was adopted and used by the researcher. However, the standardised lesson observation tool to assess teachers' performance in the teaching-learning process was used by the department heads.

3.8.2.2 Achievement test

Various types of authentic real-life problems in algebra may offer an opportunity to explore the effects of the HBPSI on grade 8 learners' achievement in the participant classrooms. The test was compiled and constructed from the grade 8 regional examinations that appeared in the years from 2014 to 2018 in such a way that the items spread over the domains of algebra in grade 8. At the same time, the relative cognitive demands of the algebra questions from low to high-cognitive level were maintained. Questions in the achievement test involved 10 multiple choice questions and five work-out questions. The total mark for the achievement test was 40, which was later converted to percentage points of 100.

3.8.2.3 Model-eliciting activities

Each activity for the intervention was designed in such a way that the development of learners' understanding of concepts was hierarchal. That is, the problem-solving process of the designed activity had an instructional sequence that explained the mental structure of learners and the method through which they attained algebra conception. Components of all six principles of the MMP were taken into account in designing each MEA. Additional activities were designed from grade 8 learners' mathematics textbook for homework to do individually and in a group. Learners were expected to be able to reflect on the preliminary genetic decomposition at its various stages. Then they were asked to discuss in each group in the next teaching session in order to bring common answers acceptable to all learners in the whole class.

3.8.2.4 Group worksheets

Real-life algebra problems of various types in grades 7 and 8 mathematics textbooks in the form of paper-and-pencil worksheets were practised by learners. Learners' responses were taken as data sources that were believed to be important in order to triangulate or validate the data collected from classroom observation and the intervention upon their answers to individual and group worksheets.

3.9 ASSESSMENT OF DATA COLLECTION TOOLS

In the following sections, assessment of data collection instruments for this study is provided.

3.9.1 Reliability and validity

Reliability deals with the consistency and dependability of the results obtained from a research. On the other hand, validity is concerned with whether a research is believable and true and whether it is evaluating what it is supposed to evaluate.

3.9.1.1 Validity of the modelling-eliciting activities

Two lecturers in the department of mathematics education at the Haramaya University validated the development of all the MEAs based on the fact that all six principles of the models and modelling perspectives were considered. Furthermore, these experts also agreed that dealing with these tasks helps grade 8 learners to develop a conception of algebra. Therefore these tasks can be used as a medium in the learning of algebra. In order to further increase validity of the

MEAs in this study they were piloted prior to the study by the researcher in a different school from the ones in which the main study were conducted to assess their suitability and feasibility for grade 8 participant learners. The sessions were audio-recorded. Furthermore, specific to this study, the findings of Ketema (2021) and Mageria (2013) were evidence for the use of MEA to explain learners' conceptual difficulties and its potential in supporting the goals of STEM learning respectively.

3.9.1.2 Reliability of the model-eliciting activities

During the pilot study, triangulation of the MEAs was used to ensure the consistency of measurements by the data collection instruments used in this study (instrument reliability) during the intervention. According to Cohen, Manion and Morrison (2011), triangulation is an attempt to present a more comprehensive description in a study of some aspect of human behaviour using two or more methods of data collection tools. Cohen et al. (2011) further suggested that triangulation is significant as it is powerful in demonstrating concurrent validity for qualitative research in particular (Cohen et al., 2011). Data collected from the group worksheet during the intervention, the transcripts of the audio recordings, the researcher's field notes and the pilot study were involved to compare and cross-check for the purpose of consistency.

3.9.1.3 Reliability of the achievement test

To compute the reliability of the achievement test using the data collected prior to the study, the researcher conducted a pilot study in a school far from the schools that participated in the main study. A pre-achievement test and a post-achievement test were administered before and after the intervention of the main study. A test-retest is a measure of the consistency of the test, and is ideally measured by administering a test twice at two different points in time (Gay et al., 2011). The Cronbach alpha coefficient was calculated using the SPSS software package.

3.9.1.4 Validity of the achievement test

The common practice of a test in preparation for the elementary school leaving examination at all regions for grade 8 learners is carried out by a group of experts in the respective regions. It is definitely believed that the selection of these experts is based on their teaching experience in mathematics, and their knowledge in learner assessment at this specific level. Furthermore, the test was given to two experienced mathematics teachers who had participated in previous regional test preparation. They were explicitly asked to write their comments. As a result of their feedback, one reworded multiple choice question was included in the test.

3.10 DATA COLLECTION AND DATA COLLECTION PROCEDURE

Data collection in this study took place in two stages; namely, in the pilot study and in the main study.

3.10.1 The Pilot Study

The purpose of a pilot study is to verify (it often works, but it is by no means a guarantee) that the data collection instruments will operate for the main study, by attempting to identify and amend problematic questions in advance. According to Creswell (2013), the purpose of a pilot study is to determine the validity and reliability of a study's instrumentation. Prior to the main study the pilot study was conducted in June 2019 in a school 20 km far from the schools that participated in the main study. 50 learners in one grade 8 classroom were participated for the pilot study. Administering the pilot study to a small group of individuals was helpful for the researcher to determine the validity and reliability of data collection instruments. Specifically, the pilot study allowed the researcher in this study to try out the test and the MEAs before the intervention. The test, which consisted of the pre- and post- segments, of a small group of learners in the pilot study were used as feedback on, (1) the clarity of the items; (2) difficulties in wording; and, (3) the time taken to complete the test. The validity of the study improved when some items in the achievement test were discarded after the pilot study. Similarly, the pilot study was also helpful to check if each MEA would possibly serve as, (1) medium of instruction aimed to develop learners' understanding of algebraic concepts at grade 8 level; (2) a tool to explain learners conceptual difficulties in their attempt to understand the context of the MEA; and, (3) a tool to explain its potential in supporting the goals of STEM learning.

The researcher administered the pre-post-test for learners participating in the pilot study. After orientating learners, they then wrote a 1½ hours achievement test at the beginning of the pilot study. After three weeks, which signaled the end of the intervention programme for the pilot study, learners wrote the same achievement test. The pre- and post-test results of participating learners in the pilot study were stored in such a manner as to protect confidentiality of records and the anonymity of participants. Similar to participant learners, the school selected for the pilot study was also protected. The results of the test coming from the pilot study in trying out the test were used to compute the test-retest reliability. The pre- and post-test in the pilot study were used to establish if there is a statistically significant difference between the mean pre-test scores and the mean post-test scores.

3.10.2 The main study

3.10.2.1 Achievement test

Similar procedure with that of the pilot study was applied to collect data for the main study. The researcher was collected the achievement test items that are aligned with algebra constructs from grade 8 regional examinations administered between 2014 and 2018. Learners in the experimental and control groups were given a pre-test. Each learner was given a unique indexed numbers as a code to write in the pre-test sheet. So anonymity was preserved. Learners were also requested to use the same indexed number during the post-test. The participant teachers administered the pre-post-test in both the control and experimental schools. Prior to the test, the researcher discussed with each participant teacher and agree on guidelines related to examination condition such as time allocated for the test, start and end time for the test. They were asked to invigilate scrupulously and to remain at their invigilation classrooms during the administration of both the pre-test and the post-test. The participant teachers were agreed to take all sorts of precautions related to the test and most importantly, the invigilation process was monitored by the researcher, which in turn ensured the similarity of the test conditions in the participant schools. The pre- and post-test results of participating learners were stored in such a manner as to protect confidentiality of records and the anonymity of participants. Similar to participant learners Identification of those involved in the study (department heads, teachers and schools) was also protected. For example, schools were known under the pseudonyms of S1, S2, S3, S4, and S5 to ensure anonymity. The pre-and post-test was separated by two months, hence it was believed that effects of recall was reduced.

3.10.2.2 Classroom observation

Classroom observation in the study was aimed at identifying grade 8 teachers' teaching methods used in the four participant classrooms and comparing the method in relation to the HBPSI intended only for learners in the teaching intervention group (see Section 4.5). The study also evaluated whether or not there were any notable similarities or differences in the quality of learning achieved in the participating classrooms. The researcher adopted and used classroom observation schedule developed by Kotoka (2012). The standardised lesson observation tool to assess teachers' performance in the teaching-learning process was used by the department heads (see, Appendix D). (See, Sections 3.4.1.1 and 3.4.1.1). In this study, the researcher conducted classroom observations in each of the four grade 8 classrooms in the control and experimental schools in October 2020. The two department heads observed each of the mathematics teachers in each of the participating classrooms based on each of the head's schedule and availability in September and October 2020.

3.10.2.3 The heuristic-based problem-solving instruction

The heuristic-based problem-solving model lesson was conducted for the experimental group by the researcher in November 2020 and December 2020. The researcher made use of the double period for mathematics (45 +45=90 minutes) schedule of the week to conduct the HBPSI in the two participating experimental classrooms. One of which is a tutorial period allocated with an aim to further consolidate the concepts taught in that particular week. Classroom teachers were agreed with the researcher to compensate the 45 minute elapsed period in the experimental classrooms in the same week. Therefore the six hour tutorial sessions for the experimental learners was interrupted during the two months intervention period. However learners in the control group were continued to attain the tutorial sessions.

Data were collected in the form of, (1) observations, (2) transcribed data excerpts from audio-recordings of learners' conversational statements, (3) written responses that appeared in the group worksheets, and (4) learners modelling activities in groups to solve the Garden Activity on a common sheet during the intervention lesson. Observation was carried out by the researcher using field notes. Observation enabled the researcher to capture information on learners' problem-solving processes and provided, "a unique example of real people in real situations" (Cohen, Manion & Morrison, 2007, p. 253). The audio recordings of observed lessons conducted by the researcher during the implementation of HBPSI were carefully analysed and subsequently transcribe. Unbeknown to the researcher, learner interactions during the heuristic–based problem-solving model lesson could be revealed through the audio recordings. The transcripts from the audio recordings uncover the preliminary observations made by the researcher during the teaching treatment and its revised version after the teaching treatment of the HBPSI. The researcher used learners' written answers that appeared on their individual and group worksheets to corroborate the transcribed recordings from the audio clips and the researcher's field notes recorded during classroom observations and the implementation of HBPSI. However classroom observation conducted by the department heads were not recorded. They were encouraged to continue with their usual style of lesson observation.

Although HBPSI was designed to investigate its effects on grade 8 learners' understanding of concepts in algebra, the researcher took advantage of learners' conversational statements during their modelling activities on the MEA to explore the difficulties that they possibly encounter in relation to conceptions of algebraic variables after these concepts have been learned from grades 5–8. It is also important to note that HBPSI was not crafted to replace subject-matter teaching that focuses on the existing curricula. In conducting the HBPSI for the experimental group, the researcher made use of a double period schedule per week in every participating classroom, one of which was a tutorial period.

3.11 DATA ANALYSIS

3.11.1 Pre-intervention classroom observation and qualitative data analysis

The qualitative data in this study had three components. The qualitative data gathered during classroom observation was aimed at identifying grade 8 teachers' teaching methods used in the four participant classrooms and comparing the method in relation to the HBPSI intended only for learners in the teaching intervention group (see Section 4.5). The analysis of the data gathered in the researcher's field notes during classroom observation and data taken by heads of departments were focused on several issues. They included 1) the teachers' lesson plan prepared on the tasks with the aim of developing learners' understanding of the topic covered; 2) attempt made to include authentic real-life tasks in the teaching-learning processes; 3) teacher-learners as well as among learners' effective communication; 4) level of learners' participation; 5) how the teacher attempted to increase success of the lesson; and finally 6) how the teacher evaluate learners' success in their learning process. The analyses on these issues were also used for triangulation purposes. Gray (2011) has argued that observation has to do with viewing actions by people, recording these actions, analysing them and finally interpreting their behaviours.

The second qualitative data taken from learners' modelling activities on the Garden Activity (MEA) and the audio-recorded learners' conversational statements during the HBPSI model lesson were assessed (Section 4.5). Only those data that exhibited the difficulties learners encountered during the HBPSI model lesson related to conceptions of variables were selected and assessed. To this end, 25 learners in five groups were selected. Two of the groups from nine groups in the first experimental classroom were randomly selected. The remaining three groups from ten groups in the second experimental classroom were also randomly selected. It was aimed at assessing the conceptual difficulties and understanding of algebraic variables that grade 8 learners have after these concepts have been learned from grades 5–8. This enabled the researcher to conclude whether or not the conventional method of instruction from grades 5–8 enabled learners to be equipped with the necessary skills to understand variables as unknown quantities involved in the Garden problem situations. With an eye to answering research question 3, the researcher was focused on finding evidence of learners' background knowledge of variables from their conversational statements from the transcripts. Those statements dealing with difficulties that arose at the beginning of the intervention and that learners were able to resolve through the guidance of the researcher were also included.

To analyse the data, the study made use of the suggestion made by Ursini and Trigueros (2001) to understand variables as unknown quantities. The learning episodes in the processes of understanding the context of the Garden Activity, mainly at the action conception stage, require learners' understandings of the concept of variables as unknown quantities (see, Section 3.5). Since the context and statement of the problem in the Garden Activity did not explicitly refer to the concept of algebraic variables, the analyses were focused on the following issues: 1) ability to recognise and identify the presence of something unknown in the context of the Garden Activity; 2) ability to symbolise relevant quantities involved in the context of the Garden Activity; 3) ability to interpret letters that appeared in the developed rule or model as representatives of specific numbers; 4) ability to substitute letters in the model with numbers to create relations that were true numerical statements; and 5) ability to determine values of relevant unknown quantities that appeared in the model.

The final qualitative data taken from learners' modelling activities on the Garden Activity (MEA) and the audio-recorded learners' conversational statements during the HBPSI model lesson (see, Section 4.5) were analysed. Only those data that possibly exhibited or elicited attributes of STEM-educated learner were selected and assessed. To this end, 10 learners in two groups were selected. One group from nine groups in the first experimental classroom were randomly selected. Another group from ten groups in the second experimental classroom were also randomly selected. With an eye to answering research question 4, the researcher was focused on finding evidence of learners' modelling activities that possibly exhibited or elicited attributes of a STEM-educated learner from the Garden Activity sheet and their conversational statements from the transcripts. The analyses were focused on the following issues: (1) asking questions during the discussion that could possibly enable them to make new connections and predictions; (2) the ability to think about two different algebraic terms that describe the same situation (for example, learners may think about the quantity that represents the number of learners in grades 5 and 8 in the school in terms of the number of learners in grades 5 and 8 that will be involved during the preparation of vegetable beds on a particular school day which led them to develop the required rule); (3) ability to recognise the need to optimise the number of learners in a single bed (for example, learners may consider different factors such as study time, examination period, and class absenteeism as relevant factors to fix the number of learners for each vegetable bed); (4) ability of learner to reflect his/her thoughts and communicate her ideas with other group members; and, (5) ability of learners to use their knowledge logically and rationally (e.g., learners may relate issues of fairness by assigning garden beds with the smaller size to grade 5 learners and beds with the larger size to grade 8 learners). The qualitative data gathered by exploring the above attributes was aimed at assessing the potential of the Garden Activity to provide problem-solving experiences that help learners develop a wide range of mathematical expertise. This enabled the researcher to conclude whether or not the Garden Activity supported the goals of STEM learning (see, Section 4.7). Morrison (2006) and Magiera (2013) have suggested these attributes described above as attributes of a STEM-educated learner (see, Section 2.7).

3.11.2 Analysis of factors that determine the effectiveness of the HBPSI for the experimental group in terms of performance

The analysis and explanation of factors that influence the success of the intervention of the HBPSI in terms of learners' performance in algebra at grade 8 level was done using data collected during the intervention. The analysis and explanation also included how this method could be implemented by mathematics teachers in middle school classrooms. Data gathered in knowledge development and evaluation studies by using APOS theory required researchers to develop preliminary genetic decomposition for that specific concept and analyse the mathematical activity needed to solve the task in terms of learners' mathematical practices and mathematical concepts activated during these practices. Two theories were used as a framework in this study in order to gather data through the HBPSI. They were the models and modelling perspective (MMP), and the APOS (Action, Process, Object, and Schema) theory. The former was used in designing an effective modelling activity (MEA). The latter was used as a guide in designing the instructional sequence conducted in this study explaining the mental structure of learners and the method through which they attained algebra conception. The researcher gathered the data that was intended to explain how learners responded to the model-eliciting activities. The data were also used to explain the learners' problem-solving process of the designed activities (MEAs) which have hierarchical components which guided the development of learners' conceptual understanding. That is, each MEA for the intervention was designed in such a way that the development of the conceptual understanding of learners in algebra at the level of grade 8 was based on the model and modelling perspective and the APOS theory.

3.11.3 Quantitative data analysis of the effects of the HBPSI on learners' achievement in algebra

Quantitative data are generally used to make quantitative assessments and can be measured numerically and the result becomes more precise. Hence the quantitative data were analysed using descriptive statistics and inferential statistics, namely the t-test and ANCOVA.

3.11.3.1 The t-test

The t-test enabled the researcher in this study to measure the success of the HBPSI. This can be achieved by identifying if there is a statistically significant difference between the mean pre-test scores of learners in the experimental group and that of the mean pre-test scores of the control group (Cohen et al., 2011)

3.11.3.2 Analysis of covariance (ANCOVA)

The expected differences in the post-test scores between participants who started with the same pre-test was estimated using ANCOVA. The analytic procedures were applied to pre-treatment scores, as a means of examining learners' problem-solving skill status before intervention and improvement scores, as a means of investigating effectiveness of the intervention.

3.12 ETHICAL CONSIDERATION

Research in education primarily focuses on human beings, and consequently the researcher was concerned about issues of ethics to look after the welfare of learners participating in the current study (McMillan & Schumacher, 2010). Thus the researcher, before the commencement of the study presented appropriate letters to all parents[5] to consent to the participation of their children in the proposed study. In the letters the researcher assured the parents that the researcher would not harm them in any way psychologically, confidentiality, voluntarily participation, anonymity and exit before the commencement of the study. An ethical clearance letter from the University of South Africa and the Province Education Office were obtained prior to the commencement of data collection of the study. Learners' names were to be kept confidential and pseudonyms were used during the data analysis.

3.13 CONCLUSION

In chapter three the research methods and methodology have been introduced, which enable the researcher to think deeply about the data collection tools and procedures to measure the effects of the heuristic-based teaching treatment on grade 8 learners. The issue of the research design, population, sample, sampling techniques, procedures to collect data and analysis of the data collected were presented. The discussion also included reliability, validity, and triangulation on the data measuring tool.

Among the issues explained were the research paradigm, the research design, sampling techniques, data collection procedures and analysis. The chapter concluded with an explanation of the ethical guidelines that the researcher used during the data collection procedures.

Chapter four presents the quantitative and qualitative findings of the study. The findings were from the classroom observation, design and implementation of the heuristic-based problem-solving instructional approach and the effect of the heuristic-based problem-solving instructional approach on the performance of learners in algebra.

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5. Letters to parents were translated into their first language (the Amharic language).

CHAPTER FOUR

DATA ANALYSIS AND FINDINGS

4.1 INTRODUCTION

This previous Chapter provided an outline of the research design, research methodology and scientific procedures employed in this study for the purpose of data collection. A comprehensive description of data collection tools and other related data sources have been discussed in Section 3.9. Furthermore, the process of data analysis for the study was presented in Section 3.10. The aim of this study was to investigate the comparative effect of heuristic-based problem-solving instruction (HBPSI) and conventional teaching approaches (CTA) on the mathematics problem-solving performance of grade 8 learners (Section 1.4). The following objectives were outlined to achieve this aim: To highlight and integrate two theories, namely the models and modelling perspective (MMP) and the action, process, object and schema (APOS) to develop the HBPSI to teach algebra in grade 8; To use MMP as an instructional tool to design the model-eliciting-activities (MEAs) to enhance, if possible, the effectiveness of HBPSI when teaching the topic of algebra in grade 8; To identify the difficulties that learners possibly encounter in relation to conceptions of variables and explain their success in their attempt to solve the designed MEA; and to compare and statistically measure the comparable effects of HBPSI and CTA on learners' mathematical problem-solving performance in the topic of algebra in grade 8 (Section 1.4).

To collect data the study employed both the qualitative and quantitative methods (see, Section 3.10). Mainly, the study involved 250 grade 8 learners in mathematics classrooms (Section 3.3). In addition, there were 4 teachers and 2 heads of departments who also took part in this study. Chapter Four provides a detailed discussion on the analysis of data collected for this study, and further presents the findings of the study from the empirical investigation. This discussion is carried out in seven distinct sections as follows.

- Section 4.2 presents data collection instruments and related research questions (Sections 3.8 & 1.5);
- Section 4.3 reports the assessment from the classroom observation before the intervention by the researcher, two heads in the department of mathematics from participating schools, one from the experimental and another from the control schools, by making use of the researcher's field notes and the data from the classroom observation schedule. Therefore, the purpose of the qualitative data analysis in Section 4.3 was aimed at identifying the teaching methods employed by grade 8 teachers in four participating classrooms and comparing the methods in relation to the HBPSI intended only for learners in the teaching intervention group;
- A concluding remark on the classroom observation is presented in Section 4.4;
- Section 4.5 presents how the new intervention involving HBPSI was designed and implemented. This section also describes the way learners are arranged in groups for the new intervention and presents the influence of the intervention in learning algebra. A brief summary of the effect of the intervention on the performance of learners in algebra is presented;
- Learners' difficulties with algebraic variables are examined in Section 4.6. With the desire to answering research question 3, participant learners (in a group of 5) were closely observed. The purpose was to find evidence of learners' background knowledge of variables from their conversational statements from the transcripts as a result of their learning from grades 5–8 using the conventional teaching approach;
- Learners' modelling activities indicated on the Garden Activity sheet and the audio-recorded learners' conversational statements that possibly exhibit or elicit the attributes of a STEM educated learner are analysed in Section 4.7; and,
- Learners' performance in the achievement test in the pre-test and post-test is analysed in Section 4.8. Therefore, the purpose of the quantitative data analysis in Section 4.8 was to use

the significant performance difference in the post-test between the experimental and control groups as contribution to the heuristic teaching treatment.

4.2 DATA COLLECTION INSTRUMENTS FOR THE STUDY

The instruments used in this study for data collection as well as the related research questions are summarised in Table 4.1.

Table 4.1: Data collection instruments and related research questions

Data collection instrument	Data source	Related research questions
Pre-intervention classroom observation	Teachers	1
Model-eliciting activities (HBPSI model lesson)	Learners	1 and 2
Learners modelling activities on the Garden Activity (conversational statements)	Learners	3 and 4
Pre-test and post-test	Learners	5

4.3 CLASSROOM OBSERVATION BEFORE THE INTERVENTION

The researcher and two department heads observed lessons conducted by grade 8 mathematics teachers in participating classes of the four schools before the intervention, that is, the pre-intervention classroom observation. The observation was conducted independently based on the common observation schedule used by the researcher and two department heads. Learners in each of the four classes were taught by incumbent teachers in control schools. Each of the teachers was observed twice. The classroom observations were aimed at identifying aspects of teaching methods and approaches employed by grade 8 teachers in the four participating classrooms, and further comparing instructional methods in relation to the HBPS, which was intended only for learners in the teaching intervention group. The conclusions drawn in the next sub-sections about the pre-intervention classroom observation were based on data from the researcher's and the two department heads' observation schedule and the researcher's field notes (see, Section 3.9).

4.3.1 Teaching methods adopted by teachers in the participating schools

During the classroom observations, the pseudonyms were used to refer to the teacher participants, such as, CT1, CT2, ET1, and ET2. For instance, the CT1 and CT2 referred to Teacher 1 (T1) and Teacher 2 (T2) in the control schools (C), and ET1 and ET2 referred ere teachers in the experimental schools (E). The teaching methods that these grade 8 mathematics teachers employed in the participating classes has been summarised and discussed individually and sequentially in the next sub-sections.

4.3.1.1 Teaching method used by CT1

The summary of the teaching method used by CT1 tended to focus on the following issues:

1) Lesson plan: CT1 prepared a weekly lesson plan that contained some activities presented for each day of the week. Although teachers were supposed to write learning outcomes in their lesson plan, it was observed that CT1 left the space provided blank. The column on the teaching aid(s) in the lesson plan contained: blackboard, colour chalk and learner textbook.

2) Lesson presentation: By introducing the topic of the lesson CT1 started to implement his lesson preparation without recalling the previous one. Although there is a 1-to-1 ratio between learner and mathematics textbook in all the participating schools, CT1 silently wrote a detailed solution of the example on the blackboard and then used a step-by-step explanation process to clarify the example to learners. Peer learning was not seen, and a dominant feature of teacher talk was observed, which tended to enhance procedural rather than conceptual learning. The learners worked independently.

3) Learners' participation: Even though the teacher (CT1) did nothing like using groups to help learners to solve problems, there was an opportunity for one learner to solve one problem on the blackboard and the teacher attempted to help other learners to learn from his errors using oral questions about points that were overlooked by the learner. Discussion among learners during the lesson was not seen. Possibly, this could have been a result of the teaching method used by the teacher. Interaction of the teacher with learners was limited, only some of the learners volunteered to answer the questions.

4) Inclusion of real-life tasks: There was not a single example (problem) during the lesson related to learners' real-life situations.

5) Lesson evaluation: The teacher used prepared questions to assess learners' understanding of the basic mathematical concepts during the lesson. CT1 made eye contact with only some learners and neglecting others. As a result there were some instances in which two learners were trying to respond frequently to questions posed by the teacher. At the end of the lesson the teacher gave homework activities from the learners' textbook. This may mean learners spent a considerable amount of time on such activities outside the school.

In summary, learners in CT1's class were busy copying everything written by the teacher on the blackboard without listening to the explanation of each step. The learners worked independently. As a result, there was no chance for learners to stay in the learning process, especially, those who could not work independently. The observation suggested that the majority of learners were passive recipients of ready-made knowledge of mathematics in the entire process of CT1's lesson presentation. Evidence documented by the researcher and the two heads on CT1's lessons showed that the teacher did not use a variety of teaching methods that would have possibly helped to enhance engagement with learners while they were learning. As a result it is concluded that the teacher did not succeed in presenting a lesson that could have helped learners to construct a meaningful understanding of the concepts. The next sub-section describes the teaching methods as observed for teacher CT2.

4.3.1.2 Teaching method used by CT2

The summary of the teaching method was focused on the following issues:

1) Lesson plan: This teacher (CT2) prepared a daily lesson plan that consisted of detailed activities of the day. Furthermore, the learning outcome was stated in the daily lesson plan of CT2. The teaching aid column of the weekly lesson plan contained: blackboard, chalk and learner textbook. The lesson presented using the learner textbook was also supported by the teacher's prepared notes.

2) Lesson presentation: The teacher started the lesson by reviewing the previous lesson. In CT2's class learners were required to copy everything written on the blackboard. This teacher (CT2) used almost the whole period to explain every step without leaving room for the participation of learners and assumed CT2 was the only responsible individual in the teaching and learning process. In addition to the learners' textbook, CT2 prepared notes to present the lesson. Group learning was not seen and a dominant feature of teacher talk was observed. There were no teacher-learners or among learners discussion even at the end of the lesson.

3) Learners' participation: There was no opportunity for learners to solve problems either in the form of groups or independently. As a result of the lecture method of teaching used by the teacher, discussion among learners during the lesson was not seen. Interaction of the teacher was limited only to some learners who volunteered towards answering simple recalling questions, which did not help learners to understand the basic concepts of the topic being taught and did not encourage higher level thinking.

4) Inclusion of real-life tasks: There was not a single example during the lesson related to learners' real-life situations.

5) Lesson evaluation: The teacher used prepared questions to assess learners' understanding of the concepts during the lesson. After a quick observation of learners' work and moving here and there in the room, the teacher provided answers to each question. Finally, before the lesson ended, the teacher gave some problems in the textbook as homework.

In summary, learners in CT2's class were busy listening to the teacher's explanation and time was given to copy the step-by-step solutions written by the teacher on the blackboard. The learners worked independently during the class work activities. It was observed that the majority of learners were passive listeners of ready-made knowledge in the entire process of CT2's lesson. As evidenced in CT2's lessons (documented by the researcher and the two heads), CT2 seemed to believe that there is no lesson in the absence of CT2's talk and chalk. As a result it was concluded that the teacher didn't use a variety of teaching methods that help to engage learners in their learning. The next sub-section describes the teaching methods as observed for ET1.

4.3.1.3 Teaching method used by ET1

The summary of the teaching method was focused on the following issues:

1) Lesson plan: This teacher (ET1) prepared a weekly lesson plan that contained some of the activities for each day of the week. Furthermore, learning outcomes that informed the learners about what was expected from them at the end of the week were stated in the lesson plan. The whole lesson was prepared by making use of the learners' textbooks as a teaching aid.

2) Lesson presentation: After a quick observation of learners' exercises book, the teacher identified those learners who had not attempted the previous homework and asked them to leave the class. Then the teacher started the lesson after a quick recalling of the previous lesson. By introducing the lesson of the day, ET1 started to implement his preparation to teach the lesson. Like CT1, ET1 used a step-by-step explanation process based on the textbook to teach concepts to learners. As a result of ET1's one-way method of teaching, like CT2's lesson, the observation suggested that the majority of learners were passive listeners of ready-made knowledge in the entire process of ET1's lesson.

3) Learners' participation: In ET1's lesson, there was an opportunity for learners to do exercises in the form of small groups in two instances. However the teacher did not facilitate the group work by encouraging each learner to share knowledge with other group members and interact with each other. It was observed that one learner in the group dominated the discussion and others did not contribute to the group. This negligence of the teacher in managing the group resulted in less contribution of other group members in the teaching-learning processes.

4) Inclusion of real-life tasks: There was not a single example or problem that dealt with learners' real-life situations.

5) Lesson evaluation: The teacher used questions that required learners to recall facts and rules as a means of assessing learners' understanding of the concepts during the lesson. As the last part of the lesson, the teacher gave homework activities in the textbook. He also warned against learners' negligence in attempting the given homework.

In summary, learners in ET1's class were busy copying the solution process of a problem written by the teacher on the blackboard. Although the learners worked in groups, the discussion in every group was dominated by one group member and the contribution of other members was not visible. As a result, the teacher did not succeed in facilitating the group discussion. The observation suggested that the majority of learners were passive receivers of ready-made knowledge of mathematics either in ET1's lesson or in the group discussion. Evidence in ET1's lessons showed that the teacher did not succeed in presenting a lesson that helped learners construct a meaningful understanding of the concepts. The next section describes the teaching methods as observed for ET2.

4.3.1.4 Teaching method used by ET2

1) Lesson plan: This teacher (ET2) prepared a daily lesson plan which contained detailed activities for the day. Furthermore the learning outcome was stated in the daily lesson plan. The teaching aid column in the weekly lesson plan contained: blackboard and learner textbook. The lesson presented was supported using the learner textbook.

2) Lesson presentation: The teacher started the lesson by recalling the previous lesson. ET2 used the whole period in explaining every step without leaving room for participation of learners. In ET2's lesson, interaction between learners during teacher presentation, for any purpose whatsoever, was considered as classroom disturbance. The dominant feature of teacher talk was observed in ET2's lesson. Group discussion was allowed at the end in the last 5 minutes.

3) Learners' participation: Discussion among learners during the lesson was not seen. The lecture method of teaching was used by the teacher. Interaction of the teacher was through simple recalling of questions which did not help learners understand the basic concepts of the topic being taught.

4) Inclusion of real-life tasks: There was not a single example during the lesson with regard to learners' real-life situations.

5) Lesson evaluation: The teacher used prepared questions as class-work to assess learners' understanding of the concepts during the lesson. Finally, the teacher gave problems in the textbook as homework.

In summary, in ET2's lesson, it seems that any discussion between two learners concerning the solution process should be at the end of the lesson. That is, ET2 had no room for any learner's difficulty during ET2's talk before the last five minutes. Learners in ET2's class were busy listening and copying the step-by-step solutions written by the teacher on the blackboard. The observation suggested that the majority of learners were passive listeners of ready-made knowledge throughout the entire process of ET2's lesson.

4.4 AN OVERVIEW ON THE PRE-INTERVENTION CLASSROOM OBSERVATION

The lesson plans prepared by teachers in the participating classes were not clear enough about the activities in the sense that they did not provide an opportunity for learners to develop their mathematical problem-solving skills on the topic being taught. In many cases, teachers gave priority to content coverage that emphasised procedures to understand worked-out examples. Teachers in the participating classes copied worked examples from the text book in their note book at home and then rewrote them on the blackboard. It seems that rewriting the same worked-out example found in their textbook into their exercise books was a waste of time for learners. The teachers provided a step-by-step explanation of the worked-out examples written on the blackboard in a way that led to passive learning rather than effective active learning. Learners were expected to practice similar problems in their textbooks at home. The mode of instruction by the four teachers did not allow active involvement in the learning process. Learners in these classrooms worked alone rather than working with learners in collaborating groups. As a result, learners passively received ready-made knowledge presented by the teacher.

Learners were not given the opportunity to discuss with their peers to share their learning experiences in constructing knowledge in a real-life context. As the teachers were the only actors in the classrooms, there was no way to facilitate learners' engagement in the teaching-learning process. The teachers did not use a variety of teaching methods that helped to engage learners in their learning. As a result, the teachers were unable to monitor learners' progress on the topic being taught. The learning process by each of these teachers was not integrated around a real-life context to explain the concepts being taught. The tasks given by the teachers did not allow learners to link mathematics inside the classroom with the mathematics outside the classroom. Therefore the lesson presented by the teacher did not facilitate connections to learners' real-life context and basic concepts of the topic taught.

Therefore the pre-intervention classroom observation, as presented in the previous sub-sections, revealed that the experimental and control schools, in terms of method of teaching of teachers, were at a comparable level. As a result, some of the extraneous variables that could influence the relationship between the independent and dependent variables of the experiment would be more likely controlled in this study. It was believed that this could measure the true effects of the new intervention in comparison to the conventional teaching approach. The next section reports on the development and implementation of the heuristic- based problem-solving instruction (HBPSI).

4.5 DEVELOPMENT AND IMPLEMENTATION OF THE HBPSI MODEL LESSON

This section describes how learners in the form of groups for the intervention were arranged and how the HBPSI model lesson was developed for learning and teaching. The description is organised into three main sections, as follows. Section 4.4.1 gives a short description of how learners in the experimental group were arranged in a group of 5; Sections 4.4.2 and 4.4.3 describe how the HBPSI model lesson can be developed and implemented respectively in the learning of algebra.

4.5.1 Rearrangement of learners in groups for the intervention

In the previous section it was noted that real-life contexts in mathematics instruction conducted by the mathematics teachers in the participating schools were not properly addressed. Learners' involvement in making sense of mathematics tasks presented by teachers was not observed and their participation in groups was weak. The participating learners in the experimental classrooms were divided into five groups each with five members, as recommended by English and Walters (2005) for MEAs. The groups were formed by the participants' mathematics teachers. Learners were assigned in this way to ensure that the groups were heterogeneous in terms of the learner's mathematical achievement in previous tests.

4.5.1.1 Justification for the rearrangement of learners for the intervention

As observed from the pilot study, the following were major obstacles in implementing the HBPSI, namely, (1) learners in some groups did not talk to each other, which may be the result of their previous learning experiences in which they could have been passive recipients of ready-made knowledge; (2) in some groups learners were off task, maybe due to communication issues and lack of understanding of basic mathematical concepts that are essential to understand the context of the problem situation; (3) group discussion was dominated by one or two learners. Most importantly, MEAs are designed to be completed by learners in groups (English & Walters, 2005). The implementation of the HBPSI in the main study was further refined by addressing the issues raised during the pilot study. It was also believed that such an arrangement could offer the researcher with the possibility of supporting more learners to learn mathematics with understanding in a real-life context, as was found effective in this study. In addition to the encouragement and support provided by the researcher for active participation of learners during the intervention, formation of such groups may create an atmosphere where each learner in their respective group has an opportunity to share ideas among themselves which foster and support group participation.

4.5.2 A Heuristic-Based Problem-Solving Instruction model lesson

Topic – Number sentence and algebraic expression for grade 8

Objective – To develop learners' understandings of concepts and procedures in number sentences and algebraic expressions at grade 8 level

Resource used – MEAs which are developed through task design principles in the lens of models and modelling perspective are used as a means of interaction during the intervention between learners and researcher.

Table 4.2: Researcher and learners' activities during the heuristic-based problem-solving instruction model lesson

Activities	
Researcher	**Learners**
In a whole class instruction setting, introduces MEA	Discuss the MEA in groups to explore the context of the problem situations.
Based on learners' day-to-day activities and their understanding of the real-life task (MEA) describes and develops algebra concepts in grade 8 by encouraging learners to model a solution of the real-life task based on the preliminary genetic decomposition.	Actively engage in processing one another's ideas in a group to make sense of the MEA and negotiate the meaning to start doing the MEA.
Supports learners through explanation to avoid language barriers in the process of understanding the MEA and facilitates the use of their strategies and approaches in order to develop concepts in algebra in groups.	Use prior knowledge of the concepts of algebra and relate with the understanding of the MEA. Then work collaboratively in the process of mathematising the MEA.

Reflection

- ✓ Individual and group homework activities which are selected from learners' text book are given.
- ✓ Activities are aimed at providing learners with an appropriate challenge to encourage them to reflect and participate in the class activities based on their real-life experiences in relation to the new intervention.
- ✓ In the following class, the activities are discussed in groups by learners and some groups are selected and encouraged to present the work in a whole class setting.

4.5.3 Description of implementation of the HBPSI

This section creates a narrative of learners' attempts to solve the MEA (Garden Activity) that describes how the HBPSI was implemented in the experimental classrooms. It describes how the learners elicited the mental constructions called for in the genetic decomposition of the MEA during its implementation. It also describes the influence of the MMP and APOS theory that guide the design of the instruction to develop learners' understanding at grade 8 level in algebra. To create this lengthy narrative of learners' attempt to solve the given MEA, the researcher used the data gathered from the audio recordings, figures of learners' answers indicated in the Garden Activity sheet and the field notes taken by the researcher.

The excerpt illustrates the way a group of five learners, referred to as "Group 1", in one of the experimental schools, attempted to solve one of the MEAs referred to as 'Garden Activity' (see Appendix A). This group was selected because the average mark of the five learners was the lowest score compared to other groups in previous mathematics tests. The purpose of this lengthy narrative was to enable the researcher to explain how the new intervention influenced the conceptual understanding of participant learners in the experimental schools in algebra at the level of grade 8. It also enabled the researcher to identify the difficulties learners encountered related to algebraic variables and how learners were able to resolve the difficulties through the guidance of the researcher. It is important to stress that each of the remaining groups in the participant classes received the same treatment.

MEA (Garden Activity)

Context and parameters

Your school director plans to prepare vegetable beds to cultivate vegetables in two separate gardens in the school using learners from grade 5 through grade 8. One of the gardens involves vegetable beds of 1.5 metres by 10 metres and the other 2 metres by 10 metres each. He also divided the learners into two groups. The learners in grade 5 - called the 'elementary group' distributed in nine sections, will prepare beds in one of the gardens and learners in grade 8 - called the 'junior group' - distributed in six sections, will prepare beds in the other garden. To prepare a single bed he decided to assign an equal number of learners, but he let learners discuss and agree on the size of a group. In order to help the director manage the gardening activities and provide the necessary tools available in the school for preparing beds, he wants you to predict the number of vegetable beds that can be prepared by learners from grade 5 through grade 8 on one of school-day.

Mathematical problem

Develop a rule that predicts the number of vegetable beds that can be prepared on a particular day depending on the number of learners from grade 5 through grade 8 in the school.

a) How many vegetable beds can be prepared using both the junior and elementary groups on a particular school-day?
b) How many vegetable beds can be prepared using only the junior group?
c) How many vegetable beds can be prepared using only the elementary group?
d) Find alternative situations so that your developed equation can be used.

The following is the way in which this section is organised: The action conception is described in Section 4.5.3.1. The interiorisation of the action to process conception is presented in Section 4.5.3.2. Through the primary genetic decomposition which has been explained in detail in Sections 2.4.2.1 and 3.6.3.2. The encapsulation of the processes conceptions to objects conceptions is presented in Section 4.5.3.3.

4.5.3.1 Action conception

After distribution of the MEA to every group, the researcher briefly mentioned the way learners in a group work collaboratively to solve the task by stressing the importance of the amount of talk produced by each member in the process of understanding the problem situation and each learner's contribution to solve the task at hand. Features of the first episode are as follows. Every member of the group read and reread the MEA silently and seemed confused to initially start the discussion. After reading the MEA repeatedly, the group started the discussion. In the following excerpt, for example, L13 represents the third learner in group 1 and L15 represents the fifth learner in group 1.

> *L13: I didn't understand what the problem is asking about.*
>
> *L11: The director plan to prepare vegetable beds in the school gardens.*

L14: He wants to prepare 2 metres *by 10* metres *beds in one of the garden and 1.5* metres *by 10* metres *beds in the other garden.*

L15: Who prepare the vegetable beds?

L12: Learners in our school.

L14: [Reread the problem] *No no.... Not all learners in the school prepare vegetable beds in the school gardens. The junior group, grades 8, prepare in one of the garden. But only grade 5 in the elementary group prepare in the other garden.*

L13: Which group prepare 2 metres *by 10* metres *beds and which one prepare 1.5* metres *by 10* metres *beds?*

L11: I don't know. But the director decides that there should be the same number of learners in each bed.

L12: Is number of learners for each bed the same for elementary and junior groups?

L15: I think so.

L14: If that is so, who will decide the number of learners for each bed?

L12: I don't know.

L14: Everybody please read the problem again.

L15: [After members of the group read the problem silently, L15 read it again loud to the group.] *It is asking about the number of vegetable beds that can be prepared by learners in grades 5 and 8 in a particular school-day.*

L14: [Reading the problem statement of the MEA again] *No no.... It is asking to decide the size of a group of learners who will be involved in preparing each vegetable bed.*

L12: How many learners are there in junior group?

L14: What about the number of learners in the elementary group?

L12: Learners in junior group are in 6 sections: 8A, 8B, 8C, 8D, 8E, 8F and those in the elementary group are in nine sections.

L13: But we have to know the number of learners who will be involved in each bed.

L14: [Reading the problem again, L14 starts talking to the group about the problem.] *It is not asking about the numbers of beds or the size in a group but it is asking about a rule to* *predict the number of vegetable beds that can be prepared by learners in grade 5 and grade 8 in a particular school-day.*

L13: What do you mean by a rule?

L14: It is just an equation (or a formula).

L12: Where do we find the equation then?

L11: If a rule is a formula, we can find it in our text book.

L11: I don't know.

L14: But I haven't seen a problem like the given problem in our textbook.

L15: How can we decide the number of learners to prepare 2 metres *by 10* metres *bed?*

L12: Don't forget there are also beds, each 1.5 metres *by 10* metres *to be prepared.*

Learners in the group were faced with a in real-life problem situation and recognised the discussion of preparing vegetable beds as something interesting for the group members and dealing with something they were familiar with at home or at school. The familiarity of the problem situation to learners increased the amount of talk produced by individual group members and created a situation where learners in the group took part in the discussion. As every member of the group shared his/her understanding of the situation of the problem with their peers, the role of the researcher was limited to being just an observer of the discussion.

As learners in the group shared ideas and commented on one another's ideas about the given context, the discussion among the group members helped to facilitate not only their understanding of the problem situation but also their transition to communicate arithmetic in their way to find the solution to the MEA. Although learners did not understand the clear picture of what a rule is, the issue raised by L13 and L14 that indicated their focus on the number of learners would be assigned to each bed shaped the discussion among the group members in the right direction for the solution processes of the MEA. It seemed that all members of the group did not have previous experience of such a mathematical problem in their real-life context that could be responded to construct relationships between key elements of the problem situation in the form of expression or equation.

L14: Please pay attention. I think the question is about a rule to predict the number of vegetable beds that can be prepared by learners in the elementary and junior groups.

L15: How can we determine the rule then?

L14: I told you that I didn't come across such kind of problem in our mathematics lesson.

L11: We have to count first the number learners in the elementary and junior groups.

L15: We have to also decide the number of learners that will be involved in each bed.

L14: How can we fix the number of learners for each bed?

L12: What do we do in preparing vegetable beds?

L14: There are different activities in preparing vegetables beds such as digging, cultivating the soil for vegetables.

L11: But how many learners should be assigned to prepare one vegetable bed?

L13: It is not given. But the director want us decide on the number of learners for each bed.

L15: There are two gardens. In one of the garden we prepare 2 metres *by 10* metres *beds and in the other 1.5* metres *by 10* metres *beds.*

L11: That is not a problem. We need to determine the number of learners for each bed.

L15: Can two learners are enough to prepare a vegetable bed.

L12: I don't know.

L14: What about three or four learners for one bed?

L11: It does not make any difference whether we assign three or four learners for each bed.

L15: If we decide on the size of the group for each bed, we need to know the number of learners in both groups.

L13: We have to ask the teacher about the number of learners in the elementary and junior groups.

L12: There are six sections in junior group.

L11: There are 49 learners in our class.

L15: What about the number of learners in the remaining five sections in the junior group?

L14: Don't forget there are learners in the elementary group; that is in grade 5.

L13: Hmm. That is a problem. We have to ask the teacher about the number of learners in the elementary group?

Although the group members focused their discussion to assign the number of learners in each bed, they did not consider relevant factors that would help them decide the size of a group of learners to prepare one vegetable bed. This is evidenced from *L11's* comment as he says "It does not make any difference whether we assign three or four learners for each bed". As evidenced in the above discussion, learners at this point wanted to immediately assign a particular number of learners to each bed and generalise the solution of the MEA by making use of the number of learners in grades 5 and 8. In the following episode, the researcher guided the discussion to help group members to explain how they decided the number of learners for each bed.

Researcher: L11 you said that "it does not make any difference whether we assign 3 or 4 learners for each bed". Would you please further explain your reason?

L11: I don't know. I simply assigned learners for one bed.

Researcher: You have two groups of learners, namely, elementary and junior groups. The vegetable beds that should be prepared are in two different gardens with different dimensions. There will be 2 metres *by 10* metres *beds in one of the garden and 1.5* metres *by 10* metres *in the other garden. Do you want to assign any arbitrary bed to any group of learners from grade 5 through grade 8?*

L14: I think it will be fair if we assign the garden with the smaller beds (i.e., 1.5 metres *by 10* metres *beds) for the elementary group.*

Researcher: Why?

L13: Because learners in the elementary group are younger compared to learners in the junior group.

L12: What is the difference?

L13: I think it is better to cover a garden with a greater portion by the elder learners.

Researcher: Do all of you agree on L13's suggestion?

[All group members agree on the issues raised by L13 and show their agreement by nodding their heads.]

Researcher: Therefore you use fairness as a relevant factor in preparing smaller beds with dimension 1.5 metres *by 10* metres *each for learners in the elementary group and the remaining beds with dimension 2* metres *by 10* metres *for learners in the junior group. Similarly, is there anything you need to consider to decide the number of learners for preparing a single bed?*

L15: The director wants us to decide the size of the group for each bed.

Researcher: L14 repeatedly told you that the question is about developing a rule to predict the number of vegetable beds learners can prepare.

L13: But I believe that it is better to get the rule from the teacher (i.e., the researcher). We will get the one that will work for our problem as we didn't come across such problem before.

Researcher: L11 what can you say about L13's suggestion?

L11: I agree with my friend's suggestion.

Researcher: Actually I am here to help or guide you develop a rule. Based on your understanding about the problem situation, I feel that you have done a lot to do so. Therefore it is better for the group to develop a rule by yourselves. To this end, you need to discuss on relevant factors that help you decide on the size of the group for each bed.

L11: It is not clear, teacher.

Researcher: [Group members keep silent] *Is there anyone who can show us 2* metres *by 10* metres *vegetable bed look like on the ground?*

L12: [Sketches a rectangle and writes 2 metres and 10 metres as its dimension in the group sheet. By pointing from one corner of the classroom to the other corner] *I think the length from that corner to that can be 10* metres*.*

L13: No no... It should be more than 10 metres*.*

Researcher: [Measures the length of the classroom by using footstep as other learners keep silent.] *It is around 7* metres*.*

L15: Hmm. Therefore the length between the two corners is small.

L12: Preparing 2 metres *by 10* metres *bed may take more time and a burden for learners.*

Researcher: Would you please explain the burden you are talking about?

L15: For example it may take more time for two or three learners.

L14: There are assignments, homework and other activities. We need time for studying and preparing for examination.

Researcher: What has all this to do with in preparing vegetable beds?

L12: We need more time to study during examination period. After preparing the beds, it may be difficult for two or three learners in cultivating the vegetables.

L15: We have also other activities outside the school.

L12: We have to give more time to study and do homework and assignments.

L14: We have to participate in vegetable cultivation in such a way that the activities should not affect our learning.

L15: [Agrees with L14's suggestion by nodding his head.] *Therefore we have to increase the number of learners for each bed.*

Although two of the group members (i.e., *L11* and *L13*) took part in limited instances in the above discussion, the remaining three members brought the mathematics into existence through their talk about the problem situation. The issue of fairness raised by L14 as a relevant factor in assigning the garden with the smaller beds for learners in the elementary group shaped the discussion of group members in considering other relevant factors to determine the size of a group of learners for a single bed. The burden of two or three learners in watering a 2 metres by 10 metres bed after preparing the beds raised by L13 play an important role in considering different factors before assigning the size of a group of learners for each bed. To bring L11 and L13 into the discussion with the remaining group members and initiate the discussion centred on every member of the group, the researcher took part in the discussion. In the following episode, the researcher guided the discussion of group members in deciding group size for preparing each bed. Learners considered different factors such as study time, examination period, and class absenteeism as relevant factors to decide on the number of learners for each bed. Such learners' discussions about the problem situation facilitate the solution processes of the MEA and guided the new intervention as intended in the heuristic instructional approach.

L11: It seems difficult for three or four learners.

L12: What about five learners in a bed?

L14: It seems reasonable to consider five learners in each bed.

L13: There are 49 learners in our class. Therefore 49 learners can cultivate nine to ten vegetable beds.

Researcher: Are you sure all learners in your class will come to school every day?

L12: Not clear teacher.

Researcher: We know that learners are often absent from class. That is, not all learners in one class are coming to school in a particular day.

L14: *Hmm. For example, there are only 44 learners present in our class today. five learners are absent.*

L13: It is difficult to predict the number of learners coming to school a particular day.

Researcher: Do you think it is better to consider one more learner in a group?

L15: Therefore let us assign exactly six learners per bed.

Researcher: When you say exactly six learners for each bed, you mean there is no possibility for less than six or more than six learners per bed.

L14: It will not be fair if the size of the group vary.

Researcher: For example, you said that there are 49 learners in your class. You are left with one after learners distributing in eight beds, six in each bed. What do you do in such cases?

L13: Hmm. It is not fair to give one bed for one learner.

L14: What do you do if four learners left in other cases?

L15: Again it is not fair even to give a bed for four learners.

L11: What do we do then?

L12: Why don't we leave this issue for the director?

Researcher: Good. There may be some groups in need of assistance due to different reasons. Therefore the director may use such opportunity to handle different problems. Do you all agree the suggestion given by L12? [Every member of the group shows their agreement by nodding their heads.]

As evidence in the following episode learners not only identified quantities associated with vegetable cultivation in the school garden but also, with the guidance of the researcher, selected some of them among others which were relevant to predict the number of vegetable beds learners in grades 5 and 8 could cultivate. Learners in the group also decided the relevant quantities as variables and constants based on their rich understanding of key elements in the problem situation that vary at any particular time or quantities that stayed the same. Finally, learners in the group represented the selected relevant quantities by any letter of their choice to develop a formula to facilitate the solution of the MEA.

Researcher: You have already fixed the number of learners that can prepare vegetables for one bed by considering situation needed to cultivate vegetables. What do you do next to predict the total number of vegetable beds that can be prepared by learners in the elementary and junior groups?

L13: I don't understand.

Researcher: To develop your rule one key element you identified in the given problem is the size of the group for each bed. Similarly there are other key elements (quantities) you need to identify.

L11: The number of learners assigned for each bed is six.

L14: We can also ask the teacher about the number of learners in the elementary and junior groups.

L11: Let us ask the teacher.

Researcher: You already mention different numerical values in the problem situation such as number of learners in your class and others. We can consider these values as quantities in the problem situation.

L15: It is not clear for me.

L13: Number of learners in our school.

Researcher: But you are asked quantities associated with the given problem.

L15: Number of learners in the elementary and junior groups.

L13: Number of learners in the elementary group in a particular school-day.

L14: Number of watering cans in the school.

L11: Number of learners in the junior group coming to school in a particular day.

L12: Number of vegetable beds that learners in the elementary and junior groups want to prepare.

Researcher: Can you tell more?

L15: Number of learners assigned to prepare vegetable bed.

Researcher: Is that all?

L11: Yes.

Researcher: Do you use all these quantities to develop your rule?

L11: I don't know.

L12: I don't think so.

Researcher: L12 would you please explain your reason to L11.

L12: We list many. Some of them are enough to predict the number of vegetable beds learners can prepare.

Researcher: Can you please select some quantities among others which are relevant in developing our rule?

L13: Number of learners in the junior group in a particular school-day.

L15: Number of learners in the elementary group in a particular school-day.

L14: Number of learners assigned to prepare each bed.

Researcher: Is that all?

L12: Yes.

L14: These three quantities are the relevant ones.

Researcher: What do you want to do using these quantities?

L11: We agree that the problem is developing a rule to predict the number of beds learners in the elementary and junior groups want to prepare in a particular school-day.

L14: Therefore, total number of vegetable beds is also relevant.

Researcher: Which of them are varying quantities and which are fixed quantities?

L13: It is not clear, teacher.

Researcher: From your selected quantities there are quantities which are variables and constant(s).

L11: I cannot see any variable in the problem?

L13: It is not also clear for me.

L15: There is no x or y in our problem. Where are the variables?

Researcher: L14, do you have something to say to L13 about L15's question?

L14: We have learned that x or y is a variable representing unknown numbers.

Researcher: Suppose x' is a variable representing positive quantities. L11, can you give any meaning to x^2*?*

L11: I don't know.

L14: x^2 *may represent area of a square.*

L13: Area of a square is s^2*.*

L11: No. The area of a square should be $A = s^2$*.*

L15: There is no difference whether you use s or x.

Researcher: Why?

L15: Both x and s are letters or symbols that stands for numbers.

Researcher: You can use any letter of your choice to describe a situation in a problem (e.g., area of a square). Therefore different letters can be used to describe the same situation in a problem. Although s and x are two different letters, they could stand for the same number.

As evidenced from the discussion learners understood one aspect of the interpretation of variables. Although group members in their previous discussion agreed about the number of learners enrolled in their class and those who come to class on a particular day as two different quantities, they did not recognise the latter as an unknown quantity. That is, they were unable to recognise the presence of unknown quantities found in the problem situation of the given MEA. Learners (L11 and L13) gave meaning to the letter s in the expression "s^2" to only the length of the side of a square instead of seeing it as an entity representing numbers with algebraic properties. For these learners, the letter "s" served as an object and restricted its usage in an expression. Furthermore, L11 had difficulty in understanding the area of a square without writing the letter "A" that stands for area of a square. In the following episode, with the guidance of the researcher, learners in the group used their understanding of the problem situation to recognise the existence of unknown quantities in the given MEA to understand the mathematical notions of variables and constants. Furthermore the researcher tried to identify the difficulties and misconceptions of learner(s) related to the different role and interpretation of unknown quantities. The researcher guided the discussion to broaden the learners' interpretation of a variable.

Researcher: As some of you said letters you are familiar with in an expression or equation in your text book represent variables to stand for numbers. Unknown quantities (variables) which can be recognised in a situation of a problem can also be represented by letters as variables. Therefore we have variables in an algebraic expression or equation. We can also recognise them in a situation of a problem. For example, is there anyone in the group who can tell us about the number of learners who will come to school tomorrow?

L11: It is 49.

L13: How do we know the actual number of learners who come to school tomorrow?

L13: 45, or 36. I don't know... may be less.

Researcher: What about the number of learners who will come to prepare vegetable beds next week in a particular school-day?

L12: It is not known as some learners may be absent during that day.

Researcher: Such unknown quantities that can be recognised in a problem situation are variables. Quantities that stay the same are constants.

L14: Therefore, number of learners in the junior group who will involve in preparing vegetable beds in a particular day is variable.

L15: The same is true for elementary group.

L12: What about the number of learners assigned to prepare each bed?

L13: This is clear. We assigned six learners. The number six is constant.

L13: The number of beds that will be prepared in a particular day is also a variable.

L15: How could it be if the number of learners for each bed is constant?

L11: Yes. Number of vegetable beds is collection of beds each prepared by a group of six learners. It is a constant.

As evidenced from the above episode, learners in the group used their understanding about the problem situation as a basis for understanding the mathematical notions of variables and constants. However some learners (L11 and L15) in the group encountered difficulty in understanding the difference between the quantities that represented the number of learners in the elementary or junior group in the school and those from the groups that would be involved during the preparation of vegetable beds on a particular day. Therefore, they interpreted the number of vegetable beds that would be prepared by the learners in terms of only the number of learners assigned for each bed without considering the unknown number of learners who were expected to come to school on any of the school-days the following week.

Researcher: L14, do you agree with L11?

L14: I don't agree. Because...

Researcher: L14, would you please tell your reason to L15.

L14: Because number of beds that will be prepared in a particular day will be more if more learners come to school during that particular day.

Researcher: Is there anything which is not clear? [See learners' understanding from their facial expressions] *Ok good.*

As recognised previously, some of the learners had difficulties in their usage of letters to represent unknown quantities. The researcher guided the discussion in the following episode, by focusing on these learners to overcome their difficulties.

Researcher: Can you please represent the relevant variables with letters of your choice and a constant by number.

After a short discussion the group represent the relevant quantities as follows:

L14: Let 'y' represent the number of vegetable beds that will be prepared by learners in the elementary and junior groups.

Researcher: L11, can you represent another relevant quantity?

L11: Let 'w' represent the number of learners in the elementary group in a particular school-day.

L13: Let 'x' represent the number of learners in the junior group in a particular school-day.

L15: We said '6' represent the group size to prepare a single bed.

As evidenced in the above dialogue L12's and L14's explanation about the number of learners involved in preparing vegetable beds on a particular day guided the explanation of the concept of the variables and constants. Learners' understanding of the problem context in the group discussion helped the researcher to explain the mathematical concepts of variables and constants on the basis of their knowledge of the situation of the MEA. This facilitated the solution process of the MEA. L13's thought about the maximum number of learners that would come to school was demonstrated by her external representation as she listed numbers such as 48, 47, 46, or 44. Although at the beginning, learners in the group experienced certain difficulties in recognising and identifying the presences of variables and constants in the problem situation, these difficulties were resolved through group discussion, with the guidance of the researcher. Learners further revealed their understanding of the variables in the problem situation by representing with letters to show their relationship in an algebraic expression that helped them develop the required algebraic rule to predict the number of vegetable beds that could be prepared and then cultivated by learners in the elementary and junior groups. The next episode explains how learners constructed an algebraic expression based on their understanding of variables and constants in the problem situation by making use of the identified relevant quantities.

Researcher: Next, using the letters you have chosen, formulate an expression that helps you predict number of vegetable beds that will be prepared in a particular day next week.

L13: It is not clear teacher.

Researcher: I mean you represent four relevant variables, among others. We've y, w, x and 6 as relevant variables. What do you do with these variables?

L11: We represent the variables to determine number of vegetable beds.

L14: w, x represent number of learners in the elementary and junior groups who come to school in a particular school-day.

L12: But y represents number of vegetable beds learners in the elementary and junior groups want to prepare.

Researcher: What do you do with w and x?

L13: Not clear teacher.

Researcher: What do you get when you apply the operational symbol(s) with w and x?

L15: When we combine them we get number of learners in both groups in a particular school-day.

Researcher: How do you combine them?

L11: When we multiplied w and x we get number of learners in both groups.

L14: No. We have to combine them.

L12: How do we do that?

L14: We have to use addition to get number of learners in both groups.

L11: Why not multiplication?

L13: I don't think multiplication works to get number of learners in both groups.

L15: If we apply multiplication the number becomes very large.

Researcher: Does it make sense to multiply w and x?

L13: It is not clear.

Researcher: [Counts the number of boys and girls in the class] *We have 24 girls and 20 boys in the class. Which operation, addition or multiplication, do you think appropriate to determine the number of learners in the class?*

L13: It should be addition.

Some group members have difficulty in constructing meaning when applying operation on the unknown variables. In the above discussion L11 was not able to see whether or not the operation he used was appropriate. He simply performed the multiplication operation on *w* and *x* so that his answer did not make sense.

L14: We need first describe number of learners in both groups in a particular school-day.

L11: The number of learners in both groups coming to school in a particular school-day should be x+w.

L14: But how can we determine number of vegetable beds that learners in the elementary and junior groups want to prepare?

Researcher: Good, you have also other variables; namely the unknown variable y and 6. How can the group show me an expression of your own that represent the number of learners in the school garden in preparing vegetable beds?

L13: We said that y represent number of vegetable beds learners in the elementary and junior groups want to prepare.

L15: We also assign six learners per bed.

Researcher: What do you do with y and 6?

L12: I don't understand.

Researcher: How can the number of learners in both groups can be described differently using letters other than w and x?

L15: We have already described number of learners in both groups using w and x.

Researcher: You can also describe the number of learners in both the elementary and junior groups that will be involved in the gardening activity using variables different from w and x. That is, we can describe the same situation in different ways.

Researcher: For example, $\frac{1}{2} and \frac{2}{4}$ *are equal.*

L13: $\frac{1}{2} and \frac{2}{4}$ *are equivalent fractions.*

Researcher: What do you mean by equivalent fraction?

L14: The two numbers are the same.

L14: Although they look different, they are the same number.

Researcher: Similarly w+x describe number of learners in both the elementary and junior groups in a particular school-day. The expression formed by y and 6, in this case; also describe the same situation in the given MEA. We need to write an expression that describe w+x differently but both should describe the same situation in the problem.

L14: Can we do this with y and 6?

Researcher: Yes.

L13: It is not clear for me.

Researcher: For example, for one bed we assigned six learners. For two beds there are twelve learners and so on.

L15: What about for three beds?

L14: It should be 18 learners.

Researcher: Therefore, 18 learners are expected to be involved in preparing 3 vegetable beds. What about for 10 beds?

L13: We have to add 6 ten times.

L14: We can also multiply 6 by 10 to get number of learners expected to prepare 10 beds.

Researcher: Can you generalise this for y beds?

L15: We have to multiply y by 6 to get number of learners expected in the school garden to prepare beds in a particular day.

Researcher: Therefore multiplying y by 6, that is 6y, will give us the number of learners that are expected in the school garden to prepare all the smaller and bigger beds in a particular school-day.

L15: Do we mean both w+x and 6y describe number of learners in both groups?

Researcher: w+x and 6y are two different expressions that describe the same situation. That is, although they look different, both describe the number of learners in both the elementary and junior groups that will come to school in a particular school-day.

As evidenced from the above discussion, learners were unable to interpret the concept of the equality of two terms in the situation of the problem. Although learners interpreted the concept of the equivalence of two terms using two fractions, their understanding of variables as meaningless symbols did not help them to interpret an expression formed by variables in the problem situation as a descriptor of a situation. Therefore they did not understand the concept of equality of two expressions which were formed by unknown variables identified in the problem situation of the MEA. In the following episode, learners use the concept of equality to use w+x and 6y as different terms that describe the same situation in the MEA. They formed an equation that could lead to deriving the required algebraic rule to predict the number of all vegetable beds which would be prepared on a particular day. As a result, learners' difficulties were resolved through group discussion, with the guidance of the researcher.

L14: But the question is about number of vegetable beds that can be prepared in a particular day by learners in both groups.

Researcher: To determine number of vegetable beds, we have to use the equal sign to connect the terms 6y and w+x.

L13: Do you mean 6y and w+x equal?

Researcher: I mean the term 6y can replace the term w+x for the purpose of describing the number of learners both in the elementary and junior groups in a particular school-day. Therefore, the equal sign that connects these two expressions is to mean what is on the left side of the equal sign is the same as what is on the right side.

L13: Therefore 6y = w+x.

L11: How do we get the number of vegetable beds then?

L14: We have to divide both sides by 6 to get the number of vegetable beds.

L13: I don't know.

L15: When we divide by 6 we get $y = \frac{w}{6} + \frac{x}{6}$.

Therefore, the number of vegetable beds that are expected to be watered is $y \text{ or } \frac{w}{6} + \frac{x}{6}$.

L13: Why do you write 6 two times. We have only one 6.

Researcher: L13, what do you think? Do you have another way of writing?

L13: [Writes the equation] *It must be like this,* $y = \frac{w}{6} + x$.

Researcher: Is that the way to divide binomial by a number? When a number crosses the equal sign to divide an expression involving more than one term it should divide every term of the given expression. Accordingly which one is the correct way of writing? Is $y = \frac{w}{6} + \frac{x}{6}$ *or* $y = \frac{w}{6} + x$?

L14: $y = \frac{w}{6} + \frac{x}{6}$ *is the correct way of writing for y.*

Researcher: In fact, $y = \frac{w}{6} + \frac{x}{6} = \frac{w + x}{6}$. *Do all of you agree?*

[Every member of the group expresses their understanding by nodding their heads.]

L11: Therefore, what is the answer? My question is where is the number for the number of beds?

L13: What is the number when we divide $w + x$ *by 6?*

Researcher: There is no numerical answer at this point because the letters w, x and y are variables.

L13: I don't know.

Researcher: To determine the number of vegetable beds that learners will prepare in a particular day, we need to know the actual number of learners who will come to school in that particular school-day. After all, the question of the problem is to develop a rule. Therefore, the number of vegetable beds is expressed in terms of the expression $\frac{w}{6} + \frac{x}{6} or \frac{w + x}{6}$.

As evidenced in the above episode some of the learners (L11 and L13) tended to use the equal sign as a symbol which separates a problem from its answer as they experienced in the elementary level. Through group discussion with the guidance of the researcher, L11 and L13 also understood the concept of the equal sign as a symbol that connect two expressions that describe the same situation of the problem. They realised that the two terms separated by the equal sign represent the same numerical value and it is possible to replace one with the other. In the next episode learners were asked to apply the rule to some problems in the MEA.

Researcher: How many vegetable beds will be prepared only by learners in the junior group if they all come to school in that particular school-day?

L15: That means we have to ask the number of learners enrolled in the junior classes.

L15: What about learners in the elementary group?

L14: We are asked to use learners in the junior group.

L13: What do we put for w in our rule?

L14: It must be zero.

Researcher: The number of learners in the junior group is 300.

Researcher: [After a short discussion of group members they wrote the answer for the problem.]

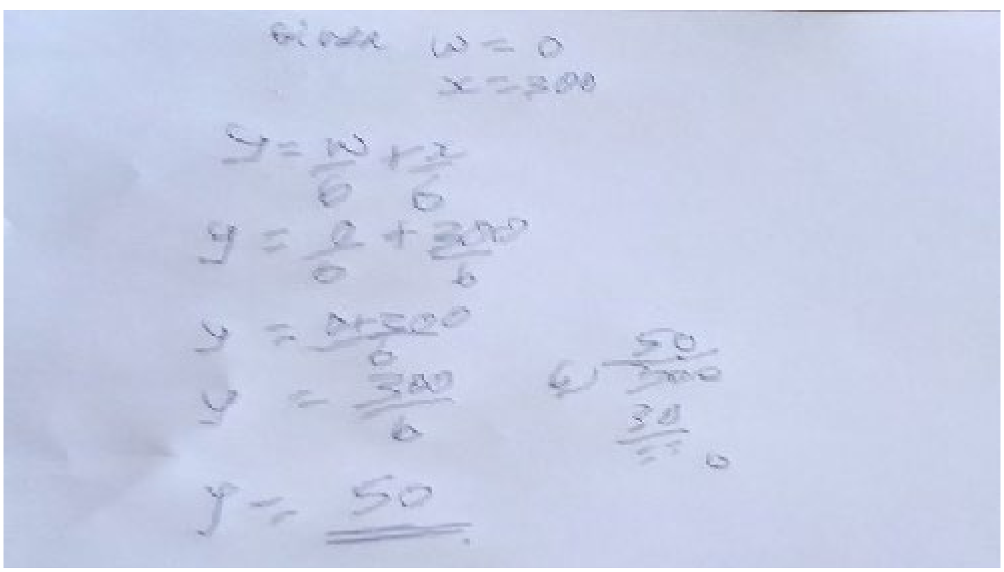

Figure 4.1: Solution given by group 1 learners

Researcher: Please explain your answer?

L14: 300 learners in a junior group can prepare 50 vegetable beds.

Researcher: In a similar way determine the number of vegetable beds for the elementary group if all the 452 learners come to school in that particular school-day?

After a short discussion of group members they wrote the answer.

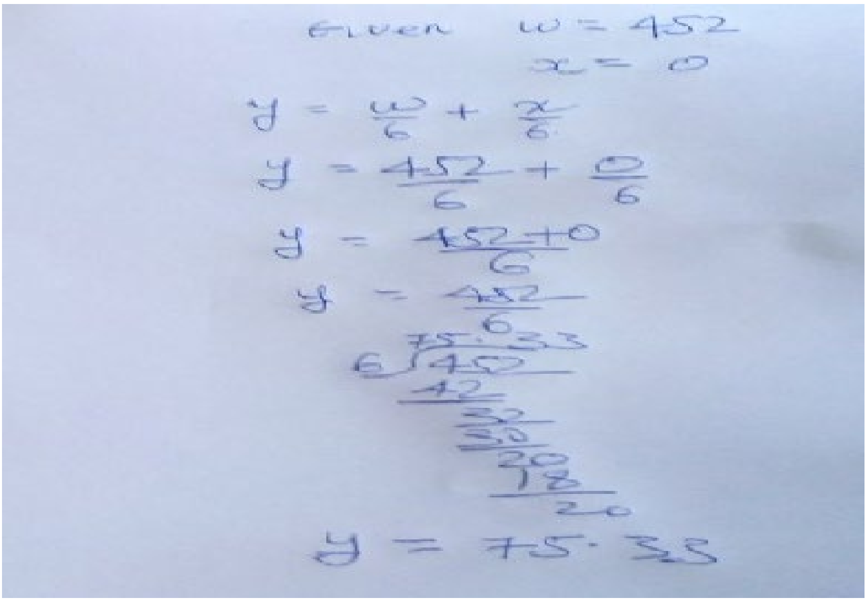

Figure 4.2: Solution given by group 1 learners

Researcher: Please explain your answer?

L15: It is 75.33 vegetable beds.

L12: No. It should be 75.

L14: For each bed we assigned six learners. Therefore in 75 beds we distributed 450 learners. Finally we are left with two learners.

L13: We agreed that we do not assign less than six learners in a bed.

Researcher: What about with learners in both groups if all learners in each group come to school in that particular school-day?

After a short discussion, group members wrote the answer.

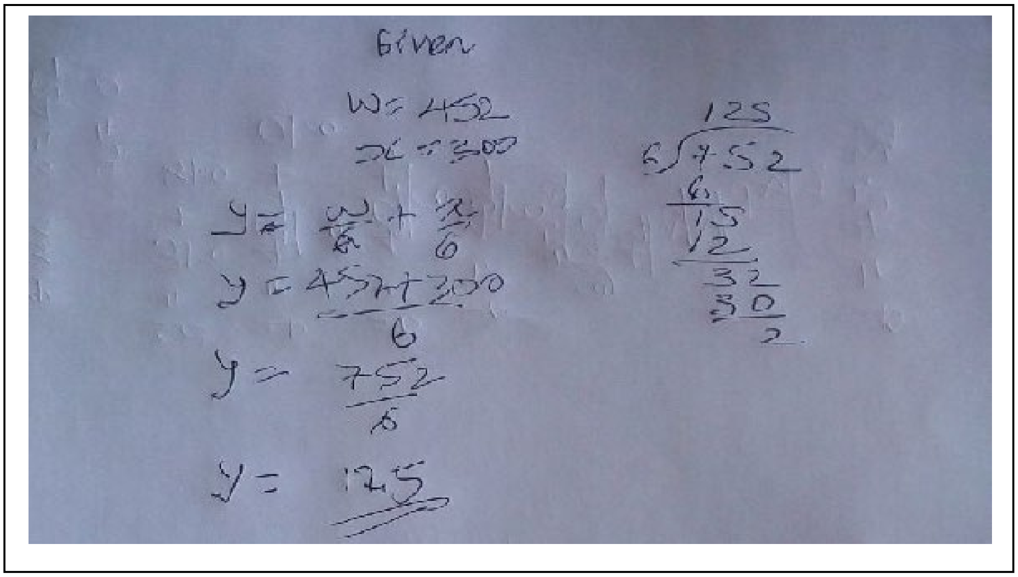

Figure 4.3: Solution given by group 1 learners

4.5.3.2 Process conception

Researcher: How many learners are expected in preparing 47 vegetable beds each 2 metres *by 10* metres*?*

L13: Did both groups participate?

L14: We decided learners in the elementary group for the smaller beds.

L15: Therefore the 47 beds are for learners in the junior group.

L12: That is, learners in the elementary group are not involved in preparing the larger beds.

L11: Yes. Therefore w is zero.

L13: Does our rule works to determine the number of learners?

L14: Yes.

L11: How does it work?

L14: [Start explaining how it works and write the answer] *We have 47 beds for y, 0 for w and 6 for the size of each group. Therefore we are left with x.*

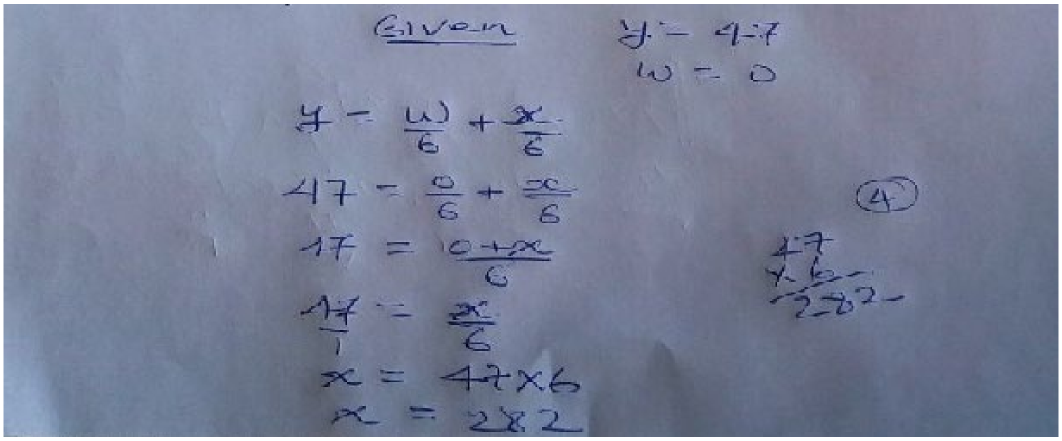

Figure 4.4: Solution given by group 1 learners

L13: 282 learners in the junior group are expected to prepare 47 beds.

Researcher: Good. You are correct. Can you determine the number of learners in each group that can prepare 100 beds, if each group prepare equal number of beds?

L13: Both groups prepare the same number of vegetable beds.

L12: Does it mean learners in the elementary and junior groups prepare 100 beds each?

L15: No. It means that each group prepare 50 beds. Altogether they prepare 100 beds.

Researcher: Can you do this with your rule? [L15 verifies his answer using the rule as follows.]

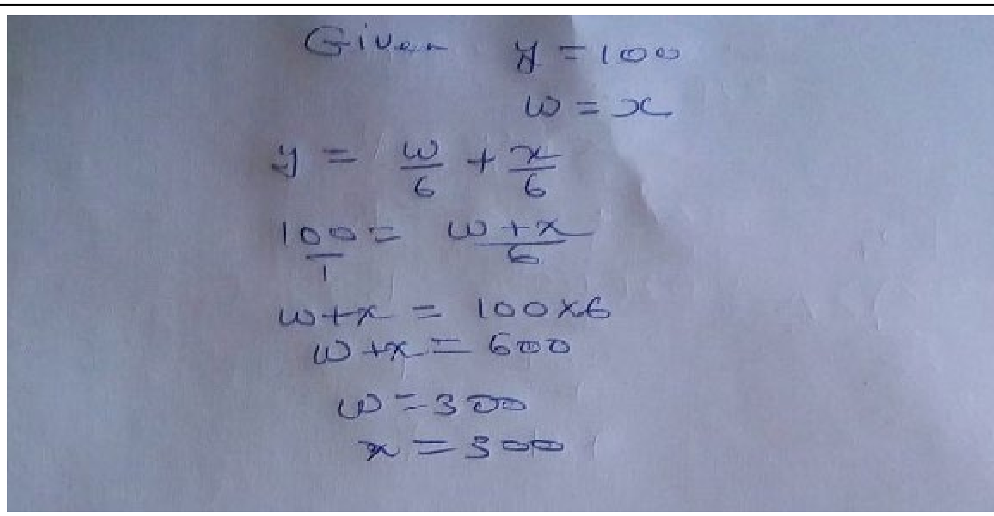

Figure 4.5: Solution given by group 1 learners

The learners in the group, in the episode above, applied the inverse of their developed rule to predict the number of learners in each group who come to school on a particular day knowing the number of vegetable beds prepared on that particular day. The learners gradually moved from calculating the output value from the input value(s) to the processes of calculating the input value(s) from the output value. The learners interiorised the inverse of their developed rule as a rule to describe a set of elements in the same situation. The learners in the group, as seen in the next section, encapsulated their processes conceptions into objects conceptions by generalising the algebraic rule they developed to other similar but different real-life situations.

4.5.3.3 Object conception

Researcher: The algebraic rule we developed was required to predict the number of vegetable beds that learners from grade 5 through grade 8 are able to prepare in a particular school day in the dry season of the academic year based on the information given on the number of learners that are assigned to prepare the smaller and bigger beds. Can we use the rule we have for other situations?

L12: I don't understand the question.

Researcher: You have considered different cases to develop the algebraic rule. Can you use the same rule but with a different situation?

L14: We consider different factors in developing the rule.

Researcher: Yes. Can you find a different situation which requires you to reconsider the factors?

Researcher: Don't forget that you will prepare the beds to cultivate vegetables in the dry season.

L14: Can the rule apply during the rainy season?

L12: What is the difference whether it is in the dry or rainy season?

Researcher: Do you think cultivation of vegetables in the rainy and dry seasons is the same?

L15: During the rainy season watering the vegetable by learners may not be needed.

L13: Why?

L11: The rain may be enough.

L14: Therefore we can reduce the number of learners in one bed.

L15: If learners need not fetch water in the water tank, three or four learners are enough to cultivate vegetables.

Researcher: Can you still use your rule to predict the number of vegetable beds that can be prepared during the rainy season?

L14: Yes.

Researcher: How do you modify the rule then?

L15: We have few learners per bed in the rainy season compared to the dry season.

L12: Let us take four learners for each bed.

Researcher: Can you modify your rule so that it can fit to the new situation?

L14: We have to replace 6 by 4 in our rule.

After a short discussion among group members, they were able to modify the rule.

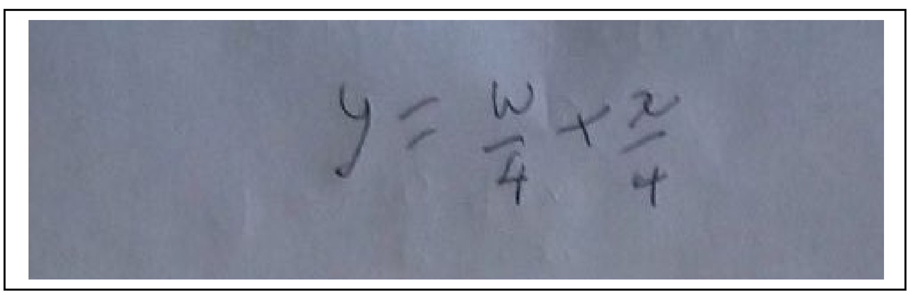

Figure 4.6: Solution given by group 1 learners

Researcher: Use your modified rule to predict the number of vegetable beds that can be prepared by both groups in a particular day if they all come to school.

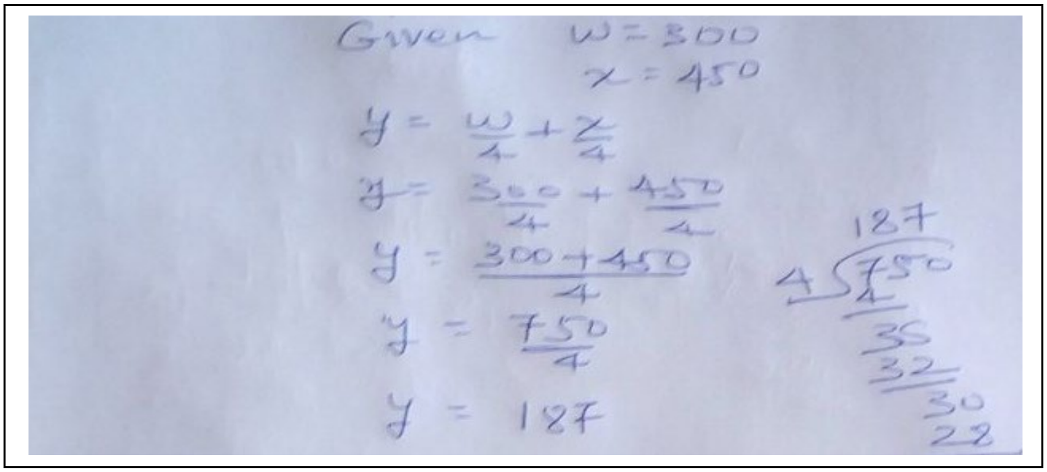

Figure 4.7: Solution given by group 1 learners

L11: Learners in both groups can prepare 187 beds in a particular day.

Researcher: What about the number of vegetable beds that can be prepared by only the junior group in a particular day if they all come to school?

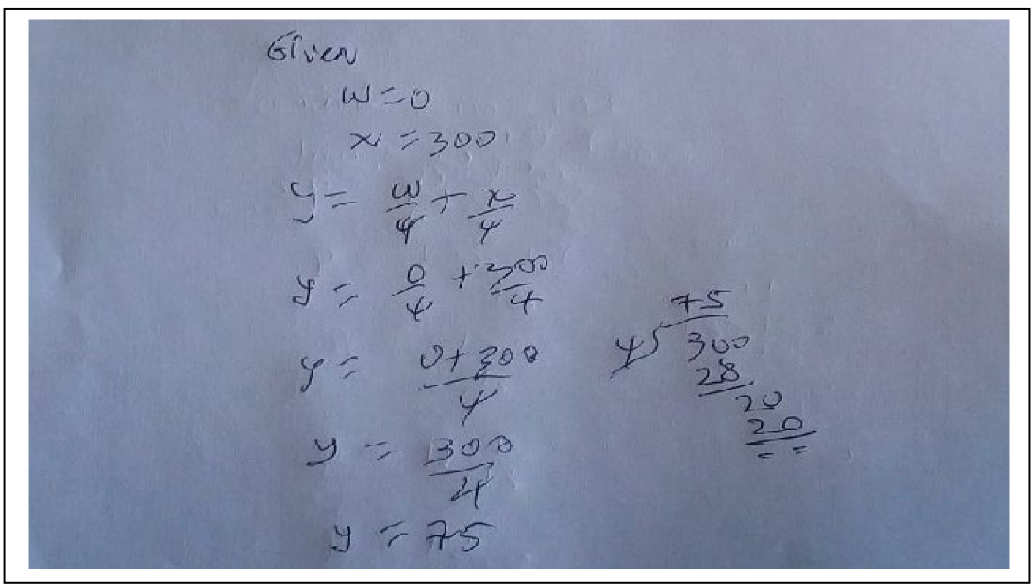

Figure 4.8: Solution given by group 1 learners

Researcher: Can you describe other situation so that our rule also works? Suppose the school arrange a visiting programme for grade 8 learners and their teachers to visit the animal farm 8 kilo metres *from your school after the national examination to leave* *primary schooling. Can you modify your developed rule to determine the number of mini-buses required for transportation?*

L11: I don't understand.

L13: Does the programme exclude learners in grade 5?

L12: In this case only grade 8 learners and their teachers are involved in the visit programme.

L15: We develop our rule to cultivate vegetables.

L11: Does it also works for visiting programmes?

Researcher: You can even flexibly use for other situations.

L13: What do we do then?

L14: We can use our rule to determine the number of mini-buses needed to transport visitors.

L13: How can we modify?

L15: We can replace grade 8 learners and their teachers by junior and elementary groups.

Researcher: L11, can you represent learners in grade 8 and their teachers by any letter?

L11: Let 'm' represent the number of grade 8 learners and 'n' be the number of their teachers.

L13: Are grade 8 learners and their teachers variables?

L11: Number of grade 8 learners is variable but that of teachers of grade 8 is constant.

L14: I don't think so.

L12: Why?

L13: Teachers may not also involve in the visiting programme.

L15: In that case both are variables.

L14: Some learners and teachers may not come to school in a particular school day.

L14: Similar situation may happen during the trip.

L12: How do you modify the rule then?

L15: Replace w by m and x by n.

L13: What would be the rule then?

Researcher: What about y in the previous rule?

L11: The unknown y represents the number of vegetable beds.

L14: But now we want to determine the number of mini-buses needed to transport visitors from school to the animal farm.

Researcher: L13 would you please represent the number of mini-buses needed to transport visitors?

L13: Let 'p' represent number of mini-buses.

L11: Now let us write the rule.

Researcher: Is that all?

L15: Yes.

Researcher: What about the constant in your previous rule?

L14: We assigned six learners to prepare a vegetable bed during the dry season.

L11: What is the number then in this case?

L13: I don't know.

L15: Six was the number of learners for one bed.

L12: We have to take the number of visitors in one mini-bus.

L11: How many people can a mini-bus accommodate?

L13: I think it is 13.

After a short discussion, L14 wrote the modified rule as follows:

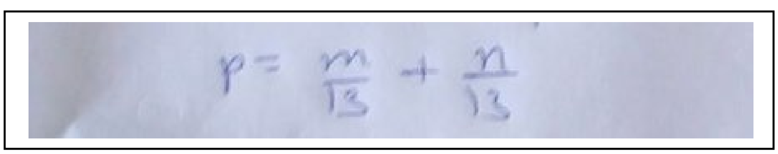

Figure 4.9: Solution given by group 1 learners

Researcher: How many mini-buses are required to transport all the invited visitors?

L13: How many learners are here in grade 8?

L11: What about the number of teachers teaching in grade 8?

L12: We can ask the teacher.

Researcher: There are 145 grade 8 learners and seven teachers.

After a short discussion, learners wrote the answer.

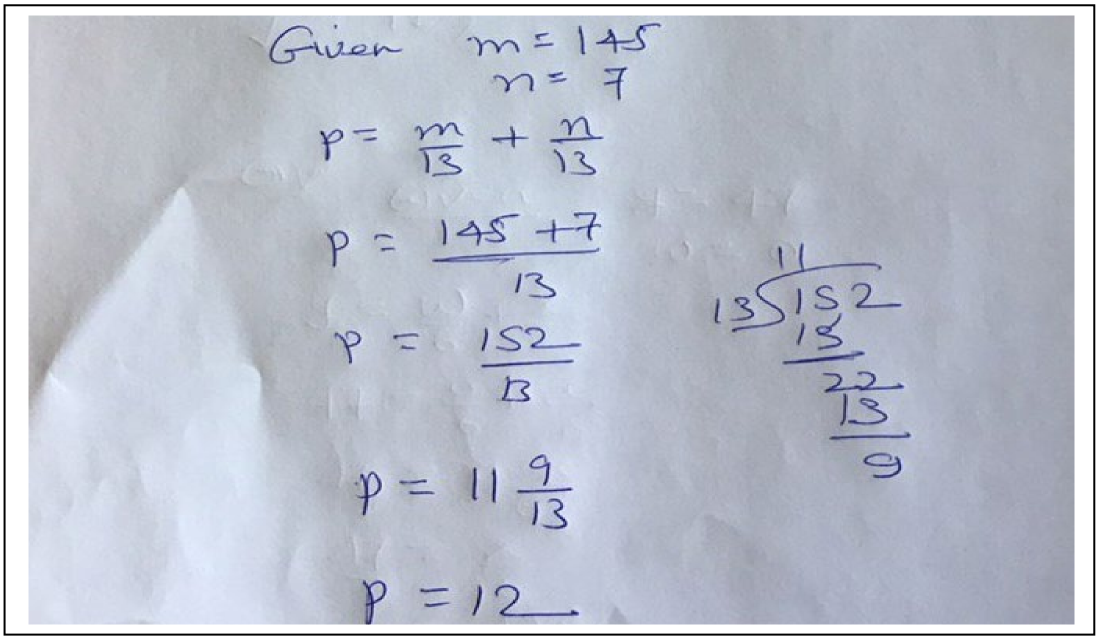

Figure 4.10: Solution given by group 1 learners

L13: There must be 11 mini-buses to transport visitors to the animal farm. But there are nine people left.

L12: It should be 12 mini-buses, one of which would take only nine visitors.

Researcher: Can you further explain your answer?

L13: Using our rule we have got 152 over 13. To transport 152 learners, we need 12 mini-buses.

With the support of the researcher through guiding questions, the new intervention lead learners to construct deep understanding of the mathematical concept involved in the given MEA based on their understanding of the problem situation which enabled them to transfer this understanding to other real-life situations. After accomplishing the process of the reconstruction of the developed algebraic rule to fit the new situation, learners in the group guided by the researcher were able to see their algebraic rule as a useful tool and adapted their previous understanding of concepts in mathematics to perform operations that satisfied the conditions of the new situation.

4.5.4 Final Remarks that Conclude the Implementation of the HBPSI

- The integration of the MMP and APOS theories allowed learners to reconstruct their previous knowledge to a deeper and more meaning full understanding of algebraic expressions.
- The guidance of the researcher during group discussion and the guiding questions to prompt learners based on learners' existing knowledge was conducted in a way that valued and encouraged participation among group members.

4.6 GRADE 8 LEARNERS DIFFICULTIES WITH ALGEBRAIC VARIABLES DURING THE MODELLING PROCESS

With an eye to answering research question 3, the researcher was focused on finding evidence of learners' understanding of variables from their conversational statements based on the Garden MEA. In this section, from the data collected during the action conception stage during the HBPSI model lesson, only those data in Section 4.5 in the HBPSI model lesson that exhibited learners' background knowledge on variables at the level of grade 8 was selected and assessed. In the following the analyses of the discussions of four groups, two in each experimental school (Group A, Group B, Group C, and Group D) are presented due to space constraints. A letter and number, for example A1, indicate the group (Group A) and the first learner in group A. Similarly, C4 would indicate the group (Group C) and the fourth learner in group C.

4.6.1 Difficulty in recognising variables in the problem situation

In responding to the Garden Activity (MEA), learners needed to recognise the presence of unknown quantities in the context of the problem. It was observed that participating learners in grade 8 have limited understanding of variables in this sense, which was exhibited in their modelling process, as shown below in the conversational statements.

Researcher: From your selected quantities in the Garden Activity there are quantities which are variables (unknown quantities) and constants.

A1: I cannot see any variable in the problem?

A3: It is not also clear for me.

A5: There is no x or y in our problem.

B4: In the given problem, are there any variables?

B2: There are variables such as x or y only in an expression or equation.

C5: Where are the variables?

C1: It is not also clear for me.

C3: I don't understand.

From the discussion, learners in the groups seemed to explain their understanding of a variable as a letter (symbol) found only in an expression or equation. From this it can be said that learners were unable to recognise and identify the presence of something unknown in a problem situation. According to Ursini and Trigueros (2001), this ability, among others, is necessary to understand the use of a variable as an unknown quantity in a problem situation. The discussion above and the learning episode of the heuristic-based teaching sequence at the action conception stage suggests that the conventional teaching methods being used by teachers in learners' previous schooling (grades 5–8) did not help them to recognise and identify the presence of unknown quantities in a problem situation.

4.6.2 Difficulty in representing unknown quantities using letters

As evidenced from the discussion, learners also encountered difficulties in representing unknown quantities in the given MEA using letters.

Researcher: Can you please represent the relevant variables you identified in the given MEA with letters of your choice and a constant by number.

A3: It is not clear teacher.
A5: I do not understand.

C1: How do we do that?
C2: I do not know.

Researcher: Can you use a letter in the English alphabet to represent the identified variables in the MEA?

A4: Let "y" represent the number of vegetable beds that will be prepared by learners in the elementary and junior groups.
A1: No! It should be "b" to represent the number of vegetable beds.

D3: Let "v" represent the number of vegetable beds that will be prepared by learners in the elementary and junior groups.
D5: It should be "s".

This finding is consistent with the mathematics education literature, which contends that learners (like A1 in our case) tend to interpret literal symbols not as representations of numbers but as abbreviated names (i.e., "b stands for bananas") (e.g., Kieran, 2007). Kinzel (1999) asserted that learners' difficulties with algebraic notation originate from narrow conceptions of variables. Following Kinzel's argument, it can be said that the conventional teaching methods used by teachers in their previous schooling were ineffective in helping learners understand the concept of variables in different contexts. The explanation given by A4 resolved A1's difficulties, as observed above.

4.6.3 Difficulty Interpreting Variables

Although the majority of participating learners in the four groups were able to represent variables in the MEA, some group members were unable to interpret variables correctly. This suggests that learners in each group had a habit of memorising procedures or algebraic rules without understanding the concepts attached to expressions involving variables in mathematics discourse from their previous schooling. This is consistent with the mathematics education literature, which contends that learners mainly have difficulty interpreting symbols when they move from arithmetic (the manipulation of numbers using basic operations) to algebra (the manipulation of variables) (Carraher & Schliemann, 2007; Kieran, 2007). These difficulties were clear from their discussion as can be seen below. The explanations given by B1 and B5 resolved learners' difficulties, as observed in the following discussion.

C2: We have already described the number of learners who are expected to come to school on a particular school day in both groups using w+x.
C1: We said 6y describes the number of learners in both groups expected to come to cultivate vegetable beds in the school garden.

Researcher: What do we put 6y and w+x mathematically?

B3: It is not clear teacher.
B1: I do not also understand.
B2: Hmm. We used the equal sign.
Researcher: How do you put this equality mathematically?
B5: 6y=w+x.

Researcher: What will 6y=w+x look like if w and x are equal?
C4: How could w and x be equal?
C1: It is also not clear to me. They are different variables.

At this point we see a misconception of variables by some of the learners in each of the four groups (B1, B3 in group B and C1, C4 in group C). For these learners, different letters mean different values. As pointed out by Stephens (2005), many learners think $h + m + n = h + p + n$ is never true because "*m*" is different from "*p*" (Stephens, 2005). The researcher guided the discussion to broaden the learners' interpretation of a variable.

Researcher: As we know, of course, the number of learners enrolled in the elementary group "w" (grade 5) and that of the junior group "x" (grade 8) are not equal. But, if equal numbers of learners in both groups come to school on a particular school day, what do you say about w and x?
B1: Teacher, still not clear.

4.6.4 Determining the Value of an Unknown Quantity

Researcher: How many vegetable beds will be prepared only by learners in the junior group if they all come to school on that particular school day?
D5: That means we have to ask the number of learners enrolled in the junior classes.
D1: What about learners in the elementary group?
D4: We are asked to use learners only in the junior group.
D2: What do we put for w in our rule?
D3: It must be zero.
Researcher: Take the number of learners in the junior group to be 300.

After a short discussion between the group members, they wrote the answer for the number of vegetable beds prepared by the junior group.

This suggests that the conventional teaching approach mainly relies on procedural skills and rote memorisation. This is consistent with the mathematics education literature, which contends that teachers have never moved away from the conventional approach of mathematics instruction (e.g., Gainsburg, 2012; Harbin & Newton, 2013).

4.6.5 Evidences for the effectiveness of the teaching treatment

From the analysis in Sections 4.6.1–4.6.4, learners' conceptual understandings of algebraic concepts (e.g., variables constructs) after being taught using the conventional teaching approach in previous schooling (grades 5–8) seems weak. In sharp contrast to the conventional teaching approach, the researcher in the HBPSI model lesson guided the discussion through guiding questions to develop a clear understanding of variable concepts embedded in the Garden Activity (MEA) (see, Section 4.5). The overall improvement in learners' understanding as observed from their conversational statements and learners' modelling activities in the action conception stage (of the learning episode in Section 4.5) was evidenced from the heuristic-based teaching sequence guided by the designed MEA, which describes the learners' mental construction in learning algebra based on APOS theory.

4.6.6 Final remark on learners' difficulties related to algebraic variables

The results revealed that the participating learners who learned algebra through the conventional teaching approach from grades 5–8 were unable to understand the concept of a variable as an unknown quantity as proposed by Ursini and Trigueros (2001). They failed to recognise the presence of something unknown in the situation of the MEA. Learners' ability to interpret variables was also limited. The participating learners seemed to depend on rote memorisation. That is, learners manipulate symbols in a meaningless and mechanical way without connecting them to their everyday activities. Teachers who employed the conventional teaching approach seemed to teach the learners using rules and procedures in order to get the correct answers, but neglected their understanding of the problem situations. An analysis of the conversational statements of learners in the four groups indicates that the HBPSI was superior to the conventional teaching approach in demonstrating algebraic concepts (see, Section 4.5). This means that the HBPSI teaching treatment has the potential to improve the learners' understanding of algebraic concepts (see, Section 4.5). In exploring learners' skills while solving a real-life problem, the designed Garden Activity (MEA) helped the researcher to explain what learners do and do not understand in terms of algebraic concepts (e.g., variables and expressions) after being taught using the conventional teaching approach in previous schooling (grades 5–8).

4.7 GRADE 8 LEARNERS MODELLING ACTIVITIES ON THE DESIGNED MODEL-ELICITING ACTIVITY

In this section, with an eye to answering research question 4, only these data in Section 4.5 from the HBPSI model lesson in Section 4.5 that possibly exhibit or elicit attributes of a STEM-educated learner were selected and assessed. To this end, the attributes of the STEM-educated learner summarised by Morrison (2006) and Magiera (2013) in an effort to link with the role of MEAs in supporting the goals of STEM learning was used. In the following analyses of the discussions, only one of the two groups (Group A) is presented due to space constraints. A letter and number, for example A1, indicate the group (Group A) and the first learner in group A.

4.7.1 Develops problem solvers

The researcher frequently observed learners in Group A, who explained the meaning of the Garden problem to themselves, seeking entry points to its solution. The learners also asked relevant questions and sought information in the MEA and the corresponding mathematical meaning, as shown below:

> *A5: Is the problem asking about the number of vegetable beds that can be prepared by learners in grades 5 and 8?*
>
> *A4: It is asking about the number of vegetable beds that can be prepared by learners in grades 5 and 8, right?*
>
> *A3: No no.... It is asking to decide the size of a group of learners involved in a single vegetable bed, is that not?*
>
> *A2: If that is so, who will decide the number of learners for each bed?*
>
> *A1: How many learners are there in grades 5 and 8?*
>
> *A4: It is not asking about the numbers of beds or the size in a group but it is asking about a rule to predict the number of vegetable beds.*
>
> *A3: What do you mean by a rule?*
>
> *A4: It is just an equation (or a formula).*
>
> *A2: Where do we find the equation then?*
>
> *A5: How can we determine the rule or equation then?*

Researcher: One key element you need to identify in the given problem is the size of the group for each bed. Similarly there are other key elements (quantities) you need to identify.

A1: What do you mean by key element?

A2: Number of learners in our school.

A4: But we are asked quantities associated with the given problem.

A3: The number of learners in grades 5 and 8.

Researcher: Are you sure all learners in grades 5 and 8 will come to school every day?

A5: How do we know the actual number of learners in grades 6 and 8 who come to school in a particular school day?

A4: Therefore number of learners in grades 6 and 8 who will involve in preparing vegetable beds in a particular day is variable.

A3: The number of vegetable beds that will be cultivated in a particular school day is also variable.

To solve the MEA, learners in Group A made new connections and predictions as they had a flexible understanding of variables. They asked questions, which helped them make new kinds of understanding of algebraic variables possible. In essence, the MEA created the opportunity for learning new concepts and helped the learners became ***problem solvers*** who could identify and use relevant mathematical concepts; collect, investigate, organise, and draw conclusions from the information involved in the MEA; and contribute to develop a rule or model, all by connecting their prior knowledge and understanding to build meaning for similar but new situations.

4.7.2 Develops innovators

Finding the rule to determine the number of vegetable beds required that Group A learners design a nonstandard procedure for quantifying the number of vegetable beds using different terms that describe the same situation in the MEA. In doing so, this MEA developed learner innovators who could express the relationships between two different algebraic terms or variables in a novel way. By critically thinking about two different algebraic terms that describe the same situation, Group A learners formed an equation that could lead to derive the required algebraic rule to predict the number of all vegetable beds which will be prepared in a particular day.

A2: w+x and 6y are two different expressions. Is that not?

A5: w+x represents the number of learners in grades 5 and 8 in the school.

A3: 6y also represents the number of learners in grades 5 and 8 who will come in the school garden to prepare vegetable bed.

A4: Although they look different, w+x and 6y describe number of learners in grades 5 and 8 that will come to school in a particular school-day.

A2: Do you mean both w+x and 6y describe number of learners in grades 5 and 8?

A3: What do we do with w+x and 6y?

A1: We have to use the equal sign to connect the terms 6y and w+x.

A5: w+x = 6y. Is that not?

As clearly seen in the episode above, Group A learners ***became innovators*** who thought about the quantity that represented the number of learners in grades 6 and 8 in the school in terms of the number of learners in grades 5 and 8 that would be involved during the preparation of vegetable beds on a particular day which led them to develop the required rule.

4.7.3 Develops inventors

A3: Are two learners enough to prepare a vegetable bed?

A1: What about three or four learners for one bed?

A2: It does not make any difference whether we assign two or three or four learners for each bed.

A4: Why not?

A5: For example, it may take more time for two or three learners.

A4: We need more time to study during examination period. After preparing the beds, it may be difficult for two or three learners in cultivating the vegetables.

A3: We have to give more time to study and do homework and assignments.

The burden of two or three learners in watering a 2 metres by 10 metres bed after preparing the beds raised by A4 played an important role in considering different factors before assigning the size of a group of learners for each bed. Learners considered different factors such as study time, examination period, and class absenteeism as relevant factors to fix the number of learners for each bed. Such learners' discussions about the problem situation facilitated the solution processes of the MEA and guided the discussion as intended by the problem-solving approach used in the current study. A follow-up discussion initiated by A1 and A3 with the group ultimately led to the formulation of an optimal number of learners in a single bed. The above discussion of learners illustrates how the group worked in the context of the MEA. In so doing, Group A learners ***became inventors*** who recognised the need to optimise the number of learners in a single bed.

4.7.4 Develops self-motivation and self-reliance

The MEA provided learners in Group A with the opportunity to try to develop problem-solving skills by applying and connecting their real-life experience to test ideas, compare and make conjectures. The MEA also provided an environment that may promote learners to explore their ways of thinking and other group member ideas to construct a shared detail responses and understanding of MEA. When asked to compare her problem-solving experiences with the MEA to her experiences with solving activities in the conventional teaching classroom, A4 shared the following:

Although I didn't come across such kind of problem in our mathematics lesson before, I recognise the discussion of preparing vegetable beds as something interesting for me and dealing with something I am familiar with at home or in school.

The MEA provided a situation in which a learner in the group is encouraged to reflect on her own thoughts to decide the size, but also communicate her ideas to other group members in ways that other groups can evaluate, reject, or accept. Therefore learners in Group A had an opportunity to learn from the different viewpoints that emerged through collaboration. In doing so, these activities ***developed problem-solving confidence***.

4.7.5 Develops logical thinkers

The MEA provided a rich context for learners' logical and rational use of their knowledge. As learners in the group shared ideas and commented on one another's ideas about the given context, learners had to critically listen and actively engage in processing one another's ideas.

> *A4: There will be 2* metres *by 10* metres *beds in one of the garden and 1.5* metres *by 10* metres *in the other garden.*
>
> *A5: I think it will be fair if we assign the garden with the smaller beds (i.e., 1.5* metre *by 10* metre *beds) for grade 6 learners.*
>
> *A3: Why?*
>
> *A2: Because learners in the grade 6 group are younger compared to learners in the grade 8 group.*
>
> *A3: What is the difference?*
>
> *A4: I think it is better to cover a garden with a greater portion by the elder learners.*

4.7.6 Concluding remark

Using the MEA (Garden Activity) designed by the researchers and the recorded conversational statements of a group of learners that arose from their problem-solving experiences, this study showed how engaging learners in MEAs supports the goals of STEM education by helping them develop a wide range of mathematical proficiencies. This result is consistent with the study results of Magiera (2013), Oferi-Kusi (2017) and Aguilar (2021). The MEA immersed the participating learners in situations in ways that encourage the generation of multiple ideas, elicit the formulation of questions, and require clarification. MEAs also develop productive dispositions, problem-solving skills, and mathematical practices and expertise that are needed for successful problem-solving experiences.

4.8 THE EFFECT OF THE INTERVENTION ON LEARNERS PERFORMANCE IN THE ACHIEVEMENT TEST IN ALGEBRA

This main section presents three statistical tools to explain the effect of the intervention on learners' performance in the achievement test in two sections. Section 4.5.1 summarizes descriptive statistics, and section 4.5.2 summarises inferential statistics.

4.8.1 Descriptive statistics

From the total of 205 participants, 178 learners in both the control and experimental groups participated fully in this study. Full participation of learners in the experimental group in this study refers to participating in solving all three MEAs, participating in solving group and individual work-sheets and taking both the pre- and post-tests before and after the intervention. Full participation of learners in the control group refers to participating in writing both the pre-and post-test stages. Although the 92 participants in the experimental group participated in solving all the MEAs and work sheets, only 81 learners wrote both the pre- and post-tests. From 113 participants in the non-intervention group, only 97 learners wrote the test (pre- and post-test) before and after the intervention. As a result 178 learners in both the control and experimental groups participated fully in this study. In this section, descriptive statistics are presented to display pre- and post-test scores of 178 learners both in the control and experimental groups. The scores are described in terms of variability, gain scores, mean scores and standard deviation.

The mean, standard deviation and the range of scores in the pre-test before the intervention are displayed in Table 4.3.

Table 4.3: The mean, standard deviation and the range of scores

Pre-test	Mean	Standard deviation	Minimum mark	Maximum mark	Number of learners
Experimental group	17.88	7.79	0	42	81
Control group	17.06	8.27	1	39	97

Before the implementation of the intervention, grade 8 learners in the experimental and control groups (n=178) wrote a pre-test that measured their problem-solving skills as their initial status. The mean score in the pre-test of the two groups differed by 0.82. This serves to provide the equivalence scores of the two groups as a baseline measure; that is ($M_{expermrntal}=17.88;$ $M_{control}=17.06$). The scores in the pre-test indicate that learners in both groups performed poorly. The slight difference in the scores of the two groups is accepted without any conclusion about learners learning experience before the intervention.

The mean, standard deviation and the range of scores in the post-test after the intervention are displayed in Table 4.4.

Table 4.4: The mean, standard deviation and the range of scores

Post-test	Mean	Standard deviation	Minimum mark	Maximum mark	Number of learners
Experimental group	28.63	15.55	10	65	81
Control group	20.06	7.93	3	42	97

After the implementation of the intervention, grade 8 learners in the control and experimental groups wrote the same test as a post-test to compare the two instructional methods namely, the HBPSI with the CTA on learners performance score in groups. The number of learners who took both the pre-test and post-test in the experimental group was 81. In the control group 97 learners took both tests. The mean score in the post-test of the two groups differed by 8.57; that is ($M_{expermrntal}=28.63;$ $M_{control}=20.06$). The scores in the post-test indicate that learners in the experimental group outperformed those in the control group. As observed (see, Table 4.4), learners' scores in the intervention group show an improvement of 8.57%. It is therefore concluded that the significant improvement seems to be the result of the participation of learners in the HBPSI. This result shows the HBPSI is better compared with the CTA at the measure of learners' performance in the algebra achievement test.

4.8.1.1 Concluding remarks on the descriptive summary of learners' scores

Analysis of research question 5 included a descriptive summary of learners' scores in the pre-test and post-test stages as presented in Table 4.3 and Table 4.4 to observe the improvement of test scores as a result of their non-participation (control group) and participation (experimental group) in the HBPSI. The deciding factor to support the research hypothesis in this study is whether learners' scores in the experimental group significantly improved in the post-test compared to the scores of learners in the control group. Analysis of mean scores of both groups suggests that learners' who participated in the HBPSI correctly scored more items than their peers in the control group in the post-test. But learners in the experimental group did not outperform the control group in their pre-test scores.

Low mean scores of learners in the control group corroborate to support the hypothesis that the significant improvement of post-test scores by learners in the experimental group is due to the implementation of the HBPSI for the experimental group. Low mean scores of learners in both the experimental and control groups in the pre-test may be an indication that factors other than the HBPSI do not impact the improved post-test scores in the experimental group. As observed in Table 4.4, a 8.58% mean score difference in the post-test between learners in the experimental and control groups reveals the effect of the HBPSI on grade 8 learners' performance in algebra who participated in this study. In the next section, inferential tests are examined to further support the claim that the HBPSI significantly improves learners' post-test scores in the experimental group. To this end, inferential statistical tools were conducted to determine if there are significant differences in the post-test scores between the experimental and control groups based on the performance of learners in the achievement algebra test at the level of grade 8.

4.8.2 Analysis from inferential statistics

This section provides the analysis of learners' scores based on the inferential tools such as t-test and ANCOVA to further corroborate the results found in Section 4.7.1.

4.8.2.1 Analysis of pre-test scores from inferential statistics of t-test

T-test was conducted to establish if there is a statistically significant difference between mean pre-test scores of learners in the experimental group and that of the mean pre-test scores of the control group. The null (H_O) and alternative (H_1) hypotheses of the t-test based on learners' pre-test performance in the experimental and control groups were formulated as follows.

H_O: There is no statistically significant difference between the pre-test scores of learners in the control and experimental groups.

$H_{1:}$:There is a statistically significant difference between the pre-test scores of learners in the control and experimental groups.

The researcher set the critical value (p-value) to 0.05 and conducted the pre-test for the control and experimental groups before the intervention with a null hypothesis stating there is no substantial difference in learners' pre-test mean scores in the experimental and control groups. As can be seen in Appendix E, the t-test has a computed t-value of 0.82 which is smaller than the t-statistic 1.64 at the 0.05 significant level of alpha. There was not a substantial difference between the two groups in their mean scores. As a result, the null hypothesis was supported from the t-test result. Therefore the difference in the mean pre-test scores of learners in the intervention group is not significantly higher than that of the mean pre-test scores of learners in the control group.

4.8.2.2 Analysis of post-test scores from inferential statistics of t-test

After the HBPSI was conducted for the experimental group, a post-test was administered for the experimental and control groups. Then an analysis of learners' post-test means scores for the experimental and control groups was performed to determine whether there was a significant difference between the mean scores of the two groups. The null (H_O) and alternative (H_1) hypothesis of the t-test based on learners' post-test performance in the experimental and control groups were formulated as follows.

H_O: There is no statistically significant difference between the post-test scores of learners in the control and experimental groups.

$H_{1:}$:There is a statistically significant difference between the post-test scores of learners in the control and experimental groups.

The difference between the means of the post-test scores showed a t-score of 4.49 with a p-value smaller than 0.05. As can be seen in Appendix E, the t-test has a computed t-value of 4.49 which is greater than the t-statistic 1.64 at the 0.05 significant level of alpha. Therefore there was a substantial difference between the two groups in their mean scores in the post-test. These results show that there was a significant difference in learners' achievement in the post-test in favor of learners in the intervention group compared to learners' scores in the non-intervention group. Therefore there was a significant difference between the mean scores for the two groups after the study. Participation in the HBPSI may account for improved learners' performance in the post-test and it was suitable in the learning of algebra at grade 8 level compared to the CTA. This is evident in the improved performance of learners in the intervention group in the post-test if their pre-test scores are taken into account. It can therefore be concluded that the mean scores of learners who participated in the HBPSI are significantly greater than the mean scores of learners in the control group who did not participate in the new intervention. As a result the null hypothesis is rejected and the alternative hypothesis was supported from the t-test result.

4.8.2.3 Analysis of learners' pre-test and post-test scores for both groups

The researcher in this study utilised the use of analysis of covariance (ANCOVA) as it is appropriate for research involving the use of inferential statistics. That is, ANCOVA is used to explain whether learners' prior knowledge before the intervention in the experimental group has an effect on their achievement in the post-test. Learners' scores in both groups were analysed using ANCOVA with the pre-test score as a covariate. The dependent variable adjusted mean scores were used to calculate the critical F-value. The null (H_O) and alternative (H_1) hypotheses are as follows.

H_O: There was no meaningful relationship between learners' algebra test scores and the heuristic approach algebra problem-solving instructional method.
$H_{1:}$: There was a meaningful relationship between learners' algebra test scores and the heuristic approach algebra problem-solving instructional method.

With a 95% confidence interval for the mean scores, the pre-test and post-test scores were compared after controlling for learners' pre-test scores. Checking the effect of learners' prior knowledge on their post-test performance involves comparing the calculated F-value with the critical F-value on the dependent variable. The decision is that if the calculated F-value is less than or equal to the critical F-value, then the HBPSI has no substantial effect on learners' performance in algebra. However, the analysis of the ANCOVA result indicated that there was a meaningful relationship between learners' post-test scores in the experimental group and the HBPSI by considering pre-test scores as a covariate. This is confirmed as $F(0.05, 1, 175)=52.64$, with the corresponding p-value 0.001, which is far less than the critical p-value 0.05 (see Appendix E). As a result the study rejected the null hypothesis as the critical F-value $F(0.05,1, 175)=3.84$ is less than the computed F-value 52.64 at a 95% confidence interval. Therefore there was a significant improvement of scores in favour of the HBPSI where learners in the intervention group had increased performance in the algebra post-test with a p-value of 0.001 which is far less than the critical 0.05 alpha value. Therefore when the impact of pre-test scores is removed, the effect of the HBPSI is significantly correlated with learners' achievement in post-test. As a result, the teaching treatment for the experimental group has a substantial impact on learners' achievement in the post-test compared to the CTA in the control schools.

4.8.3 Remarks on participants' performance in the pre-test and post-test stages

The quantitative data gathered in the pre-and-post-test stages from participant learners in this study were scored and analysed statistically in order to provide both descriptive and inferential statistics. An independent t-test was conducted to compare the mean scores between the experimental and control groups at the pre-test and post-test stages. The results and findings obtained from the descriptive statistics corroborate the results obtained using the inferential statistical tools in answering research question 5. The results from both the descriptive and inferential statistics indicate that the intervention group in the experimental schools scored higher than the comparison group in the control schools in the post-test as evidenced by improved scores in the post-test measure after the intervention for the experimental group.

There was no significant difference between the mean scores of learners in the experimental and control groups in the pre-test when compared with the t-test. This is an indication that learners in both the control and experimental groups were at a comparable level in terms of their algebra knowledge before the intervention. But the inferential t-test confirmed that there was a significant difference between the means scores of learners in the experimental and control groups in the post-test. This means that the HBPSI had a better influence on the learners' achievement in the intervention group as compared to learners' achievement in the non-intervention group who do not participant in the teaching treatment. Furthermore, ANCOVA analyses also confirmed that there was a significant improvement of scores in the post-test in favour of the HBPSI. That is, learners in the intervention group scored more items on the post-test than their peers in the control group (see, Appendix F).

CHAPTER FIVE

SUMMARY OF THE STUDY, DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

The description in this chapter is organised into three main sections, Section 5.2 gives a summary of the study; Section 5.3 discusses the findings; Section 5.4 presents limitations of the study; 5.5 presents the study contributions; 5.6 presents the concluding remarks; and Section 5.7 presents recommendations of the study.

5.2 SUMMARY OF THE STUDY

Prior to the study there were good reasons in the literature in the context of Ethiopia for the researcher to urgently find ways and means to redress poor quality as well as low achievement in mathematics. Considerable research findings in the teaching and learning of mathematics in Ethiopia indicated that mathematics is viewed by many learners as difficult to understand and having no application to the outside world. Several studies have also pointed out that activities incorporated in mathematics textbooks in Ethiopia are less supportive in developing the problem-solving ability of learners. They further pointed out that these textbooks emphasise rote learning and memorisation. Recent surveys have shown that learners had low achievement in mathematics (e.g., Bati, 2020; MoE, 2018). As pointed out by the Ministry of Education (2018), an ineffective teaching approach, poor training of teachers, ineffective professional development, ineffective leadership, low salary of teachers, focus on quantity at the expense of quality, weak economic environment, and low participation of parents, among others, are reasons for such a decline. Several other studies have different explanations for the decline of learning achievements (Bati, 2020; MoE, 2018, 2020; Tiruneh, 2020). However, all have common explanations for the decline, namely an ineffective approach to teaching learners who are learning mathematics (Bati, 2020; Tiruneh, 2020) (see, Section 1.3).

With this as a background and the literature about teaching methods that potentially improve learners' conceptual understanding; the researcher opted for the HBPSI as an approach to deepen learners' understanding of concepts to improve performance in algebra. To conduct the empirical investigation, four grade 8 classrooms were selected through convenience sampling. Learners in the experimental schools were in the intervention condition and learners from the control group were in the control condition. The number of schools in the experimental and control groups was decided intentionally to reduce (if not completely eliminate) the effects of inherent differences among the schools.

To collect data, the study used both qualitative and quantitative methods aimed to assess the effects of the HBPSI on learners' performance in algebra compared to the CTA. The research in this study involved a quasi-experimental design using a non-equivalent control group design. Both the experimental and control group learners received the pre-and post-test to measure grade 8 learners' performance in algebra as a result of their non-participation and participation in the HBPSI. Classroom observation was conducted before the intervention by the researcher and two heads from the participant schools. Participating learners' conceptual difficulties and understandings of variables were examined when they worked on a modelling activity after these concepts had been learned from grades 5–8.

A classroom observation tool developed by Kotoka (2012) was adopted and used by the researcher. However, the standardised lesson observation tool to assess teachers' performance in the teaching-learning process was used by the department heads. The analysis of the pre-intervention classroom observation revealed that the experimental and control schools in this study were at a comparable level in all-most all aspects in the teaching learning approach.

Participant learners' background knowledge on algebraic variables was also closely examined. It was aimed at assessing their conceptual difficulties and understanding of algebraic variables after these concepts have been learned from grades 5–8. To this end, learners' modelling activities on the Garden Activity sheet and the audio-recorded learners' conversational statements during the

HBPSI model lesson (see, Section 4.5) were used. The results revealed that the participating learners who learned algebra through the conventional teaching approach from grades 5–8 were unable to understand the concept of a variable as an unknown quantity as proposed by Ursini and Trigueros (2001). The study remarked that the conventional teaching approach used from grades 5–8 did not equip participant learners with the necessary skills to understand variables as unknown quantities involved in a real-life problem situation.

The potential of the designed Garden Activity to support the goals of STEM learning was also explored. To this end, data taken from learners' modelling activities on the Garden Activity sheet and their audio-recorded conversational statements during the HBPSI model lesson (see, Section 4.5.3) were used. The results revealed that engaging learners in MEAs supports the goals of STEM education by helping them develop a wide range of mathematical proficiencies suggested by Morrison (2006) and Magiera (2013) (see, Section 4.7).

Before the implementation of the intervention, grade 8 learners in the experimental and control groups wrote a pre-test that was measured their problem-solving skills as their initial status. After the intervention, participants wrote a post-test to measure the effect of the teaching treatment conducted by the researcher for the experimental group in relation to the usual teaching method conducted by classroom teachers in the control group. The analysis was based on the scores of learners in the achievement test at pre-test and post-test stages. The results of the study indicated that there was no substantial difference in pre-test scores of learners between the experimental and control groups. But there was a significant difference between the two groups in the post-test.

5.3 OVERVIEW OF THE FINDINGS OF THE STUDY IN RELATION TO THE RESEARCH QUESTIONS

The discussion on the results of this study is organised into four sections. Section 5.3.1 discusses the study results on the development and use of the HBPSI in teaching algebra at grade 8 level; Section 5.3.2 discusses the study results on learners' difficulties with algebraic concepts (e.g., variables constructs and sub constructs) which arose during their work to tackle the MEA; Section

5.3.3 discusses the learning opportunities created from the designed MEA; and Section 5.3.4 discusses the study results on the effect of the HBPSI on grade 8 learners' performance in algebra.

5.3.1 How can one design and implement the heuristic-based problem-solving instruction to grade 8 learners?

5.3.1.1 How can one design the heuristic-based problem-solving instruction to grade 8 learners?

At the beginning, the researcher noted the criticism of the conventional approach of teaching as the main obstacle to improving educational performance. This study proposes the models and modelling perspective (MMP) as an approach that agrees with the constructivists' views of learning which is opposed to the conventional teaching approach. The models and modelling perspective is aligned with the contextual aspect that has educational and psychological goals (i.e., concept introduction and development). Lesh and Zawojewski (2007) claimed, "The development of problem-solving abilities is highly interdependent and far more socially constructed and contextually situated than traditional theories have supposed" (p. 779). The models and modelling approach promotes the use of model-eliciting activities (MEAs). The six design principles as guidelines for the design of an MEA are viewed as the models and modelling perspective (Lesh et al., 2000). Any MEA promotes these principles, namely, the 1) reality principle, 2) model construction principle, 3) self-evaluation principle, 4) construct documentation principle, 5) model-generalisation principle, and 6) simple prototype principle (Chamberlin & Moon, 2005).

1) The reality principle requires the activity to help the learner to be able to interpret the situation in the given problem (Lesh & Doerr, 2003). 2) The model construction principle requires the activity to help the learner to be able to interpret mathematically the quantities, relationships, and patterns that they need to take into account (Lesh & Doerr, 2003). 3) The self-evaluation principle provides an opportunity for learners to evaluate their responses as well as those of others and to assess the adequacy of the responses (Lesh & Doerr, 2003). 4) The construct documentation principle requires learners to prepare a written document while working on the

activity to demonstrate their own thinking on the problem situation (Lesh & Doerr, 2003). 5) The model-generalisation principle requires learners to generalise the mathematical model in such a way that another person at the same level of competency could apply it to a similar situation (Lesh & Doerr, 2003). 6) The simple prototype principle requires the activity to be as simple as possible, which enables learners to produce a reasonable solution (Lesh *et al*., 2000).

The most precise way to determine whether an MEA satisfies the above six principles is to try to reply to questions that are given to be appropriate to each of the principles (Lesh et al., 2000). For the reality principle: "*Is the situation in the MEA previously encountered by the learners in their real-lives?"* For the model construction principle: *"Does the situation of the MEA lead learners to generate a model?"* For the self-assessment principle: "*If asked to further improve the MEA, can learners evaluate themselves?"* For the construct documentation principle: *"Does the learners' solution process to the MEA exhibit clearly their thinking about the problem situation?"* For the construct share ability and reusability principle: *"Is the model developed by a learner useful for other learners in the same level to solve a different MEA that has a similar situation as the given MEA?"* For the effective prototype principle: *"Does the developed model create a useful first sample (prototype) for structurally similar situations?"*

Experimental group learners' improvement in the post-test scores did appear to support the MEAs designed for this study as modelling activities that satisfy the six fundamental principles. As a result, these designed MEAs develop conceptual understanding of learners in algebra.

5.3.1.2 How can one implement the heuristic-based problem-solving instruction to grade 8 learners?

Using the data collected in the HBPSI model lesson, the researcher proposes pathways on how this approach of intervention might be employed in a regular classroom setting. This innovative method of teaching can be implemented using the APOS theory. According to Cai (2016), with regard to MEAs, learners are confronted with a mathematical problem in real life situations familiar to them although the path to the solution is not immediately obvious.

The APOS approach to solve an MEA emphasises learners' group interaction as a way for them to adopt deep understanding in mathematics learning as they engage in, revising, differentiating, and improving their thinking through communication within the group (Lesh & Zawojewski, 2007).

MEAs, which are designed for a small group, provide at least two opportunities for learners' learning development. First, during the modelling process learners share the responsibility for the construction of the mathematical model. Second, learners argue and explain their position to their peers, which in turn gives them the opportunities to develop their argumentation and communication skills and their motivation towards mathematics (English, 2003).

A genetic decomposition as a guide for teachers in designing instructional intervention is in alignment with how learners arrive at an understanding of a concept in mathematics (Arnon et al., 2014). It is a description resulting from the actions that learners need to carry out on existing mental phenomena, along with an explanation of how the exteriorisation of these actions to processes occurs (Arnon et al., 2014). The basis for the construction of a genetic decomposition of a concept starts with a hypothesis based on previous experiences accumulated in teaching the concept, the researcher's current knowledge in APOS theory and the concept.

One best possible way to connect school mathematics with real-life and to minimize difficulties in using it is the integration of MEAs into mathematics lessons (Mousoulides, Christou, & Sriraman, 2008). The MEAs should disclose the learners' views about real world situations to be modeled and help them to recognize more than one possible answer suitable to the situations (Mousoulides et al., 2008). It should be noted that, to help improve learners' comprehension skills by themselves and develop deep conceptual knowledge of a concept, MEAs should be conducted after or before direct teaching of the concept (Lesh & Zawojewski, 2007).

In several earlier and recent research findings, educators discuss the potential use of MEA to assist learners to develop deep understanding of the concept embedded in it and retain this understanding in the teaching learning of school mathematics (e.g., Aguilar, 2021; Arnon et al., 2014; Chamberlin & Coxbill, 2012; Deniz & Kurt, 2022; Lesh & Doerr, 2003; Lesh, Carmona & Moore, 2009; Oferi-Kusi, 2017). These research findings suggest the benefits of MEA for assisting learners with different capabilities to increase conceptual gain as a result of addressing real world contexts. Experimental group learners' improvement in the post-test scores did appear to support the way the MEAs used for this study were implemented. As a result learners developed understanding of concepts in algebra.

5.3.2 To what extent do grade 8 learners achieve success in utilising their knowledge of algebraic variables to engage with solving a mathematical model-eliciting activity?

In this study it was expected that (as a result of learning the various algebra concepts set for primary school level from grades 5-8) learners would possess some level of the necessary background knowledge on the concepts of variables upon which the processes of learners' background knowledge can be judged. The background knowledge in this study constitutes the action conceptions of algebraic variables. These conceptions are a set of mental constructs that might describe how the concept of variables as unknown quantities can develop in the mind of a learner. These constructs proposed are:

- The number sentence, and
- The concept of variables.

A lack of knowledge on any of the above mentioned concepts may cause serious challenges for learners when they examine relationships among variables in the designed MEA as part of their problem-solving processes. It may also cause problems in terms of their understandings of variables as unknown quantities as suggested by Ursini and Trigueros (2001).

To answer this research question, the data taken from learners' modelling activities indicated on the Garden Activity sheet and the audio-recorded learners' conversational statements during the HBPSI model lesson (see, Section 4.5) were assessed. To this end, only those data that exhibited the difficulties learners encountered during the HBPSI model lesson related to conceptions of variables were used. The results revealed that the participating learners who learned algebra through the conventional method of teaching from grades 5–8 were unable to understand the concept of a variable as an unknown quantity as proposed by Ursini and Trigueros (2001). In general, they failed to recognise the presence of something unknown in the situation of the MEA. Learners' ability to interpret variables was also limited. However, as observed in Section 4.6.4, most of the group members were able to determine the value of an unknown. This concurs with Naidoo and Naidoo (2007), who asserted that the correct answers given by learners do not necessarily prove that learners understand the concept. Also this can be said to be a result of their previous experiences in the learning of mathematics. It seems that by focusing on rote mathematical applications of procedures and rules, learners lose out on opportunities to understand concepts deeply.

An analysis of the conversational statements of learners in the four groups indicates that the modelling teaching approach was superior to the conventional teaching approach in demonstrating algebraic concepts (see, Section 4.6). This positive influence was evidenced in the recorded conversational statements made by the learners during the HBPSI model lesson (see, Section 4.6). This means that the modelling approach teaching treatment has the potential to improve the learners' understanding of algebraic concepts. In exploring learners' skills while solving a real-life problem, the designed MEA helped the researcher to explain what learners do and do not understand in terms of algebraic concepts (e.g., variables and expressions) after being taught using the conventional teaching approach in previous schooling (grades 5–8).

5.3.3 To what extent does a mathematical modelling activity create an opportunity for learning when grade 8 learners fully engage to solve the task?

Magiera (2013) describes the following three reasons how MEAs could support the goals of STEM learning: 1) the context of solving MEAs allows learners to integrate their knowledge of concepts found both inside and outside mathematics. Hence learners solving process of an MEA reveals how they are interpreting a mathematical situation through a purposeful documentation trail that promotes testing, refining, and extending their ways of thinking (Diefes-Dux et al., 2004), 2) MEAs provide the context for learning new concepts and skills because the problems often require informal explorations and discovering concepts that have yet to be formally introduced, 3) MEAs support the development of problem-solving abilities, dispositions, and expertise needed for analytical thinking.

With an eye to answering research question 4, the researcher was focused on finding evidence of learners' modelling activities that possibly exhibit or elicit attributes of a STEM-educated learner. The data used were from the Garden Activity sheet and learners' conversational statements from the transcripts. This study showed that engaging learners to solve MEAs can help learners develop a wide range of mathematical proficiencies. This result is consistent with the study results of Magiera (2013), Oferi-Kusi (2017) and Aguilar (2021). Morrison (2006) and Magiera (2013) have suggested the mathematical proficiencies created by an effective implementation of MEAs as the attributes of a STEM-educated learner. The MEAs immerse the participating learners in situations in ways that encourage the generation of multiple ideas, elicit formulation of questions, and require clarifications. MEAs also develop productive dispositions, problem-solving skills, and mathematical practices and expertise that are needed for successful problem-solving experiences.

5.3.4 What is the Comparative Effect of the HBPSI and the CTA to the Mathematical Problem-Solving Performance of Grade 8 Learners in the Topic of Algebra?

Prior to the study, the researcher and two department heads observed lessons conducted by grade 8 mathematics teachers who teach the participant classes of the four schools. The analysis of the pre-intervention classroom observation revealed that teachers' method of teaching in both the experimental and control classroom were actually comparable in many aspects. This result led the researcher to measure the true effects of the new intervention, which is different in terms of approach of teaching, group learning, inclusion of learners' real-life experience in the task in comparison to the conventional teaching approach.

Before the implementation of the intervention, grade 8 learners in the experimental and control groups wrote a pre-test that measured their problem-solving skills as their baseline performance. Using the descriptive statistics as a tool, the scores in the pre-test indicate that learners in both groups performed poorly and there was no substantial difference in their achievements. To establish whether the result found using the descriptive statistics corroborated other measuring statistical tools, a t-test was conducted. Accordingly, the study indicated there was no substantial evidence of performance difference between participant learners in the mean scores of both the control and experimental groups in the pre-test. This is an indication that learners in both the control and experimental groups were at a comparable level in terms of their algebra knowledge before the intervention.

After the implementation of the intervention, participant learners in both conditions (the control and experimental conditions) wrote a post-test to measure the effect of the HBPSI on learners in the experimental group and compare with the control group. The results in both the descriptive and inferential statistics confirmed significant performance differences in the mean scores between participant learners in both the experimental and control groups in the post-test. Furthermore ANCOVA analyses also confirmed that there was a significant improvement of scores in the post-test in favour of the HBPSI (see, Appendix F). This means that the HBPSI had a better influence on the learners' achievement in the experimental group as compared to learners in the control group who did not participant in the teaching treatment.

A closer analysis of scripts of experimental group learners' problem-solving processes in the groups and individual work sheets revealed that learners reached the right level in terms of acquisition of concepts and were able to accommodate the conceptions of knowledge in algebra and assimilated this knowledge into a new problem which was not familiar. In the heuristic-based teaching, learners are considered as an investigator instead of receiver of ready-made mathematical knowledge presented by the teacher using teacher-centred approach of instruction. In contrast to solving a problem in the conventional teaching approach using a teacher-centred approach of instruction, the MEA used in the HBPSI has the important mathematical constructs and relationships embedded within the problem context and learner elicit these as they work the problem. This seems to suggest that the teaching treatment conducted for the experimental group might have served as an effective intervention for the experimental group which contributed to the improvement in the post-test and the lack of such treatment for the control group contributed to learners' inability to improve on post-test scores.

Watkins *et al.,* (2007) highlighted that learning is an active process in which the learner relates new information (experience) to their real-life experience and may accommodate and assimilate new ideas. Through assimilation and accommodation, learners may adapt to the environment. A learner "who demonstrates a deep understanding of a concept is capable of dealing with unfamiliar and even new situations using the concept or concepts in question" (Arnon et al., 2014, p. 181).

5.4 LIMITATIONS OF THE STUDY

While the study shows effects of heuristic-based problem-solving instruction and supplements findings from previous empirical studies, it was subject to a number of potential methodological limitations which need to be considered. First, the unobserved covariates (e.g., motivation of learners, motivation of teachers, experience of teachers, teachers' mathematics pedagogy content knowledge, learning materials, time-restriction of class periods, and cognitive levels of learners) that may have influenced performance of learners in algebra were not considered. Second, a sample of only four schools and 205 learners in these schools was used for this study. Therefore, the findings of the study could not be generalised because the sample size was not an adequate representation of population learners in grade 8 classrooms and schools in North Wollo province of Ethiopia and there was no random sampling of learners and schools as they were purposively sampled. The finding of this study may not be generalizable to schools in other provinces and city administrations of Ethiopia. Therefore, caution should be taken when generalizing the results of this study.

5.5 CONTRIBUTIONS OF THE STUDY

In the last four decades, it has been observed a shift from the ready-made mathematical contents to the activity of mathematizing real-life MEAs. Despite the growing evidence of research into the relationship between several mathematics instructional approaches, varied mathematics tasks, and learners' performance, but little is known as to how MEA-guided teaching sequence could be developed to enhance learners' understanding of mathematical concepts. More recently, Loyens, Van Meerten, Shaap and Wijnia (2023) suggested that lack of clear guidelines and support for the implementation of problem-guided approaches of instruction may have a limited impact on its effectiveness. This study argued that one way to enhance learners' deep understanding of a mathematical concept in the middle school is to design a research-based MEA and develop an effective teaching sequence based on the designed MEA. Therefore, if the aim is to help learners develop their understanding of concepts and performance, teachers should know what they should implement and how they should implement a particular instructional approach.

The primary contribution of this study to literature is on the development of heuristic-based teaching sequence guided by the researcher designed MEA by making use of APOS and MMP theories with an aim to develop grade 8 learners' understanding of concepts in algebra. The study suggests that combining these theories to design a teaching sequence may enhance learners' understanding of concepts and their performance. The findings of the study also suggest that teachers can make stronger connections with contents and learners through MEA-guided instruction. As a result this study conjecture that the heuristic-based teaching sequences could be replicated or modified for other contents in different grade levels.

The second contribution of the study could be related to the researcher designed Garden MEA. The researcher enacted the Garden MEA with learners in grade 8 to help a school principal manage the gardening activity and consider watering cans needed for garden use. While there is no single best model shared by all principals, there are ways to help principals consider all the possible number of learners needed to cultivate a school garden. This study conjecture that such a model could be valuable for other social services to decide, for example, number of school buses required to transport learners from home to school.

The other important contribution of the study could be the implementation of HBPSI in order to trigger the learning process of grade 8 learners in the context of STEM. Despite the importance of launching "University STEM Outreach Programme" in all universities in Ethiopia with the aim of training middle and secondary school learners under the STEM framework, views of stakeholders indicate that there is poor mathematics participation in this programme. The HBPSI might shed light how a carefully designed MEA could be used to design instruction to develop learners' understanding of concepts in mathematics in the context of STEM.

5.6 CONCLUDING REMARKS

Prior to the study there was an urgent need for the researcher to find ways and means to redress poor quality as well as low achievement in mathematics in Ethiopia. In the region where the researcher conducted this study, recent surveys by the MoE (2018) reported that learners had low achievement in mathematics (e.g., MoE, 2018). With this background in mind, classroom observation was conducted before the intervention by the researcher and two heads from the participant schools (one from the experimental and another from the control schools) by using the field notes of the researcher and the data from the classroom observation schedule.

The analysis of the pre-intervention classroom observation revealed that the experimental and control schools in this study were at a comparable level in all-most all aspects in the teaching learning approach. The instruction in these classrooms can be characterised as teacher dominated no active involvement of learners, no collaborating groups; learners are passive rather than active receivers of knowledge, no inclusion of real-life context in the teaching process. This could provide an opportunity for the researcher to measure the true effects of the new intervention in comparison to the conventional method of teaching by controlling some of the extraneous variables in this study which influence the relationship between the independent and dependent variables of the experiment.

In this study the models and modelling perspective and the APOS theory were two main theories that guided and supported the teaching treatment conducted for the experimental group. The former was used in designing an effective modelling activity (MEA). The latter was used to guide the designing of the instructional sequence conducted in this study explaining the mental structure of learners and the method through which they attain the conception of algebra.

Advocators of the learning of mathematics through solving MEA claim that learning takes place best in a situation where learners are given opportunities and encouragement to construct their own knowledge rather than relying totally on someone else to construct it for them. Solving an MEA in a collaborative learning approach results in good learning that sharpens thinking, deepens understanding and improves achievement as the learner increases his/her involvement in the learning process, explaining his/her own idea to other group members. Scholars pointed out that bringing learners' real-life experiences to solve MEAs in the classroom provide an opportunity for them to develop multiple problem-solving strategies in familiar and unfamiliar situations, thereby making mathematics meaningful and reflective and developing conceptual knowledge (Gainsburg, 2008; Heibert & Carpenter, 1992; Matang, 2002).

In preparing an MEA the teacher should consider where the activity starts with the learners' real-life situation and ends with the processes of employing mathematics knowledge in order to find the answer to the MEA, interpreting the answer in relation to a real-world context (Yoon, Dreyfus, & Thomas, 2010). Then learners are required to test and iteratively revise the model with the given data in the MEA (Lesh & Harel, 2003). According to prominent scholars in mathematical modelling, in the usual classroom practice, teachers should note that this innovate approach to learning could not substitute the conventional method of teaching implemented in the standard classrooms.

As a starting point, teachers should note that an MEA that they intend to implement should contain four components, namely an article, readiness questions, data, and problem-solving task, challenging learners to solve the MEA through a sequence of steps. a) Learners start with an opening article; b) Learners respond to the MEA to assist them become familiar with the world around them and the situations of the MEA; c) Learners do this in the form of small groups in solving the MEA; d) Learners in a group in a whole class setting present their work; and e) Learners in their group revise and reflect on their model (Stohlmann, 2017). In the modelling process of solving problems, the teacher guides learners in a holistic approach that allows a focus in developing a conceptual understanding of learners in mathematics and their communication skills and provide a space for learners to reflect on their problem-solving processes after the solution to the MEA is invented. Researchers have claimed that success in mathematics learning through solving modelling activities in a collaborative environment in the form of a small group of learners was associated not only with receiving explanations when help was sought but also engaging in constructive activities (Webb, Troper & Fall, 1995).

5.7 RECOMMENDATIONS FOR THE TEACHING AND LEARNING OF MATHEMATICS

5.7.1 Recommendations for the teaching and learning of mathematics at school level

Based on the results and findings obtained in this study the following are recommended:

1) As a starting point, teachers may start the modelling approach of teaching using conventional word problems having no obvious solution paths as these kind of problems give an opportunity for learners to exhibit a kind of thinking that supports mathematical modelling (Czocher & Maldonado, 2015).
2) “University STEM Outreach” programme has already launched and has been conducted in 31 universities in Ethiopia for selected learners from grade 7 to grade 12 during the

summer season. Teachers can use this opportunity in teaching mathematics through modelling using MEAs.

3) There are various local activities (e.g., threshing plot called "*Awudema*" in Amharic, the local language used by farmers for harvesting crops) in Ethiopia with rich mathematical content and situated within the real-life context of all learners' at all level. Teachers can use such context rich activities to design MEAs to bring the local knowledge into viable pedagogical practice in mathematics classrooms.
4) Teachers teaching especially in the elementary and middle school levels should expose learners to different learning materials such as real objects that reinforce real situations as interpreted in the contextual task in order to activate learners' real-world knowledge when they solve problems.
5) When appropriate, teachers can use mathematical modelling which requires learners to talk and discuss in groups about real-modelling activities by imagining the concrete objects involved in the activities and provide appropriate support to solve them.

5.7.2 Recommendations for "STEM Outreach" program

1) "University STEM Outreach" programme have already launched and conducted in 31 universities in Ethiopia for selected learners from grade 7 to grade 12 during the summer season since 2012. This school-university partnership programme was launched with the aim of training middle and secondary school learners (from grade 7 to grade 12) under the STEM framework. As part of this large training program, the researcher in this study was a programme coordinator for five years for the trainees who were recruited at Haramaya University from 2013 to 2017 and witnessed the role of mathematics. What was surprising during that training was that mathematics has played only a supporting role for the other STEM areas. Although the learning materials for biology, chemistry, physics and ICT were prepared in ways that may enhance learners engagement in practical laboratory sessions, mathematics learning materials were prepared simply by selecting some of the topics from the existing curriculum (textbook) with no real-life context and no connection with the other subject areas. Therefore it can be concluded

that mathematics even in a STEM training programme is taught in isolation with an emphasis on rote skills and memorisation instead of an authentic integration with the other subject areas. This approach to train learners in a STEM framework seems unacceptable as long as evidence from literature has shown that activities incorporated in Ethiopian mathematics textbooks are less supportive to develop the problem-solving ability of learners (e.g., Buishaw & Ayalew, 2013; Gulfo & Obsa, 2015; Tesfamicael & Lundeby, 2019). Therefore, a pedagogical practice such as the HBPSI which advocates the teaching of mathematics through modelling may have room to serve such outreach programme.

2) University mathematics teachers can also adopt the models and modelling perspective and APOS theory employed for the outreach programme to teach different mathematics courses.

5.7.3 Recommendations for curriculum designers

The Ethiopian Education Development Roadmap (EEDR) (2018-2030) team suggested that the curriculum in the later phase of primary education should introduce higher order thinking skills appropriate for the level. The EEDR team and several scholars proposed the inclusion of practical, life skills and aesthetics contents that enhance the development of problem-solving into the curriculum as one possible solution among others to make the expected reform relevant. With the new curriculum currently at its initial stage of development, this research proposes pathways on how the teaching of mathematics through modelling might be employed in a regular classroom setting. Therefore, curriculum design may take into consideration the possibility of incorporating MEAs at least at the end of every chapter when appropriate.

5.7.4 Recommendations for research

Since more emphasis has been given to mathematical modelling for mathematics teaching at school level, several researches have been conducted on the use of modelling for the teaching of mathematics at various educational levels with the assumption and hope that teaching mathematics through modelling results in improved learning (Aguilar, 2021; Arnon et al., 2014; Chamberlain et al., 2019; English, 2016; Lege, 2007; Lesh & Sriraman, 2005; Lingefjard, 2005; Magiera, 2013). Lege (2007) found out that learners who develop a model for a modelling activity (MEA) are engaging in various activities such as formulating the proposed mathematical model and then validating the model. Such engagement produces a great learning effect compared to learners who performed the same problems using a formula (prearranged model). He further claimed that the difference is the degree of ownership in model enacting. Lingefjard (2005) also concluded that learners exposed to mathematical MEAs tend to better perform conventional word problems in comparison with those taught through conventional teaching approaches. Others view MEAs as a medium of instructional approach for the construction of mathematical understanding (Aguilar, 2021; Arnon et al., 2014; Chamberlain et al., 2019; English, 2016; Lesh & Sriraman, 2005; Magiera, 2013). Baker et al. (2022) and Stohlmann (2019) have suggested that in order to integrate STEM in mathematics classrooms, MEAs could offer an entry point, and they can be aligned with grade-level mathematics content and process standards.

Several researchers employed mathematical modelling via MEAs in the teaching of mathematics in lower grades and middle schools (e.g., Aguilar, 2021; Boaler, 2002; Ketema, 2021; Oferi-Kusi, 2017). These researchers pointed out that learners in lower and middle schools develop and construct models for different MEAs by making sense of real-world situations embedded in the MEAs and realise that mathematics is valuable in their daily lives. For Chinnappan and Thomas (2003), mathematical modelling helps to motivate, develop, and illustrate the relevance of particular mathematical content. According to Sriraman and English (2010), through mathematical modelling learners recognise mathematics with all its beauty and its value and usefulness. The Ministry of Education (MoE) (2020) of Ethiopia pointed out that, engaging learners to make sense of the world around them via MEAs needs to be introduced from an early age (MoE, 2020), not only because it motivates learners to learn mathematics further, but it also helps them to organise and understand mathematical concepts and their relation to the real world (English & Walters, 2005).

The APOS theory characterises learners' mathematical understanding in a particular concept as their inclination to react to situations in mathematics which are problematic by reflecting on the solution of the task in the context of the real world and the construction or reconstruction of actions, processes and objects, organising them into schemas to deal with this situation (Dubinsky, & McDonald, 2001). Dubinsky and McDonald explained that APOS theory in the understanding of various concepts in mathematics instruction assists in understanding the process of learning by providing clarification on various learners' cognitive activities during problem-solving and construction of knowledge in mathematics (Dubinsky & McDonald, 2001).

A considerable number of studies world-wide focusing on learners' progress in understanding concepts in mathematics by employing APOS theory are well documented. However, to the researcher's knowledge no research has been conducted in the learning of mathematical concepts in Ethiopia at all levels.

1) Since there is no research that employs MMP and APOS theory in the teaching of mathematics in Ethiopia, similar research should be carried out to support or challenge the findings of this study.
2) On a larger scale, other similar research should be conducted to investigate the effectiveness of the HBPSI.

   Research should be carried out on the effectiveness of mathematics instruction using MMP and APOS theory in the teaching of mathematics other than algebra.
3) Research should be carried out to design instruction that targets learners in the "University Outreach" programme using MMP and APOS theory to strengthen the programme.
4) Research should be carried out to examine teachers' problem-solving behaviours in the lens of modelling.
5) Research should be carried out on effective ways of developing in teachers the special expertise required for the HBPSI method.

6) Research should be carried out as to how HBPSI can be integrated with CTA on mathematics classrooms.
7) Research should be carried out to examine the effect of mathematics instruction using MMP and APOS theory on learners’ motivation towards mathematics learning.

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LIST OF APPENDICES

APPENDIX A: MODEL ELICITING ACTIVITIES

MEA 1 (Garden Activity)

Context and parameters

Your school director plans to cultivate vegetables in the school garden in which learners from Grade 5 to Grade 8 will participate. He divided the learners in two groups. The learners in Grades 5 and 6 are elementary groups and learners in Grades 7 and 8 are junior groups. The school garden consists of vegetable beds with dimension 2 meter by 10 meter and 1.5 meter by 10 meter. He decided to assign equal number of learners in each bed. But he let learners discuss and agree on the number of learners in each bed to water the vegetable beds twice a week just after the last period. In order to help the director prepare a schedule to manage the gardening activity and consider watering cans needed for garden use, he wants you determine number of vegetable beds that can be watered in a particular day based on your decision on size of the group in each bed.

Mathematical problem

Develop a rule that predicts the number of vegetable beds that can be watered in a particular day.

e) How many vegetable beds can be watered if both junior and elementary students will water together a particular day just after the last period.
f) How many vegetable beds can be watered if only junior students (Grades 7 and 8) will water a particular day?
g) How many vegetable beds can be watered if only elementary students (only Grades 5 and 6) will water a particular day?
h) Determine alternative situations in which your developed equation can be applied.

MEA 2

Context and parameters

The car traffic flow near your school is always overcrowded from 8:00 am to 9:00 am. One of the reasons is the overlap between office and school time in the morning. This makes learners always late for school. This problem leads to decide the school principal to start school time at 7:30 am instead of 8:30 am. To this end your principal wants you to help him determine the number of Bajaj that come to school daily from Monday to Friday, based on the number of all members of the school (including learners, teachers, principals and administrative staffs). Also note that it is common in your town that a particular Bajaj accommodate four learners (under age 16) at a time. For older people it accommodates only three.

Mathematical problem

Develop a rule that predicts the number of Bajaj that come to your school every day, depending on the number of learners and others including teachers and administrative staff members in the school.

a) How many Bajaj will arrive at school if members of the school other than learners came to school that day before 7:30 am?
b) How many Bajaj will arrive at school if only Grade 8 learners came to school that day before 7:30 am?
c) How many Bajaj will arrive at school if all members of the school (learners and others) come to school that day before 7:30 am?
d) Determine alternative situations in which your developed equation can be applied.

Adapted from Ofori-Kusi (2017).

MEA 3

Context and parameters

Due to water shortages in your school, the parent committee of your school, chaired by the school principal, has decided to buy a water tank that can store water in the school for at least a week. The water tank available on the market is either a square or circular shape. The committee decided to buy one of the two shapes. But members of the committee are unable to make an agreement on the shape of the tank. As a result they want you to help them to recommend which to buy.

Mathematical problem

Suppose both the square and circular water tanks have the same perimeter at the top and bottom surfaces with equal depth. Answer the following in a group of five.

a) Choose one of the water tanks and justify your choice with a calculation based on the amount of water it can hold.

b) Find an agreement in your group. Present your work for the whole class, and follow the presentation of other group especially different from your choice and solve the problem presented by them to confirm the correct of the answer.

c) Find an agreement in whole class format on the type of water tank you want to recommend.

d) Relate the final choice with the kind of shape farmers in Ethiopia use in their day to day experiences.

APPENDIX B: ACHIEVEMENT TEST

GRADE 8 **MARKS: 45**

TIME: 1:30 HOURS

PART I: INSTRUCTION

Each question under part I has four choices. After reading each question and the choices carefully, choose the best answer and write the letter of your choice on the separate answer sheet you are provided.

1. Almaz works two times as Abebe works in a day. Both working together had got Birr 90. How much each of them is paid?(PSLCE, 2016)
   a) Almaz is paid Birr 70 where as Abebe is paid Birr 20.
   b) Almaz is paid Birr 60 where as Abebe is paid Birr 30.
   c) Almaz is paid Birr 72 where as Abebe is paid Birr 18.
   d) Almaz is paid Birr 50 where as Abebe is paid Birr 40.
2. Hana is x years old. Her father is four times as old as Hana. Her mother is seven years younger than her father. If their ages added up to 101 years, how old is Hana?

15 years b) 18 years c) 12 years d) 9 years

3. The solution set of the equation $\frac{10x^3 - 10x^2 + 2x}{2x} = x(5x-4) - 13$ is (PSLCE, 2016)

   a) $\{-7\}$ b) $\left\{\frac{-7}{3}\right\}$ c) $\{-6\}$ d) $\left\{\frac{7}{3}\right\}$

4. The solution set of the inequality $\frac{2}{3}(x+1) \geq -2x - (5-4x) - 1$ is (PSLCE, 2016)

   a) $\{x : -1 \leq x \leq 5\}$ b) $\{x : x \geq 5\}$ c) $\{x : x \geq 4\}$ d) $\{x : x \leq 5\}$

5. What is the slope intercept form of the equation of a line that passes through (-1, 0) and (-2, -2)? (PSLCE, 2016)

   a) $y = \frac{-2}{3}x + \frac{2}{3}$ b) $y = -2x - 2$ c) $y = \frac{-2}{3}x - 1$ d) $y = 2x - 1$

6. The product of the expression $x + 7$ and $y = 2x - 3$ is (PSLCE, 2015)

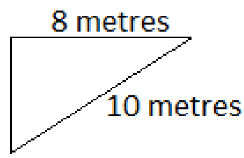
   a) $2x^2 + 11x - 21$ b) $2x^2 + 17x - 21$ c) $x^2 + 11x - 21$ d) $2x^2 - 11x - 21$

7. What value of x satisfy the equation $\frac{-4x-2}{3}=-6$ is (PSLCE, 2015)

a) -16 b) -12 c) 4 d) 0

8. Abebe wants to put a fence around an area. The fence will follow the diagram of the triangle shown below. How much fence does Abebe need? (PSLCE, 2015)

a) 48 metres
b) 24 metres
c) 18 metres
d) 480 metres



9. Simplifying $a(b+c)+(c-a)b; a,b,c \in Q(set\, of\, rational\, numbers)$ (PSLCE, 2018)

a) $c(a+b)$ b) $a(b+c)$ c) $b(a+c)$ d) $b(a-c)$

10. The two adjacent angles of a cyclic quadrilateral measure $97^0\ and\ 110^0$. What are the measures of the other two angles of the cyclic quadrilateral? (PSLCE, 2018)

a) $110^0\ and\ 97^0$ b) $83^0\ and\ 70^0$ c) $100^0\ and\ 80^0$ d) $90^0\ and\ 90^0$

11. The length of a rectangle is 4 times its width. If the width is 3.5 cm, what is the perimeter and area of the rectangle respectively? (PSLCRE 2017).

a) 56cm and 196 cm^2 b) 28cm and 12.5 cm^2 c) 14cm and 12.5 cm^2 d) 35cm and 49 cm^2

12. Which of the following is the equation of the line passing through the points $P(-2,5)$ and $Q(0,-1)$?(PSLCE, 2018)

a) $y=-3x-1$ b) $y=3x+11$ c) $y=\frac{1}{3}x-1$ d) $y=\frac{1}{3}x+1$

13. The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

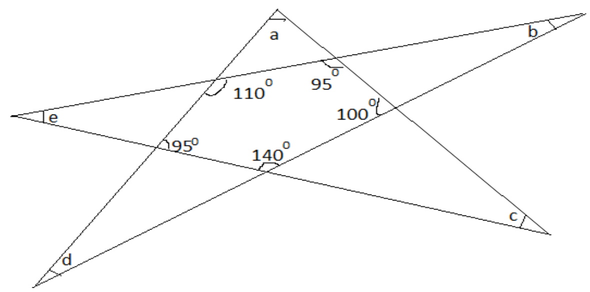
a) 55 years old b) 45 years old c) 25 years old d) 35 years old

14. Which of the following equations relates to the set of ordered pairs $\{(-1,3),(0,0),(1,-3),(2,-6),(3,-9),...\}$? (PSLCE, 2018)

a) $\{(x, y) : y = 3x\}$ b) $\{(x, y) : y = -3y\}$ c) $\{(x, y) : y = -3x\}$ d) $\{(x, y) : x = 3y\}$

15. In the figure below what is the sum of the measures of angles given by a, b, c, d and e?

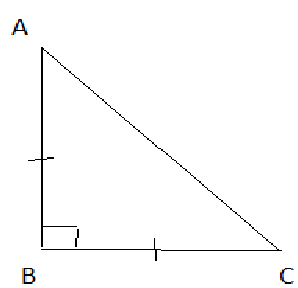
a) 180^0 b) 360^0 c) 540^0 d) 940^0



PART II: INSTRUCTIONS

1. Answer all questions on the space provided on the answer sheet.
2. Clearly show all steps.

1. A farmer has sheep and hen. The sheep and hens together have 100 heads and 356 legs. How many sheep and hens does the farmer have?
2. The time taken to cook rice is given as 20 minutes per kilogram plus 20 minutes extra. What time is required to cook rice weighing 5 kilograms?
3. In a class of 60 learners, 48% are girls and the rest are boys. Find the number of boys and girls.
4. Without using the formula for the area of a triangle, calculate the area of the triangle ABC shown in figure below with AB=BC=10cm and angle B=90^0.



5. In a group of tourists 17 came from USA, 22 came from Japan, 32 came from Germany and 9 came from South Africa. What percentage of the group came from Japan?

APPENDIX C: CLASSROOM OBSERVATION SCHEDULE

Teacher's Name ____________________________ Observer's Name_______________

Section______________________________________ Subject________________

Section______________________ Date_________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson				
2	Indication of teacher's prior preparation for the lesson				
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.				
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.				
5	Teacher's ability in simplifying difficult concepts to learners				
6	Teacher ability in encouraging learners to answer questions of another learner.				
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience				
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule				
9	Teacher ability in managing the class during the lesson				
10	Teacher assesses lessons to check possible attainment of lesson objectives				

Adapted from Kotoka (2012)

APPENDIX D: First classroom observation summary given by the researcher for experimental group 1

Teacher's Name: Teacher ET1 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Further on Algebraic Terms and Expressions

Date__________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson	**X**			
2	Indication of teacher's prior preparation for the lesson		**X**		
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.				**X**
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.		**X**		
5	Teacher's ability in simplifying difficult concepts to learners			**X**	
6	Teacher ability in encouraging learners to answer questions of another learner.			**X**	
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience				**X**
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule		**X**		
9	Teacher ability in managing the class during the lesson		**X**		
10	Teacher assesses lessons to check possible attainment of lesson objectives			**X**	

APPENDIX E: Second classroom observation summary given by the researcher for experimental group 1

Teacher's Name: Teacher ET1 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Further on Algebraic Terms and Expressions

Date__________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson		**X**		
2	Indication of teacher's prior preparation for the lesson	**X**			
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.				**X**
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.		**X**		
5	Teacher's ability in simplifying difficult concepts to learners		**X**		
6	Teacher ability in encouraging learners to answer questions of another learner.			**X**	
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience				**X**
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule		**X**		
9	Teacher ability in managing the class during the lesson		**X**		
10	Teacher assesses lessons to check possible attainment of lesson objectives	**X**			

APPENDIX F: First classroom observation summary given by the researcher for experimental group 2

Teacher's Name: Teacher ET2 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Further on Solutions of Linear Equations

Date__________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson	**X**			
2	Indication of teacher's prior preparation for the lesson	**X**			
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.			**X**	
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.			**X**	
5	Teacher's ability in simplifying difficult concepts to learners				**X**
6	Teacher ability in encouraging learners to answer questions of another learner.	**X**			
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience			**X**	
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule		**X**		
9	Teacher ability in managing the class during the lesson		**X**		
10	Teacher assesses lessons to check possible attainment of lesson objectives			**X**	

APPENDIX G: Second classroom observation summary given by the researcher for experimental group 2

Teacher's Name: Teacher ET2 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Further on Algebraic Terms and Expressions

Date___________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson	**X**			
2	Indication of teacher's prior preparation for the lesson	**X**			
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.		**X**		
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.	**X**			
5	Teacher's ability in simplifying difficult concepts to learners		**X**		
6	Teacher ability in encouraging learners to answer questions of another learner.			**X**	
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience				**X**
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule	**X**			
9	Teacher ability in managing the class during the lesson			**X**	
10	Teacher assesses lessons to check possible attainment of lesson objectives			**X**	

APPENDIX H: First classroom observation summary given by the researcher for control group 1

Teacher's Name: Teacher CT1 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Further on Solutions of Linear Equations

Date__________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson		**X**		
2	Indication of teacher's prior preparation for the lesson		**X**		
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.		**X**		
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.	**X**			
5	Teacher's ability in simplifying difficult concepts to learners			**X**	
6	Teacher ability in encouraging learners to answer questions of another learner.			**X**	
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience				**X**
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule		**X**		
9	Teacher ability in managing the class during the lesson		**X**		
10	Teacher assesses lessons to check possible attainment of lesson objectives		**X**		

APPENDIX I: Second classroom observation summary given by the researcher for control group 1

Teacher's Name: Teacher CT1 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Further on Solutions of Linear Equations

Date__________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson		**X**		
2	Indication of teacher's prior preparation for the lesson	**X**			
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.		**X**		
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.		**X**		
5	Teacher's ability in simplifying difficult concepts to learners		**X**		
6	Teacher ability in encouraging learners to answer questions of another learner.			**X**	
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience			**X**	
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule		**X**		
9	Teacher ability in managing the class during the lesson		**X**		
10	Teacher assesses lessons to check possible attainment of lesson objectives				**X**

APPENDIX J: First classroom observation summary given by the researcher for control group 2

Teacher's Name: Teacher CT2 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Cartesian Coordinate System

Date__________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson	**X**			
2	Indication of teacher's prior preparation for the lesson			**X**	
3	Teacher's ability in allowing learners to actively involved in the lesson in a group setting.		**X**		
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.	**X**			
5	Teacher's ability in simplifying difficult concepts to learners			**X**	
6	Teacher ability in encouraging learners to answer questions of another learner.			**X**	
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience			**X**	
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule		**X**		
9	Teacher ability in managing the class during the lesson	**X**			
10	Teacher assesses lessons to check possible attainment of lesson objectives			**X**	

APPENDIX K: Second classroom observation summary given by the researcher for control group 2

Teacher's Name: Teacher CT2 **Observer's Name: Researcher**

Grade: 8 **Subject: Mathematics**

Topic: Cartesian Coordinate System

Date__________________

	Teacher's style of teaching	Excellent	Good	Fair	Poor
1	Teacher's ability in linking learners' previous knowledge in a lesson	**X**			
2	Indication of teacher's prior preparation for the lesson	**X**			
3	Teacher's ability in allowing learners to be actively involved in the lesson in a group setting.				**X**
4	Teacher's ability in using teaching aids appropriately to facilitate teaching and learning process.		**X**		
5	Teacher's ability in simplifying difficult concepts to learners		**X**		
6	Teacher ability in encouraging learners to answer questions of another learner.		**X**		
7	Teacher ability in providing appropriate examples on concepts by connecting to learners' real-life experience			**X**	
8	Teacher ability in preparing lessons plan based on the curriculum and the work schedule	**X**			
9	Teacher ability in managing the class during the lesson			**X**	
10	Teacher assesses lessons to check possible attainment of lesson objectives	**X**			

APPENDIX L: CLASSROOM OBSERVATION TOOL USED BY DEPARTMENTAL HEADS

School Name:____

Teacher's Name:____

Subject:________________________

Number of learners: Male - _________ Female - _________

Semester:________ Date:___________________

No	**Teaching methods observed on:**	**Activities that need to be improved**
1	Lesson presentation	
2	Use of teaching aids	
3	Classroom participation (interaction)	
4	Documentation of learner progress (achievement)	
5	Group formation	
6	Classroom management	

Comment____

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Observer's Name____________________________ Signature_________________________

APPENDIX M: Classroom observation summary given by department head for experimental group 1

School Name: ES1

Teacher's Name: ET1

Subject: Mathematics

Number of learners: Male - _________ Female - _________

Semester: First semester Date:___________________

No	Activities	Activities that need to be improved
1	Lesson presentation	
	✓ Good blackboard usage	✓ The teacher did not revise previous lesson.
	✓ Topic of the lesson was well communicated	✓ Assessment of learners' understanding.
		✓ Alignment of lesson objective with the lesson
	✓ It was attempted to use different teaching methods	✓ Lesson preparation.
		✓ It was good if the teacher summarize the group discussions in a whole class setting.
	✓ There were attempts to support learners in each group when they get stuck.	
2	Teaching aids	
	✓ The topic does not invite the teacher to use teaching aids.	—
3	Classroom participation (interaction)	
	✓ There were attempts to encourage and support learners participate in the lesson.	✓ It was good if some learners were invited to share their idea to learners in other groups on the blackboard.
	✓ Learners within a group comment on other learners' idea.	
	✓ Learners' participate within a group	

	✓ There was an attempt to teach from mistakes made by learners during the lesson. ✓ There was an attempt to consider level of learners during participation	
4	Documentation of learner progress (achievement)	
	✓ Learners' achievements through continuous assessments documented in a mark list.	✓ There was no written document that show how the teacher form groups of learners (e.g., learners ' level of competence)
5	Group formation	
	✓ The teacher randomly selected learners to form groups.	✓ It was good if groups were formed before the lesson to maximize effective participation that contributes for better learning.
6	Classroom management	
	✓ There was good learner discipline during the lesson. ✓ Classroom management was supported by group leaders.	✓ There was no attempt made by the teacher to stop repeated disturbance from other learners outside the classroom.

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Observer's Name______________________________ Signature__________________________

APPENDIX N: Classroom observation summary given by department head for experimental group 2

School Name: ES1

Teacher's Name: ET1

Subject: Mathematics

Number of learners: Male - _________ Female - _________

Semester: First semester Date:___________________

No	**Activities**	**Activities that need to be improved**
1	Lesson presentation	
	✓ Good blackboard usage ✓ Topic of the lesson was well communicated ✓ It was attempted to use different teaching methods ✓ There were attempts to support learners in each group when they get stuck.	✓ The teacher did not revise previous lesson. ✓ Assessment of learners' understanding. ✓ Alignment of lesson objective with the lesson ✓ Lesson preparation. ✓ It was good if the teacher summarize the group discussions in a whole class setting.
2	Teaching aids	
	✓ The topic does not invite the teacher to use teaching aids.	—
3	Classroom participation (interaction)	
	✓ There were attempts to encourage and support learners participate in the lesson. ✓ Learners within a group comment on other learners' idea. ✓ Learners' participate within a group	✓ It was good if some learners were invited to share their idea to learners in other groups on the blackboard.

	✓ There was an attempt to teach from mistakes made by learners during the lesson. ✓ There was an attempt to consider level of learners during participation	
4	Documentation of learner progress (achievement)	
	✓ Learners' achievements through continuous assessments documented in a mark list.	✓ There was no written document that show how the teacher form groups of learners (e.g., learners ' level of competence)
5	Group formation	
	✓ The teacher randomly selected learners to form groups.	✓ It was good if groups were formed before the lesson to maximize effective participation that contributes for better learning.
6	Classroom management	
	✓ There was good learner discipline during the lesson. ✓ Classroom management was supported by group leaders.	✓ There was no attempt made by the teacher to stop repeated disturbance from other learners outside the classroom.

Comment____
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Observer's Name______________________________ Signature_________________________

APPENDIX O: Classroom observation summary given by department head for control group 1

School Name: ES1

Teacher's Name: ET1

Subject: Mathematics

Number of learners: Male - _________ Female - _________

Semester: First semester Date:___________________

No	Activities	Activities that need to be improved
1	Lesson presentation	
	✓ Good blackboard usage ✓ Topic of the lesson was well communicated ✓ It was attempted to use different teaching methods ✓ There were attempts to support learners in each group when they get stuck.	✓ The teacher did not revise previous lesson. ✓ Assessment of learners' understanding. ✓ Alignment of lesson objective with the lesson ✓ Lesson preparation. ✓ It was good if the teacher summarize the group discussions in a whole class setting.
2	Teaching aids	
	✓ The topic does not invite the teacher to use teaching aids.	—
3	Classroom participation (interaction)	
	✓ There were attempts to encourage and support learners participate in the lesson. ✓ Learners within a group comment on other learners' idea. ✓ Learners' participate within a group	✓ It was good if some learners were invited to share their idea to learners in other groups on the blackboard.

- ✓ There was an attempt to teach from mistakes made by learners during the lesson.
- ✓ There was an attempt to consider level of learners during participation

4	Documentation of learner progress (achievement)	
	✓ Learners' achievements through continuous assessments documented in a mark list.	✓ There was no written document that show how the teacher form groups of learners (e.g., learners ' level of competence)
5	Group formation	
	✓ The teacher randomly selected learners to form groups.	✓ It was good if groups were formed before the lesson to maximize effective participation that contributes for better learning.
6	Classroom management	
	✓ There was good learner discipline during the lesson. ✓ Classroom management was supported by group leaders.	✓ There was no attempt made by the teacher to stop repeated disturbance from other learners outside the classroom.

Comment____
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Observer's Name___________________________ Signature________________________

APPENDIX P: Classroom observation summary given by department heads for control group 2

School Name: ES1

Teacher's Name: ET1

Subject: Mathematics

Number of learners: Male - _________ Female - _________

Semester: First semester Date:___________________

No	Activities	Activities that need to be improved
1	Lesson presentation	
	✓ Good blackboard usage ✓ Topic of the lesson was well communicated ✓ It was attempted to use different teaching methods ✓ There were attempts to support learners in each group when they get stuck.	✓ The teacher did not revise previous lesson. ✓ Assessment of learners' understanding. ✓ Alignment of lesson objective with the lesson ✓ Lesson preparation. ✓ It was good if the teacher summarize the group discussions in a whole class setting.
2	Teaching aids	
	✓ The topic does not invite the teacher to use teaching aids.	—
3	Classroom participation (interaction)	
	✓ There were attempts to encourage and support learners participate in the lesson. ✓ Learners within a group comment on other learners' idea. ✓ Learners' participate within a group	✓ It was good if some learners were invited to share their idea to learners in other groups on the blackboard.

- ✓ There was an attempt to teach from mistakes made by learners during the lesson.
- ✓ There was an attempt to consider level of learners during participation

4 Documentation of learner progress (achievement)

- ✓ Learners' achievements through continuous assessments documented in a mark list.
- ✓ There was no written document that show how the teacher form groups of learners (e.g., learners ' level of competence)

5 Group formation

- ✓ The teacher randomly selected learners to form groups.
- ✓ It was good if groups were formed before the lesson to maximize effective participation that contributes for better learning.

6 Classroom management

- ✓ There was good learner discipline during the lesson.
- ✓ Classroom management was supported by group leaders.
- ✓ There was no attempt made by the teacher to stop repeated disturbance from other learners outside the classroom.

Comment____

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Observer's Name__________________________ Signature______________________

APPENDIX Q: A T-TEST OF PRE-AND POST-TEST ASSESSMENT BETWEEN THE EXPERIMENTAL AND CONTROL GROUPS

	Pre-test		Post-test	
	Experimental Group	Control Group	Experimental Group	Control Group
Mean	17.88	17.06	28.63	20.06
Standard deviation	7.79	7.93	15.55	7.93
Variance	60.61	62.95	242	62.95
Observations	81	97	81	97
Hypothesized mean difference	0		0	
df	171		171	
t Stat	1.43		15.04	
P(T<=t) one-tail	0.05		6.1(exp(-29))	
t Critical one-tail	1.66		1.67	
P(T<=t) two-tail	0.13		8.87(exp(-29))	
t Critical two-tail	1.98		1.98	

APPENDIX R: ANCOVA TEST ON THE ANALYSIS OF THE PRE-TEST SCORES AND POST-TEST SCORES BETWEEN THE EXPERMENTAL AND CONTROL GROUPS

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	25726.597[a]	2	12863.299	858.404	0.000	0.907
Intercept	13.958	1	13.958	0.931	0.336	0.005
PRETEST	22430.120	1	22430.120	1496.825	0.000	0.895
GROUP	2458.289	1	2458.289	164.049	0.000	0.484
Error	2622.397	175	14.985			
Total	130829.000	178				
Corrected Total	28348.994	177				

a. R Squared =0.907 (Adjusted R Squared = 0.906)

APPENDIX S: A SAMPLE OF GROUP WORKSHEET

1. A student take a mathematics test scores of 64 and 78. What score on a third test will give the student an average of 80?

2. The Celsius and Fahrenheit temperature scales are shown on thermometer in Figure2.6. The relationship between the temperature readings C and F is given by C= (Express F in terms of $C = \frac{5}{9}(F - 32)$.

3. If three fourth of a number is one-tenths, what is the number?

4. The sum of two consecutive integers is three times their difference. What is the larger number?

5. Can you find a number that satisfy the following property?

   a. If you multiply the number by 2 and add 4, the result you get will be the same as three times the number decreased by 7.

   b. If you increase the number by 4 and double this sum, the result you get will be the same as four times the number decreases by 6.

6. In a class there are 48 students. The number of girls is 3 times the number of boys. How many boys and how many girls are there in the class?

7. A farmer has sheep and hen. The sheep and hens together have 100 heads and 356 legs. How many sheep and hens does the farmer have?

8. 8 times a certain number is added to 5 times a second number to give 184. The first number minus the second number is -3. Find these numbers.

9. The perimeter of a rectangular field is 628m. The length of the field exceeds its width by 6m. Find the dimensions.

10. To earn grade A in a math class, Aisha must have average score at least 90 on all of her tests. Suppose Aisha has scored 80, 86, 90, 94 and 96 on her first five math tests. Determine the minimum score she needs on her sixth test to get an A in the class.
11. Eight times a natural number is increased by 4 times the number is less than 36. What are the possible value of this number?
12. Twice a number x exceed 5 by at least 4. Find all possible values of x.
13. A natural number is less than the sum of its opposite and 8. Find all such numbers.
14. Find the two smallest consecutive even integers whose sum is at least 51.
15. The perimeter of a rectangle field is 118m. If the length of the rectangle is 7m less than twice the width, what is the length of the field?
16. The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?
17. In a class there are 48 learners. The number of girls is 3 times the number of boys. How many boys and how many girls are there in the class?
18. 8 times a certain number is added to 5 times a second number to give 184. The first number minus the second number is -3. Find these numbers.
19. Twice a number x exceeds 5 by at least 4. Find all possible values of x.
20. A natural number is less than the sum of its opposite and 8. Find all such numbers.
21. Find the two smallest consecutive even integers whose sum is at least 51.
22. Four friends contribute the sum of money to a charitable organization in the ratio 1:3:5:7. If the largest amount contributed is Birr 35. Calculate the total amount contributed by the four people.
23. In a two-digit number, the sum of the digits is 12. Twice the tens digit exceeds the units digit by three. Find the number.
24. A learner scored 16 out of 25 in mathematics test. What is the learner’s score in percent?

25. In woreda election where four candidates appeared for election, the winning candidate received 36,000 votes which represented 45% of the electorate. The other three candidates received 25, 20 and 6 of votes each. How many of the electorate voted?
26. In a class where the number of girls is 36 of the total number, there are 48 boys. How many learners are there in the class?
27. The total attendance at a concert in a theatre hall was 1500, of this total 400 were children, 850 were women, and the remaining were men. Find the percent of the total attendance represented by:

    a. Children b. Women c. Men

APPENDIX T: LETTER FOR THE PROVINCIAL EDUCATION HEAD TO CONDUCT RESEARCH IN THE SCHOOLS IN NORTH WOLLO PROVINCE

PROVINCIAL EDUCATION HEAD CONSENT FORM

Title of the research: **Effects of the heuristic approach algebra problem-solving instruction in a real-life context on Grade 8 learners' performance in algebra**

Date: 09/05/2019

Mr. Shegaw Ageshen

The Provincial Education Head

P.O Box ……………………….

Woldia

Telephone: 033 331 0326/191

Fax: 033 331 1956

Dear Mr. Ageshen

I, Solomon Ketema Gebrie a lecturer at Haramaya University found in western Hararge Zone in Oromia region. I am doing a PhD research under the supervision of Dr. Joseph Jabulane Dhlamini in the College of Education, Department of Mathematics Education at the University of South Africa. It is in this vein that I request for permission to carry out research entitled: **Effects of the heuristic approach algebra problem-solving instruction in a real-life context on Grade 8 learners' performance in algebra** in four schools in your province. The schools are Melka Kole, Taytu Bitul, Kobo and Addis Amba primary and junior secondary schools. The first two are experimental and the last two are control schools. These four schools have been selected to be part of the study because they were participated in the national learning assessment conducted by MoE and UNISEF in 2015 and they are similar in terms of availability of school resources.

The aim of the proposed study is to explore the feasibility and effectiveness of a heuristic teaching instruction on Grade 8 learners' achievements in algebra.

This study will entail conducting a heuristic teaching method in the experimental schools to improve the learning of algebra by the researcher. The study will also entail observing participant teachers in all participant classrooms in the four schools. All participant learners also complete tests in their schools during school time. The study will not interfere with the normal running of participating schools.

The study may highlight pedagogical issues (e.g., educational implications) related to the learning and teaching of algebra, specifically to Grade 8 learners.

There are no expected risks in this study except in a case where mathematics teachers who are teaching in the participating classrooms will feel discomfort while they are observed by the researcher and two department heads in the presence of their learners at the beginning of the intervention. Participation of learners in this study is voluntary. However, once learners have written tests and questionnaire, it will not be possible to withdraw from the study.

There are no especial benefits that participant schools will receive owing to their participation in the study

The final findings of the research will be informed, if needed, via email solomonktm@gmail.com.

Yours faithfully

Solomon Ketema Gebrie

Researcher

APPENDIX U: PERMISSION TO CONDUCT RESEARCH FROM (DoE) OF NORTH WOLLO

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May 9, 2019

To Solomon Ketema Gebrie

University of South Africa

RESEARCH ON: EFFECTS OF PROBLEM-SOLVING HEURISTICS INSTRUCTIONAL APPROACH TO IMPROVE THE TEACHING AND LEARNING OF MATHEMATICS

With reference to your application dated 5 May, 2019 to carry out a research on the above mentioned topic in four schools in North Wollo Provience namely: Melkakole, Tayitu Bitul, Kobo, and Addis Amba Elementary and Junior Secondary Schools permission is hereby granted. I understood that we can take legal action or withdrawal from the study at any point if we feel unhappy or unfairly treated. Furthermore, you should communicate with the school principals of the experimental schools for clearance before carrying out the study.

With regards,

Shegaw Ageshen Yimam

The Province Education Head

APPENDIX V: PARENTAL CONSENT

Date: ___________________

RE: A LETTER REQUESTING PARENTAL CONSENT FOR MINORS TO PARTICIPATE IN A RESEARCH PROJECT

Dear Parent

Your child is invited to participate in a study entitled **Effects of the heuristic approach algebra problem-solving instruction in a real-life context on Grade 8 learners' performance in algebra.**

I am doing a PhD research under the supervision of Dr. Joseph Jabulane Dhlamini in the College of Education, Department of Mathematics Education at the University of South Africa (UNISA). The aim of the study is to explore the feasibility and effectiveness of a heuristic teaching instruction on Grade 8 learners' achievements in algebra.

As your child is involved in one of the selected schools where permission to conduct this study in the Province Education Head is granted, I am asking your permission to involve your child in the study that will take place in your child's school during school time. With your permission, I would like to ask your child to volunteer for this study. I expect to have other 204 children who will participate in the research. Therefore your child's participation in this study is voluntary. Your child may decline to participate or to withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly you can agree to allow your child to be in the study now and change your mind later without any penalty.

There is no negative consequence on your child in participating in this study. Your child's name will be given a pseudonym or a code number and no one, other than the researcher will know about data that will be obtained in relation to this research such as your child's responses in the pre-and post-test. Hard copies of the evaluation sheets which contain your child's performance in the pre- and post-test will be stored by the researcher in a locked filling cabinet at his place of work in his office. Therefore your child's performance and response will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

Furthermore this study will also involve the audio and video recordings of your child during the intervention. In the interest of confidentiality, neither your child's name nor any identifying information in the audio or video-recordings will be disclosed in any report. Only the research and the supervisor will listen your child's voice or view his/her picture. The researcher will maintain the recordings in a locked filling cabinet until the transcription are checked for accuracy by the research team. Then the tape containing the recordings will be erased. Transcripts of your child's recordings in the interview may be reproduced in whole or in part and make accessible during presentations or written products associated with the result of this study without disclosing your child's name and the identifying information in the recordings.

There are no potential individual benefits that you will receive owing to your child's participation in the study. However, your child may improve their performance in mathematics as the result of their participation in the new intervention.

If you have any question about this research protocol, please contact my supervisor Dr. J.J Dhlamini, at by email dhamjj@unisa.ac.za or ask me by phone or e-mail. My phone number is +251 91 198 0245 and my e-mail is solomonktm@gmail.com. Permission for the study has already been given by North Wollo Education Department and the Ethics Committee of the College of Education, UNISA.

Your signature below indicates that you have read and understand the above information and willingly do grant permission for your child to participate in this research and receive the intervention. You may keep a copy of this letter. Thank you for your time and consideration.

Sincerely

____

Name of child:

____ __________________

Parent/guardian's name Parent/guardian's signature: Date:

________________________________ ________________________________ __________________

Researcher's name Researcher's signature Date:

APPENDIX W: LETTER REQUESTING ASSENT FROM LEARNERS

A LETTER REQUESTING ASSENT FROM LEARNERS FOR PARTICIPATION IN THE STUDY IN THE SCHOOLS INVOLVED IN THE RESEARCH.

TITLE OF THE RESEARCH: Effects of the heuristic approach algebra problem-solving instruction in a real-life context on Grade 8 learners' performance in algebra.

Date_______________

Dear learner,

My name is Solomon Ketema Gebrie and I am a teacher at Haramaya University found in western Harerge Zone in Oromia region. I am asking you if I can come and do mathematics with you. I am trying to find out more about how learners do real-life mathematical problems which are familiar to them with their teachers. You may find hard to understand some of the words in this letter. You may ask me or your elder brothers and sisters or any adult to explain any of the words that you feel difficult to understand. Before deciding to participate in this study you may talk to your parents about my invitation to participate in this study.

I would like to invite you to be a very special part of my study. I am doing this study in order to improve the learning of algebra which may improve learners' performance and add aspiration to them in mathematics classrooms. By taking part in this study you and other learners of the same age in different schools may solve some of the problems that you will face in the teaching learning process of mathematics in the future.

I would like you to complete a one and half hours test in your school during school time at the beginning and end of the study. Your Mum or Dad will be asked about your participation in this study, but you are the one to make the final decision about your participation in the study. No one will be upset with you if you do not want to participate in this study or even if you change your mind later and want to stop. You are free to ask me any questions at any time whenever I am available in your school. Signing your name at the bottom means that you agree to be in this study. A copy of this letter will be given to your parents.

Although I will write a report on the study but your participation will not be known by anyone else and I will not mention your name in the report.

You will be asked to put your signature on the next page. Putting your signature indicates that you agree to take part in this study and your agreement will be communicated with your parents using the copy of this letter. If you have any question you can have your elder brother or sister or parent call me on +251 91 198 0245.

You will not receive any especial reward or incentives for agreeing to take part in this study.

Sincerely regards,

Researcher Name:______________________________ Signature _________

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school .I have understood the information about the study and I know what I will be asked to do. I am willing to be in the study.

Name	Signature	Date
Learner's name		
Witness's name		
Parent /guardian's name		
Researcher's name		

APPENDIX X: TURNITIN ORIGINALITY REPORT

Turnitin Originality Report

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EFFECTS OF HEURISTIC BASED PROBLEM-SOLVING INSTRUCTIONAL APPROACH TO IMPROVE THE LEARNING OF ALGEBRA IN GRADE 8 by SOLOMON KETEMA GEBRIE A thesis submitted in partial fulfilment of the requirements for the degree of DOCTOR OF PHILOSOPHY in MATHEMATICS EDUCATION at the UNIVERSITY OF SOUTH AFRICA SUPERVISOR: PROF JOSEPH J DHLAMINI SEPTEMBER 2023 DECLARATION In accordance with the rules and regulations of the University of South Africa, I declare that this thesis, except where explicit reference is made to the contribution of others, is the result of my own original work. It has not been previously submitted to any other institution of higher education. ____________________ Solomon Ketema Gebrie ____________________ Date i DEDICATION This thesis is dedicated to my late wife Azalech Gedefa Rise. ii ACKNOWLEDGEMENTS I would like to thank to the following people who have encouraged me to complete this thesis, without their support its culmination would not have been possible. Firstly, I would like to thank God for giving me the strength and allowing me to be in a position to complete this phase of my educational career. Secondly, I would like to express my sincere gratitude to Prof. Dhlamini, my supervisor, who gave me continuous support and constructive guidance throughout the proposal preparation right through to final preparation of the thesis. Thirdly, I would also like to thank my lovely sons Amen, Nahome and Natnael and my daughter Ruth who have understood and remained patient during my frequent absence from my responsibilities as a father while I was busy with this work. Much thanks to my wife, Mrs. Wagaye whose love and encouragement made all the early mornings and long work nights possible. Last but not least, I would like to thank Mr. Asaye, Mr. Abera, Mr. Ashagre, Mr. Wondossen who are working in North Wollo communication head office, and Mr. Amanuel, Mr. Abraham, Mr. Solomon and Mrs.

APPENDIX Y: LANGUAGE EDITING CERTIFICATE

Mrs G Hannant
28 Hillcrest Avenue
CRAIGHALL PARK
2196

Cell: 076 389 3246
gill.hannant@outlook.com

4 November 2022

TO WHOM IT MAY CONCERN

I certify that I have edited the PhD thesis

EFFECTS OF HEURISTIC BASED PROBLEM-SOLVING INSTRUCTIONAL APPROACH TO IMPROVE THE LEARNING OF ALGEBRA IN GRADE 8

By

Solomon Ketema Gebrie

However, the correction of all errors/missing information remains the responsibility of Mr Ketema.

G.C. HANNANT
BA HED

APPENDIX Z: ETHICS CLEARANCE CERTEFICATE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2019/10/16

Ref: **2019/10/16/61946605/02/MC**
Name: Mr S Gebrie
Student No.: 61946605

Dear Mr Gebrie

Decision: Ethics Approval from 2019/10/16 to 2022/10/16

Researcher(s): Name: Mr S Gebrie
E-mail address: 61946605@mylife.unisa.ac.za
Telephone: +251 91 198 0245

Supervisor(s): Name: Dr JJ Dhlamini
E-mail address: dhlamjj @ unisa.ac.za
Telephone: +27 12 429 2023

Title of research:

Effects of problem-solving heuristics instructional approach on Grade 8 learners' mathematical performance in algebra

Qualification: M. Ed in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2019/10/16 to 2022/10/16.

*The **low risk** application was reviewed by the Ethics Review Committee on 2019/10/16 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

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www.unisa.ac.za

2. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
3. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
4. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
5. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
6. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
7. No field work activities may continue after the expiry date **2022/10/16**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2019/10/16/61946605/02/MC** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof PM Sebate
ACTING EXECUTIVE DEAN
Sebatpm@unisa.ac.za

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