MATHEMATICS TEACHERS' AND LEARNERS' EXPERIENCES IN THE TEACHING AND LEARNING OF GRADE 11 EUCLIDEAN GEOMETRY: CASE OF MAN'OMBE CIRCUIT, LIMPOPO PROVINCE, SOUTH AFRICA

by

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DEDICATION

This academic thesis is dedicated to my family for their unwavering support and encouragement throughout my academic journey. Special thanks to my siblings, my husband Tinyiko, and my children Ndzalama, Nsuku, and Nhlahla, whose love and motivation have been instrumental in my achievements.

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ABSTRACT

This study explored the instructional practices and classroom experiences of mathematics teachers in teaching Grade 11 Euclidean geometry. The research adopted a qualitative approach and a case study design, employing purposive sampling to select four Grade 11 mathematics teachers from four different secondary schools. Data was collected through lesson observations, document analysis, and semi-structured interviews developed by the researcher. The findings revealed that teachers employed a range of strategies, including scaffolded verbal instruction, the use of visual aids, peer collaboration, and structured problem-solving. However, they encountered significant challenges, such as learners' weak foundational geometry skills, difficulties with geometric reasoning, low participation, and language-related barriers. The combination of scaffolded instruction and visual tools was identified as having strong potential to enhance learners' comprehension of geometric concepts. The study recommends targeted professional development for teachers, the integration of ICT and concrete teaching aids, and a stronger focus on foundational geometry in earlier grades to support more effective teaching and improve learner engagement in Euclidean geometry.

Keywords: Euclidean geometry, Grade 11 mathematics, Teacher practices, Teacher experiences, Geometry teaching challenges, Teaching strategies, Teaching techniques, Mathematics teachers, Scaffolding verbal instruction, Theorems.

ACRONYMS AND ABBREVIATIONS

Acronym	Full description
CAPS	Curriculum and Assessment Policy Statement
BDCT	Breaking Down Complex Theories
DKPG	Deficits in Key Procedural Gaps
DoBE	Department of Basic Education
DoE	Department of Education
DPKG	Deficit in Prior Knowledge from early Grades
EIAPT	Errors in Applying the Pythagoras Theorem
FET	Further Education and Training
GWPS	Group Work for Problem Solving
ICT	Information and Communication Technology
LoM	Lack of Motivation
LoLT	Language of Learning and Teaching
MBBC	Misunderstanding of Basic Geometry Concepts
NBAGBD	Negative Beliefs About Geometry Being Difficult
NCS	National Curriculum Statement
PL	Peer Learning
QTST	Question Techniques to Stimulate Thinking
SAS	Side Angle Side
T	Teaching
UCD	Use of Chalkboard Diagram
ZPD	Zone for Proximal Development

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CHAPTER 1: INTRODUCTION AND CONCEPTUALISATION OF THE STUDY

1.1 INTRODUCTION

This study investigated the practices and experiences of mathematics teachers in teaching Grade 11 Euclidean geometry within the South African curriculum context. Geometry, particularly in its Euclidean form, is a fundamental strand of mathematics that develops learners' spatial reasoning, logical thinking, and problem-solving skills (Taylor, 2023) . Yet, despite its recognised importance, it remains one of the most challenging areas for both teachers and learners, often reflected in persistent underperformance at secondary school level (Nilam, 2023).

This study was situated in the Man'ombe Circuit of the Mopani East District, Limpopo Province, South Africa, where Grade 11 learners' results in Euclidean geometry have consistently fallen below their overall mathematics performance. These outcomes raised critical questions about how teachers interpret the curriculum, select instructional strategies, and navigate the difficulties that arise in the geometry classroom. To address these concerns, this study adopted a qualitative case study approach, focusing on the classroom practices, professional knowledge, and lived experiences of four mathematics teachers.

1.2 BACKGROUND TO STUDY

Geometry, one of the oldest branches of mathematics, focuses on shapes, spatial relationships, and the properties of space (Maqoqa, 2024). At matric level, learners engage with both Euclidean and Analytical geometry, each requiring different approaches: Euclidean geometry relies on axiomatic reasoning, whereas analytical geometry makes use of coordinate systems (Johnson, 2021b).

Euclidean geometry, in particular, develops learners' deductive reasoning through proofs, which require them to move from observation to abstraction (Russell, 2025). However, both international and local studies show that learners and teachers often struggle with this strand of mathematics (Mlambo & Sotsaka, 2025; Duc, Nam & Van Trung, 2024). These difficulties create a need to investigate how teachers approach the teaching of geometry, particularly in contexts where performance remains low.

Euclidean geometry is founded on the five axioms proposed by Euclid, which underpin formal reasoning and proof (Rossella, 2021). These include principles of congruence, similarity, and circle geometry, which are core to the secondary school curriculum. The current mathematics syllabus in South Africa expects learners to master these principles, research indicates that teachers and learners alike experience difficulties in applying them, especially when proving theorems or solving geometric 'riders' (Meyer, 2022). This study examined how teachers engage with these complexities in the Grade 11 classroom.

Since 2012, the Curriculum and Assessment Policy Statement (CAPS), has required learners to reach Level 4 of the van Hieles' model of geometric thought, which focuses on formal deduction. This expectation assumes mastery of earlier levels of visualisation and analysis. Learners must also study and prove a set of prescribed theorems, including circle theorems and properties of triangles (DBE, 2015). In practice, however, these goals are often undermined by an exam-driven curriculum, low teacher confidence, and a lack of resources (Zhu & Liu, 2020). These factors limit learners' opportunities to build both procedural fluency and conceptual understanding in geometry.

Teachers' experiences in teaching Grade 11 Euclidean geometry are shaped by multiple factors, including learners' conceptual difficulties, variations in teacher training, and access to teaching resources. Zulu and Brijlall (2024) note that teachers often struggle to choose effective methods and to balance curriculum demands with learners' needs. Learner disengagement, low motivation, and language barriers further complicate classroom practice (Machisi, 2021a). Machisi (2021a) categorises teachers' professional experiences into three dimensions:

- Technical (content knowledge and instructional methods),
- Practical (day-to-day classroom performance and learner outcomes), and
- Critical (reflective thinking on effectiveness and learner needs).

These dimensions are particularly relevant to geometry teaching, where CAPS expectations may not translate easily into accessible lessons. Teachers must therefore adapt strategies to sustain learner engagement, often drawing on tools such as geo-strips or chalkboard diagrams.

Research also highlights the role of social and emotional learning in creating a supportive classroom environment. As Rio (2020) argues, integrating such practices can improve learner motivation and resilience, thereby contributing to improved outcomes in challenging subjects such as geometry.

1.3 PROBLEM STATEMENT

Through focusing on Grade 11 mathematics teachers in the Man'ombe Circuit, this study addresses a specific gap in understanding how geometry is taught in under-resourced contexts. It seeks to capture teachers' perspectives on curriculum implementation, instructional challenges, and the strategies they employ to support learners. This focus is particularly important as it provides practical, classroom-based evidence that can inform professional development, policy decisions, and curriculum design.

Geometry plays a vital role in mathematics education, fostering learners' logical reasoning, critical thinking, and visual-spatial skills. It also serves as a bridge across mathematical domains, linking arithmetic, algebra, and statistics through its strong visual and deductive elements. Despite this centrality, Euclidean geometry remains a particularly difficult area for many learners, both nationally and internationally.

In South Africa, the challenge is compounded by systemic and historical factors. For many practising mathematics teachers, Euclidean geometry was either excluded from their initial teacher education programmes or offered only as an optional component (Van Putten, Howie and Stols, 2010). As a result, several teachers currently responsible for teaching Grade 11 geometry may have engaged with the subject only superficially during their own schooling. This limited preparation has implications for their confidence and pedagogical competence.

In 2002, the National Curriculum Statement (NCS) was introduced, but continued to face many challenges from 2005. To address these, CAPS replaced the NCS in 2012, aiming to be more structural and context-sensitive, and explicitly emphasising reasoning and problem-solving (DoE, 2011). In Man'ombe circuit, Mopani district of the Limpopo province, Grade 11 learners have persistently underperformed in geometry. Between 2022 and 2024, overall mathematics scores ranges from 43-60% whereas geometry scores remained low (6.1 – 20.5%).

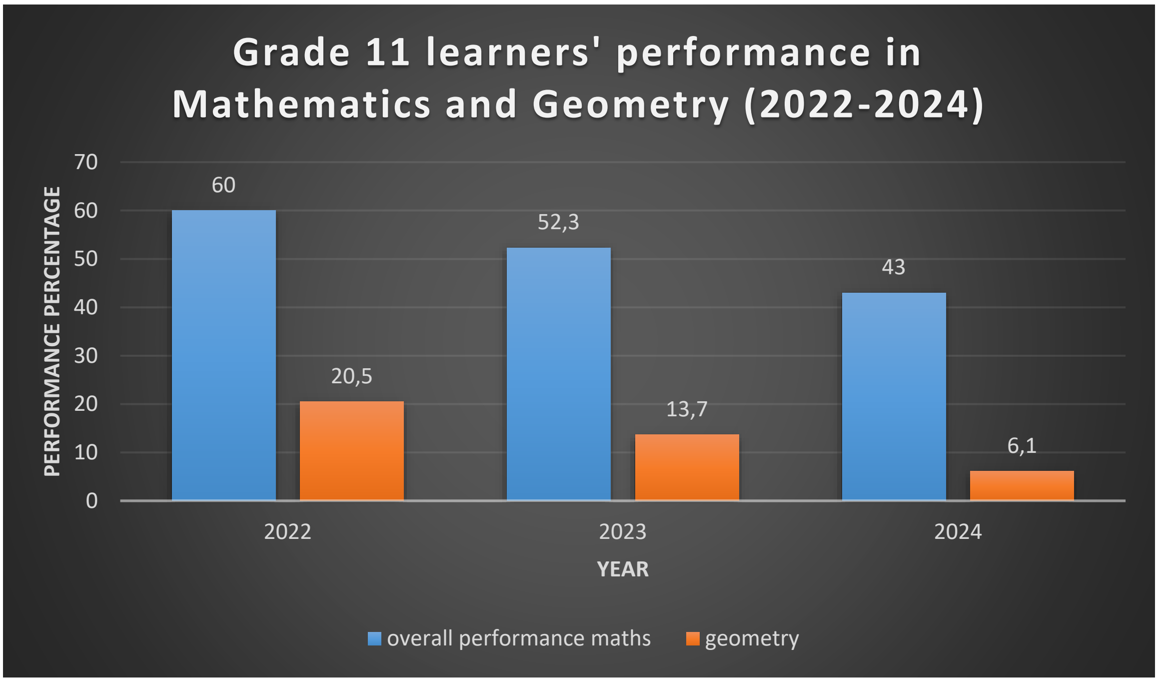


Figure 1.1: Overall Mathematics and Geometry Performance for Grade 11 learners for selected schools in Man'ombe district (DBE, 2024)

This disparity suggests that while general mathematics achievement is moderate, geometry poses an instructional challenge. Ntshangase (2023) confirms that curriculum changes have generated confusion among mathematics teachers teaching geometry, while Mahlangu (2021) stresses the urgent need for targeted teacher development following such reforms. These findings align with my own classroom experiences teaching Grade 11 revealed learner deficits in geometry, prompting support requests from the School-Based Support Team (SBST) due to insufficient teacher background instruction. There is a pressing need to understand how teachers practise and experience the teaching of Euclidean geometry. Investigating their instructional strategies, professional challenges, and classroom realities will provide insight into the factors shaping learner outcomes. The study investigated the practices and experiences of mathematics teachers in the teaching and the learners' learning experiences in grade 11 Euclidean geometry.

1.4 AIM OF THE STUDY

The aim of this study was to investigate the practices and experiences of mathematics teachers and the learners' learning experiences in grade 11 Euclidean geometry.

1.5 OBJECTIVES OF THE STUDY

To achieve the aim of the study, the following research objectives were pursued:

- To explore the teaching techniques used by grade 11 mathematics teachers in teaching Euclidean geometry.
- To identify the challenges faced by Grade 11 mathematics teachers and learners in the teaching and learning Euclidean geometry.
- To suggest possible ways in which the teaching and learning of Euclidean geometry can be improved.

1.6 MAIN RESEARCH QUESTION

The main research question is: What are the mathematics teachers' and learners' experiences in the teaching and learning of Grade 11 Euclidean geometry?

1.7 SUB-RESEARCH QUESTIONS

To address the research objectives the following corresponding sub-questions were asked:

- What techniques do Grade 11 mathematics teachers employ in teaching Euclidean geometry?
- What challenges do Grade 11 mathematics teachers and learners encounter in the teaching and learning of Euclidean geometry?
- How can the teaching and learning of Grade 11 Euclidean geometry be improved?

1.8 SIGNIFICANCE OF THE STUDY

This study contributes to both scholarly and practical debates on mathematics education. At the scholarly level, it adds to the growing body of research on Euclidean geometry by foregrounding teachers' lived experiences and instructional practices, thereby offering a nuanced understanding of the challenges and possibilities within this domain.

At the practical level, the findings are relevant to curriculum developers, policymakers, and teacher educators. They may inform the design of professional development programmes that strengthen teachers' content and pedagogical knowledge of geometry, as well as the provision of resources that support effective classroom practice. For the Department of Basic Education, the study offers evidence that can guide interventions to close the gap between curriculum expectations and classroom realities. For practising teachers, the study provides alternative strategies and insights that may enhance their own approaches to teaching geometry.

Ultimately, the study has the potential to improve learner engagement and achievement in Euclidean geometry, thereby contributing to the broader goals of CAPS and to the cultivation of mathematical reasoning skills essential for academic and professional success. Such an exploration is necessary to inform professional development initiatives, curriculum support, and pedagogical approaches that can enhance the teaching and learning of Euclidean geometry at secondary school level.

1.9 DELIMITATIONS OF THE STUDY

The study was guided by the stated objectives. The study was conducted in four selected secondary schools within the Man'ombe Circuit. From these four schools, four Grade 11 mathematics teachers were purposively selected based on their experience in teaching Euclidean geometry. The learners involved were those who the given teacher was teaching at the time of observation by the researcher.

1.10 CHAPTER ORGANISATION

The study is structured in five chapters as outlined below:

Chapter 1 provides an overview of the study and outlines the introduction, background of the research topic, problem purpose statement, purpose of the study, research questions, a preliminary literature review, and definitions of the concepts.

Chapter 2 presents the theoretical framework and a review of relevant literature. This includes recent articles, scholarly journals, key books, and dissertations published.

Chapter 3 describes the research paradigm, research design, approach, the population, sampling techniques, data collection instruments, data analysis, trustworthiness and credibility, and ethical considerations.

Chapter 4 presents and analyses data. It outlines the findings derived from classroom observations, interviews, and document analysis, and interprets these findings in relation to the literature and the theoretical framework underpinning the study.

Chapter 5 concludes the study by summarising the main findings, offering conclusions, and providing recommendations based on the study's results. It also highlights implications for teaching practice and future research.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

The previous chapter introduced the study and provided its background. This chapter offers theoretical framework and the review of the literature relevant to the teaching and learning of Euclidean geometry.

2.2 THEORETICAL FRAMEWORK

This study was guided by van Hiele's and constructivist theories which provide valuable insights into how teachers and learners experience geometry instruction.

2.2.1 Van Hieles' Theory of Geometric Thought

van Hieles' theory of geometric thought, is a framework that explains how learners progress in their understanding of geometry. The theory identifies five hierarchical levels of reasoning: visualisation, analysis, informal deduction, formal deduction, and rigour. Each level represents a qualitative shift in the way learners understand and reason about geometric concepts. A key principle of the theory is that progression through the levels is sequential: learners cannot successfully operate at a higher level without first mastering the preceding one. For example, learners who have not developed strong skills in visual recognition and analysis are unlikely to succeed in constructing formal proofs. This principle has important implications for classroom practice, as it suggests that teaching strategies must be carefully aligned to learners' current developmental stage.

van Hieles proposed instructional phases: information, guided orientation, explication, free orientation, and integration, that support learners' movement from one level to the next. These phases encourage teachers to provide structured, hands-on, and reflective learning experiences that enable learners to build their own understanding of geometric principles.

The study acknowledges that learners' difficulties in geometry may not simply be a matter of ability but may reflect a mismatch between teaching approaches and learners' cognitive readiness. Analysing teachers' practices through this lens makes it possible to identify where such misalignments occur and how they affect classroom experiences. van Hieles' theory is therefore central to this study because it provides both a diagnostic tool for understanding learner difficulties and a guide for examining teachers' instructional strategies.

Learning theories aim to explain how individuals acquire knowledge, offering a structured framework and precise vocabulary to interpret observed learning behaviours (Bandura, 2021). These theories do not prescribe fixed solutions but guide teachers' attention to pertinent variables in addressing educational challenges. In mathematics education, for example, the Van Hiele model outlines a developmental sequence of geometric reasoning, illustrating how learners move through hierarchical levels of understanding (Machisi, 2020). Constructivist theory, by contrast, emphasises that learners build new knowledge upon existing ideas through active engagement, scaffolding, and reflective thinking (Bandura 2024; Uygun & Güner, 2021).

In the context of Grade 11 Euclidean geometry, both van Hiele's and constructivist perspectives provide valuable insights into how teachers and learners experience geometry instruction. Machisi's (2020) study in South Africa shows that van Hiele-based teaching significantly enhanced learners' skills in constructing geometric proofs, with experimental groups outperforming control groups in achievement and attitudes. Similarly, Uygun and Güner (2021) argue that pre-service teachers with higher van Hiele levels were more likely to apply constructivist instructional practices. Thus, a blended approach using both van Hiele's hierarchical thinking model and constructivist methods provides a robust conceptual framework for understanding teachers' practices and experiences in teaching Grade 11 Euclidean geometry. The integration of relevant educational theories supports a deeper comprehension of learner development, teaching practices, and instructional challenges. According to Aubrey and Riley (2024), educational theories not only enhance teachers' understanding of learning processes and communication but also shape ethical conduct and professional identity. These theories help teachers to interpret learner behaviour and guide classroom decision-making. Zhao (2021) further emphasises that learning involves emotional, cognitive, and environmental factors that shape the acquisition of skills, values, knowledge, and world views.

Recent empirical research confirms that learners' proficiency in geometry is closely tied to their progression through these levels. Yalley, Armah and Ansah (2021) say that significant advancement in learners' understanding from precognition to abstraction after applying van Hiele-based instructional strategies. Similarly, Ray (2021) argues that a strong positive correlation between learners' van Hiele levels and achievement in secondary geometry classrooms in Nepal.

A study conducted by Maqoqa, (2024) on the assessment of teachers' own van Hiele levels, reveals that 97% of secondary Mathematics teachers operated at Levels 0-2, with none reaching Level 4. This finding suggests a profound need for enhanced professional development in geometric reasoning and content knowledge. In agreement, Mbusi and Luneta (2021) supports the importance of sequencing instruction through these phases to allow learners to shift from intuitive to formal reasoning. Similarly, Aldiabat and Yew (2024) report that learners who were taught geometry through these structured phases demonstrated greater conceptual understanding and deductive reasoning.

The van Hiele model thus provides teachers not only with a descriptive diagnostic tool to assess learners' geometric thought but also a prescriptive instructional guide that can be adapted to improve teaching practice and enhance learner outcomes. The model's continued use and research validation make it a critical framework for both curriculum development and teacher education in geometry.

The process of learning and teaching geometry in secondary schools can hardly be separated from the van Hiele theory, initially developed by Pierre and Dina van Hiele in the 1950s. This theory posits that learners progress through a sequence of levels or stages of geometric reasoning as they develop their understanding of the subject. According to recent studies, learners cannot transition from level 0 directly to level 2 or 3 without first mastering level 1 (Zimmerman, Greenberg and Weinstein, 2023). The advancement between levels, particularly from level 1 to level 2, is largely influenced by the nature and quality of instruction rather than the age of the learner (Taber, 2025). This view supports the claim that instruction plays a crucial role in learners' conceptual development in geometry. Ndhlovu (2023) describes this model as foundational in understanding the development of reasoning and thinking processes within geometry education, especially at the secondary level. Accordingly, progression through the van Hiele levels depends more on structured learning experiences than on the learner's age or maturity.

Moreover, the theory can be utilised to assess geometrical thinking in both learners and teachers. If a teacher and learner operate at different van Hiele levels, communication breakdowns may occur, resulting in teaching and learning difficulties (Mtshali & Ngcobo, 2021). One of the critical aspects of the theory is that instruction delivered at a level beyond a learner's cognitive stage may hinder understanding and frustrate both teacher and learner. Hence, teachers must assess and align their teaching to the learners' current level of reasoning.

This alignment is particularly vital in the South African context, where geometry remains a challenging area in mathematics education. As such, the van Hiele theory provides a valuable framework for guiding learners through the developmental stages of geometric understanding (Mahlaba, 2022).

According to Mahlaba (2021), the van Hieles' theory of the development of thinking in geometry has been influential in the arena of geometry teaching and learning. In their theory, van Hiele's proposed that learners need to pass five levels of geometric thinking order. Although these levels are numbered and named differently by different authors, this study adopts the numbering and description provided by van Putten *et al.* (2010:23). As identified by van Putten *et al* (2010:23), the levels start from 0 through to 4. Table 2.1 below presents a summary of geometric thinking as presented by van Hieles' theory. The table is drawn from Alex and Mammen (2014:1911).

Table 2.1 van Hieles' summary of geometric thinking

Levels	Known as	Description: learner will be able to
Level 0	Visualization	recognise and name figures
Level 1	Analysis	describe the attributes of shapes
Level 2	Ordering	classify and generalise by attributes
Level 3	Deduction	develop proofs using axioms and definitions
Level 4	Rigor	work in various geometrical systems

Level 0: Visualisation - at this level, the learner identifies, names, compares, and operates on geometry figures according to their appearance. The stage involves visualisation or recognition of geometric figures by their shape in which the learners use visual perception and non-verbal thinking to compare the figures with their prototypes or everyday things (Vojkuvkova, 2012:72). At this level, learners are expected to recognise, identify, name and compare geometric figures such as rectangle, square and triangle by their shapes, by their physical appearance. They should also be able to categorise them or group shapes or objects that are similar. They may not recognise properties such as reciting geometry theorems, which all learners in primary schools are assumed to operate at this level.

This is the first level that forms the foundation of the subsequent levels. Figure 2.1 below shows an example in which a learner categorises triangles from a pool of shapes.

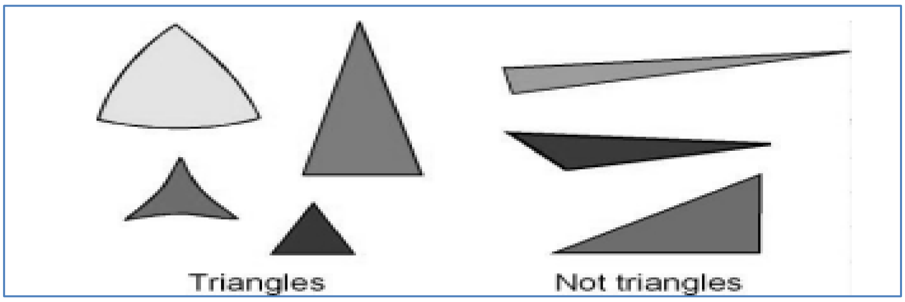


Figure 2.1: Learners at Level zero categorise triangles (Vojkuvkova, 2012:72)

Level 1: Analysis and description - At this point, the learner analyses figures in terms of their components and relationships that exist among components. It also calls for the discovery of a class of shapes empirically (Chen et al, 2023a). The stage involves the analysis and naming of properties of geometric figures. Learners do not see the need for proof of facts that were discovered empirically, but they can measure, fold, and cut paper to form shapes (Vojkuvkova 2012:73). Learners can analyse geometric figures in terms of their properties, namely, all angles of a square are right angles, and a parallelogram has equal opposite angles. Although learners have an idea of the properties, they cannot compare properties between different geometric shapes, e.g., a square and a rectangle both have four right angles.

Level 2: Informal deduction - At this level, the learner logically interrelates previously discovered properties/rules by giving or following informal arguments. At this level, learners perceive relationships between properties and figures. They are also able to create meaningful definitions and justify their reasoning by drawing maps and diagrams. Figure 2.2 presents the idea that learners at Level 2 can draw a logical map of parallelograms. Learners are capable of logically inter-relating or inter-linking various properties through informal arguments. For instance, they might explain that if the opposite sides of a quadrilateral are parallel, then its opposite angles are equal.

Additionally, they can deduce that a rectangle can be viewed as composed of two squares. This ability reflects their understanding of geometric relationships and the foundational principles that govern them. Definitions at this level are meaningful to the learners, and they are expected to follow informal arguments. Learners begin to see proofs and can follow them without necessarily writing and structuring them. By the time learners finish grade 9, they should have achieved this level since formal proofs begin from the grade 10 curriculum.

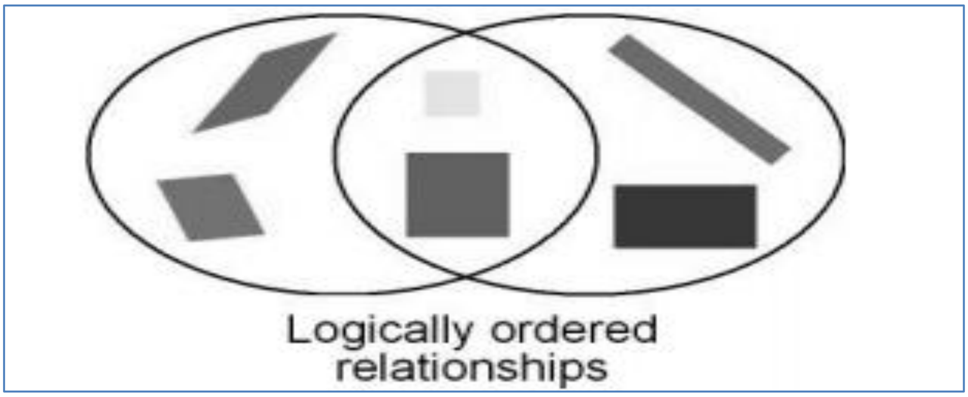


Figure 2.2: Learners at Level 2 ability to draw a logical map of parallelograms (Vojkuvkova, 2012:72)

van Hieles asserts that learners who have not yet achieved this level of geometry are most likely to be unsuccessful in the study of geometry, which involves deduction (Vojkuvkova, 2012:73).

Level 3: Deduction - This level allows the learner to prove theorems deductively and establish interrelationships among networks of theorems (Gula & Jojo, 2024). This implies that the learners would be able to differentiate between necessary and sufficient conditions of properties, the role of definitions, theorems, axioms, and proofs, while at the same time understanding the role of definitions, theorems, axioms, and proofs.

Learners can construct and prove theorems deductively. It is the beginning stage of formal deductions, and learners at this level understand the importance of deductions and the roles of theorems, postulates, and proofs. They can also structure and write proofs with understanding. By the time they finish grade 12, they should have attained this level. This level is appropriate for a typical geometry class at a high school, especially grades 10, 11, and 12.

Level 4: Rigor - the learner establishes theorems in different postulation systems and analyses and compares these systems. At this level, learners can describe the effect of adding or removing an axiom on a given geometric system. This is the final stage in which the learners are expected to have robust geometrical reasoning. Even in the absence of examples, they would be able to compare geometric results in different axiomatic systems. Learners see geometry in the abstract. They are expected to analyse and compare theorems at this level. Van Hieles acknowledged their interest in the first three levels since most of the high school geometry is taught at level 3, and it is very rare for this level to appear in secondary schools.

These levels are known to be sequential and were originally labelled from level 0 to 4 but were later changed to start from level 1 to 5 to make provision for a new level called 'pre-recognition level', which is called level 0 (Tachie, 2020). This model provides teachers with clear guidance on how to assist and support learners in progressing from one level to the next. It emphasises the design of geometry activities that are tailored to the specific developmental stages of the learners (Clement et al, 2022). The development of procedural geometrical fluency starts from level 0 to level 3, whilst from level 4 to 5 the development of conceptual understanding is displayed (Luneta, 2021). Furthermore, Luneta and Makonye (2020) add that each level must be fully developed by the learner before moving to the next level.

It is crucial to understand that, according to van Hiele (1999:310), one significant reason learners face difficulties with geometry is the mismatch between the complexity of the material presented and their current cognitive development level. When geometry concepts are taught at a level that is either too simplistic or too advanced, it can hinder learners' understanding and mastery of the subject. This implies that learners would not understand work presented by the teacher at a higher level than the level they are at, and at the same time, the teacher will not understand why the learners cannot grasp the concepts. The view is that learners tend to struggle more with mathematical reasoning and problem-solving. Consensus in the existing literature points out that low achievement in mathematics typically arises at a later stage in the learners' academic careers. The findings by Clement et al (2023) support that during secondary school, rather than in their foundational years. In a recent meta-analysis, Zhan et al (2024) found that differences in mathematical problem-solving abilities become more pronounced in high school and tertiary education rather than in primary or middle school. Van Hiele's theory must be blended with other contemporary theories to provide a more comprehensive theoretical lens through which teaching and learning can be understood. The following paragraphs present some of the key theories of teaching and learning.

One of the theories of blending van Hieles' geometry theory with other theories is to look at the idea that many learners' performance in geometry is affected by their attitude towards the subject (Khurshid, Ali & Zahra, 2021:2). From van Hieles' (2020:310), it was found that learners pass through several stages; from visualisation to rigor. Based on this theory, as discussed above, it emerged that learners who do not successfully comprehend the first level would struggle with the next level. In this case, it becomes apparent that learners who performed badly in the elementary levels are likely to struggle in the higher levels. This directly concurs with the postulation by Musa et al. (2022:191), who noted that self-efficacy in any task is built by past performances in mathematics and geometry.

The van Hiele theory holds significant implications for teaching Euclidean geometry, particularly in the Further Education and Training (FET) phase. If this theory is accepted as valid, then Grade 11 learners should at minimum attain Level 2 (Informal Deduction) to be adequately prepared for Level 3 (Deduction), where formal proofs and logical reasoning become central to the curriculum (Gaven, 2015).

However, the reality in many South African classrooms reflects a different picture. As observed by Machisi and Feza (2021), geometry teaching often follows a predictable cycle: reviewing homework, teacher-led instruction, and student practice. While this structure offers consistency, it rarely fosters the kind of deductive reasoning that van Hiele's higher levels demand. Consequently, many learners leave high school without mastering the skills necessary to construct geometric proofs independently.

The theory emphasises the importance of aligning instruction with learners' current cognitive levels. According to van Hiele (1999), a mismatch between the complexity of content and learners' developmental readiness can impede understanding. Teaching at a level that exceeds learners' current geometric thinking may lead to confusion and rote learning, while failing to challenge learners at higher levels may cause stagnation and disengagement.

Machisi and Feza (2021) argue that teachers need to diagnose and understand the level at which each learner operates to adapt their instruction appropriately. The five phases of instruction proposed by the van Hieles, Information Guided Orientation, Explicitation, Free Orientation, and Integration, offer a structured sequence that helps learners gradually transition from informal to formal geometric reasoning. For example, in the guided orientation phase, teachers should allow learners to explore and investigate geometric relationships, fostering intuitive understanding before formal proofs are introduced.

These phases also support a constructivist approach to teaching geometry, whereby learners discover properties, formulate conjectures, and justify their thinking. Alex and Mammen

(2018) highlight that learning becomes more effective when learners reconstruct geometric theorems through hands-on activities and dialogic teaching, rather than merely memorising definitions and theorems from textbooks.

In addition, Yi, Flores and Wang (2020) stress the importance of assessing learners' geometric readiness before instruction begins. If foundational concepts are lacking, teachers should first provide remediation to ensure learners are adequately prepared for formal deductive reasoning. Ignoring these developmental stages can result in superficial learning, where learners mimic procedures without truly understanding underlying principles.

The van Hiele model, therefore, offers more than just a theory of geometric understanding; it provides actionable guidance for teaching practices. With aligning instruction with learners' cognitive development and leveraging the instructional phases, teachers can support more meaningful engagement with geometric concepts. This alignment is especially critical in contexts like South Africa, where learners may struggle with both conceptual understanding and the language of instruction.

2.2.2 Constructivism Theory

Constructivist learning theory plays a foundational role in understanding how learners acquire mathematical knowledge, particularly in the context of geometry. According to Jumaah (2024), constructivism posits that knowledge is not passively received but actively constructed by learners based on their prior experiences. This view aligns with the argument by Xu and Shi (2023) that individual understanding of reality is shaped by unique experiential contexts.

In a mathematics classroom, constructivism emphasises the learner's agency in constructing meaning from active engagement, exploration, and reflection. Kurniasari (2024) notes that learners build upon their existing cognitive frameworks, and this cumulative process leads to personalised and meaningful learning experiences. Such a perspective is crucial in subjects like Euclidean geometry, where abstract concepts must be internalised through intuitive and experiential processes.

In practice, this theory encourages a shift away from teacher-directed instruction towards learner-centered pedagogies. Instead of merely transmitting knowledge, teachers function as facilitators, guiding learners in exploring, hypothesising, and reasoning through mathematical ideas (Nguyen et al., 2021). This approach allows learners to become critical thinkers and active participants in their own learning.

The constructivist model also promotes a safe and supportive classroom environment where learners feel comfortable sharing their ideas, asking questions, and making mistakes. As Charmaz (2017) and Gunderson et al. (2021) highlight, learners' misconceptions, especially in geometry, often stem from prior mental constructions. Addressing these misconceptions requires teachers to elicit learners' thinking, diagnose underlying errors, and scaffold conceptual change.

Constructivism also fosters metacognition and reflective thinking. Learners are encouraged to articulate their problem-solving processes and justify their reasoning, which in turn supports the development of deeper understanding. For example, during geometry lessons, learners can be asked to explain how they arrived at a solution or how they interpret a theorem in their own words.

Moreover, constructivist learning is enhanced through concrete and familiar contexts before transitioning to abstract reasoning. Lee (2022) and Johnson et al. (2021) observe that learners perform better when they first interact with tangible materials and real-life problems, which serve as anchors for abstract mathematical thinking. This is particularly important in teaching geometry, where visual and spatial reasoning is essential.

Ultimately, constructivist theory supports the pedagogical belief that learning is most effective when learners actively engage with content, build on prior knowledge, and are given opportunities to reflect, explore, and collaborate. These principles offer valuable guidance for teaching Grade 11 Euclidean geometry in a way that promotes deeper understanding and long-term retention.

2.2.3 Social Constructivist Theory

Social constructivism, grounded in the work of Lev Vygotsky, extends the principles of constructivism by emphasising the central role of social interaction and cultural context in the learning process. Unlike individual constructivism, which focuses on internal cognitive construction, social constructivism stresses that learning is inherently a collaborative activity shaped by dialogue, shared experiences, and guided participation (Jumaah, 2024).

At the core of this theory lies Vygotsky's concept of the Zone of Proximal Development (ZPD), the gap between what learners can do independently and what they can achieve with guidance. Effective teaching, therefore, involves identifying this zone and providing scaffolding to support learners as they acquire new skills and knowledge. Through this framework, the teacher's role becomes one of mediation, enabling learners to internalise concepts through interaction and support (Chen et al., 2023b).

In mathematics education, and particularly in the teaching of Euclidean geometry, social constructivism informs the importance of creating a classroom culture where learners engage in discourse, co-construct meanings, and negotiate understanding. As Thamae (2022) asserts, learning in geometry is not simply about mastering facts but about engaging in collaborative reasoning, discussion, and exploration.

Vygotsky's theory underscores that cognitive development is enhanced through meaningful communication, particularly when learners are exposed to diverse perspectives and encouraged to justify their reasoning. Thompson (2020) supports this view by highlighting how collective problem-solving, peer teaching, and structured group activities foster deeper conceptual understanding and improve learners' ability to apply geometric principles.

Furthermore, social constructivism encourages learner autonomy and accountability within a community of practice. Through peer interactions such as comparing strategies, challenging results, and evaluating solutions, learners actively participate in knowledge construction. These interactions help reduce misconceptions, build confidence, and create a sense of ownership over learning (Nguyen et al., 2024).

The teacher's conduct in such an environment is critical. As noted by Xu and Shi (2018), the teacher must foster an atmosphere of emotional and psychological safety where learners feel valued and understood. Listening attentively, encouraging contributions, and validating learners' ideas are key to promoting meaningful engagement.

Importantly, social constructivist classrooms are inclusive, culturally responsive, and sensitive to learners' backgrounds. Gunderson et al. (2021) warn against biased perceptions such as those related to gender or linguistic proficiency, which may limit learner participation. In creating equitable opportunities for all learners to engage, teachers support not only academic achievement but also the development of critical thinking and collaborative competencies.

In brief, social constructivist theory provides a robust framework for teaching Grade 11 Euclidean geometry. It highlights the importance of scaffolding, peer learning, dialogic teaching, and cultural responsiveness, all essential strategies for supporting learners' conceptual growth in mathematics.

2.2.4 Summary of Theoretical Framework

This study is grounded in the three interrelated theoretical perspectives: the van Hiele theory of geometric thinking, constructivist learning theory, and social constructivism. Each offers a distinct yet complementary lens through which to examine the teaching and learning of Euclidean geometry in Grade 11.

The van Hiele model provides a cognitive-developmental framework that explains how learners progress through increasingly complex levels of geometric reasoning. It emphasises the necessity of structured, sequential instruction aligned with learners' current levels of understanding. According to the theory, successful learning in geometry depends on a learner's ability to master one level before progressing to the next. This model also identifies instructional phases of Information, Guided Orientation, Explicitation, Free Orientation, and Integration, which serve as practical strategies for advancing learners' geometric thinking.

Constructivist learning theory complements this by asserting that learners construct new knowledge based on their prior experiences and conceptual frameworks. It emphasises active engagement, reflection, and scaffolding as essential components of effective instruction. In the context of Euclidean geometry, constructivism encourages learners to formulate and test conjectures, to understand relationships between shapes, and to derive meaning through hands-on exploration.

Social constructivism, rooted in Vygotsky's (1978) sociocultural theory, further enriches the framework by highlighting the role of dialogue, collaboration, and social context in learning. It posits that meaningful knowledge construction often occurs through interaction with more knowledgeable peers or teachers, within the learner's zone of proximal development (ZPD). This perspective supports instructional strategies such as peer discussion, cooperative learning, and teacher-guided inquiry, all of which are crucial in helping learners internalise complex geometric concepts.

Through integrating these three theories, this study adopts a comprehensive theoretical framework for analysing how Grade 11 mathematics teachers practice and experience geometry instruction. The van Hiele theory offers insight into learners' developmental stages, constructivism underscores the importance of active engagement, and social constructivism draws attention to the social dynamics that shape learning. Collectively, these perspectives guide both the interpretation of findings and the formulation of pedagogical recommendations aimed at enhancing the teaching and learning of Euclidean geometry.

2.3 DEFINITION OF CONCEPTS

This section introduces the key concepts that form the foundation of the study: Euclidean geometry, teachers' practices, teachers' experiences, teachers' strategies, and teaching challenges. Each concept plays an important role in understanding the difficulties and complexities involved in teaching Euclidean geometry in secondary school mathematics. Drawing on relevant literature, the section explains the meaning of these terms and shows how they relate to one another. This helps establish the theoretical and methodological direction of the study.

2.3.1 Euclidean geometry

Euclidean geometry, named after the Greek mathematician Euclid, is based on his work Elements, which presents geometry through axioms and logical reasoning (Laos, 2024). In the school curriculum, Euclidean geometry focuses on the study of plane and solid figures such as lines, angles, triangles, circles, and polygons, guided by Euclid's postulates (Maqoqa, 2024). This branch of mathematics helps learners develop spatial reasoning and deductive proof skills. In this study, Euclidean geometry is viewed both as subject content and as a structured field that requires clear instructional approaches. Because of its abstract and formal nature, it often poses teaching challenges, making it a key area for exploring teachers' practices and experiences.

2.3.2 Teachers' Practices

Teachers' practices refer to the actions, routines, and teaching methods used in the classroom (Johnson, 2021a). In this study, these include how teachers represent ideas, organise content, encourage geometric reasoning, and support learners in constructing proofs. Successful practice in Euclidean geometry requires strong subject knowledge and pedagogical content knowledge (Class, 2024). In studying these practices, the research investigates how teachers put the curriculum into action and how their teaching choices affect learners' understanding.

2.3.3 Teachers' experiences

Teachers' experiences include the professional knowledge and insights they gain through training, classroom practice, and exposure to curriculum changes (Johnson, 2021a). These experiences influence how teachers view learners' difficulties, adapt their teaching methods, and respond to the demands of Euclidean geometry. In this study, understanding teachers' experiences provides valuable context for how teaching decisions are made and how these decisions shape classroom practice.

2.3.4 Teachers' strategies

Teachers' strategies are the planned and flexible approaches they use to achieve learning goals, especially when dealing with diverse learners and curriculum expectations (Enemuoh, 2022). In Euclidean geometry, strategies may include scaffolding, structuring problems, using diagrams or models, and promoting deductive reasoning. Effective strategies show how teachers apply theoretical knowledge in practical ways, making abstract concepts more accessible to learners. In this study, strategies are used as a lens to examine how teachers adapt and respond to classroom needs.

2.3.5 Teaching challenges

Teaching challenges, in the context of Grade 11 mathematics and Euclidean geometry, refer to the difficulties that limit effective teaching. These challenges arise from a mix of pedagogical, cognitive, contextual, and systemic factors. Maqoqa (2024) highlights that many challenges come from gaps in teachers' own understanding of geometry during their schooling or training. These gaps affect teachers' confidence and ability to explain complex ideas such as proofs and spatial reasoning. External factors such as overcrowded classrooms, lack of resources, rigid curricula, and language diversity add further pressure, especially in under-resourced, multilingual schools.

In this study, teaching challenges were seen as multi-layered barriers that extend beyond the skills of individual teachers and include wider institutional and systemic issues. These challenges restrict opportunities for learner-centred teaching and limit learners' access to higher-level mathematical reasoning. Recognising these difficulties is important for designing professional development programmes and curriculum support that directly address teachers' needs in teaching Euclidean geometry.

2.3.6 Learning challenges

Machisi (2021) studied the learning experiences of 16 Grade 11 learners as regards Euclidean geometry. The learners were drawn from four high schools in South Africa. Data came from focus group discussions with learners. Learner data records were also accessed to obtain data. The study involved teaching learners using a Van Hiele theory-based approach. This method recorded and learners recorded positive or affirmative learning experiences in Euclidean geometry. The other learners were taught by conventional methods and they reported learning experiences that were negative. The study made a conclusion to the effect that the Van Hiele theory-based approach met the needs of learners' needs more appropriately compared to conventional approaches as used in learning Euclidean geometry. The study recommended preference in the use of the unconventional Van Hiele theory-based instruction when teachers teach Euclidean geometry.

The challenges faced by learners on geometric proofs have been found to emanate primarily from teachers' continued use of approaches that are traditionally teacher-centred (Abdullah & Zakaria, 2013). Teachers do not abandon the methods that were used in teaching them the Euclidean geometry no matter how ineffective these methods are (Keiler, 2018; Siyepu, 2014).

Kambila, Kyabuntu and Mbhiza (2024) aver that Euclidean geometry is one of the topics that are characterised by teaching-learning challenges as evidenced by the poor achievement associated with the topic. Their study used a non-structured classroom observation method on with 6 teachers who were teaching the Euclidean geometry topic. The observation noted how the teachers put across the Euclidean geometry concepts and principles to learners. available for the learners. The study found that the teacher utilised questions-and-answer methods to engage learners but did not provide relevant explanations in the class to guide them on how related the concepts were. This means failure to find linkages between theorems and the diagrammatic formations in Euclidean could compromise the understanding of problem-solving techniques in Euclidean geometry.

Having defined threshold concepts in this study, the next section discusses conceptualisation of Geometry.

2.4 CONCEPTUALISATION OF GEOMETRY

The term *geometry* originates from the Greek word *geometrien*, where *geo* refers to "earth" or "place" and *metria* to "measure" (Johanns, 2023). Historically, geometry developed as a practical science for measuring land and constructing buildings. Its early applications can be traced to ancient Egyptian surveyors, who used basic geometric shapes such as squares, circles, triangles, and rectangles to mark property boundaries and design structures, tasks that required more than simple visual estimation (Rubin, 2023). Later, Greek mathematicians transformed geometry into a logical system for studying shapes, sizes, and spatial relationships, thus advancing it from empirical measurement to a deductive science (Mthembu, 2023; Almeida, Teodoro & Gonçalves, 2021).

According to Patel, Singh, and Dlamini (2022), the study of geometry involves recognising and comparing different shapes and understanding their properties. Alex and Mammen (2020) emphasise that geometry is among the oldest branches of mathematics, concerned with the study of shape, size, position, and the properties of space. In this way, geometry not only reflects deep historical roots but also continues to inform both theoretical mathematics and practical applications in the modern world.

Euclidean geometry is an axiomatic system in which theorems, that is, statements proved through reasoning, are derived from a small set of axioms (Rossella, 2021). Axioms differ from theorems in that they are accepted as self-evident truths without proof, while theorems are established by logical deduction. Euclid formulated five postulates for plane geometry, expressed in terms of constructions:

- A straight line may be drawn from any point to any other point.
- A finite straight line may be extended continuously in a straight line.
- A circle may be drawn with any centre and radius.
- All right angles are equal to one another.
- The parallel postulate: if a straight line crossing two other straight lines makes the interior angles on the same side add to less than two right angles, the two lines, if extended indefinitely, will meet on the side where the angles are less than two right angles.

These axioms provide the basis for geometric proofs, some of which are particularly important in circle geometry. For instance, a line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord, and a tangent to a circle is perpendicular to the radius at the point of contact (Meyer, 2022). Learners are often expected to calculate unknown angles using such principles. For example, given that the angle in a semicircle is a right angle, learners should apply this property to determine unknown angles within the same segment.

Modern geometry can be broadly divided into *synthetic geometry*, such as Euclidean geometry, which develops logically from axioms to theorems without coordinates (Singh & Kapoor, 2021), and *analytic geometry*, which uses algebraic representation within coordinate systems (Ahmed & van der Walt, 2022). Euclidean geometry remains central, as it underpins learners' spatial reasoning, deductive proof skills, and forms the foundation for non-Euclidean systems such as hyperbolic and differential geometry (Youvan, 2024).

Beyond its theoretical significance, geometry plays an important role in applied disciplines such as architecture, engineering, computer science, astronomy, and design. For example, architects apply geometric principles to analyse space and produce blueprints, while engineers rely on them to ensure structural integrity (Smith, Brown & Patel, 2021). In the South African context, the inclusion of geometry in the mathematics curriculum not only supports academic achievement but also contributes to socio-economic development by equipping learners with skills relevant for higher education and diverse career opportunities.

Despite this, geometry is widely regarded as one of the most challenging mathematical domains for both teaching and learning. Research indicates that learners struggle with abstract reasoning and geometric proofs (Tachie, 2020; Dlamini & Mkhwanazi, 2021). To add, Danesi (2025) reports that many Grade 8 and 9 learners had difficulty expressing geometric reasoning using correct mathematical language and logical structure, reflecting a broader challenge in cultivating deductive thinking. Furthermore, the cultural dimension of geometry is often underemphasised, although it offers opportunities for learners to connect with mathematics through visual, aesthetic, and intuitive experiences (Ali & Jamil, 2022; Chen, 2021).

Geometry also makes an important contribution to learners' holistic development by fostering critical thinking, logical reasoning, visualisation, and communication skills. According to Ryu, Hannah, and Yin (2021), effective geometry teaching inspires curiosity and exploration, while its counterintuitive theorems encourage learners to question assumptions and engage in deeper inquiry. For example, geometry equips learners to interpret maps, recognise spatial properties, and understand everyday shapes and angles. Moreover, linking geometry to cultural artefacts and real-life applications supports learners' social, moral, and cultural development (Chen, 2021; Pallasmaa, 2024). This broader educational role makes geometry an essential component of a balanced mathematics curriculum.

Nevertheless, both South African and international studies report that geometry tends to receive less emphasis than other mathematical topics (Russel, 2024; Tachie, 2021). This imbalance may contribute to weaker learner performance and declining interest in the subject. Addressing this gap calls for stronger pedagogical strategies and curriculum support to raise the profile and effectiveness of geometry teaching across primary and secondary education.

2.5 EUCLIDEAN GEOMETRY IN THE CONTEXT OF CURRICULUM AND ASSESSMENT POLICY STATEMENT

The introduction of the Curriculum and Assessment Policy Statement (CAPS) in South African schools in January 2012 marked a significant shift in the structure and delivery of the mathematics curriculum. Among the most notable changes was the reinstatement of Euclidean geometry, together with its formal proofs, as a compulsory component of the Further Education and Training (FET) phase. This reintroduction reflects a national commitment to strengthening foundational mathematical skills essential for critical reasoning and advanced problem-solving (Department of Basic Education [DBE], 2011c).

In the South African context, Euclidean geometry was removed from the curriculum with the Revised National Curriculum Statement in 2006 but reintroduced through CAPS in 2012 as part of Grade 12 Mathematics Paper 2 (Alex & Mammen, 2018). However, this policy shift did not adequately address the need for retraining teachers, leaving many insufficiently prepared to teach the renewed content (Meylani, 2025).

CAPS requires learners to engage with Euclidean geometry at a high cognitive level, specifically Level 4 of the van Hiele model, which focuses on deduction through formal proofs. The van Hiele theory describes a hierarchical progression of learners' geometric understanding, from visual recognition (Level 0) to formal logical reasoning (Level 4). In line with this framework, learners are expected not only to recognise and describe geometric figures but also to formulate, prove, and apply geometric theorems across different contexts (Machisi, 2020).

Accordingly, CAPS prescribes that Grade 11 mathematics teachers introduce and guide learners in proving and applying the following key theorems:

- A line drawn from the centre of a circle perpendicular to a chord bisects the chord.
- The perpendicular bisector of a chord passes through the centre of the circle.
- The angle subtended by an arc at the centre of a circle is twice the angle subtended by the same arc at the circumference (on the same side of the chord as the centre).
- Angles subtended by a chord at the circumference, on the same side of the chord, are equal.
- The opposite angles of a cyclic quadrilateral are supplementary.
- Two tangents drawn to a circle from a single external point are equal in length.
- The tangent-chord theorem: in a circle, the angle between a chord and a tangent at one of its endpoints is equal to the angle in the alternate segment.

Learners are also expected to apply foundational theorems from earlier grades, including the properties of angles on a straight line, angles in triangles, triangle congruency, and the exterior angle of a triangle. These principles are essential for constructing multi-step geometric proofs and solving more complex problems.

Despite this structured curriculum, research shows that the teaching and learning of Euclidean geometry in South African schools remain challenging. Teachers and learners often perceive geometry as a difficult subject, owing to its abstract nature and the cognitive demands of deductive reasoning (Machisi & Feza, 2021). Many teachers report limited confidence in their own understanding of geometric proofs, which in turn affects their ability to teach effectively and support learners' development of proof-based reasoning (Baiduri, Ismail & Sulfiyah, 2020; Maqoqa, 2024).

In this regard, visualisation is increasingly recognised as a valuable pedagogical tool. Strong visual skills enable learners to identify, classify, and explore geometrical properties in dynamic and intuitive ways. The effective use of diagrams, models, and visual reasoning supports the transition from empirical observation to formal deductive logic (Parker, 2023).

The broader aims of CAPS align with South Africa's vision of promoting equitable access to quality education and lifelong learning opportunities. Geometry, therefore, is not only a curricular requirement but also a vital element of mathematical literacy and civic empowerment. Nevertheless, systemic challenges continue to hinder its effective implementation. These include teacher-related issues, such as insufficient content knowledge and low motivation, as well as external factors, such as inadequate resources, large class sizes, and language barriers (Perienen, 2020; Busch, 2023).

Consequently, strengthening the teaching of Euclidean geometry requires more than curriculum alignment. It also demands systemic support through targeted professional development, improved resource provision, and the creation of supportive teaching environments that allow educators to adapt instruction to diverse learner needs. Such measures are critical if the intended CAPS outcomes: conceptual depth, logical reasoning, and advanced problem-solving, are to be realised in the geometry classroom.

2.6 IMPORTANCE AND APPLICATION OF GEOMETRY

Geometry plays a crucial role in developing both cognitive and mathematical skills. This section outlines how engaging with geometric concepts enhances logical reasoning, spatial awareness, and problem-solving abilities, all of which are central to mathematical competence and informed decision-making in everyday life.

2.6.1 Development of cognitive and mathematical skills

Geometry provides a foundation for the development of essential cognitive and mathematical abilities that underpin learners' academic and intellectual growth. According to Ramirez and Abrahams (2022), studying geometry strengthens skills in visualisation, inferential reasoning, logical argumentation, and deductive proof. Spatial abilities such as perception, visualisation, orientation, and insight enable learners to mentally manipulate geometric figures and comprehend their relationships (Luna, Joubert & Gagné, 2021; Khoza & Msimanga, 2023). These skills facilitate the transition from empirical observation to abstract reasoning, which is critical for mastering formal proofs.

In addition, conjecturing, identifying patterns, and critical thinking expand learners' analytical capabilities, while deductive reasoning allows them to draw valid conclusions from established axioms and theorems (Singh, 2020). Such competencies are consistent with the aims of the South African mathematics curriculum, which prioritises the development of logical and spatial reasoning (DBE, 2022).

2.6.2 Cultural and educational contributions

Geometry also contributes to learners' cultural, social, and educational development. Historically, it has shaped fields such as sculpture, architectural design, and engineering, highlighting the relationship between mathematical precision and aesthetic form (Pallasmaa, 2024). Through the study of geometry, learners develop curiosity and creativity by exploring patterns, solving open-ended problems, and engaging in structured argumentation (Ali & Jamil, 2022).

The educational value of geometry extends beyond cognitive development. Chen (2021) argues that geometry supports spiritual, moral, social, and cultural growth by encouraging deep thinking, self-expression, and appreciation of structured reasoning. Furthermore, when learners articulate geometric ideas and construct proofs, they enhance their communication skills and strengthen their intellectual maturity (Ryu, Hannah & Yin, 2021).

2.6.3 Practical and societal applications

Geometry has extensive applications across disciplines, including architecture, physics, astronomy, geology, mechanical drawing, and modern computing (Ahmed & van der Walt, 2022). In practical terms, geometric principles are essential for drafting architectural plans, modelling physical structures, analysing visual data, and optimising design processes.

From a broader perspective, geometry education supports national development by preparing learners for careers in science, technology, engineering, and mathematics (STEM). It equips them with skills relevant to technological innovation and industrial advancement (Busch, 2023). Within the South African context, strengthening geometry education contributes to equity by opening access to critical career pathways, particularly in fields traditionally dominated by men (Smith, Brown & Patel, 2021).

2.6.4 Curriculum relevance and policy goals

Within the Curriculum and Assessment Policy Statement (CAPS), Euclidean geometry is not only a central element of the mathematics curriculum but also a vehicle for developing advanced deductive reasoning skills. Since its reinstatement in the FET phase in 2012, geometry has carried a strong emphasis on formal proofs and structured reasoning, reflecting the Department of Education's aim of advancing high-level mathematical competence (DoE, 2011c).

CAPS requires Grade 11 learners to investigate, prove, and apply key geometric theorems in problem-solving contexts. These expectations are consistent with van Hiele's theory, which conceptualises learners' progression in geometric reasoning from visual recognition (Level 0) to formal deduction (Level 4). Geometry, therefore, serves both as subject content and as a methodological tool for deepening mathematical understanding and meeting curriculum standards (Machisi, 2020).

2.7 TEACHER'S PRACTICES AND EXPERIENCES IN TEACHING GRADE 11 GEOMETRY

Teachers' practices and experiences in the teaching of Euclidean geometry at the Grade 11 level are shaped by multiple pedagogical, contextual, and knowledge-based factors. Research has shown that many teachers face ongoing challenges in selecting effective teaching strategies, deciding on content coverage, and utilising appropriate resources to engage learners meaningfully in this domain (Brown, 2021). As Boaler (2022) notes, to support learners in developing mathematical reasoning and problem-solving skills, teachers must provide consistent, constructive feedback and integrate assessment tools such as mock tests to monitor and support progress.

2.7.1 Instructional practices and pedagogical approaches

Instructional practices in teaching geometry involve the use of representations, logical reasoning, and guided problem-solving. Teachers are not merely conveyors of content but are also expected to facilitate learning through dynamic and context-sensitive strategies. Smith and Brown (2021) categorise teaching experiences into three types: technical (application of methods and content), practical (adaptation to classroom dynamics), and critical (reflection on equity and learner needs). In under-resourced public schools, where access to teaching materials is often limited, teachers must serve as both instructional leaders and adaptable agents of curriculum delivery (Nguyen et al., 2023).

The translation of subject knowledge into effective pedagogy is critical. According to Garcia and Lee (2022), a deep understanding of geometric concepts must be coupled with the ability to scaffold this knowledge for learners. This entails not only content mastery but also the capacity to identify learners' misconceptions and design activities that foster deductive reasoning. However, the theorems listed below are examinable (DBE, 2015). As evident below:

- The line drawn from the centre of a circle perpendicular to the chord bisects the chord.
- The angle subtended by an arc at the centre of a circle is double the angle subtended by the same arc at the circumference (on the same side of the arc as the centre).
- Angels subtended by an arc or chord of the circle on the same side of the chord are equal.
- The opposite angles of a cyclic quadrilateral are supplementary.
- The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment (DoBE, 2015).
- A line drawn parallel to one side of a triangle divides the other two sides proportionally, and
- Equiangular triangles are similar (DoBE, 2015).

2.7.2 Teachers' professional knowledge and experiences

Effective teaching of geometry hinges on teachers' content knowledge and pedagogical content knowledge (PCK). Nguyen and Tran (2023) argue that teachers' subject knowledge significantly influences learner achievement in Euclidean geometry. However, as Luneta (2022) observes, many South African teachers lack sufficient geometry content knowledge, which adversely affects their instructional effectiveness. This deficiency can render even basic aspects of geometry teaching, such as proof and reasoning, difficult for teachers to deliver with confidence.

Teachers' experiences, shaped by both pre-service training and in-service exposure, inform how they manage classroom challenges and adapt instruction. Chinn (2020) asserts that understanding mathematical content is necessary but insufficient; the ability to make this content accessible to learners is equally essential. Thus, teacher effectiveness is enhanced when subject knowledge is complemented by reflective practice and contextual awareness.

2.7.3 Contextual constraints and systemic realities

In the South African context, challenges such as large class sizes, limited access to technological resources, language barriers, and curriculum rigidity further complicate the teaching of geometry. Teachers often struggle to align the expectations of the Curriculum and Assessment Policy Statement (CAPS) with the realities of classroom instruction. These constraints, combined with gaps in professional support, exacerbate the teaching challenges experienced in the geometry strand.

Teachers' daily classroom interactions with learners often reflect a mix of technical improvisation and pedagogical adaptation. Johnson and Patel (2024) emphasise that effective mathematics teaching not only involves transmitting knowledge but also empowering learners to apply geometric concepts to real-life situations.

2.8 TEACHING CHALLENGES IN GRADE 11 EUCLIDEAN GEOMETRY

Geometry is widely acknowledged as a fascinating area of mathematics, rich with engaging problems and surprising theorems such as Morley's Miracle, the Steiner-Lehmus Theorem, and the Archimedean Spiral (Koay, 2022). These theorems allow for diverse teaching approaches and require teachers to identify engaging geometric problems, appreciate the subject's historical and cultural significance, and understand its applications across various fields. However, despite its appeal, the teaching of geometry, particularly at the Grade 11 level, presents numerous challenges for teachers.

Recent literature supports this view, emphasising that effective geometry instruction demands more than the presentation of facts (Johnson and Patel, 2024). Teachers must contextualise theorems within historical narratives and cultural frameworks to stimulate learner interest (Kaur & Sharma, 2022). Yet this pedagogical richness is often hampered by practical constraints. Mbatha and Dlamini (2024) observe that while varied teaching strategies can support learners' cognitive diversity, many teachers struggle to implement them effectively.

Studies have also noted that geometry is commonly perceived as a challenging subject, not necessarily due to cognitive limitations but because of broader systemic and pedagogical issues (Johnson & Liu, 2022). In countries such as Britain, China, France, and Germany, geometry instruction emphasises two-dimensional shapes like polygons and circles to develop learners' reasoning skills. This approach is intended to promote logical thinking and real-world modelling. However, achieving such outcomes is not always straightforward in under-resourced or multilingual educational contexts.

In South Africa, schools often assign a single mathematics teacher to cover Grades 10 through 12. According to Machisi (2023) and Tachie (2020), this staffing limitation restricts opportunities for professional collaboration, leaving teachers isolated in their lesson preparation. Furthermore, evolving curriculum requirements demand a shift from teacher-centred methods to constructivist and inclusive strategies, such as acting as a facilitator or delegator, which many teachers find difficult to adopt without adequate training or support.

One significant barrier is language. English, the language of instruction, is a second language for many teachers and learners. This impairs the development of geometric reasoning and formal proof construction (Machisi, 2020). To add, Senong (2022) shows that teachers in KwaZulu-Natal, particularly Zulu-speaking teachers, often experience anxiety when delivering geometry content in English. Coupled with frequent curriculum changes and insufficient professional development, this leads to confusion, reduced teacher confidence, and ineffective geometry instruction (Ngubane & Khoza, 2020; Li & Yu, 2022). Similarly, Setati (2008) highlights that language is not just a medium of instruction but also a resource for meaning making in mathematics classrooms, and multilingual practices, though valuable, are often constrained by school language policies that privilege English.

Multiple qualitative studies reinforce these concerns. In a Free State district case study, Tachie (2020) reports that many teachers lacked both content and pedagogical knowledge due to redeployment and minimal training. In another study, Zulu and Brijlall (2024) reveal that pre-service teachers exhibited only procedural knowledge without conceptual depth. Also, Msimeki (2021) identifies that language barriers, particularly in settings where learners are not taught in their mother tongue, often compel teachers to code-switch, though this alone is insufficient to support mathematical comprehension.

In brief, teachers often avoid complex geometric content in favour of more familiar or manageable topics. This results in a superficial engagement with geometry and denies learners the opportunity to fully develop deductive reasoning and proof skills (Danesi, 2022). Overall, these findings accentuate the need for robust teacher support, including content mastery, language-sensitive pedagogy, and targeted professional development, to enhance geometry teaching in Grade 11 classrooms.

2.9 CONVENTIONAL APPROACHES TO TEACHING EUCLIDEAN THEOREMS AND PROOFS

Conventional, teacher-centred methods continue to dominate the teaching of Euclidean geometry, often to the detriment of learner understanding. According to Machisi (2021a), many learners' difficulties with geometric proofs stem from the persistent use of these traditional approaches. Similarly, Keiler (2018) reports that many current teachers replicate the instructional methods they experienced as learners themselves. These methods typically involve the teacher presenting geometric theorems and proofs on the chalkboard while learners passively copy the content into their notebooks and reproduce it in exercises, tests, and exams (Tachie, 2020; Nguyen, 2023). In such settings, learners become passive recipients of mathematical principles, and the classroom becomes focused on rote memorisation rather than conceptual understanding.

Furthermore, teachers using traditional methods often do not verify whether learners have a solid grasp of foundational geometric concepts from earlier grades (Machisi, 2021b). As a result, geometric proofs are frequently taught as rigid, ready-made ideas to be accepted without critical interrogation or discovery. This didactic approach limits opportunities for learners to explore, hypothesise, and construct geometric knowledge themselves. In such classrooms, textbooks and teachers become the sole sources of knowledge, restricting learners' engagement with geometry as a dynamic and exploratory domain.

Although literature increasingly points to the greater efficacy of learner-centred approaches in teaching Euclidean geometry (Chen et al., 2023a), traditional methods persist for several practical reasons. Many teachers, particularly in South African contexts, have limited exposure to Euclidean geometry during their own education and are now expected to teach it without adequate training (Tachie, 2020). This often results in gaps not only in subject matter knowledge but also in pedagogical content knowledge (PCK), which is essential for effective teaching. Ndlovu (2023) states that a significant number of teachers expressed dissatisfaction with their training, reporting that it did not adequately prepare them for the complexities of teaching Euclidean geometry. Consequently, their instructional capacity remains limited, which negatively affects learners' comprehension and engagement with the topic.

2.10. CHAPTER SUMMARY

This chapter reviewed the literature through both theoretical and empirical lenses to explore teachers' practices and experiences in relation to the teaching of Euclidean geometry. The discussion highlighted the importance of geometry within the educational system, demonstrating how it fosters learners' skills in critical thinking, visualisation, intuition, spatial perspective, problem-solving, estimation, inferential reasoning, logical argumentation, and proof. The theoretical framework underpinning the teaching of geometry was examined and integrated with complementary theories of teaching and learning Euclidean geometry. Furthermore, the chapter engaged with the challenges teachers encounter in delivering geometry effectively. The subsequent chapter outlines the research methodology adopted for this study.

CHAPTER 3: RESEARCH METHODOLOGY

3.1. INTRODUCTION

Chapter 2 discussed theoretical framework and literature review. This chapter turns to research design and methodological choices underpinning the study. The chapter covers the research paradigm, design, sampling approaches, data collection techniques, and procedures for data analysis applied in the study. It also highlights the ethical principles adhered to in safeguarding and respecting participants throughout the research process.

3.2. RESEARCH PARADIGM

Hughes (2020) explains a research paradigm as the set of assumptions, values, and perspectives that shape how researchers view the world and operate within it. In addition, Mukherji and Albon (2022) emphasise that the choice of paradigm significantly influences all decisions taken during the research process. In addition, Grant and Grant (2023) note that a paradigm constitutes truth and knowledge and can also be defined as a common belief among researchers that problems should be understood before they are addressed. Researchers approach the world through their own perspectives. For this study, an interpretivist paradigm was chosen, as the focus is on exploring and interpreting the experiences of mathematics teachers in teaching Euclidean geometry at Grade 11 level. A paradigm rests on three key assumptions: ontology, epistemology, and axiology, which are elaborated on in the sections that follow (Creswell & Creswell, 2017).

3.2.1 Ontology

Ontology is about what is true. According to Killam (2023), ontology refers to the researcher's beliefs about the nature of reality and what is true. Other explanations were put forth by Moyo (2024), who describes ontology as "the way the investigator defines the truth and reality" within the research context. Khatri (2020) also adds that ontology is a branch of philosophy concerned with the assumptions made to believe that something makes sense or is real, or the very nature or essence of social constructivism under investigation.

Constructivism, as articulated by Braun and Clarke (2021), emphasises the development of subjective meanings and interpretations that individuals derive from their personal experiences concerning specific topics. This approach highlights the influence of one's social and historical context on how understandings of the world are formed and constructed. The process of meaning-making is inherently subjective, with individuals actively interpreting and negotiating their experiences to form their understanding of reality. The assumptions of reality in this study are centered around the poor performance displayed by Grade 11 learners in understanding Euclidean Geometry.

3.2.2. Epistemology

In research, epistemology is used to describe how people come to know about something, how the reality and truth known about something are known (Cohen, Manion & Morrison, 2018). This means that epistemology focuses on the nature of human knowledge and comprehension that the researcher can acquire to be able to extend, widen, and deepen the understanding of the field under investigation. In addition, Jackson (2020) asserts that epistemology is about methods of figuring out the truths. In other words, it refers to how the researcher comes to know about the truth. This study sought to know mathematics teachers' experience in the teaching of Euclidean geometry.

3.2.3 Axiology

Axiology is concerned with three branches of philosophy: ethics, aesthetics, and religion (Saunders, Lewis & Thornhill, 2019). Within research, axiological assumptions point to the values, often implicit, overlooked, or taken as universal, that shape a researcher's professional, personal, or disciplinary judgements about what is right or wrong, good or bad, and what is considered worthwhile, desirable, or beautiful (Chilisa, 2024). In the present study, participants were safeguarded from harm, and the integrity of the findings was maintained without alteration.

3.3. RESEARCH DESIGN

Creswell and Guetterman (2024) describe a research design as a structured plan outlining how a study will be carried out. It involves identifying the research problem, clarifying the intended outcomes, and establishing the logical sequence of the inquiry. Through this process, the researcher determines how data will be gathered and analysed, as well as the procedures to be followed. In essence, the research design directs the selection of methods appropriate for the study.

For the present investigation, a case study design was employed to examine the practices and experiences of teachers in teaching Euclidean geometry. This approach was deemed suitable as scholars such as Yin (2023) and Tisdell and Merriam (2025) argue that qualitative case studies enable a deeper understanding of complex phenomena, particularly when explored across multiple contexts. In this study, data were collected from teachers working in four secondary schools in the Mopani East District.

The case study method was adopted because it provided an opportunity to gain insights and deeper meaning into the challenges and experiences of teachers, which in turn influence learners' performance in Euclidean geometry. Similar approaches have been used in related studies, such as Tisdell and Merriam (2025), who investigated factors shaping geometry outcomes through case study research, and Creswell and Poth (2024), who utilised the design to explore the relationship between teachers' experiences and learners' achievement in geometry.

3.4. RESEARCH APPROACH

This study adopted a qualitative research approach. Creswell and Poth (2016) describe qualitative inquiry as a naturalistic process that aims to generate an in-depth understanding of social phenomena within their real-world contexts. Unlike approaches that emphasise outcomes, qualitative research prioritises the "why" of human behaviour and depends on individuals' lived experiences as active creators of meaning (Alase, 2017). It often draws on methods such as participant observation and in-depth interviews, which enable the collection of direct, first-hand accounts (Ravitch & Carl, 2019). Flick (2018) further explains that qualitative research seeks to interpret both individual and collective actions, beliefs, and perceptions, thereby foregrounding the perspectives of participants in understanding social realities.

The choice of a qualitative approach in this study was guided by its capacity to allow participants to articulate their experiences and viewpoints (Creswell & Guetterman, 2024). In particular, it enabled the researcher to uncover the sources of the challenges teachers face in teaching Euclidean geometry and the strategies they employ to address them. As noted by

Nowell et al. (2017), qualitative methodologies are well suited to addressing questions of “how” and “why,” offering valuable insights into the deeper mechanisms underlying social phenomena. Consequently, this approach has provided a meaningful framework for generating nuanced understandings relevant to the field.

3.5. POPULATION

Shah (2023) defines a population as the complete set of individuals or entities from whom data is required. In this study, the population comprised Grade 11 mathematics teachers across 13 secondary schools in the Man’ombe Circuit of the Mopani East District, Limpopo Province. From this group, four teachers were purposively selected for detailed data collection. These participants were chosen because their learners had produced unsatisfactory results in mathematics within the circuit, thereby allowing the study to gain deeper insights into teachers’ experiences of teaching Euclidean geometry.

3.6 SAMPLING

Etikan and Bala (2017) describe sampling as the procedure of choosing a subset or segment that reflects the characteristics of a larger population. Its primary purpose is to enhance the reliability of the conclusions drawn from the study’s findings. In this research, the sampling process began with a structured plan that identified the target population, the strategy for selection, and the size of the sample (Lakens, 2022).

3.6.1 Sampling procedure

To identify relevant participants for the study, the researcher utilised a non-probability sampling approach known as purposive sampling. As noted by Palinkas et al. (2020), purposive sampling is the most frequently employed method in qualitative research. This technique involves the intentional selection of participants based on specific characteristics that align with the research objectives (Etikan, Musa & Alkassim, 2016). Campbell, Greenwood, Prior, Shearer, Walkem, Young, Bywaters, and Walker (2020) further assert that purposive sampling entails selecting individuals who are most closely associated with the research topic. Importantly, since this method does not aim to represent any broader population, the results cannot be generalised beyond the sample itself (Campbell et al., 2020). The primary goal of sampling in this context is to collect specific cases or samples that enhance the understanding of the research question (Ahmadin, 2022).

3.6.2 Sample size

According to Tisdell and Merriam (2025), a sample is a subset of the population selected to be representative of the larger population. The sample consisted of four teachers from four secondary schools, purposely selected in Man'ombe Circuit, Mopani East District, Limpopo Province. In each school, one educator teaching mathematics in Grade 11 was selected. The researcher selected teachers who were from schools that produced poor results in Euclidean geometry and were within a radius of 45 kilometers, taking into consideration the factors around traveling and time allocated for the completion of the study. Out of the four schools selected, each one was more than 10 kilometers away from the other.

The researcher also interacted with learners who were being taught at the time of the lesson observation. The number of learners was 3 per teacher for each teacher, in total 12 students were interacted by way of checking their activity books or simple asking on tasks at hand.

3.7 DATA COLLECTION INSTRUMENTS

According to Tisdell and Merriam (2025), once a question or issue has been raised, the choice of qualitative methods falls roughly into the categories of observations, interviews, and document and artifact analysis to collect data. In this study, data were collected through field observation, document analysis, and a semi-structured interview schedule. The researcher first used observations, then followed with individual semi-structured interviews. The field observation allowed the researcher to determine teacher-learner interactions in class. The semi-structured interviews were necessary to get in-depth information on the views and allowed follow-up for more clarity on the teachers' experiences in teaching Euclidean geometry.

3.7.1 Observation

The observation in this study presented the researcher with an opportunity to gain insight into the problem through the integration of theory and practical, accurate behaviour of teachers in their classrooms before educators were subjected to individual semi-structured interviews. Through observations, the researcher was immersed in the research setting and enabled to conduct the study from the inside.

The lesson observation was conducted using video recording to facilitate a comprehensive analysis of teacher engagement during the instruction of Euclidean geometry in a Grade 11 mathematics classroom. The primary objective of this study was to investigate the pedagogical approaches employed in the teaching and learning of Euclidean geometry at this educational level. The observation concentrated on assessing the effectiveness of the teacher's instructional methods, the nature of interactions between the teacher and learners, and the strategies employed to facilitate learner comprehension of the subject matter. Walliman (2019) posits that certain research questions are best addressed through the observation of participants' behaviours and the overall classroom context.

In this study, the researcher adopted the role of a non-participant observer. Teacher participants were observed delivering lessons on Euclidean geometry to grade 11 learners, utilising a lesson observation tool. These observations aimed to address the first research question: *What challenges do teachers encounter in teaching grade 11 Euclidean geometry?* The researcher positioned herself at the back of the classroom and took field notes that documented the activities occurring within the classroom, guided by an observation framework. Each participant was observed for two lessons, following the time allocated by the Department of Education for teaching Euclidean geometry. Each school was visited two times, with each observation lasting one hour, following the established school timetable. According to the departmental work schedule, Euclidean geometry is taught over two weeks, amounting to a total of ten hours of teaching.

The researcher established protocols for analysing teaching methodologies, specifically focusing on how geometry is taught. An observation schedule was created to evaluate whether teachers utilised accurate and appropriate geometric terminology, which is essential for enhancing learners' understanding of geometric thinking (İbili, Emin, et al., 2020). This rigorous analysis aims to identify effective practices in geometry instruction and their impact on learners' conceptual grasp.

Furthermore, the observation schedule was used to observe the way teachers facilitate visualisation of the geometric mathematics problems, such as summaries of geometric theorems. The experiences of teachers in enabling understanding of the analysis, abstraction, deduction, and rigor by Grade 11 learners were observed. The observation was done for the full duration of the lesson, and the interviews were conducted thereafter.

3.7.2 Interviews

After collecting data through observations, the researcher then engaged selected participants in semi-structured interviews. Semi-structured interviews were appropriate for data collection as it could help to identify the researcher's intention to find clarity on the geometrical aspects and how teachers interpret them. According to Alase (2017), the real benefit of using interviews is that the researcher was face-to-face with the respondents, which allowed further probing on areas of interest.

In this study, Grade 11 mathematics teachers were selected for interviews conducted at their schools during breaks and after school hours. Each interview lasted approximately thirty to thirty-five minutes. The researcher employed an audio recorder and took supplementary notes to ensure that all details from the interviews were accurately captured. According to Zeyab, Alaa, Abdullah Almpdaires and Faisal Almutairi (2020), the use of an audio recorder does not supplant the need for notetaking. The notes can serve as a valuable tool for the researcher in several ways. They can facilitate the development of research questions and inquiries by providing insights and context related to the study. Additionally, documenting non-verbal communications through notes allows the researcher to capture nuances that may not be conveyed through spoken words alone, thereby enriching the overall understanding of the research subject. Utilising notes effectively can enhance data collection and analysis, making them an integral part of the research process. These elements are crucial in developing a comprehensive understanding of the research context and ensuring a thorough analysis of the data collected. With documenting both verbal and non-verbal cues, researchers can enhance the richness of their qualitative data, leading to more nuanced insights and findings. Such attention to detail is essential for producing robust and credible research outcomes. The researcher ensured the consent of participants for collecting data using the above-mentioned instruments. After interviews, the researcher listened on the audio recorder to review and reflected on the notes to identify possible gaps. Participants were given time at a later stage to verify transcriptions and to suggest corrections where necessary (Zeyab et al., 2020).

3.7.3 Development of data collection instruments

The section discusses the development of each data collection instrument used in the study. The data collection included observation and interviews in their respective form.

3.7.3.1 Observation schedule

This study used a structured classroom observation schedule (see Appendix B). The data collected from classroom visits corroborated the data gathered through other means. In the study by Mathonsi (2006), such behavior was observed by noting how learners respond when new concepts are presented in class. The observations enabled the researcher to interact with the learners. This allowed them to discuss their experiences in learning Euclidean geometry.

3.7.3.2 Semi-structured interview schedule

The researcher developed a semi-structured interview schedule (see Appendix A). A pilot study was conducted over a single day with two teachers from a different secondary school not included in the main study. The pilot primarily aimed to assess the reliability and clarity of the research instruments. The results indicated that the instruments were suitable and required no major adjustments. They were clear and comprehensible, enabling precise responses that demonstrated the participants' ability to effectively address the research questions in the main study. The pilot study resulted in the duration of the interviews being reduced from 40 minutes to around 30 minutes. This adjustment improved the efficiency of the interview process and reduced participant fatigue, potentially enhancing the overall quality of the data collected.

The semi-structured interviews obtained information on the understanding of geometrical concepts such as quadrilaterals and theorems, mastery of geometrical concepts, application of theorems, challenges that teachers faced when teaching geometry, the years of experience that the teacher had in the teaching of Euclidean geometry, effective teaching of geometry and ways in which the teaching of geometry could be improved in Grade 11.

3.8 DATA ANALYSIS

The data was analysed following the thematic method. The themes were dictated by the objectives. In qualitative research, data analysis is an iterative process that occurs throughout the study rather than solely at its conclusion. In this research, the analysis was conducted concurrently with data collection. As Thomas, Martin, Etnier, and Silverman (2023:56) emphasise, "the activity continues throughout the research process and is not a procedure to be carried out at the end of the research". Lakens (2022) notes that data analysis enables the researcher to draw interpretations from the sample and make inferences about the broader population. In this study, it facilitated an understanding of learners' performance in Euclidean geometry topics, including circle geometry and theorems, and provided insight into their comprehension of these concepts.

The analysis followed Creswell's (2014) guidelines, which encourage a reflective approach to enhance the researcher's understanding of the data. Initially, the collected information was systematically organised, arranging specific details logically and identifying key categories to aid interpretation. This approach enabled the identification of recurring themes and patterns within the data. The researcher reviewed all transcripts thoroughly, noting significant ideas and emerging themes.

Each interview transcript was analysed in detail, focusing on underlying meanings and making observational notes in the margins. Data were systematically coded by breaking it down into meaningful, labelled segments. These segments were grouped into categories and further divided into sub-categories, allowing for a comprehensive thematic analysis. Material relevant to each theme was collated to support preliminary interpretations. Finally, the data were organised to address the research questions explicitly, and the analysed findings were presented in a detailed written format.

3.9 TRUSTWORTHINESS

Creswell and Poth (2016) aver that while seeking methods that ensure truthfulness or trustworthiness in a qualitative approach, the four categories that follow may be used: credibility, dependability, transferability, and conformability.

3.9.1 Credibility

The researcher implemented rigorous procedures to assess the trustworthiness of the data in the qualitative study. Key criteria such as transferability, credibility, confirmability, and dependability were explicitly addressed. Additionally, member checks were conducted to verify the accuracy and understanding of the transcribed observations, reinforcing the reliability of the research findings. The researcher included consistency checks and credibility checks in this study. Credibility is the extent to which data can be controlled objectively and reliably to prevent inaccurate and misleading conclusions (Creswell & Poth, 2016).

To enhance the trustworthiness of the data employed in this study, several strategies were implemented:

- **Member checking:** Following data collection, the researcher transcribed, coded, and organised the data into categories, identifying key themes. The researcher subsequently returned to the participants to confirm that their views had been accurately captured and represented. Any identified inaccuracies or misunderstandings were rectified based on participant feedback.
- **Triangulation:** Triangulation refers to the use of multiple data collection methods to comprehensively explore a research question. This approach is essential for establishing the reliability and validity of both the data and the findings. In the current study, the researcher employed three interconnected instruments: classroom observations, interviews, and document analyses (Creswell and Poth, 2016). With utilising various data forms, the researcher aimed to effectively address the research question.
- **Prolonged stay in the field:** Adequate time in the field is necessary for collecting credible data. In this study, the researcher conducted two hours of observations per school while focusing on four teachers. Each teacher's lessons were observed two times, in alignment with the time allocation specified by the Department of Education for teaching Euclidean geometry. Each school was visited two times for one-hour intervals, according to the school timetable.

3.9.2 Transferability

Transferability pertains to the extent to which research findings can be applied in different contexts or by other researchers (Creswell & Poth, 2016). To enhance transferability, the research context and the foundational assumptions central to the study were described in detail.

3.9.3 Confirmability

Confirmability refers to the degree to which the results of a study can be verified and remain consistent even if conducted by different researchers (Creswell & Poth, 2016). It ensures that the findings reflect the perspectives of the participants rather than the researcher's personal bias. To achieve confirmability, the researcher maintained a transparent audit trail, documenting how data were collected, analysed, and interpreted. Reflexivity was also upheld throughout the research process, including the selection of the topic, choice of methodology, data analysis, interpretation of results, and presentation of conclusions.

3.9.4 Dependability

Dependability relates to the consistency and reliability of the research findings. The trustworthiness of a study is supported by the dependability of its results (Korstjens & Moses, 2018). In this study, dependability was ensured by grounding the findings, conclusions, and recommendations directly in the collected data. Furthermore, the researcher worked closely with the supervisor, who served as an external auditor to review the data collection, analysis, and interpretation processes. This procedure helped to verify the accuracy of the findings and confirmed that the conclusions were supported by the evidence.

3.10 ETHICAL CONSIDERATIONS

Research must be conducted in a manner that does not cause harm to participants. In this study, the researcher adhered to the ethical guidelines of the University of South Africa (UNISA). As outlined by McMillan and Schumacher (2014), ethical considerations include informed consent, avoidance of deception, confidentiality, anonymity, privacy, and a duty of care. Accordingly, access to the data and participants' identities was restricted to the researcher and the study supervisors. Participants were informed in advance about who would have access to their information. Confidentiality was maintained by ensuring that the data could not be traced back to individual participants, and all collected data were encrypted using a Microsoft password.

The researcher met with the participating teachers to explain the purpose and procedures of the study. In line with ethical principles, it was made clear that the research would not interfere with normal teaching activities. Arrangements were made to identify suitable locations for conducting the interviews, and the researcher coordinated each teacher's timetable to schedule convenient times. Letters of consent were provided to participants, who were asked to read and sign them prior to participation.

Permission to conduct the study within the schools was obtained through a formal request addressed to the manager of the Man'ombe Circuit. Prior to recording any interviews, the researcher also sought and obtained the participants' consent, and all participants signed consent forms to confirm their voluntary participation.

Ethics is closely related to morals, and ethical guidelines serve as standards that form the foundation for evaluating one's conduct in research (Head, 2020). In this study, the researcher adhered to the key ethical standards outlined by Babbie (2007). A reflective diary was maintained throughout the interviews to document the researcher's thoughts and observations. The transcribed data was securely stored to ensure confidentiality. The study began only after obtaining ethical clearance from the UNISA CEDU Ethical Committee. The approval certificate number is 2020/11/11/51412942/08/AM. (see Appendix C).

3.10.1 Protection from harm

The researcher was obliged to protect the participants from harm during the study. She ensured that they were not exposed to emotional stress, embarrassment, or loss of self-esteem because of the research, as this is against research ethics.

3.10.2 Informed consent

Leedy and Ormrod (2010) believe that when a researcher is getting information from people, he or she must inform the candidate about the nature of the study. In this study, participants were informed about the purpose and importance of the study. The researcher distributed consent forms to the participants and explained that their participation was voluntary.

3.10.3 Right to privacy

It was very important to maintain the privacy of the participants. The Constitution of the Republic of South Africa, 1996, promotes the right to privacy. Instead of mentioning the participants' names, the researcher in this study used codes to identify the participants. Participants were assured of their anonymity, and in this regard, I assured them that their names or identities and the name of the school would not be disclosed.

3.10.4 Honesty with professional colleagues

In the final report of the research findings, it is important to ensure that there is no misrepresentation of facts. There was no financial benefit for participating in research. Also, the findings of this research will be used for academic assessment at the University of South Africa.

3.11 LIMITATIONS OF THE STUDY

The study was confined to a single district in Limpopo Province, namely the Mopani East District. While the study could have encompassed additional districts, it focused solely on one circuit within the Man'ombe Circuit of the Mopani East District. Given that Mopani East comprises 13 circuits, a broader study including more circuits would have been preferable.

Furthermore, the research was limited to a sample of four schools. It should be noted that this sample may not fully represent the experiences of all mathematics teachers across Limpopo Province. Despite these constraints, the researcher maintains that the findings offer valuable insights that can support the enhancement of teachers' practices in the teaching of Euclidean geometry.

3.12 CHAPTER SUMMARY

This chapter described the research methodology employed in the study, providing justification for the selection of the research design, approach, and data collection and analysis methods as appropriate for addressing the research questions. Ethical considerations guiding the study were also outlined to ensure the credibility, trustworthiness, and integrity of the research process. In addition, the chapter detailed the strategies implemented to enhance the reliability and validity of the findings. Overall, the methodological framework presented establishes a solid basis for generating insights into the phenomenon under investigation. The following chapter focuses on the analysis and discussion of the collected data.

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND INTERPRETATION

4.1. INTRODUCTION

This chapter provides an account and interpretation of the data obtained through classroom observations, semi-structured interviews, and document analysis. The combination of these data collection methods, alongside thematic analysis, enabled a comprehensive understanding of teachers' experiences and instructional practices in teaching Euclidean geometry, while also allowing for cross-validation and comparison of findings across the different sources of evidence. The chapter begins by presenting the participants' biographical information, followed by a discussion of the themes, categories, and viewpoints that emerged from the study.

The primary purpose of the study was to explore the experiences and teaching practices of Grade 11 mathematics teachers delivering Euclidean geometry in the Mopani East District. Thematic analysis was utilised as the main approach for systematically identifying, organising, and interpreting qualitative data into coherent themes, thereby facilitating a clear understanding of participants' responses (Christou, 2022). The data was gathered to address the central research question:

How do mathematics teachers practise and learners experience the teaching and learning of Grade 11 Euclidean geometry?

From this, the study was guided by the following sub-questions:

- What techniques do Grade 11 mathematics teachers employ in teaching Euclidean geometry?
- What challenges do Grade 11 mathematics teachers and learners encounter in the teaching and learning of Euclidean geometry?
- How can the teaching and learning of Grade 11 Euclidean geometry be improved?

This chapter first explains the method used to address these research questions, followed by a description of the participants' demographic characteristics. The chapter subsequently reports the study's findings, organised according to the emerging themes and categories, accompanied by a thorough analysis. It concludes with a summary of the principal insights.

4.2. STUDY PROCEDURE

This section describes the research methods employed in the study and provides contextual information on how the research questions were addressed. As outlined in Chapter 3, the study was conducted in four secondary schools purposefully selected within the Man'ombe Circuit, Mopani East District, Limpopo Province. Four Grade 11 mathematics teachers were chosen as participants, with the schools situated within approximately 45 kilometres of one another to account for travel considerations and the time allocated for the study.

All selected participants took part in the interviews, resulting in a 100% response rate. Each teacher engaged fully in providing comprehensive answers to the interview questions. Given that the participants were mathematics teachers, it was assumed they possessed sufficient knowledge of Euclidean geometry at Grade 11 level. In addition to interviews, classroom observations were carried out to gain insights into the teachers' instructional methods and approaches in a practical teaching setting.

4.3. DEMOGRAPHIC CHARACTERISTICS OF THE STUDY'S PARTICIPANTS

This section outlines the personal and professional profiles of the four Grade 11 mathematics teachers who participated in this study. It is essential to provide the background of each participant to show their capacity and ability to teach Euclidean geometry effectively. The data collected concerning participants included information about their highest qualifications in mathematics, gender, age, teaching experience, and the type of school in which they were appointed. Table 4.1 contains the biographical information of the grade 11 mathematics teachers participating in this study. The participants were code-named as T1, T2, T3, and T4.

Table 4.1: Participants' Profiles

PARTICIPANTS	GENDER	AGE	DESCRIPTION OF RESEARCH	HIGHEST QUALIFICATION IN MATHEMATICS	TEACHING EXPERIENCE (YEARS)
T1	M	51	Rural	Bachelor of Education in Further Education and Training (FET)	12
T2	M	30	Rural	Bachelor of Education in Senior Phase and Further Education and Training Teaching (SPFT)	7
T3	M	60	Rural	Bachelor of Science in Mathematical Sciences	30
T4	F	48	Rural	PhD in Mathematics Education	14

(**Source:** Primary Data, 2023)

The data presented in Table 4.1 reveals that most participants (75%) were males, aged between 30 and 60 years, with only one female teacher, aged 48 years. The educational qualifications of the participants ranged from bachelor's degrees to PhDs, with the highest qualification (a PhD) held by the female teacher (T4). The table also provides information on the teachers' years of experience in teaching mathematics. Each participant in the study had a minimum of seven years of teaching experience. However, three educators, T1, T3, and T4, stood out with significantly longer tenures of experience, ranging between twelve and thirty years. This diversity in teaching experience contributed a wide range of pedagogical insights to the research study.

4.4 DISCUSSION OF FINDINGS

The teaching techniques used by grade 11 mathematics teachers, learners' learning experiences and the challenges in teaching Euclidean geometry.

The following subsections outline the teaching experiences, learning experiences and challenges associated with Euclidean geometry in grade 11.

Table 4. 2: The identified codes, categories, themes and teachers' perspectives

Codes	Category	Theme

Demonstration of Mathematical Instruments (i.e., use of protractor, Compass) (UMI)	Utilisation of Mathematical Instruments	Incorporation of resources and tools
Use of Chalkboard Diagrams (UCD)	Use of Visuals	
Group Work for Problem Solving (GWPS)	Group Work	Peer collaboration and engagement
Peer learning (PL)	Peer learning	
Questioning Techniques to stimulate thinking (QTST)	Use of Questioning	Scaffolded Verbal Instruction
Breaking Down Complex Theorems (BDCT)	Step-by-Step Instruction	
Deficit in Prior Knowledge from earlier grades (DPKG)	Lack of Prior Knowledge	Knowledge Gaps and Instructional Challenges
Misunderstanding of basic geometry concepts (MBBC)	Deficit in Foundational Skills	
Errors in applying the Pythagoras Theorem (EIAPT)	Errors in Basic Concepts	

Themes were developed from data through systematic analysis and interpretation, and they were revisited to ensure they accurately reflected the data (Flick, 2022). The next section discusses the categories and themes emanating from this study.

4.4.1 Incorporation of resources and tools

Teaching Euclidean geometry effectively requires teachers to use a variety of resources and tools that engage learners and enhance their understanding of abstract concepts. This theme explored how four teachers, T1, T2, T3, and T4, incorporated resources, including mathematical instruments, visual aids, and instructional strategies, into their geometry lessons.

In this section, the researcher presents findings on how teachers demonstrate the use of tools and resources in their geometry lessons.

Figure 4.1: Basic mathematical sets for use in Euclidean geometry (Primary data)

4.4.1.1 T1's Approach

On Day 1, T1 focused on revising learners' prior knowledge by introducing mathematical tools such as the compass and protractor. He demonstrated the use of these tools, showing learners how to draw a circle and construct a 90-degree angle, emphasising their foundational importance in Euclidean geometry. However, T1 noticed that 7 out of 15 learners lacked complete mathematical sets, creating a barrier to full participation in the lesson. To address this, he encouraged some learners to share tools with their peers, fostering a collaborative learning environment. T1 walked around the classroom, offering individual assistance to struggling learners and reinforcing the correct use of the instruments. This personalised support was important for learners who had difficulty with accurate constructions.

T1's strategy of tool-sharing and individualised assistance provided a temporary solution to the immediate challenge of resource shortages in the classroom. While this approach allowed all learners to participate in the lesson, it did not address insufficient access to mathematical sets. Studies, such as Machisi and Feza (2021), emphasise that the lack of personal learning tools contributes to ongoing difficulties in mastering geometry concepts. Interviews with teachers indicated that learners without their tools faced challenges completing homework and preparing for assessments, highlighting the limitations of resource-sharing as a sustainable strategy. Despite these concerns, T1's strategy ensured that all learners could actively participate in classroom activities. After the intervention, some learners demonstrated increased engagement and were able to construct accurate diagrams and apply theorems correctly. Observations revealed that those who initially struggled with using compasses and protractors gained proficiency in measuring angles and drawing geometric figures with greater precision. Additionally, classroom discussions became more interactive, with more learners asking questions and confidently explaining their reasoning. While tool-sharing provided only a temporary solution, it played a crucial role in enhancing learners' foundational geometry skills and fostering a more interactive learning environment.

During this lesson, T1 continued the lesson by drawing step-by-step diagrams of circles, chords, and angles on the chalkboard. He illustrated the properties of radii drawn from the centre O to any part of the circle, explaining that these radii are equal. A chord can be defined mathematically as a straight-line segment that connects any two points located on the circumference of a circle. In geometric terms, it represents a key concept in circle geometry, helping to understand the relationship between the radius, diameter, and the circle.

The length of a chord can vary depending on its position within the circle, and it plays a crucial role in various calculations involving circular shapes. Furthermore, he explained that a chord subtends an angle at the centre or circumference of the circle is said to be "subtended." Thus, a chord subtends an angle at the centre or circumference of the circle. To clarify this concept, the teacher illustrated it on the chalkboard with the figure he had drawn, demonstrating "perpendicular lines" and their properties.

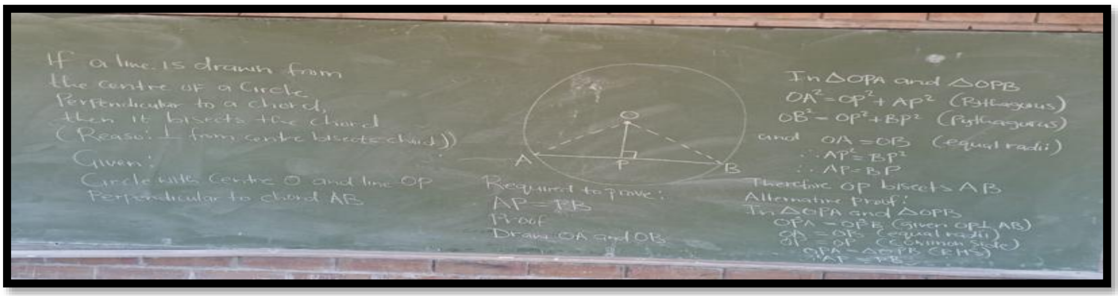


Figure 4.2: T1's illustration of 'the line drawn from the centre of a circle perpendicular to a chord that bisects the chord' theorem

The step-by-step visual illustration of procedures and solutions reduces the cognitive load for learners, allowing them to focus their cognitive resources on understanding the principles illustrated. Some learners can move at the same pace as the teacher when the topic is explained from the simple basics to the abstract concept. Observations indicated that as T1 explained the theorem using diagrams, several learners nodded in agreement, demonstrating their comprehension. Others responded to teacher prompts by correctly identifying key geometric properties, such as the diagram's congruent angles and perpendicular bisectors.

Additionally, when T1 asked learners to replicate the diagrams in their notebooks, most followed the correct procedures, reinforcing their understanding. However, a few learners initially hesitated, requiring additional clarification, suggesting that while the method was effective for most, some still needed further reinforcement. These observations confirm that the structured approach not only provided clarity but also encouraged active engagement in learning Euclidean geometry.

On the second day of observations during week two, T1 introduced the tangent theorem, a fundamental concept in geometry that details the properties and relationships of tangents to circles. This theorem is pivotal for understanding various geometric principles and applications. In exploring the characteristics of tangents, learners can gain deeper insights into circle properties and their interactions with other geometric figures. T1 explained the tangent theorem using diagrams on the chalkboard, highlighting key properties related to the theorem:

- The lengths of the two tangents are equal.
- The angles formed by the tangents and the radii drawn to the point of contact are equal in size.
- The angle formed by one tangent and the radius at the point of contact equals the angle formed by the other tangent.
- The areas of the two triangles created by the respective tangents and radii are of equal size.

The triangles formed by the tangents, radius, and chord in the context of circles exhibit right-angled properties. It is important to note that the intersection point of the two tangents demonstrates an equidistant relationship to the points of contact on the circle. This geometric principle is fundamental in understanding the relationships between tangents, chords, and radii, and plays a crucial role in various applications within the field of mathematics, particularly in the study of circle theorems. Learners were tasked with replicating these diagrams and identifying key relationships between tangents, radii, and chords. Many engaged actively, asking questions and taking detailed notes. T1's emphasis on the importance of mathematical tools led learners to recognise the need to have their protractors and compasses. This realisation is critical in fostering responsibility and preparation for geometry lessons. However, the need for learners to ask for these resources from their parents highlights systematic challenges, such as inadequate school-provided materials. Addressing these barriers is essential for ensuring equitable learning opportunities. Effective participation in mathematical activities is closely tied to having the appropriate tools.

This concurs with Machisi and Feza (2021), who assert that learners struggle to understand geometry concepts if they lack a strong grasp of foundational concepts, such as the properties and definitions of shapes. T1's use of diagrams and detailed explanations aligns with the authors' recommendation that teachers scaffold learning to address such gaps. However, the limited availability of tools in the classroom suggests the need for additional measures, such as the school's support for resource provision, to fully address those barriers.

During the interview, when T1 was asked why he began by introducing mathematical instruments, he explained:

> *I want to be sure their knowledge is adequate because I will be teaching them next year in their matriculation year.*

T1 further highlighted systematic issues, noting that many learners had a poor background in mathematics. He explained:

> *"In each class I take from grade 10 to 12, I assess their background and experience in mathematics from the lower grades. Some of them at primary school used to write class tests, and after they failed, the teacher just gave them the memorandum to copy for corrections. There was no revision with the teacher. Sometimes, the teacher gave learners the answers to the questions so they could copy them for their tests. As they progressed throughout the different grades, the attitude towards mathematics was created and nurtured on the wrong assumption that mathematics is a difficult subject".*

This statement stresses the importance of addressing foundational gaps and misconceptions early. T1's focus on reviewing instrument skills aligns with recommendations by Machisi and Feza (2021) that learners must first master basic geometric concepts before progressing to advanced topics. In addition, Shabangu, (2024) emphasise the hierarchical nature of learning geometry, where learners progress sequentially through levels of understanding. T1's focus on reviewing basic skills aligns with this approach, ensuring learners have mastered foundational concepts before progressing to more advanced theorems. This method also supports learners' personal and cognitive development, reinforcing both confidence and competence in Euclidean geometry. T1's approach to helping learners with basic concepts of geometry played a prominent role in assisting average learners to advance their understanding of the conceptual terms of geometry.

Furthermore, when T1 was asked which learning support material he employed to prepare his lessons. T1 highlighted that clear, structured diagrams and step-by-step reasoning were central to his approach to teaching Euclidean geometry, stating,

"Using visual and audio presentation, first explain the key concepts".

This strategy aligns with the goal of scaffolding learners' understanding of complex concepts. However, the persistence of resource shortages and foundational gaps suggests that instructional techniques must be refined to address these systemic challenges effectively.

4.4.1.2 T2's Approach

On the first day of lesson observation, T2 introduced the learners to mathematical sets, focusing on the use of a protractor for measuring angles. T2 began by explaining the protractor's design, highlighting the markings that indicate degrees on the tool's inner and outer edges. In addition, T2 demonstrated how to align the protractor with an angle to measure it accurately on a given diagram, providing clear examples to ensure learners understood the process.

To reinforce the concept, T2 assigned an activity in which learners practised measuring angles using the protractor. After reviewing the learners' work, T2 found that only 26 out of 55 learners (approximately 47%) had correctly measured the angles, revealing foundational gaps. The ongoing challenges faced by learners in aligning the protractor and accurately reading angle measurements highlight significant areas for instructional improvement. Despite T2's efforts in providing additional explanations to rectify misconceptions, it appears that some learners still find these skills difficult to master. This indicates that while T2's intervention helped some learners, it did not fully resolve the problem, as misconceptions persisted among those who lacked prior exposure to measurement tools.

T2's reliance on a teacher-centred approach may have also contributed to these difficulties. Observations revealed that while learners followed instructions during demonstrations, many hesitated when asked to measure angles independently. Instead of experimenting with the tool, some waited for further guidance from T2 before attempting the task, suggesting a dependence on direct instruction rather than exploratory learning. Since learners did not have access to the learning materials, this aligns with the view of Adams, Resnick and Lowrie (2023), who note that learners at the junior school level are not learning geometric concepts appropriately to prepare them for success in their high school courses.

To enhance learners' confidence in using measurement tools, it would have been beneficial to incorporate additional strategies such as guided practice and collaborative learning. These approaches foster a supportive environment where learners can explore and apply measurement concepts together, leading to deeper understanding and mastery of the tools involved. For example, T2 could have implemented structured peer-assisted learning activities, where learners worked in pairs to measure and verify angles together.

Additionally, incorporating ICT tools like GeoGebra could have allowed learners to visualise and manipulate angles dynamically before physically measuring them. These approaches would have helped bridge the gap between theoretical knowledge and practical application, fostering independent learning and problem-solving skills.

To address these challenges, T2 used the chalkboard to draw a detailed diagram illustrating Theorem Two: "The angle at the centre of a circle is twice the size of the angle at the circumference." He explained the theorem step by step, emphasising how the theorem applies to different configurations of points on a circle. T2 also used real-life examples, such as architectural and engineering designs, to contextualise the theorem and enhance learners' understanding.

This concurs with Tuong, Nam, Hau, Tien, Lavicza and Hougton (2023), who are of the view that the use of real-life examples in teaching and learning mathematics is more engaging, relevant, and approachable for learners. Learners could see how the concepts they learn in class are applied in the real world (Tuong et. al., 2023). For example, in the field of architecture, Euclidean geometry is indispensable, particularly in the design and construction of domes. Its principles facilitate the calculation of curvature, ensuring that domes maintain a smooth and consistent form. This mathematical framework is essential for determining the optimal placement and angles of supportive elements, such as arches or ribs, which are critical for achieving even weight distribution. Furthermore, a deep understanding of the relationship between central and inscribed angles equips architects with the tools to anticipate and address stress points within the dome, enhancing both its stability and durability. Beyond structural integrity, the principles of Euclidean geometry also inspire the creation of visually compelling patterns and designs on the dome's surface.

In analysing the geometric relationships between angles and arcs, architects can produce aesthetically striking and mathematically sound architectural works to illustrate geometric concepts (Ching, 2023). This strategy enhanced conceptual clarity, as some learners were able to relate abstract mathematical principles to practical applications. Observations revealed that when T2 explained the use of angles in bridge design and construction, several learners nodded in agreement and began taking detailed notes, an indication of active engagement. Others asked follow-up questions, such as how architects ensure accuracy in their measurements, demonstrating curiosity and deeper conceptual understanding.

However, while real-life examples helped learners visualise the importance of Euclidean geometry, they did not fully address practical engagement with measurement tools. Some learners still struggled with accurately using the protractor, suggesting that combining real-life examples with more hands-on activities, such as measuring angles in physical objects within the classroom, could have further reinforced learning. T2's lesson might have been more effective if it included structured, interactive activities where learners applied their knowledge beyond theoretical discussions. Figure 4.2 shows T'2 working on how he did the angle at the centre theorem.

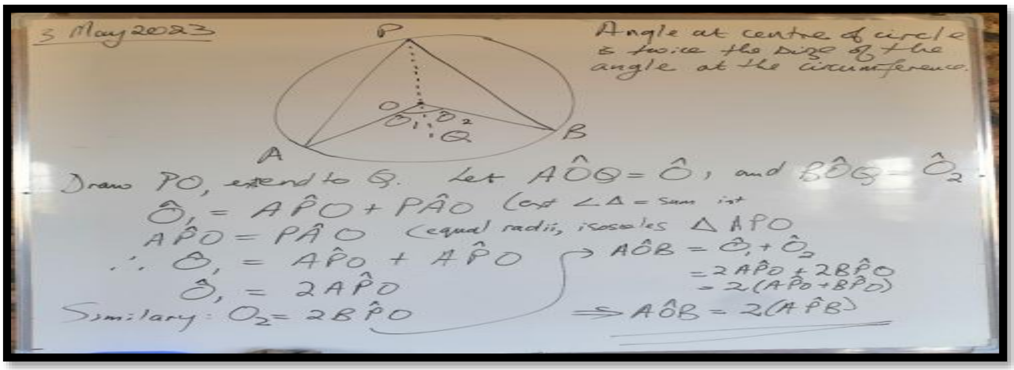


Figure 4.3: T2's illustration of the angle at the circle's centre being twice the size of the angle at the circumference

On the same day of lesson observation, T2 introduced foundational geometric terms, including "circumference," "segment," "radii," "chord," and "subtend," ensuring learners understood their meanings in the context of Euclidean geometry. T2 used questioning techniques, asking learners to identify the circumference, chord, radii, and segment on the chalkboard. Learners successfully identified circumference and radii from diagrams, demonstrating their ability to connect terms with visual representations. With the T2's guidance, learners observe that equal radii result in isosceles triangles, reinforcing their understanding of geometric properties through a practical example.

On day 2 of lesson observation on week 2, T2 drew a circle freehand on the chalkboard to illustrate the theorem "The angle between a tangent to a circle and a chord drawn at the point of contact is equal to the angle which the chord subtends in the alternate segment". T2 began by reviewing key geometric elements, radii, diameter, chord, circumference, arc, and segment, using the diagram to reinforce learners' understanding of these foundational terms. He then demonstrated subtending angles, ensuring learners grasped the relationships between the theorem and the geometric properties of the circle. Asking learners to consistently recall and reflect on topics that they have learned from the previous lesson helps learners to make connections between new and old knowledge and therefore increases the likelihood of knowledge being transferred into long-term memory. During the review, most learners were able to correctly identify geometric terms, such as circumference, chord, radii, and segment, as demonstrated on the board. However, despite this initial success, some learners continued to struggle when applying these concepts in problem-solving tasks, particularly when working with more complex diagrams.

Observations during individual tasks revealed that while some learners accurately labelled and described geometric components, others misidentified key elements or confused related terms. For example, a few learners incorrectly used the term 'diameter' instead of 'chord' when solving exercises, indicating that further reinforcement was needed.

This suggests that while T2's review of key terms was effective in activating prior knowledge, it did not fully address foundational gaps. Learners required additional scaffolding, such as guided problem-solving activities or targeted questioning, to reinforce their understanding. Therefore, while reviewing concepts at the beginning of the lesson was beneficial, it alone was not sufficient to ensure mastery, and ongoing reinforcement throughout the lesson was necessary.

Afterward, T2 instructed the learners to replicate the diagram in their notebooks and measure the angles using their protractors.

Figure 4.4: Learner 5's response from the T2 class

This hands-on activity aimed to reinforce learners' understanding of angle measurement and the relationship described in the theorem. The hands-on approach supported learner engagement as learners actively participated in constructing and measuring angles. However, its effectiveness in improving overall understanding could have been further enhanced by incorporating additional collaborative activities, such as group tasks or peer discussions. These activities might have allowed learners to exchange ideas and clarify misunderstandings collectively, thereby deepening their grasp of the theorem. As learners worked, T2 walked around the classroom, helping them individually. He emphasised the importance of precision, noting that if "dimensions are incorrectly applied or interpreted, the whole question is bound to be incorrect." T2's assistance was effective in addressing specific learner errors, particularly those related to the accurate application and interpretation of dimensions. However, integrating structured opportunities for learners to discuss common errors as a group could have added value. Such discussions might have provided a platform for learners to learn from one another, improving their conceptual understanding and problem-solving skills.

During a post-lesson interview, the researcher probed T2 to understand the way the teacher was teaching learners theorems; T2 explained:

> *"I always encourage fewer but clearer statements in solving problems in Euclidean geometry"*.

T2 acknowledged that his teaching relied heavily on traditional methods, where the teacher dictates steps and demonstrates solutions for learners to follow. T2's preference for simplicity in teaching aligns with the structured approach often associated with traditional teaching methods. However, this reliance on such methods might limit learners' ability to explore and apply different methods and geometry concepts independently. While simplicity ensures clarity and order, it could restrict the development of critical thinking and problem-solving skills that are crucial for effective geometry learning. This observation resonates with Khoza's (2015) findings that traditional methods dominate classrooms, where teachers direct the learning process, often leaving minimal room for learner autonomy.

Similarly, Tachie (2020) highlights that teachers predominantly rely on chalkboard diagrams for learners to copy. While this ensures structured learning, it limits learners' active participation and prevents them from verifying the accuracy of diagrammatic techniques independently. However, structured explanations and step-by-step demonstrations, as seen in T2's approach, provided some benefits. Some learners were able to follow clear instructions, which were given to them by T2 when drawing free-hand diagrams of angles using a protractor, which helped them understand foundational concepts such as scales, measurement, and accuracy with minimal confusion. This was particularly helpful for learners who relied on direct instruction to grasp geometric properties.

Despite these benefits, classroom observations revealed that T2's teacher-centered method hindered independent problem-solving. During a lesson on the tangent-chord theorem, learners could replicate diagrams accurately but hesitated when required to apply the theorem in problem-solving tasks. Many waited for T2's guidance instead of attempting the problems independently, suggesting that they lacked confidence in applying geometric concepts without direct instruction. This reliance on passive learning suggests that, while T2's structured explanations provided clarity, they did not foster deeper conceptual engagement or independent reasoning.

The teacher should act as a guide in the learning process, thereby supporting learner-centred learning. Incorporating constructivist teaching strategies could address these limitations. For instance, exploratory tasks and the use of ICT tools can enhance learners' engagement and promote active participation. Such methods align with Khoza's (2015) recommendation for more learner-centred approaches, which empower learners to take ownership of their learning. This is also consistent with Tachie (2020), who emphasises that leveraging innovative tools and activities can encourage learners to experiment with and apply geometric concepts more effectively. Through integrating these strategies, T2 could maintain the clarity of structured explanations while also fostering independent learning. Combining teacher-led instruction with inquiry-based tasks would provide learners with the opportunity to develop critical thinking and problem-solving skills in Euclidean geometry.

4.4.1.3 T3's Approach

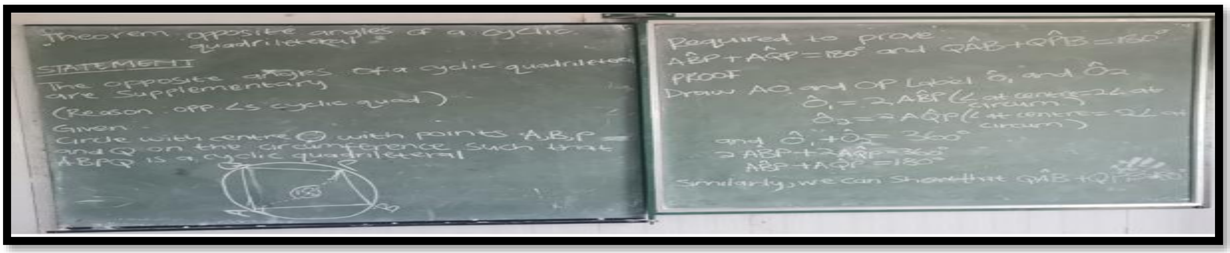


Figure 4.5: T3's illustration of 'the opposite angles of cyclic quadrilaterals being supplementary' on the chalkboard

On day 1, T3 began the lesson by drawing a diagram representing the theorem on the chalkboard regarding "opposite angles of a cyclic quadrilateral being supplementary." He emphasised that the corners of the quadrilateral lie on the circumference of the circle. T3 explained that the opposite angles in a cyclic quadrilateral add up to 180^0, the interior angles add up to 360^0, that opposite sides are not parallel, and the diagonals intersect at the centre of the circle. He clarified that cyclic quadrilaterals have unique properties distinguishing them from regular quadrilaterals, such as their dependence on the circle's circumference. These distinctions are important for learners as they help in categorising and solving problems more effectively. Understanding the supplementary nature of opposite angles, for instance, provides learners with a clear strategy for solving angle-related problems. Additionally, recognising the relationship between the quadrilateral and the circle aids in visualising geometric properties and applying them to diverse problem-solving contexts. When the researcher asked T3 why he did not focus on the use of mathematical sets, T3 mentioned that he did not focus on using them because, in his view, learners had enough time to practice this skill in the previous grade.

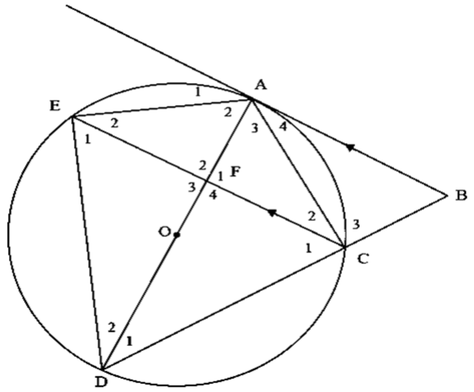
Upon careful consideration and analysis, it appears that the assumption made by T3 regarding learners' prior mastery of mathematical sets in the preceding grade lacks substantiation. Observational data from the classroom revealed that several learners faced difficulties connecting the geometric properties inherent to cyclic quadrilaterals with broader mathematical concepts. This challenge may arise from insufficient engagement with relevant tools during instruction. Such an oversight indicates that neglecting the application of mathematical sets may have contributed to the identified skill deficiencies among the learners.

Furthermore, the utilisation of structured tools, particularly mathematical sets, has been demonstrated to significantly enhance learners' understanding of geometric relationships. Research conducted by Chiphambo and Feza (2020) supports this assertion, highlighting the role of effective teaching strategies in the realm of mathematics education. These strategies not only facilitate deeper comprehension but also aid in the development of essential problem-solving skills among learners. Therefore, it is imperative to incorporate such methodologies to promote a more robust grasp of mathematical concepts and foster overall skill enhancement in learners.

On the second day of lesson observation in week 2, T3 shifted the focus to test preparation. Learners revised and the practising questions such as the one in figure 4.4 which was helping them prepare for their upcoming test practiced using previous examination papers in groups, and there was no learning and teaching taking place due to the follow-up program agreed on by the researcher and T3, so the researcher did not observe anything. T3 assisted them by offering guidelines on how to approach and solve the questions. Before the end of the Euclidean geometry lesson, T3 assigned an individual activity (see Figure 4.4) to the learners, despite their prior group work. While the source of the examination questions was not confirmed, the activity exposed learners to real exam scenarios. The integration of group work and individual tasks in T3's lesson seemed to enhance learner engagement and comprehension to some degree. Collaborative group work facilitated discussion, idea-sharing, and collaboration among learners, likely deepening their grasp of the material. The subsequent individual task reinforced this learning by allowing learners to independently apply their knowledge and evaluate their understanding. Nevertheless, T3 could enhance learning outcomes by incorporating more interactive strategies, such as peer review. This method aligns with Mohamad and Tasir (2023), who encourages reflective learning by enabling learners to analyse and provide constructive feedback on one another's work, which helps identify misconceptions and strengthens overall comprehension. The implementation of peer review would further enhance learner engagement by promoting critical assessment of peers' approaches while simultaneously reinforcing personal understanding. While the individual task provided a method of self-assessment, integrating peer review with group work could foster a more dynamic learning environment that balances collaboration, reflection, and independent problem-solving. This approach would strengthen learners' abilities to critically evaluate both their work and that of their peers, ultimately leading to a more comprehensive grasp of the subject matter. The activity below was asked by T3, illustrated in Figure 4.5.

Question 5

In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B.



5.1 Show $\hat{C}_2 = \hat{D}_1$

Figure 4.6: Tan-chord theorem question from T3 class

In Figure 4.5 above, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced; cuts tangent BA at B. Learners were required to show $\hat{C}_2 = \hat{D}_1$. Figure 4.7 below is the response from learner 4 observed in T3's lesson.

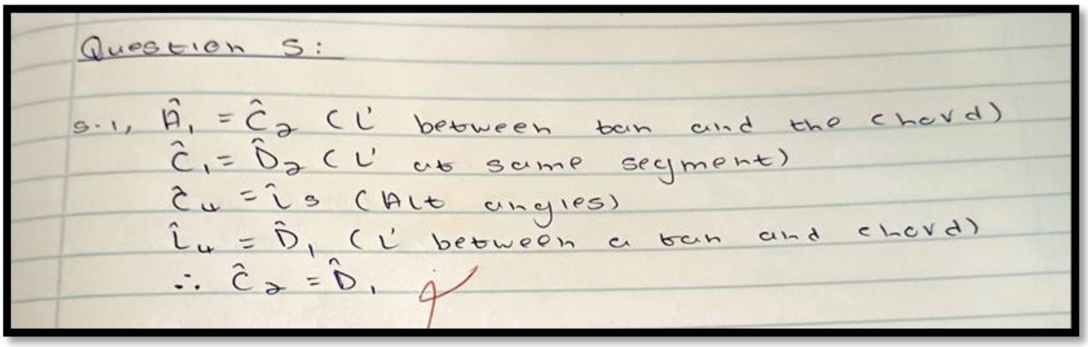


Figure 4.7: Learner 4's response from T3 class

In analysing question 5 (referenced in extract 4.3), it became apparent that Learner 4 encountered difficulties in comprehending the tan-chord theorem. The researcher observed that numerous learners failed to employ appropriate methods or applications, particularly neglecting the critical step of constructing a diagram before attempting to solve the question. T3 highlighted that the tangent-chord theorem, which delineates the relationship between a tangent and a chord at their point of intersection, is essential to various geometric principles.

This theorem underpins broader concepts in geometry, including angle properties, circle theorems, and the interplay between tangents and radii, all of which are pivotal for advanced problem-solving within the discipline.

Some learners' difficulties with the tan-chord theorem may stem from weak foundational knowledge in geometric constructions and pattern recognition. The researcher observed that many learners struggled to draw accurate diagrams, a key skill in geometry, leading to incorrect answers. This issue may be linked to T3's omission of mathematical sets, which could have supported learners in visualising and constructing geometric figures more effectively. The lack of such tools suggests a need to assess prior knowledge and ensure foundational skills are in place. Tessema, Michael and Areaya (2024) states that hands-on tools can significantly enhance spatial reasoning and improve learners' diagrammatic accuracy.

Furthermore, the researcher's observation was limited to certain aspects of the lesson, as other theorems were covered in her absence. The researcher conducted observations in the first week and again in the second week of teaching Euclidean geometry. When asked about the learners' struggles, T3 stated:

> *"Schools tend to hire teachers who are not well trained in mathematics for the lower grades. This is the reason why learners fail to apply the basic knowledge of diagram construction when answering questions. Instead, schools place well-trained teachers in the FET Phase to ensure that Grade 12 results are satisfactory, so learners can meet the minimum entrance requirements for the university".*

The systemic issue of underqualified mathematics teachers in lower grades creates challenges for both learners and teachers. Some learners progress to higher grades without mastering foundational skills, such as diagram construction, making it difficult for them to engage with complex geometric concepts, creating challenges in higher-level geometry. This learning gap forces FET phase teachers to reteach the content that learners should have learned in the lower grades, rather than focusing on advanced mathematics, increasing their workload, and slowing down overall curriculum progress. For teachers, this issue leads to higher stress levels, time constraints, and instructional inefficiencies, as they must balance remediation with new content delivery.

For learners, the lack of early exposure to key mathematical concepts hinders their confidence, problem-solving abilities, and overall performance in high school mathematics. To address these challenges, interventions such as targeted teacher training and bridging programs are essential. Providing foundational mathematics training for lower-grade teachers can ensure early conceptual understanding, reducing content gaps in later grades. Research conducted by Machisi and Feza (2021) emphasises that investing in teacher training programs is crucial for improving instructional quality in foundational education. This improvement significantly influences learners' long-term success in mathematics.

When the researcher asked how T3 handles difficulties in understanding geometrical concepts, T3 emphasised the importance of using models. He stated:

> *"Making models of the shapes with papers in addition to the basics of geometry makes learners understand the concepts".*

T3 explained that visual aids like diagrams and models help learners understand complex ideas. T3 highlighted,

> *"I use apps, textbooks, instruments, and study guides" to connect learners with geometry concepts".*

T3's use of a variety of resources, including ICTs, highlights his effort to create an engaging learning environment despite systematic challenges. While T3's use of various resources, including ICT tools, is commendable, it is important to consider whether these resources effectively address learners' foundational gaps in geometry. Research suggests that ICT tools are most effective when combined with structured, hands-on learning activities, as they provide interactive visualisations while reinforcing procedural fluency (Lee, Wu, Lin, Wang and Huang, 2024).

A combination of traditional methods and ICT tools should include guided practice sessions where learners first explore geometric relationships using dynamic software, such as GeoGebra, and then apply their understanding through hands-on construction with mathematical sets. This approach allows learners to transition from virtual representations to physical problem-solving, ensuring they develop both conceptual understanding and practical skills.

On Day 1, his explanation of cyclic quadrilaterals focused on theoretical clarity but did not incorporate hands-on activities to engage learners practically. On Day 2, test preparation activities provided learners with practice opportunities but revealed gaps in fundamental skills, particularly diagram construction.

The systemic issues highlighted by T3, including inadequate training for teachers in lower grades, underscore the need for long-term interventions. While T3 used diverse resources, such as ICT tools, to support learning, a stronger emphasis on foundational skills and interactive methods could enhance learners' understanding of Euclidean geometry. T3's strategies, particularly the use of ICT tools, contributed to learning, but their effectiveness in addressing foundational gaps remains uncertain. Strengthening the focus on core mathematical skills and interactive teaching methods could have further improved learners' understanding of Euclidean geometry. This concurs with who emphasise that systemic factors, including teacher training and resource allocation, greatly impact curriculum implementation.

4.4.1.4 T4's Approach

On day 1, T4 used a mathematical set to demonstrate angles, 90°, 180°, and 360°. Before introducing the theorem that states "subtended angles in the same segment of a circle are equal," she demonstrated these angles on the protractor, engaging learners in hands-on measurement activities.

Figure 4.8: T4's demonstration of angles on the protractor

The learners actively engaged with protractors, taking notes and asking questions, which enhanced their understanding of the topic. Among the 46 learners, 35 had protractors, which increased curiosity and prompted numerous questions. For example, when a learner asked, “How can we measure 270 ?°” T4 responded by demonstrating the process, effectively fostering exploration and understanding.

Figure 4.9: Learner 10 measuring angles using a protractor in T4 class

T4’s use of hands-on tools, such as protractors, actively engaged learners and helped bridge gaps in understanding. Through directly interacting with these tools, learners were encouraged to explore measurement concepts and ask questions, reinforcing their comprehension. During the lesson, learners were instructed to measure different angles on diagrams provided by T4. As they worked, several learners asked clarifying questions about aligning the protractor correctly, while others compared their measurements with peers to verify accuracy. T4 moved around the classroom, offering guidance and correcting errors in measurement technique, ensuring that learners grasped the correct procedures. This aligns with constructivist methods, which emphasise that learners should construct their understanding while the teacher facilitates the learning process. This concurs with Elijah (2024), who notes that hands-on tools promote deeper learning by allowing learners to engage directly with the mathematical concepts, fostering exploration and problem-solving. This was apparent in the lesson, where learners engaged actively by using protractors, taking notes, and posing insightful questions, such as how to measure 270°. Through guided facilitation, the teacher promoted this active engagement, illustrating that the use of hands-on tools can significantly enhance conceptual understanding in mathematics.

When the researcher asked why T4 began by teaching measurement, T4 explained that:

“When teaching mathematics, it is important to build on learners’ prior knowledge to ensure a solid understanding, even if this approach is not explicitly stated in the Annual Teaching Plan (ATP)”

After introducing angles, T4 continued the lesson by drawing the theorem “subtended angles in the same segment of a circle are equal” on the chalkboard as illustrated in Figure 4.6.

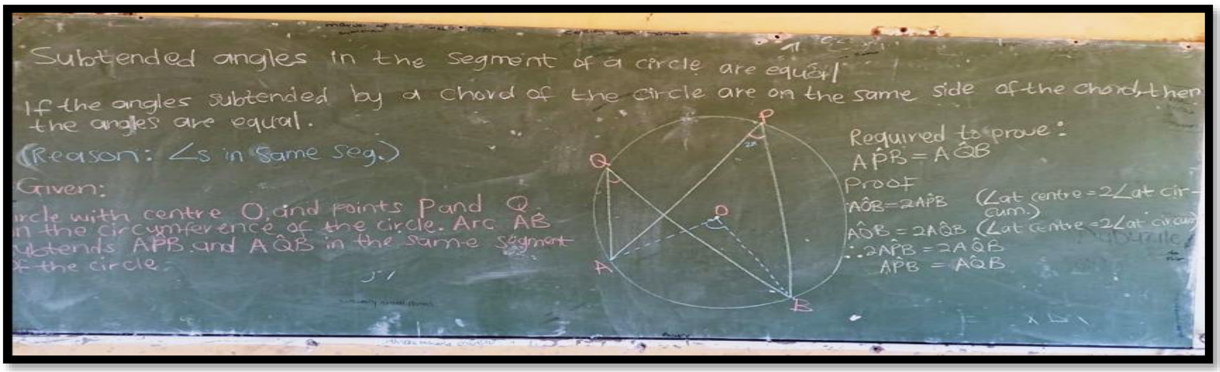


Figure 4.10: T4 illustrating that subtended angles in the same segment of a circle are equal

T4 drew the circle with centre O, radius, and the chord, defining each term. She reminded learners about the earlier concept that the “angle at the centre is twice the angle at the circumference” and elaborated on subtended angles in the same segment. She explained that equal angles subtend the same arc, reinforcing this with visual aids. T4’s reliance on diagrams contributed to learners’ conceptual understanding by providing clear visual representations of the key geometric concepts. By drawing the circle with centre O, radii, and the chord, while defining each term, she reinforced the relationships between angles and arcs, making abstract ideas more tangible.

This method effectively linked prior knowledge to new concepts, particularly when she reminded the learners that ‘the angle at the centre is twice the angle at the circumference’, and elaborated on subtended angles in the same segment. However, while the visual aids, such as using different colours to highlight key concepts and definitions, were beneficial, it is uncertain whether learners actively engaged with the material or if they passively absorbed the information. T4’s explanations provided necessary scaffolding, but deeper engagement may have required more interactive strategies, such as having learners construct their diagrams or apply the concepts independently. This additional scaffolding could have ensured that all learners comprehended the material rather than relying solely on T4's demonstrations.

When the researcher asked T4 to reflect on teaching methods used in the class, T4 highlighted the importance of adapting teaching strategies to meet learners' needs through practical engagement. She stated:

"I prefer that learners learn from hands-on experience".

She believes that using instruments helps learners strengthen their understanding of geometric relationships. This aligns with Tachie (2020), who emphasises that effective mathematics teaching requires sound content knowledge, strong instructional skills, and practical strategies to deliver lessons effectively. T4's hands-on approach aligns with Killen and O'Toole, (2023) recommendations, as it incorporates practical strategies that engage learners and reinforce their understanding of geometric relationships.

In emphasising hands-on learning and the use of instruments, T4 demonstrates strong instructional skills and the ability to make abstract concepts more tangible, which Killen et al highlight as essential for effective mathematics teaching. However, while these methods provided valuable hands-on experience, they may not have fully addressed all learners' challenges. Some learners might still struggle to comprehend key concepts without additional scaffolding. Incorporating group activities or peer collaboration could reinforce conceptual understanding by allowing learners to discuss, clarify, and apply ideas together. These strategies would offer extra support for those who require more guided practice, ensuring that all learners benefit from the hands-on approach. Thus, while T4's approach was engaging and aligned with best practices, integrating additional collaborative learning opportunities could further enhance learners' understanding and retention of geometric concepts.

When T4 asked about the learning support material she uses to prepare the lesson, T4 emphasised:

"Using diagrams and different colours to differentiate different shapes".

The use of colour coding and visual aids such as diagrams is recognised as an effective strategy for clarifying complex ideas, especially within educational settings (Pettersson, 2020). This approach helps to enhance understanding and retention by providing visual differentiation and context, making it easier for learners to grasp intricate concepts. Teachers who implement such techniques often find that they facilitate better engagement and comprehension among learners, thereby improving overall learning outcomes. For instance, T4's implementation of colour coding and diagrams significantly enhanced the understanding of geometric concepts, thereby facilitating a deeper grasp of material that might otherwise be overwhelming. Through integrating these visual tools, teachers can effectively bridge the gap between abstract theories and practical understanding, fostering an inclusive learning environment.

T4's lessons demonstrated the effectiveness of hands-on learning and interactive teaching strategies in Euclidean geometry. Her use of mathematical sets, protractors, and visual aids helped engage learners actively, reinforcing key concepts like angles and subtended arcs. This approach aligns with Elijah's (2024), constructivist view, which emphasises the importance of personal experiences in the construction of knowledge, as learners engage with tangible materials to make sense of abstract concepts.

However, while her methods fostered curiosity and participation, some learners struggled to apply these concepts during independent activities. This gap between comprehension and application highlights the need for additional scaffolding or differentiated instruction to support diverse learner needs. T4 could address this by incorporating collaborative tasks, where learners work together to solve problems, share insights, and support each other's understanding. This aligns with Elijah's (2024) constructivist approach, which emphasises that learners construct knowledge through personal experiences and social interaction. Through collaboration, learners engage with concepts actively, reinforcing their understanding through discussions and peer explanations. Furthermore, step-by-step problem-solving exercises can help learners bridge the gap between comprehension and application. By guiding learners through problems in a progressive manner, T4 can reinforce key concepts and offer structured support, enabling learners to build confidence in applying their knowledge independently.

T1 and T4 prioritised hands-on learning, encouraging learners to manipulate instruments like protractors and compasses to develop practical skills in Euclidean geometry. This approach effectively engaged learners in active problem-solving, allowing them to apply theoretical knowledge in a tangible context. Experiential learning plays a crucial role in fostering both conceptual understanding and engagement in mathematics. Xiu (2024) emphasises the importance of hands-on learning in educational contexts, suggesting that this approach enables learners to actively engage in the construction of their own knowledge.

It is important to state that rather than merely passively absorbing information, learners are encouraged to participate in direct experiences that facilitate deeper understanding and retention of concepts. This active engagement is crucial for fostering critical thinking and problem-solving skills, positioning hands-on learning as a vital component of effective pedagogical practices. This approach helps learners develop a deeper understanding of mathematical relationships by physically interacting with tools and models. This aligns with Killen and O'Toole, (2023), who emphasises that experiential learning promotes engagement by making abstract mathematical concepts more tangible, which enhances retention and application in problem-solving.

In this study, T4's use of hands-on tools, such as protractors, aligns with these perspectives, as it provided learners with an opportunity to explore geometric measurements through direct manipulation. Observations revealed that learners who engaged in hands-on activities demonstrated greater curiosity and willingness to ask questions, reinforcing their conceptual grasp of angles and measurements.

However, integrating more collaborative tasks or step-by-step problem-solving exercises could further support learners who struggle to connect these practical skills with theoretical concepts, ensuring a deeper understanding of geometric principles. Allowing learners to manipulate and experiment with the tools fostered a deeper understanding of the subject matter. T1's emphasis on revising prior knowledge of mathematical instruments and scaffolding learners through practical tasks created opportunities for collaboration and deeper conceptual clarity. Similarly, T4's focus on interactive methods, such as measuring angles and using protractors, encouraged curiosity and active participation.

In examining pedagogical methods for teaching geometry, it is essential to assess the effectiveness of various instructional strategies. T4's approach, which emphasises collaborative tasks and step-by-step problem-solving exercises, has the potential to address gaps observed in traditional methods. Unlike T1's structured, teacher-led framework, T4's integration of hands-on tools facilitates greater engagement among learners. This engagement is evident as learners actively participate in measuring and verifying angles, thereby enhancing their learning experience and promoting a deeper understanding of geometric concepts.

Observations showed that learners in T4's class were more willing to experiment with protractors and discuss their findings with peers, whereas in T1's lessons, some learners relied heavily on teacher guidance before attempting problems. However, despite this increased engagement, some learners in T4's class still struggled with accurately aligning the protractor and reading angle measurements correctly. These additions could allow learners to apply their knowledge in a supportive, peer-driven environment, facilitating stronger connections between theory and practice. To further bridge the gap between engagement and understanding, T4 could integrate structured group problem-solving sessions. In these sessions, learners take turns explaining their reasoning, which reinforces both conceptual clarity and collaborative academic skills. This method encourages deeper comprehension as learners discuss and dissect problems together, allowing for diverse perspectives and approaches to emerge. By fostering an environment where learners actively participate and communicate, T4 can enhance critical thinking and improve overall learning outcomes.

T2 and T3 relied more heavily on traditional teaching methods, focusing on structured diagrams and theoretical explanations to enhance conceptual clarity. While these methods can provide a solid foundation of knowledge, they may not fully foster active learner participation. As Tachie (2020) highlights, traditional approaches often limit opportunities for learner engagement, which can hinder the development of deeper understanding. T2 and T3's methods, while effective for presenting information, may benefit from incorporating more interactive elements or hands-on activities to better align with current effective teaching practices that emphasise active participation and critical thinking. Their approach focused on providing learners with clear and accurate representations of concepts, but limited opportunities for hands-on engagement. While diagrams helped clarify theoretical content, the lack of practical tasks may have exacerbated gaps in foundational skills, such as diagram construction. Without opportunities to apply the concepts through active learning, learners may struggle to fully internalise the skills necessary to construct and interpret diagrams independently. This contrast in teaching strategies illustrates a distinct variation in educational approaches within the classroom.

T1 and T4 emphasised the development of practical skills through experiential learning, while T2 and T3 focused on delivering theoretical knowledge and precise diagrammatic techniques. Combining practical and theoretical approaches might better address the challenges identified in Euclidean geometry instruction. With integrating hands-on activities with clear theoretical explanations, learners could gain a deeper understanding of geometric concepts, bridging the gap between abstract theory and practical application. This approach could provide a more comprehensive learning experience, reinforcing foundational skills while fostering active engagement. T1 and T4's methods seemed to engage learners actively, fostering curiosity and improving their confidence with mathematical instruments. However, while T2 and T3's strategies ensured clarity in geometric concepts, their heavy reliance on traditional methods was limited.

T3's reliance on Xitsonga helped learners grasp the material during instruction, but created barriers when learners attempted to answer questions in English. This highlights the importance of aligning instructional methods with assessment requirements to ensure learners are adequately prepared. This concurs with Xiu (2024), who discusses the challenges of bilingual instruction in mathematics, emphasising the delicate balance between making content accessible in learners' home language and preparing them for assessments in a second language. Ensuring that instructional methods support both comprehension and exam readiness is crucial for effective learning in bilingual settings.

4.4.2 Peer collaboration and engagement

In this section, the researcher presents finding on peer collaboration and engagement during geometry lessons.

4.4.2.1 Group work

The researcher did not observe any group work during T1, T2, and T4, except for Day 2 of T3, when learners were revising for an upcoming test. During T3, the learners were divided into groups of four, where they exchanged ideas and collaborated to come up with solutions. T3 actively moved around the classroom to assist them by providing clarification on concepts learners were struggling with, as well as asking guiding questions, which helped in stimulating deeper thinking on the topic, thus encouraging discussion. This approach aligns with Graven (2015), who emphasises that group work enhances problem-solving skills and deepens conceptual understanding by fostering communication, critical thinking, and peer support.

T3's use of group work facilitated these elements, enabling learners to engage more deeply with the material and support each other's learning. During the group activity, most learners appeared more engaged compared to when they worked individually. They discussed different approaches to solving problems, explained concepts to one another, and asked questions within their groups. However, not all learners participated equally; some appeared disengaged or hesitant to contribute, relying on their peers to take the lead. This uneven participation suggests that while group work fosters collaboration, some learners may benefit from clearer roles or more support to become actively involved. According to Gillies (2016), the effectiveness of group work is enhanced when learners are assigned structured roles and guided through collaborative norms, which can help ensure balanced participation and shared responsibility.

When the researcher asked the participants about the strategies they use to engage these learners, which is crucial for effective teaching, T3 shared the following approach to teaching Euclidean geometry:

> *"I sometimes place the learners in groups of three or four to work on corrections together while teaching one another how to solve problems. This method helps their peers understand the solutions better. After explaining the first theorem twice and grouping them for common activities on the perpendicular bisector, I understood the concept. If I assess it to be necessary, I will regroup them for the topic of cyclic quadrilaterals".*

This concurs with Graven (2015), who asserts that learners are required to engage actively in their educational experiences through activities such as debating, arguing, exchanging ideas, and collaboratively addressing real-life challenges. This approach emphasises the importance of interaction and collaboration in the learning process, fostering critical thinking and problem-solving skills essential for academic and personal growth. This engagement allows learners to construct meaning in mathematics classrooms. T3's approach aligns with this view, fostering a collaborative environment where learners can support one another. However, the lack of group work observed with T1, T2, and T4 suggests a missed opportunity to enhance engagement across classrooms.

These findings are underpinned by constructivist and social constructivist learning theories. Constructivism, as advocated by Piaget, posits that learners actively construct their own understanding and knowledge of the world through experiences and reflection. T3's practice of allowing learners to collaborate and make meaning through discussion and shared problem-solving aligns with this theory, as it enables learners to build upon their prior knowledge.

Furthermore, Vygotsky's social constructivism highlights the importance of social interaction in the development of cognition, where learning is mediated through engagement with more knowledgeable others. T3's facilitation of peer-to-peer teaching and collaborative exploration reflects Vygotsky's Zone of Proximal Development (ZPD), in which learners develop higher-order thinking skills through social engagement. The emphasis on communication, scaffolding, and guided participation observed in T3's classroom supports the principles of both theories and underscores the educational value of group work in enhancing learner understanding and engagement in mathematics.

4.4.2.2 Peer learning

During the observation, T3 implemented peer learning (PL) by asking learners to form groups to discuss the questions. However, no learner from the groups presented what was discussed. The lack of group presentations may have limited accountability and the opportunity for learners to reinforce their understanding by articulating their ideas to the class. Without the expectation to present their group's findings to the class, some learners appeared less motivated to participate actively in discussions, often allowing more confident peers to lead without contributing.

Additionally, the absence of public sharing may have enabled certain learners to remain passive, knowing they would not be required to explain their thinking in front of others. Presentations could have encouraged deeper engagement and helped learners clarify their ideas through structured speaking, which is a key component of meaningful learning. Structured peer learning, such as group presentations, could enhance outcomes by encouraging learners to take ownership of their understanding of knowledge. Presenting their ideas to the class not only reinforces individual learning but also promotes peer-to-peer teaching. This collaborative approach allows learners to clarify concepts for each other and address any misunderstandings that may arise. Encouraging such interactions can enhance overall comprehension and create a more dynamic learning environment.

This approach aligns with collaborative learning, which emphasises active learner participation, shared responsibility, peer interaction, and the co-construction of knowledge through dialogue and problem-solving. Presentations could have encouraged deeper engagement and helped learners clarify their ideas through structured speaking, which is a key component of meaningful learning. Structured peer learning, such as group presentations, could enhance outcomes by encouraging learners to take ownership of their understanding of knowledge. This approach aligns with collaborative learning, which emphasises active learner participation, shared responsibility, peer interaction, and the co-construction of knowledge through dialogue and problem-solving.

This observation can be framed within the theoretical foundation of Vygotsky's Social Constructivist Theory (1978), which emphasises that learning is a social process whereby individuals construct knowledge through interaction with more capable peers or adults. Specifically, the Zone of Proximal Development (ZPD) suggests that learners can achieve higher levels of understanding when supported by others within their learning environment. In this context, peer learning becomes an essential scaffold for knowledge acquisition, enabling learners to bridge the gap between what they can do independently and what they can accomplish through guided interaction. Furthermore, Bandura's Social Learning Theory (1977) supports the idea that learners often model the behaviours, attitudes, and outcomes of their peers. Therefore, incorporating structured group presentations not only fosters active engagement but also leverages observational learning as a tool for deeper cognitive processing.

During the interview, the researcher asked which method or technique proved to be more successful and why. T3 elaborated on their approach, stating:

> *"Learners can assist each other by collaborating to solve challenging theorems. When learners work together in mathematics classrooms, they actively create meaning".*

T3's perspective aligns with social constructivist principles, which emphasise the importance of collaboration in constructing knowledge. However, the absence of group presentations and insufficient scaffolding in T3's observed classroom practices indicate a gap between theory and practice, like that observed with T2. Further, T2, during their interview, highlighted the benefits of peer teaching, explaining:

> *"Sometimes I use a brighter learner to teach them. I realised that learners learn better when their peers teach them".*

However, the researcher did not observe this method being implemented in T2's classroom during the observation period. This raises questions about the consistency between T2's stated beliefs and their actual instructional practices. While T2 acknowledges the theoretical benefits of peer teaching, its absence during observations may indicate challenges such as time constraints, lack of training, or difficulties managing diverse classroom dynamics.

Furthermore, both T2 and T3's stated beliefs about peer learning align with Vygotsky's (1978) theory of the Zone of Proxima Development, where learners learn from the guidance of peers, parents, teachers, and other individuals who have more experience. Collaborative learning is further supported by Naidoo and Mkhabela (2019), who argue that peer teaching enables learners to co-construct knowledge through collaborative problem-solving. However, these benefits are contingent on careful planning and the teacher's ability to scaffold the peer learning process. (Graven, 2015).

4.4.3 Scaffolded verbal instruction

Learner engagement and interest in class are essential conditions for active learning. Teachers play an important role in ensuring that learners actively participate in teaching and learning, particularly in challenging subjects like Euclidean geometry.

4.4.3.1 Use of questioning

During the lesson observation, T1, T2, T3, and T4 effectively engaged learners by asking questions that were relevant to the content being taught. While most classes demonstrated active participation, learners in T3's class, where instruction was primarily in the mother tongue, often appeared confused, struggling to understand many of the questions. This suggests that while the use of the mother tongue in instruction may have supported comprehension initially, it could have created barriers when learners needed to engage with geometry concepts in English.

It was observed that several learners appeared to struggle with mathematical terminology such as "perpendicular," "segment," and "chord," which were not always clearly understood when presented in English. In some cases, some learners misinterpreted questions or gave incomplete responses, possibly due to confusion arising from the differences between everyday language use and formal academic language in English. These language-related challenges may have hindered their ability to apply concepts correctly and communicate their reasoning during problem-solving tasks. This aligns with Xiu (2024), who highlights that bilingual instruction in mathematics can create difficulties when learners must transition between their home language and formal mathematical discourse in English. However, research also shows that bilingual instruction can be effective when properly scaffolded.

Ali, Ihsan and Sherazi (2023) emphasise the importance of strategic code-switching and language support to help learners navigate mathematical meaning across languages, while Moschkovich (2021) argues that learners can engage in sophisticated mathematical reasoning in multilingual classrooms when provided with opportunities to discuss and make sense of concepts in both languages. In T3's case, reliance on the mother tongue may have initially made content accessible, but the absence of structured transitions to English may have hindered learners' ability to fully grasp and apply geometric concepts in the language required for assessments. This underscores the importance of balancing language accessibility with exam preparedness, ensuring that learners can effectively navigate both instructional and assessment contexts.

To address these challenges, participants were encouraged to share the questioning techniques they implemented in their classes. T1 shared an effective strategy:

> *"I encourage learners to draw a diagram of the model or object they are studying within a specific theorem or axiom. This helps them visualise the concept, and I then ask them to measure to check if their answers are consistent. This method not only enhances their understanding but also demonstrates the real-world significance of their learning".*

T2 added another valuable perspective:

> *"Working through a significant volume of questions related to a specific subtopic can greatly enhance learners' understanding and retention of concepts. Active engagement with these questions allows learners to apply what they've learned, reinforcing their grasp of the material. However, it's essential to recognise that in larger classes, providing individual attention can be more challenging, which may limit some learners' participation. Finding ways to*

support all learners in these settings can help enhance their engagement with the content and promote deeper understanding".

These strategies reflect scaffolded verbal instruction by breaking down complex concepts into manageable steps, providing learners with opportunities to visualise, apply, and refine their understanding. The instructional methods employed by T1 and T2 effectively facilitate learner engagement and skill development. T1's approach, which incorporates diagramming, and consistency checks, actively involves learners in the learning process. Evidence of this can be seen in the enhanced participation during questioning, where learners articulated the steps in problem-solving, such as drawing free-hand diagrams using a protractor with increased confidence and detail. Moreover, the ability of several learners to independently apply the diagramming technique to novel problems indicates significant progression in both accuracy and consistency of their work.

Conversely, T2's strategy focuses on the reinforcement of comprehension through repetitive practice. This method has led to observable increases in learner engagement, as evidenced by a rise in the number of clarifying questions posed during problem-solving sessions. Such inquiries reflect a deeper understanding of the underlying concepts. Additionally, learners demonstrated greater autonomy in solving problems, showcasing an improvement in their capability to apply concepts independently, with less reliance on teacher interventions. Overall, the integration of repetition and practice is evidenced to solidify comprehension and enhance problem-solving abilities among learners.

This aligns with the views of Clements and Samara (2020), who emphasise the importance of strategic questioning in mathematics teaching, an approach effectively employed by both T1 and T2. Observations revealed that questioning techniques prompted learners to engage in deeper thinking, particularly when they were asked, "How might you approach this problem differently?" In response, several learners paused to reflect and began exploring alternative methods, thereby shifting from a surface-level understanding to more critical and reflective engagement. Such questioning not only encouraged learners to move beyond memorised procedures but also fostered a deeper conceptual grasp. Furthermore, this approach promoted active participation while simultaneously strengthening understanding, as learners were required to articulate and justify their reasoning.

However, while T2's method of working through multiple questions reinforced learning, T2's ability to provide individual support was constrained by the larger class. The observation indicates that certain learners tend to exhibit disengagement when they are not actively participating in discussions or answering questions directly. This suggests that implementing supplementary strategies, such as structured peer discussion or small-group problem-solving sessions, may enhance overall participation and engagement in a variety of learning environments. Such approaches could facilitate deeper interaction among learners and cater to diverse learning needs more effectively.

When asked about the techniques for teaching Euclidean geometry, T2 stated:

> *"In the case of these two learners and others who scored below 10%, I devote time to speak to them to identify what challenges they are having with this section of mathematics".*

T2's individualised approach aligns with scaffolded verbal instruction by addressing specific learner challenges through targeted support. This strategy reflects effective differentiation, as its tailors' instruction to the needs of learners who may be struggling. By providing one-on-one explanations and prompting learners with guiding questions, T2 helps clarify misconceptions and gradually build their understanding. For example, T2 was observed assisting learners who struggled with measuring angles by demonstrating step-by-step alignment of a protractor and asking strategic questions to reinforce accuracy. This form of direct, personalised guidance is a key element of differentiated instruction. However, similar strategies were not observed among other teachers, as most relied on whole-class instruction without individualised scaffolding. While whole-class methods provide structure, they may not adequately address the needs of struggling learners who require additional guidance. T2's approach could serve as a model for enhancing learner participation and understanding through more personalised, scaffolded support in diverse classroom settings.

Another point that was of interest in the observed lessons was the questioning technique used in the Man'ombe circuit, which the teachers incorporated into their Euclidean geometry classes. This aligns with Killen and O'Toole, (2023), who outlined that teaching without good strategic skills and sound knowledge of the concept usually puts the teacher in a challenging situation, as learners tend to lose interest in the topic. In the observed practices, some teachers demonstrated strong strategic questioning that actively engaged learners. For example, T1 and

T2 used structured, step-by-step questioning to help learners build understanding. Their questions often followed a scaffolded approach, beginning with guided questions that prompted learners to recall foundational concepts before moving to open-ended questions that encouraged critical thinking. For instance, T1 would ask,

"*Can you explain what this theorem holds*?" before transitioning to a more challenging question like, *"How might we apply this theorem to solve a different type of problem*?". Learners responded actively and confidently, as evidenced by their quick verbal responses and eagerness to volunteer answers. In contrast to earlier lessons where learners hesitated, the structured questioning led to increased participation and more thoughtful, independent answers. Figure 4.11 shows a question that was asked by T3.

Question 1: *Solve for x in the diagram where OQ = 4 units is a perpendicular bisector and chord PR = 10 units.*

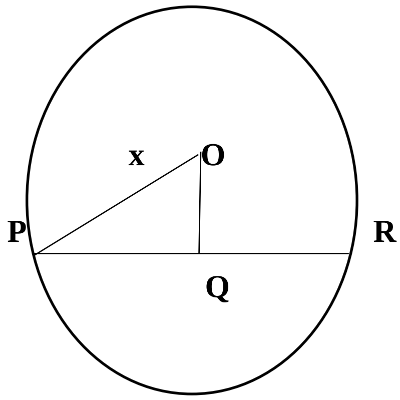


Figure 4.11: A question posed by T3 in class to solve for x in the given diagram

T3's questions, although aimed at clarifying concepts, often lacked follow-up prompts or scaffolding, making it difficult for learners to engage deeply. Many of T3's questions were closed-ended and focused on factual recall rather than encouraging conceptual reasoning or discussion. For example, T3 posed a question based on a diagram in Figure 4.11, asking learners *What is the value of x in this diagram*?, without prompting learners to explain their thought process or consider why a particular method would be used. As a result, learners often provided brief, factual responses without demonstrating a deeper understanding of the concept. This approach led to memorisation rather than reasoning, as evidenced by one instance where learners were able to state the correct statement but struggled to apply it to a different problem when asked to justify their answers.

Additionally, because there was little follow-up questioning, learners hesitated to elaborate on their responses or engage in meaningful discussions, hindering their ability to connect concepts and apply them critically. Below is an example of one learner's response observed.

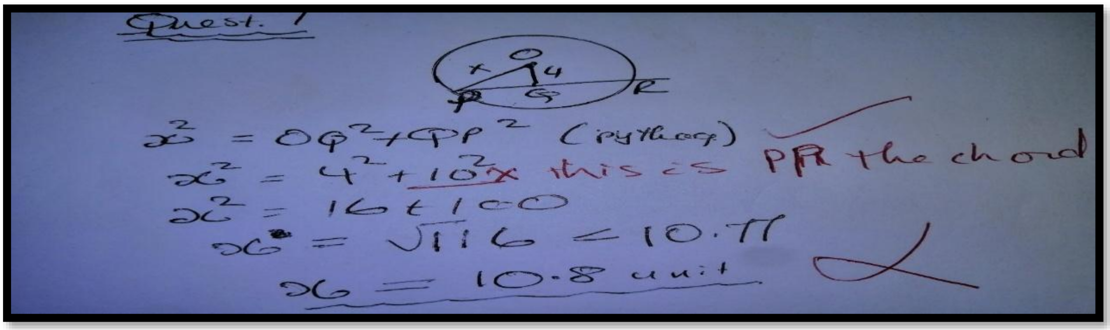


Figure 4.12: Learner 7's response from the T3 class

This suggests that while some teachers showed strong strategic questioning, there were areas where more targeted or varied questioning could have better supported learners' understanding and participation. For instance, teachers could benefit from using more probing questions, such as asking learners to justify their responses or explain how they arrived at a solution. Additionally, encouraging learners to compare different problem-solving approaches could deepen their conceptual clarity and foster critical thinking. The researcher inquired how the teacher helps learners identify relationships between figures and shapes, as well as the methods they believe provide a clear understanding of the concepts being taught. T4 stated:

> *"It is important to involve learners in their learning and not treat them as empty vessels waiting to be filled with information by their teachers. Teachers should employ teaching and learning approaches that enable learners to create knowledge. The teacher's role should be that of a facilitator or mediator in the learning process".*

This aligns with Clements and Samara (2020), who emphasises that effective geometry requires teachers to act as facilitators, empowering learners to construct knowledge actively. During the lesson, T4's facilitation was evident, as learners were encouraged to engage with the material through hands-on activities, group discussions, and structured questioning. For example, learners were asked to interact with visual aids, such as geometric diagrams, to explore relationships between angles and shapes.

T4's questioning techniques included guiding learners to justify their reasoning and apply prior knowledge, which helped many learners make meaningful connections between concepts. However, while the philosophy of learner-centred facilitation was evident, the level of participation varied. Some learners actively responded to T4's questions and contributed to discussions, but others struggled to actively participate. This suggests that while T4's approach was effective in encouraging learner involvement, additional strategies, such as peer collaboration or differentiated tasks, could further enhance participation and understanding, particularly for learners who require more guidance.

Effective teaching methods are vital to improving learners' understanding of Euclidean geometry, which constitutes a significant portion of the mathematics curriculum (20% in Grade 10 and 26.7% in Grades 11 and 12; DBE, 2011). In the observed lessons, the methods employed by teachers varied in their effectiveness. T1 and T4 incorporated hands-on activities and encouraged active participation, which helped learners engage with geometric concepts. For example, T1 frequently used physical models and guided learners through measuring angles with protractors, reinforcing procedural understanding. T4 similarly used interactive demonstrations to illustrate geometric properties.

However, despite this engagement, some learners struggled to apply geometric concepts independently when solving problems. This suggests that while hands-on methods improved participation, they may have required additional scaffolding, such as incorporating guided practice, where the teacher models problem-solving steps before allowing learners to try independently, could have helped bridge the gap between demonstration and application. Structured group work enhances peer learning by promoting interaction, collaborative problem-solving, and articulation of thought. Additionally, using think-aloud strategies during teacher modelling could have made the reasoning process more transparent and accessible to learners.

In contrast, T2 and T3 relied more on theoretical explanations and diagrams, which provided clarity but lacked interactive elements. While their structured approach helped learners recognise key geometric properties, some learners found it challenging to translate these theoretical concepts into problem-solving contexts. Incorporating exploratory tasks, such as asking learners to investigate geometric relationships through open-ended problems, could have provided more opportunities for critical thinking and application.

The integration of interactive technology, specifically dynamic geometry software such as GeoGebra, can significantly enhance learner engagement. By facilitating real-time manipulation of geometric figures, learners are allowed to test conjectures and observe the impact of altering various conditions on properties. This hands-on approach not only deepens understanding but also promotes critical thinking and problem-solving skills in the mathematical context. These approaches could support deeper conceptual understanding and improve learners' ability to apply geometric principles independently.

Considering the findings, it is evident that while the instructional approaches employed provided considerable value, they did not consistently address the varied needs of the learners. To foster a more profound comprehension and application of Euclidean geometry, an integrated approach that incorporates active participation, structured guidance, and differentiated support appears essential. This multifaceted strategy may enhance learner engagement and facilitate the development of critical thinking skills within the geometric context.

Therefore, strides must be made to improve the understanding of geometry concepts by enhancing the learners' geometry language and communication techniques and using teaching methods to enhance learners' attitudes toward this problem. Scaffolded verbal instruction, such as the questioning techniques used by T1 and T2, has the potential to address these gaps by engaging learners in meaningful dialogue and encouraging active participation. For example, T1's use of questions that prompted learners to visualise and measure geometric objects helped to clarify abstract concepts and foster communication in the context of geometry. Similarly, T2's individualised questioning approach allowed for targeted verbal explanations, which contributed to deeper learner understanding.

However, while these methods were effective in some instances, the observed predominance of teacher-centred methods in some classrooms suggests that further emphasis on scaffolding is necessary. This agrees with Killen and O'Toole (2023), who highlights that effective teaching requires not only sound content knowledge but also the use of strategic questioning to actively involve learners in constructing knowledge. Thus, teachers could benefit from increasing their focus on scaffolded verbal techniques to further enhance learners' communication and understanding of geometry. Improving learners' understanding of geometry is important not only for school performance but also for success in tertiary studies, particularly in the fields of mathematics, architecture, engineering, and health sciences, where geometry is foundational (Thamae, 2022). The long-term implications of weak geometry skills highlight the need for mathematics teachers to be fully equipped with sound knowledge and effective instructional strategies. As Luneta (2014) asserts, teaching without strategic skills and concept mastery can disengage learners and compromise their understanding.

4.4.4 Knowledge gaps and instructional challenges

T1 to T4 observed significant gaps in learners' prior knowledge, which impacted their ability to engage with Euclidean geometry. In T1's class, the researcher observed that learners lacked a deep understanding of foundational concepts. While some learners could provide correct statements with valid reasons, others offered correct statements with incorrect reasoning or even incorrect statements and reasons. This observation aligns with Van Hiele's theory, which suggests that learners must progress through structured levels of geometric understanding before mastering complex theorems (Machisi & Feza, 2021).

According to van Hiele, learners must first develop competency at Level 1 (Visualisation), where they recognise shapes by appearance, and Level 2 (Analysis), where they understand properties of geometric figures. However, in T1's class, several learners struggled at the tailors' levels, particularly in correctly identifying geometric relationships and justifying their reasoning. For example, some learners could recognise that a line drawn from the centre of a circle perpendicular to a chord bisects the chord but incorrectly justified their reasoning by referring to unrelated properties of triangles. This suggests that they had not yet reached Level 3 (Informal Deduction), which is necessary for reasoning with geometric proofs.

The presence of these knowledge gaps highlights the challenge of teaching advanced theorems when learners have not fully mastered lower-level geometric reasoning, as outlined in Van Hiele's theory. While T1 employed a structured teaching approach, there was limited evidence of explicit scaffolding to guide learners through the earlier van Hiele levels. For instance, during lessons on circle geometry, T1 introduced complex theorems without first revisiting foundational concepts such as identifying angle types or properties of basic shapes. This suggested an assumption that learners were already operating at higher reasoning levels. As a result, some learners struggled to grasp the logic behind the theorems, often resorting to rote memorisation rather than meaningful understanding.

To enhance the progression of learners through the Van Hiele levels, T1 could have integrated more guided questioning techniques. This approach would encourage learners to articulate their understanding of mathematical properties and relationships. Additionally, incorporating hands-on exploration using diagrams or manipulatives would provide valuable, tangible experiences that deepen comprehension and engage learners more effectively. Structured reasoning exercises, where learners justify each step in a proof or problem, might also help bridge the gap between recognition and analytical understanding, ultimately enabling them to apply theorems more confidently and independently.

Learners often recited theorems without truly demonstrating conceptual understanding, highlighting significant gaps in prior knowledge. This lack of understanding was evident when learners struggled to justify their answers or explain the reasoning behind the theorems they stated. For example, when asked to apply the angle at the centre theorem, several learners could recall the rule but were unable to explain why it worked or when it should be used. Others misapplied geometric properties, such as confusing the angle in a semicircle with the angle at the circumference, indicating that they were relying heavily on memorisation rather than reasoning through the relationships.

In T2's class, learners encountered challenges in applying fundamental geometric principles related to the theorem stating that "the angle at the circle's centre is twice the size of the angle at the circumference". This was particularly evident during exercises focused on the properties of angles. For instance, when tasked with identifying the circumference, chord, and segment of a circle, many learners struggled to provide explanations, despite being able to articulate the theorems. Additionally, some learners exhibited confusion, underscoring a significant lack of conceptual clarity regarding these geometric concepts. Like in T1's class, learners in T2's class struggled to apply geometric concepts during problem-solving, relying on rote memorisation of theorems without grasping their foundational principles.

T1 primarily used direct instruction, focusing on having learners memorise and recite geometric theorems. While this approach helped some learners recall theorems, it did not foster a deeper understanding of how to apply them. In contrast, T2 employed a more hands-on approach, incorporating tools like compasses and protractors to encourage learners to explore geometric concepts practically. T2 also focused on problem-solving with a variety of questions, aiming to strengthen learners' conceptual understanding.

However, this approach did not fully address learners' struggles with conceptual application. Despite the use of instruments and varied question types, many learners continued to struggle with logical reasoning and diagram interpretation. For example, some learners could measure angles accurately but could not explain the geometric relationships involved, such as why certain angles were equal or supplementary. Others had difficulty constructing logical arguments or justifying each step in multi-step problems, indicating that their reasoning skills and ability to connect concepts remained underdeveloped.

In T3's class, learners demonstrated poor construction skills and an inability to integrate multiple theorems to solve problems. Some learners frequently made errors in diagram construction, such as incorrect placements of points or lines when constructing perpendicular bisectors. When asked to apply the properties of isosceles triangles in solving problems, learners failed to integrate multiple theorems effectively, often making errors in their geometric reasoning and diagramming. Learners faced challenges in connecting questions to their prior knowledge, which further impeded their problem-solving processes. T3's teaching strategies did not fully help learners make connections between geometric concepts.

Unlike the more teacher-centred methods used by T1 and T2, where content was delivered through step-by-step explanations and guided practice, T3 adopted a more learner-centred approach by encouraging learners to work independently or in groups to explore geometric problems. However, this approach lacked sufficient structured guidance. Learners were often expected to construct diagrams or solve complex problems involving multiple theorems without first being scaffolded through simpler tasks. As a result, many struggled with diagram construction, often misplacing key elements like equal angles or parallel lines. They also had difficulty integrating multiple theorems, such as using both the angle at the centre and cyclic quadrilateral properties within one problem. The absence of step-by-step modelling or explicit questioning left learners without a clear framework, which hindered their ability to apply concepts effectively.

According to Van Hiele's theory, learners need to progress through levels of geometric understanding. However, T3's classroom observations suggested that many learners were still operating at Van Hiele Level 1 (Analysis), where they could identify properties of shapes but struggled to understand relationships between those properties or apply multiple theorems in a connected way. In some cases, their reasoning even reflected Level 0 (Visualisation), where understanding is based primarily on the appearance of shapes rather than their attributes. This limited their ability to link concepts and engage in higher-order problem-solving tasks.

In T4’s class, the learners’ prior knowledge influenced the lesson’s pacing. T4’s use of multiple problem-solving procedures clarified key concepts and theorems, supporting deeper conceptual understanding. T4’s teaching methods appeared more effective in addressing prior knowledge gaps than T1 and T3. T4 used a variety of problem-solving methods, incorporating hands-on activities and allowing students to actively engage with geometric theorems using compasses and protractors. This varied approach seemed to clarify complex concepts and support deeper understanding. In contrast, T1’s more rote-memorisation-based approach and T3’s difficulty in guiding learners to make connections between concepts were less effective in bridging learners’ gaps.

The researcher conducted interviews to gather teachers’ perspectives on the factors contributing to learners’ difficulties in geometry classes. The researcher’s interview questions were:

“What causes poor performance of the learners in the geometry classes?”

T1 emphasised:

“Learners fail in proving concepts and giving appropriate reasons.”

To address this, T1 encouraged learners to review unclear material and seek assistance. However, classroom observations suggested that these self-review strategies were not structured enough to support learners in overcoming reasoning challenges. While T1 promoted independent learning, there was limited evidence of explicit, teacher-led scaffolding to develop learners’ reasoning skills. For instance, learners were often left to rework problems on their own without guided questioning or modelled reasoning processes that could help them understand why certain steps were taken. This lack of structured support made it difficult for learners to move beyond memorisation and begin constructing logical, step-by-step arguments when solving geometric problems. T2 highlighted:

“A lack of understanding of concepts and negative attitudes toward the topic”.

To overcome these, T2 suggested hands-on learning activities involving tools like compasses and protractors, enabling learners to explore geometric ideas practically.

T3 explained that:

"Learners often lack a foundational understanding of shapes and figures".

To strengthen the learning experience, T3 could consider incorporating follow-up activities that build more explicitly on learners' prior knowledge. For example, guided practice sessions where students apply sketching techniques to increasingly complex shapes could reinforce their understanding. Structured feedback, either through peer review or teacher-led critique, would also support learners in refining their skills. Additionally, collaborative problem-solving tasks, such as group drawing challenges or real-world observation projects, could help bridge new concepts with what students have previously learned, fostering deeper connections and greater confidence. T4 shared that one big reason learners often struggle with Euclidean geometry is their belief, as expressed below:

"That the topic is inherently difficult".

This mindset can lower motivation, making it tough for them to engage with the basics. Additionally, T4 mentioned that some teachers feel "*uncomfortable with the topic*" themselves, which might be adding to the problem by leaving certain knowledge gaps unaddressed. This was evident in T4's class, where, despite reviewing foundational concepts, some learners remained disengaged.

The issue of knowledge gaps extends beyond individual classrooms. Both T2 and T3 highlighted these difficulties stemming from gaps in earlier phases of education.

Mathematics progressively builds, like a bricklaying process, where each concept depends on a strong foundation. Without this, learners struggle particularly when dealing with geometric proofs that require higher-order reasoning. van Hiele's theory supports this, emphasising that before learners can engage with formal proofs, they must first develop a deep understanding of basic shapes and properties. To bridge these gaps, mathematics teachers must assess learners' current geometric thinking levels and implement targeted interventions (Yi, Flores & Wang, 2020).

4.4.5 Deficit in prior foundational skills

Teachers observed deficits in learners' foundational mathematical skills, which hindered their ability to engage with advanced geometry topics. T1 highlighted that many learners lack foundational skills, a challenge often referred to as Deficits in Key Procedural Gaps (DKGP). T1 explained:

> *"At times, I must move forward with new concepts after identifying that learners lack understanding of previously taught material from lower grades. I then address the specific challenges each learner faces when marking their activities".*

T1 provided an example of this deficit, noting that some learners struggled with squaring numbers, an essential skill for applying Pythagoras' Theorem. For instance, some learners mistakenly multiplied the squared number by two instead of squaring it. This procedural error illustrates a broader issue in learners' conceptual understanding of mathematical operations. To address learners' errors, T1 employed individualised feedback by identifying specific misconceptions in each learner's work and providing targeted corrections. Instead of relying solely on whole-class instruction, T1 closely monitored individual progress and offered direct, personalised support to address mistakes as they occurred.

This approach ensured that learners received immediate clarification, preventing them from reinforcing incorrect procedures. Luneta (2014) emphasises that misconceptions in mathematics, if not addressed early, become deeply ingrained and hinder future learning. Research suggests that individualised feedback plays an important role in correcting these errors by reinforcing accurate mathematical reasoning. Observations suggest that this strategy contributed to a noticeable improvement in learners' understanding. Some learners who initially misapplied the rules of squaring were later able to correctly apply Pythagoras' Theorem in follow-up exercises. This demonstrates that immediate, personalised feedback helped reinforce both procedural accuracy and conceptual understanding

Both T2 and T3 observed that learners struggled with basic calculations, including fractions and conversions, which significantly hindered their ability to engage with advanced geometry concepts. T2 noted that learners often failed to recognise that all radii in a circle are equal, a fundamental concept when working with chords and arcs. This misunderstanding affected their ability to apply theorems correctly, leading to errors in geometric reasoning. In T2's classroom, difficulty with the equal radii concept led to mistakes in solving chords and arcs. For example, some learners were unable to recognise that triangles formed by two radii and a chord are isosceles, which caused errors when calculating missing angles or proving that two chords were equal. This misunderstanding also affected their ability to apply circle theorems accurately, leading to confusion when identifying congruent segments or arcs.

Similarly, in T1's classroom, weak arithmetic skills, especially in fractions and conversions, affected learners' ability to construct accurate geometric diagrams. This was evident in tasks requiring precise measurements and multiple geometric theorems applications. For instance, some learners struggled to substitute values correctly into formulas, made calculation errors when applying the Pythagorean Theorem, or failed to accurately find the perimeter and area of composite shapes. These arithmetic weaknesses often led to incorrect final answers, even when the geometric reasoning was sound, ultimately hindering their overall problem-solving effectiveness. T4 echoed these concerns, emphasising that learners' inability to perform basic algebraic manipulations often prevented them from solving geometry problems effectively.

In the context of geometric studies, particularly during instruction on the properties of tangents, it was observed that learners faced significant challenges when it came to rearranging equations to isolate unknown variables. This skill is essential for effectively solving problems related to line segments and angles, as it enables learners to manipulate mathematical expressions and arrive at desired solutions. Addressing these difficulties is vital for fostering a deeper understanding of geometric principles and enhancing overall problem-solving abilities in mathematics. This difficulty in algebraic manipulation created barriers when applying geometric theorems, as learners were unable to progress beyond setting up equations. Recognising these challenges, T4 implemented step-by-step scaffolding, incorporating visual aids and prior knowledge checks to reinforce learners' understanding. For instance, before introducing new theorems on angles in circles, T4 used diagrams to review basic angle properties and had learners label known angles.

During problem-solving tasks, T4 broke down complex problems into smaller, manageable steps, such as first identifying known values, then selecting the correct theorem, and finally applying it. This approach helped learners build confidence and apply concepts more accurately. This approach aligns with Van Hiele's Stage 2 (Analysis Level), where learners begin to recognise geometric relationships and develop procedural fluency. At this stage, learners analyse properties of geometric figures but may still struggle with formal deductive reasoning.

This concern regarding deficits in foundational skills is emphatically highlighted by Tachie (2020), who asserts that the inconsistent introduction of Euclidean geometry in the South African mathematics curriculum presents a significant challenge. Many schools cannot effectively teach geometry, leaving learners with weak mathematical foundations. This aligns with the observations made in T1 and T4 classrooms, where learners struggled with basic calculations, algebraic manipulations, and diagram construction. For example, learners in T1's class demonstrated difficulty squaring numbers, while those in T4's class faced challenges isolating unknowns in algebraic equations. These systematic deficiencies underscore the necessity for actions by the Department of Basic Education (DBE).

Providing teachers with specialised training and access to teaching materials is crucial for enhancing the quality of geometry instruction. The provision of resources such as standardised teaching guides, professional development workshops, and technological support has the potential to address foundational gaps and enhance learners' preparedness for advanced geometry.

4.4.6 Errors in basic concepts

Errors in basic concepts were prevalent across all observed classes, reflecting gaps in foundational understanding. T1 began the lesson by revising the Pythagoras theorem using a grade 10 textbook example. Learners were tasked with calculating the hypotenuse of a right-angled triangle, given that the other sides are 9 cm and 5 cm.

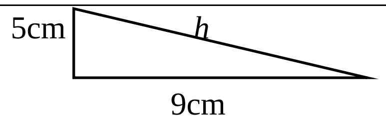


Figure 4.13: The Pythagorean theorem question from T1 class

Only 10 out of 20 learners managed to solve the problem correctly. Common errors included some learners incorrectly multiplying the hypotenuse by two instead of squaring it, while others failed to add the squares of the two shorter sides correctly. These errors reflect the foundational gaps in arithmetic skills identified in section 4.1, such as a weak understanding of squaring and addition. Below is an example of one learner's response that was observed.

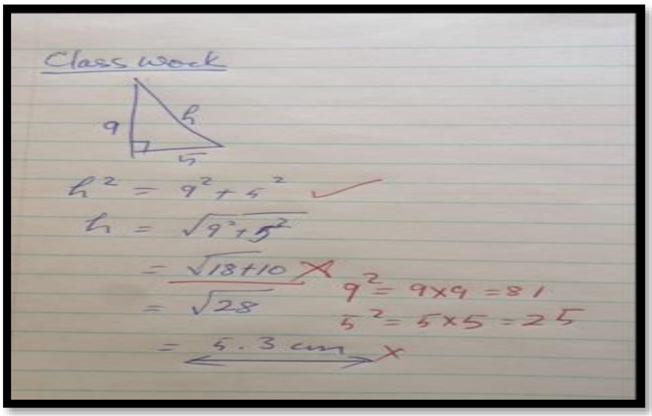


Figure 4.14: Showing Squaring posed a significant challenge with learners

The teacher's narrative is reflected in the learners' work in Extract 4.6. T1's feedback addressed errors by focusing on misconceptions related to squaring numbers. For example, the learner initially attempted to solve for the hypotenuse using the Pythagorean Theorem but incorrectly wrote $\sqrt{9^2 + 5^2} = \sqrt{18 + 10}$, showing a misunderstanding of squaring and addition rules. In response, T1 provided clear, step-by-step feedback directly on the learner's work, demonstrating that 9^2= 9x9 = 81 and 5^2= 5x5 = 25, and that these should be added to give 106. This visual correction helped clarify the squaring process. In follow-up tasks, learners showed improved accuracy, indicating that T1's targeted feedback helped correct the misconception and reinforced the correct procedure.

Scaffolding could help further by providing structured support, such as step-by-step guidance when applying the Pythagorean theorem. For instance, learners could be given a guided template that breaks the process into smaller parts: identifying the lengths of the perpendicular sides, writing out the formula $a^2+b^2=c^2$, substituting the correct values, performing each squaring step separately, and finally calculating the square root of the sum. Additionally, using colour-coded diagrams to highlight the sides of the triangle or providing a worked example with annotations would help reinforce understanding. This structured approach would help learners avoid errors in squaring and substitution. This resonates with Luneta (2014) who emphasises that step-by-step guidance reduces cognitive overload and allows learners to focus on key procedural elements, making abstract concepts more accessible.

Although learners effectively used calculators to perform operations, they often relied on the tool without understanding the underlying concepts. For instance, some learners entered incorrect operations, such as multiplying instead of squaring, which resulted in errors. This suggests that while calculators can aid in computation, an overreliance on them without conceptual understanding may reinforce misconceptions rather than clarify mathematical principles. To address these issues, strategies such as visual aids, peer teaching, and step-by-step scaffolding can help learners develop a stronger conceptual foundation while refining their procedural fluency. For example, visual aids like diagrams or number lines can make squaring numbers more concrete. This helps learners understand the concept without relying too much on the calculator.

Peer teaching gives learners the chance to explain the process to each other, which reinforces their understanding and helps clear up any misconceptions about how squaring works. And scaffolding breaks the squaring process into smaller steps, guiding learners along the way so they can grasp both the "how" and the "why" behind the procedure, not just the mechanics of using a calculator. According to Smith et al. (2018), such strategies help solidify foundational understanding and can be particularly effective for geometric concepts, where learners often struggle to see the connections between definitions and theorems.

On day 2 of the observation with T2, the class worked on the theorem regarding the angle between a tangent to a circle and a chord drawn at the point of contact. This theorem states that the angle between a tangent and the chord is equal to the angle subtended by the chord in the alternate segment. The class was given an activity to apply the tangent-chord theorem, which is presented in Figure 4.15.

Find the values of the unknown letters, stating the reasons.

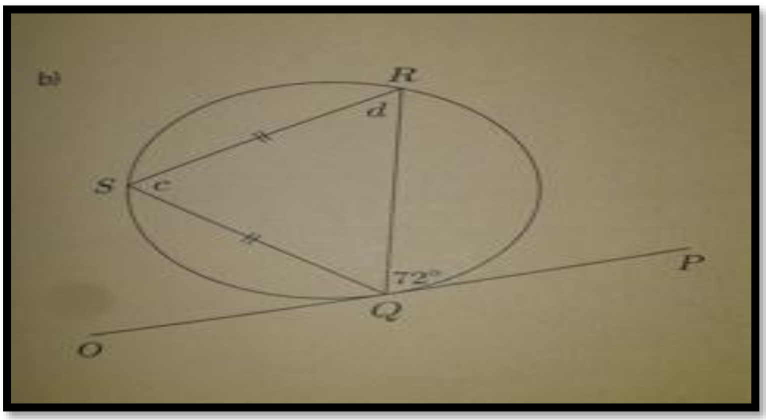


Figure 4.15: Tan-chord theorem question from T2 class

Most learners demonstrated an understanding of the tangent-chord theorem. However, one learner, as illustrated in Extract 4.16, struggled to grasp concepts and complete the task accurately.

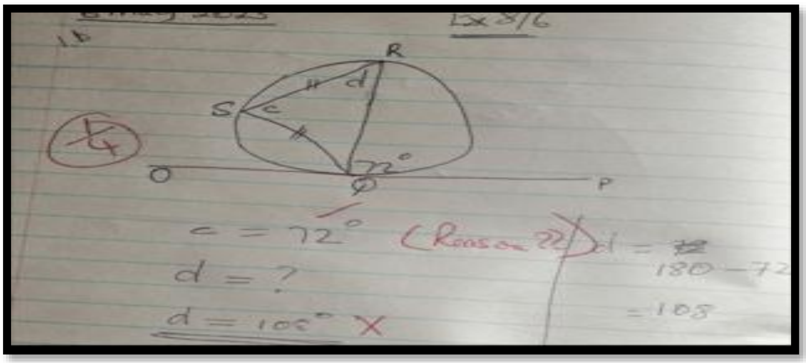


Figure 4.16: Learner 1's response from the T2 class

The learner in Extract 4.16 failed to provide a reason for their solution and struggled to recognise equal radii, which are fundamental in solving problems related to circles. Understanding tangents, radii, and chords is essential for grasping the relationship within a circle. This task aligns with Levels 2 and 3 of the van Hiele levels of geometric thought, emphasising understanding relationships between geometric definitions and engaging in formal reasoning. However, the errors observed suggest that some learners, such as the one in Extract 4.16, were operating at level 1. At this level 1, learners focus on the recognition of geometric figures by their appearances. At Level 3, learners begin to explore the relationships between definitions, theorems, and their corollaries and to engage in formal proofs, which require logical reasoning and conceptual understanding.

During the review of learners' work, T2 offered encouraging feedback to learners who explored multiple methods for solving problems. He highlighted the value of their diverse approaches and encouraged them to consider adopting more streamlined techniques. T2 actively guided learners through questioning, prompting them to reflect on why certain methods were more efficient than others. For instance, he asked questions such as;

"What happens if you approach this problem differently?" and "Can you explain why this method works?"

This kind of questioning helped learners reach higher levels of reasoning by considering alternative strategies. Additionally, T2 provided worked examples to scaffold learners' understanding, demonstrating step-by-step how to simplify their approaches while deepening their conceptual understanding. Through these interactions, T2 encouraged learners to think critically and develop more efficient problem-solving skills.

As noted by T2 and T4, the negative attitude towards Euclidean geometry may be specific to this branch of mathematics but can also reflect broader challenges in mathematics education. Learners may struggle with other areas due to a general lack of confidence in mathematical problem-solving. The researcher asked T2 about their approach to allowing learners to use multiple procedures when solving problems. T2 explained:

> *"As long as learners can navigate the solution without overwhelming themselves with too many steps, I see no issue. I encourage them to focus on clear and concise reasoning when tackling problems in Euclidean geometry".*

T2's strategy of offering feedback on multiple solution methods was intended to help learners develop clearer reasoning and more efficient problem-solving approaches. In practice, some learners were able to apply multiple procedures effectively, demonstrating logical reasoning and clear, step-by-step solutions. However, others struggled with selecting the most efficient method, sometimes including unnecessary steps that complicated their reasoning. This suggests that while T2's strategy of offering feedback on multiple solution methods encouraged flexibility, additional scaffolding may be required to help learners refine their problem-solving strategies. As Luneta (2014) discusses, learners often need structured guidance to bridge the gap between exploring multiple methods and developing a streamlined approach.

Luneta (2014) emphasises that addressing misconceptions and providing opportunities for conceptual clarification are essential in mathematics instruction. Without targeted feedback, learners may continue reinforcing inefficient problem-solving habits, leading to persistent errors. In the context of T2's classroom, where learners explored multiple solution methods, reinforcing reasoning strategies through structured questioning and guided reflection is crucial. This approach not only helps learners clarify their understanding but also encourages them to recognise and correct misconceptions in their problem-solving processes. Through guiding learners to consider why certain methods are more effective, T2 can help them strengthen their conceptual foundation and develop more efficient, accurate solutions.

T2's perspective highlights an important consideration: learners may become confused when employing too many steps in their problem-solving process, which can confuse their understanding of the underlying concepts (i.e. misidentified equal radii, failing to provide a reason for their solution, inability to connect relationships between tangents and subtended angles). Problem-solving methods need to align with a learner's understanding of the concepts. However, some learners may believe that using fewer steps means they have not fully addressed the question, which can lead to unnecessary complexity. Encouraging learners to explore different strategies, such as scaffolding and individualised feedback, teachers suggested incorporating strategies like guided practice, interactive activities, and frequent revision sessions. Observations revealed that lessons integrating interactive elements, such as group problem-solving, engaged learners, and promoted better understanding.

On the very same day 1 with T3, after teaching the theorem that states opposite angles of a cyclic quadrilateral are supplementary. Using the chalkboard, T3 provided a clear and comprehensive explanation of the theorem, ensuring that learners understood its principles. Learners were actively engaged during the lesson, with many posing questions and responding to inquiries from the teacher, which indicated their interest in the topic. While learners' lack of confidence often influenced their engagement with Euclidean geometry, this was most evident in their struggles to apply the theorem accurately. Some learners had difficulty applying the theorem correctly, frequently making errors in their reasoning or steps. For instance, when asked to justify their approach, many hesitated, suggesting that they had not fully internalised the logic behind the theorem. However, their questions, such as "What if this side of the triangle changes?" indicated a curiosity and an attempt to grasp the deeper relationships between the geometric elements, suggesting potential gaps in their understanding. Despite these struggles, some learners demonstrated improved performance when guided through these questions, indicating that further engagement with structured feedback could enhance their ability to apply the theorem more confidently and accurately.

As part of the class activity, the learners were assigned Question 1(a) from Exercise 8 in their Siyavula textbooks as shown in Figure 4.16 below, which required them to apply the theorem to solve problems involving cyclic quadrilaterals. Figure 4.17 below presents the selected example of the learners' work, highlighting the varying levels of performance. Errors observed in learners' work, such as misidentifying equal radii or failing to calculate angles accurately, suggest that foundational gaps persist. Out of the six books reviewed, errors were found in two, with extracts provided as evidence. These errors highlight the learners' struggle to progress beyond van Hiele Level 2, accentuating the need for targeted intervention.

Question 1 (a)

In the following diagram, calculate the angles represented by a and b

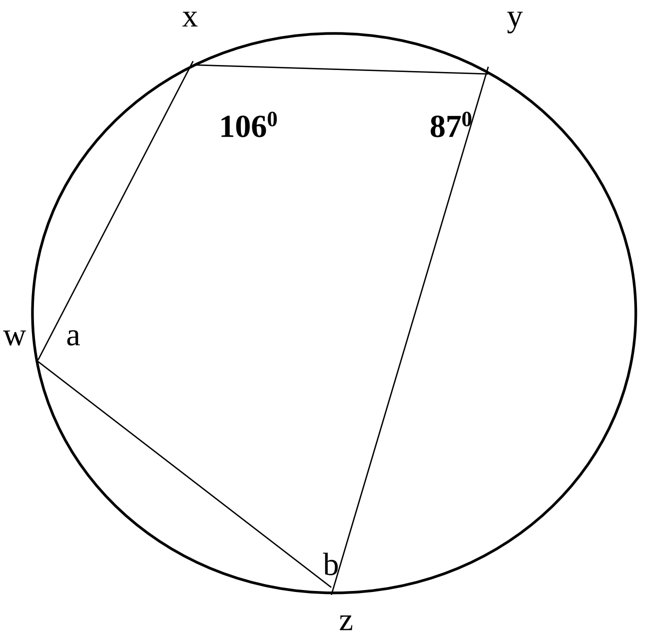


Figure 4.17: Cyclic Quadrilateral question from T3 class

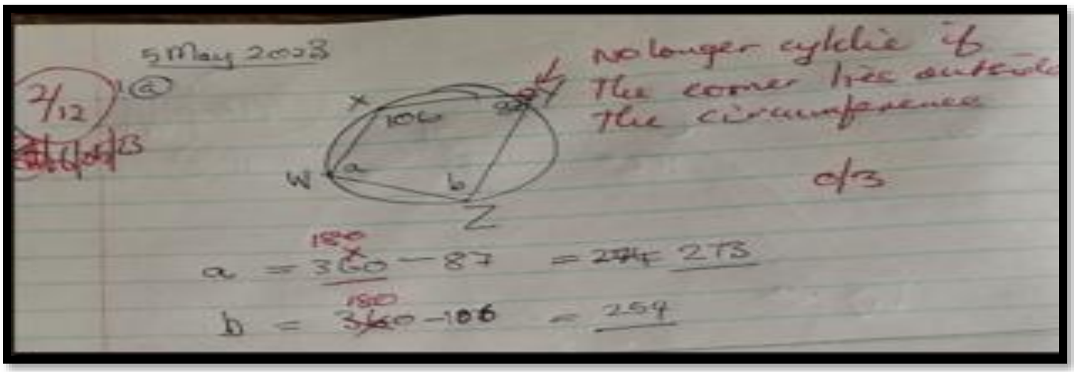


Figure 4.18: Learner 10's response from the T3 class

While some learners accurately identified and calculated supplementary angles, others did not use compasses, resulting in irregular shapes, as shown in Figure 4.17.

One learner showed a lack of precision in constructing the cyclic quadrilateral, drawing angle XYZ outside the circumference of the circle, which disqualified the figure as cyclic. Diagrams must be constructed neatly and accurately so that they conform to the geometric description provided, even if they are not drawn to scale. For accurate representation, circles must be drawn as true circles, not ellipses, and quadrilateral sides should be straight and constructed with a ruler.

During T3's lesson on cyclic quadrilaterals, learners were instructed to reproduce diagrams accurately using compasses and rulers. However, many failed to use these tools, resulting in irregular shapes and misaligned angles. This lack of precision affected their ability to correctly apply geometric properties. For example, an inaccurately drawn cyclic quadrilateral may lead learners to misidentify key relationships, such as the sum of opposite angles or the alignment of intersecting chords. Inaccurate diagrams can cause incorrect angle measurements, which in turn affect the application of theorems, such as the inscribed angle theorem.

These inaccuracies may lead learners to incorrectly conclude that angles are equal when they are not, or fail to recognise parallel lines, thus hindering their problem-solving ability and understanding of geometric relationships. While learners were expected to use compasses, it was unclear whether T3 explicitly reinforced or demonstrated their correct use. This suggests that additional emphasis on geometric tool application could improve accuracy in diagram construction. Explicit instruction and repeated practice can help learners develop precision in their geometric representations. Teachers can incorporate guided demonstrations on proper instrument use, structured exercises focusing on constructing accurate diagrams, and technology-based tools such as GeoGebra to provide dynamic visualisations of geometric properties.

The findings align with studies such as Yi, Flores, and Wang (2020), which emphasise the need for addressing foundational gaps and misconceptions to improve learners' engagement and performance in geometry. Strategies like scaffolding, diagnostic assessments, and teacher modelling of correct diagram construction could mitigate these challenges and reinforce conceptual understanding through accurate representations. Despite the inaccuracies in diagram construction, most learners demonstrated a good understanding of the concept. Out of six activity books reviewed, only one learner did not get the correct answer.

Figure 4.17 shows that the learner incorrectly used 360 degrees instead of 180 degrees to calculate supplementary angles, misunderstanding the difference between the sum of supplementary angles and the internal angles of a quadrilateral. This error suggests that the learner probably did not verify their reasoning or take advantage of resources like the textbook to confirm their calculations. Encouraging learners to cultivate the habit of reviewing their work and utilising available resources can significantly enhance their understanding during practice.

In one of the books reviewed by the researcher, a learner miscalculated the angles by mistakenly summing them to 180 degrees, which produced the following answers:

Angle a: $180 + 87 = 267^{\circ}$

Angle b: $180 + 106 = 286^{\circ}$

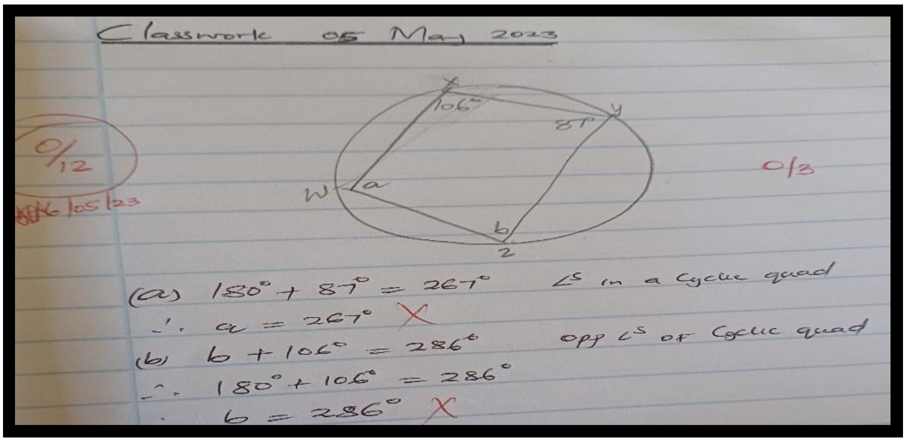


Figure 4.19: Learner 3’s response from the T3 class

The answers of this learner show a lack of understanding of the properties of cyclic quadrilaterals, especially the sum of the angles. This aligns with Van Hiele’s theory of geometric thinking, suggesting that the learner is likely operating at Level 2 (analysis), where they struggle to understand the relationships between properties of shapes. At this stage, learners can recognise individual properties of the shape but struggle to relate them systematically or apply them to solve problems effectively. For example, when working with cyclic quadrilaterals, learners may correctly identify that opposite angles should be supplementary, but they may struggle to connect this property to the diagram or apply it in problem-solving. They might recognise that an angle is part of the cyclic quadrilateral but fail to correctly use the relationship between angles to determine the missing angle, reflecting their difficulty in analysing how properties interact systematically, which is characteristic of Level 2 thinking.

The errors made by this learner demonstrate that they did not fully understand the requirements of the task or the properties of cyclic quadrilaterals. Providing additional support and guidance, such as step-by-step scaffolding or modelling accurate diagram construction, could enhance learners' comprehension and performance.

Observing during T4's lesson, the focus was on the theorem related to angles subtended by the same chord in the same segment. Learners were assigned Exercise 8-3, Question 2 as shown in Figure 4.20, which asked them to calculate unknown angles based on relationships between chords and subtended angles.

Exercise 8 – 3

(2) *Find the value of the unknown angles.*

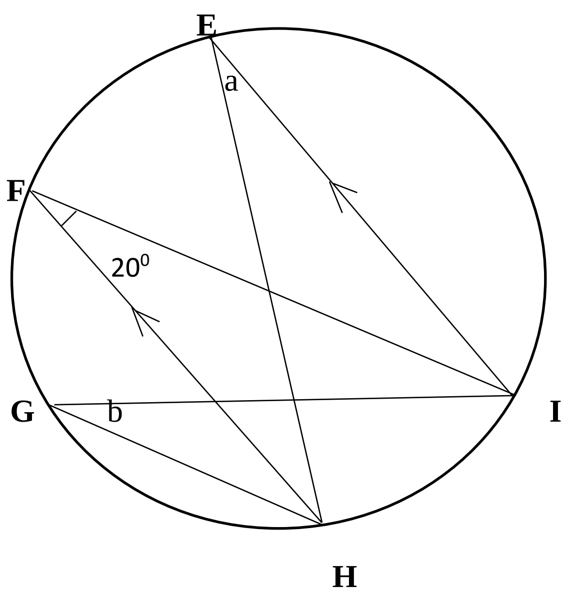


Figure 4.20: Angles Subtended by the same chord question from the T4 class

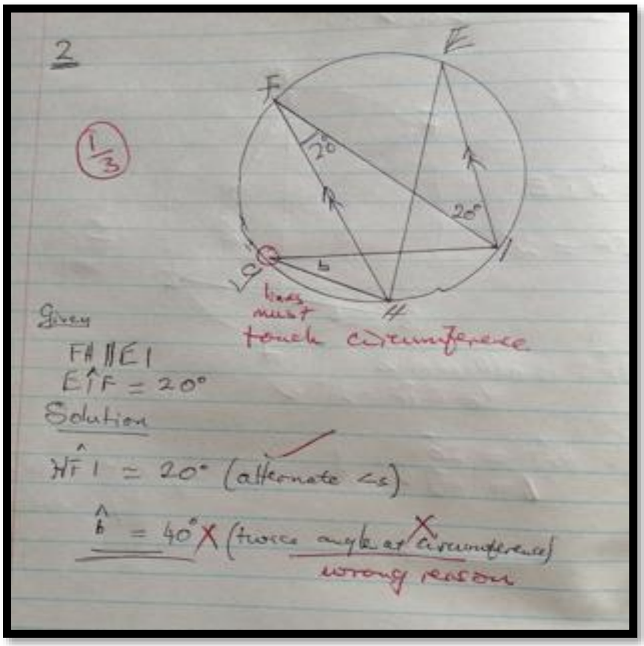


Figure 4.21: Learner 4's response from the T4 class

One learner encountered two challenges while attempting to solve the problem. The arc labelled HI subtended three angles within the same segment at points G, F, and E. According to the subtended angle theorem, these angles should be equal. However, the learner mistakenly concluded that angle G (denoted as b) measured 20 degrees and offered an incorrect justification for this conclusion. Despite this error, the learner demonstrates a solid understanding of the theorem when consulting another resource, successfully solving related problems with confidence.

To support learners, T4 guided them to identify the relevant arcs that subtend the angles accurately. The teacher emphasised that chord HI subtends angles at points F and E, while chord FE subtends angles at points H and I, helping learners make connections between chords and subtended angles. T4's approach of clarifying relationships in the diagram helped some learners refine their understanding, though persistent errors in identifying arcs suggest a need for further scaffolding.

4.5 DISCUSSION OF FINDINGS: POSSIBLE WAYS IN WHICH THE TEACHING AND LEARNING OF EUCLIDEAN GEOMETRY CAN BE IMPROVED

During interviews, T4 and other teachers suggested strategies: 1. encourage learners to actively redraw diagrams, measure angles, and verify their answers to ensure consistency in their work. 2. Provide ample opportunities for practice with similar problems to help reinforce understanding and build their confidence. 3. Foster peer collaboration through group work or pair activities where learners can discuss and compare their approaches, helping to clarify misunderstandings. 4. Use real-world applications of geometric concepts to make the material more relatable and demonstrate its practical value. 5. Incorporate formative assessments that allow teachers to identify specific gaps in understanding and provide targeted interventions. For example, T1 shared:

> *"I encourage my learners to draw diagrams of the models or objects they are working with concerning the theorem or axiom at hand. Then, I prompt them to measure and verify whether their answers are consistent. This approach helps them see the real-world application of what they are learning".*

T2 mentioned:

> *"Doing a lot of questions related to the subtopic".*

Also, T4 noted:

> *"I will check learners' prior knowledge on the basics of the theorem like when looking at the Tan-chord theorem one needs to check if learners know the sum of angles in a triangle, the relationship between the tangent and the radius or the diameter, etc, then I build the concepts from there going forward".*

Incorporating such strategies into classroom practice, alongside structured practice tasks, could help learners develop stronger conceptual understanding and improve their confidence in applying the subtended angle theorem.

The errors made by some learners demonstrate that they did not fully understand the requirements of the task or the properties of cyclic quadrilaterals. Providing additional support and guidance, such as step-by-step scaffolding or modelling accurate diagram construction, could enhance learners' comprehension and performance.

Some errors committed by learners suggest that they probably did not verify their reasoning or take advantage of resources like the textbook to confirm their calculations. Encouraging learners to cultivate the habit of reviewing their work and utilising available resources can significantly enhance their understanding during practice of Euclidean geometry.

There were observations of incomplete work and lack of participation from some learners which suggested that negative attitudes toward Euclidean geometry may have played a role in their struggles. Learners who lacked confidence in the subject were more hesitant to engage in problem-solving tasks, reinforcing the perception that Euclidean geometry is inherently difficult. To counteract these attitudes, teachers can integrate practical applications of geometry and share success stories of other learners' understanding to inspire learners, making the subject more relatable and approachable.

One specific improvement strategy involves incorporating collaborative group activities. These activities could involve learners working together to construct geometric diagrams, solve problems, or discuss reasoning processes, promoting peer learning and fostering a deeper understanding of the subject.

The integration of Information and Communication Technology (ICT) tools, such as dynamic geometry software like GeoGebra, can significantly enhance learner engagement by offering interactive and visually engaging representations of geometric principles. These tools allow learners to experiment and visualise geometric relationships dynamically, making abstract concepts more accessible and reinforcing their comprehension (Ntshangase, Ndlovu & Oladele, 2024).

4.6 CHAPTER SUMMARY

This chapter examined and interpreted the data collected through classroom observations, semi-structured interviews, and document analysis, with the purpose of exploring teachers' practices and experiences in teaching Euclidean geometry to Grade 11 learners. The findings illuminated the instructional techniques employed by teachers, the challenges they face, and the strategies they adopt to enhance learner engagement and conceptual comprehension. Analysis of the data revealed key themes that highlighted both the strengths of current classroom practices, such as the effective use of questioning and visual representations, and ongoing challenges, including limited learner participation, time pressures, and curriculum demands.

The insights gained provide a basis for considering ways to further develop and support teaching practices. The following chapter draws on these findings to present a synthesis in relation to the research questions, the theoretical framework, and the existing literature, thereby contributing to a more comprehensive understanding of how teachers' experiences and instructional approaches influence the teaching of Grade 11 Euclidean geometry.

CHAPTER 5: FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION: SUMMARY OF STUDY

Chapter 1 introduced the study, outlining the research problem, the rationale for the investigation, the research questions, and considerations of trustworthiness and ethics. Chapter 2 offered a comprehensive review of the literature concerning teachers' experiences and strategies in teaching Euclidean geometry. This chapter also explained the theoretical framework underpinning the study and discussed the challenges faced by teachers, alongside various pedagogical approaches to teaching Euclidean geometry. Chapter 3 detailed the research methodology and design, including the data collection methods, which comprised classroom observations and semi-structured interviews with Grade 11 mathematics teachers. Chapter 4 presented the analysed findings from the data, structured around emergent themes and illustrated with direct quotations from the participants, reflecting their perspectives on teaching Euclidean geometry to Grade 11 learners. The chapter concludes with a summary of the findings, recommendations, and the overall conclusions of the study.

Poor learner performance in geometry adversely affects overall achievement in mathematics. Geometry functions as a central theme within the mathematics curriculum and provides a valuable means of visualising arithmetic and algebraic concepts (Siyepu & Kizito, 2022). However, several teachers were not formally trained in teaching Analytical or Euclidean geometry, as the subject had been removed from the curriculum during their training period. In this context, the study examined the experiences and instructional practices of mathematics teachers in teaching Grade 11 Euclidean geometry.

The study was guided by Van Hiele's theory, which outlines the stages through which learners develop their understanding of geometry. The theory identifies five levels of geometric thinking: visualisation, analysis, abstraction, formal deduction, and rigour. Progression occurs sequentially, and learners generally struggle to grasp a higher level without mastering the preceding stage.

Using a qualitative approach, it investigated the experiences and instructional practices of mathematics teachers teaching Euclidean geometry in Grade 11. Data were gathered through semi-structured interviews and classroom observations from a purposively selected sample. Thematic analysis based on the objectives was employed to examine the data.

The study was guided by the following sub-questions:

- What techniques do Grade 11 mathematics teachers employ in teaching Euclidean geometry?
- What challenges do Grade 11 mathematics teachers and learners encounter in the teaching and learning of Euclidean geometry?
- How can the teaching and learning of Grade 11 Euclidean geometry be improved?

The findings of the study are outlined in the next section.

5.2 SUMMARY OF THE STUDY FINDINGS

5.2.1 Techniques used by Grade 11 mathematics teachers employ in teaching Euclidean geometry

i. The teachers began by evaluating what learners had retained regarding the use of mathematical instruments and engaged in remedial teaching to make every learner knowledgeable.

ii. Teachers revisited concepts from previous grades that were relevant to the new theorems, thereby enabling learners to build upon their existing knowledge.

iii. Some teachers encouraged rote learning by requiring learners to memorise and recite theorems aloud. At the end of each lesson, learners participated in activities to practise what they had learned.

iv. Not all teachers require learners to prove the theorems covered. For others who did so they said this helped them internalize both concepts and procedures.

v. A logical approach was emphasised in written activities. Learners were instructed to identify the facts in each diagram, articulate the information provided, and determine the missing value to be calculated.

vi. Some teachers adopted peer-teaching strategies, grouping learners in threes or fours to collaborate on corrections. This informal interaction facilitated shared learning, as learners taught one another how to approach challenging problems, thereby strengthening collective understanding.

vii. One teacher adopted an examination-oriented approach, resulting in a rushed delivery of theorems. This practice ultimately disadvantaged learners who required more time to fully grasp the concepts.

5.2.2 Challenges that Grade 11 mathematics teachers and learners encounter in the teaching and learning of Euclidean geometry

i. Many learners struggled with using a compass and accurately reading a protractor when measuring angles, skills introduced in the Grade 6 curriculum but critical for subsequent understanding. A lack of foundational knowledge from earlier grades thus posed a major obstacle.

ii. Classroom observations revealed limited learner engagement, as few sought clarifications during lessons.

iii. Learners also found it difficult to identify relationships between angles and chords. Although many could recognise geometric shapes based on their properties, nearly half were unable to transfer this knowledge from basic diagrams to exercise questions. Consequently, they struggled to prove concepts or provide appropriate justifications in problem-solving.

iv. The absence of diagrams in some questions exacerbated this issue, as learners often overlooked key features necessary for accurate calculation of angles or distances. Many lacked a solid understanding of shapes and figures.

v. The complexity of questions, coupled with a lack of clarity in instruction, compounded these difficulties.

vi. The use of English in textbooks posed a language barrier, further impeding comprehension. Classroom observations indicated that learners often struggled to communicate and prove geometric concepts, highlighting their unfamiliarity with geometric language.

vii. The majority of learners showed weak conceptual understanding and negative attitudes towards the subject. Many perceived Euclidean geometry as difficult and were unmotivated.

viii. Some teachers themselves expressed uncertainty or lack of confidence about teaching the topic effectively.

5.2.3 How can the teaching and learning of Grade 11 Euclidean geometry be improved

The study identified effective methods for improving the teaching of Euclidean geometry.

i. It is crucial for teachers to emphasise the teaching of geometric proofs, enabling learners to use precise language and notation.

ii. Actively engaging learners from the outset is essential, and using colour-coded or shaded diagrams to distinguish shapes and components proved effective.

iii. Allowing learners to manipulate carved shapes or physical models further enhanced understanding, as these hands-on experiences facilitated comprehension of theorems.

iv. Teachers who integrate physical models as teaching aids enable learners to develop a more meaningful understanding of geometric concepts and theorems.

v. Transforming certain theorems into folk songs was also recommended as a mnemonic device to aid memorisation.

5.3 CONCLUSIONS

From the findings of the study, it became clear that learners lacked prior knowledge of concepts and shapes drawn by mathematical instruments, which are foundational to Euclidean geometry. The teachers' skills show need for updating on how best to deliver effective lessons in Euclidean geometry. There were observations of incomplete work and lack of participation from some learners which suggested that negative attitudes toward Euclidean geometry may have played a role in their struggles.

The findings of this study reveal that by the time learners reach Grade 11, they often display significant deficiencies in their mathematical knowledge. While schools frequently employ well-trained teachers for the Further Education and Training (FET) phase, a weak foundation established in the earlier grades substantially undermines learners' interest in and comprehension of the subject of mathematics.

5.4 RECOMMENDATIONS OF THE STUDY

The Department of Education should ensure that district offices appoint qualified teachers to teach mathematics effectively at the primary school level. Euclidean geometry, which focuses on the study of shapes, is particularly important. Learners require a strong grasp of all concepts introduced in preceding grades to successfully engage with this branch of mathematics. To address this challenge, continuous professional development in Euclidean geometry is essential, particularly as some teachers may not have received adequate exposure to the subject during their own secondary education.

Within the classroom, the use of tangible shapes and figures should be prioritised to strengthen learners' conceptual understanding and appreciation of geometry. Evidence from this study suggests that teachers who integrate physical models as teaching aids enable learners to develop a more meaningful understanding of geometric concepts and theorems.

To support such practices, teacher workshops should be organised to provide training in effective instructional approaches for Euclidean geometry. The study underscores that the teaching of Grade 11 Euclidean geometry is significantly shaped by the quality of mathematics instruction in earlier grades. Thus, establishing a strong foundation in mathematical skills at primary school level is vital. Equally, the availability of well-trained teachers is crucial to ensure learners acquire a sound understanding of mathematics during their formative years.

5.5 SUGGESTIONS FOR FURTHER STUDIES

Based on the literature review and findings of the study, the following areas are suggested for further research:

i. An investigation into the pedagogical methods employed for teaching Euclidean geometry in university teacher training programmes.

ii. Investigating Further Education and Training (FET) Phase learners' perceptions of the teaching and learning processes associated with Euclidean geometry.

5.6 CONCLUSIONS OF THE STUDY

Teaching geometry effectively requires a holistic approach that combines sound instructional strategies, active learner participation, and the use of suitable support materials. Through employing such strategies, teachers can foster a classroom environment that promotes learners' conceptual understanding and enhances their ability to articulate ideas using the language of Euclidean geometry. Ongoing professional development is vital in supporting educators to maintain effective teaching practices. Teachers need to continually refine their pedagogical approaches to meet the diverse learning needs of their students while also deepening their own understanding of complex mathematical concepts. Effective classroom management is essential for creating conditions that support meaningful learning. Encouraging learners to take ownership of their learning not only increases engagement but also develops critical thinking and independent learning skills. By focusing on these elements, teachers can make a substantial impact on learners' achievement, motivation, and overall success in geometry.

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APPENDIX A: INTERVIEW SCHEDULE

NAME OF SCHOOL: ______________________________

DATE OF VISIT: ______________________________

TIME OF INTERVIEW: _____________________________

1. What is your gender?
2. How old a126re you?
3. What is your highest academic achievement?
4. How many years of experience do you have in teaching geometry in grade 11?
5. Have you studied geometry in grade 12?
6. How many years of experience do you have in teaching geometry in Grade 11?
7. Have you done geometry in your Grade 12?
8. How do you motivate learners to learn geometry with ease and in an understandable way?
9. Learners tend to experience problems in understanding geometrical concepts such as quadrilaterals and theorems. How can you turn the situation around?
10. Geometry riders tend to combine several figures like triangles, circles, rectangles, etc. What method can a teacher use to assist learners see relationships in these figures?
11. Which method /technique proves to be more successful and why?
12. *W*hat, in your opinion causes learners to fail in Euclidean geometry?
13. On what learning theory is your geometry teaching based?
14. Do you incorporate learning theory/theories in your teaching?
15. If so, how?
16. What strategies would you use to develop a clear understanding of the application of a theorem?
17. What learning support materials would teachers use to help learners master geometrical concepts?
18. Does the curriculum change the way you teach geometry? If so, how do you cope with the new curriculum?
19. One of the problems with many geometry learners is their weakness in the language of geometry. What can be done to improve this situation?
20. What are the biggest challenges that you face when teaching geometry?

21. What are the strategies that you can suggest that can help in effective teaching of geometry?
22. Suggest ways in which the teaching of geometry can be improved in Grade 11?

APPENDIX B: LESSON OBSERVATION SCHEDULE

Subject: Mathematics **Topic: Euclidean geometry**

Date…………………. Grade: 11 Number of learners……………

Gender of the teacher observed………. Time…… School……………

Errors and Imprecision

- Major Mathematical Errors or Serious Mathematical Oversights

  ………

  ………

- Imprecision in Language or Notation

  ………

  ……..

- Lack of Clarity

  ………

  ………

- Overall, Errors and Imprecision

  ………

  ………

Richness- capture the depth of the mathematics offered to learners

Meaning of Facts/ Procedures

- Linking/ Connections

  ………

  ………

- Explanations

  ………

  ………

Focus on Mathematical Practice

- Multiple Procedures or solution methods

  ………

  ………

- Developing Mathematical Generalizations

……

……

- Mathematical Language

……

……

- Overall Richness

……

……

Learner Participation

LEARNER PATICIPATION

- Learners provide explanations

……

………..

- Learner Mathematical questioning and reasoning

……

………..

- Enacted task cognitive activation

……

……

- Overall Learner Participation in Meaning-Making and Reasoning

……

……

Teacher and learner's interaction:

WORKING WITH LEARNERS

- Responding to Learner Mathematical Productions in Instruction

……

……

- Remediation of Learners' Errors and Difficulties

……

……

- Overall Working with Learners

  ……

  ………..

- General comments:

……………………………………………………………………………………………

……………………………………………………………………………………………

………………………………………

APPENDIX C: Ethical clearance certificate

UNISA university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2020/11/11

Ref: **2020/11/11/51412942/08/AM**
Name: Mrs MB Nkosana
Student No.: 51412942

Dear Mrs MB Nkosana

Decision: Ethics Approval from 2020/11/11 to 2023/11/11

Researcher(s): Name: Mrs MB Nkosana
E-mail address: 51412942@mylife.unisa.ac.za
Telephone: 072 7960 925

Supervisor(s): Name: Ms E. Makwakwa
E-mail address: makwaeg@unisa.ac.za
Telephone: 0124294575

Name: Prof ZMM Jojo
E-mail address: jojozmm@unisa.ac.za
Telephone: 012 429 6627

Title of research:

Teachers' experiences in the teaching of grade 11 euclidean geometry

Qualification: MEd in Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2020/11/11 to 2023/11/11.

*The **low risk** application was reviewed by the Ethics Review Committee on 2020/11/11 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.

University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.
3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2023/11/11**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2020/11/11/51412942/08/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

Prof AT Motlhabane
CHAIRPERSON: CEDU RERC
motlhat@unisa.ac.za

Prof PM Sebate
EXECUTIVE DEAN
Sebatpm@unisa.ac.za

Approved - decision template – updated 16 Feb 2017

University of South Africa
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www.unisa.ac.za

APPENDIX D: REQUEST FOR PERMISSION TO CONDUCT RESEARCH - SCHOOL

ENQ: NKOSANA M.B
STUDENT NO: 51412942
CELL: 072 796 0925
e-mail: nkosana.mash@gmail.com

P O BOX 2092
GIYANI
0826

Dear Sir / Madam

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN THE SCHOOL

TITLE OF THE RESEARCH: MATHEMATICS TEACHERS' PRACTICES AND EXPERIENCES IN THE TEACHING OF GRADE 11 EUCLIDEAN GEOMETRY: The case of Mopani District in Man'ombe Circuit, Limpopo Province

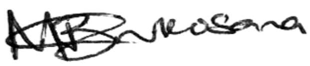
My name is Nkosana Mashudu Bridget. I am a master's student under the supervision of Dr E Makwakwa and Prof ZMM Jojo in the Department of Mathematics.

I request permission to conduct the research study in the sampled schools. The study is entitled: Mathematics teachers' experiences in the teaching of grade 11 Euclidean geometry. This is my part towards fulfilling the requirements for the master's degree at the University of South Africa. The study aims to investigate teachers' practices and experiences in the teaching of Euclidean geometry to identify gaps that contribute to the geometry performance in Mopani District schools. The study will entail collecting important information through observation of the lesson and an interview with the teacher, which could assist in terms of improving geometry performance in schools.

The benefits of this study are to improve learners' performance in geometry and teachers' attitudes in schools and to encourage teachers not to look down upon themselves. There will be no potential risks to the participants. There will be no reimbursement or any incentives for participation in the research. The findings and the recommendations of the study will be sent to all the participants.

COVID-19 protocols will be adhered to. Screening of participants and sanitation will be done before each lesson and interview starts. Participants and the researcher will be requested to always wear masks. Clean, sanitised classrooms will be used all the time. Social distancing will always be observed. No sharing of instruments, pens, pencils, and calculators will be allowed.

Yours faithfully



NKOSANA M.B. (MRS)

APPENDIX E: LETTER FROM LIMPOPO DEPARTMENT OF EDUCATION

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

CONFIDENTIAL

Ref: 2/2/2 Enq: Makola MC Tel No: 015 290 9448 E-mail:MakolaMC@edu.limpopo.gov.za

Nkosana MB
P O Box 2092
Giyani
0826

RE: REQUEST FOR PERMISSION TO CONDUCT RESEARCH

1. The above bears reference.
2. The Department wishes to inform you that your request to conduct research has been approved. Topic of the research proposal: **"MATHEMATICS TEACHERS EXPERIENCE IN THE TEACHING GRADE 11 EUCLIDEAN GEOMETRY. "**
3. The following conditions should be considered:

3.1 The research should not have any financial implications for Limpopo Department of Education.

3.2 Arrangements should be made with the Circuit Office and the School concerned.

3.3 The conduct of research should not in anyhow disrupt the academic programs at the schools.

3.4 The research should not be conducted during the time of Examinations especially the fourth term.

3.5 During the study, applicable research ethics should be adhered to; in particular the principle of voluntary participation (the people involved should be respected).

3.6 Upon completion of research study, the researcher shall share the final product of the research with the Department.

REQUEST FOR PERMISSION TO CONDUCT RESEARCH : NKOSANA MB Page 1

Cnr 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X 9489, Polokwane, 0700
Tel:015 290 7600/ 7702 Fax 086 218 0560

The heartland of Southern Africa-development is about people

4 Furthermore, you are expected to produce this letter at Schools/ Offices where you intend conducting your research as an evidence that you are permitted to conduct the research.

5 The department appreciates the contribution that you wish to make and wishes you success in your investigation.

Best wishes

Mashaba KM
DDG: CORPORATE SERVICES

25/04/2022
Date

Cnr 113 Biccard & 24 Excelsior Street, POLOKWANE, 0700, Private Bag X 9489, Polokwane, 0700
Tel:015 290 7600/ 7702 Fax 086 218 0560

The heartland of Southern Africa-development is about people

APPENDIX F: REQUEST FOR PERMISSION TO CONDUCT RESEARCH - DISTRICT

ENQ: NKOSANA M.B
STUDENT NO: 51412942
CELL: 072 796 0925
e-mail: nkosana.mash@gmail.com

P O BOX 2092
GIYANI
0826

Dear Sir / Madam

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN MOPANI DISTRICT SCHOOLS

TITLE OF THE RESEARCH: MATHEMATICS TEACHERS' PRACTICES AND EXPERIENCES IN THE TEACHING OF GRADE 11 EUCLIDEAN GEOMETRY: The case of Mopani District in Man'ombe Circuit, Limpopo Province

My name is Nkosana Mashudu Bridget. I am a master's student under the supervision of Dr E Makwakwa and Prof ZMM Jojo in the Department of Mathematics.

I request permission to conduct the research study in sampled schools in your District. The study is entitled: Mathematics teachers' experiences in the teaching of grade 11 Euclidean geometry. This is my part towards fulfilling the requirements for the master's degree at the University of South Africa. The study aims to investigate teachers' practices and experiences in the teaching of Euclidean geometry to identify gaps that contribute to the geometry performance in Mopani District schools. The study will entail collecting important information that could assist in terms of improving geometry performance in schools. I will also video record all the actions that the teachers do in class during those lessons. I will stick to their actions, capture what and how they teach, without revealing their identities.

The benefits of this study are to improve learners' performance in geometry and teachers' attitudes in schools and to encourage teachers not to look down upon themselves. There will be no potential risks to the participants. There will be no reimbursement or any incentives for participation in the research. The findings and the recommendations of the study will be sent to all the participants.

COVID-19 protocols will be adhered to. Screening of participants and sanitation will be done before each lesson and interview starts. Participants and the researcher will be requested to always wear masks. Clean, sanitised classrooms will be used all the time. Social distancing will always be observed. No sharing of instruments, pens, pencils, and calculators will be allowed.

Yours faithfully

____

NKOSANA M.B. (MRS)

APPENDIX G: REQUEST FOR PERMISSION TO CONDUCT RESEARCH-EDUCATORS

LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

MOPANI EAST DISTRICT
CONFIDENTIAL

REF : 2/2/2 ENQ: Nkanyani H.G EMAIL: NkanyaniHG@edu.limpopo.gov.za DATE: 30.11.21

TO : NKOSANA MASHUDU BRIDGET

PERMISSION TO CONDUCT RESEARCH: MATHEMATICS TEACHERS' EXPERIENCES IN THE TEACHING OF GRADE 11 EUCLIDEAN GEOMETRY

1. The above matter refers.

2. The Department wishes to inform you that your request to conduct research on the above mentioned Topic has been approved.

3. Your focus should only be limited to the identified Schools i.e.

NAME OF SCHOOL	NAME OF CIRCUIT
Hivuyeriwile	Man'ombe
Macema	Man'ombe
Ndengeza	Man'ombe
Rithavile	Man'ombe

4. The following conditions should be considered to:

4.1. Arrangement should be made with the selected Schools.

4.2. The research should not be conducted during working hours.

4.3. During research, applicable research ethics should be adhered to, in particular the principle of voluntary participation (the people involved should be respected).

4.4. Upon completion of the research study, the researcher shall share the final product of the research with the Department.

PERMISSION TO CONDUCT RESEARCH: NKOSANA M.B

DEPARTMENT OF EDUCATION
MOPANI EAST DISTRICT, Private Bag X 578 GIYANI, 0826
Tel 015 811 7803

The heartland of Southern Africa – development is about people

4.5. The research should not have any financial implications to the Department of Education Limpopo Province.

5. Furthermore, you are expected to produce this letter to any s and offices where you intend to conduct your research since it will serve as proof that you have been granted permission to conduct the research.

6. The Department appreciates the contribution that you wish to make and wishes you success in your research.

	30.11.2021
....	....
DISTRICT DIRECTOR	**DATE**

APPENDIX H: CONSENT TO PARTICIPATE IN THIS STUDY

CONSENT TO PARTICIPATE IN THIS STUDY

I, ________________________________ (participant name), confirm that the person asking my consent to take part in this research has told me about the nature, procedure, potential benefits and anticipated inconvenience of participation.

I have read (or had explained to me) and understood the study as explained in the information sheet.

I have had sufficient opportunity to ask questions, and I am prepared to participate in the study.

I understand that my participation is voluntary and that I am free to withdraw at any time without penalty (if applicable).

I am aware that the findings of this study will be processed into a research report, journal publications, and/or conference proceedings, but that my participation will be kept confidential unless otherwise specified.

I agree to the recording of the interview where I will respond to semi-structured questions related to a critical exploration of School Governing Bodies' legal mandates, which involves policy interpretation and implementation in schools under Mopani District.

I have received a signed copy of the informed consent agreement.

Participant Name & Surname ………………………………. (Please print)

Participant Signature ………………………………… Date …………………

Researcher's Name & Surname: Nkosana Mashudu Bridget Date: …………..

Researcher's Signature ……………………………. Date: ………….

APPENDIX I: LANGUAGE EDITING CERTIFICATE

Nendauni editing hub
Let us perfect your document.

COPY-EDITING ENDORSEMENT

To whom it may concern,

This certifies that the dissertation whose title appears below, has undergone editing for the purpose of ensuring correct English language grammar, punctuation, spelling, and overall style. However, it remains the researcher's obligation to consider and implement suggestions provided by the editor.

TITLE

Mathematics Teachers' Practices and Experiences in the Teaching of Grade 11 Euclidean Geometry

RESEARCHER

Nkosana Mashudu Bridget

DATE EDITED

19 AUG 2025

Signed

Dr Lutendo Nendauni
Copyeditor
Nendauni Editing Hub
Cape Town
South Africa

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APPENDIX J: SIMILARITY INDEX

Similarity Report

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MASHUDU BRIDGET NKOSANA

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