AN EXPLORATION OF THE USE OF VIRTUAL MANIPULATIVES IN ENHANCING UNDERSTANDING OF CIRCLE GEOMETRY: A CASE OF LEVEL 4 NATIONAL VOCATIONAL STUDENTS

by

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Submitted in accordance with the requirements
for the degree of

MASTER OF EDUCATION

in the subject

MATHEMATICS EDUCATION

at the

UNIVERSITY OF SOUTH AFRICA

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AUGUST 2025

DECLARATION

I, Marcus Anthony Palayandi, hereby declare that:

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DEDICATION

This work is lovingly dedicated to the memory of my beloved daughter, **Marie Louise Mabena**, whose life, though brief, continues to inspire strength, purpose and perseverance in me each day. Her memory has been the silent companion through every late night and challenge in this journey.

I also dedicate this thesis to my devoted wife, **Mary-Ann** and my son, **Ehud** and daughter **Leo-lize Palayandi**, as well as my precious grandchildren, whose love and encouragement have been a constant source of support.

A special word of thanks to **Professor Jojo** and **Dr. Eva Makwakwa**, who, during the most painful time of my life, provided the academic and emotional support I needed to persevere and complete this degree. Your belief in me will never be forgotten.

To all of you – this achievement is as much yours as it is mine.

ACKNOWLEDGEMENTS

Firstly, I give all honour and glory to my Lord God, my Redeemer and the Perfecter of my faith. Deep within this journey, He taught me that it is the power of the process that forms and shapes us. Truly, He is the God of the finish.

I wish to extend my deepest gratitude to the following individuals and institutions whose support, encouragement, and contributions were instrumental in the completion of this study:

To my wife, children and their families – thank you for your unwavering love, patience and unselfish support throughout this journey.

To my colleagues in the Mathematics Department, your ongoing encouragement, insightful discussions and generous assistance meant so much to me.

To the South West Gauteng Technical and Vocational Education and Training (TVET) College, including the Campus Manager at Molapo, the Chief Executive Officer, Deputy CEO and all campus authorities, thank you for granting me permission to conduct this research at your institution.

A heartfelt thanks to the fundamental head, education specialists, campus head, lecturers and students who participated in this study – your cooperation was invaluable.

To my supervisor, Dr. Eva Makwakwa, and co-supervisor, Professor Z.M.M. Jojo, thank you for your patience, guidance and critical insight. Your support during times of uncertainty, even during your personal and family time, especially weekends, will always be remembered with deep gratitude.

To the Department of Higher Education and Training (DHET), thank you for granting me permission to conduct this study at the selected public TVET college in the North West Province.

To my immediate and extended family – thank you for your sacrifice, interest and encouragement. Your support allowed me the time and space to pursue this dream, even when it came at the cost of time we could have shared together.

A special thank you to Nicole Maans for her selfless support, impact and commitment

To each one of you – thank you for walking this road with me. This milestone is a shared achievement.

ABSTRACT

This study explored the impact of virtual manipulatives (VMs), specifically GeoGebra, on enhancing National Certificate Vocational (NC(V)) Level 4 students' understanding of circle geometry. It investigated how technology-integrated instruction can address learning challenges in Euclidean Geometry. The research was guided by three key objectives: (i) to identify the challenges students face in understanding circle geometry (ii) to examine the nature of VM-based activities that promote comprehension, and (iii) to determine how virtual tools help minimise learning barriers.

The study was framed within the Pirie-Kieren theory of the growth of mathematical understanding, which views learning as a dynamic and recursive process. A spiral action research methodology was employed, progressing through three instructional phases: conventional teaching, practical hands-on activities and VM-based learning using GeoGebra.

Thirty Level 4 students from Molapo Campus, located in the South West Gauteng TVET College, participated in the study. Qualitative data was collected through semi-structured interviews, questionnaires, as well as pre- and post-tests structured in line with the Pirie-Kieren theory of the growth of mathematical understanding.

The final results showed that the baseline assessment revealed significant gaps in students' foundational understanding of circle geometry. In particular, many students struggled to recognise and apply key properties of circles, especially when these properties needed to be used in combination with triangles and straight lines. Although the introduction of practical instruction improved student engagement—most notably during lessons involving cyclic theorems—its impact was limited by time constraints and students' difficulties in accurately using measurement instruments. Furthermore, several students reported that they had minimal exposure to mathematics outside the classroom, which restricted opportunities for revision, practice, and consolidation of newly learned concepts.

The introduction of GeoGebra significantly improved conceptual clarity, visualisation and interactive learning. Students demonstrated enhanced confidence, better problem-solving skills and a deeper grasp of geometric relationships. VM-based learning allowed learners to explore, test theories and connect geometric ideas more effectively than through traditional methods.

This study highlights the transformative potential of digital tools in mathematics education, especially in bridging conceptual gaps in challenging topics like circle geometry. It advocates for the integration of VMs into NC(V) instructional strategies to promote deeper understanding, active engagement and improved academic performance in mathematics.

Key words:

Virtual Manipulatives (VMs)

GeoGebra Circle Geometry

NC(V) L4

Spiral Action Research

Conceptual Understanding

Euclidean Geometry

Technology-Integrated Instruction

Student Engagement

Visualisation

Problem-Solving Skills

Digital Tools

Instructional Strategies

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ABBREVIATIONS

Abbreviation	Full Description
ACE	Advanced Certificate in Education
CEO	Chief Executive Officer
DHET	Department of Higher Education and Training
DoE	Department of Education
ERD	Engineering and Related Design
L1	Lecturer One
NC(V)	National Certificate (Vocational)
NCTM	National Council of Teachers of Mathematics
NQF	National Qualifications Framework
Q1	Question One
Q2	Question Two
Q3	Question Three
RE1	Response to Question 1
S1	Student 1
TIMSS	Trends in International Mathematics and Science Study
TVET	Technical and Vocational Education and Training
W1	Week 1
IN1	Intervention 1

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CHAPTER 1: INTRODUCTION

1.1 INTRODUCTION

This chapter presents an overview of the study, followed by the background, problem statement, purpose of the study, the rationale for the study, a summary of literature review and theoretical framework, then followed by the research questions that guided this study, and the definition of terms. Furthermore, it also presents a summary of the methodology used in the study.

1.2 BACKGROUND OF THE STUDY

The significance of geometry in real life makes it a fundamental section of mathematics that students need to master and grasp. Quarshie (2023) assert that understanding geometry is an important mathematical skill, since the world in which we live is ‘inherently geometric’. It is inherently geometric because geometry focuses on the development and application of spatial concepts through which children learn to represent and make sense of the world (Wardhani et al., 2023).

Geometry forms 30% of the syllabus content done by NC(V) Level 4 (L4) students. Failure to understand geometry is a major contributor to the poor performance in mathematics at the NC(V) Technical and Vocational Education and Training. This poor performance in mathematics ultimately deprives the L4 students of obtaining NC(V) certification.

According to research reports released by the South West Gauteng College (SWGC) Academic Board for 2021–2023, the NC(V) L4 students experienced difficulties in mathematics and therefore performed inadequately in this subject over those three years. Table 1.1 shows the results.

Table 1.1: NC(V) L4 percentage pass rate: 2022–2023 (Research Campus)

Classification of college	**Mathematics Percentages Pass Rate 2021–2023**		
	2021	2022	2023
Research Campus	38.4%	39.3%	41%

At NCV Level 4 the policy pass rate for Mathematics is 30%. It can be noticed from Table 1.1, that there was an increase of only 2.6% in the mathematics pass percentage between 2021 and 2023. Although the pass percentage seems to be increasing, it is still far from the 60% target set by the SWGC for L4 mathematics students. Students' average geometry scores ranged from 0% to 10%, demonstrating a serious deficiency in conceptual knowledge and practical application abilities.

The researcher suspects that the low pass rate among SWGC NC(V) Level 4 mathematics students may be linked to the fact that many of these students enter the TVET college directly after completing Grade 9, a phase in which they are generally not exposed to circle geometry within the standard curriculum. This suspicion is informed by both institutional performance data and classroom evidence. Historical NC(V) mathematics results at SWGC consistently show that students who enter with only Grade 9 mathematics perform poorly in geometry-related assessments, with many achieving between 0–10% in the circle geometry component. Additionally, baseline assessments conducted at the beginning of this study revealed that the majority of participating students lacked basic conceptual understanding of geometric properties, confirming the gap suggested by their prior learning.

Although the researcher acknowledges that this issue may warrant a focused investigation on Grade 9 entrants specifically, the purpose of the current study was not to compare different entry pathways but to explore how virtual manipulatives could support conceptual development in circle geometry among Level 4 students as they are currently constituted. Students who joined the NC(V) programme after Grade 9 (e.g., those who re-entered after dropping out or transitioned from other pathways) did not constitute a large enough subgroup to allow a meaningful comparative analysis. Institutional academic records also show that students entering with qualifications beyond Grade 9—though limited in number—tend to perform marginally better, but the sample was too small to draw statistically defensible conclusions.

The available evidence, therefore, supports the researcher's suspicion that the lack of exposure to circle geometry prior to NC(V) enrolment contributes to low performance. However, this study remained focused on interventions within the NC(V) Level 4 cohort rather than on comparative entry-level analysis, which could be pursued as a recommendation for future research.

As previously stated, the overall weighting of geometry, which includes circle geometry, contributes 30% to the final examination mark. Furthermore, circle geometry is usually covered in Grade 11 in mainstream education. It is also worth mentioning that the Curriculum and Assessment Policy Statement guideline states that Grade 9 students are only exposed to the fundamental ideas of Euclidean geometry, with a focus on:

- Drawing and constructing a wide range of geometric figures and solids using appropriate geometric instruments,
- Developing an appreciation for the use of constructions to investigate the properties of geometric figures and solids,
- Developing clear and more precise descriptions and classification categories of geometric figures and solids,
- Solving a variety of geometric problems, drawing on known properties of geometric figures and solids (Thamae, 2022)

Circle geometry forms part of Euclidean geometry in the L4 NC(V) TVET, curriculum. At this level, the curriculum states that students are expected to state and apply major theorems such as:

- The angle at the centre is twice the angle at the circumference,
- The angle in a semicircle is a right angle,
- Angles in the same segment are equal,
- Opposite angles in a cyclic quadrilateral sum to 180°.

The angle between the chord and the tangent is equal to the angle in the alternate segment" (Department of Higher Education [DHET], 2013: 25). However, it has been observed that many students usually memorise the theorems through rote learning and cannot apply them correctly (Chand, 2021). Maharani et al. (2019) reveals that numerous students experience difficulty in recognising and naming geometric shapes and their attributes. These challenges were linked to a weak grasp of underlying concepts, inadequate reasoning abilities and minimal familiarity with formal definitions of geometric terms. It has also been noticed that they incorrectly apply properties of parallel lines to any chords drawn in a circle. In my experience as a mathematics lecturer, I have also observed that students are unable to apply properties of a right angle to any triangle in a circle. In some extreme cases, students usually guess answers. Birkhoff and Beatley (2024) argues that when the emphasis is placed on rote memorisation of information in learning, it usually leads to misconceptions about geometric concepts. Furthermore, Jeyabal et al. (2016) concur that students fail to follow procedures correctly because of rote learning of mathematics, and that they learn algorithms without connecting them to the underlying semantic information.

Overall, the research indicates that, with careful planning and implementation, technology can significantly enhance the effectiveness and accessibility of mathematics instruction. Motseki and Jojo (2022) conducted a quasi-experimental study to investigate the effectiveness of using the Geometer’s Sketchpad (GSP) to enhance NC(V) Level 4 students’ understanding of circle geometry. The Geometer's Sketchpad is a dynamic geometry software program designed for constructing, exploring, and visualising geometric concepts through interactive manipulation.

In their quasi-experimental design, the researchers compared two intact NC(V) Level 4 classes: an experimental group that was taught using GSP-based instruction and a control group that received traditional, teacher-centred instruction. Pre- and post-tests were administered to both groups to measure their conceptual understanding of specific circle geometry concepts, particularly properties of tangents and chords. The results showed that students in the GSP-supported class demonstrated significantly higher post-test achievement and deeper conceptual understanding than those taught through conventional methods, thereby confirming the positive impact of virtual manipulatives on learning outcomes.

Furthermore, the critical outcomes of the NC(V) L4 requires students to communicate effectively using visual, symbolic, and/or language skills in various modes, and to use science and technology effectively (DHET, 2007). Saal et al. (2020) found that incorporating educational technology in under-resourced communities can be successful, as long as there is sufficient infrastructure and strong support systems for educators. Overall, the research indicates that, with careful planning and implementation, technology can significantly enhance the effectiveness and

accessibility of mathematics instruction. Motseki and Jojo (2022) investigated the effectiveness of using GSP to enhance NC(V) L4 students' understanding of circle geometry. The Geometer's Sketchpad (GSP) is a dynamic geometry software program made specifically for exploring and visualizing geometrical concepts. Through interactive geometric figure construction and manipulation, it fosters a deeper conceptual understanding through visual discovery.
The quasi-experimental study found that students who received instruction with GSP showed significant improvements in understanding properties of tangents and chords compared to those taught through traditional methods.

To date, the use of visual skills at public colleges in South Africa seems to be minimal. Although lecturers in these colleges make use of technology such as the Academic Support Tutor (AST), it is considered to be static and concrete in nature (Motseki & Jojo, 2022). AST tutor is a visual computer software program that uses animation characters to teach mathematics. It gives exercise activities after each lesson with answers. However, though the program itself helps with pacing or assisting new lecturers on mathematics content it still leaves the learning environment teacher-centred. Students do not discover and explore the concepts on their own; thus, teaching and learning still follow the traditional approach where the teacher remains the central figure in the learning process (Smith et al. 2022).

Although the primary focus of this study is on students' understanding of circle geometry, it is important to acknowledge that their learning does not occur in isolation. NC(V) L4 students are expected to "communicate effectively using visual, symbolic and language skills" and to "use science and technology effectively." However, students can only meet these outcomes if their lecturers provide opportunities to engage with visual and technological tools such as virtual manipulatives (VMs). The concern raised by the examiner—that lecturers may not fully explore these critical outcomes—helps to explain *why* students continue to experience conceptual difficulties. Students rely on lecturers to introduce and guide the use of VMs; therefore, limited lecturer exploration does not shift the focus of the study but rather highlights a contextual factor affecting student learning. The study therefore investigates how VMs enhance students' understanding**,** while recognising that lecturer practice influences this process.

Recent studies highlight the value of combining visual, symbolic, and linguistic modes with technological tools to improve learners' comprehension and engagement in science and mathematics education. Cao et al. (2023) developed a multi-modal instructional system using generative artificial intelligence to convert abstract science, technology, engineering and mathematics (STEM) concepts into visual metaphors, which substantially enhanced students' conceptual understanding and involvement.

Visual manipulatives is a dynamic software that focuses on methods of learning, shifting the emphasis from the teacher to the student, a more collaborative way for students to learn. When using this software, the lecturer becomes a facilitator that models instructions, provides feedback and answers questions when needed. The involvement of the student manipulating the content on the computer with a click of a mouse, may contribute to self-exploration that brings about a greater understanding of circle geometry.

A more recent definition of VMs is provided by Moyer-Packenham and Bolyard (2016) who describe them as "interactive, technology-enabled visual representations of dynamic mathematical objects, including all of the programmable features that allow them to be manipulated, that present opportunities for constructing mathematical knowledge" (p.16). However, VMs are much more than just a computer software technology. Rather, VMs provide students with opportunities to combine pictorial and symbolic representations; and with the actions that they perform on them, enhancing opportunities to explore concepts and construct the meaning of theorems involving circle geometry. Furthermore, a recent study by Andrade et al. (2022) examined the relationship between pre-service teachers' mathematics self-efficacy and their intention to use virtual math manipulatives. The study found a significant association between self-efficacy and the behavioural intention to use VMs, suggesting that increased confidence in mathematical abilities can enhance the likelihood of adopting such technological tools in teaching. The researcher believes that students' confidence in their skills will improve their ability to exert control over their existing circumstances, which has a beneficial impact on their life as they study circle geometry.

1.3 RATIONALE FOR THE STUDY

The examination reports from different years about the seven TVET colleges in South Africa paint a dismal picture on student performance in mathematics especially Euclidean geometry (DHET, 2021). The recent report for example released by the DHET highlights, that many programmes that involve Mathematics do not perform very well. They indicate that as a result very low certification and completion rate, projected in Table 1.2.

Table 1.2: Number of students in TVET who registered, wrote and completed the NC(V) L4 qualification, by programme and gender, 2021

Qualification Category	Female			Male			Total			
	Number registered	Number wrote	Number completed	Number registered	Number wrote	Number completed	Number registered	Number wrote	Number completed	Completion rate
Report 190/1 N3	9 576	7 891	3 523	10 720	8 565	4 168	20 296	16 456	7 691	46.7%
Report 190/1 N6	37 462	33 792	21 548	13 524	11 859	7 137	50 986	45 651	28 685	62.8%
NC(V) Level 4	19 498	16 745	8 719	7 233	5 826	2 655	26 731	22 571	11 374	50.4%
Total	66 536	58 428	33 790	31 477	26 250	13 960	98 013	84 678	47 750	56.4%

Source: (National Examination s Database, November 2021 p.:48))

One of the contributing factors to the low certification and completion of these NC(V) L4 mathematics programmes, could have been students' underperformance in Paper 2 during the November Examinations which includes the topic of Euclidean geometry. As already, indicated Euclidean geometry is introduced at NC(V) L4. Although several lecturers at the college, including the researcher, introduced remedial periods and extra classes to improve student performance in algebra and trigonometry at Levels 2 and 3, these efforts were independent interventions and not part of this study. They are mentioned only to provide context about existing practices at the institution. This research remains a classroom-based action research conducted solely by the researcher with his own class. The reference to colleagues does not imply collaboration but highlights that, despite general remedial efforts, little attention had been given to the teaching methodology of Euclidean Geometry, which motivated this focused investigation.

Although the study focuses on circle geometry, many NC(V) L4 students struggle because they lack foundational Grade 9 geometry. Colleagues' comments simply highlight this context. It does not shift the problem; rather, it explains why students revert to memorising theorems, reinforcing the need for improved teaching approaches.

This is evidenced by the consistently poor performance in Paper 2, which in turn, negatively impacts the certification and completion rates of NC(V) L4 students.

In addition, many studies have been conducted on geometry at secondary schools (Chimuka, 2015; Dongwi, 2018; Govender, 2016; Gweshe, 2024; Machisi & Feza, 2021; Masilo, 2018; Motseki, 2018; Shadaan & Leong, 2013; Shonhiwa, 2019). Many of those studies focused on formal deduction through pre-visualisation analysis, the use of computer-assisted intervention to teach circle geometry (Masilo, 2018). Some focused on teaching circle geometry programmes based on Van Hiele's phase instruction (Dongwi, 2018). Others like Govender (2016) focused on virtual spaces that could be used for teaching and learning of Euclidean geometry. Ntshangase (2024) focused on the effectiveness of using software like GeoGebra on Grade 11 students' understanding of circles.

According to Mudhefi (2022) students often advance from one grade to the next with large gaps in their geometry knowledge and skills primarily due to inadequate teaching of Euclidean geometry across grade levels. As a result, many students enter the Further Education and Training (FET) phase unprepared.

A recent study by Maqoqa (2024) investigated Grade 10 learners' comprehension of Euclidean geometric concepts, the study employed the Van Hiele model to evaluate students' geometric reasoning and revealed that the majority were functioning at the basic visualisation stage, which is significantly below the expected level of geometric understanding for their level. This observation is equally applicable to students in Technical and Vocational Education and TVET colleges whose curriculum aligns with the geometry content of the FET phase.

Traditional instructional methods alone may not be sufficient to address the challenges described above. Both teachers and students urgently need innovative and engaging learning experiences. Ontong and Bruwer (2020) found that first-year students often rely on past exams to predict future tests, which limits their critical thinking and reflects weak deductive skills. Moreover, existing research Mohanraj (2024) suggests that the use of virtual environments can support the development of spatial thinking and foster metacognitive strategies, such as self-regulation questioning. However, TVET students have largely not been exposed to such instructional approaches, highlighting a gap that warrants further exploration.

While several studies have examined the use of dynamic geometry software such as GeoGebra in improving the performances of Grade 11 learners there is no evidence of similar research being conducted at SWGC.

This leaves a vacuum, which indicates a paucity of research on the use of VMs to enhance students' understanding in circle geometry at TVET colleges. Therefore, the present study is guided by the research question: How can VMs enhance NC(V) L4 students' understanding of circle geometry? The current study assumes that incorporating VMs as an instructional model could help bridge the existing knowledge gaps and promote deeper geometric understanding among the students.

1.4 SIGNIFICANCE OF THE STUDY

The necessity for this study also emerged from the researcher's experience in lecturing NC(V) L4 mathematics students. Through years of facilitating mathematics instruction, the researcher observed that students often struggle to understand and apply key conceptual and procedural knowledge – particular in Euclidean geometry, with circle geometry posing major challenges.

This study therefore hopes to explore the ability of using VM to enhance NC(V) L4 students' understanding of circle geometry. In doing so, the researcher endeavours

- To identify the challenges experienced by NC(V) L4 students understanding of circle geometry.
- To determine the nature of virtual manipulative activities that enhance the understanding of circle geometry.
- To determine how the use of virtual manipulation can minimise the challenges experienced by NC(V) L4 students understanding of circle geometry.

1.5 PRELIMINARY LITERATURE REVIEW

This section briefly discusses the concepts reviewed in Chapter 2. The concepts include definition of circle geometry, the nature of virtual manipulation, and use of

GeoGebra as a virtual manipulative to enhance students understanding of circle geometry

1.5.1 Circle Geometry

Circle geometry is the study of the properties of angles and lines within, on and outside circles. Hibi (2024) explores key circle properties and theorems, highlighting their essential place in the structure of Euclidean geometry. As part of the DHET's (2007) curriculum, TVET L4 mathematics students learn circle geometry. Circle geometry forms part of Paper 2 in the South African NC(V) Mathematics examination, yet many students still encounter significant difficulties with this topic. According to Brijlall and Abakah (2022) students often lack the foundational understanding and struggle to integrate geometric ideas, which impairs their problem-solving abilities in circle geometry. As a result, the purpose of this research is to explore how the use of virtual manipulation enhances NC(V) L4 students' understanding of circle geometry

1.5.2 Virtual Manipulatives

Moyer-Packenham and Bolyard (2016) expanded on the earlier definition by describing a virtual manipulative as a technology-based, interactive visual tool with programmable features that support learners in building mathematical understanding. VMs allow learners to interact with mathematical ideas through both visual and symbolic formats, promoting deeper exploration and understanding – particularly in areas such as circle geometry. Unlike physical tools like blocks, which are limited to hands-on demonstration, VMs provide enhanced, dynamic features that support richer engagement and more effective knowledge construction (Moyer-Packenham & Bolyard, 2016). VMs create a unique learning environment in which the properties of many shapes and compositions are integrated within the virtual programme. In comparison to physical manipulatives, this allows students to apply a variety of representational forms in dynamic and varied ways. Moreover, Byrne (2023) examines the difficulties of incorporating physical manipulatives into teaching, emphasising that educators face challenges in ensuring all students have access to the materials and that the items are well-maintained. The study also points out that physical manipulatives are prone to being lost or damaged, which can interfere with the learning process. In addition, Akbıyık and Tavil (2024) examined the role of VMs in mathematics education, emphasising that they offer interactive, dynamic visualisations of mathematical concepts. This enables students to engage with and manipulate these representations, facilitating the construction of mathematical knowledge. Unlike traditional physical manipulatives, VMs provide new ways for students to visualise and explore mathematical ideas.

Recent research of Ahmad and Siller (2024) has emphasised that although VMs resemble physical ones, they provide distinct benefits that improve mathematical learning. VMs offer dynamic, interactive visualisations of mathematical concepts, enabling students to manipulate these representations to build understanding. This method allows for a deeper exploration of mathematical ideas in ways that physical manipulatives cannot facilitate. Furthermore, VMs can be customised to meet individual learning needs, offering personalised support and feedback that enhances problem-solving and critical thinking abilities (Akbıyık & Tavil, 2024; Yakubova et al., 2024). As a result, the user’s capacity for academically significant responsibilities increases, which has an influence on their learning. Students’ challenges with circle geometry are exacerbated by the reality in which students are taught procedural knowledge while studying circle geometry rather than focusing on a student-centred approach to conceptual understanding in teaching mathematics (Abakah, 2023). Moreover, recent studies emphasise the importance of VMs in promoting student-centred learning. These digital tools allow students to interact with mathematical concepts, reinforcing their understanding in real time. By manipulating visual representations, VMs cultivates the development of critical thinking and problem-solving skills, enabling students to take control of their learning and deepen their conceptual understanding of mathematical topics (Akbıyık & Tavil, 2024; Yakubova et al., 2024). Furthermore, recent research also highlights that increased use of VMs significantly enhances students’ perceptions of their learning, motivation and academic success. Studies show that engaging with these interactive tools leads to improved confidence and understanding, as students feel more capable of grasping mathematical concepts. This heightened self-assurance, along with greater motivation, often results in better academic performance, demonstrating how VMs foster a positive shift in students’ learning experiences and outcomes (Haji Ismail, Shahrill, & Asamoah, 2023; Moyer-Packenham & Westenkow, 2013).Students come to appreciate learning and comprehending mathematical concepts using VMs since it is more engaging than the traditional “Chalk and Teach” approach formerly employed by common Mathematics lectures (Moyer-Packenham & Bolyard 2016)). In South African Grade 11 Euclidean geometry, various symbols are used to represent key elements of circle geometry. The symbol **o** represents a circle, while **A, B, C, α, β** are used to denote points and angles on the circle, with **A, B, and C** being points and **α** and **β** representing the angles between them. The symbol ⊥ indicates perpendicular lines, such as when a radius is perpendicular to a chord, and ∥ is used for parallel lines, which are commonly encountered when discussing tangents and secants. These symbols are essential for understanding and solving circle geometry problems (Siyavula, 2011.).

VMs can be very useful for improving the students' understanding of circle geometry's abstract symbolic language. By creating links between visual and symbolic representations, to aid students who commonly struggle to describe what they have learned in circle geometry, in clarifying their thinking and enable them to share their ideas with others.

1.5.3 The Use of GeoGebra As a Virtual Manipulative to Enhance Students Understanding of Circle Geometry

GeoGebra is an interactive mathematics tool that unifies various mathematical fields such as geometry, algebra and calculus within a single dynamic environment. Initially created by Markus Hohenwarter in 2001 during his master's studies at the University of Salzburg, the software has since advanced into a powerful resource for teaching mathematics at multiple educational levels. Its intuitive design promotes concept visualisation and exploration, enriching both instructional practice and student understanding (Bayaga, 2024). GeoGebra, according to Prodromou (2015) may promote active student-centred problem-solving by enabling mathematical experiments, interactive explorations and discovery teaching.

GeoGebra provides a platform for users to explore, interpret and model various mathematical ideas and their connections. For example, while examining the circle theorem that states "the angle at the centre is twice the angle at the circumference," students can interactively move points along the circle's edge and observe corresponding changes in angle measures. This dynamic interaction enhances conceptual understanding by enabling learners to visualise and engage with geometric relationships in real time (GeoGebra, n.d.). Furthermore, GeoGebra can significantly enhance students' comprehension of the tangent-chord theorem by visually demonstrating the relationship between the tangent and the alternate segment, an area where many students often struggle conceptually. By interactively manipulating elements within the applet, learners are able to see how the angle formed between a tangent and a chord equals the angle in the alternate segment, thereby clarifying this frequently misunderstood concept (GeoGebra, n.d.). Students would through the use of GeoGebra experience that moving the point around the circle simultaneously changes the angle in the opposite segment as well as the angle between the tangent and the cord.

According to Chimuka (2017), who conducted a study on the effect of integration of GeoGebra software in the teaching of circle geometry on Grade 11 student achievement, the use of GeoGebra improves students' achievement, improves students' geometric thinking, and motivates students to learn circle geometry.

1.6 THEORETICAL FRAMEWORK

This study was framed using Pirie and Kieren (1989)'s Model of growth of mathematical understanding. The theory is illustrated, by eight nested circles and consists of eight distinct layers of activities that explain one's knowledge development (see Table 1.3) (Pirie & Kieren 1989). All previous levels are contained in each layer, and development moves outward (Wright, 2014). The model's outer layers represent higher levels of understanding. When students apply mathematical material or recall prior knowledge to connect new concepts, they go back and forth between these layers (Wright, 2014). Pirie and Kieren (1994, p.173) dubbed the term "folding back" to express the need to return to an inner layer when faced with a challenge understanding an outer layer level. The authors suggest that in order to improve or deepen understanding at a more advanced or external level (the "outer layer"), it is necessary to return to and strengthen foundational or core knowledge (the "inner layer"). This implies that meaningful progress in complex concepts often requires reinforcing earlier, more basic understandings. The theory provides a framework for learners' thinking about and revising their understanding as well as the process of acquiring knowledge (Wright, 2014). The Pirie-Kieren theory investigated students' comprehension of their knowledge structure as a layered, nonlinear, recursive process (Codes, 2013).

The eight layers in this model according to Pirie and Kieren (1994) may appear on any subject for each person at various stages of their understanding development. These layers consist of: (1) primitive knowledge; (2) Image-making (3) image-having; (4) Image getting; (5) formalising; (6) observing; (7) structuring; and (8) Inventing. Even though the Pirie-Kieren theory has, eight layers (see Figure 3). However, just the first six levels were examined in this study. This is due to the study's time constraints, as well as the fact that most L4 students lack a fundamental understanding of Euclidean geometry and hence must update Grade 9 content of work. According to Maqoqa (2024), the majority of L4 students in South Africa lack a foundational understanding of Euclidean geometry, making it necessary for them to revisit Grade 9 material to bridge critical knowledge gaps. The descriptions of the six layers in terms of circle geometry is presented in Table 1.3.

Table 1.3: Layers of Understanding in Circle Geometry

Layers	***Descriptions in terms of circle geometry***
Primitive knowledge	The cognitive knowledge that is required to structure newly learned notions. In the case of circular geometry, for example. Students would need to recall their understanding of congruency to come up with a concept for using chord and radii in visuals connected to circular geometry. Primitive knowledge, on the other hand, does not refer to a lack of mathematical ability, but rather to all of the information that a learner brings to a learning situation, including both essential and inadequate, if not wrong, knowledge.
Image-making	Refers to the activities that students engage to gain a better comprehension of an idea. The students are using various images to try to uncover the connections between pattern pieces, as well as extending new constructs to further images. Students perform actions manipulating a diagram in order to decide its result example drawing a chord and then connecting the chord with a line from the centre of a circle. They make conjectures or observations about the result based on visual clues on the screen.

Layers	***Descriptions in terms of circle geometry***
Image-having level	The learner has a conceptual method that he or she can use without having to engage in specific tasks. It is not necessary to rely on one's intuition. This is referred to as "Don't Need" borders, and it occurs between three levels of cognition (the darker lines in the model). This layer reflects a student's mental image of the outcome of an action, such as in the midpoint theorem, a line is drawn from the circle's centre being perpendicular to a chord. Therefore the angles formed are equal to 90° (degrees).
Property noticing	The stage at which the learner may see links and distinctions between the visuals he or she has created and the properties of the concept learned. For instance, in the case of the quadrilateral theorem, the students can identify those opposite angles of a cyclic quadrilateral are equal to 180.°
Formalising	The learner generalises the properties he or she has found based on earlier displays without referring to a specific action or image and can describe how he or she does so (Pirie and Kieren, 1994, p.171). Consequently, the student can, for example, construct a formal proof showing that a parallelogram is a quadrilateral whose opposite angles are supplementary (i.e., equal to 180°).
Observation	The learner creates a link between representations and a theorem. Symbolic and verbal representations are the most common.

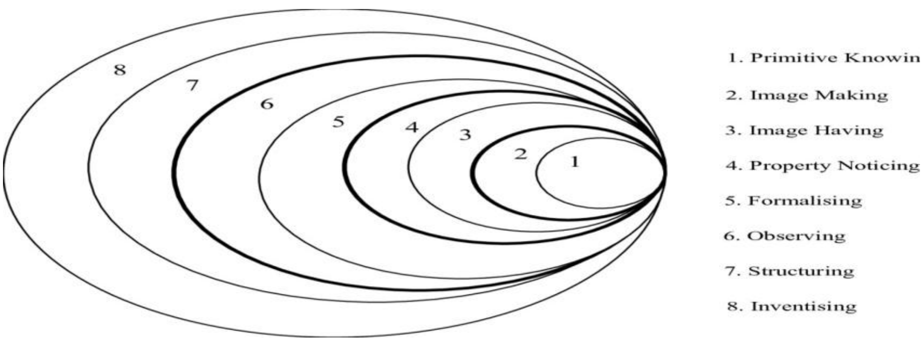


Figure 1.1: Pirie-Kieren model of growth of mathematical understanding

Source: (Pirie & Kieren, 1994, p. 175).

1.7 PROBLEM STATEMENT

Poor performance of students' Euclidean Geometry has been a concern to Mathematics teachers, parents, and the government (Thamae, 2022). The total weighting of this topic contributes 30 percent towards the total mark of Paper 2 of the final external examination. NC(V) Level 4 students consistently perform poorly in circle geometry, which is a major component of the mathematics curriculum. This weak performance significantly lowers their overall Mathematics results, and in many cases contributes to students not meeting the minimum requirements for NC(V) certification**.** The central problem, therefore, lies in students' difficulties with circle geometry.

In 2019, the Trends in International Mathematics and Science Study (TIMSS) confirmed that South Africa continues to perform among the lowest globally, reflecting persistent weaknesses in mathematics teaching and learning. Many Grade 9 learners who later enter TVET colleges bring inadequate understanding of basic Euclidean geometry, partly due to traditional, teacher-centred strategies that rely heavily on memorisation rather than conceptual exploration. Research shows that virtual manipulatives (VMs) offer dynamic, visual, and interactive representations that can strengthen conceptual reasoning in geometry. **Therefore, this study seeks to explore how the use of VMs enhances NC(V) Level 4 students' understanding of circle geometry.**

1.8 AIMS AND OBJECTIVES

This study aims to explore how the use of virtual manipulation enhances NC(V) L4 students' understanding of circle geometry. The following are the objectives of this study:

- To identify the challenges experienced by NC(V) L4 mathematics students' understanding of circle geometry;
- To determine the nature of virtual manipulative activities that enhance the understanding of circle geometry;

- To determine how virtual manipulation can be used, to minimise the challenges experienced, by NC(V) L4 mathematics students in their understanding of circle geometry.

1.9 RESEARCH QUESTION

The main research question of the study is: How can virtual manipulatives enhance NC(V) L4 mathematics students understanding of circle geometry?

In the study, the following sub-questions will be asked:

- What are the challenges experienced by NC(V) L4 mathematics students understanding of circle geometry?
- What is the nature (understanding how something works, reacts, enfolds and unfolds) of virtual manipulation activities that enhance the understanding of circle geometry?
- How can virtual manipulation be used to minimise the challenges experienced by NC(V) L4 mathematics students understanding of circle geometry?

1.10 METHODOLOGY

1.10.1 Research Paradigm and Approach

This investigation was guided by the interpretive paradigm. This was because a qualitative research method was employed in this study since it highlighted individual interactions as well as the capacity to include specific data and provide insight into students' viewpoints (Leko et al., 2021).

1.10.2 Research Design

The researcher adopted an action research design in this study. Action research, according to Creswell (2014), develops through the self-reflective spiral cycle of planning, acting, observing, reflecting and then re-planning, further implementing, observing and reflecting.

The process involves:

- Researcher's planning to initiate change. (Plan on the model)
- Researcher's implementing the changes (act)

- Observing the process of implementation and consequences (observe)
- Reflecting on processes of change and re-planning (reflect)

The main characteristic traits of action research, relates to the collaboration between researcher and participants to solve an identified problem such as exploring the use of Virtual manipulations to enhance the student's understanding of circle geometry.

1.10.3 Sampling

The population of this study involved 30 L4 TVET mathematics students from Molapo, South West Gauteng TVET College. The convenience sampling method was used. Two classes comprising of 30 students in total was selected from Molapo, South West Gauteng TVET College. Convenience sampling was chosen because it is the home campus for the researcher (Creswell, 2015). Johnson (2019) recommends that in action research, the sample size depends on the setting and the strategy chosen for incorporation within the activity investigate cycles. Even though there were four L4 NC(V) classes, the researcher only taught two of them, hence the decision to only included the two classes currently taught by the researcher in the study.

1.10.4 Data Collection Instruments

This study used a baseline test, a post-test, observation schedule, an open-ended questionnaire with tasks and semi-structured interviews to collect data. The researcher developed all of the instruments.

1.10.4.1 Credibility and trustworthiness

To determine whether there were any discrepancies in the data, the researcher looked for ways to triangulate the findings, making sure that the responses from the interviews matched what the researcher had observed and were similar to the test.

The researcher further attempted to generate believable and compelling results while also presenting negative or contradictory findings that contributed to the research's credibility.

The researcher had his supervisors and three experts in mathematics look at the baseline test, post-test and open-ended questionnaire to make sure it was suitable for measuring circle geometry understanding by using virtual manipulation and that the content validity was preserved.

1.10.4.2 Data analysis

The qualitative data was analysed using the content analysis method. Content analysis is a data analysis method that allows one to make correct conclusions from verbal, visual, or written data in order to define and quantify certain occurrences in a systematic and objective way (Kuckartz & Radikar, 2023). The purpose of content analysis is to organise and extract meaning from the information gathered, as well as to derive reasonable conclusions from it (Bengtsson, 2016). In qualitative content analysis, data is presented in the form of words and themes, allowing certain conclusions to be drawn from the findings. After identifying and labelling themes, the researcher organised them into categories and subcategories. At this stage, the researcher began linking these categories to the theoretical framework underpinning the study, interpreting how the emerging patterns reflected or diverged from the concepts within the theory. This theoretical connection helped to explain the meaning of participants' responses beyond mere description, providing a deeper analytical interpretation of the data

1.10.5 Validity and reliability

The validity, according to Andersson et al. (2024), relates to the accuracy of results and conclusions. Furthermore, the validity of the study was validated through the use of a procedure known as respondent validation.

This merely includes double-checking the initial results with the participants to see if they are still valid. Data or findings from participants (students) were returned to ensure correctness and consistency with their personal experiences. Verification addressed the co-constructed character of knowledge by allowing participants to interact with and add to semi-structured interviews and interpreted material after their semi-structured interviews (Birt et al, 2016).

When we talk about instrument reliability, we imply that if the same instrument is used at different periods or given to different individuals from the same population, the results will be the same (Heale & Twycross, 2018).

The researcher went through two procedures to verify reliability. The first step was to enter the data into a table to offer an overall review of the data gathering process as well as an updated evaluation of the results as they become available. The researcher was able to rapidly analyse the results based on the records of each responder and observe how far the study had advanced by using a table for data recording. Triangulation was the second procedure. Triangulation, according to Flick (2018), is a way of assessing the reliability of the findings. The researcher obtained data from several sources, including a baseline test, a post-test, semi-structured interviews and observation of events, persons and procedures.

1.10.5 Ethical Consideration

All researchers are subject to ethical concerns, regardless of their chosen research methodology (Donkoh & Mensah, 2023). Ethics addresses what is right and wrong, and what should or should not be done in a study (Cohen et al., 2018). In this study, ethical considerations included: right of access**,** voluntary participation**,** anonymity and confidentiality**,** and ethical clearance from UNISA. The section below explains how each principle was applied.

1.10.5.1 Right of Access / Authorization

Formal authorization to conduct the study was granted by:

- The UNISA Research Directorate,
- The Department of Higher Education and Training (DHET), and
- Molapo TVET College, the approved research site.

Access was granted only after the researcher submitted a synopsis of the proposal to the institution’s research committee, which reviewed and approved the study.

1.10.5.2 Voluntary Participation and Informed Consent

All participants were informed that participation in the study was entirely voluntary. They were assured that they could withdraw from the study at any time without penalty. Students were also informed of the study’s purpose, procedures, and their rights before signing the consent form. Written permission was obtained from all students to participate in the research and to be interviewed if clarification of their written responses was required.

1.10.5.3 Anonymity and Confidentiality

Participants were assured of complete anonymity and confidentiality. Fictitious names (pseudonyms) were used to protect the identities of students. No identifying information was included in the data, analysis, or reporting. All information collected was kept secure and only used for academic purposes.

1.10.5.4 Ethical Clearance

Ethical clearance was obtained from the University of South Africa (UNISA**)**. The clearance confirmed that the study met ethical standards for research involving human participants and ensured that the rights, safety, and dignity of students were protected throughout the research.

1.11 Definition of terms

An interactive, technology-enabled visual representation of a dynamic virtual mathematical object that allows for the construction of mathematical knowledge manipulatives by including all of the programmable elements that allow it to be altered. Nature Defining understanding as to how something works, reacts, enfolds and unfolds.

1.11 CHAPTER SUMMARY

This study explored how virtual manipulatives, particularly GeoGebra, could enhance students' understanding of circle geometry. The background highlighted challenges in mathematics education and the need for innovative tools. The rationale stemmed from observed difficulties learners faced in grasping geometric concepts. The study was significant in addressing gaps in teaching methods and contributing to improved student outcomes. A review of literature focused on circle geometry, virtual manipulatives, and the application of GeoGebra. The PKM theoretical framework supported a constructivist view of learning. The problem statement outlined learners' limited conceptual understanding. The research aimed to improve comprehension through virtual tools. A qualitative, interpretivist approach guided the methodology, which included action research, purposeful sampling, and multiple data collection tools. Ethical considerations were adhered to throughout the research process. The next chapter presents the literature review.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This section provides a brief introduction to geometry, specifically circle geometry and discusses the concepts that are covered in this chapter. These concepts include the challenges encountered by students in understanding circle geometry, the nature of virtual manipulative activities that enhance circle geometry understanding and how virtual manipulatives can be used to reduce the challenges encountered by students in understanding circle geometry. The researcher further presents the theoretical framework that underpins the study in the same chapter. The chapter then concludes with a summary of the literature reviewed and its implications.

2.2 THEORETICAL FRAMEWORK FOR CIRCLE GEOMETRY

This study was framed using Pirie and Kieren's (1989) theory of the growth of mathematical understanding, commonly known as the Pirie and Kieren model or PKM. The theory is illustrated, by eight nested circles and consists of eight distinct layers of activities that explain one's knowledge development as shown in Figure 2.1.

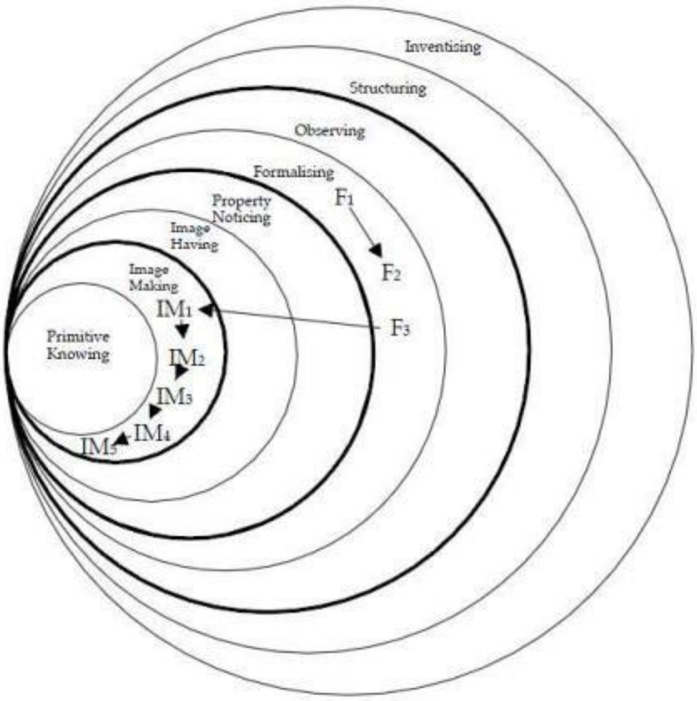


Figure 1.2: Pirie and Kieren's nested circles

Source: Meagher (2005, p. 151)

All previous levels are contained in each layer and development moves outward (Wright and Hilton 2024). Wright and Hilton (2024) expand on the PKM by highlighting that mathematical understanding is an ongoing, cyclical process. Students continuously revisit and refine their ideas – a process called “folding back” – as they learn new concepts. This repeated reflection helps deepen their understanding and strengthens connections between new and existing knowledge, resulting in a more comprehensive and adaptable grasp of mathematics. Pirie and Kieren (1994, p. 173) dubbed the term “folding back” to express the need to return to an inner layer when faced with a challenge understanding an outer layer level. The requirement to enhance understandings in the outer layer coincides with revisiting an inner layer.

Rexhepi and Makasevska (2024) conducted a study in Kosovo showing that teaching fractions using the PKM significantly improved Grade 3 students’ understanding compared to traditional methods. Using a quasi-experimental design with pre- and post-tests, their findings suggest that the PKM effectively supports deeper mathematical comprehension in elementary students. It enables the observation of a specific student’s or group’s growth in mathematical understanding, according to Pirie and Kieren (1994). This theory does not capture a single instance, but rather allows for representations of understanding to grow over periods ranging from minutes to hours, days, weeks or even years. Furthermore, the theory provides a framework for learners’ thinking about and revising their understanding as well as the process of acquiring knowledge (Wright, 2024). Irvine (2023) re-examines the PKM, highlighting its connection to complexity theory and its view of learning as a nonlinear, recursive and evolving process. He focuses on the idea of “folding back,” where students repeatedly revisit and improve their understanding throughout their learning journey. This ongoing, layered process reflects the intricate and ever-changing nature of how students build and develop their mathematical knowledge. The eight layers in the PKM may appear on any topic for each student at various stages as their understanding develops. However, just the first six levels were observed in this study. This is due to the time constraints related to the study, as well as the fact that most L4 students lack a fundamental understanding of Euclidean geometry and, as a result, must update Grade 9 content of work based on personal experience. These layers are described in detail in the following sections.

2.2.1 Primitive Knowledge

Primitive knowledge does not refer to a lack of mathematical ability, but rather to all of the information that a learner brings to a learning situation, including both essential and inadequate, if not wrong, knowledge. Primitive knowledge is the cognitive knowledge that is required to structure newly learned notions. In the case of circular geometry, for example. Students would need to recall their understanding of congruency to come up with a concept for using chord and radii in visuals connected to circle geometry. The theorem illustrates the importance of primitive knowledge (The line drawn from the centre of a circle perpendicular to a chord bisects the chord). Students are expected to demonstrate that line AP = BP or angle P = 90°, and they have to use prior knowledge of congruency, chords and radii to answer questions.

2.2.2 Image-Making

Image-making refers to the activities that students engage in to gain a better comprehension of an idea. The students use various images to try to uncover the connections between pattern pieces, as well as extending new constructs to elaborate images. Students perform actions manipulating a diagram to decide its result example drawing a chord and then connecting the chord with a line from the centre of a circle. They make conjectures or observations about the result based on visual clues. Students are expected to distinguish between prior knowledge and its application in new situations (Pirie & Kieren, 1994). At this level of understanding, students do something mentally or physically to gain an understanding of a concept.

2.2.3 Image-Having

Image-having refers to the activities that students engage in to gain a better comprehension of an idea. The students use various images to try to discover the connections between pattern pieces, as well as extending new constructs to elaborate images. Students perform actions manipulating a diagram to decide its result; for example, drawing a chord and then connecting the chord with a line from the centre of a circle. They make conjectures or observations about the result based on visual clues. If a student creates an image in his or her mind about a topic, we can say that the learner is at the image-having level (Pirie & Kieren, 1994). Single-activity images are replaced with mental images at this level. This means that when dealing with mathematics, students do not need to engage in any physical activities (Pirie & Kieren, 1992; 1994).

2.2.4 Property-Noticing

The properties of the constructed image are identified at the fourth level. This is referred to as property-noticing. At this level, the student can manipulate or combine image properties to obtain context-specific relevant properties (Pirie & Kieren, 1994). This would be the stage at which the learner may see links and distinctions between the visuals they have created and the properties of the concept learned. For instance, in the case of the quadrilateral theorem, the students should identify that the opposite angles of a cyclic quadrilateral are supplementary and add up to 180°.

The learner has a conceptual method that they can use without having to engage in specific tasks. It is not necessary to rely on one's intuition. This is referred to as "Don't Need" borders, and it occurs between three levels of cognition (the darker lines in the model). This layer reflects a student's mental image of the outcome of an action, such as in the midpoint theorem: a line is drawn from the circle's centre being perpendicular to a chord, therefore, the angles formed are equal to 90° (degrees)

2.2.5 Formalising

A method, rule or property is generalised from the properties at the formalising level (fifth level). Students are expected to derive a method or a common characteristic from the properties of previously held images (Pirie & Kieren, 1994). At this level, noticed properties are used to create class-like objects. At this stage, the student may see links and distinctions between the visuals that they have created and the properties of the concept learned. For instance, in the case of the quadrilateral theorem, the students can identify that opposite angle of a cyclic quadrilateral are supplementary.

2.2.6 Observing

In the observing level, the learners verbalise and express themselves about the formalised concept. At this level, students can reflect on formal activities and express their thoughts as theorems (Pirie & Kieren, 1994). At this level, a method, rule or property is generalised from the properties. Learners are expected to draw a method or common quality from the properties of previously held images (Pirie & Kieren, 1994). The Pirie–Kieren Model explains that, as learners progress through the levels of understanding, they begin to generalise the properties they have observed without depending on a specific diagram or manipulation (Pirie & Kieren, 1994). At this stage, they can explain why a property holds and justify it verbally or symbolically. For example, a student may observe several dynamic representations of a quadrilateral and then generalise that a parallelogram's opposite angles are supplementary. Here, the learner is not merely recalling a rule but is **connecting visual, symbolic, and verbal representations** to construct a theorem. This illustrates how understanding grows from concrete manipulation to abstract reasoning.

2.3 CONCEPTUAL UNDERSTANDING OF GEOMETRY

2.3.1 A Definition of Conceptual Understanding

According to Wiggins (2016), Conceptual understanding in mathematics refers to learners' ability to recognise important ideas, make meaningful inferences about them, and appreciate how these ideas function within mathematical relationships. Because facts and methods learned through understanding are linked, they are easier to remember and apply and they can be recalled if forgotten. As a result, students are better prepared to use these inferences—such as recognising relationships between angles, identifying properties of geometric figures, and connecting visual patterns to formal theorems—strategically to solve problems, particularly non-routine ones. Conceptual understanding, or the ability to grasp ideas in a transferable manner, enables students to apply what they learn in class across different mathematical domains, including circle geometry.

2.3.2 The Origin of Geometry

Geometry originates from ancient Greek knowledge, where it was known as “earth measure” and was used to measure land in the past (Hvidsten, 2016). Numerous historical beliefs, according to Nazarovich and Kurudirek (2024), established a form of geometry appropriate to the relationships between lengths, areas and volumes of physical objects. In addition, Thamae (2022) asserts that there are numerous forms of geometry, including plane geometry, elliptic geometry, analytical geometry, coordinate geometry, Euclidean geometry and others. School curriculum designers select a few types to be studied at school level, such as transformation, analytical geometry and Euclidean geometry.

Euclidean geometry is a branch of mathematics that provides a comprehensive visual resource for understanding arithmetic, algebraic and statistical concepts (Mensah-Wonkyi & Adu, 2016). Furthermore, Euclidean geometry is important in everyday life because it can be found in the structure of the solar system, geological formations such as certain rocks and crystals, plants, and flowers, and even animals (Magnani, 2022). To be specific, circle geometry is taught at L4 TVET college students as part of their Euclidean geometry curriculum.

2.3.3 Circle Geometry

Circle theorems are regarded as an important feature of geometry and are particularly useful for ship navigators. It is no wonder that geometry is always covered in international examinations because it is so fundamental in everyday life. TIMSS, West African Senior School Certificate Examination and other assessments demonstrate this. Furthermore, circle geometry is regarded as the study of the properties of angles and lines within, on and outside circles. In addition, circle geometry is a discipline of Euclidean geometry that includes theorems, theorem converses, corollaries and axioms. As part of the DHET's (2012) curriculum, NC(V) L4 mathematics students learn geometry that entails the circle geometry theorems dealing with angles in a circle, cyclic quadrilaterals and tangents. Table 2.1 presents the list of theorems, converse theorems and their corollaries that NC(V) L4 mathematics students are expected to study. The proof of the theorems is not examined in the November final examinations.

Table 2.1: Theorems, converse theorems and their corollaries

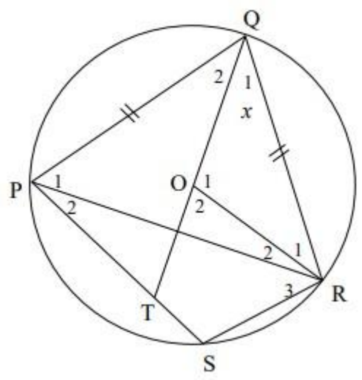
No	Component	Description of theorem converse or corollary
1	Theorem	The line drawn from the centre of a circle perpendicular to a chord bisects the chord; (line from centre to midpoint of a chord)
	Converse	The line from the centre of a circle to the midpoint of a chord is perpendicular to the chord. (Line from centre ⊥ to chord)

No	Component	Description of theorem converse or corollary
		The perpendicular bisector of a chord passes through the centre of the circle (perpendicular bisector of a chord)
2	Theorem	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circumference circle (on the same side of the chord as the centre) (∠ at centre = 2 ×∠ at circumference)
	Corollary	Angle in a semicircle is 90° (∠s in a semicircle) Angles subtended by a chord of the circle, on the same side of the chord, are equal (∠s in the same seg) Equal chords subtend equal angles at the circumference (equal chords; equals) Equal chords subtend equal angles at the centre (equal chords; equals) Equal chords in equal circles subtend equal angles at the circumference of the circles. (Equal circles; equal chords; equals)
	Corollary Converse	If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter. (converse ∠s in a semicircle) If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic
3	Theorem	The opposite angles of a cyclic quadrilateral are supplementary (opposites of the cyclic quad)
	Converse	If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic. (opposite ∠s quad sup OR converse opposite ∠s of cyclic quad)
	Corollary	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral (exterior ∠ of the cyclic quad)

No	Component	Description of theorem converse or corollary
4	Theorem	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact. (tangent ⊥ radius)
	Converse	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle (line ⊥ radius)
5	Theorem	Two tangents drawn to a circle from the same point outside the circle are equal in length. (Tangents from common point OR Tangents from the same point)
6	Theorem	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. (Tangent-chord theorem)
	Converse	If a line is drawn through the endpoint of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. (Converse tangent-chord theorem OR ∠ between line and chord)

All the preceding theorems are integrated into one or two questions and one concept is linked to other concepts in circle geometry. The following is an example of an examination question that tests the students' understanding of circle geometry and problem-solving capabilities.

In the diagram below, O is the centre of the circle, and P, Q, R, and S are points on the circumference. Line TOQ is a straight line such that T lies on PS. If $PQ = QR = x$, find:



Calculate, with reasons, $\hat{P}_1$ in terms of x. (3)

Show that TQ bisects P$\hat{Q}$R. (3)

Show that STOR is a cyclic quadrilateral. (3)

The preceding question demonstrates how various concepts can be incorporated into a single examination question. Students must apply prior knowledge of angles such as an isosceles triangle and its properties, for example, to the above question. Students must also identify equal angles within the same diagram and provide reasons for their choices. Students must also demonstrate, using the same diagram, why STOR is a cyclic quadrilateral or how to apply the midpoint theorem to demonstrate that TQ bisects angle PQR.

According to Pirie and Kieren (1994), there is a growing interest in teaching and learning mathematics with understanding, as evidenced by recent curriculum revisions in several countries. Yang et al. (2021) expand on the idea originally presented by Hiebert and Carpenter (1992) that true understanding in mathematics is essential for effective problem-solving. Their study explores how students develop this understanding by linking new mathematical ideas to at least two existing concepts in their minds – one broader, overarching concept (superordinate) and one more adaptable concept (convertible) that helps generate specific examples. This dual connection process helps students build a more organised and integrated mathematical knowledge structure.

Understanding mathematics involves more than carrying out procedures; it requires the ability to justify methods and explain why a process works, a view supported by mathematics education researchers who emphasise reasoning, justification, and explanation as core components of mathematical proficiency (Kilpatrick, Swafford & Findell, 2001).

. In other words, full knowledge of a mathematical concept is achieved when proofs are used to teach it (Wiggins, 2016). Sierpinska (1994, p.12) begins her book on mathematical understanding by asking, “How should I teach so that my students understand?” What exactly do they not comprehend? “What and how do they comprehend?” This implies that meaningful mathematical understanding develops when learners can connect new ideas to existing mental structures and recognise how previously learned concepts support new reasoning. In this sense, prior knowledge forms the foundation upon which deeper conceptual understanding is built.

2.4 PROBLEMS WITH TEACHING EUCLIDEAN GEOMETRY

According to research (Kyabuntu & Mbhiza, 2024), the lecture method or the traditional axiomatic approach is used to teach Euclidean geometry in the South African context. Furthermore, research has shown that the traditional axiomatic approach results in rote and memorisation learning. According to Chukwu and Ogunkunle (2021), most teachers do not use methods that enable students to acquire conceptual knowledge through a thorough understanding of geometric concepts or giving a clarifying explanation of concepts. Students make more procedural errors when solving problems and are unable to determine geometric relationships.

Furthermore, Chukwu and Ogunkunle (2021) ascribes student inconsistencies and difficulties to the traditional medium of instruction. I contend that traditional teaching methods, such as memorisation, do not allow students to learn geometric problem-solving skills. When using the traditional normative approach, students are unable to acquire knowledge and understanding of growth beyond image-having, as outlined in Pirie and Kieren’s (1994) theory of the growth of mathematical understanding.

Furthermore, circle theorems are regarded as one of the most complex topics for most students. Recent studies, such as Susuoroka et al. (2022), show that students often memorise circle theorems without understanding them conceptually, largely due to traditional teaching methods. Theorems serve as the foundation for circle geometry but students memorise theorems rather than determining which theory applies to a given geometric situation. This way of acquiring mathematics and drawing skills is limited.

Poor performance in circle geometry among NC(V) Level 4 students is strongly associated with limited conceptual understanding, as learners who do not grasp foundational geometric relationships cannot meaningfully apply circle theorems. Ineffective teaching approaches, such as rote demonstration, further restrict opportunities for visual reasoning. As a result, students often remain unfamiliar with core circle geometry ideas (Alex & Mammen, 2016).In the absence of proof or evidence, such as two tangents meeting at the same distance from the circle, students cannot know or understand circle geometry theorems. In addition, according to Nirode and Boyd (2021), the process of proving theorems commonly requires students to use conclusions from several aspects of geometry and algebra.

Furthermore, Alex and Mammen (2016) assert that South African NC(V) L4 mathematics students have an inadequate foundation in Euclidean geometry. Having an inadequate foundation makes it difficult for students to achieve the level of image-making and beyond in terms of development and growth in understanding. Furthermore, according to the PKM, lectures at TVET colleges teach at the level of image-having and beyond. This would be true even if students lacked the necessary conceptual skills, such as image-making, or were unable to apply the necessary primitive knowledge, which is cognitive knowledge required to structure newly learned concepts. For example, students might need to recall their understanding of congruency to come up with an idea for using chord and radii in circle geometry illustrations.

It has also been noticed that students are also neither innovative nor logical when it comes to solving circle geometry difficulties (Abakah, 2023). Students typically rely solely on their textbook and lecturer for information. According to Abakah (2023) learning circle geometry from a textbook affects students' problem-solving abilities as well as the development of their spatial thinking, analysis and conceptualisation of circle geometry principles. The way students respond to questions suggests that they have forgotten the fundamental application of circle geometry from previous years. They are unable to justify each assertion made in response to a problem, some students still struggle with key theory proofs. Furthermore, many students have difficulty providing correct proofs for their responses because they guess or give incorrect reasons. For example, if a question asks students to determine the magnitude of an angle while providing reasons for their answer, students frequently provide the correct magnitude but the incorrect proof or reason.

Furthermore, students struggle to understand geometry principles, reasoning and problem-solving. Those principles include theorems dealing with angles in a circle, cyclic quadrilaterals and tangents.

In addition, when dealing with circle geometry, students struggle to connect different geometric concepts (Brijlall & Abakah 2022). Many students struggle to recognise the appropriate theorem to use in answering questions in examinations as well as writing the appropriate mathematical statement. Research consistently shows that many South African students struggle with the foundational concepts required to understand and apply circle theorems. Studies report that learners experience difficulty identifying relationships between angles in a cyclic quadrilateral, interpreting tangent–radius properties, and recognising alternate and corresponding angle relationships when solving geometric proofs (Alex & Mammen, 2016). Similar challenges are noted in detecting angles subtended at the centre and circumference, as well as interpreting the semicircle theorem, which contributes to persistent underperformance in Paper 2 of the NC(V) Mathematics curriculum. These difficulties indicate that students often lack the conceptual understanding needed to justify geometric reasoning, resulting in varied levels of proficiency across different circle geometry theorems.

Thus, the purpose of this study is to explore how the use of virtual manipulatives supports NC(V) Level 4 students in developing a deeper conceptual understanding of circle geometry.

2.3 COMPUTER TECHNOLOGY

One possible strategy to increase students' learning and accomplishment in circle geometry is to integrate virtual technology into the teaching and learning of the topic.

Many studies (Korenova, 2020; Susuoroka et al., 2022; Zengin, 2017) support the use of computer technology applications, such as GeoGebra (virtual manipulative), in mathematics teaching and learning. In addition, Howard et al. (2021) contend that computer technology helps students make sense of the learning material and that the interactive effects of sound, animation, narration and additional definitions provided by technology (computers) appeal to today's learners, motivating them to concentrate better and achieve higher average scores. In addition, according to Nkomo et al. (2021), using computer technology to teach educational materials helps students acquire and develop knowledge, uncover alternative answers, participate actively in the learning process and improve problem-solving abilities. Because technology is used for presenting knowledge, testing assessment and delivering feedback, computer-based teaching makes instructional techniques significantly more successful than traditional teaching methods. Furthermore, computer technology aids in the promotion of innovation, problem-identification and problem-solving abilities, as well as helping to personalise education by inspiring students to participate actively in the learning process.

A study by Algarni and Lortie-Forgues (2023) however, found that using technology in mathematics teaching and learning has no significant impact. In a study Spencer-Smith and Hardman (2014), the impact of computer contribution on school leavers' senior certificate mathematics scores was examined across 31 schools in the Education Management and Development Centre, East Education District of Cape Town, South Africa. This was accomplished by comparing the performance of two groups: one that served as a control group and one that served as an experimental group.

The experimental group (14 high schools) began using computers in 2001, while the control schools received computers between 2006 and early 2007. The experimental schools were likely to be more technologically advanced than the control schools. There was no significant difference in final senior certificate mathematics results between schools with and without computers; no substantial difference in results after the Khanya labs for the experimental group was installed; no huge disparity in the percentage of students who passed senior certificate mathematics; and no noteworthy improvement in mathematics enrolment rates.

These findings highlight the importance of exercising caution when implementing information and communication technologies into schools as a potential miracle cure for mathematical failure in our context. Spencer-Smith and Hardman (2014) suggests that more research should be conducted to provide a more comprehensive picture of information on communication technology. Spencer-Smith and Hardman (2014) suggestion, on the other hand, allows for the incorporation of different software, approaches or methodologies regarding the use of computer technologies.

2.4 VIRTUAL MANIPULATIVES

The term “virtual” is derived from the Latin word virtualis, which is derived from the Latin word virtus, which means “excellence, or worth.” This leaves us with the question of whether virtual manipulatives can be described as excellent and worthwhile for mathematics instruction. VMs are furthermore described as an interactive web-based visual representation (computer technology) of a dynamic object that provides choices for increasing mathematical knowledge (Gullion, 2024).

Virtual manipulatives permit students to combine graphical and symbolic representations, as well as the activities they interact with the software, expanding opportunities for investigating concepts and developing the meaning of circle geometry theorems. In contrast to physical manipulative, which involves lecturers handing out blocks (tangible objects) to students to demonstrate mathematical concepts, virtual manipulatives go beyond what physical manipulatives can do. For example, many virtual manipulatives, unlike physical manipulatives, include built-in constraint systems that support sensemaking and make mathematical ideas more explicit as the student engages with the tool. Virtual tools respond to the learner’s actions and provide prompts and guidance to help the user focus on the mathematics in the task, whereas physical manipulatives do not provide specific and directed feedback and interaction.

According to Vágová (2021) VMs can be thought of as an externalised representational form. Externalised representations are most often a more natural representation of structure than mental representations because they allow for the computation of more clear and specific encoding of information (Finn et al., 2018). VMs, as an externalised representational, are a form because they enable the construction of arbitrarily complex structures, lower the cost of controlling thought and aid in the coordination of thought. VMs may be considered a significant type of representation or a combination of several types of representation because a representation is defined as a set of signs, characters, icons or objects that represent something else (Mainali, 2021). VMs create a unique learning environment in which the properties of many shapes and compositions are integrated within the virtual programme (Clements & Sarama, 2016). As a result, VMs allow students to use a diverse range of representational forms in dynamic and varied ways. Some VMs, for example, include links between enactive, iconic and symbolic notations to help learners connect these forms of representation. Some VMs allow students to change the shape of an on-screen object or mark it with mathematical notations.

Working with different representational forms improves users' capacity for mathematically significant activities, which influences user learning. Students' challenges with circle geometry are exacerbated by the typical reality in which students are taught procedural knowledge while studying circle geometry rather than focusing on a student-centred approach to conceptual understanding in teaching mathematics (Awaji, 2021). As a result, using a VM is a student-centred method because it immediately instils a conceptual understanding of any mathematical subject in the students' minds.

Given recent advancements in computer technology, mathematics lecturers can now use contemporary technology such as computers to teach virtual manipulation (Engelbrecht & Borba, 2024). Furthermore, students appreciate learning and comprehending mathematical concepts using virtual manipulation since it is more engaging than the traditional "chalk and teach" approach formerly used by traditional mathematics lectures (Adegboyega et al., 2023). VMs can be very useful for students who want to improve their understanding of circle geometry's abstract symbolic language. In circle geometry, increased use of VMs improves students' opinions regarding their learning, motivation and academic achievement (Moyer-Packenham & Westenskow, 2016). Symbols can be found: o represents a circle, letters such as A, B, C, β, α, represent angles formed by two vertical lines, || represents parallel lines, and ⊥ is the symbol for two perpendicular lines. Students who struggle with circle geometry often struggle to make connections between visual and symbolic representations, but VMs could perhaps help.

Furthermore, students frequently struggle to describe what they have learned in circle geometry, but VMs could also help them clarify their thinking and share their ideas. VM tools like Geometer's Sketchpad, GeoGebra and Cabri have been introduced to help students with visualisation and result in positive learning achievements (Clements

& Sarama, 2016). To avoid any misconception, students need to be familiarised with the use of technology and the mathematical content of circle geometry

2.4.1 Advantages of Virtual Manipulatives

The use of VMs in teaching and learning has numerous advantages. Mendiburo et al. (2014) found, for example, that VM instructions were just as effective as physical manipulative instructions. VM instruction, on the other hand, was more effective in teaching fractions because physical objects cannot be changed in the same way that virtual objects can (for example, in circle geometry, the size of angles can be changed to reinforce theorems). In many cases, this allows students to create more examples than they could with physical objects. Many VMs also provide hints and feedback to students, allowing them to practice on their own without the lecturer's help.

Virtual manipulatives (VMs) have been shown to enhance students' conceptual understanding by supporting visualisation, exploration, and active engagement in mathematical ideas. Moyer-Packenham and Westenskow (2016) found that VMs promote deeper reasoning and meaningful connections between representations, which is essential for understanding geometry. Their work also highlights that visual interaction helps learners recognise patterns, construct meaning, and make sense of abstract ideas. These benefits are particularly relevant to circle geometry, where learners must investigate relationships, test conjectures, and interpret geometric properties dynamically. Additional studies support this view, noting that VMs encourage discovery learning, improve engagement, and strengthen students' ability to justify mathematical relationships (Zacharia & Michael, 2019; Mudaly & Mpofu, 2020). Thus, using VMs may help TVET students build coherent conceptual structures in circle geometry by connecting new ideas with prior knowledge. According to Fiorella and Zhang (2018), visuals allow students to analyse, hypothesise and communicate ideas to others by sketching or drawing images. In addition to those skills, visuals may help to capture NC(V) L4 mathematics students' attention and engage them in their learning. Engaging students in their learning is a difficult task, especially in circle geometry, and the use of visualisation strategies as a teaching tool may help to achieve the goal of student engagement in the classroom.

Furthermore, Mardiana (2020) states that access to technology is insufficient and that both the lecturer and the curriculum must play critical roles in mediating the use of technological tools. As a result, lecturers and curriculum developers must be knowledgeable decision-makers who can determine when and how technology can be used to enhance students' learning appropriately and effectively. As a result, TVET colleges and mathematics programmes must ensure that students and teachers have access to instructional technology, such as classroom hardware, handheld and lab-based devices. These devices should be loaded with mathematical software and applications, as well as web-based resources. Furthermore, adequate training needs to be provided to ensure effective use of mathematical software and applications.

However, using VMS alone does not guarantee that students understand concepts and processes. According to Zhang and Cutler (2024) lecturers must use a variety of teaching strategies, including the use of various types of VMs and hybrid classes to represent ideas and the use of technology, such as the internet and computers, as a teaching and learning aid.

2.4.2 Disadvantages of Virtual Manipulatives

Although virtual manipulatives have numerous advantages for improving mathematical comprehension, there are limitations and difficulties with their use that lectures need to be aware of.

2.4.2.1 Lectures' ability to apply technology practices in the classroom

When discussing the disadvantages of virtual manipulatives, it is crucial, to begin with disadvantages that are directly related to the use of technology. Much of the technology-related challenges we face have to do with the prevalence of lectures' ability to apply technology practices in the classroom (Dinçer, 2024). Lecturers with limited software technology skills are frequently intimidated by a lack of technological proficiency and these inadequacies frequently leave lecturers feeling overwhelmed. One of the technological challenges that TVET college lecturers face is a lack of training to develop their proficiency with technology software such as Sketchpad and GeoGebra.

2.4.2.2 Students understanding of the concept of virtual manipulatives

Students are unfamiliar with the concept of VMs and lecturers who are familiar with this resource are attempting to introduce it to their students. However, it is critical to understand the distinction between VMs and what is simply virtual (Moyer-Packenham & Westenskow, 2016). The key distinguishing feature between VMs and virtual students is that with VMs, students can interact with dynamic software, these interactions provide opportunities for acquiring mathematical knowledge, such as using a mouse to manipulate objects in the programme. However, simply having students read virtual articles, fill out worksheets on the screen, or answer questions in front of a pictorial object does not qualify as VMs.

2.4.2.3 Lecturers are ill-informed concerning the benefits of technology in teaching circle geometry

Furthermore, according to Ghawail et al. (2021), lecturers may regard the internet as untrustworthy or that a website did not prove to be advantageous, or they may be hampered by a lack of technological resources and support. The use of virtual manipulation at TVET colleges seems to be severely limited due to a lack of technological software resources. As a result, for virtual manipulatives to truly demonstrate their learning and cognitive benefits, programmes like GeoGebra must be used by every student, not just in a lecturer demonstration, a significant barrier to students' access to virtual manipulation is a lack of available software technology. In addition, academic staff has several misconceptions about VMs, such as the belief that manipulatives do not aid in the understanding of mathematical concepts and those VMs are better used in lower grades than secondary or TVET colleges (Keldgord & Ching, 2022).

2.4.2.4 Virtual manipulatives do not support collaborative learning

According to Keldgord and Ching (2022), VMs are also not designed to support collaborative learning, a major issue for socio-cultural constructivist educators. Mahmoud (2023) further claim that using manipulatives in the classroom is not as simple and that good application necessitates carefully defining the role of the teacher, as well as the goals and potential tasks involved. This may necessitate a significant amount of effort on the part of the educator, causing some students to abandon the use of VMs entirely.

However, these facts, are debatable because it takes a dedicated lecturer to determine what is beneficial and what is not in using VMs to teach circle geometry. Despite the disadvantages listed above, it is safe to say that if one can overcome the initial apprehension associated with the introduction of technology, the benefits of VMs will far outweigh any disadvantages.

2.4.3 Comparison of Virtual and Physical Manipulatives

Physical and virtual manipulatives are tools of representation used to scaffold students' learning of mathematical concepts. Virtual and physical manipulatives have distinct advantages and disadvantages. For example, many virtual manipulatives have explicit symbolic visual links, whereas physical manipulatives involve sensory senses.

According to Moyer-Packenham and Westenskow (2016), the power of virtual manipulatives lies in their ability to combine multiple representations in ways that assist students in connecting multiple aspects of mathematical concepts and ideas. Sullah et al. (2017) compared virtual and physical manipulatives to determine which type of manipulative is more appropriate for teaching geometry. According to their findings, there is no significant difference in achievement scores between the virtual and physical groups. However, the mean score in the virtual group students was higher, indicating that it was more appropriate to use for teaching geometry.

Furthermore, physical manipulatives can be difficult to integrate, according to Siller and Ahmad (2024), because lecturers must keep an eye on each student and manipulative pieces, or blocks can become lost or broken. Furthermore, computer technology can create visual representations of mathematical concepts that are as meaningful as physical manipulatives (Moyer-Packenham & Westenskow 2016). Integrating computers or internet resources into mathematics instruction is important because we are entering a technological age and computers provide an efficient and diverse method of learning.

On the other hand, Bouck et al. (2020), used virtual and physical manipulatives in a quasi-experimental study to assess students' problem-solving abilities. Although there were no statistically significant differences in problem-solving ability between the two groups, students who used VMs expressed more exploration of VM-inputs and enhanced students' high achievement and attitudes, both of which require additional attention and research. Students were encouraged to use VMs on a variety of topics.

Correspondingly, Sarama and Clements (2016) conclude that the flexibility of VMs allows students to explore geometric figures in ways that physical shape sets do not allow. Students, for example, can change the size of the computer shapes, altering all or just some of them. According to Moyer-Packenham and Westenskow (2016), the flexibility of the VMs had several positive effects on kindergartners' patterning. When using VMs, they created more patterns and used more elements in their patterns than when using physical manipulatives or drawing. Finally, students only created new shapes while working on the computer (e.g., by partially occluding one shape with another). As a result, a richer network of integrated concrete knowledge is developed by students, potentially more than physical manipulatives.

2.5 GEOGEBRA AS A VIRTUAL MANIPULATIVE TOOL FOR CIRCLE GEOMETRY LEARNING

GeoGebra was designed to integrate geometry, algebra and calculus into a single dynamic environment. Markus Hohenwarter invented GeoGebra, a free mathematics programme, in 2002 (Radović et al 2020). The most recent version of the programme is available for download from the GeoGebra website (Zengin et al., 2012) Radović et al, (2020) describe GeoGebra as a dynamic virtual mathematics programme that provides an interactive learning environment in which users may build and interact with mathematical objects. Users of GeoGebra – primarily lecturers and students – can use this platform to investigate, explain and model mathematical topics and their relationships.

Awaji et al. (2025) evaluated the success of learners using GeoGebra through a pre-test-post-test experimental design. The findings demonstrated that VMs, such as GeoGebra, can be effectively used in learning environments to foster higher-order thinking and enhance knowledge retention. Furthermore, following a non-equivalent comparison group design, Ogbonnaya and Chimuka (2016) evaluated the effect of incorporating GeoGebra in the teaching of circle geometry on learners' motivation to study and enjoyment of the session. Two Grade 11 mathematics classes from two separate schools participated in the study. Because of the availability of a computer lab, an experimental group was used. This group was taught using GeoGebra, whereas the other group was taught without using GeoGebra. The questionnaire data were analysed, and it was found that GeoGebra did indeed inspire students to study mathematics while making the sessions interesting.

The study by Ernie et al. (2023) examines how engaging students in mathematical reasoning can challenge their existing beliefs about mathematics. The authors highlight that when students are encouraged to construct mathematical explanations, they lay a foundation for deductive reasoning, which is essential for deeper mathematical understanding. The author noticed that the dynamic nature (drag feature) of the software alters the structure of the explanation and that the students were able to generalise the solution and respond appropriately. Also, Tamam and Dasari (2021) found that GeoGebra is an effective tool for teaching mathematics, as it helps simplify abstract concepts – especially in geometry, algebra and statistics – through interactive visualisations. Their study shows that allowing students to manipulate mathematical objects enhances understanding, boosts engagement and makes learning more enjoyable.

Carriazo-Regino et al. (2024) found that using GeoGebra significantly improved Grade 10 students' trigonometry skills, including reasoning, communication and problem-solving, compared to traditional teaching methods. This suggests GeoGebra effectively enhances student understanding and performance in trigonometry.

2.6 THE SIGNIFICANCE OF THE LITERATURE REVIEW FOR THIS RESEARCH

Circle geometry is an important concept in mathematics that our students do not fully grasp. These studies and those on which they report were searched for evidence of VM explorations to enhance students' understanding of circle geometry. While many studies of students' understanding of mathematics are related to this study, few acknowledged the use of VMs to improve TVET college students' conceptual understanding of circle geometry or to reduce the challenges students face when learning circle geometry.

2.7 CHAPTER SUMMARY

In this chapter, the researcher provided an overview of the literature on exploring the use of VMs to enhance the students' understanding of circle geometry. The researcher looked at the benefits and drawbacks that students may encounter when using VMs to understand concepts, properties and the importance of instructional design in general when doing circle geometry in this study. Following this discussion, it became clear that new knowledge should be incorporated into existing related networks for current understanding to facilitate future learning. Key aspects of the use of VMs have been investigated. The chapter concluded by reflecting on the implications of the research literature. The next chapter presents the research methodology.

CHAPTER 3: RESEARCH METHODOLOGY

3.1 INTRODUCTION

In Chapter 2 the researcher presented literature on concepts such as the challenges/difficulties encountered by students when learning circle geometry, how virtual manipulatives are used to improve circle geometry understanding, and how virtual manipulatives can be used to reduce the challenges faced by students when learning circle geometry. In that chapter, Pirie and Kieren's (1994) theory of the growth of mathematical understanding was presented as the theoretical lens.

The purpose of this study is to explore how virtual manipulatives support NC(V) Level 4 students' conceptual understanding of circle geometry.In this chapter, I outline the research paradigm, research design, research approach sampling and data collection instruments used in this study. In addition, this chapter discusses ethical considerations, credibility and trustworthiness.

3.2 RESEARCH PARADIGM

Takdir and Arif (2022) highlight how Kuhn's concept of paradigms – shared systems of thought can guide how problems are understood and solved. This idea is also relevant to mathematics education, where shifts in teaching approaches reflect evolving paradigms in how students develop mathematical thinking and understanding. This involves critically examining their views on reality (ontology) and knowledge (epistemology) to ensure consistency in research approach and analysis. This view supports Wisker's (2018) notion of a paradigm as a foundational set of assumptions that guide how research is structured and interpreted. George et al. (2025), in the AMEE Guide No. 181, highlight that a researcher's philosophical framework, encompassing ontology, epistemology and axiology, is fundamental in guiding the research process. This perspective corresponds with Creswell's (2014) view that these three core assumptions form the basis of a research paradigm, shaping how researchers perceive reality, gain knowledge and assign value to their work. In this study, the researcher followed the interpretivist paradigm. The interpretivist paradigm enabled me to see the world through the eyes and experiences of NC(V) L4 mathematics students.

3.2.1 Ontology

Ornellas and Engelbrecht (2021) discuss ontology as a philosophical inquiry into the nature of reality and the fundamental qualities of social phenomena, focusing on how individuals' assumptions shape their understanding of what is real and meaningful. This aligns with the broader view of ontology as examining the nature of reality within educational contexts, which informs how teaching, learning, and mathematical understanding are interpreted and studied in mathematics education research. Furthermore, according to Fernandez (2018), ontology is the essence of reality. Ontology, therefore, determines the nature of the existence of a certain phenomenon. This study explores how virtual manipulatives support NC(V) Level 4 students' understanding of circle geometry within an interpretivist paradigm. Interpretivism assumes that reality in educational settings is constructed through learners' experiences and interactions, and that meaning is understood through participants' perspectives rather than through objective measurement. In this study, students' learning processes and the meanings they attach to geometric relationships are therefore interpreted through the lens of their lived classroom experiences. This positioning ensures consistency between the study's purpose, research questions, and methodological choices.

Ontology is the philosophical study of the nature of existence or reality, of being or becoming, as well as the underlying categories and connections of objects that exist. Based on the researcher's underlying belief system, ontology analyses the essence of existence and being. It investigates the researcher's assumptions for believing that something makes sense or is legitimate, as well as the basic character or content of the phenomenon of using VMs to understand circle geometry for NC(V) L4 mathematic students. As a result, the researcher obtained a better understanding of the structure and character of reality, as well as his beliefs about what can be said about it. This included how students interacted with reality while using virtual manipulatives to learn about circle geometry, how individual students behaved and how interactions shaped relationships while using GeoGebra.

To interpret the information, the researcher had to understand philosophical ideas about reality. The researcher's assumptions, concepts, or claims assisted him in orienting his thinking about how VMs are used to improve understanding of circle geometry, and how VMs can be used to reduce the challenges faced by students when learning circle geometry.

Ontology served as the basis for the researcher's paradigm since it provided an understanding of the things that made up the world (Kamal, 2019). Furthermore, ontology aims to discover the true nature of the basic concepts that constitute themes that the researcher might investigate to make sense of the meaning hidden in study data. It allowed the researcher to investigate whether the use of virtual manipulatives to learn about circle geometry is a reality or a figment of his imagination.

Philosophical notions, concerning the nature of reality, were required to comprehend how the researcher understood the data he collected. These assumptions, ideas or claims helped to orient his thinking about the topic under study, its relevance, and how he could approach it to answer the research question, grasp the situation at hand and contribute to its resolution.

3.2.2 Epistemology

The term epistemology is derived from the Greek episteme, which means "knowledge." Samosamo (2023) defines epistemology as the study of how we learn, how we know the truth or reality. The study of knowledge refers to its nature, forms, acquisition and transmission. Moreover, it focuses on the nature of human knowledge and comprehension that one may be able to achieve as a researcher or knower to extend, develop and deepen understanding in his field of study.

Furthermore, epistemology can be seen as a component of the research process since it affects how the researcher perceives the world around him as well as how he differentiates between right and wrong (Ryan, 2018). Interpretivists also aim to enrich the data that has already been collected (Riyami, 2015). Interpretivism assumes that humans are independent of knowledge due to relativist ontology and subjective epistemology. Consequently, according to Pilarska (2021), interpretivist epistemology, is a study of the relationship between research and the research subject, and it focuses on the meaning, voice, position, experience, ideas and feelings of individuals.

This study leaned toward practical epistemology and the belief that knowledge is best learned through sensory experiences and provable, objective truths. Objective truth refers to a fact that cannot be disputed or debated – it is a universal truth with a definitive, unchanging answer .

3.2.3 Axiology

Axiology refers to the ethical considerations that must be addressed when designing a study. Finnis (2017) examines the philosophical approach to making worthwhile or accurate decisions. Concepts of appropriate and inappropriate conduct in the research must be developed, examined and comprehended. We use it to assign weight to various aspects of our study, including participants, data and the audience to whom we present our results. Therefore, the researcher was guided by four ethical conduct criteria: teleology, deontology, morality and fairness (Kivunja & Kuyini, 2017).

Teleology is a moral philosophy that emphasises pursuing what is intrinsically good or desirable. Within this study, the researcher applied a teleological stance by ensuring that the research produced meaningful and ethically responsible outcomes. This was achieved by designing learning activities that genuinely supported students' understanding, using virtual manipulatives in ways that added educational value, ensuring that students were not disadvantaged by participating, and presenting findings honestly and transparently so they could inform future teaching practice. In this way, the study's processes and outcomes aimed to maximise educational benefit for both the participants and the broader teaching community.

Deontology enabled the researcher to recognise that every action taken during the study carries ethical consequences for participants, the researcher, and the academic community (Kivunja & Kuyini, 2017). This awareness guided the researcher to act with duty-based responsibility—for example, by protecting participants' rights, ensuring informed consent, avoiding any form of coercion, safeguarding confidentiality, reporting data truthfully, and adhering strictly to UNISA's ethical procedures. Through these actions, the researcher upheld the deontological obligation to do what is ethically required, regardless of convenience or outcome. Furthermore, it allowed for greater flexibility in dealing with individual participants or observations.

The morality criterion refers to the inherent moral values that were upheld throughout the study. The researcher ensured that the data interpretation was correct. Finally, the fairness criterion highlighted the importance of treating all NC(V) Level 4 participants equitably throughout the study. In this context, fairness meant ensuring that every student—regardless of prior exposure to technology, academic ability, or confidence in geometry—received the same level of support when using virtual manipulatives. The researcher ensured that instructions were explained uniformly, additional assistance was provided to students with limited computer experience, and no participant was advantaged or disadvantaged during data collection. This approach safeguarded equal opportunity for meaningful learning and ensured that the research process upheld respect and justice for all participants.

3.3 WHY INTERPRETIVIST PARADIGM WAS CHOSEN FOR THIS STUDY?

This study was guided by an interpretivist paradigm, which seeks to understand how individuals make meaning of their experiences within specific social and educational contexts. Interpretivism assumes that reality is subjective, socially constructed, and best understood through the perspectives of participants rather than through objective measurement (Creswell & Poth, 2016; Cohen, Manion & Morrison, 2018). In qualitative educational research, interpretivism emphasises understanding learners' thoughts, actions, interactions and interpretations as they engage with learning activities. Within this study, the interpretivist paradigm was appropriate because the researcher sought to explore how NC(V) Level 4 students at Molapo TVET College constructed understanding of circle geometry when engaging with virtual manipulatives. This approach allowed the researcher to interpret students' reasoning, perceptions and learning processes as they interacted with GeoGebra and other hands-on activities.

The social construction of reality is a concept that contends that rather than being a 'natural' entity, people's reality is created and shaped by their interactions. That reality also includes the interaction using VMs to make sense of geometric theories such as circle geometry. Moreover, the interpretivist paradigm enabled the researcher to take detailed field notes and richly describe the students' experiences, allowing readers to immerse themselves in the same cultural context encountered by the researcher. Consequently, the researcher had two roles to perform: using his unique knowledge and engagement to develop findings, and convincing the scientific community that behavioural features based on participants' experiences could be applied in various contexts. Using this paradigm also allowed the study to focus on a single issue, whether virtual manipulatives would help NC(V) L4 students understand circle geometry (Moustakas, 1994; Remenyi et al., 1998).

3.4 RESEARCH DESIGN

In this study, the researcher used an action research design. According to Creswell (2014), action research involves planning, implementing, observing, reflecting and then re-planning, further implementing, observing and reflecting.

3.4.1 Reasons for Using an Action Research Design

Action research is widely used in educational settings because it enables practitioners to investigate challenges in their own classrooms and implement practical solutions in a cyclical, reflective manner (Creswell, 2014; McNiff et al., 2012). In the context of this study, action research was selected because the researcher sought to understand and improve NC(V) Level 4 students' learning of circle geometry through the use of virtual manipulatives. The researcher had observed persistent learner difficulties with geometric reasoning, proof, and conceptual understanding, and required a research approach that allowed him to introduce targeted interventions, observe their effects, and refine the teaching strategy systematically.

Action research was also appropriate because it aligns with the researcher's dual role as both lecturer and reflective practitioner. This design enabled the researcher to examine his own instructional practices, evaluate how students engaged with virtual manipulatives during the intervention cycles, and make informed adjustments to promote deeper conceptual understanding. In addition, the cyclical nature of action research—moving through stages of planning, acting, observing, and reflecting—matched the study's intention to iteratively improve teaching methods rather than test a fixed treatment.

Furthermore, action research supports classroom-based innovation by allowing researchers to introduce pedagogical tools, such as GeoGebra and other virtual manipulatives, and evaluate how these tools function within authentic learning environments. Because NC(V) students have varied backgrounds, technological exposure, and prior knowledge, an approach was needed that allowed the researcher to respond flexibly to emergent classroom challenges. Action research provided the structure needed to trial VMs, determine how students made sense of circle geometry concepts, and adjust the instructional approach based on real-time feedback.

Thus, action research was chosen because it offered a suitable, context-driven method for understanding how virtual manipulatives could enhance students' conceptual understanding of circle geometry while simultaneously allowing the researcher to refine his own teaching practice.

3.4.2 What is Action Research Design?

The study of social issues by Lewin is widely regarded as a watershed moment in the evolution of action research as a design(Hendrick 2019). Furthermore, McAteer (2021) note that the beginnings of action research in the United Kingdom can be traced to the Schools Council's Humanities Curriculum Project (1967–72), which emphasised an experimental curriculum and re-conceptualisation of curriculum creation.

Rabgay (2021) describes action research as a collaborative and participatory process. The author also asserts that action research is situational and contextual. This implies that the researcher collects data from the participants and explains the results as an intervention to influence ongoing activities (Young et al., 2016). Additionally, based on the participants' interpretations, the action research design promotes the contemplation of learning and understanding circle geometry.

As there is always room for improvement in teaching and educating students, action research is quite popular in education. Although there are numerous ways to teach in the classroom, action research is particularly effective because its cycles allow the lecturer to reflect continuously on their instructional practices and how these practices influence students' understanding of circle geometry. Action research can also be conducted in areas of teaching practice that need to be explored, such as the current study, which examined the use of VMs in enhancing the understanding of circle geometry among NC(V) L4 students, or in settings in which continual improvement was emphasised. In this study, the spiral model of action research was followed as depicted in Figure 3.1.

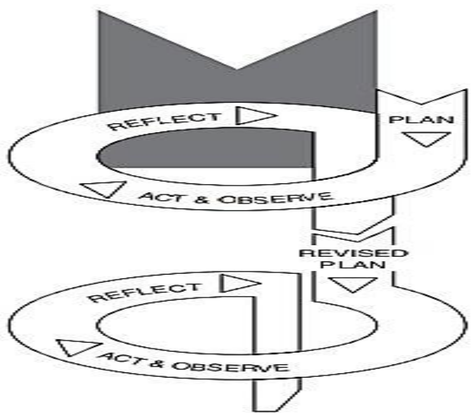


Figure 3.1: Kemmis and McTaggart's action research spiral

Source: Kemmis and McTaggart (2000, p. 595)

Figure 3.1 displays Kemmis and McTaggart's (2000) spiral model of action research, which was adopted for this study. Action research is a flexible spiral method that enables both actions in the form of change and improvement and research (understanding, knowledge) to be accomplished concurrently. Understanding enables more informed change while also being informed by that change.

The research was carried out in three cycles from 20 February to 10 March 2023, which included aspects of planning, action, observation and reflection. Identifying issues, developing a learning and implementation plan for each action activity, student worksheets, evaluation and media tools and tools and materials required for learning are all part of the research planning process.

During the first cycle, a baseline test was used to assess circle geometry prerequisites for NC(V) L4 students. Based on the preliminary evaluation findings, conventional teaching lessons were developed and implemented. Observation of learning activities is carried out during the implementation of teaching and learning activities in order to determine the course of learning.

During the reflection phase, the researcher was able to "analyse, synthesise, evaluate, explain, and draw conclusions" about the lessons. The information was gathered during the participants' normal class lessons. The lessons were taught during the first cycle, and all NC(V) L4 mathematics students were allotted six periods (4.5 hours) per week.

3.4.2.1 Planning stage

Before beginning the action research cycles in class, data were acquired from Umalusi mathematics examiners' reports to identify students' challenges with circle geometry.

Informal interviews were also performed with four mathematics lecturers from different campuses to obtain their perspectives on students' challenges and the teaching of circle geometry. Based on the findings of the reports and interviews, a baseline test was developed and given to students to diagnose the prerequisites for circle geometry.

- Lesson 1
- The first meeting with students took place at the beginning of Cycle 1, where the researcher administered the baseline test to determine learners' initial understanding of circle geometry concepts. The second meeting followed in the same week and was used to review the baseline test outcomes and identify specific areas of weakness. During the third meeting, the researcher introduced the foundational concepts that students needed before beginning circle geometry theorems. In the fourth meeting, the researcher taught the midpoint theorem and the quadrilateral theorem using traditional chalkboard instruction, without any technological tools or learning aids. The fifth meeting concluded Cycle 1, during which students completed a 45-minute post-test that the researcher evaluated to determine changes in understanding.
- Interviews

The researcher conducted structured interviews with five students at the sixth meeting, which was videotaped.

- Reflection 1

After the sixth meeting with the participants, the researcher briefly consulted a colleague who is also a mathematics specialist to reflect on and evaluate the conventional teaching approach used in Cycle 1. This colleague was not involved in data collection and did not participate in the action research process. Rather, the discussion served as an informal professional reflection to help the researcher refine his own practice. The study is therefore not a collaborative action research project but an individual classroom-based action research design, with the colleague's role limited strictly to informal feedback. The reflection was an attempt to assess what happened or what was not completed in a previous step or attempt. The outcomes of the reflection were used to take additional steps toward achieving research objectives and preparing the way for intervention in the second cycle.

- Intervention plan 1 (second cycle)

  After completing the first cycle in the fifth meeting and conducting reflective analysis in the sixth meeting, the seventh meeting marked the beginning of Cycle 2. In this session, the researcher introduced a practical, hands-on teaching approach in which students constructed physical geometric models to support their understanding of circle geometry concepts.

  Lesson 2

Students were given a worksheet that instructed them on how to draw solids and how to use a compass, protractor, pencil and ruler to measure various radii, chords and angles. Following that, students were asked leading questions to define their discoveries. These inquiries included their observations made while building and measuring the solids. With the assistance of a mathematics specialist colleague, the researcher observed how students interacted with the task that was given to them was done during the eighth and ninth meetings with the students. During the tenth meeting, the students had to once again complete a 45-minute post-test, which was marked by the researcher.

- Interviews (second cycle)

Following the completion of the post-test meeting, the researcher conducted structured interviews with five students during the 11th meeting, who were once again videotaped. Students were asked to compare the first method of teaching with the second method of constructing solids during the interview.

- Reflection (second cycle)

Engaging in professional conversations with a qualified Mathematics colleague enabled the researcher to reflect on his personal biases, examining and rethinking his perspectives, while taking into account all aspects of the student's experiences with practically constructing solids. The researcher's curiosity about the changes that resulted from the initial intervention plan grew during this period of reflection.

- Intervention plan 2 (third cycle)

During the 11th meeting, the researcher intervened once more, this time by introducing them to the GeoGebra programme, which was done on the computer in the computer lab.

- Lesson 3 (Third cycle)

Following the discussion with a colleague, students were given a task-oriented questionnaire with instructions like, how to change lines on the programme, attempting to improve students' understanding of developing circle geometry theorems. Once again the researcher with the assistance of a colleague observed the participation of the students during the 12th and 13th meetings. During the 14th meeting, the students had to once again complete a 45-minute post-test, which was marked by the researcher.

- Interviews (Third cycle)

During the 15th meeting, the researcher conducted the third semi-structured interview with five students. Interviews we once again recorded with a video camera.

- Reflection (third cycle)

Following the third cycle of interviews, the researcher engaged in another professional conversation with a qualified mathematics colleague. This discussion examines the impact of using VMs versus practical solid construction. The goal of this conversation was to reach a valid and reliable conclusion about whether the use of virtual manipulatives can improve NC(V) L4 mathematics students' understanding of circle geometry.

3.5 RESEARCH APPROACH

According to Winaldo et al. (2022), a research technique is a method through which data is acquired and examined. Qualitative research methods, quantitative research methods and mixed methods research methods are the three categories of research methods. A qualitative research approach was selected for this research. The reasons for choosing a qualitative approach are summarised in the following section.

3.5.1 Reasons for Following a Qualitative Approach

The qualitative method was chosen since it is often used to comprehend concepts and perceptions. It offers viewpoints on a variety of themes and assists in the creation of thoughts or hypotheses for potential quantitative research. The qualitative approach, which uses analysis to dive deeper into problems, assists in the development of novel ideas and individual points of view (Carol, 2016).

According to Creswell and Poth (2016), qualitative research aims to gain a comprehensive and contextualised understanding of particular situations or occurrences by investigating and comprehending the meaning that individuals or groups assign to a social or human problem. Thus, the goal of qualitative research in this study was to describe, interpret and explore how the use of virtual manipulatives can improve the understanding of NC(V) L4 mathematics students, systematically from the perspective of the population (students) being studied, and to generate new concepts and theories.

As Creswell and Poth (2016) documented in the literature, using a qualitative approach can be advantageous because of its natural sensitivity to contextual setting, direct data collection, rich narrative description, complexity, process orientation, induction and participation perspectives. The researcher considered the natural contexts and settings in which NC(V) L4 mathematics students functioned since the goal was to give an in-depth grasp of real-life problems; hence, the natural setting of the study was done at a TVET college.

This study did not focus on statistics, but rather on how students responded to the given tasks when using VMs. While exploring the use of VMs for enhancing circle geometry understanding, the researcher had to adapt to various ideas, concepts or discoveries. In this study, the researcher used a qualitative method to understand individual interactions and gain insight into students’ perceptions of the use of virtual manipulatives (Anderson, 2017).

3.5.2 Qualitative Approach

According to Creswell and Poth (2016), qualitative inquiry begins with assumptions, worldviews and theoretical lenses that enable researchers to understand the meaning individuals attach to social and educational experiences. In this study, a qualitative approach was essential because the researcher sought to explore how NC(V) Level 4 students make sense of circle geometry concepts when learning with virtual manipulatives. Rather than reducing learning to scores or numerical outcomes, qualitative methods allowed the researcher to investigate students' reasoning, perceptions, misconceptions and cognitive processes in depth. This holistic perspective made it possible to understand not only *what* students struggled with, but also *why* they experienced difficulties and *how* their interactions with virtual manipulatives shaped their understanding. Thus, the qualitative paradigm served the purpose of capturing the complexity of students' mathematical thinking within authentic classroom contexts.

According to Creswell (2016), qualitative research is a method for investigating and comprehending the significance that individuals attach to a social human situation. The research method in this study entailed developing questions and procedures, collecting data in the NC(V) L4 mathematics students' settings; evaluating the data, inductively, moving from particular to general themes, and producing interpretations of data meaning.

3.6 SAMPLING

The population for this study consisted of 30 NC(V) L4 mathematics students from a TVET college in the South-West Gauteng district of South Africa. A convenient sample of five NC(V) L4 mathematics students was selected after the conclusion of each cycle. Convenience sampling is a type of non-probability sampling that is commonly applied in clinical and qualitative research. This sampling strategy chooses clinical cases or people who are easily accessible (Stratton, 2021). Therefore, because of the researcher's proximity to these student cohorts and their willingness to participate in the study, convenience sampling was chosen (Lau & Mang, 2023). Furthermore, the researcher used convenience sampling because it allowed him to collect basic data and trends about his study without dealing with the challenges of a randomised sample. The sample size in action research is determined by the environment and the approach used for inclusion within the activity investigation cycle, according to (Cohen, et al, 2018) .

3.7 DATA-COLLECTION INSTRUMENTS

The data collection instruments in this study were administered in a deliberate sequence to ensure comprehensive measurement of both student learning outcomes and experiences. According to McMillan et al. (2018), data collection instruments are tools used to explore phenomena of interest. In line with this, the study employed a baseline test, post-test, task-based open-ended questionnaire, semi-structured interviews, and classroom observations.

The population for this study consisted of 120 NC(V) Level 4 mathematics students from a TVET college in the South-West Gauteng district of South Africa. Although two classes of approximately 15 students each participated in the baseline activities and intervention cycles, a purposive sample of five students was selected to participate in the interviews. These five students were chosen because they demonstrated varying levels of understanding during the cycles and were therefore able to provide rich and diverse insights into their learning experiences.This test aimed to assess students' prior knowledge and establish a reference point for evaluating progress. Following the instructional segment using virtual manipulatives (VMs), a post-test was conducted to measure learning gains and the effectiveness of the programme. The post-test results also informed the design and focus of subsequent semi-structured interviews.

Open-ended, task-based questionnaires were administered to enable students to elaborate on their learning experiences in their own words. This qualitative data provided richer insights into students' perceptions and helped contextualise their test performance.

Semi-structured interviews, conducted face-to-face with nine students selected based on their test performance, allowed the researcher to probe more deeply into individual experiences, clarifying aspects of learning that quantitative measures alone could not reveal (Creswell, 2014).

Finally, classroom observations were conducted in the same setting as the baseline test. These observations enriched the data by capturing students' engagement, interactions with the VMs, and learning behaviours, supporting the triangulation of findings.

The sequential administration of these instruments—from baseline assessment to post-test, followed by qualitative questionnaires, interviews, and observations—ensured that both quantitative and qualitative aspects of learning were systematically captured, allowing for a robust analysis of the impact of VMs on students' understanding of circular geometry.

3.7.1 Baseline test

Lazarus et al. (2022) assert that a well-designed baseline assessment helps lecturers understand students' knowledge, abilities, and attitudes, set realistic targets, and track progress. To determine whether NC(V) L4 mathematics students possessed foundational knowledge for circle geometry, **a** baseline test was administered, assessing prior understanding of circle properties, the Pythagoras theorem, congruency, and quadrilateral features. Example items included identifying congruent triangles, calculating sides of right-angled triangles, and stating properties of parallelograms. This quantitative data profiled students' readiness and highlighted learning challenges. Following instruction with virtual manipulatives, a post-test measured learning gains through items on circle theorems, sector areas, and angle relationships. Complementary open-ended questionnaires captured students' experiences, challenges, and strategies qualitatively, while semi-structured interviews with nine students (Creswell, 2014) probed deeper into understanding and reasoning. Classroom observations recorded engagement and interaction with VMs. The sequential use of these instruments ensured comprehensive quantitative and qualitative data to evaluate the impact of virtual manipulatives on learning.

3.7.2 Post-test

To assess students' understanding of circle geometry theorems taught through various instructional methods, the researcher developed post-test questions based on NC(V) Level 4 final national examination papers. These questions were initially selected by the researcher and subsequently reviewed by experienced TVET mathematics lecturers to ensure they met the standards and taxonomy prescribed by the Department of Higher Education and Training (DHET).

During the study, students completed three post-tests, each following a different instructional cycle: traditional teaching, physical manipulatives, and virtual manipulatives using GeoGebra. All cycles covered the same core circle geometry content, including theorems, their converses, and corollaries, such as: opposite angles of a cyclic quadrilateral are supplementary; the exterior angle of a cyclic quadrilateral equals the interior opposite angle; the angle between a tangent and a chord equals the angle subtended by the chord in the alternate segment; and tangents from a common external point are equal in length.

The first post-test followed traditional teaching, where concepts were delivered through lectures, board demonstrations, and worked examples, emphasising procedural understanding. The second post-test assessed understanding after instruction using physical manipulatives, including protractors, rulers, and printed diagrams, which allowed students to measure angles, draw and construct figures, and explore geometric relationships concretely. The third post-test was administered after instruction using GeoGebra, where students interacted with dynamic virtual manipulatives to visualise, manipulate, and test geometric properties.

Although the content remained consistent across the three tests, the post-tests differed in instructional context and cognitive processes emphasised. The traditional cycle focused on procedural learning, the physical manipulatives promoted hands-on exploration and measurement, and the GeoGebra cycle fostered dynamic reasoning, visualisation, and deeper conceptual understanding. Consequently, while the questions were similar, students' strategies, interpretations, and depth of understanding were expected to vary according to the instructional approach used in each cycle.

3.7.3 Observation schedule

Classroom observation, the third method of data collection, was used to monitor NC(V) Level 4 mathematics students' behaviours and engagement during different teaching approaches, including traditional instruction and the use of virtual manipulatives (GeoGebra). Observations were conducted over a two-week period during regular mathematics periods at a conveniently located TVET College in the South-West Gauteng district, with 15 students per class. The primary aim was to understand how students asked questions, participated in class, and interacted with GeoGebra to construct and explore circle geometry theorems. Over the two weeks, a total of 12 contact sessions were held, with 6 sessions per week**,** allowing sufficient time for both instruction and observation.

During cycle 2, which focused on the use of physical manipulatives, lessons lasted 45 minutes each, during which students engaged with protractors, rulers, and diagrams to explore geometric relationships concretely. In cycle 3, also consisting of 45-minute lessons**,** students worked in the computer lab with GeoGebra, constructing circle theorems and exploring dynamic properties. The researcher gathered field notes and used a video camera to record students' interactions during both physical manipulatives and GeoGebra-based lessons, enabling a comparison of engagement levels, problem-solving approaches, and conceptual understanding across the different instructional methods.

Naturalistic observation, as described by Major (2018), was applied to study authentic interactions between students and instructors, as well as patterns in student behaviour. Creswell (2015) highlights the value of observation as an accurate record of what people do and say in real-life settings. Observations continued until the researcher and students agreed that critical mass – or data saturation – had been reached.

The researcher also attempted to maintain as much neutrality as was feasible in this study. As a result, the researcher was interested in how students used computer software to complete mathematical tasks as opposed to teaching and learning through conventional methods. In addition, using the PKM, the researcher devised an observation schedule that classified each activity level (1-6) as weak, moderate or strong, allowing him to comment on each activity level. They can be summarised as follows:

- Level 1: The recognition of qualities of congruency was an example of primitive knowledge. Pythagoras, triangles and quadrilaterals were identified by students who understood and created patterns. By using any representation, the student could remember previous material.

- Level 2: Students who created images could continue patterns without requiring them to do specific activities, like drawing images. Students understood how to apply visual representations to the topic at hand.
- Level 3: Property-noticing could be used to develop a formula that describes all pattern interactions by generalising the pattern rule. By using representations to convey mathematical meanings, the students could construct patterns and algorithms.
- L4: The most common types of representations are symbolic and verbal. By extrapolating the pattern rule, students could create a general formula that describes all pattern interactions at the formalising level. Students could use mathematical representations to construct patterns and algorithms. The most common representations were symbolic and verbal.
- L5: Students in the 5th level developed a general formula for extending patterns to greater stages and relating patterns to a range of mathematical subjects such as algebraic expressions, equations and geometry. The students connected representations and theorems. The majority of the time, symbolic and verbal representations were used.
- Level 6: Finally, at the structuring level, students strived to develop a fundamental knowledge of circle theorems and their significance.

3.7.4 The open-ended questionnaire with tasks

An open-ended question, as opposed to a list of possible replies, allows participants to respond on their terms and helps the respondent to comprehend the issue in their way (Creswell and Poth, 2018). For example, students were asked: "How is your understanding of circle theorems involving cyclic quadrilaterals after using GeoGebra?" These semi-structured questions featured a list of prepared questions that acted only as a guide and followed up on pertinent remarks given by the students.

The researcher designed a questionnaire with open-ended for students for self-monitoring and self-reporting. Each participant completed this questionnaire in class after the second cycle.

Research study topics were specifically addressed in the questionnaire-related task and included semi-structured questions that addressed all the sub-questions. The following questions were asked to select the activities that are most likely to encourage learning using virtual manipulatives when teaching Circle Geometry at NC(V) L4:

Some of the questions included:

- Were there any activities that you found difficult? Explain.
- Did using GeoGebra (VM) help you grasp cycle geometry? If so, please explain how.
- Do you think using GeoGebra has minimised your challenges with circle geometry?

The tasks required students to establish the various theorems of circle geometry using virtual manipulatives (GeoGebra). As an example, students were instructed to use GeoGebra software to determine that “an angle in the centre of a circle is twice the size of the angle it occupies at the circumference”.

- The students were instructed to draw a circle on the computer with O as the centre and BC as the chord.
- The students were then requested to draw a line perpendicular to the chord BC from point O to point D.
- Students were subsequently instructed to calculate the measures of angles BDO and CDO and to determine the length of segment DB.
- The students calculated the diameters of angles BDO and CDO as well as measured the lengths of BD and CD.

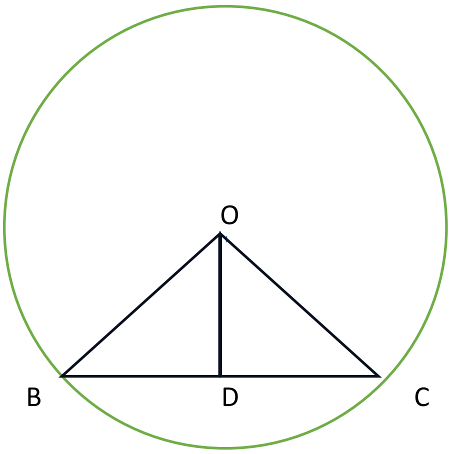


Figure 3.2: Task: A circle when line drawn from the centre is perpendicular to the chord

In addition, the questionnaire included questions about students' progress in understanding the activities and their experience with GeoGebra.

3.7.5 Semi-structured interviews

Semi-structured interviews are frequently open-ended, which allows for flexibility. Setting questions in a predetermined order allows for easy comparison between respondents, but it can be restrictive. Less structure can help the researcher see patterns while still allowing them to compare responses from different students.

A list of prepared questions guided the semi-structured interviews, which followed up on pertinent comments made by the interviewees. Interviews were used in this study as a crucial research tool to learn more about the comprehension and cognitive processes of the pupils. Before gathering data, the researcher developed a series of open-ended questions to direct the interviews and elicit thorough answers from participants. Five students were specifically chosen for interviews in order to elucidate and elaborate on their written responses, which were derived from assignments, exercises, and a post-test. In order to guarantee a range of viewpoints, selection criteria included students' willingness to participate and representation of different performance levels.

Interviews were conducted after each instructional cycle, typically after school in the researcher's office, providing a focused and uninterrupted setting. Each interview was structured to allow participants to explain their reasoning in detail, and follow-up probing questions were used to clarify ambiguous or incomplete responses. The data collected were analyzed using a meta-analytic approach, which enabled the identification of recurring themes, patterns, and areas requiring further observation during classroom sessions.

The primary aim of the interviews was to assess higher-order cognitive skills, including abstract thinking, critical thinking, insight, conceptualisation, and imagination. Indicators of these skills were identified from students' explanations, reasoning strategies, and problem-solving approaches. By using pre-prepared open-ended questions alongside flexible probing techniques, the researcher ensured that participants' thought processes were thoroughly explored, allowing for a rich and nuanced understanding of their conceptual learning.

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The participants were able to articulate their views on the use of VMs (GeoGebra) while answering questions about circle geometry. Using open-ended questions enabled students to comment on what they had found to be exceptional, unusual or basic that the researcher did not consider when developing the related tasks. This presented the researcher with a plethora of relevant information for this experiment.

This was a qualitative data collection strategy in which the researcher asked students a series of carefully crafted, predefined yet open-ended questions. As an example, the researcher asked students, “What difficulties did you encounter when completing the tasks?” This was done after the post-test was completed in the researcher’s office. The interviews assisted the researcher in explaining, better understanding, and exploring the students’ opinions, behaviour, experiences, phenomena and the like.

As part of the semi-structured interviews, the researcher had the opportunity to probe the student for further information and to pursue a line of inquiry that the researcher had initiated. A video recorder was used to record all interviews, and the video recordings were later transcribed into notes.

3.7.6 Sequence in which the instruments were administered

Before getting taught, circle geometry, students were given a baseline test to determine their level of knowledge. During the first cycle, the researcher taught circle geometry using conventional methods. After that, the researcher administered a post-test, and five students were chosen for a semi-structured interview.

The researcher implemented his first intervention plan during the second cycle to achieve adequate results. The intervention was based on theorems of circle geometry and practical solid construction. Students were required to use a compass, protractor and ruler to draw and measure diagrams. Furthermore, students were given a task sheet with circle geometry questions about their practical experience to complete while the researcher and colleague observed their reactions. Following the practical application, students took a post-test and five were selected for a semi-structured interview.

During the third cycle, the researcher introduced students to GeoGebra as a virtual tool to support their understanding of circle geometry. Students completed a task sheet containing circle geometry questions using the software while the researcher observed their interactions. Afterwards, they completed a questionnaire reflecting on their experience with virtual manipulatives. A post-test was then administered, and five students were purposively selected for semi-structured interviews based on their responses and engagement during the cycle.

3.8 DATA ANALYSIS

Baseline tests, questionnaires, post-tests, semi-structured interviews and lesson observations were used to gather data from the 30 NC(V) Level 4 students at the South West Gauteng TVET College’s Molapo Campus. The baseline test assessed whether students possessed the foundational knowledge required for circle geometry and addressed the first research question. Qualitative data analysis followed a systematic process of organising, interpreting and generating meaning from the multiple data sources collected during the action research cycles.

The analysis began with repeated reading of all raw data to gain familiarity and identify initial impressions. This step enabled the researcher to summarise information, note recurring ideas, and distinguish similarities and differences in student responses. From this process, meaningful segments of data were identified and coded. Coding involved breaking down large amounts of textual information into smaller, conceptually meaningful units that represented emerging ideas in the data. Creswell (2016) describes codes as interpretive labels assigned to segments of text to help develop broader explanations, while Cooper et al. (2025) refer to coding as the process of tagging data according to relevant concepts.

Once codes were established, they were grouped into categories and subcategories to form higher-order themes. This hierarchical organisation allowed the researcher to capture a “big picture” view of students’ understanding, challenges, and interactions with circle geometry and virtual manipulatives. Constructs from existing literature were used to guide the naming and refinement of themes, supporting alignment with theoretical perspectives and enabling extension of pre-existing ideas. Categories were further examined for their characteristics and dimensions; for instance, the category “understanding” was analysed along a continuum ranging from weak to strong comprehension.

A qualitative content analysis approach guided the interpretation of the data. Content analysis involves identifying patterns, key concepts, and relationships within textual material to uncover meaning and draw conclusions (Kristiana, 2023). Its purpose is to organise and interpret data systematically, enabling the researcher to extract meaning and derive insights grounded in participants’ lived experiences (Bengtsson, 2016). Using content analysis, the researcher examined students’ words and actions to identify how they reasoned through geometric ideas, interacted with virtual manipulatives and constructed or failed to construct conceptual understanding.

This integrated process of coding, categorising and theme development provided a robust, rigorous and transparent analytical framework suitable for an interpretivist study aimed at understanding students’ learning processes in circle geometry.

Table 3.1: Examining students’ mathematical understanding of patterns by the PKM

Layers	**Codes**
Primitive knowledge	Recognition of properties of congruency Pythagoras, triangles and quadrilaterals know what patterns are and create patterns. Any representations can be used by the student to recall earlier information.
Image-making	Trying to find the relationship between items of patterns using various images and trying to further images by extending new constructions. The student can relate the representations to the problem under consideration.
Image-having	Furthering patterns without the need for engaging in particular activities such as drawing images. The student can connect the visual representations to the problem at hand.
Property-noticing	Indicating a correlation between pattern items. The student can relate the representations to mathematical concepts and compare representation properties.

Layers	Codes
Formalising	Creating a general formula that describes all patterns' interactions by generalising the pattern rule. The student can build patterns and algorithms by using representations to explain mathematical meanings. Symbolic and verbal representations are primarily used.
Observing	Developing a general formula for extending patterns to larger stages and connecting patterns to a variety of mathematical fields, such as algebraic expressions, equations and geometry. The student establishes a link between representations and a theorem. Symbolic and verbal representations are primarily used.
Structuring	Attempting to build a basic understanding of how to formulate circle theorems and make logical observations on their significance
Inventing	New pattern-related questions are being developed.

In this study, data coding analysis was used. As shown in Table 3.1, coding was based on the students' use of different representations, as well as their levels of knowledge, and matching behaviours. The representation type of each question was classified based on the student's use of it; in addition, each student's representation type and level of comprehension were numbered to generate an understanding map. Therefore, a question could have several answers depending on the student's response. The representation types, understanding levels and movement from each level were coded to analyse the data. Abbreviations were used to indicate the levels and types of representation see Table 3.2.

Table 3.2: Coding scheme used in analyses

CODE	DESCRIPTION
P	Primitive knowing
IM	Image-making
IH	Image-having

PN	Property noticing
F	Finalising
0	Observing

3.9 THE ETHICS OF RESEARCH

All researchers are subject to ethical responsibilities regardless of the methodology they use (Lucas et al., 2018). This study followed strict ethical standards to ensure that the rights, dignity and safety of all participants were protected. The ethical principles observed are detailed below.

3.9.1 Ethical Clearance and Authorisation

Formal ethical clearance was obtained from the University of South Africa (UNISA) Research Ethics Committee, confirming that the study met institutional requirements for research involving human participants. Additional permission to conduct campus-based research was granted by the Department of Higher Education and Training (DHET) (see Appendix B) and by South West Gauteng College (SWGC), Molapo Campus, where the study was conducted. Access to the participants was granted only after the proposal synopsis had been approved by the institutional research committee.

3.9.2 Informed Consent

All students received detailed information about the purpose of the study, procedures, time commitments and their rights before agreeing to participate. Since all participants were over the age of 18, parental consent was not required. Students signed informed-consent forms confirming voluntary participation and permitting the researcher to use their responses for academic purposes, including follow-up interviews where clarification was needed.

3.9.3 Voluntary Participation

Participation in this study was strictly voluntary. Students were informed that they had the right to withdraw at any point without providing a reason and without experiencing any negative consequences. No student was coerced, pressured or incentivised to participate.

3.9.4 Confidentiality

Confidentiality was ensured by storing all data securely and limiting access to authorised academic personnel only. Raw data, interview recordings and written responses were handled responsibly and were not shared beyond academic requirements.

3.9.5 Anonymity

To protect participants' identities, no names or identifiable personal information were used in transcripts, analysis or reporting. Pseudonyms or participant codes were assigned, ensuring anonymity throughout the research process and the final dissertation.

3.9.6 Beneficence

Beneficence refers to the responsibility to act in ways that promote participants' well-being. In this study, beneficence was upheld by ensuring that the learning activities involving GeoGebra added educational value and supported students' understanding of circle geometry. The study was designed so that participation benefitted students by enhancing their exposure to meaningful mathematical tools.

3.9.7 Non-Maleficence

Non-maleficence requires researchers to avoid causing any form of harm. The researcher ensured that no student was disadvantaged during the intervention. Because some students had limited exposure to computers, all participants were first taught how to use GeoGebra to ensure fair access. No psychological, academic or physical harm occurred, and all data were reported honestly without falsification.

3.9.8 Integrity and Honesty in Reporting

All findings presented in this dissertation accurately reflect the data collected. No manipulation, exaggeration or misrepresentation of results occurred. The researcher maintained transparency throughout the research process by documenting procedures, decisions and interpretations carefully

Use of Pseudonyms

Pseudonyms, ranging from S1 to S30, were assigned to the 30 students. Younis (2016) defines a pseudonym as a fictitious name used to identify a person while differing from their real name. Pseudonyms are intended to preserve students' anonymity and protect their identity without revealing it, as noted by Boughey (2016).

By implementing pseudonyms, the researcher ensured that students' identities remained confidential while still allowing for a structured and systematic analysis of their responses. As explained previously, the researcher alternated between the codes to seek similarities before looking for common themes. Through these processes, the challenges identified were categorised into the following theme with its subthemes.

3.10 TRUSTWORTHINESS

There are several terms used in the literature to describe the validity and reliability of qualitative research. While some writers use the validity and dependability of the terms interchangeably, others Merriam and Grenier (2019) use the terms trustworthiness and credibility. This researcher used the term trustworthiness.

Enworo (2023) proposed four facts to enhance the credibility of qualitative research: credibility, dependability, confirmability and transferability.

3.10.1 Credibility

Credibility refers to the authenticity of the facts or the viewpoints of the participants, as well as the researcher's interpretation and portrayal of them (McGill et al., 2023). By engaging participants for a lengthy period and consistently studying them, the researcher established the credibility of the study. Several unique questions were posed about circle geometry and competence. Participants were asked to provide examples to support their claims, and the researcher followed up with probing questions. The researcher examined the facts from their raw interview material until a theory was formed that provided the researcher with an understanding of the extent of the topic under investigation.

Additionally, the researcher used triangulation to improve the process of qualitative research. In triangulation, a researcher uses more than one data collection technique to reveal various aspects of a research topic, such as in-depth interviews, discussions and field notes. To assist with the observation technique, the researcher solicited the assistance of another subject specialist from the same college. A second observer was included to provide an additional qualitative perspective on the students' engagement and to enhance the credibility of the observations. Rather than measuring statistical agreement, the second observer supported the trustworthiness of the findings by confirming key patterns, behaviours, and interactions noted during the lessons. All sessions were video recorded since it provides trustworthy documentation, allows the researcher to double-check his data, and allows for systematic feedback to be obtained through participant interviews.

3.10.2 Dependability

Dependability refers to the constancy of the data over similar conditions (McGill et al., 2023). Moreover, the action research approach allowed the research team to act as partners in the process, with all participants offering their views and contributing to the change process according to their expertise and knowledge. According to the researcher's methods and descriptions, a study would be considered reliable if the findings were replicated with similar individuals under similar conditions (Koch et al., 2018).

As part of this study, the researcher ensured reliability by maintaining a clear audit trail of the research path, from raw data to the literature review, research questions, theoretical framework, methodology, data collection tools and data interpretation. The audit trail included a comprehensive collection of notes on all decisions made during the study.

3.10.3 Confirmability

Confirmability refers to the researcher's ability to establish that the data accurately represents the replies of the participants and not the researcher's prejudices or perspectives (Nowell et al. 2017). The researcher can demonstrate confirmability by detailing the method used to draw conclusions and interpretations of the data and demonstrating that the data used to obtain the conclusions was obtained solely from the data. This can be demonstrated through participant statements that reflect each emerging topic in qualitative study reporting. Throughout this study, the researcher-maintained confirmability by recording processes and reviewing and rechecking data. Performing the role of devil's advocate, looking for or explaining negative occurrences that contradicted or discarded assumptions with queries like "what if...", the researcher also used probing techniques and judgements.

3.10.4 Transferability

The term "transferability" refers to results that may be applied to other situations or people (Nowell et al. 2017). In qualitative research, this criterion is satisfied if the findings are meaningful for those who did not participate in the study and readers can relate the findings to their own experiences.

By providing enough information about the informants and the research setting, the researcher attempted to give readers and researchers a sense of how transferrable or applicable the findings were.

3.11 CHAPTER SUMMARY

This chapter outlined the research methodology that guided the study. It began with an introduction to the overarching approach, followed by a detailed discussion of the research paradigm. The study adopted an interpretivist paradigm, which aligned with its emphasis on understanding students' experiences and behaviours in their natural educational environment.

The discussion of ontology, epistemology, and axiology provided the philosophical foundations underpinning the research. Ontologically, the study assumed that reality is subjective and shaped by individual perceptions. Epistemologically, it valued co-constructed knowledge between researcher and participants, and axiologically, it recognised the researcher's values and their influence on the study.

An interpretivist paradigm was chosen to enable rich, contextualised insights into the students' learning experiences with virtual manipulatives. The action research design supported this choice, as it allowed iterative reflection and practical interventions aimed at improving teaching and learning. The chapter explained what action research entails and why it was suitable for this educational context.

A qualitative approach was followed to capture in-depth, descriptive data. This approach was justified by the study's goal to explore the meaning and interpretation behind student behaviours and responses rather than to quantify outcomes.

Sampling involved selecting NC(V) L4 mathematics students from a conveniently located TVET college. The sample included two classes of 15 students each.

A variety of data collection instruments were used: a baseline test to assess prior knowledge, post-tests after each instructional cycle, an observation schedule, a task-based open-ended questionnaire, and semi-structured interviews. The instruments were administered in a specific sequence to align with the research cycles and provide complementary data sources.

Data analysis followed a qualitative approach, focusing on patterns, themes, and interpretations drawn from the students' responses and observed behaviours.

Ethical considerations were addressed throughout the study, including informed consent, confidentiality, and voluntary participation. The chapter concluded with a discussion on trustworthiness, ensuring the study met criteria for credibility, dependability, confirmability, and transferability.

The next chapter presents to data analysis and interpretation of the findings.

CHAPTER 4: DATA ANALYSIS AND INTERPRETATION OF FINDINGS

4.1 INTRODUCTION

The previous chapter provided a comprehensive overview of the research paradigm, design and data collection instruments employed in this study. This chapter presents a detailed analysis of the results related to the learning difficulties experienced by NC(V) L4 mathematics students in understanding circle geometry.

The investigation was conducted through an action research design, unfolding in three cycles. Each cycle aimed to address different aspects of the learning process and evaluate the effectiveness of interventions. The baseline test administered served as a diagnostic tool to assess students' prior knowledge and identify areas of weakness in geometry. This test covered fundamental properties related to circles, straight lines, triangles, quadrilaterals, and Pythagoras' theorem, providing valuable insights into students' foundational understanding of geometry.

Following the baseline assessment, circle geometry theorems were taught using convetional methods. The central research question guiding this study was:

- How can virtual manipulatives enhance NC(V) L4 mathematics students' knowledge of circle geometry?
- To address this question, the following sub-questions were formulated:
- What are the challenges experienced by NC(V) L4 mathematics students in understanding circle geometry?
- What is the nature of virtual manipulation activities that enhance the understanding of circle geometry?
- How can virtual manipulation be used to minimise the challenges experienced by NC(V) L4 mathematics students in understanding circle geometry?

By analysing students' responses, the study introduced interventions such as practical demonstrations and virtual manipulatives to enhance learning experiences and comprehension of circle geometry concepts. The subsequent sections of this chapter present an analysis of students' responses, drawing on theoretical frameworks and empirical evidence to elucidate the complexities of teaching and learning circle geometry.

4.2. PRESENTATION AND INTERPRETATION OF THE FINDINGS

4.2.1 Themes and Subthemes

Themes, as defined by Ayres et al. (2022), function as attributes, descriptors, elements and concepts. They assist researchers in addressing the study's core issues by serving as implicit subjects that organise recurring concepts. Themes include codes with a high degree of generality, unifying ideas related to the research topic under a single reference point (Dawadi, 2020). Dawadi further describes a theme as a thread of underlying meaning revealed through interpretive analysis and in students' subjective understandings.

To identify patterns in students' responses and develop a deeper understanding of the data, each theme may encompass several subthemes (Maguire & Delahunt, 2017). According to Warren (2020), a large volume of text is coded and summarised into categories. In this study, codes exhibiting similar patterns were combined to form broader categories. By grouping these codes, the researcher was able to create a systematic organisation of concepts. The primary objective of this analysis was to examine students' responses from Tests and semi-structured interviews to answer the research question.

Through qualitative coding, the researcher effectively structured data collected through qualitative methods, such as semi-structured interviews, into coherent theories. By alternating between codes to identify similarities and then searching for common themes, the researcher systematically organised and interpreted the data. Ultimately, the challenges identified were categorised into specific themes and their associated subthemes.

4.2.2 Themes

An overview of the codes, categories, and overarching themes found through the examination of students' experiences and difficulties studying circle geometry is given in Table 4.1.

Table 4.1: Overview of the identified codes, categories and themes

Codes	**Categories**	**Themes**
Language difficulty (LD) Inaccurate calculations (IC) Unfamiliar with concepts (UFC) Confusion about theorems (CT) Struggling to apply theorems (SAT)	Difficulties in Understanding Key Concepts • Inability to grasp essential concepts and properties related to circles and their geometric properties • Confusion and uncertainty when faced with questions related to circle geometry due to gaps in understanding • Language limitations that prevented comprehension from being expressed	Barriers to comprehension of circle geometry concepts
Lack of engagement (LE) Limited focus (LF) Difficulty paying attention (DPA) Interaction with tools (IT) Participating in class discussions (PCD)	Classroom Engagement and Participation • Limited engagement with the subject outside of school contributed to their struggles • Overconfidence or complacency regarding their abilities, resulting in a lack of practice	Barriers to active participation in the classroom environment

Codes	Categories	Themes
Positive feelings (PF) Increased confidence (IC) Reduced anxiety (RA) Uncertainty during tasks (UT) Frustration with mistakes (FM) Satisfaction with progress (SP)	Emotional Responses and Motivation • Student's feelings and motivation engaging with geometric concepts • Positive attitude towards practical activities	Emotional reactions and motivational factors influencing learning
Learning through experimentation (LTE) Understanding through practical tasks (UPT) Hands-on problem-solving (HPS)	Practical Experiences with Geometry • Exploration empowers students to construct their own understanding	Hands-on activities and their role in promoting geometric understanding
Difficulty with instruments (DWI) Challenges with measurements (CWM) Improved spatial reasoning (ISR)	• Practical engagement fosters understanding and proficiency • Demonstrating resilience and perseverance during challenges • Benefits of practical activities	Hands-on activities and their role in promoting geometric understanding
Positive interaction with GeoGebra (PIG) Visualisation of theorems (VOT) Improved comprehension through technology (ICT)	Effectiveness of virtual tools • Visualisation of geometric relationships, fostering active problem-solving.	Impact of virtual tools on learning geometry concepts

Codes	Categories	Themes
GeoGebra vs traditional methods (GTM) User-friendly interface (UFI) Reduction in learning challenges (RLC)	• Comparison of traditional teaching methods with using virtual tools	

4.3 PRESENTATION AND INTERPRETATION OF THE FINDINGS

A baseline test was administered to assess students' prior knowledge. The first question required students to recall fundamental information about circles, including their names and characteristics. Students needed to identify terms such as tangent, chord, diameter and radius.

Question 2 focused on basic angles and lines in geometry. Specifically, Questions 2.1 and 2.2 asked students to compute unknown angles on straight lines and in triangles, respectively, using the exterior angle's value as a guide.

In Question 3, students were tasked with demonstrating that a given figure, BCDE, is a cyclic quadrilateral. Question 4 required students to compute the length of one side of a right triangle using the Pythagorean theorem.

Understanding students' starting points is crucial for any educational intervention. This section presents the results of the baseline test administered to NC(V) L4 mathematics students, which assessed their prior knowledge and identified areas of weakness in circle geometry. The results shed light on the challenges students faced in grasping fundamental geometric concepts.

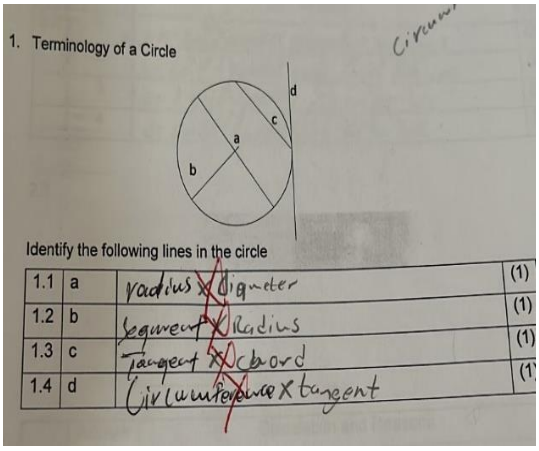
The baseline test questions were designed to align with the first level of the PKM, "Layer 1: The recognition of qualities of congruency as an example of primitive knowledge." This level includes understanding and creating patterns with Pythagoras, triangles and quadrilaterals. At this stage, students are expected to recall and apply previously learned material using various representations.

The research assumption that the low pass rate in L4 mathematics at South West Gauteng College might be related to Grade 9 students enrolling in TVET college without prior exposure to circle geometry appears supported by the data. Participant responses indicated difficulties with basic geometric definitions and a lack of familiarity with foundational concepts.

Student	Mark%	Student	Mark%
Student 1	7	Student 15	14
Student 2	20	Student 16	13
Student 3	11	Student 17	13
Student 4	13	Student 18	11
Student 5	13	Student 19	19
Student 6	9	Student 20	16
Student 7	12	Student 21	9
Student 8	7	Student 22	7
Student 9	2	Student 23	5
Student 10	8	Student 24	6
Student 11	9	Student 25	3
Student 12	14	Student 26	6
Student 12	19	Student 27	8
Student 13	2	Student 28	8
Student 14	3	Student 29	8
Student 14	7	Student 30	4

Table 4.1 summary of students scores for baseline test

Out of the thirty students, only one passed the test. Four students scored between 16% and 20%, while the remaining students scored between 2% and 14%. The following extracts illustrate the challenges students faced in completing the baseline test. Figures 4.1 and 4.2 presents an example response from S1 and S2 addressing a question about the terminology of a circle.



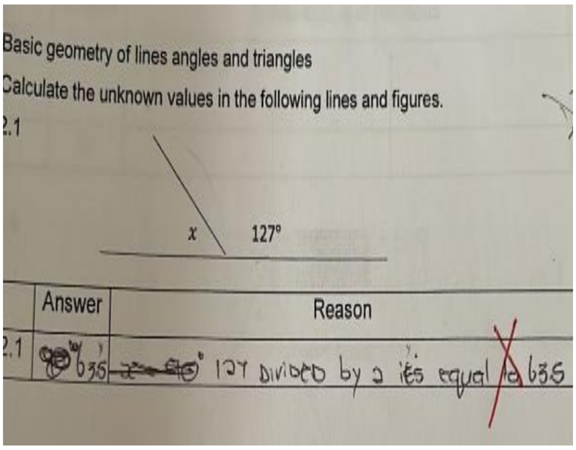


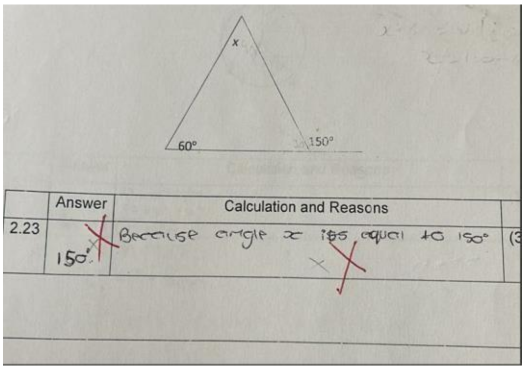
Figure 4.1: Student S1 response

Figure 4.2: Student S2 response

The question required students to recall fundamental details about circles, such as their names and properties. Despite some understanding of geometric concepts, they struggled to articulate their knowledge and were unfamiliar with the fundamental aspects of circle geometry. For example, some responses revealed that students were unaware that an angle on a straight line is supplementary and equals 180°, as illustrated in Figure 4.2. This highlights their struggles with basic geometric definitions and foundational concepts, which can hinder engagement with more advanced topics.

This finding aligns with Juman et al. (2022), who observed that students often face significant difficulties in learning geometry, such as naming parts of geometric structures or drawing diagrams for given problems. According to Pirie and Kieran (1994), many students in this study would be classified as functioning below the first layer, where they are expected to recognise primitive knowledge such as congruency, triangles and quadrilaterals. While some students could comprehend and produce patterns using representations like Pythagoras, triangles and quadrilaterals, many were unable to recall prior information or apply it effectively.

A significant challenge revealed by the baseline test was the misunderstanding of the properties of straight lines and inaccuracies in calculations involving triangles. Students S2 and S3 did not grasp that the exterior angle of a triangle equals the sum of the two opposing interior angles, or that the sum of the interior angles in a triangle is 180°, as shown in Extract 4.3. This demonstrates their struggle with applying fundamental geometric principles, emphasising the need for targeted interventions.



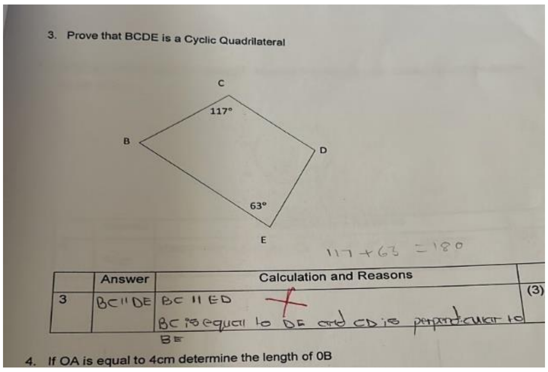


Figure 4.3: Student S2 response

Figure 4.4: Student S3 response

The inability of some students to demonstrate that figure BCDE is a quadrilateral indicates a lack of fundamental geometric knowledge. Additionally, the test revealed that students lacked the prerequisite understanding to recognise shapes as quadrilaterals, as evidenced by S3's response in Figure 4.4. This shortcoming suggests a poor grasp of geometric shapes and their characteristics. It is consistent with Musfiratul et al. (2023), who note that students often struggle with understanding the types of triangles, solving angular problems and comprehending special lines in triangles.

Furthermore, S2 and S3 would be categorised as not functioning at the first level of the PKM, as they had difficulty identifying the characteristics of external angles of triangles and quadrilaterals. This lack of comprehension and inability to generate patterns indicates a failure to recall and apply prior knowledge – a critical foundation for classifying geometric figures and applying geometric theorems effectively.

The baseline test was crucial in uncovering these challenges in grasping fundamental circle geometry concepts. The observed lack of prerequisite knowledge among NC(V) L4 mathematics students echoes findings from previous research. For example, Hassan et al. (2023) found that many students in Sokoto, Nigeria, struggled with fundamental geometric concepts and spatial reasoning. Similarly, Yunus et al. (2019) observed that year-five students in Malaysian public schools lacked proficiency in geometric reasoning and the application of geometric principles.

These findings are consistent with existing literature on mathematics education, which highlights that these misconceptions in geometric thinking lead to poor performance (Ordiz et al., 2022). This underscores the need for targeted interventions to address misconceptions and build a solid foundation in geometry. By identifying common patterns of errors, educators can better support student learning and facilitate deeper engagement with circle geometry concepts (Mensah et al., 2023).

More than 70% of the participants did not operate at Layer 1 of the PKM, as they could not recognise the fundamental defining features of quadrilaterals and triangles. This observation prompted the researcher to introduce intervention strategies to minimise this knowledge gap.

4.4 RESEARCHER'S APPROACH TO TEACHING CIRCLE GEOMETRY: CYCLE ONE

In this section, the outcomes of traditional instruction in circle geometry are presented. Students' responses, perceptions, and learning outcomes are examined, and the action research cycles followed in this study are summarised in Figure 4.5.

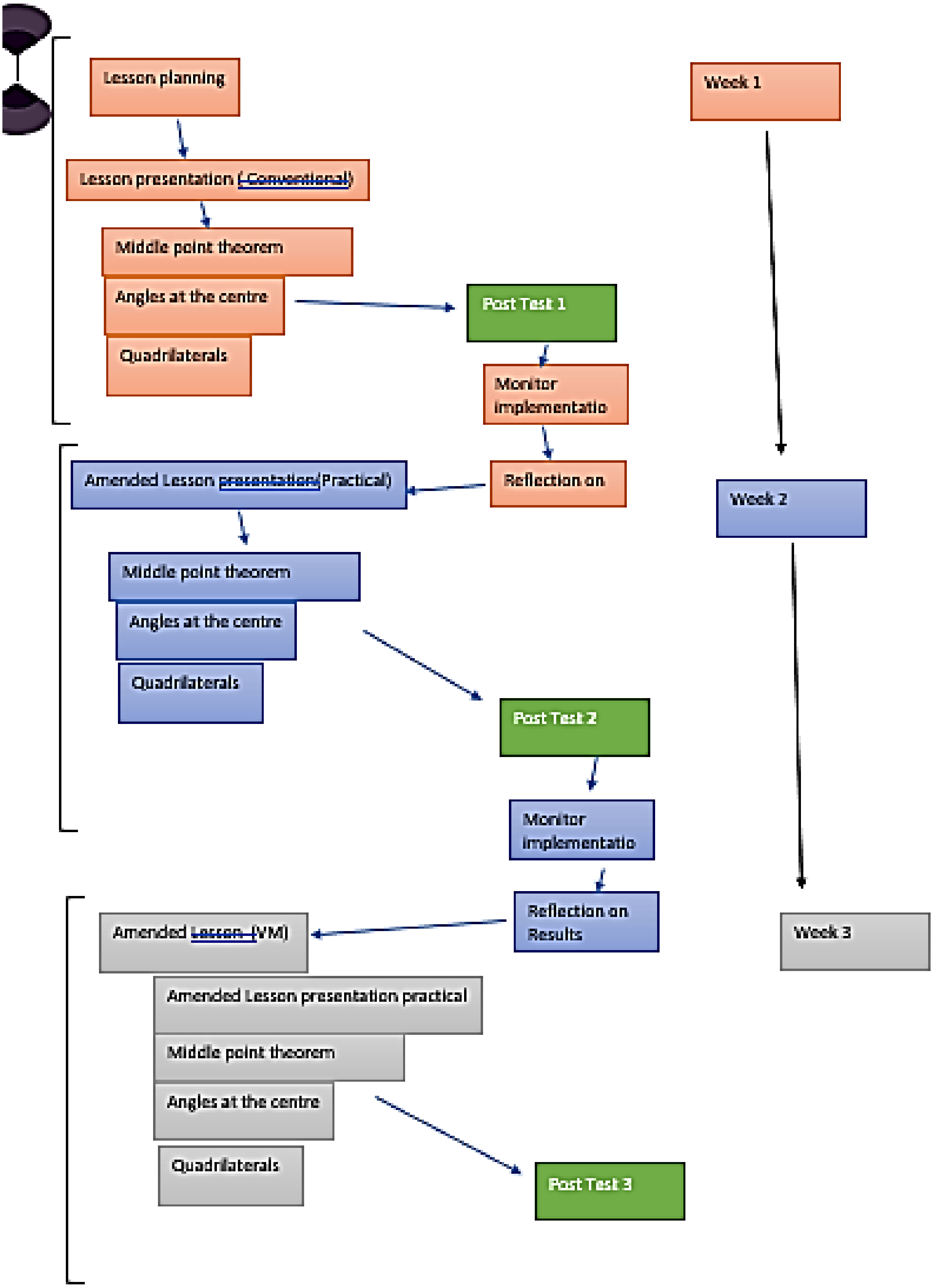


Figure 4.5: Elliott's action research model

Source: Adapted from Elliott (1991)

4.4.1 Circle Geometry Taught Using the Traditional Method – Cycle 1

Traditional instruction involves conventional teaching strategies, such as lectures and textbook-based learning (Gull & Shehzad, 2015), particularly when teaching geometric concepts. In the first cycle, the researcher employed chalk-and-talk methods to teach circle geometry. Interactions during this phase were limited, with students typically responding only to what they understood. Notably, there was a lack of spontaneous responses from the students, despite the researcher's attempts to engage them.

For example, when asked to explain the midpoint theorem, a few students provided satisfactory answers; however, the majority struggled. This indicates a disconnect between their learning experiences and the instructional strategies employed (Sariyasa, 2017). According to the PKM, most students are assumed to operate at Layer 2, meaning they understand how to apply visual representations to the topic without engaging in specific geometric activities.

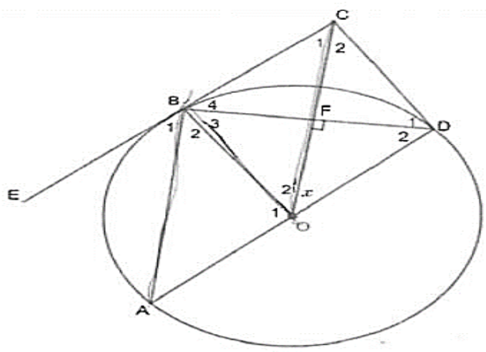
4.4.2 Post-Test Evaluation

Following the teaching phase, students participated in an hour-long post-test evaluation during Cycle 1, Week 1, to gauge their learning outcomes. The post-test was designed to assess the effectiveness of conventional instruction. Students were required to apply concepts such as the tan-chord theorem, the midpoint theorem, congruency, and the properties of parallel lines to demonstrate geometric properties (see Figures 4.6 and 4.9).

For example, in Figure 4.6 AD is the circle's diameter with centre O, BE is tangent to the circle, and OC is perpendicular to BD.

After the test, five students were selected for semi-structured interviews based on their responses. These interviews provided further insights into the students' perspectives on traditional instruction and its impact on their learning experiences in circle geometry. The challenges they faced are presented under the category "Difficulties in Understanding Key Concepts."

1.3 In the figure below AD is the diameter of the circle with centre O. BE is a tangent to the circle and OC is perpendicular to BD. $F\hat{O}D = x$.



1.3.1	Prove that OC is parallel to AB (OC‖AB).	(2)
1.3.2	Prove that $F\hat{O}B = x$.	(2)
1.3.3	Hence prove that OBCD is a cyclic quadrilateral.	(3)

Figure 4.6: Test question

Students were asked to refer to the diagram in figure 4.2 and answer the following questions:

Question 1.3.1: Demonstrate that OC and AB are parallel (OC II AB). Students had to demonstrate that angles $\mathrm{B2} + \mathrm{B3} + \mathrm{BFO} = 180°$ to demonstrate the co-interior angles' further nature.

Question 1.3.2: Prove that $F\hat{O}B = x$. This required students to apply the midpoint theorem and their knowledge of congruency from the Grade 9 geometry curriculum.

Question 1.3.3: Prove that OBCD is a cyclic quadrilateral. The student needed to apply the tan-chord theorem and their understanding of parallel lines and corresponding angles found in the diagram. They also had to use information derived from Question 1.3.1

4.5 BARRIERS TO COMPREHENSION OF CIRCLE GEOMETRY CONCEPTS

Students' responses indicated that their approaches to applying and reasoning with circle theorems varied significantly. Some responses revealed barriers to their understanding of geometric concepts. After analysing all 30 scripts, 21 students met the criteria for "Not Competent" on at least three of the targeted theorems (midpoint theorem, tan–chord theorem, Pythagorean theorem, and angle properties of parallel lines). This represents 70% of the sample (21 ÷ 30 × 100).

4.5.1 Difficulties in Understanding Key Concepts

The following extracts from students' manipulation of the test questions (Questions 1.3.1 to 1.3.3) and semi-structured interviews highlight the difficulties students experienced in understanding key concepts of circle geometry.

Students S27 and S2's responses (see Figures 4.7 and 4.8) to the questions in Figure 4.2 indicate that they had difficulty with applying the midpoint theorem, understanding congruence and interpreting diagrams. This suggests a challenge in connecting prior knowledge to more advanced problems and constructing logical geometric proofs. For example, the work of S27 and S2 illustrates these issues clearly.

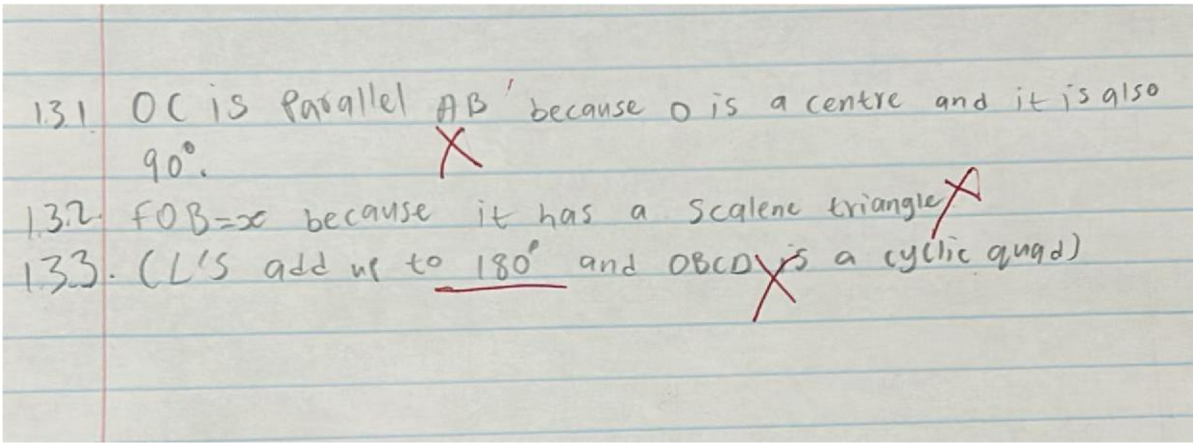


Figure 4.7: S27's manipulation of the test question 1.3.1-1.3.3

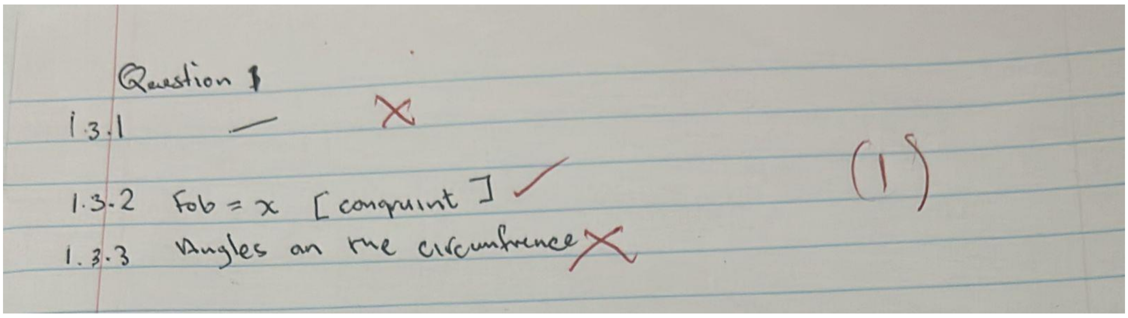


Figure 4.8: S2's manipulation of the test question 1.3.1-1.3.3

When asked about the difficulties encountered with the test (question 1.3 in Figure 4.6), S4, S2 and S27 stated that they were unable to recall the characteristics of triangles and parallel lines, which prevented them from answering the questions.

S4's Reaction: I had difficulty remembering the characteristics of parallel lines and triangles, which made it hard for me to choose the correct answer.

S2: It was difficult for me to remember the characteristics of triangles and parallel lines.

S27: I was not able to recall information about triangles and parallel lines in the test.

These responses suggest a gap in students' understanding of fundamental geometric principles, hindering their ability to reason and apply circle theorems in problem-solving contexts (Hassan et al., 2023). The challenges observed in students' performances further highlight the limitations of traditional instruction in fostering deep conceptual understanding and promoting active engagement in mathematical reasoning, as established in the literature (Hodkowski, 2024). Although S27 had one answer correct, her overall performance on the test suggests that she is operating at lower levels in the PKM – such as "primitive knowledge" or "image-making" where students struggle to understand fundamental ideas due to a lack of prior knowledge.

This gap included an insufficient understanding of basic concepts and characteristics related to circles and their geometric properties. Communication difficulties, compounded by language barriers, further exacerbated these issues, as evident from the above extracts. The observed challenges stem from a general lack of comprehension: students struggle to grasp basic ideas, rules or processes in circle geometry. This finding aligns with Sulistiowati et al. (2019), who noted that students often face difficulties in solving geometry problems due to their inability to comprehend problem statements, identify suitable problem-solving strategies and accurately execute mathematical procedures.

A similar problem was encountered by students when solving Question 3.1 in Figure 4.9. In that question, students were required to express angles in terms of x, prove that triangle TDE is an isosceles triangle, and demonstrate that points B, A, T and D form an cyclic quadrilateral.

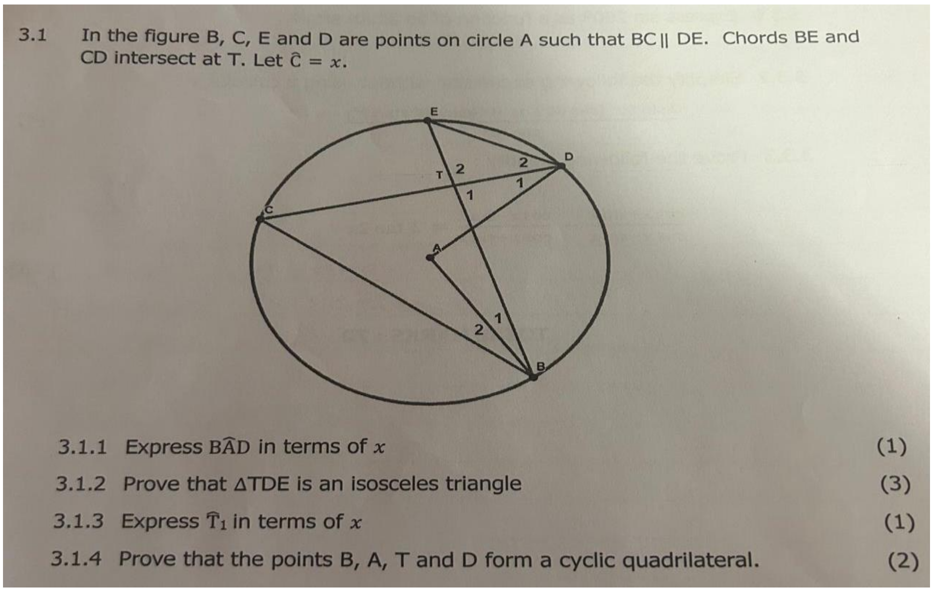
Students S11, S13, and S17's answers indicate significant difficulties in comprehending and applying certain circle geometry theorems. Specifically, they struggled with:

- Applying circle theorems:

Identifying and applying the rule “Angles Subtended by the Same Arc at the Circumference Are Equal” proved challenging. This theorem states that any angle subtended by the same arc on the circumference of a circle is congruent. o Understanding that in a cyclic quadrilateral (a quadrilateral whose four vertices lie on the circumference of a circle), the opposite angles are supplementary. Students were unsure whether a quadrilateral was cyclic or if the supplementary property could be applied effectively.

- Algebraic representation and visualisation

Representing unknown angles algebraically (as x) was a fundamental skill that many struggled with. Students had difficulty finding an angle whose value could be expressed as x and then using equations to determine its value based on geometric relationships. Visualising relationships among arcs, chords and angles was another challenge, hindering their ability to apply the theorems correctly.


3.1 In the figure B, C, E and D are points on circle A such that BC || DE. Chords BE and CD intersect at T. Let $\hat{C} = x$.

3.1.1	Express $B\hat{A}D$ in terms of x	(1)
3.1.2	Prove that ΔTDE is an isosceles triangle	(3)
3.1.3	Express $\hat{T}_1$ in terms of x	(1)
3.1.4	Prove that the points B, A, T and D form a cyclic quadrilateral.	(2)

Figure 4.9: Question 3.1

The student S11’s response in Figure 4.10 reveals a gap in comprehension of key geometric concepts such as expressing an angle in terms of x and characteristics of a cyclic quadrilateral, suggesting that the traditional instructional approach may not have adequately addressed these foundational areas. This can be seen in Figure 4.10:

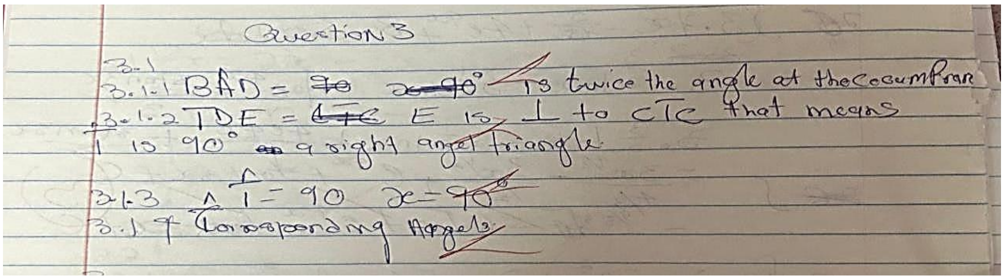


Figure 4.10: S11 Response to Figure 3.1.1 - 3.1.4

Furthermore, Figure 4.11 shows that S13 had trouble understanding the question, which resulted in answers that were not pertinent. A small attempt at memory is made, but it is completely out of context, suggesting either confusion, a lack of conceptual understanding, or perhaps a lack of practice and study at home. This can be seen in Figure 4.11:

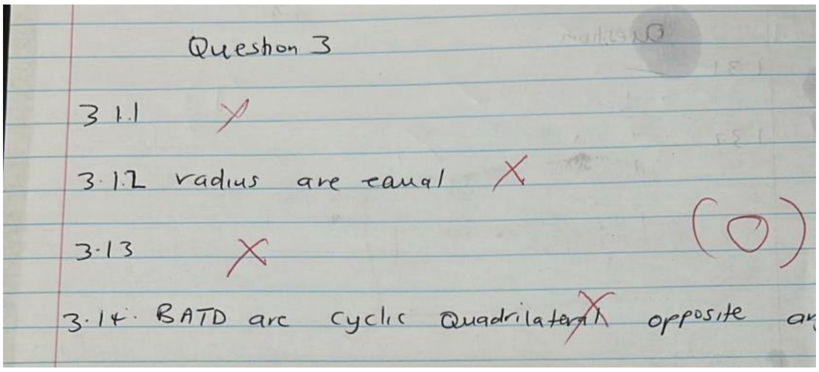


Figure 4.11: S13 Response to Figure 3.1.1 - 3.1.4

Additionally, the data in Figure 4.12 reveals that some participants, such as S17, did not engage with the question. There could be several reasons for this lack of interest, such as a lack of knowledge about the subject, a lack of faith in their skills, or perhaps trouble understanding the question.

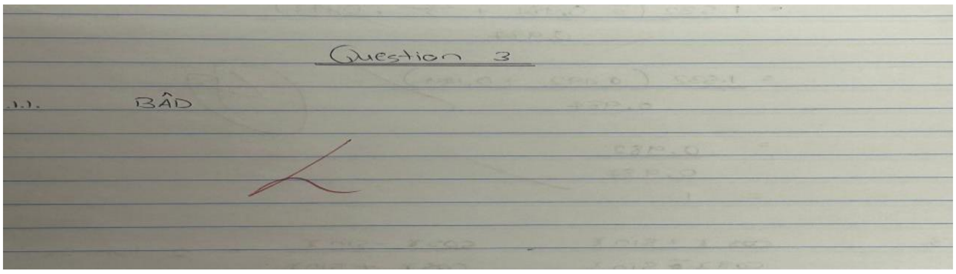


Figure 4.12: S17 Response to Figure 3.1.1 - 3.1.4

During interviews, when the researcher asked S4, S13 and S17 to provide more details about their experience taking the test can be seen below:

- S4's Reaction*: "I was unable to remember the characteristics of parallel lines and triangles. This made it difficult to give correct answer to the question.*
- S11 said, *"The test was complicated because there were some aspects of geometry that I did not understand."*
- S17 said, "*I did not understand the questions because I have not done geometry like this in a long time."*

Students S11, S4 and S17 attributed their struggles to a lack of understanding of core principles, such as the relationships between radii, chords, arcs and angles, as well as difficulties in applying circle theorems to problem-solving situations.

4.5.2 Confusion and uncertainty related to Circle Geometry due to gaps in Understanding

Students faced confusion during tests due to difficulties in circle geometry concepts and theorems, as noted by Hassan et al. (2023). Students (S1 and 14) expressed the following sentiments:

- S1 said, "*When I saw the test, it was tricky, I was confused because I did not understand some of the things."*
- S14 said, "*The way the questions were put was in a very tricky manner, it was difficult and the way the questions were asked and I had no idea how to answer the question."*

These statements suggest that students struggled with specific concepts and theorems relevant to circle geometry, leading to confusion during the test. This concurs with Hassan et al.'s (2023) findings, which similarly observed that students often have difficulty visualising the concept of a circle and applying related theorems in assessments. In the context of Pirie and Kieren's model of growth, students appear to be operating at lower layers, such as "primitive knowledge" or "image-making," where they struggle with basic concepts. This lack of foundational understanding can prevent them from advancing to higher layers, such as formalising or observing, where they can apply abstract reasoning to solve problems.

4.5.3 Inability to Grasp Essential Concepts and Properties related to Circles and their Geometric Properties

The challenge that students do not grasp the concept that the exterior angle of a triangle equals the sum of the opposing interior angles, or that the total of the interior angles of a triangle equals 180°, suggests a fundamental misunderstanding of geometric principles (Also, see Figure 4.13, and Figure 4.14).

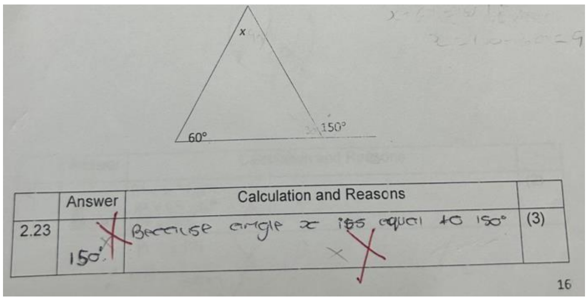


Figure 4.13: Student S6 Response

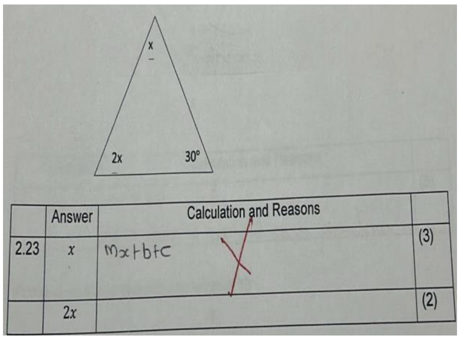


Figure 4.14: Student S14 Response

When asked which test questions they found most challenging, S6 and S14 gave the following answers:

- S6 said, *"I did not know the answers related to triangles*……*I did not understand that all the angles inside the triangle were 180°"*
- S14's response was similar: *"I struggled with the triangle question. I had forgotten that a triangle's internal angles always add up to 180°."*

In this vein, the interview responses elucidated the reasons behind the lack of understanding observed among students. This finding illustrates the inability of the students to grasp essential concepts and properties related to circles and their geometric properties.

4.5.4 Language Limitations That Prevented Comprehension from Being Expressed

Being able to communicate through language makes us human, as it serves as the primary means of modifying our ideas, personalities and creative expression

(Alejandro 2024). Language can be transformed into symbols, numbers and equations in mathematics.

It necessitates knowledge beyond language. The statement “Mathematics is the language in which God has written the universe” is attributed to Galileo Galilei (n.d.). It highlights the fact that reading mathematics is a skill. The topic of language in mathematics has been the subject of numerous investigations. For instance, a study by Sarabi and Kunnathodi (2017) revealed various linguistic challenges students encountered when learning mathematics. The following extract shows that the student could not answer the question; he may have left it blank due to language difficulty.

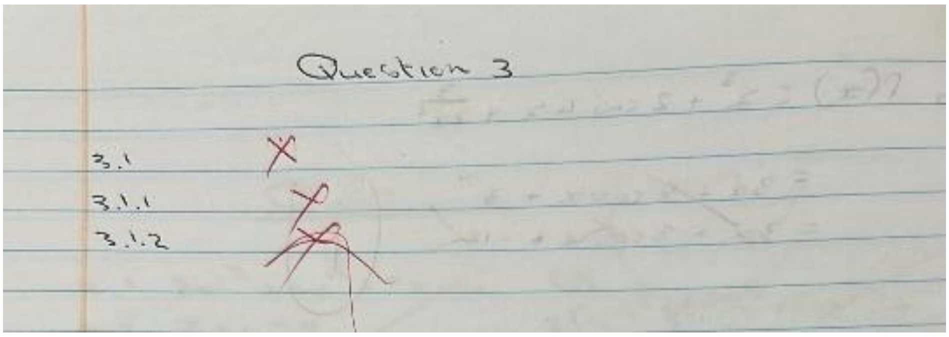


Figure 4.15: Student S10

When (S10, S15, and S18) were asked to provide more details about their experience writing the test, their response during the semi-structured interview were:

- S10 said, “*I did not understand and could not answer it because I am not good with English*.”
- According to S15, *“I got confused by the wording of the questions and did not understand what they were asking.”*
- S18 stated, *“The way the questions were written made it hard for me to understand what to do.”*

S10’s response indicates how the limitation of understanding the language used in the question paper affected his ability to provide an appropriate response. The complexity of the language made it difficult for students to understand the requirements accurately, as S15 expressed. Furthermore, S18 emphasised that language complexity and unfamiliar phrasing hindered their comprehension of the test. English is regarded as a second or third language for most students at SWGC. Many students coming from high schools with low English proficiency were adversely affected in acquiring mathematical skills because they developed these skills in their first language. It is common for students to be impacted by information delivered in a language other than their own. Various factors contribute to these issues, including students' linguistic backgrounds, inattention, reliance on translation and challenges in deciphering word problems. Mathematics students must have both the mental skills for performance and the linguistic comprehension necessary for effective communication (Rusdi, 2020). This was clearly demonstrated by Figure 4.15 which displays S10's answer to the question. For students to apply theorems, they first need to understand what each theorem entails.

4.6 THEME 1 BARRIERS TO ACTIVE PARTICIPATION IN THE CLASSROOM ENVIRONMENT

Barriers to active participation in the classroom were identified through data on students' difficulties in understanding geometric concepts. These barriers relate to classroom engagement, limited engagement with the subject outside of school, and lack of practice.

Tracking of Data: Example of Code–Category–Theme Application

Student Evidence / Observation	Code	Category	Link to Theme
Few students could explain the midpoint theorem; most gave short or incomplete answers.	PCD	Classroom Engagement and Participation	Shows students were not actively participating.
Minimal interaction during chalk-and-talk instruction.	LE	Classroom Engagement and Participation	Indicates low engagement with instruction.
Students responded only with pre-known information; no spontaneous contributions.	LF	Classroom Engagement and Participation	Reflects limited cognitive involvement.

4.6.1 Classroom Engagement and Participation

During the first cycle, the researcher taught circle geometry using conventional chalk-and-talk methods. In this phase, interactions were minimal, with students typically responding only with what they already understood. Despite the researcher's efforts to involve them, there were no spontaneous reactions. For instance, when asked to describe the midpoint theorem, only a small percentage of students could provide an adequate response. This suggests a gap between the instructional strategies used and the students' actual engagement and learning experiences (Sariyasa, 2017).

Moreover, according to Op 't Eynde (2004), learning occurs when students actively engage with the language, norms and procedures that guide activities in the mathematics classroom. This study reinforces that concept, as teacher-centred practices were prevalent – a pattern also noted in Nigerian classroom practices (Abah et al., 2017).

4.6.2 Limited Engagement with the Subject

Participants indicated that limited engagement with the subject outside of school contributed to their struggles. This includes limited revision and practice of key geometric principles beyond formal lessons.

For instance, S5 explained his test performance by saying:

> *"I was not paying attention or practising at home. I only practised at school, but not much at home."*

Similarly, S8 stated:

> *"I didn't spend enough time practising outside of class. I only did the exercises in class,"*

and S12 admitted:

> *"I don't usually do extra work at home, and I think that's why I didn't do well."*

Cesaria and Heerman (2019) argue that mastering geometry requires consistent practice, which is reflected in these responses. Students who do not practise at home fail to interact meaningfully with the material, putting them at a disadvantage.

4.6.3 Lack of Practice and Overconfidence

Lack of practice was evident when S1 and S12 were asked what went wrong in their test performance. S1 stated:

"I could not answer questions 2 and 3. I struggled with circle theorems because they required a lot of practice,"

while S12 said:

"*I did not study for the test. I thought the test would be easy."*

These responses indicate that students recognised the importance of practice in mastering circle geometry but did not dedicate sufficient time to it. Overconfidence also played a role, as illustrated by S7, who said:

"Practising was a bit difficult. I did not look at the theorems to understand them. I went in with a blank mind, thinking that I knew everything."

Research has shown that regular practice correlates with higher achievement and better problem-solving abilities (Aktas & Unlu, 2017; Lehtinen et al., 2017). In terms of the PKM, students at the image-making stage must engage in active visualisation of theorems and relationships. Failure to do so impedes their ability to form robust mental representations of mathematical concepts. The students' responses from both semi-structured interviews and the post-test clearly indicated that conventional teaching did not sufficiently lead to an understanding of circle geometry, prompting the researcher to consider alternative strategies.

4.7 THEME 2 PRACTICAL APPLICATION OF GEOMETRY INSTRUMENTS: INTERVENTION PLAN – CYCLE 2

Building on the challenges identified in Cycle 1, the second intervention focused on practical applications using geometry instruments to address students' comprehension errors and lack of practice. This intervention was implemented over a one-week period, during which students engaged in hands-on construction, measurement, and verification activities designed to strengthen their understanding of circle theorems.This phase emphasised hands-on learning with compasses, protractors and rulers. Such tools are fundamental in visualising and reinforcing geometric principles, allowing students to directly construct and measure geometric figures.

In Cycle 2, students were tasked with drawing and measuring diagrams related to circle geometry theorems. They used a ruler, protractor and compass to draw a circle, a chord and a line that bisected the circle perpendicularly from its centre. After constructing these figures, they measured the angles formed and summarised their results. S5 and S9’s responses to this instruction are captured in Figures 4.16 and 4.17, respectively.

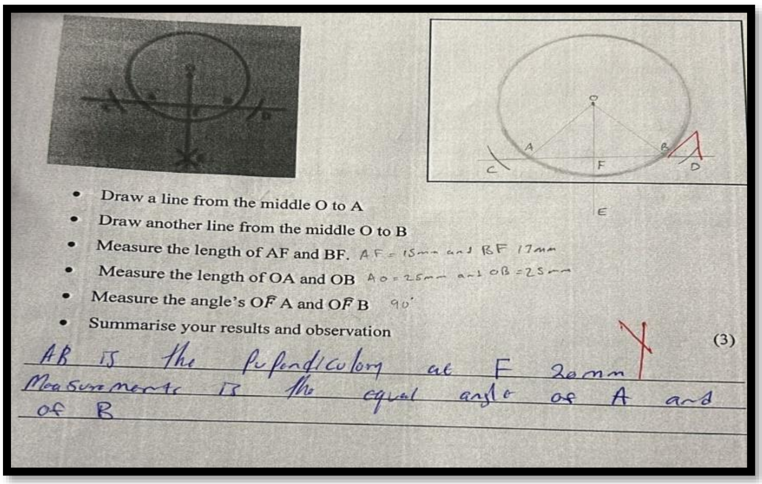


Figure 4.16: Student S5 Response

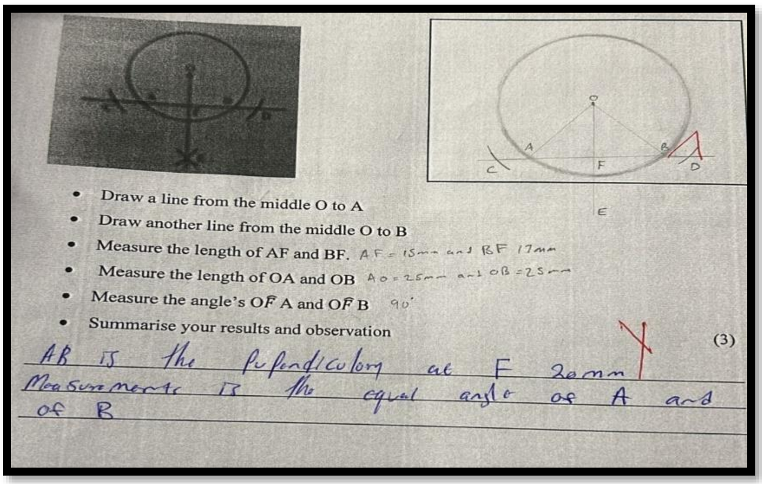


Figure 4.17: Student S9 Response

As stated by students S5 and S9, many challenges in circle geometry were attributed to inaccurate measurement and an inability to effectively summarise responses. When asked to describe their practical experience, S5 said:

> *“I struggled, but I measured my answers wrong,”*

while S9 responded:

"Drawing the circles and lines was okay, but I did not know how to summarise my answers."

While most students found drawing circles, radii and chords relatively easy, many struggled to measure angles accurately. Some observed the formation of angles during construction, yet their measurements were often imprecise – likely due to unfamiliarity with the measuring tools. Furthermore, summarising results and articulating findings proved difficult for many, with only a few students successfully connecting their constructions to established theorems. This observation aligns with Hendroanto and Fitriyani (2019), who noted that even experienced mathematicians sometimes face challenges using geometric tools.

Despite these difficulties, the practical approach led to increased student engagement compared to the traditional teaching methods employed in Cycle 1. Students showed greater enthusiasm for using geometry instruments, although the time limit for the exercise may have been too restrictive for mastery. The researcher observed that with more time and practice, students grew more confident in their use of the measuring tools.

To assess student engagement and understanding during this intervention, the researcher employed the PKM – which classifies learning into six layers. Each level was categorised as weak, moderate or strong based on student performance during the intervention. Table 4.2 summarises these observation results.

Table 4.2: Observation results of cycle 2

Layer	**Weak**	**Moderate**	**Strong**
Layer 1: The recognition of qualities of congruency was an example of primitive knowledge. Pythagoras, triangles and quadrilaterals are examples of students who understood and created patterns. By using any representation, the student could remember previous material	X		
Layer 2: Students who created images could continue patterns without requiring them to do		X	

Layer	Weak	Moderate	Strong
specific activities, like drawing images. They understood how to apply visual representations to the topic at hand			
Layer 3: Property notice could be used to develop a formula describing all patterns' interactions by generalising the pattern rule. By utilising representations to convey mathematical meanings, the students can construct patterns and algorithms.		X	
Layer 4: The most common representations are symbolic and verbal. Students could extrapolate the pattern rule to create a general formula that describes all pattern interactions at the formalising level. A student could use mathematical representations to construct patterns and algorithms.		X	
Layer 5: Students in this level developed a general x formula for extending patterns to greater stages and relating patterns to various mathematical subjects such as algebraic expressions, equations and geometry. The student connected representations and theorems. They connected representations and theorems, mostly using symbolic and verbal representations.	X		
Layer 6: At the structuring level, students worked to develop a fundamental understanding of circle theorems and their significance.	X		

Table 4.2 shows that students demonstrated moderate engagement and understanding at lower layers (1–4), while their performance was weaker at higher layers (5–6). This suggests that students struggle with abstract reasoning and applying

complex geometric concepts, likely due to limited exposure and insufficient hands-on practice in earlier stages of their education. To close these gaps, students can be better prepared for success in advanced mathematics and real-world problem-solving by receiving focused support for developing higher-level thinking abilities, in addition to the ongoing integration of practical tools into instruction.

4.8 THEME 3: THE IMPACT OF PRACTICAL ENGAGEMENT ON STUDENT MOTIVATION, COMFORT, AND UNDERSTANDING IN CIRCLE GEOMETRY

This theme explores how students responded to practical, hands-on activities during the intervention cycles and how these activities influenced their motivation, comfort, and conceptual understanding of circle geometry. These constructs relate directly to the research questions because the study seeks to understand how virtual manipulatives and practical engagement support students' learning processes. Students' perceptions of usefulness, enjoyment, participation, and ease of engagement help explain *how* practical experiences contributed to their developing understanding of circle geometry and *why* such activities may facilitate improved reasoning, visualisation, and confidence. Thus, Theme 3 provides insight into the mechanisms through which instructional strategies influenced students' conceptual growth during the study.

Tracking of Student Responses Using the Coding Framework

Student Response	Code(s) Applied	Category	Link to Theme
S5: "Yes, I was comfortable despite making a few mistakes."	PF, IC	Emotional Responses & Motivation	Shows comfort, confidence, and positive feelings during practical work.
S20: "I was nervous at first… but I started to enjoy the practical work as I started to understand."	UT, RA, PF, SP	Emotional Responses & Motivation	Shows growth mindset—moving from uncertainty to enjoyment and progress.
S12: "Yes, I was comfortable despite making a few mistakes."	PF, IC	Emotional Responses & Motivation	Reflects comfort and confidence despite errors.
S13: "I was comfortable because we are doing engineering…"	PF, IC	Emotional Responses & Motivation	Shows confidence influenced by prior exposure.

4.8.1 Emotional Responses and Growth Mindset in Practical Circle Geometry

This sub-theme examines how students' emotional reactions—such as increased confidence, reduced anxiety, and willingness to persist with challenging tasks—as well as the development of a growth mindset influenced their engagement with practical activities in circle geometry. These constructs relate directly to the research questions because understanding how students *feel*, *persist*, and *respond* during practical and virtual manipulative-based learning helps explain *how* these instructional strategies support or hinder their conceptual understanding. Emotional readiness and a growth-oriented attitude shape the ways students interact with tasks, interpret feedback, and construct meaning, thereby revealing key mechanisms through which practical engagement influences their learning of circle geometry.

When asked about their practical experience using tools such as protractors, rulers and compasses, S5 shared:

> *"Yes, I was comfortable despite making a few mistakes."*

Similarly, S20 remarked:

> "*I was nervous at first because I made mistakes, but I started to enjoy the practical work as I started to understand."*

These responses reflect a growth mindset, with students viewing mistakes as part of their learning journey. This aligns with Boaler's (2022) assertion that students who embrace mistakes typically make rapid progress. It suggests that educators can nurture such a mindset by encouraging students to view errors as opportunities for learning rather than as failures.

However, comfort and confidence levels varied among students in their use of geometry tools, influenced by their familiarity and proficiency. For example, S12 said,

> "*Yes, I was comfortable despite making a few mistakes*,"

while S13 explained:

"*Yes, I was comfortable because we are doing engineering, and those are the tools used for measuring."*

These responses indicate that individual factors – such as prior experience, spatial reasoning skills and fine motor coordination – play a role in how easily students adapt to using these tools. Mabena et al. (2020) emphasise that attitudes such as comfort, motivation and willingness significantly influence students' engagement and mathematical achievement. Students with prior experience or innate aptitude may quickly become adept at using tools like protractors and compasses, which enhances their confidence. In contrast, others may initially struggle, requiring more time and practice to develop proficiency.

For these students, hands-on instruction and repeated practice with geometry tools were particularly beneficial. Over time, such activities helped them understand the purpose of each tool and its correct usage. This consistent practice gradually increased their comfort and confidence, allowing them to participate more effectively in geometry tasks. Addressing skill gaps through targeted practice might enable all students to achieve success and feel more empowered in their mathematical learning.

4.8.2 Motivation and Engagement in Practical Geometry

This category examines how practical activities influenced students' motivation, willingness to participate, and overall engagement during the learning process. These constructs are directly related to the research questions because the study seeks to understand *how* practical and virtual-manipulative-based instruction supports students' conceptual understanding of circle geometry. Motivation and engagement shape the extent to which students interact meaningfully with geometric tasks, persevere through difficulties, and explore relationships within the circle theorems. Increased motivation and comfort, therefore, help explain *why* practical experiences enhanced students' reasoning, visualisation and understanding of circle geometry concepts during the intervention cycles.

When asked about their practical experience using tools such as protractors, rulers and compasses, S8 reacted:

"I liked using the ruler and protractor because it was like solving a challenging task. At first, I had difficulties getting the angles exactly perfect, but after attempting a few times, I got the hang of it."

S15 said:

"I liked it, but at first, I was nervous. But after a while, I felt at ease working with the instruments."

S19 commented:

"*At first, I was nervous about using tools, but class practice gave me more confidence. Angles and forms were easier to see when using a protractor and compass, particularly when working with circle geometry."*

This theme further explores students' attitudes and motivation during practical exercises with geometry instruments. Their responses demonstrate how these activities influenced their learning experiences and comfort with geometric concepts.

These questions were repeated several times to capture varied responses. Overall, S8, S15, and S19 expressed positive reactions to using practical tools like rulers, protractors and compasses during their circle geometry tasks. They acknowledged the importance of hands-on practice for preparing for tests and developing a deeper understanding of geometry. This reflects the relevance of practical activities in reinforcing theoretical concepts and offering effective problem-solving strategies. Their statements align with the PKM, where students begin to form positive associations through hands-on experiences. Engaging in practical work may stimulate curiosity and lay the foundation for further exploration and understanding. Research suggests that practical activities and real-world applications enhance students' motivation and engagement with mathematical concepts (Toxirjonovich & Abduvaliyevich, 2020).

Students' motivation and engagement during practical geometry activities were further highlighted by S9, who shared:

"I felt great because we were exploring new things, and it was a fun exercise."

This response suggests a positive attitude toward learning geometry through practical activities. The student enjoyed the process of discovery, emphasising the importance of maintaining a sense of enjoyment and exploration in circle geometry. This concurs with research indicating that fostering positive attitudes enhances students' engagement and willingness to learn (Berger et al., 2020).

4.8.3 Positive Attitude Toward Practical Activities in Circle Geometry

Students often expressed satisfaction and enjoyment with hands-on geometry activities, indicating increased engagement and a positive shift in their attitudes. When asked about their practical experience using tools such as protractors, rulers and compasses, S2 said:

> "*I found it very good and amazing. It was not bad, and the experience was good."*

S5 commented:

> *"It was far better than just learning it from a book."*

S8 added:

> "*I enjoyed working with protractors and compasses."*

These responses reflect a positive attitude toward practical geometry activities. Students expressed satisfaction and enjoyment, suggesting high engagement and interest in using geometry tools. Such activities foster a positive outlook on learning and enhance students' willingness to explore and interact with mathematical concepts.

4.8.4 Hands-on Learning as a Bridge Between Theory and Practice

This research further explores the impact of hands-on activities on students' comprehension of theoretical concepts, highlighting how active participation enhances their ability to connect abstract ideas with practical applications.

When students were asked to describe their practical experience in learning circle geometry using a protractor, ruler and compass, their responses were as follows:

> S2: *"At first, I had no clue what I was doing. But when we started the practicals, I realised that I was starting to understand what I was doing."*

S21: "*I like using the protractor, ruler and compass since it made it possible for me to interact practically with the material. Compared to simply listening to the teacher describe everything, it made the topics easier to understand."*

S16: "*I struggle to measure my angles correctly but after practicing I got it right*."

S10: "*The experience was quite different from the normal lessons. I like solving problems on my own. The lesson was very interactive, but I still need assistance to understand some of the procedures*."

From these responses, several key aspects of hands-on circle geometry learning emerged. S2 and S21 emphasised how practical exercises encourage active engagement, thereby enhancing understanding. In S21's case, using tools like a protractor and compass proved more effective than passive listening, while S2's experience showed that practical work built confidence and overcame initial uncertainty. Through repeated practice, both S16 and S10 improved their ability to measure angles, underscoring the importance of a consistently effort-driven approach to learning.

Together, these responses highlight the importance of experiential learning tools in enhancing comprehension and demonstrate the need for practice and support to address individual challenges. They also reveal a shift from confusion to understanding, emphasising the positive impact of actively engaging with geometry tools.

Though practical work can be difficult, students recognise its learning potential. According to Visokolskis and Gerván (2022), practical activities can bridge the gap between theory and practice and improve student engagement and understanding, while Toxirjonovich and Abduvaliyevich (2020) suggest that experiential learning helps students gain a concrete grasp of abstract concepts.

Further illustrating this point, student S8 stated:

"*At first, it was challenging because we weren't accurate with how to use the instruments. Although I didn't know how to use the instruments, the lesson was helpful and I learned how to use them."*

S8's admission of initial challenges coupled with eventual mastery shows that while using the instruments was difficult at first, perseverance led to improved skills and a positive learning outcome.

4.8.5 Benefits of Practical Learning and Skill Development

Students noted that hands-on activities not only helped cultivate essential skills for academic achievement, such as precision and problem-solving, but also improved their test preparation. When asked about their practical experience with tools such as protractors, rulers and compasses, the responses from S3, S13, S14, and S19 were as follows:

> S3: "*Yes, the practical work helped me in the test; it gave me some basic ideas on how to tackle geometry.*"
>
> S13: "*Yes, it helped because we drew the circle and used the ruler and protractor to show the angles of the triangles and circles.*"
>
> S14: "*I was better prepared for the test than before*."
>
> S19: *"At first, I had no clue what I was doing. But when we started the practicals, I realised that I was starting to understand what I was doing; it helped me with the test."*

These responses highlight the practical benefits of hands-on learning in geometry. Students acquired skills that enhanced both their spatial reasoning and test performance. This observation concurs with research indicating that hands-on learning activities positively influence students' problem-solving abilities and spatial reasoning skills (Noreen & Rana, 2019; Sariyasa, 2017; Toxirjonovich & Abduvaliyevich, 2020).

Incorporating experiential learning into geometry teaching is essential for promoting long-term engagement and success in mathematics. Although some improvement was noted in the circle geometry test results, it was insufficient. Consequently, the researcher decided to include virtual manipulation as the next strategy.

4.9 USE OF VIRTUAL MANIPULATION TO FOSTER UNDERSTANDING OF CIRCLE GEOMETRY

In the third cycle, the researcher conducted a second intervention by introducing GeoGebra, a virtual tool designed to aid students' understanding of circle geometry, and this cycle was carried out over a duration of one week.

Offline tutorial videos were used to teach students how to navigate and use GeoGebra's functionalities. The students were given time to interact with the GeoGebra calculator suite selecting tools and exploring various geometric designs. An entire session was dedicated to explaining the functionality of the GeoGebra software before the students engaged with the programme themselves.

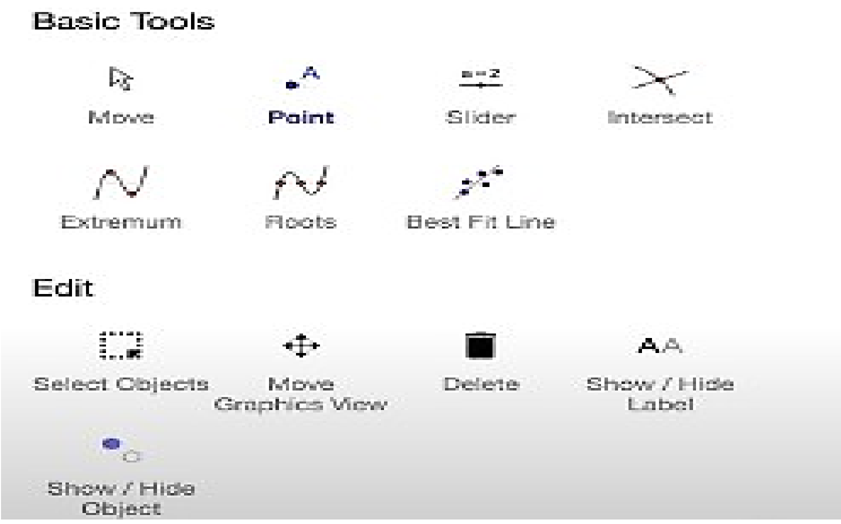


Figure 4.18: Basic tools of GeoGebra

To explore circle theorems, the researcher demonstrated how to apply basic tools such as the slider, pointer and intersection.

Figure 4.19: Basic tools of GeoGebra used to construct figures

Additionally, the researcher explained how to use GeoGebra's Construction programme, which allowed students to select tangents, angle bisectors, centres and perpendicular lines. The first task involved drawing the midpoint theorem. Students used the circle tool in GeoGebra to draw a circle. They were instructed to open Graphics View and Algebra View on their computers. Using the Circle with Centre through Point feature, they drew a circle by positioning the cursor in the Graphics View. Moving the pointer toward the centre, they determined the radius.

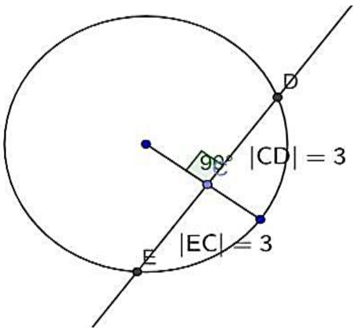


Figure 4.20: Radius perpendicular to a chord bisects the chord

During the following class period, the students constructed a chord within the circle and created a perpendicular bisector using the line segment tool. By moving the chord around, students observed how the perpendicular bisector behaved.

Figure 4.21: Student in computer lab using Geogebra

The image shows Level 4 students from Molapo College working in a computer lab while using GeoGebra to explore circle-geometry concepts. The students shown in the provided photograph, are seated at desktop computers and appear fully engaged in manipulating points, observing geometric relationships, and testing theorems dynamically. This image reflects the authentic learning environment during the intervention sessions, where Molapo TVET students used virtual manipulatives to deepen their understanding of angles, arcs and the behaviour of circle theorems.In the following session, students interacted with the software to explore another theorem: that the angle formed by the chord in the alternate segment is equal to the angle formed by the tangent at the point of contact with the chord.

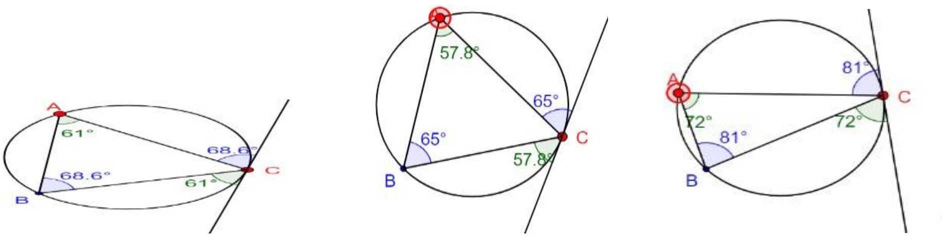


Figure 4.22: Tangent-chord theorem

Using GeoGebra, students manipulated points on the circle, testing the theorem across various configurations. For instance, they would adjust. Each student had to drag Point A in order to create a specific angle (e.g., ∠ C = 65°) and then drag Point

C to increase ∠ C to 81°, observing how the angle in the alternate segment changed to match this value. Students were required to explain the relationship between the angle formed in the alternative segment and the angle formed by the chord and tangent.

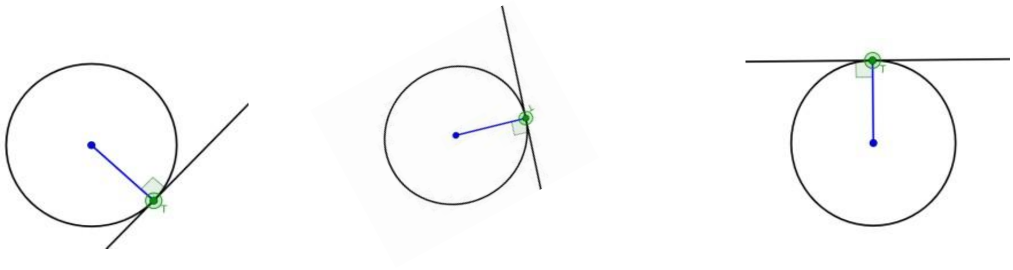


Figure 4.23: Tangent at any point on a circle is perpendicular to the radius at that point

In the fourth session, students explored the theorem that the tangent at any point on a circle is perpendicular to the radius at the point of contact. Students used GeoGebra to drag Point T around the circumference and observed the interaction between the tangent and radius at different points of contact. This dynamic exploration helped solidify their understanding of the theorem.

Following a post-test administered by the researcher, five students were selected to participate in a semi-structured interview. After completing this interactive session, students were assigned a task sheet and completed a questionnaire reflecting on their experience with virtual manipulatives. While the researcher watched, the students worked with the GeoGebra programme. Students were given a task sheet to complete in GeoGebra that contained questions about circle geometry as shown in Figure 4.23. The worksheet required students to study a diagram that would test the student's understanding of the following circle theorems.

The tan-chord theorem states

- Angle at the circumference subtended by same arch
- A line drawn from centre to a tangent is perpendicular

Furthermore, the worksheet required the application of knowledge about radii forming triangles within a circle.

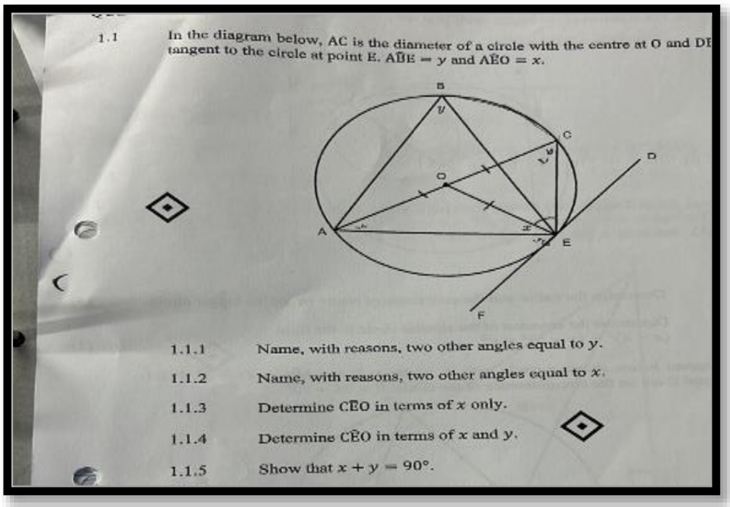


Figure 4.24: GeoGebra-based questions on circle geometry

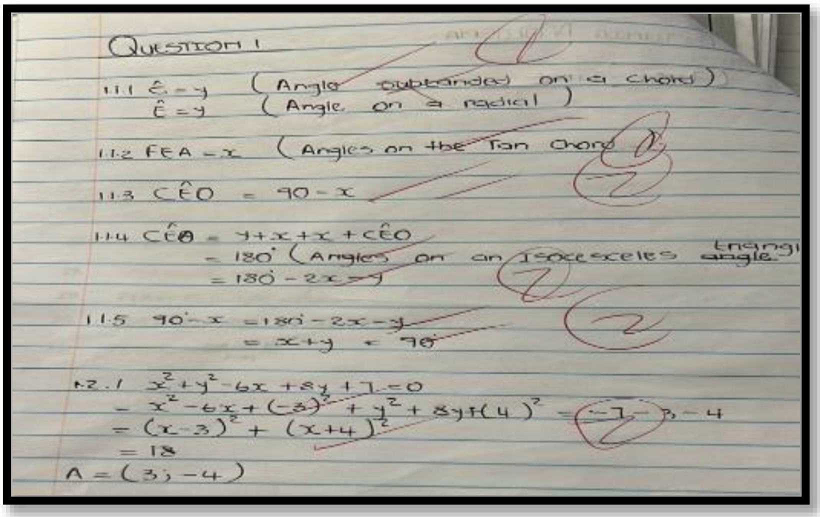


Figure 4.25: Student S18's Response

Student	Mark%	Student	Mark%
Student 1	30	Student 15	64
Student 2	91	Student 16	53
Student 3	47	Student 17	43
Student 4	23	Student 18	51
Student 5	39	Student 19	87
Student 6	29	Student 20	16
Student 7	54	Student 21	39
Student 8	27	Student 22	37
Student 9	22	Student 23	45

Student 10	60	Student 24	36
Student 11	39	Student 25	23
Student 12	76	Student 26	6
Student 12	84	Student 27	48
Student 13	22	Student 28	28
Student 14	13	Student 29	58
Student 14	71	Student 30	34

Thirty students participated in the test, with three achieving exceptional scores of 84%, 87%, and 91%, respectively. Nineteen students scored within the range of 30% to 76%, while nine students scored between 19% and 30%. Overall, these results reflect significantly better performance compared to the two previous cycles.

An example of student S18's response to the questions in Figure 4.24 is shown in Figure 4.25. This extract reflects a deeper understanding of circle properties after being introduced to GeoGebra. S18 accurately identified angles and demonstrated her understanding of the relationships between angles and chords (CEÔ and other angles in terms of x, and expressed ∠CEO in terms of both x and y), even expressing complex ideas like $x+y$=90°. This illustrates the effectiveness of using GeoGebra in improving students' problem-solving skills in geometry. After using GeoGebra, S18 showed better performance on the test, indicating that the tool supported her learning process effectively. She was able to grasp complex geometric concepts such as accurately identifying angles and demonstrating her understanding of quadrilaterals and circle theorems more easily than with traditional teaching methods. These findings highlight how GeoGebra can be an effective tool in teaching geometry, supporting students in achieving better understanding and academic performance.

The researcher then administered a post-test semi-structured interview with five students. Here are the responses of S7, S12, and S18 to the question, "How did GeoGebra help you with the test you wrote?":

> S7: "*I found that GeoGebra made it easier to see some of the problems, particularly those that involved angles and circles. However, because it was the first time that I used GeoGebra, I still made mistakes. If I practised more, it would be better.*"

S12: "*As I practiced with GeoGebra, it became clearer to me how some shapes relate to one another. I could understand how angles function in a cyclic quadrilateral, for instance. Additionally, my test result increased from the other two tests I wrote.*"

S18: "*GeoGebra was very helpful. Using it, I was able to test different concepts and create accurate diagrams. It was simpler for me to solve problems through practice, and these practices helped me during the test. I could clearly identify relationships like tangents and radii.*"

Next, the students were given a questionnaire to reflect on their experience with virtual manipulatives, particularly GeoGebra. This questionnaire aimed to gather insights into their perceptions, experiences and learning outcomes related to using GeoGebra.

The following questions were posed during the interviews:

Were there any activities that you found difficult using GeoGebra? Explain.

- Did using GeoGebra (VM) help you grasp circle geometry more easily? If so, please explain how.
- Do you think using GeoGebra has minimised your challenges with circle geometry?
- How has your understanding of circle theorems, especially involving cyclic quadrilaterals, changed after using GeoGebra?
- How would you explain the difference between conventional (traditional) teaching and teaching using GeoGebra?

The following section (Section 4.10) reports on students' experiences with VMs, particularly their use of GeoGebra. It presents an analysis of responses from an open-ended questionnaire designed to capture students' reflections on their experiences with these tools.

4.10 THEME 5: IMPACT OF VIRTUAL TOOLS ON LEARNING GEOMETRY CONCEPTS

This theme presents participants' experiences with using GeoGebra, a virtual tool, as compared with traditional teaching techniques. It highlights the benefits of technology-enhanced learning and investigates how effectively virtual tools help students understand geometric relationships and facilitate active problem-solving.

4.10.1 Effectiveness of Using Technology (GeoGebra): Lessons from Observer's Notes

It was clear from examining the baseline test students lacked the fundamental understanding required to interact with circle geometry. They experienced difficulties in identifying circle properties and understanding the properties of triangles and straight lines. While a few students demonstrated some understanding of Pythagoras' theorem, their ability to apply this knowledge within geometric contexts was limited. To address this, it was recommended that these core concepts be reviewed before starting Cycle 1.

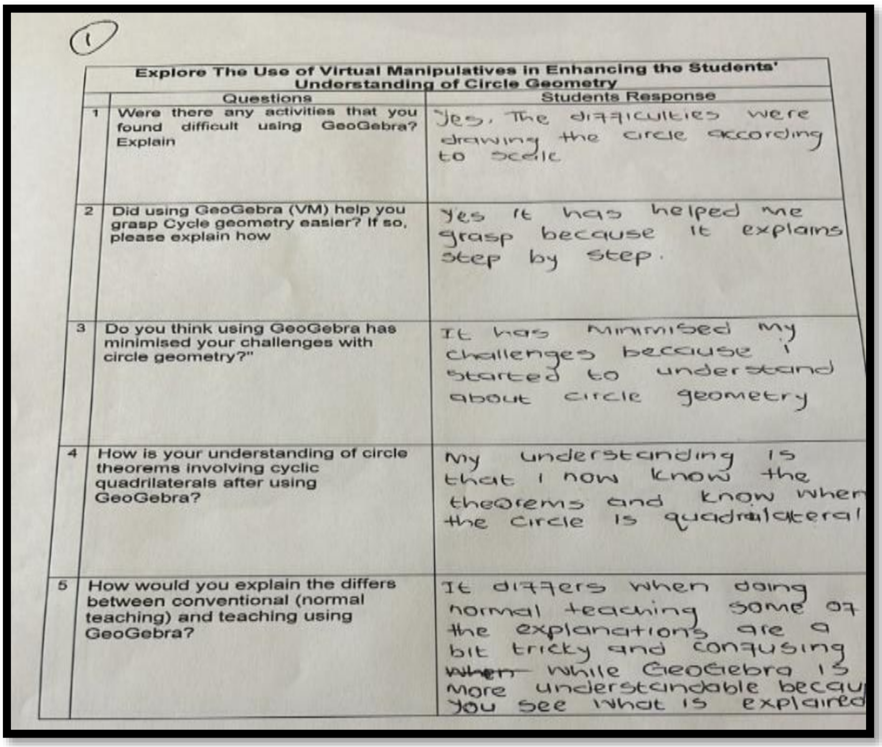
During Cycle 1, the students appeared disengaged from the lesson, possibly due to traditional teaching methods or their unfamiliarity with the topic. The observer noted limited interaction, with students providing minimal responses to questions posed by the researcher. Although some students remembered concepts like equal radii and perpendiculars, they struggled to explain the midpoint theorem and apply it to examples efficiently. Consequently, test results after Cycle 1 ranged from average to poor, indicating that traditional teaching methods were insufficient for addressing students' learning needs.

In Cycle 2, when cyclic theorems were introduced through practical activities, there was a notable increase in student energy and interaction. However, difficulties with using physical measuring instruments led to time constraints and hindered the smooth progression of the lesson. Despite these challenges, students began to develop their understanding of the theorems under the guidance of the researcher. Their performance on the post-test improved compared to Cycle 1, suggesting that the practical approach helped engage students more effectively, even though technical difficulties with instruments persisted.

The third cycle marked a significant shift in student engagement and understanding as they were introduced to GeoGebra. The researcher noted that students followed instructions with ease and demonstrated enthusiasm in using the computer program. Many students had prior experience with computers, which facilitated their active participation and experimentation with geometric concepts. Using GeoGebra, they were able to manipulate angles and develop their own theorems through a hands-on, interactive approach. This led to deeper learning, as students were able to draw connections between their experimental findings and established theorems in geometry.

In summary, these observations underscore the effectiveness of integrating technology, such as GeoGebra, into the curriculum to enhance students' engagement and understanding of circle geometry. The transition from traditional teaching methods to practical, hands-on activities and ultimately to digital tools like GeoGebra, resulted in improved student performance and a more comprehensive grasp of geometric principles. The use of GeoGebra not only alleviated challenges associated with manual drawing and measurement but also created an interactive and explorative learning environment conducive to deeper understanding.

To complement the researcher's notes, semi-structured interviews were conducted with students to gauge their perceptions of GeoGebra and its impact on their learning experiences in circle geometry. The responses collected from students benefits. Below are some of the key reflections from S3.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Explore The Use of Virtual Manipulatives in Enhancing the Students' Understanding of Circle Geometry

	Questions	Students Response
1	Were there any activities that you found difficult using GeoGebra? Explain	Yes, The difficulties were drawing the circle according to scale
2	Did using GeoGebra (VM) help you grasp Cycle geometry easier? If so, please explain how	Yes it has helped me grasp because it explains step by step.
3	Do you think using GeoGebra has minimised your challenges with circle geometry?"	It has Minimised my challenges because I started to understand about circle geometry
4	How is your understanding of circle theorems involving cyclic quadrilaterals after using GeoGebra?	My understanding is that I now know the theorems and know when the circle is quadrilateral
5	How would you explain the differs between conventional (normal teaching) and teaching using GeoGebra?	It differs when doing normal teaching some of the explanations are a bit tricky and confusing ~~when~~ while GeoGebra is more understandable becau you see what is explained

Figure 4.26: Student S3 Response

From Figure 4.26, Student S3 found GeoGebra helpful in addressing challenges like accurately drawing circles to scale. This observation reflects a reduction in overall challenges and a clearer understanding, consistent with studies suggesting that virtual tools like GeoGebra can enhance conceptual understanding and alleviate difficulties in geometry learning (Zulnaidi et al., 2020).

S3 further compared GeoGebra with conventional teaching methods, highlighting the advantages of technology-enhanced instruction. GeoGebra's visual representations provided clearer explanations and deeper insights into circle theorems, especially those involving cyclic quadrilaterals. However, the student also noted that applying theorems to quadrilaterals remained somewhat tricky. This reflection aligns with existing literature, which emphasises the benefits of interactive visualisation for promoting understanding (Sudihartinih & Purniati, 2019).

4.10.2 Visualisation of Geometric Relationships Encouraging Active Problem-Solving

When students were asked about their experience using GeoGebra, the responses included the following:

> S21*: "GeoGebra is quite interesting, and I felt good using it because it gives the answers."*
>
> S1: "*When you get the GeoGebra examples, you can see what they want, and you can usually take answers from GeoGebra without using a pencil, ruler or protractor. It is straightforward because you can see this is parallel to this, and this is parallel to this, etc. It is recommended that the upcoming generation use GeoGebra."*

The positive reception of GeoGebra by students validates that virtual tools can provide an engaging and interactive learning environment, fostering a deeper appreciation for mathematical concepts. While virtual manipulatives can facilitate learning, continued practice and reinforcement are essential for achieving mastery and overcoming challenges, as indicated in Figure 4.24. It is therefore imperative for educators to leverage the strengths of virtual tools while providing ample opportunities for hands-on practice and application to promote holistic mathematical development.

Student S21's experience with GeoGebra captures her appreciation for the ease and accuracy that GeoGebra offers compared to traditional methods. Instead of spending time using manual instruments for drawing and measuring, she found that GeoGebra provided immediate visual confirmation of relationships, such as parallel lines, thereby enhancing her learning experience. Student S25 echoed this sentiment, highlighting the accuracy and reliability of technology over manual tools:

> *"Using technology is much better because it is more accurate than when we are using the instruments to draw the circles and measure. Technology was more accurate because even if it changes, we are able to see if the values remain the same or change. So, I say technology was far better."*

Here, the emphasis is on GeoGebra's precision in geometric tasks – a crucial element when understanding theorems that rely on exact measurements and constructions. The student noted how the dynamic nature of the tool enabled them to observe changes and verify relationships, providing a deeper and more flexible understanding of geometry.

Student S29 found the experience engaging and fun, stating:

> "*The experience was good because we were using laptops and technology, which was fun and easy."*

This simple yet powerful reflection highlights the student's positive attitude toward technology-enhanced learning. By making geometry fun and accessible, GeoGebra likely contributed to greater student motivation and interest in the subject.

Similarly, student S24 expressed satisfaction with their experience using GeoGebra:

> "*I found it very easy working with GeoGebra, and I enjoyed it*."

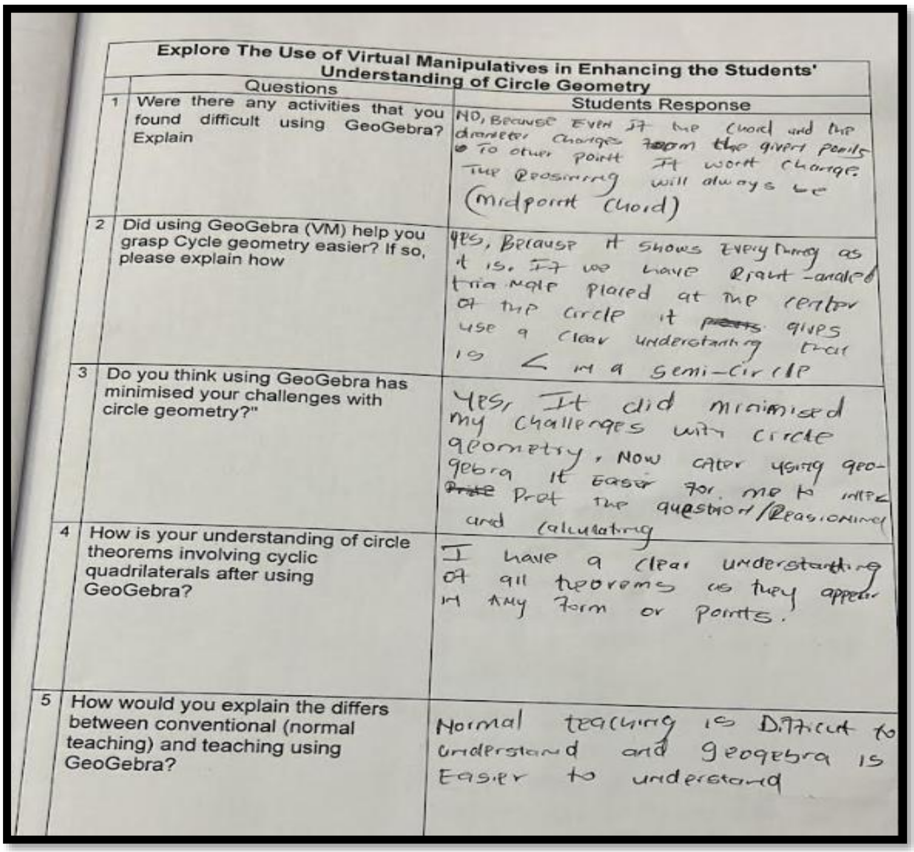
This feedback further underscores the ease with which students were able to interact with GeoGebra, allowing them to focus on understanding geometric concepts rather than struggling with the tools themselves. These reflections align with literature emphasising the importance of interactive tools like GeoGebra in enhancing conceptual understanding. For instance, Condori et al. (2020) acknowledge that tools such as GeoGebra serve as cognitive instruments that support and enhance students' problem-solving skills and overall cognitive development in mathematics. Research also suggests that exposure to diverse examples and interactive demonstrations can enhance students' conceptual understanding and problem-solving skills (Noreen & Rana, 2019). By providing students with opportunities to interact with virtual manipulatives, educators can bring about a deeper appreciation for mathematics and promote active engagement in learning (Kontrová & Šusteková, 2020).

Student S23’s response underscores the transformative potential of GeoGebra. According to S23, using GeoGebra has reduced her challenges with circle geometry, making it simpler to prove theorems and improving her vocabulary and reasoning on this topic. Thus, by leveraging the interactive and dynamic features of GeoGebra, educators can create enriching learning experiences that empower students to develop a robust understanding of circle geometry and improve their problem-solving skills.

Finally, from Figure 4.26 Student S3 found GeoGebra helpful in addressing challenges like accurately drawing circles to scale. S3 further compared GeoGebra with conventional teaching methods, highlighting the advantages of technology-enhanced instruction. GeoGebra’s visual representations provided clearer explanations and deeper insights into circle theorems – especially those involving cyclic quadrilaterals. However, the student also noted that applying theorems to quadrilaterals remained somewhat tricky. This reflection aligns with existing literature, which emphasises the benefits of interactive visualisation for promoting understanding (Sudihartinih & Purniati, 2019).

4.10.3 Comparison of Traditional Teaching Methods With Using Virtual Tools

Figure 4.27 presents excerpts from a student questionnaire exploring learners’ perceptions of GeoGebra as a virtual manipulative in understanding circle geometry. The responses highlight students' experiences, including their perceived ease of grasping geometric concepts, reduction in challenges, and comparisons between conventional instruction and technology-enhanced learning.



Explore The Use of Virtual Manipulatives in Enhancing the Students' Understanding of Circle Geometry

	Questions	Students Response
1	Were there any activities that you found difficult using GeoGebra? Explain	NO, Because Even if the chord and the diameter changes from the given points To other point It won't change The Reasoning will always be (midpoint chord)
2	Did using GeoGebra (VM) help you grasp Cycle geometry easier? If so, please explain how	Yes, Because it shows Everything as it is. If we have Right-angled triangle placed at the center of the circle it gives use a clear understanding that is ∠ in a semi-circle
3	Do you think using GeoGebra has minimised your challenges with circle geometry?"	Yes, It did minimised my challenges with circle geometry, Now after using geogebra it Easer for me to interpret the question/Reasoning and calculating
4	How is your understanding of circle theorems involving cyclic quadrilaterals after using GeoGebra?	I have a clear understanding of all theorems as they appear in Any form or points.
5	How would you explain the differs between conventional (normal teaching) and teaching using GeoGebra?	Normal teaching is Difficult to understand and geogebra is Easier to understand

Figure 4.27: S21's response

Building upon observations from previous cycles, the effectiveness of integrating GeoGebra into the curriculum was assessed. When S21 compared technology with traditional teaching, she stated:

> *"GeoGebra is quite interesting, and I felt good using it because it gives the answers. When you get the GeoGebra examples, you can see what they want, and you can usually take answers from GeoGebra without using a pencil, ruler or protractor (instruments). It is straightforward because you can see this is parallel to this, and this is parallel to this, etc. It is recommended that the upcoming generation use GeoGebra."*

This response captures the student's appreciation for the ease and accuracy that GeoGebra offers compared to traditional methods. Instead of spending time using manual tools to measure and draw, the student found that GeoGebra provided immediate visual confirmation of relationships – such as parallel lines – which enhanced her learning experience.

S25 echoed this sentiment, highlighting the accuracy and reliability of using technology over manual tools:

> "*Using technology is much better because it is more accurate than when we are using the instruments to draw the circles and measure. Technology was more accurate because even if it changes, we are able to see if the values remain the same or change. So, I say technology was far better.*"

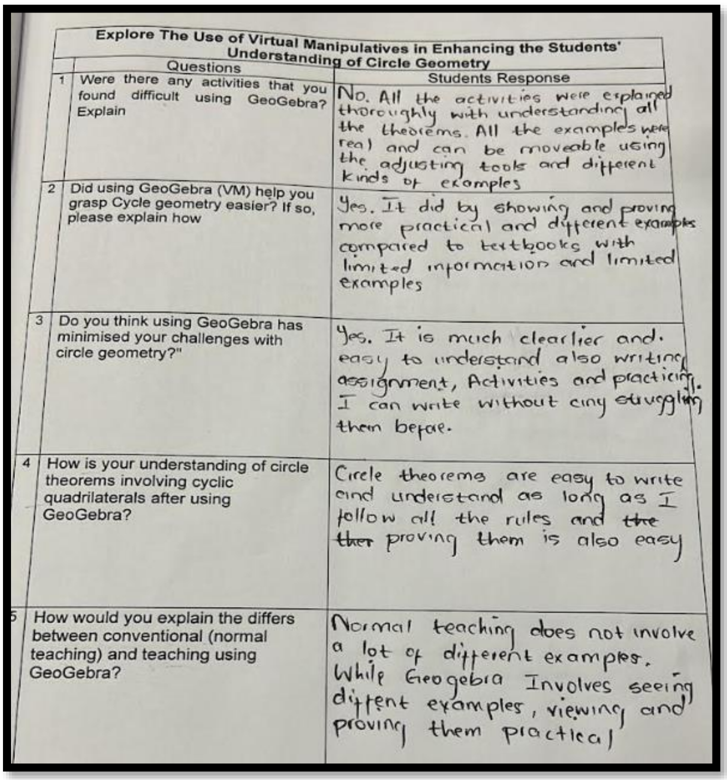
Here, the emphasis is on GeoGebra's precision in geometric tasks, a crucial element when understanding theorems that rely on exact measurements and constructions. The dynamic nature of the tool allowed students to observe changes and verify relationships, providing a deeper and more flexible understanding of geometry.

S26 added to this by reflecting on GeoGebra's ability to facilitate the exploration of geometric theorems through rotations and transformations – capabilities that were difficult to achieve with traditional methods.

When participants were asked, "What would you say is the difference between using GeoGebra and the other previous methods used?" S7's response was as follows:

> "*The difference between the first one is that we did not have extra numbers to rotate the shape of the circle, but with the technology, we could rotate the circle, and it showed that the same theorem could be applied.*"

This insight illustrates how GeoGebra's interactive features, such as rotation, allowed students to explore multiple representations of geometric theorems. Such flexibility in exploring different orientations and forms of geometric shapes was previously unavailable using conventional teaching methods.



Explore The Use of Virtual Manipulatives in Enhancing the Students' Understanding of Circle Geometry

	Questions	Students Response
1	Were there any activities that you found difficult using GeoGebra? Explain	No. All the activities were explained thoroughly with understanding all the theorems. All the examples were real and can be moveable using the adjusting tools and different kinds of examples
2	Did using GeoGebra (VM) help you grasp Cycle geometry easier? If so, please explain how	Yes. It did by showing and proving more practical and different examples compared to textbooks with limited information and limited examples
3	Do you think using GeoGebra has minimised your challenges with circle geometry?"	Yes. It is much clearlier and. easy to understand also writing assignment, Activities and practicing. I can write without any struggling then before.
4	How is your understanding of circle theorems involving cyclic quadrilaterals after using GeoGebra?	Circle theorems are easy to write and understand as long as I follow all the rules and ~~the~~ ~~ther~~ proving them is also easy
5	How would you explain the differs between conventional (normal teaching) and teaching using GeoGebra?	Normal teaching does not involve a lot of different examples. While Geogebra Involves seeing diffent examples, viewing and proving them practical

Figure 4.28: Student S7's Response

In addition to the observations made by the above student, another student (S7) emphasised the clarity of understanding gained through the use of GeoGebra. S7 remarked:

> *"I have a clear understanding of the theorems as they appear in any form or point."*

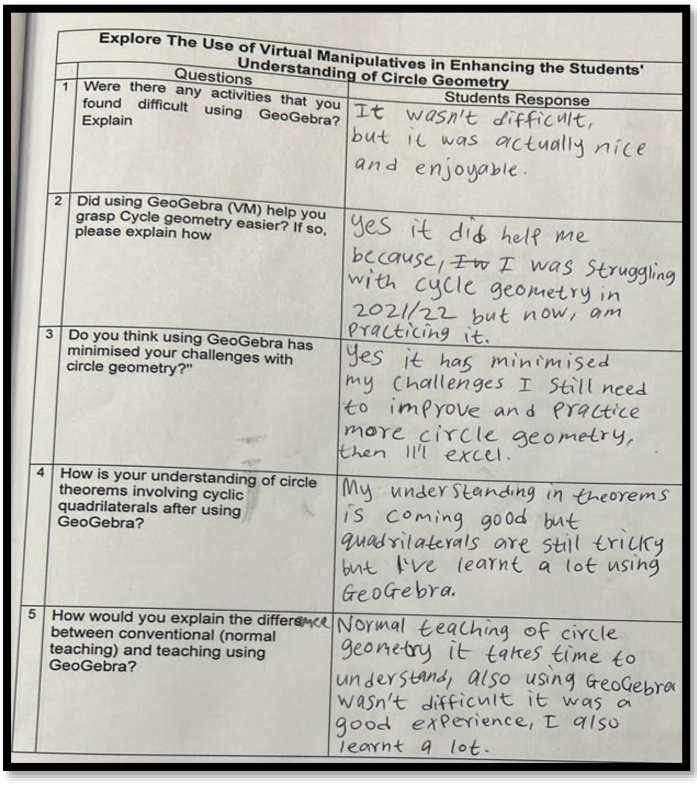
Furthermore, S7 noted the comparative ease of understanding when using GeoGebra compared to traditional teaching, as shown in Figure 4.28. S7's assertion of having a clear understanding of theorems in various forms or points highlights the dynamic nature of learning facilitated by GeoGebra. Unlike traditional teaching methods – which often present static examples and abstract explanations – GeoGebra allows students to explore mathematical concepts in diverse contexts and representations (Bhagat & Chang, 2015; Zulnaidi et al., 2020). In addition to the observations made by the above student, another student (S7) emphasised the clarity of understanding gained through the use of GeoGebra. S7 remarked:

> "*I have a clear understanding of the theorems as they appear in any form or point*."

This statement reflects S7's confidence in GeoGebra's ability to present mathematical concepts dynamically. Unlike traditional teaching methods – which often offer static examples and abstract explanations – GeoGebra enables students to explore theorems across diverse contexts and representations.

Furthermore, S7 noted the comparative ease of understanding when using GeoGebra, as shown in Figure 4.28. This assertion underscores the dynamic nature of learning facilitated by GeoGebra. The tool's interactive features allow students to manipulate geometric figures and observe real-time changes, ultimately leading to a deeper comprehension of complex theorems. Such findings align with the literature (Bhagat & Chang, 2015; Zulnaidi et al., 2020), which highlights the benefits of interactive visualisation in enhancing mathematical understanding.

Together, these observations suggest that GeoGebra not only simplifies the learning process but also cultivate an environment where abstract concepts become more accessible and engaging.



Explore The Use of Virtual Manipulatives in Enhancing the Students' Understanding of Circle Geometry

	Questions	Students Response
1	Were there any activities that you found difficult using GeoGebra? Explain	It wasn't difficult, but it was actually nice and enjoyable.
2	Did using GeoGebra (VM) help you grasp Cycle geometry easier? If so, please explain how	yes it did help me because, ~~I w~~ I was struggling with cycle geometry in 2021/22 but now, am practicing it.
3	Do you think using GeoGebra has minimised your challenges with circle geometry?"	yes it has minimised my challenges I still need to improve and practice more circle geometry, then I'll excel.
4	How is your understanding of circle theorems involving cyclic quadrilaterals after using GeoGebra?	My understanding in theorems is coming good but quadrilaterals are still tricky but I've learnt a lot using GeoGebra.
5	How would you explain the difference between conventional (normal teaching) and teaching using GeoGebra?	Normal teaching of circle geometry it takes time to understand, also using GeoGebra wasn't difficult it was a good experience, I also learnt a lot.

Figure 4:29: Student S13 Response

Another student, S13, echoed similar sentiments (Figure 4.29). When asked in the questionnaire, “How would you explain the difference between conventional (normal) teaching and using GeoGebra?” the student responded:

> “*Normal teaching does not involve a lot of different examples, while GeoGebra allows you to see different examples and view and prove them practically.”*

Traditional teaching methods often rely on static examples and abstract explanations, which may limit students’ ability to visualise and apply mathematical concepts (Kontrová & Šusteková, 2020; Noreen & Rana, 2019). In contrast, GeoGebra offers dynamic and interactive representations, allowing students to explore multiple examples, visualise geometric relationships and engage in hands-on problem-solving.

This response emphasises the contrast between traditional methods and GeoGebra’s dynamic approach. The software enabled students to explore multiple examples and visualise geometric relationships, thereby fostering active problem-solving.

In addition to these student questionnaire reflections, the researcher classified the layers of engagement based on the PKM. The classification of each activity across six levels (1–6) as weak, moderate or strong revealed that all six layers showed moderate engagement, as summarised in Table 4.3.

Table 4.3: Observation Schedule (Cycle 3)

Layers	**Weak**	**Moderate**	**Strong**
Layer 1: The recognition of qualities of congruency was an example of primitive knowledge. Pythagoras, triangles and quadrilaterals are examples of students who understood and created patterns. By using any representation, the student could remember previous material		X	
Layer Students who created images could x continue patterns without requiring them to do specific activities, like drawing images. Students		X	

Layers	Weak	Moderate	Strong
understood how to apply visual representations to the topic at hand			
Layer 3: Property notice could be used to develop a formula that describes all patterns' interactions by generalising the pattern rule. By utilising representations to convey mathematical meanings, the students can construct patterns and algorithms.		X	
Layer 4: The most common types of representations are symbolic and verbal. By extrapolating the pattern rule, students could create a general formula that describes all pattern interactions at the formalising level. A student can use mathematical representations to construct patterns and algorithms. The most common representations are symbolic and verbal		X	
Layer 5: Students in this level developed a general formula for extending patterns to greater stages and relating patterns to various mathematical subjects such as algebraic expressions, equations and geometry. The student connected representations and theorems. They connected representations and theorems, mostly using symbolic and verbal representations.		X	
Level 6: Finally, at the structuring level, students strived to develop a fundamental knowledge of circle theorems and their significance.		X	

Initially, Layers 5 and 6 – which involved extending patterns and developing knowledge of circle theorems – were classified as weak. However, following the introduction of GeoGebra, significant improvement was observed, with both layers now classified as moderate. This shift underscores the effectiveness of VMs in promoting student engagement and enhancing the understanding of complex geometric concepts. Building upon observations from previous cycles, the effectiveness of integrating GeoGebra into the curriculum was further assessed.

4.11 CONCLUSIONS

This study used action research to investigate the effectiveness of virtual manipulatives in enhancing NC(V) L4 mathematics students' understanding of circle geometry. It explored the challenges faced by students in mastering circle geometry, the nature of virtual manipulative activities that enhance understanding, and how these tools can help minimise learning difficulties.

To evaluate student reactions and analyse the challenges of teaching circle geometry, the research used data from pre-tests, post-tests, interviews and observations. A baseline test was administered to assess students' prior knowledge and identify areas of weakness. Initial instruction employed traditional teaching methods to address these weaknesses and improve understanding. Results from the baseline test revealed that students lacked basic geometric knowledge – they struggled to identify forms such as circle components, triangle properties and quadrilateral figures. Although some students demonstrated a partial understanding of geometric principles, they found it difficult to articulate their knowledge, highlighting gaps in their foundational understanding.

The researcher also employed the PKM to assess student engagement and understanding during the intervention, categorising their learning as weak, moderate or strong. Traditional teaching methods served as the initial intervention; however, student interactions were limited, with most students only responding to content they partially understood, revealing a disconnect between their learning experiences and the instructional strategies used.

After this first intervention, students began to actively engage in the learning process. Experimentation, practical assignments and hands-on problem-solving tasks improved comprehension. However, challenges persisted in using various tools – particularly in measuring angles – even though students showed improved spatial reasoning. According to the PKM, most students progressed to moderate layers of understanding (Layers 1–4).

In the third cycle of the study, the integration of GeoGebra significantly enhanced students’ interest and understanding of circle geometry. This interactive, practical approach fostered deeper engagement, allowing students to shift perspectives and develop their own theorems. Students demonstrated enthusiasm and adaptability, achieving a more thorough understanding of mathematical concepts.

The progression from traditional teaching methods to hands-on activities, culminating in the use of GeoGebra, led to marked improvements in students’ comprehension and performance. The PKM reflected these advancements, with students surpassing their previous levels of understanding and performance.

4.12 CHAPTER SUMMARY

This chapter presented the data collected and analysed during the study, using qualitative procedures and thematic coding while preserving participant anonymity through pseudonyms. Findings were organised into themes reflecting the students' experiences across two instructional cycles. Initially, traditional teaching methods revealed significant barriers to understanding circle geometry, including conceptual confusion, language limitations, and low classroom engagement. The second cycle introduced practical tools and virtual manipulatives like GeoGebra, leading to increased motivation, improved comprehension, and more positive attitudes. The practical and visual elements of virtual tools enabled students to better grasp abstract concepts, bridging the gap between theory and practice. The chapter highlighted the transformational impact of technology on student learning in geometry. The next chapter presents a summary of the thesis, conclusions and recommendations.

CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

Chapter 5 serves as the culmination of this research, providing an in-depth interpretation of the findings, their implications and the broader significance of the study. This chapter revisits the research objectives and evaluates how the results align with the initial aims of the study.

This research study addresses the following question: How can virtual manipulatives improve the understanding of circle geometry among NC(V) L4 mathematics students?

To further explore this main question, the following sub-questions were asked:

- What are the challenges experienced by NC(V) L4 mathematics students in understanding circle geometry?
- What is the nature (understanding how something works, reacts, unfolds, and develops) of virtual manipulation activities that enhance the understanding of circle geometry?
- How can virtual manipulation be used to minimise the challenges experienced by NC(V) L4 mathematics students in understanding circle geometry?

The purpose of this chapter is threefold: to interpret the findings in relation to the existing literature and the theoretical framework; to identify the implications of these findings in academic and practical contexts; and to make recommendations for future research and application. Additionally, this chapter addresses the limitations encountered during the research process, offering a balanced perspective on the study's contributions and constraints.

By synthesising the study's outcomes and positioning them within the broader field, this chapter highlights the key contributions of the research while pointing to potential avenues for future exploration.

5.2 SUMMARY OF THE RESEARCH METHODOLOGY AND DESIGN

This study employed a qualitative approach within the framework of a participatory action research design to gain an in-depth understanding of the students' responses and learning experiences. The rationale for adopting this research design and strategy was outlined in Sections 3.4. and 3.5.

The qualitative approach enabled the researcher to collect rich, detailed data through tools such as baseline tests, semi-structured interviews, tests and questionnaires, which were applied iteratively in a spiral process. This iterative approach allowed the researcher to refine the intervention until satisfactory results were achieved. Importantly, the purpose of these tests was not to measure correct or incorrect answers but to identify and analyse misconceptions and barriers to understanding.

To ensure the accuracy and reliability of findings, the research used multiple data collection methods and conducted the study in the students' authentic learning environment. These measures enhanced the validity of the results by situating the research within the context of real-world classroom dynamics.

5.3 SUMMARY OF LITERATURE REVIEW

The literature review examined three key concepts: (i) circle geometry; (ii) virtual manipulatives; and (iii) the use of GeoGebra in enhancing students' understanding of circle geometry. From the literature reviewed, it was established that circle geometry is a branch of Euclidean geometry that focuses on the properties of angles and lines in relation to circles. Despite its inclusion in the NC(V) L4 mathematics curriculum and its assessment in Paper 2, South African students continue to struggle with this topic. This research explored the role of VMs in improving students' comprehension of circle geometry concepts.

Virtual manipulatives, such as interactive web-based visual representations, provide an engaging and dynamic learning environment. Unlike physical manipulatives, virtual tools allow for enhanced exploration of mathematical concepts through graphical and symbolic representations. Studies suggest that VMs improve conceptual understanding, supporting student-centred learning and positively impact motivation and academic performance. They also help bridge the gap between procedural knowledge and conceptual understanding, which is often lacking in traditional teaching approaches.

In this study, the GeoGebra, a free, dynamic mathematics software program that integrates geometry, algebra and calculus, allowed students to explore mathematical relationships interactively. The software enables learners to visualise and manipulate circle geometry theorems, such as the “angle at the centre is twice the angle at the circumference” and the tangent-chord theorem, promoting deeper conceptual understanding. Research indicates that incorporating GeoGebra in teaching improves student achievement, enhances geometric thinking and increases motivation to learn circle geometry.

These insights support the study’s objective of investigating how VMs, particularly GeoGebra, can enhance students’ understanding of circle geometry in the NC(V) L4 mathematics curriculum.

5.4 SUMMARY OF THE FINDINGS OF THE STUDY

The findings of this study reveal critical insights into the progression of student understanding of circle geometry, evaluated across three instructional cycles: (i) conventional teaching; (ii) practical activities; and (iii) virtual manipulatives. These cycles illuminate the challenges faced by students, the impact of different teaching approaches, and the effectiveness of integrating technology.

5.4.1 Challenges Experienced by NC(V) L4 mathematics students in Understanding Circle Geometry

From the baseline test results, it was evident that students lacked a foundational understanding of circle geometry, including the ability to recognise circle properties and connect those with related concepts in triangles and straight lines. While some students demonstrated a basic grasp of Pythagoras’ theorem, their application of it in geometric contexts was limited. These gaps underscored the need to review fundamental concepts prior to the introduction of advanced topics.

5.4.2 Instructional Cycles and Key Observations

Cycle 1: Conventional Teaching

The conventional teaching approach highlighted students’ lack of engagement and interaction. Students demonstrated minimal interest in the lesson, potentially due to unfamiliarity with the topic and the passive nature of traditional methods. While some students recalled basic concepts, such as equal radii and perpendiculars, they struggled to articulate and apply the midpoint theorem. Test results following this cycle ranged from mediocre to subpar, reflecting the limitations of traditional instructional methods.

Cycle 2: Practical Activities

Practical instruction led to a notable improvement in student engagement. During lessons on cyclic theorems, students participated more actively and began formulating their own hypotheses. However, time constraints and challenges with measuring instruments hindered the full potential of this method. Despite these challenges, students showed improved post-test scores compared to Cycle 1, indicating that hands-on activities were more effective than conventional teaching.

Cycle 3: Virtual Manipulatives (GeoGebra)

The introduction of GeoGebra in the third cycle marked a significant shift in student learning. Students were highly engaged, demonstrating enthusiasm and confidence in using the digital tool. They experimented with angles, created original theorems and made connections between their experiences and established geometric principles. This interactive, technology-driven approach promotes a deeper understanding of circle geometry and resulted in the highest test scores among all cycles.

5.4.3 Key Findings

The gradual shift from traditional methods to practical and then technological approaches demonstrated the increasing effectiveness of active and interactive learning strategies. GeoGebra emerged as a powerful tool for enhancing student engagement and understanding by enabling students to visualise, experiment with and explore geometric concepts independently. Semi-structured interviews with students further confirmed the positive impact of GeoGebra, with participants highlighting its role in simplifying complex concepts and making learning enjoyable.

The findings highlight the challenges students faced with traditional methods and the transformative potential of integrating technology into mathematics instruction. The progression through the instructional cycles underscores the effectiveness of combining practical activities and digital tools to facilitate active learning and improve conceptual understanding of circle geometry.

5.4.4 Interpretation of Findings

This study sought to answer how virtual manipulatives, specifically GeoGebra, could enhance NC(V) L4 mathematics students' understanding of circle geometry. The phased approach of the spiral action research revealed that virtual tools have a transformative impact on learning by enabling exploration, experimentation and visual representation, which encourages deeper conceptual clarity.

The findings confirm that traditional teaching methods alone are insufficient for engaging students or addressing their challenges with foundational and advanced geometric concepts. Practical activities provided some improvement in engagement, but the integration of GeoGebra as a virtual manipulative was instrumental in overcoming many of the barriers identified in earlier cycles.

The use of GeoGebra not only bridged gaps in prior knowledge but also provided a dynamic and interactive platform for learning. Students were able to create, manipulate and explore geometric theorems in ways that were not possible with traditional approaches. This aligns with existing literature highlighting the efficacy of technology in enhancing mathematics instruction and confirms the critical role of active learning strategies in improving student outcomes.

5.4.5 Relating Findings to the Research Questions

In responding to the main Research Question: How can virtual manipulatives enhance NC(V) L4 mathematics students' understanding of circle geometry?

GeoGebra proved to be a powerful tool for enhancing students' understanding of circle geometry by:

- Allowing students to visualise and manipulate geometric concepts interactively, leading to greater engagement and comprehension,
- Enabling students to construct theorems, test hypotheses and see immediate results of their manipulations, which deepened their understanding, and
- Providing a fun and user-friendly learning environment that motivated students to participate actively in lessons.

The transition from conventional methods to VMs showed that incorporating technology can significantly improve conceptual understanding, as students were better able to relate abstract geometric principles to practical applications.

Sub-Question 1: What are the challenges experienced by NC(V) L4 mathematics students in understanding circle geometry?

The findings identified two primary barriers:

- Barriers to active participation in which *s*tudents struggled to engage in classroom environments where traditional methods were dominant. Factors such as lack of interaction, passive instruction, and insufficient prior knowledge contributed to their disengagement; and
- Barriers to comprehension in which *s*tudents lacked a foundational understanding of basic geometric principles, such as the properties of triangles and straight lines, which hindered their ability to grasp more complex concepts like circle theorems.

These challenges were exacerbated by limited practice, minimal extracurricular engagement and language barriers.

Sub-Question 2: What is the nature of virtual manipulation activities that enhance the understanding of circle geometry?

The study demonstrated that virtual manipulation activities using GeoGebra are highly interactive and exploratory, which directly contribute to enhanced learning. Key aspects revealed include:

- Dynamic visualisation: Students could manipulate geometric figures, such as moving points on a circle or modifying angles, to test theorems and observe relationships in real-time;
- Exploration and experimentation: GeoGebra allowed students to create their theorems and experiment with different configurations, fostering independent learning and hypothesis testing; and
- Motivational factors: Students found the technology engaging, enjoyable and easy to use, which increased their confidence and interest in geometry.

Student feedback highlighted the accuracy, speed and interactivity of GeoGebra as advantages over traditional tools, making it an effective medium for understanding complex geometric concepts.

Sub-Question 3: How can virtual manipulation minimise the challenges students face in understanding circle geometry?

It was found in this study that virtual manipulation, particularly with GeoGebra, addressed the challenges identified by:

- Providing a hands-on, engaging learning environment: Students became active participants in their learning, moving beyond passive instruction to a more exploratory approach,
- Enhancing comprehension: GeoGebra's visual and interactive capabilities helped students visualise static theorems, bridging the gap between abstract concepts and practical understanding, and
- Building confidence: Students reported feeling more confident and less intimidated when using technology, leading to improved performance.

By addressing gaps in foundational knowledge and promoting deeper exploration, VMs significantly reduced the challenges students faced

5.4.6 Unexpected Results

There were some unexpected results that emerged in this study. These include:

- Rapid engagement with GeoGebra: Students adapted quickly to GeoGebra, likely due to their prior familiarity with computers. This level of enthusiasm was unexpected, considering their earlier disengagement in traditional and practical instruction.
- Overestimation of foundational knowledge: The initial assumption that students had a firm grasp of basic geometry concepts was disproven, as many struggled with fundamental ideas such as the sum of angles in a triangle. This highlights the importance of revisiting foundational knowledge before advancing to more complex topics.

These findings emphasise the transformative potential of virtual manipulatives like GeoGebra in enhancing the teaching and learning of circle geometry. By addressing foundational gaps, fostering active engagement and providing dynamic visualisation, virtual tools offered a possible solution to the challenges faced by NC(V) L4 mathematics students. The study reinforces the need to integrate technology into mathematics education to promote deeper conceptual understanding and improve overall student outcomes.

5.6 IMPLICATIONS OF THE STUDY

5.6.1 Theoretical Implications: Contributions to Existing Theories

This study offers significant contributions to the field of mathematics education, particularly in the teaching and learning of geometry, by integrating VMs within a participatory action research framework. The findings of this study support and expand upon existing theoretical frameworks in the following ways:

- Constructivist learning theory: This study aligns with constructivist principles, which emphasise active learning through exploration and problem-solving. The use of GeoGebra as a virtual manipulative provided students with opportunities to actively engage in the construction and exploration of geometric theorems. By enabling students to visualise geometric relationships and experiment with geometric figures, the study contributes to a deeper understanding of how technology can facilitate active, hands-on learning, as highlighted in Piagetian and Vygotskian perspectives on cognitive development.

- PKM: The results also corroborate the PKM, particularly in demonstrating how students' progress through various levels of understanding in mathematics. By introducing virtual tools like GeoGebra, students advanced from lower levels of understanding (image-making and image-having) to higher levels (theorems and patterns). The study suggests that virtual manipulatives can catalyse this progression by providing visual and interactive learning opportunities that deepen students' conceptual knowledge.

- Technology-enhanced learning frameworks: The integration of GeoGebra into the study contributes to the growing body of literature surrounding technology-enhanced learning in mathematics. It demonstrates that digital tools can offer more than just a medium for presenting information; they can actively engage students in constructing mathematical knowledge. This reinforces the view that technology, when properly integrated into the curriculum, can transform the learning experience, particularly in abstract subjects like geometry.

5.6.2 Practical Implications: Relevance and Application of Findings to Practice or Policy

The findings from this study have several practical implications for educators, curriculum developers and policymakers, particularly in the context of teaching geometry to NC(V) L4 mathematics students:

- Enhanced instructional practices: The study highlights the importance of incorporating virtual manipulatives into the mathematics curriculum, especially for students who struggle with traditional teaching methods. GeoGebra, as demonstrated in the study, facilitates active engagement and visual understanding of geometric concepts, thus improving comprehension. Educators can leverage similar digital tools to support students' learning by offering interactive, exploratory and student-centred learning experiences.
- Addressing misconceptions and gaps in knowledge: The study showed that students struggled with foundational geometry concepts, which often hindered their understanding of more complex topics like circle geometry. This study revealed that teachers can use virtual tools to diagnose and address these misconceptions by offering immediate feedback and opportunities for students to experiment and test their own hypotheses. Additionally, teachers could incorporate pre-lesson exercises using virtual manipulatives to ensure that students have a solid grasp of the necessary foundational concepts before moving on to more advanced topics.
- Curriculum reform and policy development: This study suggests the potential for incorporating technology-driven learning into vocational education curricula. Given the positive impact of GeoGebra on students' engagement and understanding, policymakers and curriculum designers should consider integrating virtual tools into the NC(V) mathematics syllabus. The results of this study could inform policy decisions on the importance of technology in education, advocating for greater inclusion of digital tools in classrooms to enhance both teaching effectiveness and student learning outcomes.

- Teacher professional development: The study underscores the importance of professional development for educators to equip them with the knowledge and skills necessary to integrate digital tools effectively into their teaching. Educators who are comfortable using VMs are more likely to design innovative lessons that encourage student engagement and deeper learning. Providing teachers with the appropriate training and resources to implement technology in the classroom should be a priority for education stakeholders.
- Student motivation and confidence: Practically, the findings also indicate that technology has a positive effect on student motivation and confidence. By incorporating GeoGebra into lessons, educators can create more engaging learning environments that increase students' interest in mathematics. This is particularly important in the context of vocational education, where students may not traditionally view mathematics as relevant or interesting. The study suggests that technology-enhanced instruction can help students gain confidence in their abilities, leading to improved learning outcomes and higher levels of academic achievement.

In conclusion, this study not only contributes to theoretical knowledge in mathematics education but also provides valuable insights into how virtual manipulatives can be applied in practice, particularly in improving the understanding of circle geometry for NC(V) L4 mathematics students. By enhancing instructional methods and supporting curriculum reforms, these findings have the potential to improve both teaching practices and student learning outcomes in vocational education.

5.7 LIMITATIONS OF THE STUDY

The researcher used convenience sampling for this study because he worked with students chosen by him, as stated in Section 3.6. This constituted the study's initial limitation. Moreover, this study only included 30 students. This small sample, which was chosen to improve the richness of the qualitative data, might not be sufficient to generalise the findings.

The second limitation of the study was the constrained time available to complete each intervention while simultaneously managing the extensive amount of data generated through the action research design. The researcher needed to cover a significant volume of instructional content within a short period, which required completing interventions quickly while still aiming for the expected performance outcomes. This challenge was compounded by the three-week duration allocated for the exploration of virtual manipulation using GeoGebra, during which the researcher could only utilise a one-hour daily tutorial period. As a result, repeating interventions to ensure mastery was both time- and labour-intensive, making the data analysis process more complex and demanding.

The third limitation concerned the constant use of English as the primary language of communication throughout the research, as the researcher was unable to translate mathematical theorems written in English into the different languages spoken by the students. Because of this, the researcher lost confidence in his ability to ensure that students who performed poorly grasped a number of crucial cycle geometry concepts.

The final limitation was the time and effort required to repeat each intervention until the desired and expected performance was attained, and a lot of data was gathered using an action research design which made the data analysis complex and time-consuming.

5.8 RECOMMENDATIONS

The recommendations in this study are divided into three categories: practice, policy and future research.

5.8.1 Practice

Based on the findings, this study recommends that:

- Technology should be integrated into the curriculum such that educators incorporate virtual manipulatives like GeoGebra into daily mathematics instruction to enhance students' conceptual understanding and engagement.
- Teachers should receive professional development training to effectively use digital tools in the classroom.
- Traditional and virtual methods should be combined such that a blended teaching approach that includes both hands-on practical activities and virtual tools can maximise student comprehension.
- Educators must focus on foundational skills that address gaps in students' prerequisite knowledge, such as basic geometric principles, to ensure readiness

for more complex topics like circle geometry. Policymakers should allocate resources to enhance technological infrastructure in schools and TVET colleges by providing computers, reliable internet and educational software like GeoGebra.

5.8.2 Policy

Mathematics curricula should incorporate VMs for teaching geometry, with policies promoting interactive and dynamic teaching strategies.

- Educational policies should support multilingual teaching resources to make mathematical concepts accessible to non-native speakers.
- Policymakers should work to close the digital divide by ensuring equal access to technology and training for under-resourced schools and students.

5.9 FUTURE RESEARCH

Future studies should examine the long-term impact of virtual tools like GeoGebra on students' mathematical understanding and academic performance across different levels.

Research should compare the effectiveness of virtual manipulatives with traditional and blended teaching methods to determine the most effective approach.

Further investigation is needed into how language barriers affect mathematical comprehension and how multilingual resources can help bridge these gaps, particularly within technology-blended approaches**.** Exploring how digital tools, virtual manipulatives and multilingual technological platforms can support conceptual understanding may provide deeper insights into how linguistic diversity interacts with mathematical learning in TVET contexts.

Future research could focus on how interactive technologies can promote independent and self-directed learning in mathematics.

By implementing these recommendations, the educational landscape can further adapt to modern challenges, equipping both educators and students with the tools needed to thrive in a digital and globalised world.

5.10 CONCLUDING REMARKS

A synopsis of the review of the research questions for this study was provided in this chapter. In addition to recommendations for future NC(V) L4 TVET College practices in using virtual manipulatives to teach circle geometry, the researcher presented findings interpreted from the interviewees' responses, tests, observations and recommendations for additional research.

The study found that NC(V) L4 mathematics students improved their understanding of circle geometry through virtual manipulatives. After the second intervention, students could formulate circle theorems independently. This enhanced skill is expected to further boost their mathematics performance. The findings suggest that mastering circle theorems increases student confidence and success in other math areas. The study highlights the potential impact on teaching strategies for circle geometry. TVET college leaders have the authority to implement initiatives that improve mathematics education. Reviewing these findings can help college representatives make informed decisions for better student outcomes.

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DECLARATION ON THE USE OF AI

I acknowledge the following uses of GenAI tools in this thesis:

- I declare that I have referenced use of GenAI outputs within my thesis in line with the University referencing guidelines.

APPENDICES

APPENDIX A: ETHICAL CLEARANCE

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2021/08/11

Dear Mr MA Palayandi

Decision: Ethics Approval from 2021/08/11 to 2024/08/11

Ref: 2021/08/11/67124798/42/AM
Name: Mr MA Palayandi
Student No.: 67124798

Researcher(s): Name: Mr MA Palayandi
E-mail address: 67124798@mylife.unisa.ac.za
Telephone: 0736972530

Supervisor(s): Name: Dr E. G. Makwakwa
E-mail address: makwaeg@unisa.ac.za
Telephone: 012 429 4575

Name: Prof ZMM Jojo
E-mail address: jojozmm@unisa.ac.za
Telephone: 012 429 6627

Title of research:

AN EXPLORATION OF THE USE OF VIRTUAL MANIPULATIVES IN ENHANCING UNDERSTANDING CIRCLE GEOMETRY: CASE OF LEVEL 4 NATIONAL VOCATIONAL STUDENTS

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2021/08/11 to 2024/08/11.

*The **medium risk** application was reviewed by the Ethics Review Committee on 2021/08/11 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

APPENDIX B: REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN SCHOOLS

03 December 2021

The principal

South West Gauteng TVET College

Head office

For attention: Mr. Monyamane

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN SCHOOLS

Dear Mr. Monyamane

My name is Marcus Anthony Palayandi, and I am a MED student at UNISA The research I wish to conduct for my Master's dissertation involves "an exploration of the use of virtual manipulatives in understanding circle geometry by L4 national vocational students". This project will be conducted under the supervision of Dr Makwakwa, Eva (South Africa) and Professor Zingiswa Mybert Monica Jojo(South Africa).

I am hereby seeking your consent to conduct my research at the Molapo Campus with The L4 students. I have provided you with a copy of my dissertation proposal which includes copies of the measure and consent and assent forms to be used in the research process, as well as a copy of the approval letter which I received from the UNISA Research Ethics Committee.

Upon completion of the study, I undertake to provide the Department of Education with a bound copy of the full research report. If you require any further information, please do not hesitate to contact me on cell no 0736972530 or email address 67124798@mylife.unisa.ac.za.

.

Thank you for your time and consideration in this matter.

Yours sincerely,

Marcus Palayandi

Appendix E Application form from Students to conduct Research in Public colleges

higher education & training
Department: Higher Education and Training
REPUBLIC OF SOUTH AFRICA

DHET 004: APPENDIX 1:

APPLICATION FORM FOR STUDENTS TO CONDUCT RESEARCH IN PUBLIC COLLEGES

1. APPLICANT INFORMATION

1.1	Tittle	Mr.		
1.2	Name and surname	Marcus Anthony Palayandi		
1.3	Postal Address	78 Brandvlei Crescent Eldorado Park Ext 4		
1.4	Contact details	Tel		
		Cell	073 6972 530	
		Fax		
		Email	69124798@mylife unisa.co.za	
1.5	Name of Institution where enrolled	UNISA		
1.6	Field of study	MED EDUCATION MATHEMATICS		
		Please tick relevant options		
		Doctoral degree (PhD)		
		Master's Degree		X
		Other (please specify)		

2 DETAILS OF STUDY

2.2	Title of study
An exploration of the use of virtual manipulatives in enhancing understanding circle geometry: case of L4 national vocational students	

2 DETAILS OF STUDY

2.3	Purpose of study
This study aims to explore how the use of virtual manipulation enhances L4 NC(V) students' understanding of circle geometry.	

Please indicate the types of research activities you are planning to undertake in the College, as well as the categories of persons who are expected to participate in your study (for example, lecturers, students, College Principals, Deputy Principals, Campus Heads, Support Staff, Heads of Departments), including the number of participants for each activity.			
3.1	Complete questionnaires	Expected participants	Number of participants
		a) Students	30
		b) Lecturer	1
		c)	
		d)	
		e)	
3.2	Participate in individual interviews	Expected participants	Number of participants
		a) Students	30
		b) Lecturer	1
		c)	
		d)	
		e)	
3.3	Participate in focus group discussions/ workshops	Expected participants	Number of participants
		a)None	
		b)	
		c)	
		d)	
		e)	

3.4	Complete standardised tests (e.g. Psychometric Tests)	Expected participants	Number of participants
		a) Student	30
		b)	
		c)	
		d)	
		e)	
3.5	Undertake observations Please specify	Students will be observed while working on the computers	
3.7	Other Please specify	None	

4. SUPPORT NEEDED FROM THE COLLEGE

Please indicate the type of support required from the College (Please tick relevant option/s)			
Type of support		*Yes*	No
4.1	The College will be required to identify participants and provide their contact details to the researcher.		X
4.2	The College will be required to distribute questionnaires/instruments to participants on behalf of the researcher		X
4.3	The College will be required to provide official documents. *Please specify the documents required below*		X
4.4	The College will be required to provide data (only if this data is not available from the DHET). *Please specify the data fields required, below*		X
	NONE		
4.5	Other, please specify below		X

	NONE

5. DOCUMENTS TO BE ATTACHED TO THE APPLICATION

The following 2 (two) documents must be attached as a prerequisite for approval to undertake research in the College	
5.1	Ethics Clearance Certificate issued by a University Ethics Committee
5.2	Research proposal approved by a University.

6. DECLARATION BY THE APPLICANT

I undertake to use the information that I acquire through my research, in a balanced and a responsible manner. I furthermore take note of, and agree to adhere to the following conditions:

a) I will schedule my research activities in consultation with the said College/s and participants in order not to interrupt the programme of the said College/s.

b) I agree that involvement by participants in my research study is voluntary, and that participants have a right to decline to participate in my research study.

c) I will obtain signed consent forms from participants prior to any engagement with them.

d) I will obtain written parental consent of students under 18 years of age, if they are expected to participate in my research.

e) I will inform participants about the use of recording devices such as tape-recorders and cameras, and participants will be free to reject them if they wish.

f) I will honour the right of participants to privacy, anonymity, confidentiality and respect for human dignity at all times. Participants will not be identifiable in any way from the results of my research, unless written consent is obtained otherwise.

g) I will not include the names of the said College/s or research participants in my research report, without the written consent of each of the said individuals and/or College/s.

h) I will send the draft research report to research participants before finalisation, in order to validate the accuracy of the information in the report.

i) I will not use the resources of the said College/s in which I am conducting research (such as stationery, photocopies, faxes, and telephones), for my research study.

j) Should I require data for this study, I will first request data directly from the Department of Higher Education and Training. I will request data from the College/s only if the DHET does not have the required data.

k) I will include a disclaimer in any report, publication or presentation arising from my research, that the findings and recommendations of the study do not represent the views of the said College/s or the Department of Higher Education and Training. l) I will provide a summary of my research report to the Head of the College/s in which I undertook my research, for information purposes. I declare that all statements made in this application are true and accurate. I accept the conditions associated with the granting of approval to conduct research and undertake to abide by them.	
SIGNATURE	
DATE	03/12/2021

APPENDIX C: STATEMENT AND CONSENT FORM

STATEMENT AND CONSENT FORM

“EXPLORING THE USE OF VIRTUAL MANIPULATIVES IN UNDERSTANDING CIRCLE GEOMETRY”: MARCUS PALAYANDI

I, ____ (participant’s full name) have read and understood the attached participant information sheet entitled: The contact details of the researcher have been given to me for me to direct any further questions that I may have about this study.

I understand that while information gained during this study may be published, I will not be identified and my personal details will not be given. I understand the research team will have access to the data and results. I understand that I am free to withdraw from this study at any stage without academic or any other penalty.

I recognise that my participation in this study will assist with research into the experiences of students who are mothers at university, and that I may not benefit personally from the research.

I understand that this research will be conducted in a way consistent with the national statement on ethical conduct in research involving humans (2007) and the Privacy Act (1998).

____________________________________ ________________

Participant’s signature Date

I, ____ (researcher’s full name) have explained the study to the signatory who stated that she understood the purpose and process of this study.

____________________________________ ________________

Researcher’s signature Date

APPENDIX D: WRITTEN EXPLANATION OF THE STUDY

AN EXPLORATION OF THE USE OF VIRTUAL MANIPULATIVES IN UNDERSTANDING CIRCLE GEOMETRY BY L4 NATIONAL VOCATIONAL STUDENTS

ABOUT THE RESEARCHER

My name is Marcus Palayandi and I am enrolled in Masters Studies at UNISA. This study explores the use of virtual manipulatives in understanding circle geometry by L4 national vocational students, and my motivation to conduct research in this area comes from my own experience as a lecturer teaching Mathematics for TVET college students in L4.

INTRODUCTION

You are invited to participate in this study looking at exploring the use of virtual manipulatives in understanding circle geometry by L4 national vocational students. This study aims to seek an alternate way to learn and understanding circle geometry. This is important because students performed very poorly in the Geometry and in specific circle Geometry.

PURPOSE OF THE STUDY

The purpose of the study is to explore the effects of virtual manipulatives in teaching Circle Geometry for mathematics students in L4. This study therefore sought to:

1.6.1. Determine the activities that will favour learning using virtual manipulatives in teaching L4 (NVC) Circle Geometry.

1.6.2. Determine the nature of improvement of learning using virtual manipulatives in teaching L4 (NVC) Circle Geometry.

Determine how virtual manipulatives in teaching and learning Circle Geometry will improve the results of L4 (NVC) students?

PARTICIPANTS

All the students participating in this study will be currently studying an approved course on- campus at Molapo South West Gauteng college. They will also all be L4 students doing Mathematics fulltime.

THE STUDY PROCESS

This study involves information being collected in three ways. Firstly, all the participants will be asked to complete a questionnaire. This questionnaire, consisting of a number of questions, aims to give the researcher an idea of each student's commitments both on and off campus. After the completion of this questionnaire you participate in either an individual interview with the researcher. You are free to ask the researcher any questions relating to this study process, or the question being asked, at any time.

The individual interviews will be conducted first. During these interviews you will be asked to tell me about your experience at college. Although it is intended that each student have one interview, more than one may be required. These interviews are expected to go for around 30 minutes. They will take place at a place and during a time that is agreeable to both the researcher and student. The interviews will be recorded on a digital voice recorder for analysis by the researcher.

PARTICIPATION IS VOLUNTARY

Participation in this study is completely voluntary. You are free to withdraw from this study at any time without any academic penalty or other disadvantage. There are no risks associated with your involvement in this research. You may choose not to answer some questions. You may ask for the digital voice recorder to be turned off at any time. If you withdraw from the study, any personal details and data collected will continue to be stored securely with the information from the other participants of the study. Alternatively, you may ask for your data to be destroyed.

CONFIDENTIALITY

Participation in this study is strictly confidential. Your personal information will be identified using a participant identification number and your information will not be given to anyone who is not part of this study. The research report may include parts of interviews; you will not be identified in any way. Any publications arising from this study will not identify any of your personal information. All documents and notes taken will be stored in a secure location at the university until the completion of the study This research will be conducted in a way that is consistent with the national statement on ethical conduct in research involving humans (2007) and the Privacy Act (1998).

CONTACTS

If you wish to ask any questions or discuss any part of the research, I can be contacted at 073 6972 530.

APPENDIX E: QUESTIONNAIRE

QUESTIONNAIRE Explore the use of Virtual Manipulatives in Enhancing the students Understanding of Circle Geometry		
Question		Students Responses
1	Which activities, if any, did you find challenging when using GeoGebra? Please explain.	
2	Did using GeoGebra (virtual manipulatives) help you understand Circle Geometry better? If yes, please explain how.	
3	Do you feel that using GeoGebra reduced your difficulties with Circle Geometry? Please elaborate.	
4	How has your understanding of circle theorems involving cyclic quadrilaterals changed after using GeoGebra?	
5	In your opinion, how does learning Circle Geometry through GeoGebra differ from traditional teaching methods?	

APPENDIX F: QUESTIONNAIRE SEMI-STRUCTURED INTERVIEW QUESTIONS

Sub-question	**Example Question**	**Participant's Response**
Determine the activities that will favour learning using virtual manipulatives in teaching L4 (NVC) Circle Geometry.	Which activities did you enjoyed the most? Explain Were there any activities that you found challenging? Which ones and why?	
Comparing the different methods of teaching	Students were asked to How would you compare the first method of teaching with the second method of constructing solids during the interview.	
Determine the nature of improvement of learning using virtual manipulatives in teaching L4 (NVC) Circle Geometry	Did the use of GeoGebra (VM) improve your understanding of Cycle geometry? If so, explain how.	
Determine how virtual manipulatives in teaching and learning Circle Geometry will improve the results of L4 (NVC) students?	Did your performance in cycle geometry improve because of the use of virtual manipulatives? If so by how much? Explain. What other suggestions do you have to improve the quality in using VM? How can the use of GeoGebra be improved on to better students' performances?	

APPENDIX G: BASELINE TEST

CIRCLE GEOMETRY PRE-TEST

This test will serve as a baseline test to ascertain the students' prior knowledge to do Circle geometry.

Total Marks	25
Time Allocation	45 min
Researcher	Palayandi MA

Question	Content	Mark Allocation	Student mark
1	Terminology	4	
2	Straight lines	2	
3	Parallel lines	5	
4	Triangles	8	
5	Quadrilaterals	3	
	Total	25	

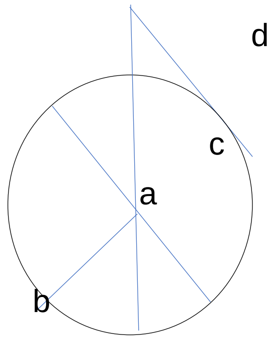
Student Surname and Name	
Student Contact Details	

Student Signature	
Researcher's Signature	

Comments and Reflections

QUESTIONS

Terminology of a Circle



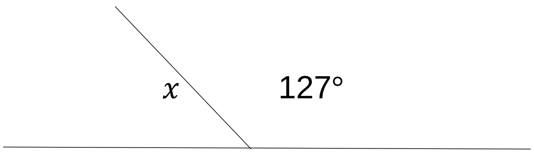
Identify the following lines in the circle

1.1	a		(1)
1.2	b		(1)
1.3	c		(1)
1.4	d		(1)

Basic geometry of lines angles and triangles

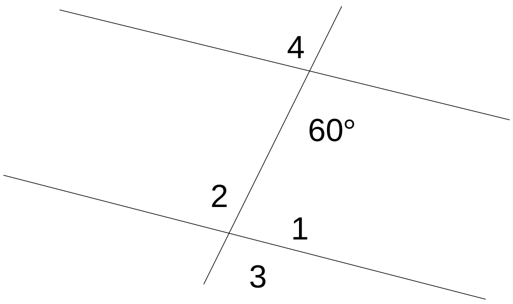
Calculate the unknown values in the following lines and figures.

2.1



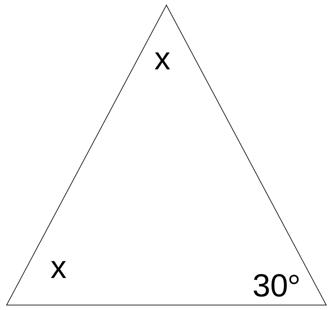
	Answer	Reason	
2.1			(2)

2.2



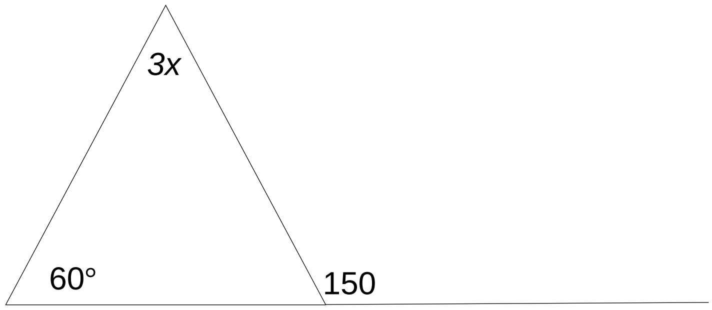
	Answer	Reason	
2.2	1		(2)
	2		(2)
	3		(2)
	4		(2)

2.3



	Answer	Calculation and Reasons	
2.23	x		(3)
	$2x$		(2)

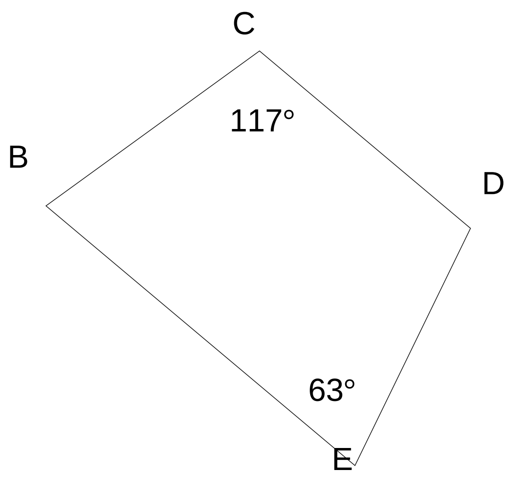
2.4



	Answer	Calculation and Reasons	
2.23	x		(3)

Question 3

Prove that BCDE is a cyclic Quadrilateral



3		(3)

TOTAL [25]

APPENDIX H: POST-TEST

TEST L4 (MED RESEARCH)

Question 1

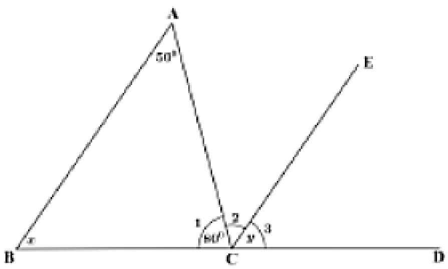
State whether the following statements are true or false

Angles on a straight line add up to 180 degrees (1)

An angle formed in a semicircle is always a right angle (90°). (1)

The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment. (1)

In the diagram below, side BC of ΔABC is a produce to D CE bisect ACD $\hat{A}$ = 50° $\hat{C}$ =80°. Let $\hat{B} = x \; and \; \widehat{C_3}$= y

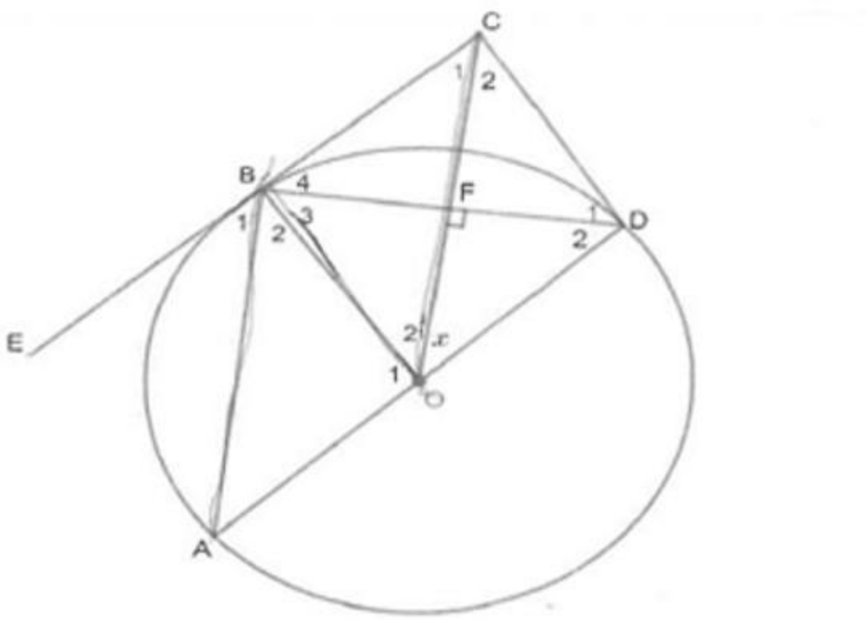


Calculate x (3)

Calculate y (3)

Is CE//BA? Give reason. (2)

In the figure below AD is the diameter of the circle O. BE is the tangent to the circle and OC is perpendicular to BD. F$\hat{O}D$= x



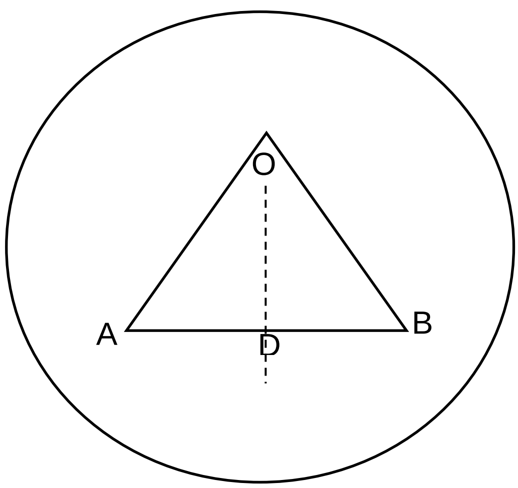
1.3.1: Demonstrate that OC and AB are parallel (OC II AB). (4)

1.3.2: Prove that $F\hat{O}B = x$. (4)

1.3.3: Prove that OBCD is a cyclic quadrilateral. (4)

Question 2

In the diagram below, O is the centre of the circle. Line segment AB is a chord, and OD is perpendicular to AB such that OD ⊥ AB. D is the point where the perpendicular from O meets AB. The length of AB is 10 cm, and OD = 6 cm.



2.1 What is the length of AD? (1)

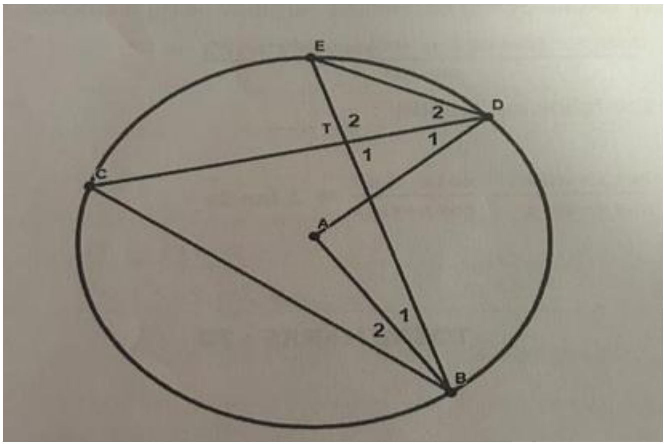
2.2 What is the length of DB? (1)

2.3 Prove that triangle ODA is a right-angled triangle. (1)

2.4 Use the Pythagorean Theorem to calculate the radius OA of the circle. (2)

Question 3

3.1 In the figure B, C, E and D are points on the circle A such that BC ll DC. Chords BE and CD intersect at T. Let $\hat{C} = x$.



3.1.1 Express B$\hat{A}$D in terms of x. (1)

3.1.2 Prove that Δ TDE is an isosceles triangle (3)

3.1.3 Express T_1 in terms of x (1)

3.1.4 Prove that the points B,A,T and D form a cyclic quadrilateral (2)

TOTAL [30]

APPENDIX I: TURNITIN REPORT

APPENDIX J: CONFIRMATION OF PROFESSIONAL EDITING

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19 July 2025

Declaration of editing

AN EXPLORATION OF THE USE OF VIRTUAL MANIPULATIVES IN ENHANCING UNDERSTANDING CIRCLE GEOMETRY: CASE OF LEVEL 4 NATIONAL VOCATIONAL STUDENTS

By

Marcus Anthony Palayandi

I declare that I have edited and proofread this thesis. My involvement was restricted to language usage and spelling, completeness and consistency.

I am qualified to have done such editing, being in possession of a Bachelor's degree with a major in English, having taught English to matriculation, and having a Certificate in Copy Editing from the University of Cape Town. I have edited more than 600 Masters and Doctoral theses, as well as articles, books and reports.

As the copy editor,

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APPENDIX K: TRANSLATED INTERVIEW TRANSCRIPTS

The following section presents the translated versions of the semi-structured interviews conducted with NC(V) Level 4 Mathematics students after each intervention cycle. Although the interviews were conducted in a mix of English and informal spoken expressions, the responses have been translated and standardised into clear academic English for coherence, readability, and alignment with UNISA requirements.

D1. Translated Interview Extracts – Cycle 1 (Traditional Teaching)

Question:

"What difficulties did you experience when answering the test questions?"

S4 – Translated Response:
I struggled to remember the properties of parallel lines and triangles. This made it difficult for me to select the correct answers.

S2 – Translated Response:
I found it difficult to recall the characteristics of triangles and parallel lines needed to solve the problem.

S27 – Translated Response:
I was unable to remember the information about triangles and parallel lines, so I could not answer the questions.

Question:

"Tell me more about your experience writing the test."

S4 – Translated Response:
I could not recall the properties of parallel lines and triangles, which made it difficult to answer correctly.

S11 – Translated Response:
Some parts of the geometry were difficult for me to understand, so the test felt complicated.

S17 – Translated Response:
I did not understand some questions because I had not done this type of geometry in a long time.

D2. Translated Interview Extracts – Cycle 2 (Practical Activities)

Question:

"Describe your experience using tools such as a ruler, protractor and compass."

S5 – Translated Response:
I tried, but my measurements were wrong because I struggled with accuracy.

S9 – Translated Response:
Drawing the circles and lines was manageable, but I did not know how to summarise my findings.

S20 – Translated Response:
At first, I felt nervous because I made mistakes, but as I continued practising, I started to enjoy the activities.

S12 – Translated Response:
I felt comfortable even though I made a few mistakes while measuring angles.

S13 – Translated Response:
I was comfortable using the instruments because we also use them in engineering.

S8 – Translated Response:
I enjoyed using the protractor and ruler. I found it challenging at first, but after practising several times, I improved.

S15 – Translated Response:
I was nervous at first, but after some time, I felt more confident working with the instruments.

S19 – Translated Response:
I was unsure about using the tools at first, but classroom practice increased my confidence. Using the tools helped me visualise angles and shapes better.

D3. Translated Interview Extracts – Cycle 3 (GeoGebra Intervention)

Question:

“How did GeoGebra help you with the test?”

S7 – Translated Response:
GeoGebra helped me see the problems more clearly, especially those involving angles and circles. I still made mistakes because it was my first time using the software, but I believe more practice will help.

S12 – Translated Response:
As I worked with GeoGebra, it became easier to see how the shapes and angles were related. My test score improved compared to the previous cycles.

S18 – Translated Response:
GeoGebra helped me test different ideas and create accurate diagrams. Practising with it helped me understand tangents and radii better and made the test easier.

Question:

“What differences do you notice between traditional teaching and using GeoGebra?”

S21 – Translated Response:
GeoGebra makes learning more interesting. It is easier to see relationships such as parallel lines without using physical instruments.

S25 – Translated Response:

Technology is more accurate than drawing by hand. Even if the shape changes, GeoGebra shows whether the values stay the same or change.

S29 – Translated Response:

Using laptops was enjoyable and made the activities easier to follow.

S24 – Translated Response:

It was easy to work with GeoGebra, and I enjoyed the lessons.

S7 – Translated Response:

Using GeoGebra allowed us to rotate the circle and still see that the same theorem applies in different positions. This helped me understand the theorem better.

S13 – Translated Response:

Traditional teaching does not always show different examples, but GeoGebra allows you to view many examples and test them practically.

Question:

"Has GeoGebra reduced your challenges in learning circle geometry?"

S23 – Translated Response:

Yes, GeoGebra helped me understand the theorems more clearly. It made it easier to prove results and improved my reasoning and vocabulary in geometry.