LEARNER’S PROBLEM SOLVING ABILITIES IN GRADE 11 EUCLIDEAN GEOMETRY TASKS

by

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DECLARATION

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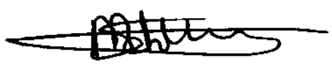
Learner's problem solving abilities in Grade 11 Euclidean geometry tasks

I declare that the above dissertation is my own work and that all the sources that I have used or quoted have been indicated and acknowledged by means of complete references. I further declare that I submitted the dissertation to originality checking software and that it falls within the accepted requirement for originality. I further declare that I have not previously submitted this work, or part of it, for examination at Unisa for another qualification or at any other higher education institution.



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DEDICATION

I dedicate this dissertation to my heavenly Father, the Almighty God. He granted me wisdom, strength, and courage to persevere and carried me through to the conclusion of this project. May the glory and honour be unto the Lord.

I also dedicate this work to my husband Emmanuel, my mother Sarah, and my daughters, Ntokozo and Charis.

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ABSTRACT

The aim of this study was to explore Grade 11 learners' problem-solving abilities that are manifest when they solve Euclidean geometry tasks. Additionally, this study evaluated the levels of problem-solving ability they operated within. The investigation was underpinned by the Polya problem-solving model. Through this model, the learners' problem-solving abilities that guided them to solve the related problem-solving task were analysed and interpreted. The study used a qualitative method of enquiry and followed a case study research approach. Participants consisted of 63 Grade 11 mathematics learners from two different schools which were selected through convenience sampling. This study used two instruments to collect data, namely, the Euclidean geometry task and semi-structured interviews. The study revealed that 69.8% of participants had low problem-solving abilities, while 28.6% of participants had average problem-solving abilities. Only 1.6% of participants had high problem-solving abilities. In addition, the researcher was able to identify participants' misconceptions and errors. It is anticipated that this study will make a valuable contribution to the enhancement of learners' proficiency in problem-solving skills, specifically in the domain of Euclidean geometry, as well as their overall performance in mathematics.

KEY TERMS:

Mathematics; Geometry; Euclidean geometry; Problem-solving; Problem-solving abilities; Learner performance; Mathematics achievement; Mathematics cognitive processing; van Hiele levels of geometric thought; Polya problem-solving approach

LIST OF ABBREVIATIONS

ANA	Annual National Assessments
CAPS	Curriculum Assessment Policy Statements
CLT	Cognitive Learning Theory
DBE	Department of Basic Education
DOE	Department of Education
EGT	Euclidean Geometry Task
FET	Further Education and Training
GDE	Gauteng Department of Education
LTM	Long-Term Memory
NCTM	National Council of Teachers of Mathematics
NSCE	National Senior Certificate Examination
STM	Short-Term Memory
TIMSS	Trends in International Mathematics and Science Study
WM	Working Memory
VHL	van Hiele Levels

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CHAPTER ONE

A THEORETICAL OVERVIEW OF THE STUDY

1.1 CONTEXTUAL BACKGROUND OF THE STUDY

In South Africa, there are concerns regarding student performance in mathematics (Department of Basic Education [DBE], 2020). There are indications from the Trends in International Mathematics and Science Study (TIMSS) (2019) that there is a lack of growth in mathematics proficiency among Grade 8 mathematics learners in South Africa when compared to other countries. Out of 39 countries that participated in the TIMSS eighth grade mathematics assessment, South Africa performed second last, the first being Singapore and the last being Morocco (TIMSS, 2019). Locally, data from the Annual National Assessments (ANA) has also raised concerns about students' suppressed proficiency in mathematics. ANA has indicated that, even as early as primary school, the mathematical performance of learners in South Africa falls below the minimum outcomes and standards that have been set (van der Berg, 2015).

At the time of conducting this study, the minimum level at which the mathematics learners were expected to achieve the content, assessment, and promotion requirements in South Africa had been set at 40% for Grades 1 to 9, and 30% for Grades 10 to 12 (DBE, 2011). Consequently, the passing criteria for mathematics in South Africa has been such that the learner who achieves a 40% in Grade 9 mathematics, or a learner who has achieved a 30% in Grade 12 mathematics, would be considered to have passed the subject. When ANA was administered in Grade 6 in South Africa, only 11% of learners passed mathematics in 2012. In 2013, only 27% of learners passed the ANA tests in mathematics and only 35% in 2014 (DBE, 2014). In terms of the Curriculum and Assessment Policy Statement (CAPS), a 50% is recognised as a decent grade for passing Grade 6 mathematics. It seems learners at this grade level are continuing to fall short (see DBE, 2014, 2011). In the last five years, Grade 12 learners' mathematics performance in the National Senior Certificate Examination (NSCE) has been average, with little or no improvement (see Table 1.1).

Table 1.1: Grade 12 end-of-the-year mathematics performance

Candidates	2016	2017	2018	2019	2020
No. wrote	265 810	245 103	233 858	222 034	233 315
No. achieved 30% & above	135 958	127 197	135 638	121 179	125 526
% Achieved	51.1	51.9	58.0	54.6	53.8

Source: DBE (2020, p. 57)

Table 1.1 shows that about 50% of learners who wrote mathematics achieved a mark of at least 30% between 2016 and 2020. Moreover, according to the NSCE Reports, mathematics was the least performed subject in Grade 12 when compared to other subjects (see DBE, 2020, 2019, 2018, 2017 & 2016). Building upon these findings, the primary objective of this study was to probe into the specific content area or topic in mathematics that may have significantly contributed to the subpar performance of learners, as reflected in the academic years specified in Table 1.1.

This being said, it must be noted that throughout the years, the most challenging and poorly performed content area in mathematics has been Euclidean geometry (Mamali, 2015; Ngirishi, 2015; Cassim, 2006; TIMSS, 2006/07; Usiskin, 1987; Usiskin, 1982). The South African National Diagnostic Reports for the National Senior Certificate Grade 12 has also identified Euclidean geometry as the least performed content area in mathematics examinations (DBE, 2017, 2016, 2015). Mamali (2015) indicated that learners' performance in Euclidean geometry was below average, attributing it to a lack of positive attitude towards learning geometry, as well as a deficiency in willingness and readiness to comprehend geometric concepts.

On this matter Ngirishi and Bansilal (2019) stressed that Euclidean geometry is seen as the most complicated strand of the mathematics curriculum. These authors proceeded to highlight that learners incorrectly assume that the Euclidean geometry is irrelevant to their daily lives. The history of studying Euclidean geometry in South Africa has never been substantive. In 2006, when the South African curriculum was revised, Euclidean geometry was rendered a non-compulsory section of the mathematics curriculum for students in Grade 10, 11, and 12 (Department of Education [DOE], 2006). Hence in the Grade 12 assessment sessions, the paper three, which ordinarily would consist of geometric topics, was treated as a non-compulsory assessment activity (DOE, 2006). This change in curriculum may have deprived learners an opportunity to engage in constructive and productive geometry discourses in their classrooms.

In 2011, the CAPS was used as a constructive tool to review the decision to reintroduce geometry into the mainstream mathematics syllabus in the Further Education and Training (FET [1]) band. The latter process would once again render geometry as an additional educational avenue that learners at these grade levels would be able to explore (DBE, 2011). It was the researcher's view that this background showed that there could still be a gap in the teaching and learning of geometric concepts at a high school level,[2] and possibly, insufficient acquisition of geometry specific problem-solving skills. South African education may not be providing essential foundational teaching and learning expertise for geometry. This could raise reasonable educational concerns because the tasks in geometry are largely designed to cultivate and enhance learners' mathematical problem-solving skills. Learners' poor performance that is commonly observed in Grade 12 could be attributed to the insufficient acquisition of multiple skills needed to promote problem-solving activities in geometry. Ordinarily, such essential problem-solving skills would have been learnt and developed in previous grades, and prior to Grade 12.

Alex and Mammen (2014) conducted a study with Grade 10 students learning mathematics using the van Hiele model of thinking. Alex and Mammen (2014) found that most study participants were at Level 0, which indicated a concerning state of teaching and learning of geometry. These results meant that study participants would only manage to recognize, name, and group shapes according to their appearance (Alex & Mammen, 2014). According to CAPS, learners in Grade 10 are supposed to have accomplished the levels 0 to 2 of the van Hiele levels of reasoning ahead of entering the FET phase (see DBE, 2011; see also, Masilo, 2018). The van Hiele model of thinking recognizes an arrangement of five progressive reasoning levels (0-4) in a way that learners attain knowledge of geometric concepts. They are as follows:

1. Level 0 – Visualization: The identification, naming and grouping of shapes according to their appearance.
2. Level 1 – Analysis: Describing the attributes of shapes.

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1 In the South African education system, the Further Education and Training (FET) involves the Grade 10 to Grade 12 levels. The educational offerings at these levels also include career-oriented education and training offered in technical colleges, community colleges and private colleges. In this arrangement Grade 12 becomes the exit point for high school education and the exiting learner may join the higher education learning institutions.

2 In this study, the word high school referred to an educational institution offering tuition to learners from Grade 8 to 12.

3. Level 2 – Informal deduction: Classifying and generalizing by attributes.
4. Level 3 – Formal deduction: Developing proofs using axioms and definitions.
5. Level 4 – Rigor: Working in various geometrical systems. (See Alex & Mammen, 2012; 124)

Prior to conducting this study, the researcher was concerned that learners were performing poorly in mathematics, especially in Euclidean geometry. At this time, the researcher had accumulated five years of experience in teaching high school mathematics and was familiar with teaching and learning in the South African education system. Hence, the researcher observed troubling instances of perennial poor mathematical performance in Grades 8 to 12. It was clear that in most cases, some learners would opt to not write the Euclidean geometry section of the Paper 2 in an examination. Paper 2 of the mathematics examination paper is designed to cover all work that is done in geometry for a particular grade level. While writing an examination of Paper 2, some learners filled the answer sheet with non-relevant information without providing correct responses that were required of them. Given these observations, the researcher undertook this study to document constraining concepts in geometry that posed potential challenges to Grade 11 learners.

1.2 RATIONALE FOR THE STUDY

Evidence shows that poor performance in Euclidean geometry has persisted for several years (for examples, see Gauteng Department of Education [GDE], 2003, 2002, 2001, 1995; Mathematics, Science and Technology Report, 2003; DBE, 2017, 2016, 2015). At the time of conducting this study, the performance of learners in Euclidean geometry was below average (see DBE 2020, 2019 & 2018). It is therefore important for mathematics teachers, teacher educators, and researchers in the field of Mathematics Education to resort to finding empirically tested interventions to elevate mathematical instruction in geometry lessons.

Innovative and effective methods of continuous assessment and testing should be explored to improve learners' response to geometry instruction. For example, the integration of technology and media in the teaching and learning of geometry is one educational avenue that could be explored to enhance learners' performance in geometry (Leendertz et al., 2013; Mosia, 2016; Swam & Dixon, 2006). Misconceptions among learners can have a negative impact on their ability to employ skills and strategies when solving geometry problems. This study argues that addressing this issue can require further engagement with learners to ascertain ways in which learners engage with geometric knowledge that promote problem-solving skills. This engagement could reveal the challenges learners experience and their cause. The primary aim of this study was to make a meaningful contribution towards the improvement of learners' problem-solving skills in Euclidean geometry, with the goal of improving teacher instruction and pedagogical practice.

1.3 STATEMENT OF THE PROBLEM

Globally, geometry is recognised as a fundamental branch of mathematics that plays a vital role in developing students' critical thinking and problem-solving skills (Bankov, 2013; Serin, 2018). Da Zhou (2019) stressed that the primary goal of mathematics teaching and learning is to develop a learner's ability to solve complex problems. According to CAPS, one of the specific aims of learning mathematics is to develop problem-solving abilities and cognitive skills (DBE, 2011). Therefore, geometry is an area of study that is given prominence in mathematics education. According to CAPS, the assessment weighting for Euclidean geometry in Grade 12 is about one third (33%) of Paper 2 (DBE, 2011: 55). At the time of conducting this study, the Grade 12 mathematics examination Paper 2 was allocated a total mark of 150 in which 50 ± 3 marks were allocated to Euclidean geometry.

Relatively, this would be the highest mark allocated to a topic in the mathematics examination in both Paper 1 and Paper 2 in Grade 12. With this kind of weighting, it would mean that the challenges associated with poor performance in Euclidean geometry could be a major contributing factor to learners' under-achievement in mathematics. Given these realities in the mathematics curriculum, the researcher had also observed that only a portion of Euclidean geometry content was taught in Grade 12. Most of the geometry work that is assessed in Grade 12 would have been taught in lower grade levels, with a relatively large portion of circle geometry being allocated to a Grade 11 class (see Table 1.2).

This could suggest that the learners' mathematical challenges that are observable in Grade 12 could be indicative of the challenges experienced in previous grade levels. Hence, the researcher argued that learners reach the Grade 12 class with some gaps in content knowledge of geometry. Consequently, the challenges that are documented in the National Senior Certificate Grade 12 National Diagnostic Reports may be attributed to learners' low problem-solving abilities[3] that are seen when they solve Euclidean geometry tasks (DBE, 2018, 2017, 2016, 2015).

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3 The word *ability* was conceptualized in this study to refer to one's inner means or skills to execute something. Hence in some instances the words ability and skills have been used interchangeably in this study.

Table 1.2: National Senior Certificate (Grade 12) Mathematics Exam paper 2

Euclidean geometry	2017	2018	2019	2020
Total mark and percentage allocated	51 (34%)	48 (32%)	48 (32%)	49 (32.7%)
Total mark and percentage allocated to lower grades content (majorly circle geometry)	33 (64.7%)	31 (64.6%)	28 (58.3%)	25 (51%)
Total mark and percentage allocated to Grade 12 content	18 (35.3%)	17 (35,4%)	20 (41.7%)	24 (49%)

Source: DBE (2020, 2019, 2018, 2017)

The term "learners' problem-solving abilities" in this study is conceptualized as encompassing the cognitive capabilities and skills that learners demonstrate when engaging in mathematical tasks related to Euclidean geometry. Ordinarily, these problem-solving skills and abilities could be tied to the nature of instruction[4] that describe the nature and form of pedagogical interactions in the classroom. Ruseffendi (2006) noted that learners' ability to solve Euclidean geometry problems is influenced by their level of ability in terms of low, average, or high ability that one possesses (see Section 1.4). Therefore, this would mean learners should acquire and possess certain abilities or skills needed to make headways when solving mathematical problems. It was on this ground that this study was commissioned to probe learners' problem-solving abilities that are manifest when solving problems in Grade 11 Euclidean geometry tasks.

Being the second level in the FET phase, Grade 11 should present educational opportunities to build learners' problem-solving abilities. This could rightfully prepare learners to perform well in Grade 12 examinations. According to Masilo (2018), mathematics learners in Grade 11 should have gained sufficient experience in mathematical problem-solving activities. At this grade, learners should have acquired the knowledge, understanding, and spatial abilities to demonstrate innovativeness and basic speculation in critical thinking for problem-solving (Masilo, 2018).

4 The word *instruction* was also used in this study to refer to the activities of teaching and learning in the classroom. Hence the phrase mathematics instruction would refer to the way in which mathematics is taught and learnt in the classroom, and the teacher is the driver of this process.

CAPS has stated that Euclidean geometry requires learners to show proficiency in doing formal proofs that were learned properly (DBE, 2011). Ideally, at the end of a circle geometry lesson, learners should be able to solve riders and construct proofs of related theorems. According to the van Hiele model of thinking, the ability to construct and understand proofs is linked to Level 3. However, Cassim (2006) stressed that learners are less capable of reaching this level of innovative thinking, comprehension, and information because they have not gained mastery at the lower levels of the model (i.e., Level 2, 1, 0). This could make the development process of learners' mathematical problem-solving abilities slow and inadequate (Kistian & Verawati, 2020).

1.4 UNPACKING THE NOTION OF LEARNERS' PROBLEM-SOLVING ABILITIES

A problem is generally considered to be a hindrance when there are discrepancies between what is expected and what occurs. The situation is considered a problem if one does not have a readily available solution. Learners are required to have the ability to solve various problems that they may encounter in their everyday life, such as problems related to mathematics and those that they will encounter in their future careers and society (Pasaribu, 2021). In the context of this study, mathematical problems are regarded as questions that have real-life relevance and are within the cognitive reach of learners' thinking abilities. These problems do not have immediate algorithms or procedures that learners can directly apply, and that this poses challenges that require critical thinking, reasoning, and creative problem-solving skills (Simamora et al., 2019). Da Zhou (2019) stressed that the primary goal of mathematics teaching and learning is to develop a learner's ability to solve complex problems. According to CAPS, one of the specific aims of learning mathematics is to develop problem-solving abilities and cognitive skills (DBE, 2011). Therefore, problem-solving is central to and a principal component of mathematics (Dhlamini, 2012). Dahar (1989) noted that:

> "Problem-solving is a human activity that combines previously acquired concepts and rules and not as a generic skill. This definition implies that when someone has been able to solve a problem, then that person already has a new ability. This new ability can be used to solve relevant problems. The more problems a person can solve, the more abilities he will have that can help him navigate his daily life" (pg. 38).

It is reasonable to think that problem-solving abilities are not just inherent or naturally inbuilt mechanisms, but they can be acquired, developed, improved, or enhanced. Several studies have been conducted to improve learners' problem-solving abilities using different approaches and models of teaching instruction (for examples, see Anggraini & Fauzan, 2020; Eva Fulop, 2021; Kistian & Verawati, 2020; Pasaribu, 2021; Pasaribu & Suyanto, 2020; Son & Ditasona, 2020; Sutrisno et al., 2020; Wahab, Saragih, & Siman, 2020). This means that learners may possess different problem-solving abilities. Learners may have different and varying levels of problem-solving abilities, when compared with other learners (Sutrisno et al., 2020).

Kistian and Verawati (2020) interpreted a problem-solving ability as the effort an individual makes when solving a problem, and the new ideas that follow in order to achieve the expected goals. The problem-solving process assumes that the learner knows and can apply a variety of problem-solving strategies (Éva Fülöp, 2021). A strategy is generally an approach, model, a way, or a series of steps that an individual can use to solve problems or to find answers to the question asked (Éva Fülöp, 2021; Kistian & Verawati, 2020). Polya (1945) formulated four well-known steps that an individual can use to solve mathematical problems. They include: (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and (4) looking back.

In terms of the description of a strategy, learners' mathematical problem-solving abilities may be conceived as cognitive abilities or the inner able-ness or the quality of being able to do, which then aids a learner in understanding and being able to respond to a problem. Meaning, when faced with a problem-solving situation, these cognitive abilities or qualities of able-ness are automated and become activated to guide the learner on how to respond to the problem. Having acquired these problem-solving skills, a learner is perceived to have developed desirable strategies to solve the problem. Problem-solving abilities may be cultivated from continued exposure to problem-solving situations. With this exposure the learner could develop desirable problem-solving abilities and mastery, which get automated in an event of a problem-solving demand or expectation. In this study, the learner's mastery of the problem-solving abilities was meant to signify a quality of being a skilful problem solver. Hence, these abilities help the learner to generate the problem-specific skills that are usable for a particular problem-solving event. Therefore, problem-solving abilities are evident when learners successfully apply their problem-solving skills.

In academic contexts, measuring a learners' problem-solving abilities normally requires administering a test (NCTM, 1989). According to Wahab, Saragih, and Siman (2020), learner's test or assessment of their problem-solving abilities in mathematics would be a test that is used to measure a learner's ability to apply their knowledge and skills to solve problems. In this study, a mathematics task-based worksheet that addressed some aspects of geometry was used to measure learner's problem-solving abilities and skills (see Appendix A). Given the regular closeness and thinness of the lines between certain terms and constructs, this study treated the terms ability and skill as almost synonymous constructs, hence they have been often used interchangeably in this text. The process of evaluating the learners' task-based worksheet was largely informed by Polya's (1945) problem-solving model, which was modified by the researcher for the current study.

1.5 RESEARCH QUESTIONS

This study was guided by the following primary research question (RQ):

1.5.1 What is the level of problem-solving ability at which Grade 11 learners are operating when solving mathematical tasks in Euclidean geometry?

In answering the primary question, three levels of problem-solving ability were established, namely low, average, and high level. In addition, to answer the primary question of the study the following questions were asked:

1.5.1.1 How do the problem-solving abilities of Grade 11 learners manifest when they solve mathematical tasks in Euclidean geometry?

1.5.1.2 To what extent do learners apply the knowledge and skills required to solve Euclidean geometry tasks?

1.6 ADDRESSING THE ASSESSMENT OF THE LEVELS OF PROBLEM-SOLVING ABILITY

To answer sub-RQ1, the problem-solving abilities that learners demonstrated during a problem-solving task in Euclidean geometry were examined. Learners' problem-solving abilities that guided them to solve the related problem-solving task were analysed by the researcher.

To answer sub-RQ2, the study explored the knowledge and skills that the learners employed to solve the Grade 11 Euclidean geometry task. The extent to which the learners applied the required knowledge and skills to solve the given task was determined through the analysis of the learner's performance in difference cognitive levels of mathematics assessment and the van Hiele levels of geometric thought.

To answer the primary research question, the problem-solving ability scoring rubric was developed and used to analyse the learner's written responses where the scores were allocated to individual learners and the problem-solving ability percentage determined for the entire group. The levels of problem-solving ability were further categorized in terms of low, average, and high problem-solving ability using the scores obtained from the problem-solving ability rubric.

1.7 A SNIPPET PREVIEW OF THE RESEARCH METHODOLOGY

This section presents a brief preview of the research methodology of the study, while a detailed discussion is provided in Chapter Three. This study utilised a qualitative interpretive paradigm and a case study design. The data collection process was carried out with special consideration to the Covid-19 regulations and protocols, provided that the study was conducted during the Coronavirus pandemic period. At the time, South Africa was at an alert Level 1 lockdown where most normal activities could take place with precautions and health guidelines followed in accordance with the Disaster Management Act regulations (South African Government, 2021). Issues of trustworthiness, credibility, dependability, confirmability, and transferability were ascertained.

1.8 AIM AND OBJECTIVES

The aim of the study was to explore Grade 11 learners' problem-solving abilities in the context of Euclidean geometry tasks. This study also evaluated learners' problem-solving ability in terms of low, average, and high ability. The findings may serve as a reference for teachers in improving the teaching and learning of Euclidean geometry, thereby contributing towards the improvement of learners' problem-solving skills in Euclidean geometry. The following objectives were set out:

1. To establish if learners are able to demonstrate understanding of the problem-solving tasks given to them in Euclidean geometry.
2. To examine the mathematical problem-solving strategies, if any, that learners use when solving tasks in Euclidean geometry.
3. To identify forms of knowledge and skills that the learners demonstrate when solving tasks in Euclidean geometry.
4. To determine if learners are able to relate their problem solution to the originally given problem.

1.9 SIGNIFICANCE

The aim of an effective mathematics instruction is to serve as a foundation to the meaningful learning of other subjects. Mathematics, as a school subject, is a pre-requisite for enrolment into most critical career choices (e.g., medicine, engineering studies). For university entry, learners are expected to pass mathematics with a predetermined average percentage that varies from one institution to another. At the tertiary level, students who perform well in mathematics have a higher probability to succeed in their future vocations. It is subsequently fundamental that learners at school must perform well in mathematics to achieve the desirable passing grades.

1.10 LIMITATIONS

After serious consideration of the research methodology of the study and the context in which the study took place, the following limitations were observed:

1. The actual investigation was restricted to two schools in the city of Pretoria, under the Tshwane South district, in the Gauteng province of South Africa. Though this was considered a limiting factor, conducting the study within the confines of this location allowed the researcher to access the target schools, and these schools fell within a reasonable traveling demand from the researcher's place of residence.
2. Only a certain group of Grade 11 mathematics learners were selected for participation in the study. The selection of this group turned to render the study findings not generalizable to other grade levels within the Tshwane South district.
3. The study solely focused on learners and thus excluded an examination of teachers' pedagogy. It is the researcher's belief that teachers' pedagogy may have influence on the nature of learners' acquired problem-solving abilities. Hence this study would not have managed to study the problem holistically, which could have generated more representative and comprehensive findings.

1.11 DEFINITION OF KEY CONCEPT IN THE STUDY

Definitions for the following terms used in the current study are provided below.

1.11.1 Mathematics

DBE policy documents largely describe mathematics as a language that utilizes symbols and notations to represent the relating numerical, geometric, and graphical associations (e.g., see DBE, 2011). According to Elaine (2014, as cited in Mamali, 2015), mathematics is a structured science that manages rationale of shapes, amount, and patterns. This study took a position that looks at mathematics as a respected and critical subject to which learners must continuously strive to acquire the required aptitudes, skills and knowledge that enhance their understanding and proficiency in dealing with real-life problem-solving situations.

1.11.2 Geometry

Generally, geometry could be described as a process of learning mathematics relating to the material earth that surrounds us. This simple description is in consonance with the view of Ngirishi (2015), who defined geometry as a branch of mathematics that can relate mathematics with the actual world. On a more specific level, Louridas and Rassias (2013) have defined geometry as the numerical examination of the measure, the properties and connections of points, lines, angles, figures, surfaces, and solids. Some sections of Grade 11 geometry caught the attention of this, and this investigation covered the topic such as circle geometry focusing on cyclic quadrilateral, tangent, and centre theorems. These were concepts in Grade 11 geometry that were thought to be posing challenges to learners.

1.11.3 Euclidean geometry

Euclidean geometry is characterized as the investigation of plane and solid figures based on axioms and theorems utilised by the Greek mathematician Euclid in 300 BC (www.britannica.com). Additionally, Euclidian geometry requires the genuineness of knowledge in mathematics, spatial perception and understanding, critical thought for applying theorems to special cases, a capacity to sum up from well-established realities, and an emphasis on the significance of proof.

1.11.4 Problem-solving

Problem-solving is an intellectual practice that encourages one to think decisively and innovatively in search of alternative ideas and definite approaches to face each hindrance (In'am, 2016). According to Dahar (1989), problem-solving is a human activity that combines previously acquired concepts and rules and not as a generic skill. Problem-solving is therefore an activity in which there is a search for solution to a problem (Dhlamini, 2012).

1.11.5 Problem-solving abilities

Kistian and Verawati (2020) operationalized problem-solving ability as the effort an individual makes when solving a problem along with the innovative ideas that follow. According to Polya (1973), a learner's mathematical problem-solving skills is judged by their ability to develop and use strategies to solve the problem and interpret the answers according to the given problem.

1.11.6 Learner performance

Learners' performance may be defined as the ability of the learner to meet the educational standards that are set by the National Policy Pertaining to the Programme and Promotion Requirements (NPPPPR) (DBE, 2011). According to Wesslen and Maria (2005, as cited in Mamali, 2015), learners' performance pertains to the capacity of a student to display comprehension by taking part in schoolwork, writing tests, presenting, and engaging in class dialogue. In this research study, when considering a learner's mathematical ability, all the afore-mentioned definitions are taken into consideration, including a learner's school attendance, motivation, class preparation and involvement.

1.12. ORGANISATION OF CHAPTERS IN THE STUDY

Chapter One explores the theoretical concepts and constructs related to this study. The contextual background of the study is first discussed, along with rationale for why this study was conducted. Section 1.5 raises several research relevant to the aims and objectives of the study (see also Section 1.7).

Chapter Two explores the relevant literature associated with Euclidean geometric problem-solving. Chapter Three includes the current study's research methodology and design. Additional issues covered in this chapter include descriptions of the study population and sample along with the data collection instruments (i.e., how they were developed and used). Chapter Three also addresses issues of rigor and trustworthiness to enhance the authenticity of data collection instruments. The study used task-based testing worksheets and interviews for data collection.

Chapter Four focuses on the analysis and interpretation of the data. Chapter Five provides the final report of the study. Some of the concepts that are covered in this chapter include an introduction of the synopsis of discoveries, significance of the examination, hindrances, suggestions, and a personal reflection.

CHAPTER TWO

LITERATURE REVIEW AND THEORETICAL PERSPECTIVE

2.1. INTRODUCTION

In exploring and evaluating the mathematical problem-solving abilities of learners in Grade 11 Euclidean geometry tasks, the study reviewed the literature on problem-solving. The chapter first explores what problem-solving in mathematics entails. A systematic description of cognitive processing in Euclidean geometry problem-solving is further discussed. The connection between problem-solving and cognitive processing is explored to gain insight into the process of learners' acquisition of problem-solving abilities and skills. Therefore, in this study, the notion of cognition was conceptualized in terms of how learners acquire geometric problem-solving skills.

To understand how learning takes place, this chapter further explores learners' information structures and their cognitive architecture under the notion of information processing. The knowledge of how learners process information, particularly geometric information, is deemed significant in this study bearing in mind that when the instructional designers or teachers know how learning takes place, they will develop and implement effective instructional methods, and thereby contribute towards the improvement of learners' problem-solving skills in Euclidean geometry. The last section of this chapter comprehensively covers work by Polya (1945) on problem-solving leading to the current study's theoretical framework. Polya's theory provides a framework for the study's results.

2.2. PROBLEM SOLVING

Several scholars have observed that the last several decades have brought about a surge of remarkable technological advancements. The advancement of technology in the Fourth Industrial Revolution (4IR)[5] has changed the way we live, our ways of communicating, and how we work (Schwab, 2016). We have further observed that some of the jobs that people

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5 The 4th Industrial Revolution (4IR) is a fusion of advances in artificial intelligence, robotics, the internet of things, genetic engineering, quantum computing, and more technologies (McGinnis, 2020).

used to occupy have now been replaced by new technologies, with some company branches downsizing or closing because of new ways of doing the work. According to Gray (2016), complex problem-solving is the most required skill to thrive in the 4IR, as listed by the World Economic Forum in 2015. This means that most industries require employees with advanced problem-solving expertise. Dhlamini (2012) maintains that problem-solving is a major intellectual practice mostly required in daily life activities and in a professional environment. Based on these facts, there is a need for the education system to produce skilful problem solvers who will be able to meet current and future career and economic demands. It appears that the South African school mathematics curriculum considered this problem-solving requirement, thus placing problem-solving abilities and cognitive skills as one of critical specific aims in education (DBE, 2011).

It is clear that problem-solving is a critical skill for learners to acquire, and it is through the learning of mathematics that learner's problem-solving abilities and skills can be developed (see DBE, 2011; Dhlamini, 2012). Da Zhou (2019) concurs that the main objective of teaching and learning mathematics is to develop a learner's ability to solve complex problems. According to the National Council of Teachers of Mathematics (NCTM) (2000), solving problems is a primary goal and a major means of learning mathematics. Given this background, it is possible to conclude that problem-solving is central to and a principal component of mathematics (Dhlamini, 2012).

This section explores the notion of problem-solving as a primary goal for learning mathematics and as a critical skill for learners to develop. Various definitions of problem-solving in mathematics are explored from the existing literature to provide a better understanding of problem-solving abilities. The section further provides a description of mathematics problem-solving abilities, the factors influencing them, and how they can be enhanced.

2.2.1. What is problem-solving in mathematics?

To understand what problem-solving in mathematics entails, it would be better to start by defining what a problem is. Generally, a situation is considered a problem when one does not have a readily available solution. If one knows exactly what to do when they come across a particular situation, then that situation would not be regarded as a problem. The literature definitions of the term "problem" seem to assume that it is non-routine and non-predictive in its solution process. According to Bransford and Stein (1993), "a problem exists when an obstacle separates the initial state and the goal state and there is no ready-made solution for the problem solver" (pg. 7).

Given the definition of what a problem is, according to Simamora et al. (2019), mathematical problems are questions that arise from real-life situations and challenge learners' thinking abilities. These problems do not have readily applicable algorithms or procedures for learners to use. Based on this definition, mathematical problem-solving involves the learner encountering a problem for which there is no immediate solution process. The learner actively explores and employs a variety of approaches to seek a solution to the problem (Dhlamini, 2012).

Moreover, problem-solving is a process whereby the learner draws from previously encountered experiences, acquired knowledge, skills, and understanding to meet the demands of unfamiliar situations (NCTM, 2000; Dahar, 1989; Krulik & Rudnick, 1980; Polya, 1973). This indicates that learners go through a long process to find solutions to problems that they have not encountered before and use the knowledge, skills and understanding they previously learnt (Ruliani et al., 2018). Thus, the learner's prior knowledge and skills are important in solving new problems and developing new mathematical knowledge and understanding.

According to NCTM (2021), the term problem-solving refers to mathematical tasks that can provide cognitive challenges for enhancing learner's mathematical understanding and development. This suggests that through these mathematical tasks, learners engage in an intellectual practice that encourages them to think decisively and innovatively in search of alternative ideas and definite approaches to tackling problems (In'am, 2016). In simple terms, problem-solving is therefore a cognitive activity in which there is a search for a solution (Dhlamini, 2012) and a process of discovering new knowledge (Wahab et al., 2020).

The current study focused on problem-solving in Euclidean geometry. Euclidean geometry is viewed as a major branch of mathematics that has a potential to develop learner's critical thinking abilities and problem-solving skills (Serin, 2018). Learner's reasoning and justification skills are developed through geometry (NCTM, 2000). Ruliani et al. (2018) noted that multiple representational skills are required to communicate mathematical problems. Euclidean geometry is one of the branches of mathematics that utilizes these representational skills such as visual representation in the form of diagrams or riders, while learners may be required to use their representational abilities such as symbolic representations (mathematical statements/notations, numerical/algebraic symbols) and verbal representations (written texts/reasons) (Ruliani et al., 2018).

2.2.2. Problem-solving models

According to Siregar (2017) models are one of the answers to solving problems. McNeese (2021) noted that a problem-solving model provides a problem solver with a road map (directions) to follow for continuous progress in solving problems until the problem is solved. On the same note, Polya (1981) stated that problem-solving is a process that begins from the second the learner is faced with the problem until the problem is solved. There are many proposed problem-solving models in the literature developed to explain problem-solving processes. Table 2.1 shows some problem-solving models that could be used either in a mathematics classroom or outside the classroom, but only two of them are discussed in some detail.

Table 2.1: Problem-solving models

POLYA (1945)	**KRULIK & RUDNICK (1980)**	**SCHOENFELD (1985)**	**BRANSFORD & STEIN (1993)**
Understanding the problem	Read	Reading Analysis	Identify problems and opportunities
Devising a plan	Explore	Exploration Planning	Define goals
Carrying out the plan	Select a strategy	Implementation	Explore possible strategies
Looking back	Solve	Verification	Anticipate outcomes and Act
	Review and extend		Look back and learn

The literature suggests that Polya's model is the most well-known mathematical problem-solving model from which most of the current models were developed (see Peranginangin & Surya, 2017; Saman & Chin, 2016; NCTM, 2014; Dhlamini, 2012). In his book *How To Solve It*, Polya formulated four steps that an individual can use to solve mathematical problems: understanding the problem, devising a plan, carrying out this plan, and looking back.

The model's first step requires learners to understand the problem, whereby they are encouraged to read and visualize the problem. They then identify the elements that are unknown, state the data and the conditions. The learner may draw a diagram or introduce relevant notations. The second step requires devising a plan, whereby learners can develop a problem-solving strategy. They use prior or acquired knowledge and experiences in problem-solving to find connections between the data and the unknown. Depending on the topic at hand, the question for the learner could be, "do you know a theorem that could be useful?" or the learner may formulate an equation as a strategy to solve the problem. Some strategies may include guess and check, looking for a pattern, making an orderly list, working backwards, using a model, drawing a picture, using a formula (Polya, 1957), visualisation, organising data (Éva Fülöp, 2021) or determining what is known, being asked, and the information required (Kistian & Verawati, 2020).

The third step entails carrying out a plan, whereby learners implement the strategy they have developed in the previous step, such as an equation. In this step the learner will solve the equation. The fourth step: looking back, has the learner examine the solution that has been obtained. For instance, using the previous example, the learner will substitute the solution obtained in the equation and see if the equation is balanced. Polya's model and strategies have been recommended for use by secondary school mathematics teachers to teach mathematics (Dhlamini, 2012). Additionally, they can be used as an assessment tool to evaluate students' problem-solving skills (Peranginangin & Surya, 2017).

The IDEAL approach to problem-solving is another example of a problem-solving model which was conceptualised by Bransford and Stein (1993). In their book *The IDEAL Problem Solver*, Bransford and Stein (1993) designed five components in their model which are represented by the acronym IDEAL. Each letter stands for an aspect of thinking that is important for problem-solving process (Bransford and Stein, 1993), namely, **I** → Identify problems and opportunities; **D** → Define goals; **E** → Explore possible strategies; **A** → Anticipate outcomes and Act; and **L** → Look back and Learn.

The first component of the IDEAL model is a way of viewing problems as opportunities for finding the solution. It is an active attempt to identify problems or steps that others may have overlooked. The second component emphasises the importance of considering alternative goals when attempting to solve a problem. The goals are to be clearly outlined. The third component has to do with exploring relevant approaches that will address each specific goal for solving the problem. At this stage, whatever one does reflects their approach to a problem. For instance, the physical and/or mental escapes from problem situations are coined “the let-me-out-of-here approach” by Bransford and Stein (1993). This type of strategy is observed when learners do not make any attempt to solve the problem, for example, leaving an answer space to a question blank. Some strategies highlighted by Bransford and Stein (1993) are systematic analysis, breaking down a problem into parts, using external representations, or working a problem backwards.

The fourth component emphasises anticipating the results of the strategies before acting on them and the final component of the IDEAL model focuses on looking at the effects of the implemented strategy and learning from the problem-solving experience. Unlike other problem-solving models which are fixed in a step-by-step orderly way, the IDEAL model may also be applied in a flexible way where the problem solver may move backwards first to redefine some aspects before the final phase (Bransford and Stein, 1993). According to Dhlamini (2012) the IDEAL model could become more beneficial in a teaching environment that promotes the operation of daily life context to facilitate learning.

Based on Polya’s model, it is possible to conclude that mathematical problem-solving is a cognitive activity of defining the problem, developing a problem-solving strategy, implementing the strategy to solve the problem, and verifying the solution obtained. It is also reasonable to assume that to be a proficient problem solver, a learner must demonstrate the ability to effectively execute the above problem-solving steps. In its aim to explore and evaluate the learner’s problem-solving abilities, the current study adopted Polya’s model as a study framework (see Section 2.4).

2.2.3. Problem-solving instruction

There has been a shift worldwide in the last two decades towards the teaching and learning situation in the mathematics classroom so that problem-solving becomes the basic way of learning mathematics (Stramel, 2020). As a result, the importance of effective problem-solving instruction cannot be overemphasised. Through solving mathematical problems, leaners develop critical ways of thinking/ analytical skills, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom (NCTM, 2000) and they are empowered to be more analytical in making decisions in their daily lives (Syaiful et al., 2020). According to DBE (2011) mathematical problem-solving teaches us to think creatively and enables us to understand the world around us. Son and Fatimah (2020) further emphasise that through problem-solving learners find new ways of applying their mathematical skills, develop a deeper understanding of mathematical ideas and feel the experience of being mathematicians. This would be made possible through efficient and effective problem-solving instruction.

Huitt (2003) define instruction as the purposeful direction of the learning process. In this study the word “instruction” was used to refer to activities of teaching and learning in the classroom. Hence the phrase mathematics instruction would refer to the way in which mathematics is taught and learnt in the classroom, whereby the teacher is the driver of this process. Therefore, problem-solving instruction in mathematics refers to a teaching and learning approach that uses problem-solving tasks to facilitate mathematics learning (Dhlamini, 2012). According to Roh (2003) the approach can be viewed as a classroom strategy that organises mathematics teaching and learning around problem-solving activities which affords learners opportunities to think critically and present their own creative ideas. In this approach problems are the ones which drive the learning process. The lesson usually begins with a problem that learners must solve (Roh, 2003). By so doing, learners are guided to learning a new skill that they will use to solve the problem (Dhlamini, 2012).

According to Gordon (2009), problem-solving is an approach that form part of the constructivist teaching and learning methods. Contrary to the traditional teaching and learning approach which perceive a learner as a passive recipient of mathematical knowledge, a teacher as a transferor of knowledge to a learner like an empty vessel to be filled, where the prior knowledge of a learner is not necessary to learning and in which mathematics is learned through procedural learning (Ertner & Newby, 2013). A constructivist approach posits that learning mathematics involves learners actively creating, interpreting, and reorganising knowledge in unique ways (Gordon, 2009). This implies that teachers should create a learning situation that is learner-centred whereby the learning process is focused on a learner’s intellectual processes and their active involvement in the construction of their knowledge (Son & Ditasona, 2020). The tasks that teachers create or select must provide opportunities for learners to build on their knowledge and interests through active engagement in meaningful problem-solving (Artzt & Armour-Thomas, 2007).

The significance of lesson planning and preparation is very crucial in problem-solving instruction (Wahab, Saragih & Siman, 2020). It is important for a teacher to take some time to plan and prepare for mathematics lessons. This is the phase where the creation or the selection and sequencing of problems is done (Artzt & Armour-Thomas, 2007). The problems to be selected are the ones that will help learners to construct meaning and develop understanding of the concept of the desired outcome. They must be tasks that can facilitate the learning of mathematical content covered in lessons. Correct selection of these problems is especially important and to also ensure that the tasks are appropriate and relevant to the level, age, and grade of the learners. This could be done by planning in consultation with the relevant Curriculum Assessment Policy Statement.

In simple terms, a problem-solving instruction in mathematics can be viewed as an approach where learners not only learn from solving problems, but also from collaborating with each other, listening to one another and sharing and defending their points of views (Gordon, 2009). They also discover that making mistakes is a learning process. This approach in the teaching and learning of mathematics is learner-centred and not teacher-centred. The learner takes responsibility for his own learning, while the teacher is a facilitator of the learning process. However, the current study focused on a learner's mathematical problem-solving skills and abilities only without examining the teacher's pedagogy. Normally, these problem-solving skills and abilities could be tied to the nature of instruction that is prevalent in the classroom. Thus, this have been noted as a limitation of the study (Section 1.10).

2.2.4. Research on a learner's mathematical problem-solving abilities

2.2.4.1. What is problem-solving ability?

Dahar (1989) noted that problem-solving is not a generic skill but a human activity that relates and uses the concepts, rules and experiences that were encountered before to solve current and future challenges. NCTM (2000) attest that during the problem-solving process, learners can apply and adapt the strategies they develop to other situations and in other contexts. This implies that when a learner has successfully solved a problem, then that learner has a new ability that can be used to solve relevant problems. The more mathematical problems the learners can solve, the more abilities the learners will have that can help to solve more complex problems and navigate his daily life (Dahar, 1989).

Based on the preceding paragraph, it means therefore that the ability to solve problems (problem-solving abilities) can be acquired, developed, and improved or enhanced (Section 1.4). Moreover, it is also reasonable to assume that learners may possess different problem-solving abilities and that they may have different and varying levels of problem-solving abilities when compared with other learners (Sutrisno et al., 2020). This would mean that problems are subjective to individual learners. Thus, what appears to be a problem to one learner, might not be a problem to another (Fuadi et al., 2017).

Dhlamini (2012) established a link between mathematics performance and problem-solving abilities. The author noted that “performance in mathematics is a reflection of one’s problem-solving abilities” (pg.10). Furthermore, Ruseffendi (2006) alluded that a learner’s ability to solve problems is influenced by the level of ability in terms of low, average, or high ability that one possesses. This means that learners should acquire and possess certain abilities or skills needed to make headways when solving mathematical problems. It was on this ground that this study was commissioned to probe learner’s problem-solving abilities that are manifest when solving problems in Grade 11 Euclidean geometry tasks (see Section 1.8).

Son and Ditasona, (2020) argue that mathematical problem-solving involves cognitive and meta-cognitive strategies because it requires the learner to create new strategies to find solutions to problems. According to Kistian and Verawati (2020) the learner’s skills that they use to develop a strategy are an ability that the teacher should notice. A strategy is generally an action plan, an approach, a model, a way, method, or a series of steps that an individual can use to solve problems or to find answers to the questions asked (Éva Fülöp, 2021; Kistian & Verawati, 2020). A problem-solving strategy is usually general, flexible, and overarching (Éva Fülöp, 2021) (for examples, see Section 2.2.2). Éva Fülöp (2021) maintains that the strategy belongs to the cognitive part of the problem-solving process, while the method is a bridge between the cognitive and doing part.

Therefore, in terms of the description of a strategy, learners’ mathematical problem-solving abilities may be conceived as cognitive abilities, or inner able-ness, or the quality of being able to do, which then aid learners in understanding and responding to a problem. Meaning, when faced with a problem-solving situation, these cognitive abilities, or qualities of able-ness, they are automated and get activated by a learner, and guide them on how to respond to a problem. Having acquired these problem-solving abilities, a learner is perceived to have developed desirable strategies to solve the problem, use the selected strategies to solve the problem, and interpret the answers according to the given problem (Polya, 1973). Problem-solving abilities may be cultivated from continued exposure to problem-solving situations. With this exposure, a learner can develop desirable problem-solving abilities and mastery, which get automated when encountering a problem.

In this study, the learner’s mastery of the problem-solving abilities was meant to signify a quality of being a skilful problem-solver. Hence, these abilities help the learner generate the problem-specific skills that are usable for particular problem-solving events. Therefore, a learners' problem-solving ability is evident when learners successfully apply their problem-solving skills. The next section outlines problem-solving indicators that are usually used to assess learners' mathematical problem-solving abilities.

2.2.4.2. Assessing mathematical problem-solving ability

The concept of ability is abstract, just as the concept of life is abstract. To determine whether something is alive in simple biology, there are seven series of actions called "life processes" that are used to indicate whether "things" are alive. Namely, whether they can grow, reproduce, move, excrete, require nutrition, or have sensitivity and transpiration. All living things must demonstrate these seven processes to be classified as living things (Claybourne, 2012). Similarly, in mathematics, there are problem-solving indicators in which learners must demonstrate in order to possess the ability to solve problems (NCTM, 1989).

In a series of studies, learners' written responses have been analysed and interpreted against the following problem-solving indicators: (1) understanding the problem; (2) devising a plan; (3) carrying out the plan; and (4) looking back (Zanzali & Nam, 2020; Syaiful et al., 2020; Ruliani et al., 2018; Peranginangin & Surya, 2007; Fuadi et al., 2017; Rosli et al., 2013). The authors used these indicators to identify, assess, profile, analyse, or evaluate learners' mathematical problem-solving abilities. Furthermore, Sutrisno et al. (2020) and Wahab, Saragih and Siman (2020) used the same indicators to measure the impact of the application of different approaches and models of teaching instruction on learners' problem-solving abilities.

Academically, to measure learners' problem-solving abilities, a test will usually be administered to learners (NCTM, 1989). According to Wahab, Saragih, and Siman (2020), a test or assessment of problem-solving ability in mathematics is used to gauge a learner's aptitude in applying their knowledge and skills to effectively solve mathematical problems. A problem-solving ability scoring rubric is usually used to allocate scores to individual learners. The scores are used to determine the ability of the learners to solve problems (e.g., Fuadi et al., 2017; Peranginangin & Surya, 2017; Rosli et al., 2013; Ruliani et al., 2018; Syaiful et al., 2020; Sutrisno et al., 2020; Wahab, Saragih & Siman, 2020).

Rosli et al. (2013) noted that a problem-solving rubric is an appropriate tool when examining learners' ability to solve problems. According to Kulm (1994, as cited in Rosli et al., 2013) the usual traditional method of grading learner's written responses based on correct or incorrect items is better substituted by the scoring technique which is based on a performance rubric.

This method of grading a learner's work focuses on the content and process abilities demonstrated through the written responses rather than just relying on the scores of correct responses (Wiggins, 1990). Scores obtained from performance rubrics provide a rich information regarding learner's concept understanding and their learning development (Rosli et al., 2013). Kistian & Verawati (2020) attest that the learner's correct answer is not an absolute standard measure, but that the process the learner uses to obtain the answer is the most important. Similarly, in the current study the process of evaluating the learners' task-based worksheet was largely informed by Polya's problem-solving model using a performance rubric.

2.2.4.3. Factors influencing mathematics problem-solving ability

According to DBE (2011) problem-solving involves higher order reasoning and processes, therefore, learners will experience difficulties if they are not well-prepared during the mathematics learning process (Son & Ditasona, 2020). Kistian and Verawati (2020) argued that the deficiency of the utilisation of appropriate teaching and learning approaches which develop mathematical problem-solving abilities is the cause of learners' low problem-solving abilities in most of the schools today. These statements highlight the significance of the teaching approach employed by the teacher in influencing the effectiveness of the learning process (Dhlamini, 2012) and in enhancing the learner's ability to solve mathematical problems as well as real-world problems beyond the classroom.

Based on observations made by several scholars, most teachers in schools today are still using a traditional teaching approach to facilitate learning in the classrooms (Anggraini & Fauzan, 2020; Kistian & Verawati, 2020; Wahab, Saragih, & Siman, 2020). This approach is more teacher-centred, whereby the teacher talks, demonstrate or do examples, while learners listen, copy notes, and complete repeated exercises. This approach renders the learners to be passive participants in the learning process whereby they simply memorise methods used by the teacher without a deep understanding of the concepts (Wahab, Saragih & Siman, 2020).

According to Anggraini and Fauzan (2020), knowledge alone is not the sole determinant of learners' mathematical problem-solving abilities. The authors assert that there are several factors which determine the ability of learners to solve problems, including their belief in problem-solving. Literature in educational psychology refers to this belief as self-efficacy (Bandura, 1989). Bandura (1989) defines self-efficacy as one's judgements of his capabilities to perform on a task. Therefore, self-efficacy in mathematics has to do with learners' belief in their own ability to solve mathematical problems. Self-efficacy can predict whether learners will attempt to solve a given task, the amount of effort that will be put in solving the problem and how likely are they to persist to reach the solution stage (Bandura, 1989). This belief provides self-confidence and motivation in the completion of mathematical tasks (Anggraini & Fauzan, 2020). Putri and Prabawanto (2019) found that learners who possess high levels of self-efficacy are more committed and effective in mathematics learning goals than learners with low levels of self-efficacy.

2.2.4.4. Enhancing mathematics problem-solving ability

Learners' mathematical problem-solving abilities can be acquired, developed, and improved, or enhanced (see Section 2.2.4.1). To develop problem-solving and cognitive skills, DBE (2011) maintains that teaching should not be limited to "how" but should rather feature the "when" and "why" of problem types. Indicating that learners will be left ill-equipped in utilising their knowledge when there is a demand if learning only features procedures and proofs without a good understanding of why they are important. This is because rote or procedural learning in mathematics only facilitates surface rather than a deep understanding of mathematical concepts. Topics learnt may be easily forgotten as a result of learners not attaching meaning to what they learn. To eliminate this challenge in teaching and learning, teachers should consider Bloom's taxonomy which is the most recognised learning theory that was designed to target the depth of learning that learners must achieve (Anderson & Krathwohl, 2001). For instance, the CAPS document for mathematics provides a taxonomy of four cognitive levels used as a guideline for instruction and for constructing all assessment tasks in mathematics: knowledge, routine procedures, complex procedures, and problem-solving (DBE, 2011). According to DBE (2011), an assessment task should include questions in all cognitive levels. Cognitive Level 4 is intended to facilitate the development and improvement of learners' problem-solving and cognitive skills.

Wahab, Saragih, and Siman (2020) identified teacher quality as one of the key factors in the improvement of learners' problem-solving abilities and skills. Artzt and Armour-Thomas (2007) defined quality practice as teacher behaviours that are consistent with findings about how student learning is best facilitated. This means that teachers should be life-long students who continue to improve their teaching approaches, knowledge, and skills. The South African Department of Education has recognized the need for improvement in the quality of teachers, and as such, many programmes are carried out every year to improve the quality of teachers. However, the effectiveness of these efforts hinges on teachers as agents of change and implementers of the mathematics curriculum in improving their own quality. Without such improvement, the government's endeavours may be futile.

Recently, several studies have been conducted to improve learners' problem-solving abilities and skills using different approaches and models of teaching and learning instructions. These include the Realistic Mathematics Education (RME) approach; Problem-Based Learning (PBL) learning model; Inquiry-Based Leaning (IBL); Computer Assisted Instruction (CAI); Context-Based Problem Solving Instruction (CBPSI) and Polya problem-solving instructional approach coupled with social constructivist instructional approach (e.g., Abakar, 2019; Anggraini & Fauzan, 2020; Dhlamini, 2012; Eva Fulop, 2021; Gweshe, 2015; Kistian & Verawati, 2020; Masilo, 2018; Pasaribu, 2021; Pasaribu & Suyanto, 2020; Son & Ditasona, 2020; Sutrisno et al., 2020; Wahab, Saragih & Siman, 2020).

One of the teaching approaches found to be effective in overcoming the learner's low ability in solving problems is the Realistic Mathematics Education (RME) approach (Anggraini & Fauzan, 2020; Wahab, Saragih & Siman, 2020). In a study conducted with Grade 8 learners, Anggraini & Fauzan (2020) found that learners who learned using the RME approach had higher mathematical problem-solving abilities than those who learned using conventional learning. Moreover, Wahab, Saragih and Siman (2020) observed that the learning tools (e.g., lesson plans, textbooks, activity sheets and tests) developed based on RME approach effectively improved the mathematical problem-solving abilities of fifth grade learners.

Kistian and Verawati (2020) considered the Problem Based Learning (PBL) as a learning model that is suitable in helping learners to improve their problem-solving abilities and motivation in learning mathematics. The authors found that the average results of learners' mathematics problem-solving ability taught through PBL learning model was higher than that of the learner's mathematics problem-solving ability taught through Expository learning model. Similarly, in the study conducted with the Grade 10 learners, Dhlamini (2012) discovered that Context-Based Problem Solving Instruction proved to be effective in enhancing the learner's problem-solving performance as compared to the conventional problem-solving instructions.

Depending on the mathematics topic in focus, some instructional approaches and models are more effective than the others. To enhance the learner's problem-solving skills in Euclidean geometry, Masilo (2018) implemented an Inquiry-Based Learning (IBL) through Inquiry-Based Facilitation (IBF) on Grade 11 learners. The study revealed that IBL promotes deeper understanding, reliable, heuristic, self-guided learning that enhances problem-solving skills in

Euclidean geometry. Another view on the same topic, Gweshe (2015) considered that the contributing factors to low performance in solving circle geometry tasks could be caused by the lack of learner confidence and motivation in mathematics. In the study conducted to investigate the effect of using Computer Assisted Instruction (CAI), the results indicated that the use of the computer software, GeoGebra, in the teaching and learning of Circle Geometry improved the performance and motivation of Grade 11 learners. On the same note, Abakar (2019) used Polya's problem-solving instructional approach coupled with social constructivist instructional approach as an intervention strategy to teach circle geometry in Grade 11. It emerged from the study that the research intervention evoked learners' desire and interest to learn circle geometry. Also, the research intervention improved the study participants' performance and problem-solving skills in circle geometry concepts. Thus, the researchers recommended the adoption of their instructional approaches in mathematics classrooms.

2.3. COGNITIVE PROCESSING IN EUCLIDEAN GEOMETRY PROBLEM SOLVING

Abakah (2019) describes problem-solving as a cognitive activity that is a manifestation of learners' cognitive abilities. He further alludes that the learner's ability to solve problems is dependent on their level of cognitive ability to be able to think as they explore solutions to mathematical problems. Therefore, problem-solving in Euclidean geometry is perceived in this study as a cognitive activity. Thus, this section of the literature review is centred on the concept of cognitive processing. Cognitive processing is a term coined in cognitive science which has to do with the active functionalities of the mind. To understand cognitive processing in terms of how learning takes place, this section begins by exploring the cognitive learning theory by zooming into the concepts of information processing and cognitive load. Since the cognitive processing in focus is on problem-solving in Euclidean geometry, the conceptual understanding needed for geometric concepts is unpacked from the existing literature. The section explores cognitive processing according to the van Hiele theory with a special interest because of its development on geometric thought and how it contributes to the teaching and learning of geometry. At the finish of this section some learner's misconceptions and errors in Euclidean geometry are tabled from the existing literature.

2.3.1. Cognitive processing according to Cognitive Learning Theory (CLT)

Cognitive learning theory emanated a different learning perspective in history. The limitations of the behaviourist tradition kindled a cognitive revolution in the late 1950s (Ertner & Newby, 2013). However, from the philosophical foundations, the roots of cognitive theory can be traced as far back from the works of Plato in 400 BC (Grider, 1993). According to Grider' paper, in psychology, Earnest's attempts in exploring and understanding the functions of the mind began in Germany in the late 1800s. Since the 19th century, many psychologists have contributed significantly towards cognitive theory. Furthermore, the development of the cognitive learning theory was credited to the well-known educational psychologist, Jean Piaget who coined the concept in 1936 (Brown, 2019). Consequently, some cognitive theorists continued to build their work upon Piaget's ideas (Grider, 1993). However, the current study's use of cognitive theory was informed by several theorists.

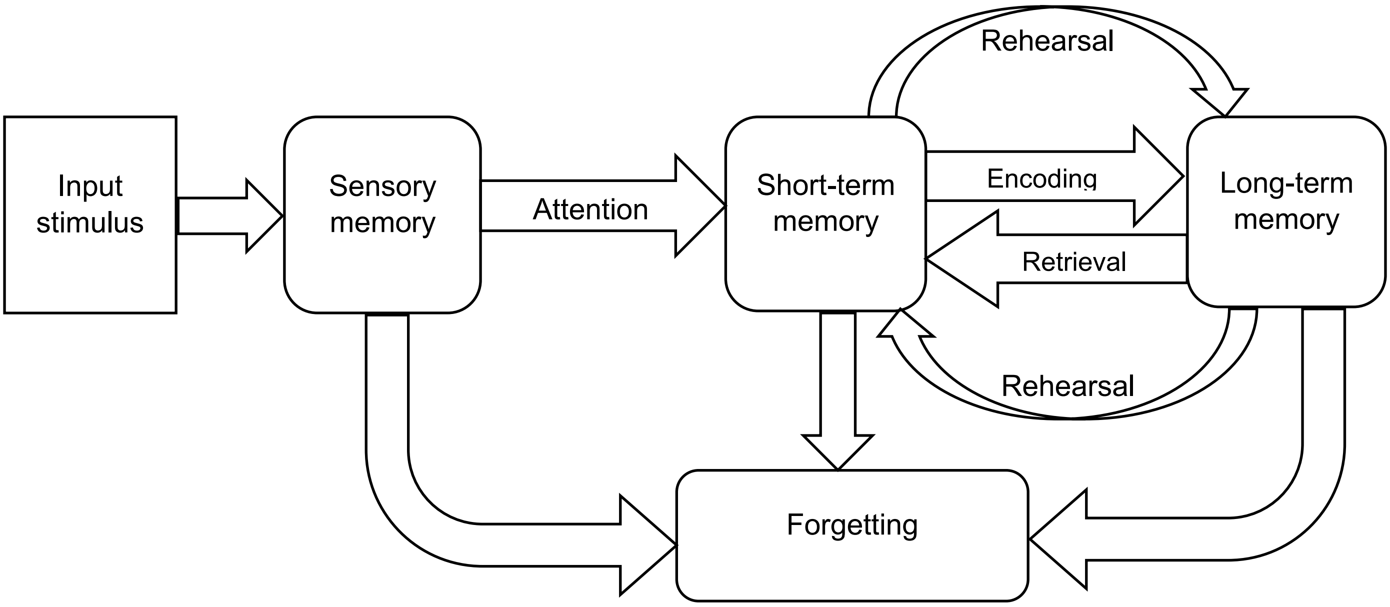
CLT is a broad theory that is concerned with mental activities. It stresses the acquisition of knowledge and internal mental structures (Anderson & Dron, 2011). In simple terms, cognitive learning theory is all about understanding the human mind and how learning occurs in the brain (Brown, 2019). The focus is on the conceptualisation of learners' learning processes where the issues of how information is received, organised, stored, and retrieved by the mind are emphasised (Ertner & Newby, 2013). Contrary to the behaviourist learning theory which focuses on the observable behaviour resulting from one's interaction with the environmental stimulus (Anderson & Dron, 2011), CLT focuses on the unobservable activities of the mind and how they influence behaviour (learning). In cognitive learning theories learners are perceived as active seekers and processors of information (Schunk, 2012). According to Yilmaz (2011) the instructional design must be aligned with the learner's cognitive architecture for effective learning to occur.

The theory efficiently explains the internal mental structures and the operations of internal knowledge representation behind the cognitive process of problem-solving (Wang & Chiew, 2010). Schunk (2012) attests that cognitive theory is generally regarded more fitting for clarifying complex forms of learning such as reasoning, problem-solving and information processing. In the current study, the theory provides the basis for understanding the cognitive structures of the brain and its processes in order to determine the learner's predispositions to problem-solving activities. Most importantly, learners' existing mental structures and cognitive processing are determined to maximize the effectiveness of learning through relevant instructional design (Ertmer and Newby, 2013).

2.3.1.1 Information processing

There are several theories that inform information processing. The term does not signify an identity of any specific theory alone. According to Schunk (2012) it is a broad term used in reference to theoretical perspectives focusing on the sequential and execution of cognitive events. However, George Armitage Miller became the first theorist to contribute his ideas of information processing (Bouchrika, 2021). The theory describes learning in terms of storage systems (Akdeniz et al., 2016). It deals with how the information is received by the mind, how it gets to be stored in the memory and how it is retrieved when needed. It is assumed that information processing takes place in phases that mediate between stimulus reception and stimulus production (Schunk, 2012). Furthermore, the brain or mental process is likened to a computer system and its functions. Although various information processing models have been developed, they share similar elements such as memory stores, cognitive processes, and executive cognition (Bouchrika, 2021). Figure 1 below shows an illustration of the multi-store model of memory put forward by Atkinson and Shiffrin (1968).

Figure 2.1: Atkinson and Shiffrin (1968) Multi-Store Memory Model



Retrieved from ResearchGate https://images.app.goo.gl/Hu1MiaYHTL9jzbkV8

Atkinson and Shiffrin (1968) in their model put forward three memory stores, the sensory memory, the short-term memory (STM) or working memory (WM) and the long-term memory (LTM). They also included some cognitive processes involving attention, rehearsal, encoding and retrieving. According to the model, data processing begins from the input stimulus from the environment through the senses of the body (sight, sound, touch, taste, and smell). The input stimulus is briefly registered into the sensory memory. A person is exposed to a lot of incoming information at one time. It would be very difficult to pay attention to all competing sensory inputs at the same time. Therefore, an individual would have to try to ignore or acknowledge incoming input stimulus (Michela, 2018). These sensory memories vanish quickly. It is suggested that visual records last for about half a second while audio records last for about three seconds in the sensory register (Akdeniz et al., 2016). It is in the sensory memory where perception takes place. The assigning of meaning to the input stimuli is largely depended on what is already known (Michela, 2018; Schuck, 2012). An individual's attention plays a key role in determining the information that will be processed or forgotten. Therefore, attention is selective (Michela, 2018). The information input that is attended to and perceived gets transferred into the STM or WM.

Schunk (2012) describes short-term memory (STM) as working memory (WM). He uses the terms interchangeably like some other researchers do. However, Lynch (2021), Dhlamini (2012), and Cowan (2008) provides a distinction between the two concepts. Though, the terms are not too distinct from each other. According to the authors, STM only signifies the faculties of a human mind that temporarily stores information, while the WM includes the STM and more. The WM is a system responsible for active processing and structuring of information. Given this background, this study will use the term "working memory".

According to Dhlamini (2012), the WM is the section of the mind that deals with active processing of information entering the cognitive system through the sensory memory. Schunk (2012) describes the WM as a place of awareness or "what one is conscious of" at a particular moment (pg. 166). The WM is limited with respect to capacity and duration (Schunk, 2012). The limitation in terms of the capacity involves the "space", how much information can be held at one time in the WM. Miller (1956) proposed that the WM can store up to 7 ± 2 units or chunks of information that could be processed at one time. A unit or chunk is any item which makes sense to an individual such as a number, letter, word, image, or common expressions (Dhlamini, 2012; Schunk, 2012). In connection to this, an example of grouping telephone numbers into chunks to facilitate memory is used more appropriately. Whereas the limitation in terms of the duration involves the "time", how long the information can be held active in the WM. Jeroen, Merrienboer, and Sweller (2005) emphasised that new information can be stored in the WM for a period of 20-30 seconds and that if not rehearsed it is forgotten. The authors noted that the limitations of the WM in capacity and duration only apply to new information acquired through the senses and not to the information retrieved from the LTM.

From the WM the information is then encoded into long-term memory. Encoding is a process of transferring novel information into the information processing system and preparing it for storage in the LTM (Schunk, 2012). Michela (2018) describes encoding as a phase where learning truly takes place. When new information is given meaning and integrated with existing information in the LTM. Schunk (2012) points out three factors influencing encoding, namely, organisation, elaboration, and schema structures. Schunk suggests that learning and recalling is made easier when (1) the learning material is well-organised; (2) new information is elaborated (i.e., linked with the known); (3) a schema structure “organizes large amounts of information into meaningful systems” (pg. 189).

Michela (2018) noted the description of memory as capable of storing information for later retrieval as ideas, images, and thoughts. This description generally refers to the long-term memory. Unlike the WM, LTM is unlimited in duration and capacity (Schunk, 2012). Akdeniz et al. (2016) maintains that LTM is a more permanent storage where information can be kept asleep for a long period of time until is it retrieved back into conscious. Simply put by Dhlamini (2012) is that all the knowledge and skills an individual have acquired resides in more or less permanently accessible form in the LTM. He further alludes that all we know is stored in the LTM and awaits activation. Akdeniz et al. (2016) argues that the information stored in the LTM cannot be forgotten, but rather, an individual might fail to access or retrieve it. Schunk (2012) suggests that consistent rehearsal of data input leads to easy retrieval of the information. On the same note, Akdeniz et al. (2016) suggest that the use of clues aid in the retrieval of stored information in the LTM. In addition, the dual-code theory maintains that the LTM embodies knowledge in two ways: knowledge is represented in a verbal system which is expressed in a language and an imaginal system which is expressed in visual and spatial information (Schunk, 2012).

According to the cognitive load theory knowledge is stored in the LTM in the form of schemas (Sweller, 1994). Song (2011) defines a schema as a “knowledge structure that represents a class of things, events and situations” (pg. 16, as cited in Dhlamini, 2012). In simple terms, a schema can be anything that has been learnt and is treated as an entity (Dhlamini, 2012). According to Sweller (1994), a schema organises the elements of information according to the way in which they will be applied. For example, there are schemas for dealing with problems (problem-solving schemas). As a result, a schema for the category of algebra problems will enable an unlimited variety of expressions or equations incorporated in this category to be dealt with (Sweller, 1994). These problem-solving schemas like any other knowledge structures are stored in the LTM.

Given this background, and relevance to the current study, problem-solving skills can be actively processed in the WM and stored in the LTM. It is also correct to assume that learners need to acquire problem-solving schemas for dealing with Euclidean geometry in order for them to efficiently solve related problems. In Grade 11 circle geometry, the consistent rehearsal of the theorems may aid in the facilitation of storage in the LTM. Furthermore, the use of clues may help in the retrieval of stored theorems in the LTM for application in solving associated problems. This means instead of trying to recall all the nine circle geometry theorems at once. A learners may group these theorems into three parts: those that require a centre as centre theorems; those that require all four points to be on the circumference of a circle as cyclic quad theorems; and those that require a tangent as tangent theorems (Basson et al., 2020).

2.3.1.2 Cognitive load in Euclidean geometry

The storage of information in the mind for later retrieval is a process that takes time, and it involves multiple cognitive processes (Schunk, 2012). Given the preceding information processing section, it is clear that the memory system can only process a limited amount of information at a given time. This is due to the limited attentional capacity of individuals to capture all the incoming environmental stimulus (Schunk, 2012), moreover, the capacity limitation of the working memory (Paas et al., 2003). Consequently, not all the information received through the senses is processed by the brain and stored in the mind. As a result, cognitive load theory was developed to account for the information processing limitations in the design of instructions (Schunk, 2012).

It was in the 1980s when John Sweller and his colleagues developed the cognitive load theory. The theory received attention around the world and was extensively expatiated upon in the 1990s by other researchers (Paas et al., 2003). Cognitive load theory focuses on instructional design principles based on the interaction between information structures and the human cognitive architecture that enable learners to process information (Paas et al., 2003 and Jeroen et al., 2005). It illustrates how that the information about how human brains learn and use knowledge can be utilised to create more effective instructional design and facilitate techniques that will maximise learning (NSW Department of Education, 2017).

According to Jeroen, Merrienboer, and Sweller (2005) cognitive load theory is founded on the idea that the working memory is limited in capacity and duration, particularly in dealing with new information. Which means, the working memory can only process a limited amount of new information at one time and that new information is active in the working memory for only a few seconds. Consequently, “when learning complex cognitive tasks, learners are often overwhelmed by the number of interactive information elements[6] that need to be processed simultaneously before meaningful learning can commence" (Paas et al., 2010: 115, quoted in Dhlamini, 2012). Dhlamini (2012) defines cognitive load as a mental burden experienced by a learner while executing a task. Due to the limited number of information elements, the working memory can process at a given time; the author emphasises that whenever learners are "bombarded with information and, if the complexity of their instructional materials is not properly managed, the working memory limit will be exceeded, and cognitive overloading will result" (pg. 74). Consequently, If the working memory is overloaded, there is a high possibility that the learners may not grasp the content being taught, they may be confused or may misinterpret the information, and it will not be effectively encoded in the long-term memory (NSW Department of Education, 2017).

Given the cognitive load definition above, it is reasonable to think that certain problem-solving tasks in mathematics can impose cognitive load on a learner's working memory (Dhlamini, 2012). Sweller (1994) raised concerns regarding the difficulties experienced in learning mathematics. He attests that these difficulties concern cognitive load in mathematical problem-solving. Moreover, Gupta and Zheng (2020) maintain that cognitive load play a key role in a learner's ability to solve problems in mathematics. This could suggest that cognitive load might have an influence on the low performance recorded in the Grade 12 learners' mathematics performance in the National Senior Certificate Examination (see Table 1.1). On the other hand, some teachers may argue that the current number of topics in mathematics as outlined by the Curriculum and Assessment Policy Statement (CAPS) is excessive. These topics include (1) Algebra, (2) Number patterns, sequence, and series, (3) Functions, (4) Differential calculus, (5) Financial mathematics, (6) Counting and probability, (7) Statistics, (8) Analytical geometry, (9) Trigonometry, and (10) Euclidean geometry (DBE, 2011). These teachers may advocate for a more streamlined approach that focuses on essential mathematical concepts to ensure better comprehension and mastery among students. Thus, each topic consists of several concepts under it, and they are expected to be taught at a limited period. Masilo (2018) suggest that committing mathematical errors during problem-solving is evidence that learners experience difficulties.

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6 An element in this statement is defined as any information that needs to be learned. Therefore, information elements interact if they are related in a manner that requires them to be assimilated simultaneously (Sweller,1994). For instance, in order for a learner to be able to simplify a fractional algebraic expression, they may need to apply several elements (information learnt) (1) laws of fractions, (2) laws of exponents, (3) factorisation etc.

In Grade 11 Euclidean Geometry, learners require competencies to do formal proof that were learned properly (DBE, 2011). Ideally, at the end of a circle geometry lesson, learners should be able to solve riders and construct proofs of related theorems. However, several learners struggle to remember the theorems and therefore cannot apply them in a problem task. According to the van Hiele model of thinking, the ability to construct and understand proofs is linked to Level 3. Yet, Cassim (2006) stressed that learners are less capable of reaching the level of innovative thinking, comprehension, and information because they have not gained mastery at the lower levels of the model (i.e., Level 2, 1, 0) (see Section 2.3.3). Masilo (2018) maintains that the visualisation as well as the analysis and informal deduction levels of the van Hiele model of thinking play a major role in building fundamental knowledge and understanding of Euclidean Geometry. Thus, it may be argued that without this fundamental knowledge, Euclidean Geometry problem-solving at more advanced levels such as formal deduction and rigour may impose cognitive load on the learner's working memory.

Paas et al. (2003) posits that the way in which the content is presented to learners and the learning activities required of learners can also impose cognitive load. They further allude that many conventional instructional techniques impose cognitive load because most of them are designed without any knowledge or consideration of learners' information structures or cognitive architecture. Some researchers have found that many teachers are guilty of imposing cognitive load on learners in the teaching and learning of mathematics (Anggraini & Fauzan, 2020; Kistian & Verawati, 2020; Wahab, Saragih & Siman, 2020). In the teaching of Euclidean geometry, in Grade 11 and 12 in particular, the instruction is pitched at a very abstract formal deduction level without considerations of the learners cognitive processing readiness.

Given the preceding statement, it is essential for instructional designers or teachers to be knowledgeable and to make considerations of the learner's structure of information or cognitive architecture when developing instruction. This means that both the learning task requirements and the current cognitive abilities of the learner must be considered. The purpose for these considerations is to develop the most efficient instruction that will lessen the cognitive load (Michela, 2018). For instance, in the teaching and learning of Euclidean geometry, teachers are advised to first determine the learner's geometric level of thinking and then design a sequence of activities that will enable learners to progress according to the van Hiele levels (Masilo, 2018). van Merrienboer et al. (2003, as cited in Schunk, 2012) also suggested the use of simple-to-complex sequencing of the learning material to minimize cognitive load. The authors further suggest the use of authentic tasks where the teacher begins the instruction with the case that the learners have encountered in the real world or their environment. For example, teachers could conduct tasks that will involve the learner's reaction, collation of figures and processes on objects familiar to learners (Masilo, 2018). Ideally, learners should be able to identify spatial figures, describe the attributes of the figures, and generalise the figures by their attributes before they can efficiently perform at formal deduction level where they can develop proofs using axioms and definitions (Alex & Mammen, 2014).

The main idea of cognitive load theory is that the instructional method should minimize the cognitive load (Schunk, 2012). Dhlamini (2012) encouraged teachers to acknowledge that learners operate using a limited working memory. He further alluded that the instruction should be tailored to inspire learners to efficiently use their limited working memory. Some suggested instructional strategies range from organisational to sequencing where information can be provided in manageable pieces, linking of new information with prior knowledge, presenting information both visually and aurally, the use of cues, timeous feedback, questioning techniques, etc. (Dhlamini, 2012; Michela, 2018).

One of the key techniques of overcoming the limitations of the WM according to the cognitive load theory is by schema acquisition and automation (NSW Department of Education, 2017; Sweller, 1994). Sweller (1994) use the works of other researchers on novice-expert differences to demonstrate the significance of schema acquisition. Evidently, the work on novice-expert differences suggests that differential access to a large store of schemas is an essential characteristic of skilled performance. This means that what appears to be a difficult problem to a novice problem solver, an expert may solve it with ease. The author noted that the problem-solving schemas that an expert have acquired on a particular subject or trade serves as building blocks for intellectual skills. These skills can be applied spontaneously. With extensive practice, automation occurs (NSW Department of Education, 2017; Dlamini, 2012; Sweller, 1994). Automation is a process whereby problem-solving can be performed automatically with minimal conscious effort (Sweller, 1994). These processes reduce the cognitive load in that they can bypass the restrictions of the WM. Since there are only few units or elements that can be accommodated in the WM at a given time, a schema constitutes a single unit in the WM. This means that although the number of units is restricted in the WM, through the presence of a single schema the amount of information in the WM may be enormous. Consequently, capacity restrictions are bypassed. Similarly, when schemas are automated, less WM space is required. As a result, the WM capacity is freed-up to process novel information (Sweller, 1994). Section 2.3.3 provides further clarity as to how these schemas can be acquired in Euclidean geometry.

In the current study, it is reasonable to assume that the learners' poor performance that is commonly observed in Euclidean geometry could be attributed to the insufficient acquisition of schemas needed to promote problem-solving activities due to the content cognitive load experienced by learners. Based on cognitive load theory assumptions, it is also sensible to think that the instructional methods that the teachers adopted in Euclidean geometry might have led to the challenges that are documented in the National Senior Certificate Grade 12 National Diagnostic Reports (see DBE 2020, 2019 & 2018). Therefore, the findings of the study to explore and evaluate the Grade 11 learners' problem-solving abilities that are manifest when they solve Euclidean geometry tasks will serve as a reference for teachers in improving the teaching and learning of Euclidean geometry, thereby contributing towards the improvement of learners' problem-solving skills in Euclidean geometry.

2.3.2. Conceptual understanding of Euclidean geometry

Masilo (2018) quoted Hamilton & Ghatala's definition of concept as mental representation of a category of some kind (things, actions, situations etc.) where it allows people to sort stimuli with similar characteristics into categories. Therefore, conceptualization refers to the action or process of thinking in order to formulate a concept or idea of something. According to Simon (2017) a concept is an element of understanding and knowledge. The author further allude that a mathematical concept is knowledge of the mathematical necessity of a particular mathematical relationship. This means knowing the "why" of mathematics (knowing the workings behind the answer). For instance, to efficiently solve geometry tasks, a learner needs to understand geometric concepts. In other words, the learner should understand the actual reasoning behind every written statement or procedure to meaningfully engage with geometry concepts. Thus, conceptual understanding refers to the comprehension of mathematical concepts, operations, and relations and being able to link, interpret and apply them correctly to a variety of situations (Engelbrecht & Potgieter, 2005; Rittle-Johnson & Schneider, 2014).

In terms of Euclidean geometry, the aim is for learners to develop an understanding of spatial concepts and relationships (Department of Education, 2003). It is through these spatial concepts and relationships where students learn to represent and make sense of the world (Alex & Mammen, 2014). Thus, it is recommended that in order to develop essential mathematics skills in Euclidean Geometry, the learner should apply spatial skills to identify, pose, use properties of shapes, apply critical thinking, and creativity to advance problem-solving (DBE, 2011). In addition, the DBE (2011) highlighted that through acquiring geometric problem-solving skills, learners are prepared to handle abstract concepts and improve their reasoning in problem-solving as a result of the experience they have acquired. This is because Euclidean geometry consists of a complex network of interconnected concepts which demand representation systems and reasoning skills in order to conceptualize and analyse not only physical but also imagined spatial environments (Alex & Mammen, 2014). Hence, most concepts in geometry require learners to visually perceive objects and identify their properties by comparing them with their previous experiences involving similar objects (Mamali, 2015).

This study focuses on Grade 11 Euclidean geometry concepts which puts an emphasis on circle geometry. The reasoning behind this section of geometry involves formal conceptual systems to study shape and space (Alex & Mammen, 2014) and learners are less capable of reaching this level of high reasoning, creative thinking, and information because they have not gained mastery at the lower levels of the van Hiele model of geometric thought, that is the visualization, analysis, and informal deduction levels (Cassim, 2006; Alex & Mammen, 2014). The understanding of spatial concepts and relationships is well explained by the van Hiele levels of geometric thought (Alex & Mammen, 2014) discussed in the next section.

2.3.3. Cognitive processing according to the van Hiele theory of geometric thought

The van Hiele theory of geometric thought has been developed and applied to provide clarity for the difficulties experienced by many learners with respect to higher order cognitive processes, particularly proof, essential for achievement in high school geometry (Usiskn, 1982). The model was created in the 1950s by the two Dutch mathematics instructors, Pierra van Hiele and Dina van Hiele-Geldof. The van Hiele model of mathematical idea arose out of their doctoral works in 1984 which were finished all the while at the College of Utrecht. Since Dina died not long after completing her exposition, it was Pierre who explained, changed, and progressed the theory. The exploration’s centre of attention was on thought and concept development in geometry (Crowley, 1987). As per Makhubele (2015) the van Hiele levels of thought in geometry is the most celebrated and conspicuous model utilised in the educating of geometry. In practice, it can be used to guide instruction as well as assess a learner’s abilities (Crowley, 1987). On the same note, De Villiers (1996, as cited in Makhubele, 2015) further underlined that it is the most popular framework as of now accessible for considering instructing and learning measures in geometry. Furthermore, numerous research studies have substantiated the validity of the Van Hiele theory in elucidating the developmental stages of a learner's geometric understanding (Bleeker, 2011). This theory has significantly influenced mathematics instruction worldwide and is deemed highly relevant to the current study. The van Hiele model of geometric thinking recognizes three aspects: the existence of levels, the characteristics of the levels, and the advancement from one level to the next (Usiskin, 1982). The model endorses an arrangement of five progressive reasoning levels (0-4) in a way learners attain geometric concepts beginning with the identification, naming, and grouping of shapes according to their appearance and coming full circle in having the option to compose a formal proof in geometry (Crowley, 1987; Usiskin, 1982; van de Walle et al., 2013). The five van Hiele geometric thinking levels, all together, are as per the following:

• Level 0 – Visualization (recognizing and naming geometric figures; where learners can group shapes based on their visual characteristics).
• Level 1 – Analysis (describing the attributes of shapes; where learners can group shapes based on their properties such as sides, angles, and symmetry).
• Level 2 – Informal deduction (classifying and generalizing shapes by attributes; where learners can make deductions about the properties of shapes and follow a logical argument in simple deductive reasoning).
• Level 3 – Formal deduction (developing proofs using axioms and definitions; where learners identify problems, pose exploration questions, and construct proofs based on their understanding from informal deduction).
• Level 4 – Rigor (working with various geometrical systems; where learners can make comparisons and contrasts between different axiomatic systems of geometry).

The South African geometry school curriculum is designed to address the van Hiele levels according to the learner's progression in terms of the phases. Although each phase (especially in the lower grades, grade R-9) maintains the same van Hiele level, there's progression in space and shapes content from one grade to another. For instance, in the foundation phase and the intermediate phase new shapes, properties and features of shapes and objects are added on what was done in the previous grade in order to achieve progression from one grade to another. Table 2.2. below shows the build-up of the geometry concepts from grade R through to Grade 12 and the associated van Hiele levels.

Table 2.2: South African geometry school curriculum

PHASE	GRADE	TOPIC	CONCEPTS	VAN HIELE LEVEL
Foundation phase	R-3	Space and shapes	1. Position, orientation, and views 2. 3-D objects 3. 2-D shapes 4. Symmetry	0
Intermediate phase	4-6	Space and shapes	1. Properties of 2-D shapes 2. Properties of 3-D shapes 3. Symmetry 4. Transformations 5. Viewing of objects 6. Position and movement	1
Senior phase	7-9	Space and shapes	1. Geometry of 2-D shapes 2. Geometry of 3-D objects 3. Geometry of straight lines 4. Transformation geometry 5. Constructions of geometric figures	2
Further Education and Training phase (FET)	10	Euclidean geometry	1. Revise basic results established in earlier grades regarding lines, angles, and triangles, especially the similarity and congruence of shapes. 2. The midpoint theorem 3. Properties of quadrilateral: kite, rectangle, rhombus, parallelogram, square and trapezium 4. Solve problems and prove riders using the properties of parallel lines, triangles, and quadrilaterals	3
	11	Euclidean geometry	1. Accept results established in earlier grades as axioms 2. Investigate and prove the theorems of the geometry of circles. 3. Use the theorems and their converses, where they exist to solve riders.	3 &4
	12	Euclidean geometry	1. Conditions for polygons to be similar 2. Prove the similarity and proportionality of shapes and the Pythagorean theorem by similar triangle	3 &4

The Euclidean geometry concepts in the FET phase, in Grade 12 particularly, is an integration of all the concepts in the lower grades. For learners to understand the concepts in Grade 11 and Grade 12, they require a good comprehension of geometric figures, shapes, angles, and line geometry as developed from the early grades through to Grade 10 (Abakah, 2019). Below is an illustration of the summary of the five hierarchical levels that describe the thinking processes in geometry concepts depicting a link from one level to another.

Figure 2.2: The van Hiele Model of geometry thinking

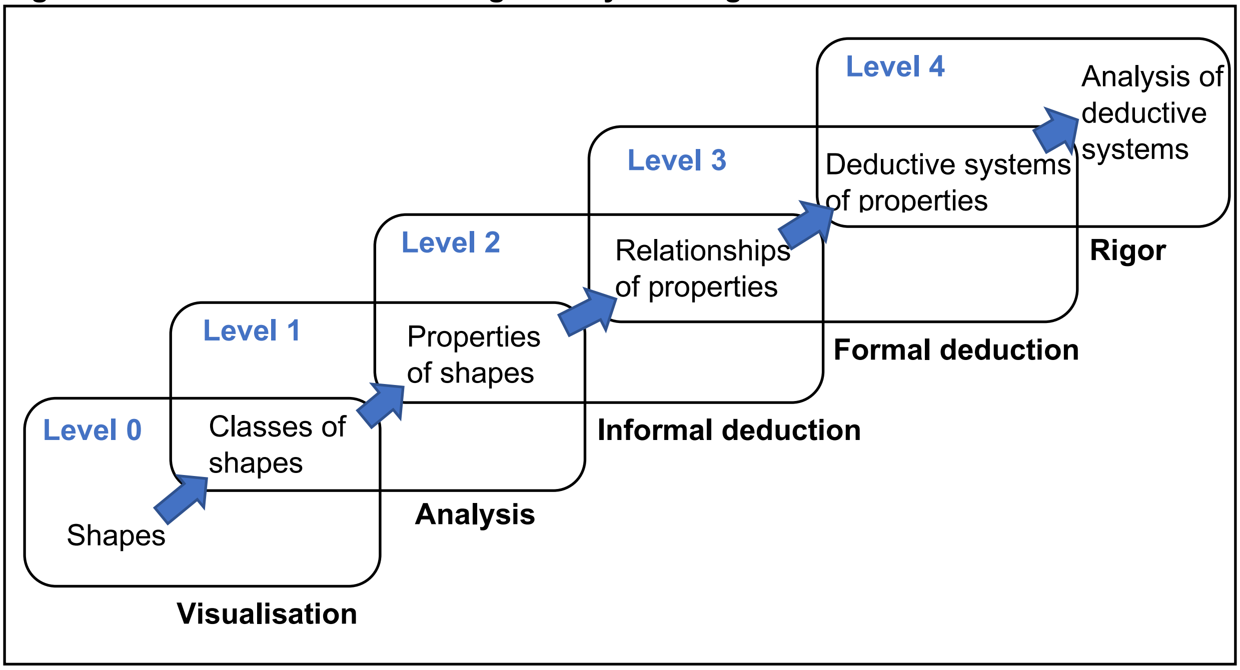


Figure 2.2 demonstrates five important characteristics of the van Hiele levels as follows:

1. Order: The levels are sequential. A learner must first develop visualisation schemas which serves as building blocks for the analysis level and so on. It has been documented that learners who experience difficulties in processing geometric concepts are taught at a higher van Hiele level than they are at or ready for (Usiskin, 1982).
2. Intrinsic and extrinsic: The basic objects in the lower level become the objects of study in the next level. For instance, a shape must be perceived and recognized in Level 0, then its properties can be analysed in Level 1.
3. Advancement: The progression of the levels is not age dependent. Rather, the instructional design and methods utilised by the teacher has direct influence on the progress (or lack of it) from level to level. For example, van de Walle et al. (2013) noted that some learners and adults forever remain in Level 0 of the van Hiele model.
4. Mismatch: The desired learning and progress may not take place if the instruction is delivered at a different level than the level at which the learner currently functions. This means, if the teacher and the learner function at different levels of the van Hiele model, they will not understand each other. The learner may not be able to follow through the

teacher, instructional material, content, vocabulary and so on, if they are at a higher level than the learner.

5. Linguistics: Each level is characterised by its own linguistic symbols and its own systems of relations connecting these symbols. What is said to be a fact in one level may be modified in another level. For instance, a square is recognized with only one name in Level 0; but may also be referred to as a rectangle or parallelogram in another level.

The importance of the van Hiele levels and accompanied characteristics cannot be overemphasised. When we look at Euclidean geometry in Grade 11 and 12 it poses a threat to most learners. They fail to cope with higher thinking processes in these higher grades because of the lack of geometric knowledge and understanding that should have been acquired in the lower grade levels (Masilo, 2018). Several studies conducted within the South African context proved that learners are progressed to the FET phase with lack of geometric conceptual understanding, knowledge and experience from the lower grades that should serve as a building block for higher order reasoning and creative thinking. Alex and Mammen (2014) conducted a study with the Grade 10 mathematics learners using the van Hiele model of thinking. Alex and Mammen (2014) found that most study participants were at Level 0, which indicated a concerning state of teaching and learning of geometry. These results meant that study participants would only manage to recognize, name and group shapes according to their appearance (Alex & Mammen, 2018). Also, Ngirishi (2015) explored the Grade 10 and 11 learners' understanding on basic geometric concepts using the van Hiele model of thinking. The findings of the study revealed that most participants were operating at Level 0 and 1 with a few participants able to reason at Level 2 of the van Hiele model. According to CAPS, learners in Grade 10 are supposed to have accomplished Levels 0 to 2 ahead of entering the FET phase and to perform at Level 3 of the van Hiele levels of reasoning (see Table 2.2 and DBE, 2011).

Given these challenges, the instruction in the lower grade levels led learners to learn by memorisation because they cannot recall the knowledge at higher grades (Masilo, 2018). Unfortunately, these challenges persist because Euclidean geometry instruction in Grade 11 and 12 is pitched at a very formal abstract deduction level despite the content gaps experienced by learners (Alex & Mammen, 2014). As per the characteristics of the van Hiele model, the advancement of a learner from one level to another is largely dependent on the instruction in place than on the age or maturity of a learner. In other words, the instruction plays a major role in the cognitive processing and schema acquisition of geometric concepts.

Thus, the instructional design and the way the content is presented are essential elements of pedagogical concern. It was on this basis that the van Hiele proposed five successive phases of learning (see Table 2.3). They assert that instruction designed in line with these sequential phases promotes the schema acquisition of a level (Crowley, 1987). Therefore, the phases in Table 2.3 provides teachers with guidelines for aiding learners to progress from one level to the next.

Table 2.3: van Hiele's phases of learning

No.	Phase of learning	Description
1	Inquiry or information	The teacher establishes the learner's prior knowledge by engaging them in a discussion about the objects of study in the level. The level-specific vocabulary is introduced.
2	Directed orientation	The teacher actively engages learners in activities that will help them explore the geometric concepts involving the properties and relationships, e.g., by constructing or measuring.
3	Explication	The learners are allowed to express their views on their learning experience. The teacher introduces new terminology after learners have had an opportunity to familiarise themselves with the concepts and ensures that the appropriate language is used.
4	Free Orientation	Learners apply acquired knowledge to solve more complex, open-ended tasks.
5	Integration	The teacher integrates and gives learners an overview of all that they have learnt without presenting anything new. This affords the learners an opportunity to review and summarize acquired information.

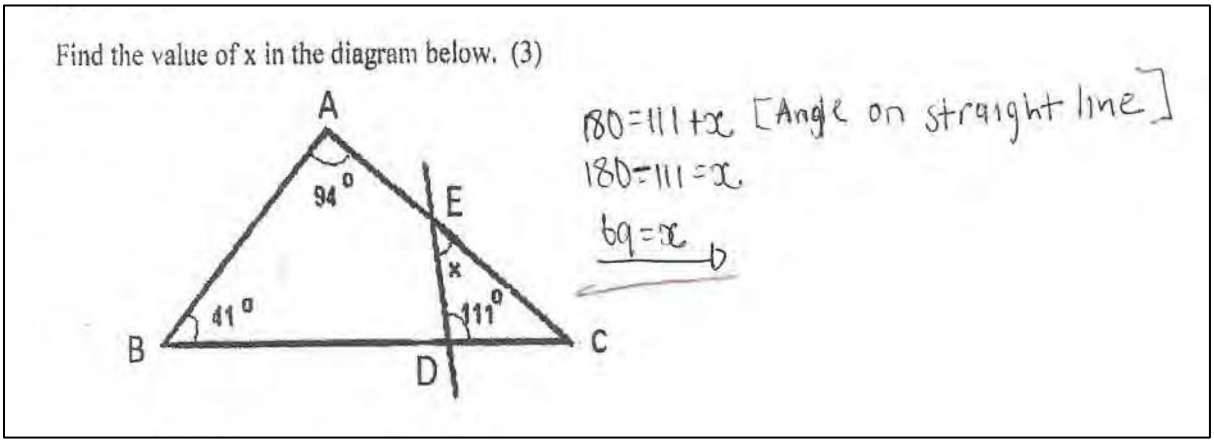
2.3.4. Some learner's misconceptions and errors in Euclidean geometry

One major problem that leads to very serious learning difficulties in mathematics is learner misconception. The Encarta online dictionary defines misconception as a mistaken idea or view resulting from misunderstanding of something (www.encarta.com). There are many definitions of misconception in literature. Michael (2001) defined misconceptions as lack of understanding of content that may prevent the learner from understanding the subject matter.

According to Smith, Disessa, and Roschelle (1993) a misconception is a conceptual structure constructed by the learner, which makes sense in relation to his/her current knowledge, but which is not aligned with conventional mathematical knowledge. Therefore, a misconception is a kind of misinterpretation of the concept.

Misconceptions may result from learner's previous inadequate teaching, informal thinking, or poor remembrance. Ay (2017) concurs that misconceptions emerge as a result of experiences and wrong beliefs of individuals. According to Mestre (1987), as cited in Ay (2017) every individual has a unique thinking system which is used in sense-making and expressing the world. He emphasises that if these thinking systems are faulty or deficient, they constitute the basis for misconception. This is the reason why misconceptions will always be part of the learning process. Learners have many misconceptions in geometry in particular. Sweller (1994) stresses that when learners are supposed to learn and understand so many concepts and activities at the same time, they may experience a cognitive overload which results in inefficient learning. Hence, the many misconceptions arising from learners in geometry. An example of one of the misconceptions identified by Ngirishi (2015) in his study with the Grade 10 and Grade 11 learners on geometry involved the meaning of angles lying on a straight line, where the learner stated that $D\hat{E}C$ and $E\hat{D}C$ sum up to $180°$ because they lie on the same straight line (see Figure 2.3).

Figure 2.3: An example of learners' misconception on angles lying on a straight line



Source: Ngirishi (2015)

According to Ngirishi (2015), the learner lacked conceptual understanding of geometric concepts. In this case the learner is aware that angles lying on the straight line sum up to $180°$, however, the concept is interpreted and applied incorrectly. Hence, there is a shift now from angles on a straight line to adjacent supplementary angles.

Misconceptions and errors though closely connected; there is a difference between the two. Misconceptions if not rectified will lead to errors. Harper (2010), as cited in Ngirishi (2015) defined an error as a deviation from accuracy or correctness (to wander or go astray); a mistake, as in action or speech belief in something untrue; holding of mistaken opinions. Errors are witnesses of learners' misunderstanding or habits. Luneta (2008) concurs that errors are signs of the challenges experienced by learners in a learning process. Meaning, therefore, errors are symptoms of the misconceptions that learners have. To better understand the common errors committed by learners in mathematics problem-solving, particularly in Euclidean geometry. Newman in Abdul (2015) established an error analysis model consisting of five sources of errors in mathematics problem-solving, in which when viewed properly, the model is in line with Polya problem-solving model (see Table 2.4 and Section 2.4.1).

Table 2.4: sources of errors in mathematics problem-solving

No.	Sources of errors	Description
1	Reading errors	An inability of a learner to read the given mathematical problem and to identify mathematical symbols and sentences used in constructing the problem.
2	Comprehension errors	An inability of a learner to understand mathematics problems
3	Transformation errors	An inability of a learner to come up with a suitable method of mathematical solution
4	Process skill errors	An inability of a learner to correctly process the solution to mathematics problems
5	Encoding errors	An inability of a learner to write the encoding error according to the given question

Source: Abdul (2015)

What Newman coined as reading errors, some authors view it as language barrier (Cassim, 2006; Mamali, 2015; Ngirishi, 2015). For instance, for a learner to read the given mathematical problem and to identify mathematical symbols and sentences used, he/she must understand the geometry language. This is because the vocabulary in Euclidean geometry is specific and carries meaning, descriptions and even properties. Geometric terms such as point, line, angle, right angle, bisect, diagonals, congruent, similar, parallel, perpendicular, tangent, circle, triangle, square and rectangle help us communicate our ideas to others with precision (Alex & Mammen, 2018). Therefore, a lack of understanding regarding geometric language may lead some learners to hold misconceptions. For example, any line that touches a circle may be perceived by some learners as a tangent.

More often than not, some learners do not make any attempt to solve Euclidean geometry problems, they would just leave a blank space or skip the question. This is highlighted in an overview of the diagnostic reports on National Senior Certificate Examination where some learners opt not to write the Euclidean geometry section or part of the section of the Paper 2 in an examination (DBE, 2018, 2017, 2016, 2015). Some authors view this case as the learners' inability to understand geometric problems (Cassim, 2006; Ngirishi, 2015). While Bransford and Stein (1993) refer to such instances as the physical and/or mental escape from problem situations strategy which they coined "the let-me-out-of-here approach".

In his exploration of the South African Grade 12 learners' understanding of Euclidean geometry concepts, Cassim (2006) examined the strategies that learners employed to solve geometric problems. Based on his observation of learners' responses to a given task, it was noted that most learners made incorrect use of theorems to solve the problem (rider). Instead, their responses were primarily based on the visual appearance of the diagram, and they attempted to "force" a solution even when it was not appropriate at that stage. Furthermore, the learners encountered difficulties when dealing with proof-type problems. One example is also highlighted in Ngirishi's (2015) work. In his exploration of the Grade 10 and Grade 11 learners' understanding of geometry concepts. He expressed a concern about the learners' responses to proof type questions. He put forward the evidence of the lack of logic and connections in most of the learners' responses. The diagnostic reports on National Senior Certificate Examination (DBE, 2020, 2019, 2018, 2017, 2016, 2015) noted the very same concerns regarding proof type questions. An indication is made that students fail to make a link to solve some problems with the majority having no idea where to start. In some cases, students begin with the statement they are trying to prove. Some of the errors and misconceptions noted amongst others are the tendency of making an assumption of properties based on the visual appearance of a diagram, and writing correct statements and reasons which are not related to the questions asked. Given these challenges, it may be necessary to continually examine learners' common errors and misconceptions of geometrics concepts to come up with suggestions for improvement.

2.4. THEORETICAL FRAMEWORK

2.4.1. Polya's theory of problem-solving

This study defines problem-solving as a cognitive activity in which there is a search for a solution to a problem (Dhlamini, 2012) and a process of discovering new knowledge (Wahab et al., 2020). Similarly, In'am (2016) established problem-solving as an approach to solving a problem that involves processes and strategies. He further recognises problem-solving as a systematic process where specific steps are applied in exploring alternative ideas to finding patterns leading to the solution of a problem. Abakah (2019) emphasise on these patterns by stating that mathematics seeks to understand patterns that might facilitate the resolution of the problems at hand. The author maintains that to solve mathematics problems, learners must explore appropriate patterns that will navigate them to the relevant solutions. On this basis, several problem-solving models or methods have been developed to facilitate problem-solving processes (the exploration of appropriate patterns) for problem solution (see Section 2.2.2).

The current study is underpinned by Polya's theory of problem-solving. Polya's theory is expressed in a model of problem solution. Abakah (2019) adopted Polya problem-solving hypothesis as a theoretical framework for his study and utilised Polya problem-solving instructional approach to teach circle geometry which he coupled with social constructivist instructional approach. In'am (2016) used Polya method as an instructional technique as well as an indicator to analyse the solutions of the learner's Euclidean geometry problems. Other authors used the Polya method as an instructional design and found that the application of the theory can improve the learner's mathematical problem-solving abilities (Daulay & Ruhaimah, 2019).

Euclidean geometry is viewed as a major branch of mathematics that develops learners' critical thinking abilities and problem-solving skills (Serin, 2018). According to In'am (2016) the aim of Euclidean geometry is to provide learners with knowledge and skills to guide them to understand a problem and come up with a plan to solve it through logical and systematic stages on the basis of reliable bases, theorems, and definitions. However, the author's concern is that most learners were found to exhibit no plan in solving problems. Therefore, Table 2.5 below is a presentation of the four steps proposed by Polya with the focus on Euclidean geometry.

Table 2.5: Polya steps for problem-solving

No.	Problem-solving steps	Action
1	Understanding the problem	**Identify the elements that are known and those asked for** 1. READ the information statement next to the diagram. 2. TRANSFER/MARK all the information on the diagram. E.g., parallel lines, equal sides (radii), equal angles or right angles
2	Devising a plan	**Develop a problem-solving strategy.** 1. Analyse the diagram by using KEYWORDS to find applicable properties and theorems (unpack the theorems). E.g., Centre, Diameter, tangent(s), cyclic quad, parallel lines, triangles, equal sides, or angles. 2. Use different colours to highlight and mark off with reasons.
3	Carry out the plan	**Implement the strategy to solve the problems** 1. LINK the information obtained. 2. Develop a rough proof.
4	Looking back	**Interpret the results according to the problem of origin** 1. Rewrite a formal proof. 2. Provide reasons for all statements and the solution declaration.

Source: www.theanswer.co.za

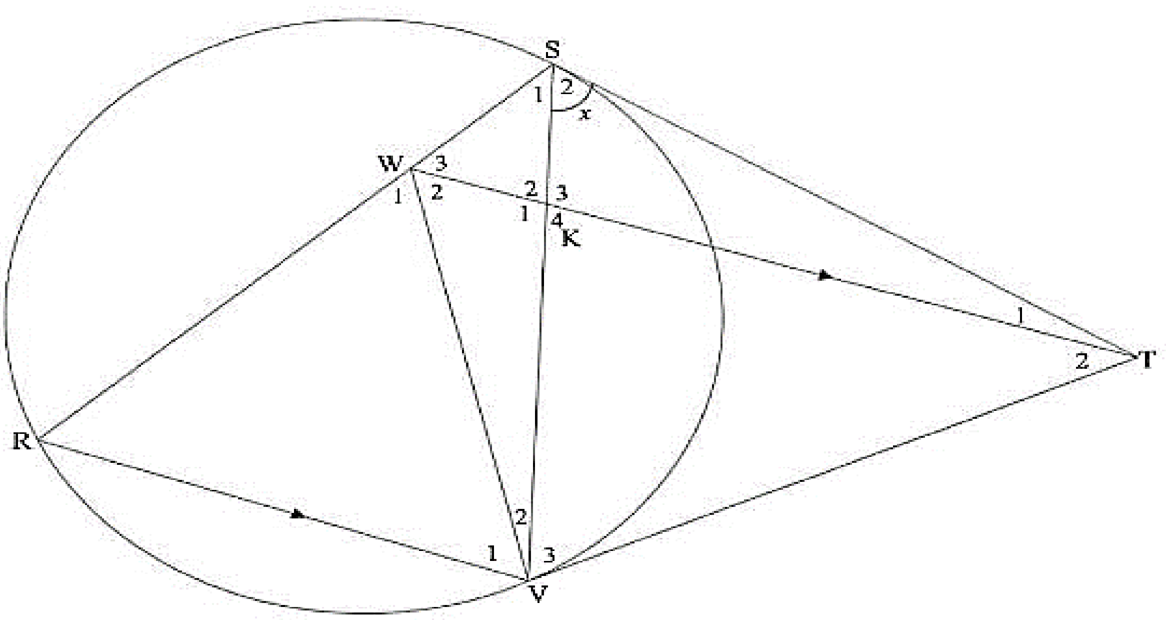
Mathematical problem-solving indicators suggested by NCTM (1989) are based on Polya method. Several researchers have utilised Polya's steps of problem-solving as problem-solving indicators to analyse learners' mathematical problem-solving abilities or skills (Daulay & Ruhaimah, 2019; Fuadi et al., 2017; In'am, 2016; Peranginangin & Surya, 2017; Rosli et al., 2013; Ruliani et al., 2018; Sutrisno et al., 2020; Syaifuli 2020; Wahab & Siman, 2020; Zanzali & Nam, 2020). Peranginangin and Surya (2017) conducted a qualitative descriptive study to analyse the Grade 7 learners' mathematics problem-solving ability on quadrilaterals using a problem-solving ability test. The learners' written responses were analysed and interpreted against the Polya model. The author's findings revealed that the learners' mathematical problem-solving ability in terms of these indicators were not complete. For instance, the first indicator showing the learners' understating of the problem was good with 75%, the second indicator showing the learners' planning was good enough with 66%. However, the third indicator where the learners had to carry out the plan was 29% and 24% in the fourth indicator where learners had to confirm their answers and that was very less. Similarly, other researchers utilised these indicators, some to identify, others to assess, to profile, to analyse or to evaluate learners' mathematical problem-solving abilities (see Section 2.2.4.2).

2.4.2. Considerations of the framework in the study

The current study was underpinned by Polya model. Through the model, the problem-solving abilities that learners demonstrated during a problem-solving task in Euclidean geometry were examined. Learners' problem-solving abilities that guided them to solve the related problem-solving task were analysed by the researcher. Learners' written responses were analysed and interpreted against the Polya model. This model is best suited for problem-solving in geometry concepts, especially in Euclidean geometry (Abakah, 2019; In'am, 2016). For instance, in Grade 11 Euclidean geometry curriculum (circle geometry, also assessed in Grade 12), a typical problem as seen in Figure 2.4 below:

Figure 2.4: Circle geometry problem

In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



Source: National Senior Certificate Examination Grade 12 Mathematics Paper 2, DBE (2019)

Require the learner to first read the given statement and understand the nature of the problem and the related diagram. Secondly, the learner will need to think critically of the plan to solve the problem which might involve analysing the diagram by using keywords (e.g., tangents, parallel lines) to find applicable properties, theorems and proves. Then use the identified properties, theorems and proves to link the information obtained and finally, rewrite a formal proof in accordance with the given question along with reasons for all statements provided. All this is better achieved with the consciousness of Polya model for problem-solving.

2.5. CONCLUSION OF THE CHAPTER

The current study dealt with problem-solving from a cognitive standpoint. In other words, problem-solving in Euclidean geometry is perceived in this study as a cognitive activity. Thus, this chapter attempted to describe the concepts of problem-solving and cognitive processing in Euclidean geometry and further featured a problem-solving theory to frame the study. Section 2.2 focused on problem-solving and the establishment of an understanding of the concept of problem-solving abilities. The section provided a clear description and explanation of what problem-solving abilities are, how they can be assessed, the factors influencing them, and how they can be enhanced. Section 2.3 was centred on cognitive processing with special interest on Euclidean geometry. Cognitive processing was viewed in terms of how learning takes place. Therefore, an account of cognitive learning theory was laid out with a further explanation of information processing and cognitive load in Euclidean geometry. The discussion in this section includes the conceptual understanding needed to engage in geometric tasks, cognitive processing according to the van Hiele theory of geometric thought, and some of the learners' misconceptions and errors in Euclidean geometry. Finally, Section 2.4 provides an overview of the theoretical perspectives of the study. The next chapter describes the methods used in the current study.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1. INTRODUCTION

This chapter presents the methods used to conduct this study. The researcher starts by describing the organising principles this study used. The researcher's strategy in conducting this study is laid out as far as the research design and the associated research processes. The chapter outlines the utilization of qualitative case study and the determination of the research site and the sample. The chapter further provides a description of the data collection instruments, a rationale for their use, and their subsequent construction. Associated with the latter is the researcher's narrative on the methods that the study used to collect information in relation to this investigation. As this chapter draws to a close, the information gathering procedures and the research morals that largely informed the conduction of this study are discussed in detail.

3.2. THE RESEARCH PARADIGM

Paradigms provide the foundation for different types of educational research (McMillan & Schumacher, 2010). Nieuwenhuis (2007) defines a paradigm as a set of assumptions or beliefs about the fundamental aspects of reality which give rise to a particular worldview. These worldviews, or paradigms, may represent a 'framework that guides research and practice in a field' (Willis, 2007, p. 8). Hence Nieuwenhuis (2007) further alludes that a research paradigm serves as a lens or organising principles by which reality (research data) is interpreted. In other words, a paradigm reflects the researcher's beliefs about the nature of reality (ontology), and the nature of knowledge (epistemology) (Krauss, 2005). Types of paradigms listed by Gupa and Lincoln (1994) are, namely; positivist, post-positivist, interpretivist and critical theory. However, McMillan and Schumacher (2010) highlight post-positivist and interpretivist/ constructivist as two major paradigms which influence research approaches, quantitative and qualitative approach, respectively.

This study is founded on the qualitative interpretive or interpretivist paradigm. This paradigm is also called the constructivist paradigm because of its obsessive pursuit to seek deeper understanding, and subsequently making sense of "the world of human experience" (see

Cohen & Manion, 1994; Fard, 2012; Fazliogullari, 2012). In the same view, Creswell (2003) relates interpretivism with social constructivism. According to Scotland (2012), interpretivist paradigm views reality as being subjective and individually constructed and subject to interpretation (Cohen et al., 2007). Wahyuni (2012) indicates that "interpretivists subscribe to constructivism as they believe that reality is constructed by actors and people's perception of it" (pg. 71). Researchers in the interpretivist paradigm seek to understand a situation rather than to explain the situation under investigation (Makhubele, 2015).

In this research approach the interpretivist knowledge gaining would be largely reliant upon understanding the "participants' views of the situation being studied" (Creswell, 2003, p. 8), that is, striving to understand reality from the perspective of the research participant. Hence one could position the aim of an interpretivist research as that of striving to offer the perspective of a social phenomenon and to analyse the phenomenon under investigation to provide deep insight into the way in which the participants make sense of the phenomena they encounter (Nieuwenhuis, 2007). This means that in an interpretivist framework the reality would be understood from the standpoint of individuals who are participating in it (Cohen, Manion & Morrison, 2007). Participants are mainly encouraged to generate their own interpretation and meaning of reality or the world that surrounds them. According to interpretivism, meaning is not discovered but is constructed through the interaction between consciousness and the world (Scotland, 2012). Figure 3.1 represents how the interpretivist community perceive reality, demonstrates their approach towards a phenomenon and how it is perceived, showing the methods they use to analyse reality, and what constitutes their intends of the product or results of the study.

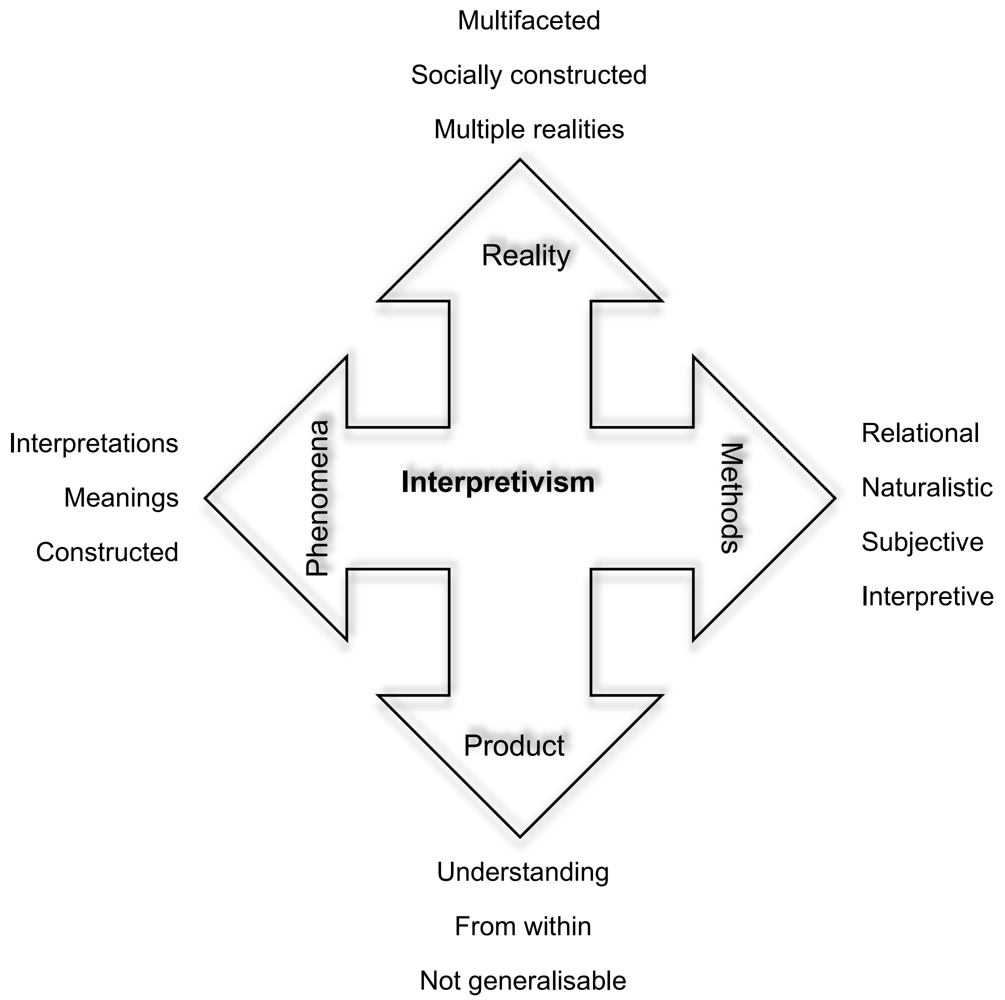


Figure 3.1 Representation of interpretivism as proposed by Nieuwenhuis (2007)

The current study adopted an interpretive paradigm with the aim to understand the learner's construction of Euclidean geometry concepts by exploring the extent of their problem-solving abilities that are manifested when solving Euclidean geometry tasks. This is made possible because the interpretivist paradigm study individuals with their unique characteristics (Cohen, Manion & Morrison, 2007). The closeness and the richness of the researcher-participant engagement opened privileged opportunities to gain understanding of learners' preferred problem-solving dispositions, and why they preferred certain problem-solving strategies over the other. Also, this moment of engaging helped the researcher to assess the extent of learners' problem-solving abilities and to document skills that learners tended to use when solving Euclidean geometry problems in Grade 11. In addition, the researcher's extended interaction with the learners accorded an extra opportunity to study the mathematical problem-solving strategies that learners used when solving tasks in Euclidean geometry.

3.3. RESEARCH DESIGN

According to Nieuwenhuis (2007), a research design is a plan or strategy which moves from the underlying philosophical assumptions to specifying the selection of respondents, the data gathering techniques to be used and the data analysis to be done. McMillan and Schumacher (2010) identify quantitative and qualitative research approaches as predominantly used methods of research with many researchers recently using a combination of the characteristics of both quantitative and qualitative designs called mixed method. The difference between quantitative and qualitative approaches lies within the nature of the design, how data are collected and analysed, and the types of conclusions derived from the data (McMillan & Schumacher, 2010). Each approach consists of several distinct research designs. The types of research designs used in quantitative approach include experimental designs, causal-comparative designs, correlational designs, survey, ex post facto and secondary data analysis, and the data obtained through these designs is primarily presented in the language of numbers (McMillan & Schumacher, 2010). Moreover, inferential, and descriptive statistical techniques are generally utilised in analysing the data collected in quantitative approach.

On the other hand, qualitative designs Include ethnography, phenomenology, case studies, grounded theory, and critical studies. The data obtained through these research designs is primarily presented in the language of words. According to McMillan & Schumacher (2010), qualitative data analysis is majorly an inductive process of arranging information into categories and identifying patterns and relationships among the categories. However, Cassim (2006) emphasises that it does not mean that calculations are not used at all in analysing collected data, but that they are not the major form in which the results are presented (p. 48).

This study used a qualitative method of enquiry. This approach was employed due to its ability to enable the researcher to collect rich descriptive data with respect to a particular context with the intention to develop a deep understanding of the phenomena being studied in its natural setting (Maree, 2007). According to Fetters, Curry and Creswell (2013), qualitative methodologies are utilised in studies that aim to explore how and why a phenomenon occurs so as to understand and describe the nature of an individual's experience. Therefore, qualitative research approach was deemed appropriate for this study as the researcher aimed to explore, understand, interpret and report on the learner's mathematics performance in terms of their problem-solving abilities when doing Euclidean geometry tasks in Grade 11.

The predominant research approach for the current study was a case study design. In'am (2016), Bleeker (2011), Mateya (2008) and Cassim (2006) in similar studies used the case study research design. As indicated by these studies, a case study design was chosen as the most proper plan for their investigations, given that the main quality of this sort of exploration is the delimitation of the object of study. This means in a case study the boundaries of the research sample, location and research focus can be set by the researcher to align to the aims and objectives of the study. Using case study helped the researchers to determine the setting, the duration, and their sample size. Case study was used to examine a specific case in order to gain insight into an issue. In addition, In'am (2016) referred Merriam (1988) study that also used a case study in the field of education. According to Maree (2007), case studies allow in-depth study on a particular educational situation. Similarly, in this study the focus was on one aspect of a problem studied in some depth within a restricted time. In this case, the focus was on a learner's problem-solving abilities in Grade 11 Euclidean geometry tasks. However, it may not imply that one site only is studied (see Schumacher & McMillan, 1989, as cited in Nieuwenhuis, 2007).

Cohen, Manion and Morrison (2007) listed three types of case studies that were first identified by Yin (1984) as exploratory, descriptive, and explanatory. Subsequently, Merriam (1988) referred to Yin (1984) case study types as descriptive, interpretive, and evaluative. It was the researcher's view that the current study would follow the model of exploratory, interpretive, and evaluative approaches. This research approach would satisfy the aim of the research enquiry to explore, understand and interpret the Grade 11 learners' problem-solving abilities that are evident when they solve Euclidean geometry tasks and to evaluate the levels of problem-solving ability in terms of low, average, and high level ability they operate within.

3.4. SAMPLING

Sampling refers to a process of selecting individuals who will participate in the study and from whom the study data will be collected (Fraenkel & Wallen, 2009). This section provides a description of the study population from which the sample was selected and further elaborates on the sampling procedures that were followed.

3.4.1. Population

Fraenkel and Wallen (2009) defines population as the larger group to which the results of the study are hoped to be applied. The population of this study consisted of secondary school Grade 11 learners who were enrolled in pure mathematics in public[7] schools in the Tshwane

7 Public schools in South Africa are schools that are regulated and funded by the government.

South district of the Gauteng province in South Africa.8 The Tshwane South district is located in the City of Tshwane, previously called Pretoria. At the time of conducting the current study, the information from the Department of Basic Education confirmed that there were 58 public secondary schools in the Tshwane South district.

When this study was operationalized, the Tshwane South district was divided into five educational circuits, ideally in accordance with their proximity of their geographical areas. Circuit 1 was made up of schools located around centurion up to Olieven, while schools from Laudium to Atteridgeville and Pretoria West formed circuit 2. Circuit 3 consisted of schools around Pretoria Central and Pretoria East. Circuit 4 consisted of schools around Mamelodi West and schools around Mamelodi East formed circuit 5. Table 3.1 shows the number of schools in each of the five educational circuits in the Tshwane South district. In addition, Table 3.1 provides the number of learners who were taking mathematics in Grade 11 in each of the schools when this study was conducted.

Table 3.1: Schools in the Tshwane South district that comprised the population of the study

Circuit	Number of public secondary schools in a specified circuit	Number of learners taking mathematics in Grade 11
1	13	1511
2	9	528
3	14	2596
4	9	484
5	13	873
Total	**58**	**5992**

(Adapted from Mphuthi, 2020)

The information used to compile Table 3.1 was collected from the database of the Tshwane South district mathematics FET (Grades 10–12) curriculum advisers who directly provide special educational support to the 58 schools in Table 3.1. As indicated in Table 3.1, the number of schools in each circuit varies and so is the number of Grade 11 mathematics learners in each school. The difference in number goes as far as the number of learners in each class. The recorded number of learners doing mathematics in the same grade level at

8 Gauteng is one of the nine provinces of south Africa. It is also the smallest province.

the beginning of the year usually decreases by the end of the same year as learners who find mathematics to be a challenging subject turn to choose mathematical literacy as an alternative subject, and eventually this reduces the number of mathematics takers. Therefore, the total number of learners taking mathematics in Grade 11 in the entire Tshwane South district was recorded to be 5,992 in May 2021.

3.4.2. Sample

A sample in research refers to a group of individuals taken from the larger population from which data is collected (Fraenkel & Wallen, 2009). The sample in the current study consisted of 63 Grade 11 mathematics learners from two schools located in the same circuit. The researcher focused on Grade 11 learners because circle geometry theorems are introduced, explored, and thoroughly taught in this grade level in accordance with the mathematics curriculum in South Africa (DBE, 2011). Most importantly, the larger portion of Euclidean geometry assessed in Grade 12 final examination paper 2 is the circle geometry taught in Grade 11 (see Table 1.2). In addition, it was easier to gain access to Grade 11 learners than the Grade 12 learners because of the school's or teacher's fear that the activities of the research might interfere with the normal plan of teaching and learning activities resulting to delays in syllabus coverage. The research study focused on only one class of Grade 11 mathematics learners from each of the two schools to work with approximately the same number of learners in both schools. In this study the two schools were referred to as School A and School B.

School A had only one classroom that catered for mathematics in Grade 11 and consisted of 35 learners. There were two types of learners that were identified in the Grade 11 class that offered mathematics in School A, and these types represented subject streams. The first stream consisted of 25 learners who were doing pure mathematics and science and the second stream consisted of 10 learners who were taking pure mathematics and accounting as subjects. Each time during a mathematics period, learners from the two streams would combine in one mathematics classroom. At the time this study was conducted, two learners from School A were absent. One of the learners absent was from the subject stream that consisted of pure mathematics and science while the other learner was from the pure mathematics and accounting stream. This meant that only 33 learners participated in the study from School A.

School B consisted of two Grade 11 classrooms in which learners were taking mathematics as a subject. In School B there were 31 learners in one Grade 11 classroom and 12 learners in another classroom. The classroom with 31 learners consisted of learners who were doing pure mathematics and science stream only, while the classroom with 12 learners consisted of learners doing pure mathematics and accounting stream only. Therefore, the school consisted of a total of 43 learners who were doing mathematics in Grade 11. Ideally, as part of the researcher's initial plan only one class of learners would be selected from two Grade 11 classes in School B. The selected group happened to be the classroom with the learners doing pure mathematics and science stream as the number was suitable for the study when compared to the other mathematics classroom. In School B one learner was absent at the time of administering the Euclidean geometry task and therefore, there were 30 learners who participated in the study. Table 3.2 shows a summary of the sample from which data were collected for the study.

Table 3.2: Summary of participants in the study

school	**Number of Grade 11 participants who were taking mathematics as a subject**			**Mean age (in years)**
	Boys	Girls	Total	
A	13	20	33	16.5
B	13	17	30	16.6
Total	**26**	**37**	**63**	**16.6**

(Adapted from Mateya, 2008)

Table 3.2 shows that from schools A and B there was a total of 26 boys and a total of 37 girls, summing up to 63 total number of Grade 11 mathematics learners who participated in the study. The mean age of learner participants in Table 3.2 was 16.6.

Both schools were public and classified as quintile[9] four schools, implying that they are fee-paying schools and located in more developed, resource-intensive communities (Western Cape Education Department, 2013). Given this status, both schools were administered through the same governmental policies, rules, and regulations.

3.4.3. Sampling procedures

McMillan and Schumacher (2010), Fraenkel and Wallen (2009) and Maree and Pietersen (2007) have identified probability and nonprobability as two major categories of sampling

9 All South African public ordinary schools are categorised into five groups, called quintiles, largely for purposes of the allocation of financial resources. Quintile one is the poorest districts, while quintile five is the least poor.

techniques. According to Maree and Pietersen (2007), probability sampling methods are based on the principles of randomness and probability theory, while nonprobability methods do not subscribe to this sampling criterion. Nonprobability sampling is the most commonly used method in educational research and qualitative studies are generally based on nonprobability methods of sampling rather than random sampling approaches (see McMillan & Schumacher, 2010; Nieuwenhuis, 2007).

In this study the researcher employed convenience sampling, which is a nonprobability sampling method. This sampling method was used to select two secondary schools from 58 schools in the Tshwane South district. Several researchers have conducted educational research studies using convenience sampling approach (for examples, see Mphuthi, 2020; Masilo, 2018; Gweshe, 2015 & Dhlamini 2012). Convenience sampling may be applicable in situations when a group of individuals are selected because they are accessible and conveniently available, or there are already existing relations between the researcher and the targeted sample (Fraenkel & Wallen, 2009; Maree & Pietersen, 2007). The two schools were selected because of their proximity in relation to the researcher's duty station. This assisted the researcher in avoiding excessively incurred travelling costs. Moreover, prior to conducting this study the researcher had established a relationship with mathematics teachers of these two schools. This relationship was built from attending the same mathematics training workshop organised by the Department of Basic Education, Tshwane South district.

There are well documented advantages and disadvantages of convenience sampling. The advantages may include a relatively easier accessibility of participants and the relatively cost-effective nature of using this method. The major disadvantage of convenience sampling is that it is usually considered to be less representative of the actual study population, a shortfall that may lead to the sample being biased (McMillan & Schumacher, 2010; Fraenkel & Wallen, 2009; Maree & Pietersen, 2007). In this case, Fraenkel and Wallen (2009) underscored the importance of replicating the study, which involves repeating the research with different groups of subjects in varying situations to validate the findings. In response to the latter, the researcher selected two different schools where the learners are taught by different mathematics teachers and therefore, assumed to be exposed to different methods of teaching and learning styles.

Following the administering of the Euclidean geometry task, only 3 learners were purposefully selected from each school for follow-up semi structured interviews. McMillan and Schumacher (2010) explain that purposeful sampling is a strategy of selecting small groups or individuals likely to be knowledgeable and informative about the phenomenon of interest. Subsequently, the selection of learners for interviews was based on their performance in the Euclidean geometry task. The learner who obtained the highest, average, and lowest score in the Euclidean geometry task according to the marking guideline participated in the interviews. The interview respondents were mainly sampled to seek further clarity to gain deeper insights into learners' understanding of Euclidean Geometry and the strategies that they would have employed in solving geometry tasks. This sampling technique was employed by the researcher to ensure that learners with different levels of problem-solving abilities participated in the interviews.

3.5. THE INSTRUMENTATION PROCESS

Instrumentation is a process of building research instruments[10] that could be utilised suitably in gathering information on the investigation (www.slideshare.net). According to Fraenkel and Wallen (2009), the term instrumentation may generally refer to the entire process of preparing to collect data. Subsequently, this section of the study discusses the purpose and the development of each data collection instrument, how the researcher responded to the notions of trustworthiness, dependability, triangulation, credibility, and transferability that were inherently embedded in conducting this study. This study used two instruments to collect data, namely, Euclidean geometry task and semi-structured interviews (see Appendix A and C, respectively).

3.5.1. The purpose of the data collection instruments

In this sub-section the researcher provides a justification for choosing each instrument for the purpose of data collection in this study.

3.5.1.1. Euclidean geometry task

A task-based worksheet is a research instrument consisting of a written list of questions for the purpose of gathering information from the respondents (McLeod, 2018). According to Parahoo (2006, as cited in Ngirishi, 2015), an individual's way of thinking may be best revealed in the documents they produce. Therefore, the sourcing of information from learners using the Euclidean geometry task was the main data-gathering event in this study. Mainly, this study aimed to explore, understand, and interpret the Grade 11 learners' problem-solving abilities that they manifested when solving Euclidean geometry tasks and evaluate the levels of

10 The device the researcher uses to collect data is called an instrument.

problem-solving ability in terms of their low, average, and high level (see Section 1.8). In this regard, Wahab, Saragih and Siman (2020) have alluded that to measure the learner's problem-solving abilities, a test is usually used in which the learners' ability to apply their knowledge and skills to be able to solve problems is assessed. Therefore, the geometry task in this study provided an opportunity for learners to express their knowledge and understanding of geometry concepts. The task also allowed them to execute their preferred problem-solving strategies and demonstrate their problem-solving skills when responding to a problem. Consequently, the researcher was able to answer the research questions of the study (see Section 1.5.1.1; Section 1.5.1.2). In addition, the researcher was able to identify the learners' misconceptions and errors which largely showed themselves in the learners' responses, and eventually, the feedback from the geometry task assisted the researcher in sampling learners who participated in subsequent interviews.

3.5.1.2 Semi-structured interviews

An interview is a two-way conversation organised with the purpose of gathering information (Nieuwenhuis, 2007). A research interview involves an interviewer, who asks questions and an interviewee, who responds to those questions. In this study, Interviews were used as a follow-up data collection technique to corroborate data that emerged from the learners' written response, that is, from administering the Euclidean geometry task to Grade 11 learners. According to Nieuwenhuis (2007), the aim of a qualitative interview is mostly to obtain rich descriptive data to help the researcher to understand the participant's construction of knowledge and social reality. Given the exploratory nature of the study, semi-structured interviews were conducted with three learners from each of the selected schools using open-ended questions to gain insight into the learners' knowledge of Euclidean Geometry and get a sense of the strategies that they employed when solving geometry problems in response to the written task. Dhlamini (2012) is also of the view that interviews afford participants with an opportunity to put into words and give voice to their problem-solving thoughts and ideas.

3.5.2 The development of the data collection instruments

This sub-section explains, in detail, the process of developing or constructing each one of the data collection instruments that are mentioned in Section 3.5.1.

3.5.2.1 Constructing the Euclidean geometry task

The task was developed in line with the guidelines of the Further Education and Training Phase (Grades 10–12) Mathematics Curriculum and Assessment Policy Statement (CAPS) on the Grade 11 syllabus covering the topic on Euclidean geometry (DBE, 2011). Table 3.3 shows the content coverage of the Euclidean geometry topic included on the task.

Table 3.3: Grade 11 syllabus coverage on Euclidean geometry topic

Concept	Questions
Prior knowledge e.g., properties of triangles and parallel lines	Q: 1.2; 1.3; 1.5; 2.1.2; 2.2.1; 2.2.2; 2.2.3
The angle subtended by the arc at the centre and the circumference of a circle.	Q: 1.3; 2.1.1
Angle in the semi-circle	Q: 1.2
Angles subtended by a chord of the circle	Q: 1.4; 2.2.2
Opposite angles of a cyclic quadrilateral	Q: 1.4
Exterior angle of a cyclic quadrilateral	Q: 1.1; 1.2; 1.4; 1.5
Two tangents drawn to a circle from the same point	Q: 2.2.2
The angle between the tangent to a circle and the chord	Q: 1.3; 1.5; 2.1.3; 2.2.1; 2.2.2; 2.2.3
Tangent perpendicular to radius	Q: 1.3; 2.1.3

As shown in Table 3.3, the fairness of the task regarding the coverage of the Grade 11 circle geometry content and consideration of prior knowledge is well accounted for. Only the theorem stating the line drawn from the centre of a circle perpendicular to a chord bisects the chord. Its converse was not included in the task.

The CAPS document also provides a taxonomy of four cognitive levels used as guidelines in the construction of all assessment tasks in mathematics: knowledge, routine procedures, complex procedures, and problem-solving. In line with the Gauteng Department of Education (2021) School Based Assessment (SBA) for Grade 11 mathematics, the cognitive levels were rephrased for this study. They included: knowing, applying routine procedures, multi-step procedures, and reasoning and reflection. An approximate percentage for each cognitive level has been determined as 20%, 35%, 30%, and 15%, respectively. According to DBE (2011), an assessment task should include questions in all cognitive levels. However, in this study the level one questions were minimized and added to level four questions due to the nature of the study that focussed on the exploration of learners' problem-solving abilities. Table 3.4 shows the analysis of the cognitive levels for each question in the task.

Table 3.4: Euclidean geometry task cognitive analysis grid.

	MARKS			
Question	**Level 1 (20%)** Knowing	**Level 2 (35%)** Applying routine procedures	**Level 3 (30%)** Multistep procedures	**Level 4 (15%)** Reasoning and reflection
Section 1				
1.1	3			
1.2		3		
1.3		3		
1.4				3
1.5			3	
Section 2				
2.1.1		2		
2.1.2		3		
2.1.3		2		
2.2.1			4	
2.2.2			4	
2.2.3				5
Total (35)	**3**	**13**	**11**	**8**
Percentage (100)	**9%**	**37%**	**31%**	**23%**

In cognitive level one questions, learners were required to recall information and use mathematical facts. Cognitive level two questions required learners to use well known procedures, simple applications and calculations which might involve few steps. In cognitive level three questions learners required conceptual understanding to solve problems involving complex or multistep calculations where there was no obvious route to the solution. Cognitive level four questions required learners to apply higher order reasoning and process to integrate their knowledge or break the problem into pieces to identify what must be solved (DBE, 2011).

The structure of the task was adapted from Cassim's (2006) exploratory study that focussed on Grade 12 learners' understanding of Euclidean Geometry with special emphasis on cyclic quadrilateral and tangent theorems. In the current study the task was divided into two parts, Section 1 and Section 2. Section 1 consisted of multiple-choice questions with five items, while Section 2 consisted of open-ended questions with six items. Usually, learners are not asked multiple choice questions in Grades 10–12 in an examination question paper. However, as emphasised by Cassim (2006), in as much as the answer lies in front of the learner, they are required to work out the answer to determine the correct option. Each question was accompanied with four seemingly valid solution options, of which only one was the correct answer to the question. The open-ended questions involved numeric questions where the learners had to calculate the sizes of angles and prove type questions. The task consisted of typical examination type questions in which some of them may have been encountered by the learners before.

3.5.2.2 Constructing the semi-structured interview schedule

A list of questions or a schedule was prepared in advance to ensure that each interview was presented with the same questions in the same order (Nieuwenhuis, 2007). The interview questions were designed to probe deeper into learners' task responses, with a specific focus on exploring mathematical problem-solving strategies, if any, that they used when solving problems in the Euclidean geometry task.

3.5.3 Ensuring trustworthiness in the data collection process and analysis

Lincoln and Guba (1985) suggest that the trustworthiness of a research study is significant to evaluating its worth. Here the trustworthiness of the instruments is discussed. In line with Connelly (2016), the trustworthiness of the instruments in the current study referred to the degree of confidence in methods used to ensure the quality of these instruments. Instead of relying on terms such as reliability and validity, which are largely associated with quantitative research, terms such as credibility, dependability, confirmability, and transferability are used (Lincoln & Guba, 1985). This section describes how these key criteria of trustworthiness were achieved in this study during the data collection process and analysis. Also, issues of triangulation have emerged. Connelly (2016) have recommended that in qualitative research issues of reliability & validity could be addressed as follows:

	Reliability	**Validity**
Internal	Dependability	Credibility
	Consistency	Accuracy
External	Confirmability	Transferability
	Neutrality	Applicability

3.5.3.1 Dependability

Lincoln and Guba (1985) suggest that dependability concerns the stability of data over time. This may refer to a situation where the research's findings are consistent and could be repeated. Mphuthi (2020) maintains that the notion of replicability might be viewed as depicting the nature of external validity in quantitative studies. External validity prioritises the generalisability of the results, a situation which may not appear to be possible in qualitative studies (see Figure 3.1). According to Wahyuni (2012) dependability involves the process of taking into account all the changes that transpire in the research site and how these influences the manner in which the research is carried out. In agreement, Moon et al. (2016) emphasise on transparency and clear documentation of all research processes in the establishment of dependability in qualitative research so that other interested researchers may be able to track the same procedures. In this study, the researcher provided a comprehensive description of the entire research process. In addition, the participant's original written work and the interview recordings can be used to ascertain the dependability of the study. This ensures that the data was not fabricated.

3.5.3.1.1 Dependability of the Euclidean geometry task

One of the ways to establish dependability in qualitative studies is through inquiry audit (Lincoln & Guba, 1985). Thus, to enhance the elements of dependability of the Euclidean geometry task following the process in Section 3.5.2.1, mathematics experts were involved to ascertain whether the items included in the task were in line with the requirements of the Grade 11 mathematics syllabus and that they were dependable in exploring and measuring the learners' problem-solving abilities that are manifest when solving the task concerned. The mathematics experts involved two senior mathematics teachers who had been teaching mathematics in Grades 10 – 12 for more than 15 years at the time of conducting this study, one head of department for mathematics at school level and one FET mathematics curriculum adviser at district level. They all worked separately using a modified school level assessment subject specific pre-moderation tool (see Appendix E). Furthermore, the task together with the interview schedule were reviewed by the study supervisor who was a professor in mathematics education.

3.5.3.1.2 Dependability of the semi-structured interviews

Similarly, the same mathematics experts who worked on the Euclidean geometry task evaluated the interview schedule. The interview schedule being a follow-up research collection tool, was assessed in relation to the Euclidean geometry task to check whether the list of questions would afford respondents an opportunity to put into words and give voice to their problem-solving thoughts and ideas. The evaluation went as far as touching the issues of language, grammar, clarity of questions, comprehensibility to the learners and relevance to the study.

3.5.3.1.3 Feedback from mathematics experts

The feedback from the mathematics experts highlighted various aspects of the Euclidean geometry task, the marking guideline, and the interview schedule. The comments that were made ranged from the language, the phrasing of statements, the mark allocation, clarity, and relevance of questions. For instance, the researcher was advised to increase the mark allocation for Section 1 multiple choice questions from one mark to three marks per item, reason being the questions require the application of several theorems and steps in reaching the correct solution. The comments went as far as touching the cognitive levels of the questions. The researcher was further advised to minimize the percentage on level one (knowledge) questions and add to level four (problem-solving) questions due to the nature of the study that focussed on the exploration of learners' problem-solving abilities.

3.5.3.1.4 Adjustment of the research instruments

After the evaluation or validation process, some adjustments were made in the mark allocation of the items, editing of language, and rephrasing of statements or questions both in the task and interviews schedule. In addition, a problem-solving question was formulated and incorporated in Section 2. In the final draft of the Euclidean geometry task, the fairness of the task regarding the coverage of the Grade 11 circle geometry content and consideration of prior knowledge was well accounted for (see Table 3.3). The final revised interview schedule was compiled with recast questions (see Appendix C).

3.5.3.2 Credibility of the data

Credibility of data refers to the accuracy of the data and whether it reflects the observed phenomena (Wahyuni, 2012; Yilmaz, 2013). In agreement, Merriam's (1991) meaning of credibility in qualitative research entails the congruency of the findings of a research study with the observed phenomena. While Lincoln and Guba (1985) define credibility as the confidence in the truth of the data and analysis. The authors further describe several techniques that can be employed to establish credibility which includes triangulation and member checking.

According to McMillan and Schumacher (2010) triangulation is one of the techniques used to seek patterns in the data obtained. It involves the use of “multiple researchers, multiple theories for data interpretation, multiple data sources to corroborate data and multiple disciplines to widen one’s understanding of the method and the topic of interest” (Janesick, 1998, as cited in McMillan & Schumacher, 2010: 331). In this study, the Euclidean geometry task administered to learners served as the primary data-gathering tool while the semi-structured interviews were used as a follow-up data collection technique to corroborate data that emerged from the learners’ written response. In addition, the technique like member checking was employed to authenticate the results. Lincoln and Guba (1985) suggest that member-checking is the most important technique in qualitative studies for establishing credibility. In the current study, the researcher made copies of the learners’ written worksheets of the Euclidean geometry task to avoid learners tampering with the original response and used them to provide feedback to the learners. The Grade 11 mathematics teachers also verified the researcher’s marking on the original learners’ written worksheets and the mathematics heads of departments of the participating schools moderated a sample of learners’ written tasks. Moreover, after transcribing the learners’ interview responses, the researcher went back to the respondents to allow them to verify if the transcriptions represented their actual responses or what they said.

To further ensure credibility in the study, the researcher presented some of the learner’s written responses in the data analysis section. The learner’s written responses and comments made during the interviews were a point of focus with special attention given to bracketing. In this way, the researcher ensured that the participant’s experiences, words, and descriptions were faithfully presented without omissions or additions during the entire process of analysis.

3.5.3.3 Confirmability

Confirmability refers to the degree of neutrality or the extent to which the findings of the research are inspired by the participant’s experience and not researcher interest, bias, or motivation (Cohen & Crabtree, 2006). According to Moon et al. (2016), to establish confirmability the study must demonstrate that the findings are clearly connected to the conclusions in a path that can be tracked, and the procedures repeated. The significance of confirmability to research appears to be the same as that of credibility. In the current study, the researcher provided data records, detailed data analysis and well documented description of results.

3.5.3.4 Transferability

To ensure the extent that the findings are transferrable to other situations the researcher used thick descriptions of events in reporting the findings (Wahyuni, 2012). A thick and rich description of the research site is intended to allow readers to 'be moved' to the actual research setting or to get a sense of how the research activities played out in the research space (Creswell, 1994). According to Lincoln and Guba (1985), the thick descriptions in research may assist the researcher to achieve external validity, which may also refer to the relatedness of the study findings to the actual reality at which the research took place. Hence the responsibility of researchers is to "provide sufficiently rich data for the readers and users of research to determine whether transferability is possible" (p. 316).

3.6 THE USAGE OF CODES TO IDENTIFY SCHOOLS AND PARTICIPANTS

The privacy of participants in a research study must be maintained (McMillan & Schumacher, 2010). Given that the anonymity and confidentiality of the research participants is of outmost importance, each learner who participated in the study was given an identifying pseudonym or code name to safeguard their identity and that of the schools. In this study the two schools were referred to as School A and School B. The learner's responses from School A were coded LA1 – LA33. Whilst the learner's responses from School B were coded LB1 – LB30.

The identification for School A and School B describes the order of school visits. The first school from which the researcher collected data was denoted as School A and the second school from which the researcher collected data was denoted as School B. In the codes LA1 – LA33 and LB1 – LB30 the letter "L" stands for learner, A or B stands for School A or School B and the numbers corresponds to the sequence of the names of the learners as they appear on the recording mark sheet used to capture their scores for the written task. The construction of code names for the participating schools and learners made it easy for the researcher to use the same code names for the geometry task responses and for the interviews for the selected learners to avoid double naming which might result in confusion.

3.7 EMBARKING ON A DATA COLLECTION PHASE OF THE STUDY

Data collection is the process or the research method of gathering information to answer the research question(s) (McMillan & Schumacher, 2010). In this section, the process of administering the Euclidean geometry task on learners who were taking mathematics in Grade 11, and the subsequent sessions of the conduction of semi-structured interviews with learner respondents is discussed.

3.7.1 The administrating of the Euclidean geometry task to learners

The actual conduction of this study took place in 2021, at the time when the entire world was grappling with the devastating and pandemic effect of the novel coronavirus (Covid–19). In South Africa, the country had just passed through the second wave of Covid-19 infections. At the time of intending to administer the Euclidean geometry task to learners in participating schools, the country was at the adjusted alert level one lockdown, which lasted for a period of three month from the 1st of March 2021 to the 30th of May 2021 before another lockdown level adjustment. At that time, most of the normal activities could take place with precautions and strict adherence to state pronounced health guidelines that were largely informed by the Disaster Management Act regulations (South African Government, 2021). Therefore, human movement and interaction were strictly regulated by government institutions and agents, and this would subsequently have impact on the data collection experience of this study.

Given this new and imposing reality, the researcher ensured that all data collection activities were characterized with strict observance of the Covid-19 protocols as set out by the state in the Disaster Management Act in relation to the alert level one lockdown regulations (South African Government, 2021). The researcher always wore a face mask covering both the nose and the mouth. Despite the placing of sanitization stations in the school's entrance, offices and classrooms, the researcher kept possession of a personalized sanitizing kit to clean the hands. During all research related meetings/ gatherings, and in the classrooms during the implementation of all research activities, the requirement of social distancing was strictly observed enforcing the 1.5 metres requirement between each person. While in the classroom the windows were made to remain opened to allow fresh air and ventilation to regulate. Learners who were placed in the care of the researcher were encouraged by the researcher to sanitize their hands regularly and to ensure that they wore their masks properly over the mouth and nose. During the administration of the task, the researcher sanitized the hands before distributing the worksheets and after receiving the written scripts from learners. The learners also sanitized their hands before receiving the task and after submitting the scripts to the researcher.

After receiving the consenting forms from the two schools and parents, and the signed assent forms from the learners to participate in the study, the researcher arranged a meeting with the Grade 11 mathematics teachers regarding the administering of the task. The meeting also addressed issues of the date and time of researcher's visitations to schools, and issues relating to the rampaging coronavirus were also covered. Consequently, learners were informed by their respective teachers about the schedule that was intended to outplay the research activities. The arrangement was that the visitations would take place in the first term of the school calendar because it was anticipated at that time that the classroom treatment of the topic on Euclidean geometry would be completed in both schools. However, to allow the learners to fully complete the topic on Euclidean geometry, the task was administered at the beginning of the second term with the first week of schools reopening dedicated to School A. The visitation to School A commenced on May 5, 2021 and the second week was allocated to visiting School B on May 10, 2021. Finally, learners were able to write the task in the afternoon of visitations and the writing of the task lasted for an hour from 14h15 to 15h15, which were time frames that fell outside the official tuition time in both schools. It was the researcher's desire not to interfere with official tuition in participating schools. Teachers assisted in organising the venues where the task would be written. All learners who participated in the writing of the task were subjected to assessment conditions that simulated those of examination sessions, and the researcher played the role of an invigilator.

The researcher marked all learners' scripts using a marking guide (see Appendix B). After marking, the researcher made copies of the learner's written worksheets to avoid learners from tampering with the original responses and used them to provide feedback to the learners (see Section 3.5.3.1). The Grade 11 mathematics teachers were given the original learners' written worksheets to verify if the researcher's marking of learners' tasks was correct and accurate. The heads of department of mathematics in the participating schools were also requested to moderate at least 10% of the sample of the learners' written tasks.

3.7.2 The interview process with learners

The researcher conducted the semi-structured interviews with learners telephonically. To conduct interviews telephonically was largely opted to avoid frequent physical contact with the schools and the learners as the country was still in alert level one lockdown due to the coronavirus pandemic (see Section 3.6.1). It was initially indicated on the parent's/ guardian's consent and learners' assent letters that there would be telephonic interviews conducted with the selected learners. For this reason, the reply slip required the parents and the learners to provide their telephone numbers. Therefore, with prior arrangements, all interviews were conducted after the official tuition time in both schools when the learners were considered to be free at home. Each interview session lasted for a duration of almost 15 minutes maximum. The interview questions were asked using the same words, in the same order in which they appeared in the interview protocol (Nieuwenhuis, 2007). All interview calls were recorded with prior consent for later analysis.

The interviews were conducted with a sample of three learners in each of the two participating schools. The sampling of learners for the interviews was based on their performance in the Euclidean geometry task (Section 3.5.2.1). The learners who participated in the interviews represented three performance categories, namely, those that obtained the highest mark, the average mark, and the lowest mark. Table 3.5 below shows the actual learners' marks obtained in the Euclidean geometry task.

Table 3.5: Marks of the task for learners who were selected for interviews.

Category	Code name	Mark out of 35	Percentage
Highest	LA19	20	57
	LB7	22	63
Average	LA28	16	46
	LB6	14	40
Lowest	LA13	2	6
	LB4	2	6

This categorical selection was meant to ensure that learners with different levels of problem-solving abilities would participate in the interviews to get a sense of how learners in these performance categories interacted with mathematical problem-solving instances (see Section 3.4.3). Mainly, the semi-structured interviews aimed to probe learners' task responses to gain more insight and knowledge of, (1) learners' understanding of Euclidean Geometry; and (2) learners' strategies that they observably employed when solving problems in the geometry tasks.

3.8. DATA ANALYSIS

Qualitative data analysis tries to establish how respondents made sense of a particular phenomenon by analysing their understanding, perceptions, views, knowledge, feelings, attitudes, values, and experiences (Nieuwenhuis, 2007). According to McMillan and Schumacher (2010), qualitative data analysis is an inductive process of arranging information into categories and identifying patterns and relationships among the categories. The process of inductive analysis is relatively a systematic way of coding, categorising, and interpreting data to provide meaning to a matter of interest (McMillan & Schumacher, 2010). Figure 3.1 illustrates the steps in qualitative analysis showing the inductive process that researchers follow.

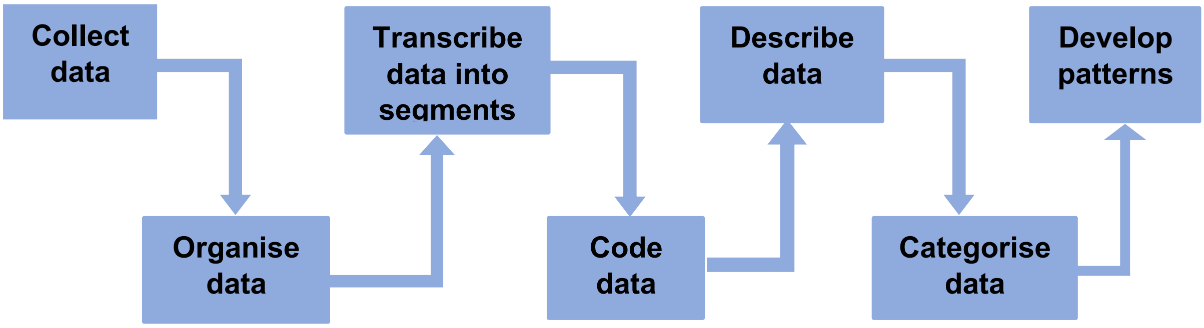


Figure 3.2 Steps in analysing qualitative data as proposed by McMillan and Schumacher (2010).

Through the process of inductive analysis of qualitative data, the research findings emerge from the dominant themes arising from the raw data. While on the other hand, a more deductive technique of data analysis categorises the information required from the data in advance from the literature on the topic (Nieuwenhuis, 2007).

3.8.1 Analysis of the Euclidean geometry task

Firstly, each learner who participated in the study was given an identifying pseudonym or code name to maintain the anonymity of the learners and their schools (see Section 3.6). The researcher followed an inductive process of analysing data at the beginning and adopted a deductive approach towards the end of the analysis. After the data collection, the learner's scripts were marked, and the responses were grouped into different categories according to the outcomes of individual items. Then the researcher established an open coding system to track the learner's responses (see Table 4.2). The categories were analysed and organised into prevalent themes constituting the learner's knowledge and skills/ strategies they employed when solving the Euclidean geometry task. Furthermore, the extent to which the learners applied the required knowledge and skills to solve the given task was determined through the analysis of the learner's performance in different taxonomy of mathematics' cognitive levels and the van Hiele levels of geometric thought.

On a more deductive approach of data analysis, the learners' problem-solving abilities that guided them to solve the related problem-solving task were analysed and interpreted against the following problem-solving indicators in line with Polya's model. The problem-solving indicators were used to develop a problem-solving ability scoring rubric (see Appendix F). The rubric was utilised to facilitate the allocation of scores to individual learners and the rating of learners' problem-solving abilities in terms of the allocation of a percentage. The levels of problem-solving ability were further categorized in terms of low, average, and high problem-solving ability using the scores obtained from the problem-solving ability rubric (Appendix F).

3.8.2 Analysis of the semi-structured interviews

The learners who participated in the interviews were identified as LA13, LA19, LA28, LB4, LB6 and LB7 with "L" standing for learner, A or B for School A or School B and the numbers corresponds to the sequence of the learners as they appear on the recording mark sheet used to capture their scores for the written task (see Section 3.6 and Table 3.5). The interview audio recordings were transcribed word by word by the researcher. The data from the transcribed interviews were therefore grouped based on the questions that were asked during the interview session. Similar themes were constructed and categorised into patterns of similarities.

3.9 ETHICAL CONSIDERATIONS

Firstly, the research ethics clearance was granted by the institutional Ethics Review Committee for this study to be conducted. Then, the permission to conduct a research study at schools in the Tshwane south district was obtained from the Department of Basic Education provincial level and district level. Consent letters were sent to the principals of the targeted schools to request for permission to conduct research at their schools and informed consent forms were signed and stamped for approval. Furthermore, consent letters were sent to parents and assent letters were given to the learners inviting them to participate in the study of which they gave their approval by completing, signing, and returning the reply slips.

All participants were clearly informed about the intentions, procedures, and benefits of the research and guaranteed that their names and all personal details would be treated with strict confidentiality. It was explicitly stated in the invitation letters and letters of informed consent or assent that participation in the study would be voluntary and that withdrawal from the study would be accepted at any stage without prejudice or penalty. The participants were further assured that there were no potential risks in this study, and with honesty the researcher informed participants that they would not be reimbursed, or they would not receive any incentives for their participation in the research. Participants were granted an opportunity to ask questions and seek clarity regarding their participation in the study by using the researcher's telephone number provided to them on the letters.

3.10. CONCLUSION

The research methodology and design of this study was described in this chapter by first discussing the research paradigm that underpinned and influenced this study, namely, the qualitative interpretive paradigm. The chapter presented in detail a qualitative case study as a research design employed in this investigation. The sampling techniques, the instrumentation and the data collection processes were outlined with special consideration and compliance to the realities of Covid-19 and associated state published regulations as the study was conducted during the pandemic period. In addition, issues of trustworthiness relating to credibility, dependability, confirmability, and transferability were presented in this chapter. A brief account of how this study treated issues of data analysis processes has also been given attention in this chapter. Chapter Three concluded by providing a brief commentary on issues relating to ethical considerations that were inherent to the study. Chapter Four provides a rich narrative on the actual segment of the data analysis process for this study.

CHAPTER FOUR

DATA ANALYSIS AND FINDINGS

4.1. INTRODUCTION

This chapter discusses the relevant data analysis and presents the findings of the study, which were collected using qualitative methods. Subsequently, the researcher used qualitative methods to analyse the data. Firstly, the learner's written responses for the administered Euclidean geometry task were analysed and the results presented. Thus, the results featured the learner's performance based of the marking guideline, the learner's performance to the items in Section One and Two separately, and the item-by-item analysis of learners' responses. Through the item-by-item analysis of learners' responses, the learner's knowledge, and skills that they employed to solve problems in Euclidean geometry were further explored. Secondly, the learner's responses to the semi-structured interviews were analysed to corroborate data that emerged from the learner's written responses from the Euclidean geometry task. Thirdly, the learners' problem-solving abilities that guided them to solve the related problem-solving task were analysed by the researcher.

4.2. DATA COLLECTION INSTRUMENTS AND RESEARCH QUESTION

In the current study, a written task (Euclidean geometry task) was administered to the Grade 11 mathematics learners who participated in the study. Subsequently, semi-structured interviews were conducted as a follow-up data collection technique to corroborate data that emerged from the learner's written responses. Table 4.1. summarises the purpose of each data collection tool in relation to the question(s) answered in the study (see also, Section 1.6).

Table 4.1: Data collection instruments and related questions

No.	Research Question (RQ)	Data source	Data collection instrument(s)	Analysis indicators
1	How do the problem-solving abilities of Grade 11 learners manifest when they solve mathematical tasks in Euclidean geometry?	Grade 11 learners	Euclidean geometry task and interview schedule	Problem-solving ability indicators according to Polya (1945)
2	To what extent do learners apply the knowledge and skills required to solve Euclidean geometry tasks?	Grade 11 learners	Euclidean geometry task	Mathematics cognitive levels and van Hiele levels of geometric thought
3	What is the level of problem-solving ability at which Grade 11 learners are operating when solving mathematical tasks in Euclidean geometry?	Grade 11 learners	Euclidean geometry task and interview schedule	Problem-solving ability score rubric

4.3. ALLOCATING CODES TO LEARNERS

Table 4.2 shows the information of the learners who participated in the study and the codes used to identify them. The letter "L" stands for learner, while letter A or B stands for School A or School B and the number corresponds to the sequence in which the written tasks were collected from the learners. For instance, LA1 stand for the first learner from which the answer script was collected from School A, while LB1 stand for the first learner from which the answer script was collected from School B and so on. The allocation of identity codes was done to maintain the anonymity of the learners and the schools (Section 3.6).

Table 4.2: Codes for Grade 11 mathematics learners who participated in the study

Learner	School A			School B		
	Gender	Age	Code	Gender	Age	Code
L1	F	16	LA1	M	17	LB1
L2	F	16	LA2	M	18	LB2
L3	F	16	LA3	F	16	LB3
L4	M	17	LA4	F	17	LB4
L5	M	19	LA5	M	16	LB5
L6	M	16	LA6	F	16	LB6
L7	F	16	LA7	M	16	LB7
L8	M	16	LA8	M	16	LB8
L9	F	16	LA9	F	16	LB9
L10	M	16	LA10	F	16	LB10
L11	F	16	LA11	M	16	LB11
L12	M	17	LA12	F	16	LB12
L13	M	16	LA13	F	16	LB13
L14	F	16	LA14	F	17	LB14
L15	F	16	LA15	M	16	LB15
L16	M	18	LA16	F	17	LB16
L17	F	16	LA17	M	17	LB17
L18	F	16	LA18	F	16	LB18
L19	F	16	LA19	F	17	LB19
L20	M	18	LA20	F	16	LB20
L21	M	16	LA21	F	18	LB21
L22	M	18	LA22	F	15	LB22
L23	F	16	LA23	F	18	LB23
L24	F	16	LA24	M	17	LB24
L25	F	16	LA25	M	16	LB25
L26	M	18	LA26	F	17	LB26
L27	F	17	LA27	M	18	LB27
L28	F	16	LA28	M	16	LB28
L29	F	16	LA29	M	16	LB29
L30	F	17	LA30	F	16	LB30
L31	F	16	LA31			
L32	M	18	LA32			
L33	F	16	LA33			
TOTAL		**33 Learners**			**30 Learners**	

4.4. ANALYSIS OF RESULTS FROM THE EUCLIDEAN GEOMETRY TASK

4.4.1. Learners' performance according to the marking guideline

Firstly, the learner's written task was marked according to the marking guideline prepared for the associated task (see Appendix B). The learner's raw marks for the task were recorded out of 35. The marks were then converted into a percentage from which the performance of the learners was analysed using the CAPS grading code (DBE, 2011). Table 4.3 shows the statistical performance analysis of learners from School A and School B.

Table 4.3: Levels and percentages for statistical performance analysis of learners

Rating levels	**Description of competence**	**Percentage**	**Number of learners**	
			School A	School B
1	Not achieved	0-29	17	16
2	Elementary achievement	30-39	8	4
3	Moderate achievement	40-49	6	5
4	Adequate achievement	50-59	2	3
5	Substantial achievement	60-69	0	2
6	Meritorious achievement	70-79	0	0
7	Outstanding achievement	80-100	0	0
	TOTAL		33	30

Source: DBE (2011)

As per the statistical analysis of learners' performance shown in Table 4.3, it emerged that 17 (51.5%) of learners from School A did not achieve and 16 (48.5%) of learners achieved, with 8 (24%) of them scoring between 30% to 39%, 6 (18%) of them scoring between 40% to 49%, 2 (6%) of them scoring between 50% to 59%, and no one scored 60% or more. In School B, 16 (53.3%) of learners did not achieve and 14 (46.7%) of learners achieved, with 4 (13%) of them scoring between 30% to 39%, 5 (17%) of them scoring between 40% to 49%, 3 (10%) of them scoring between 50% to 59%, 2 (7%) of them scoring between 60% to 69%, and no one scored 70% or more. Further analysis show that 31 (94%) of learners from School A performed below 50%, while only 2 (6%) of them achieved a percentage above 50%. In School B, 25 (83.3%) of learners performed below 50%, while only 5 (16.7%) of them achieved a percentage above 50%. In the next sections, how the learners answered each question item is scrutinised.

4.4.2. Learners' performance to items in Section One

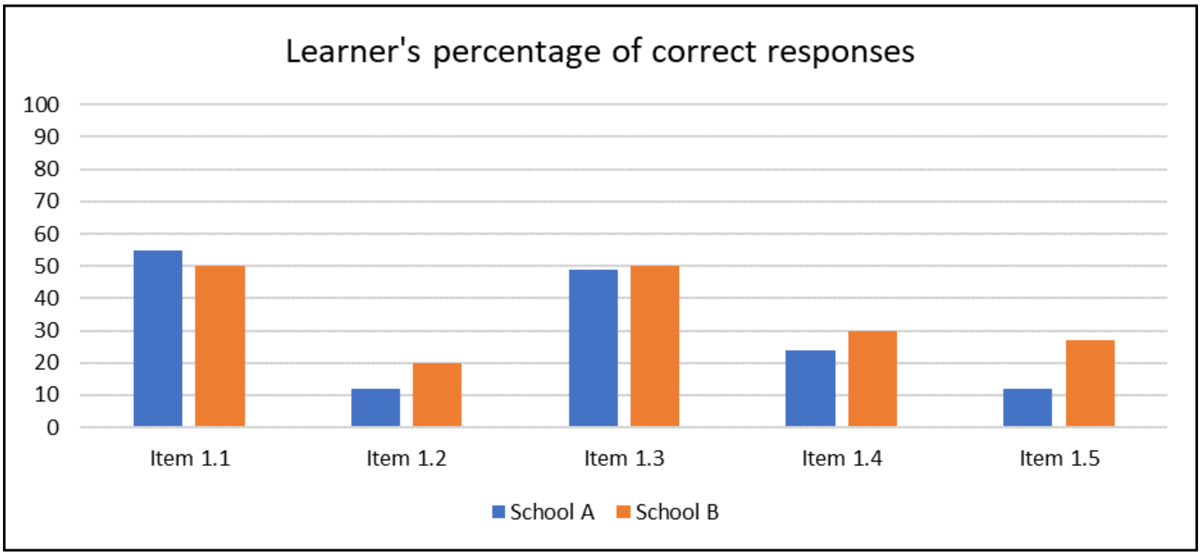
Section one of the administered Euclidean geometry task comprised of five multiple choice items. Table 4.4. represents the choices (A-D) of learners' responses to the items in Section One of the task. The number of learners who selected each given option is indicated. Also, the number of learners who did not attempt to answer any item is indicated.

Table 4.4: Analysis of learners' responses to items in Section One of the EGT

SCHOOL A							
Items	**A**	**B**	**C**	**D**	**No response**	**Total**	**Percentage of correct responses**
	Number of learners' responses per item						
1.1	4	**18**	5	4	2	33	55%
1.2	**4**	0	0	27	2	33	12%
1.3	4	11	2	**16**	0	33	49%
1.4	4	9	**8**	10	2	33	24%
1.5	10	10	**4**	8	1	33	12%
SCHOOL B							
Items	**A**	**B**	**C**	**D**	**No response**	**Total**	**Percentage of correct responses**
	Number of learners' responses per item						
1.1	6	**15**	3	6	0	30	50%
1.2	**6**	2	3	19	0	30	20%
1.3	3	9	3	**15**	0	30	50%
1.4	2	4	**9**	11	4	30	30%
1.5	7	4	**8**	10	1	30	27%

The shaded cells in Table 4.4 above represents the correct options to the items in Section One of the Euclidean geometry task, while the bold number in the shaded cells indicates the number of learners who selected the correct options in the respective items. The last column of the Table indicates the percentage of the correct responses by the learners, also illustrated on Figure 4.1.

Figure 4.1: Analysis of learners' performance in Section One of the EGT



The performance of learners in School A and School B in Section One of the Euclidean geometry task is more or less similar with item 1.1 and 1.3 achieved at average percent, followed by item 1.4, then item 1.5 and 1.2. Section 4.4.4 provides a question-by-question analysis to explore the learners' responses further.

4.4.3. Learners' performance to items in Section Two

Section two of the Euclidean geometry task consisted of open-ended questions which involved cognitive level one and two numeric questions where the learners had to calculate the sizes of angles and also included cognitive level three and four proof type questions. In answering this section of the task to solve the problems (riders), learners had to write a statement and provide a reason for the given statement. Therefore, Table 4.5 classified the learner's responses focusing on the learner's written statements and reasons provided.

Table 4.5: Classification of learners' responses in Section Two of the EGT

RESPONSE	CLASSIFICATION
Solved problem	Sound Understanding (SU) Correct statement with valid reasons Better Understanding (BU) Correct statement with some valid reasons
Partial solved problem	Partial Understanding (PU) Some correct statement(s) with valid reasons Specific Depiction (SD) Correct statement with invalid reason Specific Misconception (SM) Incorrect statement with valid reason
Un-solved problem	No Understanding (NU) Incorrect statement with invalid reason
No attempt made	No Response (NR) No attempt to solve the problem

Adapted from Chusinkunawut et al. (2018) and Cassim (2006).

The classification criteria formulated in Table 4.5 for learners' responses was adapted from Chusinkunawut et al. (2018) and Cassim (2006) and modified to suit the study's context. Then the researcher analysed the learner's responses in accordance with the classification criteria in Table 4.5. Also, the learner's responses were classified according to the van Hiele Levels (VHL) of geometric thinking (see Section 2.3.3). Table 4.6 indicates each learner's response to the given item in Section Two of the Euclidean geometry task.

Table 4.6: Learners' responses according to the criteria outlined in Table 4.5

SCHOOL A								SCHOOL B							
Learner	2.1.1	2.1.2	2.1.3	2.2.1	2.2.2	2.2.3	VHL	Learner	2.1.1	2.1.2	2.1.3	2.2.1	2.2.2	2.2.3	VHL
LA1	SU	NR	SU	NU	SD	NU	1	LB1	SU	BU	SU	PU	NR	NR	1
LA2	SU	PU	PU	NU	NU	NU	1	LB2	SU	SU	NR	NR	PU	NR	1
LA3	NU	BU	SU	NU	NU	SD	1	LB3	SU	BU	SU	PU	NU	SD	1
LA4	SU	PU	NU	NU	NU	SD	1	LB4	SU	NR	NR	NU	NR	NU	0
LA5	SU	SU	SU	NU	PU	NU	2	LB5	SU	BU	SU	NU	NU	NU	1
LA6	SM	PU	NU	NU	NU	NU	0	LB6	SU	SU	SU	PU	NU	SD	1
LA7	SU	SD	NU	NU	NU	SD	1	LB7	SU	SU	SU	NR	NU	NR	0
LA8	NU	NU	NU	NU	NU	SD	1	LB8	SU	BU	SU	NR	NU	SD	1
LA9	SU	BU	SU	NU	NU	SD	1	LB9	SU	BU	SU	NU	NU	NU	1
LA10	NU	NU	NU	NU	NU	NU	0	LB10	SU	BU	SU	NU	NU	NU	1
LA11	SU	BU	SM	NU	NU	PU	1	LB11	SU	PU	SU	NU	NU	SD	1
LA12	NR	SD	NR	NU	NU	NU	1	LB12	SU	BU	NU	NU	NU	SD	1
LA13	SM	NR	NR	NU	NU	NU	0	LB13	SU	NU	NU	NU	NU	NU	0
LA14	SU	NU	NU	NU	NU	NU	0	LB14	SD	NU	NU	NU	NU	NU	0
LA15	PU	NU	NU	NU	NU	NU	0	LB15	NU	NU	NU	NU	NU	NU	0
LA16	PU	SD	NR	NR	NU	NR	0	LB16	SU	NU	NU	NU	NR	NU	1
LA17	SU	BU	SU	PU	NU	SU	3	LB17	SU	SD	NU	NU	NU	NU	1
LA18	SM	PU	NU	NU	NU	SD	0	LB18	SU	NU	NU	NU	PU	SD	1
LA19	SU	SU	SU	NU	SD	PU	1	LB19	SU	BU	NR	NR	NR	NR	0
LA20	SU	SD	SU	NU	SD	SD	1	LB20	SD	NR	NR	NU	NR	NU	1
LA21	SU	NU	SD	PU	NU	SD	1	LB21	SM	SD	NU	NU	NU	NU	0
LA22	SU	SU	NU	NR	NU	NR	1	LB22	SU	BU	NU	NU	NR	NU	1
LA23	SU	SU	SU	PU	NU	PU	2	LB23	SU	SU	SU	NU	NU	NU	1
LA24	SU	SD	NU	NU	NU	PU	1	LB24	SU	BU	SD	NR	NU	NR	1
LA25	SU	BU	SU	NU	NU	NU	1	LB25	SU	BU	NR	NR	NR	NR	0
LA26	SU	BU	SU	NU	NU	NU	1	LB26	SM	NU	NU	NR	NU	SD	1
LA27	SD	NU	SD	NU	NU	SD	0	LB27	NU	NU	NU	NR	SD	NR	0
LA28	SU	SU	SU	NU	NU	PU	1	LB28	SU	SU	NU	NU	PU	NU	1
LA29	SU	NU	SD	NU	NU	NU	0	LB29	SU	BU	NU	NR	NU	NR	0
LA30	SU	SU	NU	NU	NR	NU	1	LB30	SU	SU	SU	NU	PU	SD	1
LA31	NU	NU	NU	NR	PU	NU	0								
LA32	SD	SD	NU	NU	NU	NU	0								
LA33	SU	SD	NU	NU	NU	SD	1								

Table 4.6 was utilised to compile the analysis of the participant's responses to Section Two of the Euclidean geometry task (see Table 4.7). Additionally, the data obtained in Table 4.7 was illustrated in Figure 4.2 for School A and Figure 4.3 for School B.

Table 4.7: Analysis of learners' responses to Section Two of the EGT

SCHOOL A					
Item	**Solved problem**	**Partial solved problem**	**Un-solved problem**	**No attempt made**	**Total**
2.1.1	21	7	4	1	33
2.1.2	12	11	8	2	33
2.1.3	11	5	14	3	33
2.2.1	0	3	27	3	33
2.2.2	0	5	27	1	33
2.2.3	1	15	15	2	33
SCHOOL B					
Item	**Solved problem**	**Partial solved problem**	**Un-solved problem**	**No attempt made**	**Total**
2.1.1	24	4	2	0	30
2.1.2	18	3	7	2	30
2.1.3	11	1	13	5	30
2.2.1	0	3	18	9	30
2.2.2	0	5	18	7	30
2.2.3	0	8	14	8	30

Figure 4.2: graphic representation of School A learner's performance in Section Two of the EGT

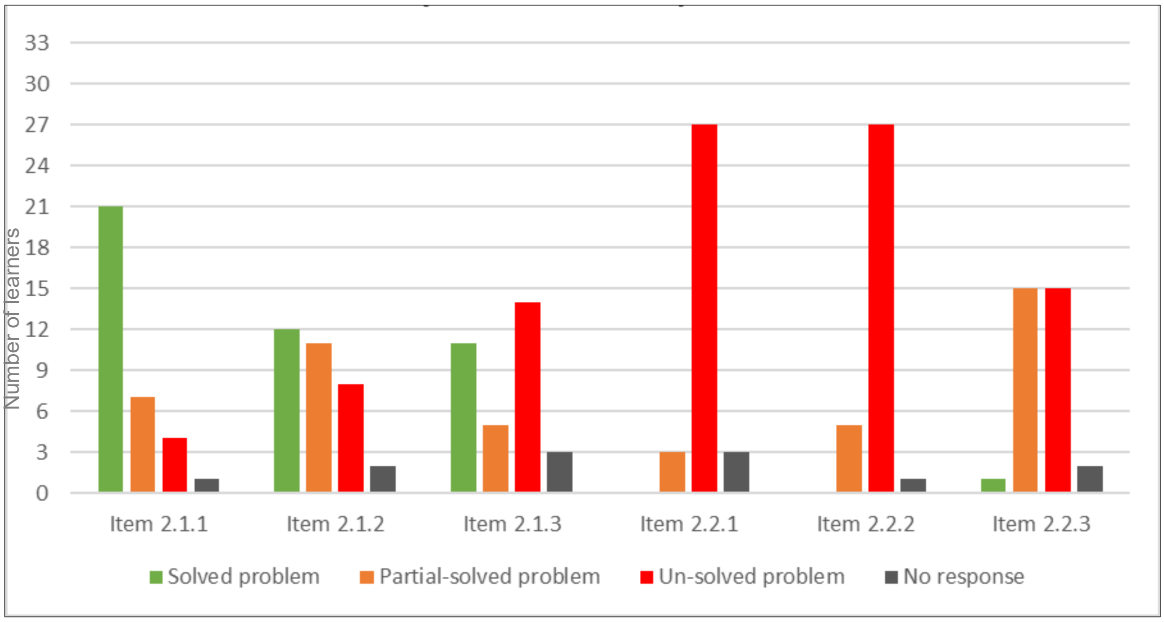


Figure 4.3: graphic representation of School B learner's performance in Section Two of the EGT

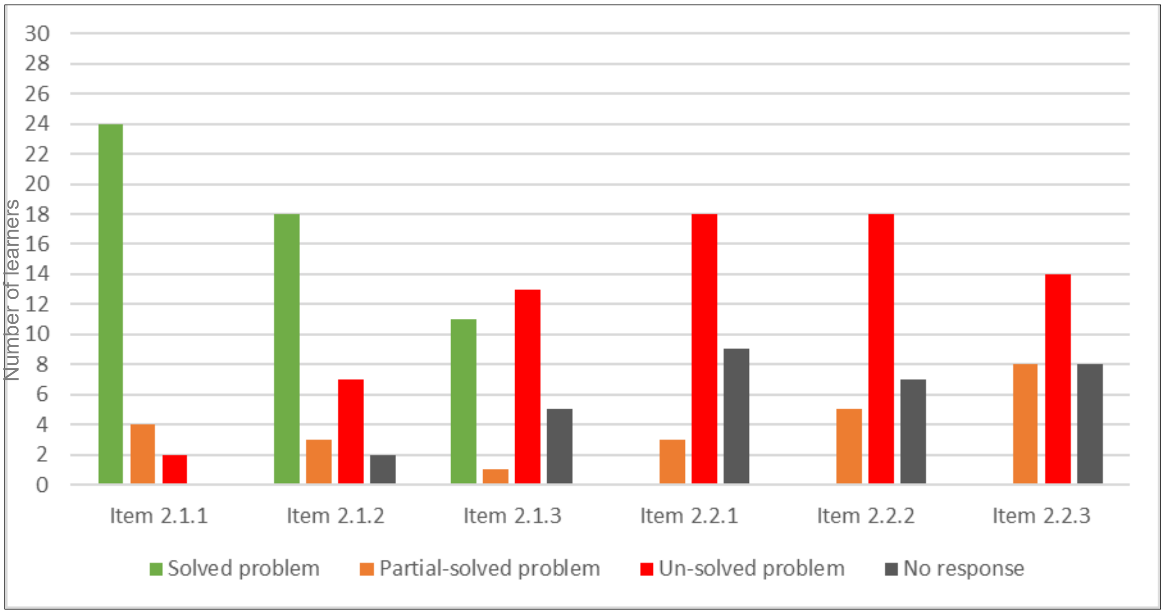


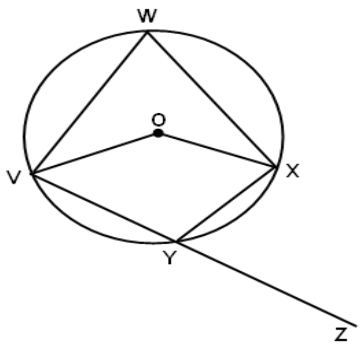
Table 4.7 which is graphically represented in Figure 4.2 and Figure 4.3 shows that most participants were able to respond correctly to cognitive Level One and Two items in 2.1 than cognitive Level Three and Four items in 2.2. Additionally, more participants made no attempts

in responding to the items in 2.2 than the items in 2.1. The next section focuses on item-by-item analysis of learners' responses.

4.4.4. Item-by-item analysis of learners' response

This section explores the learner's responses to items in both Section One and Section Two of the Euclidean geometry task. Table 4.4 and Table 4.7 were used to discuss the learner's responses in greater details with a focus on each item of the task at a time. Learner's samples of correct and incorrect responses were provided and scrutinised. The scrutinization was focused on the common solution approaches and the reasoning behind the presentation of responses in each item.

4.4.4.1. Item 1.1 (Section 1)



This item is a cognitive Level One question (see Table 3.4). A learner's understanding of the exterior angle of a cyclic quadrilateral was tested. It was stated that VWXY is a cyclic quadrilateral. O is the centre of the circle. Line VY is produced to point Z outside the circle, and that from the diagram alongside, one can prove that $\widehat{W} = X\hat{Y}Z$. Therefore, learners had to choose an option that revealed their knowledge of "when" this theorem was applicable. Possible options were as follows:

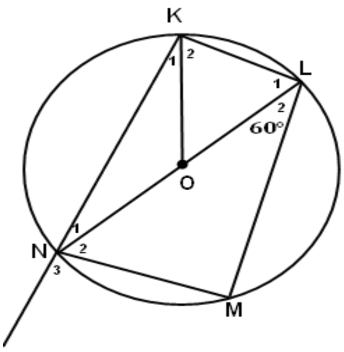
A. Only in this cyclic quadrilateral can we be sure that $\widehat{W} = X\hat{Y}Z$.
B. Given any cyclic quadrilateral VWXY with VY produced to Z, then $\widehat{W} = X\hat{Y}Z$.
C. Only when the quadrilateral VWXY looks like a kite can we be sure that $\widehat{W} = X\hat{Y}Z$.
D. Given any quadrilateral, VWXY with VY produced to Z, then $\widehat{W} = X\hat{Y}Z$.

Table 4.8: Summary of learners' responses to item 1.1

School	A	B	C	D	No response	Total
	Number and percentage of learners' responses per option					
A	4	**18**	5	4	2	33
	12%	**55%**	15%	12%	6%	100%
B	6	**15**	3	6	0	30
	20%	**50%**	10%	20%	0%	100%

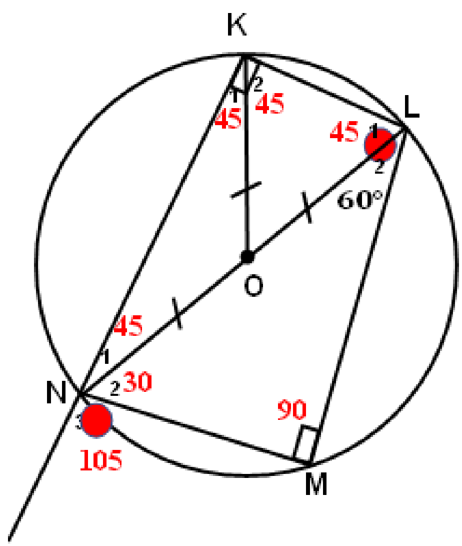
Option B was the correct answer because in any given cyclic quadrilateral with one of its sides extended, the exterior angle will always be equals to the interior opposite angle. This was one of the highest correctly answered items in Section One of the Euclidean geometry task. About a half of the learners in both schools correctly answered this item. 18 (55%) of the participants from School A selected the correct option and 15 (50%) of the participants from School B selected the correct option. This is concerning considering that this item is a cognitive level one (knowledge) question. It appears that about a half of the participants from both schools still lacks the conceptual understand of the exterior angle of a cyclic quadrilateral. This item required no workings; therefore, samples of the participant's responses were not provided.

4.4.4.2. Item 1.2 (Section 1)



This item is a cognitive Level Two question (see Table 3.4). It required learners to use well known procedures, simple applications and calculations which might involve few steps. The item tested the learner's current and previous knowledge on the application of the properties related to the exterior angle of a cyclic quadrilateral, the diameter, and the isosceles triangle in determining the size of an angle. It was stated that KLMN is a cyclic quadrilateral of a circle with centre O. Line OK bisects $N\hat{K}L$ and line NL is a diameter. The size of angle L_2 was given as $60°$. Therefore, learners had to determine the size of angle N_3. Possible options were (A) 105^0, (B) 100^0, (C) 95^0 (D) 90^0.

Solution to item 1.2 (Option A)



$\hat{N}_3 = \hat{L}_1 + \hat{L}_2$ Ext ∠ of a cyclic quad

The aim is ∴ to find $\hat{L}_1$

In ΔNKL; $\hat{K} = 90°$ ∠ in semi-circle

$\hat{K}_1 = \hat{K}_2 = 45°$ Given ($\hat{K}$ is bisected)

$\hat{K}_2 = \hat{L}_1 = 45°$ Radii

$\therefore \hat{N}_3 = 45° + 60° = 105°$

OR

$\hat{N}_1 + \hat{N}_2 + \hat{N}_3 = 180°$ Adj supp $\angle s$

Find $\hat{N}_1$ and $\hat{N}_2$

In ΔNKL; $\hat{K} = 90°$ ∠ in semi-circle

$\hat{K}_1 = \hat{K}_2 = 45°$ Given ($\hat{K}$ is bisected)

$\hat{K}_1 = \hat{N}_1 = 45°$ Radii

In ΔNML; $\hat{M} = 90°$ ∠ in semi-circle

$\therefore \hat{N}_2 = 30°$ $\angle s$ of Δ

$\hat{N}_3 = 180° - (45° + 30°) = 105°$

Table 4.9: Summary of learners' responses to item 1.2

School	A	B	C	D	No response	Total
	Number and percentage of learners' responses per option					
A	**4**	0	0	27	2	33
	12%	0%	0%	82%	6%	100%
B	**6**	2	3	19	0	30
	20%	7%	10%	63%	0%	100%

This was the least correctly answered item in both schools. Only 4 (12%) of the participants from School A selected the correct option and only 6 (20%) of the participants from School B selected the correct option. Angle N_3 is an exterior angle of a cyclic quadrilateral, and it is also one of the adjacent supplementary angles (i.e., angles lying next to each other on a straight line add up to 180^0). Participants had two option methods of determining the size of the angle.

LA11 is one of the participants who correctly answered this item. In the response shown in Figure 4.4, the participant used the second method that is indicated on the solution to the item above. They were aware that angle N_1 together with angle N_2 and angle N_3 are adjacent supplementary angles. To determine the size of angle N_3, participant LA11 had to find the other two angles first. They identified angle K as an angle in the semi-circle[11] which equals 90^0. They further equated angle K to angle M with a reason that opposite angles of a cyclic quadrilateral are supplementary, if angle K is 90^0, so is angle M. But again, angle M is also an angle in a semi-circle. However, without following up on what was started. Participant LA11 then indicated that angle K_1 is equals to angle N_1 with radii as a reason, which also indicates that angles opposite equal sides of a triangle are equal. Both angles were equated to 45^0. Though 45^0 is indicated with a pencil on the diagram, there was no evidence of how it came about on the participant's response. After identifying angle N_1 to be 45^0, the participant then made a follow-up on the previous statement to find the size of angle N_2 by using the sum of the interior angles of a triangle which adds up to 180^0 given that the size of angle L_2 in triangle NML is equals to 60^0. The participant exhibited sound knowledge and understanding of geometric concepts, but lacked an ability to make orderly connections in representing this knowledge.

11 The angle subtended at the circumference of the circle by a diameter is a right angle (Phillips et al., 2012).

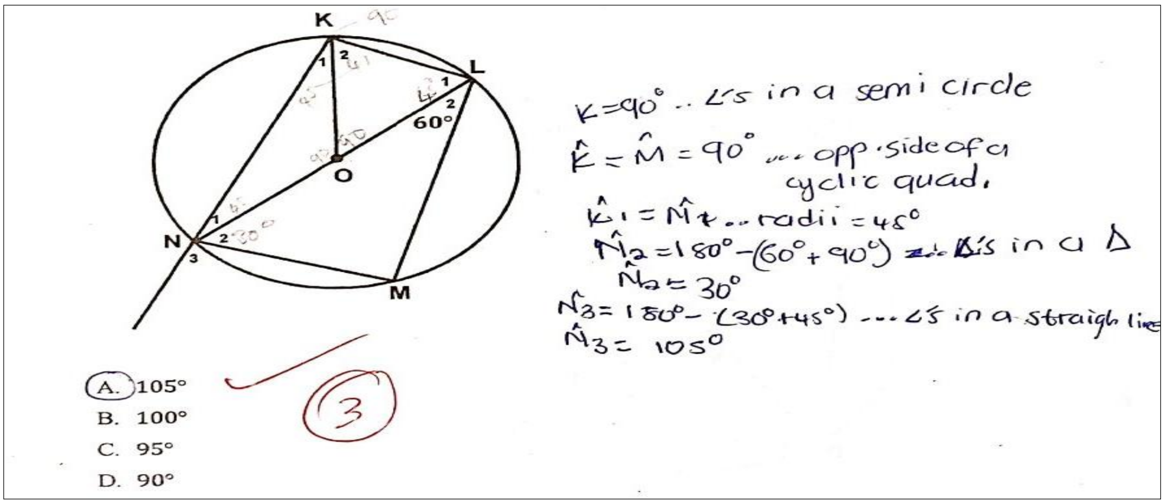


Figure 4.4: LA11 response to item 1.2

Most participants across schools selected option D, which suggests that angle N_3 is equal to 90^0. There were 27 (82%) of the participants from School A and 19 (63%) of the participants from School B who incorrectly selected this option. Several of them who selected option D assumed that line KN and line LM were parallel lines. They used the property that states that the angles lying on a straight line adds up to $180°$ ($\hat{N}_1 + \hat{N}_2 + \hat{N}_3 = 180°$ adjacent supplementary angles). They were able to identify angle M as an angle in a semi-circle and determined angle N_2 to be 30^0 from the sum of angles in triangle LNM. But they erroneously identified angle N_1 to be 60^0 because of alternate angles, without any stated or proven parallel lines. This is one of the errors learners mostly commit. Just about anywhere where the Z shape is formed, learners assume alternate angles. LB28 portrayed good insight in solving the problem in item 1.2 except that he assumed alternate angles where there were no parallel lines. Circled in red on Figure 4.5 below, the participant stated that angle N_1 is equal to 60^0 with a reason that angle L_2 and angle N_1 are alternate angles. However, participant LB28 did not indicate the lines which are parallel because there were no parallel lines stated or proven. The participant seemingly assumed that the angles alternate because they are founded on a Z shape.

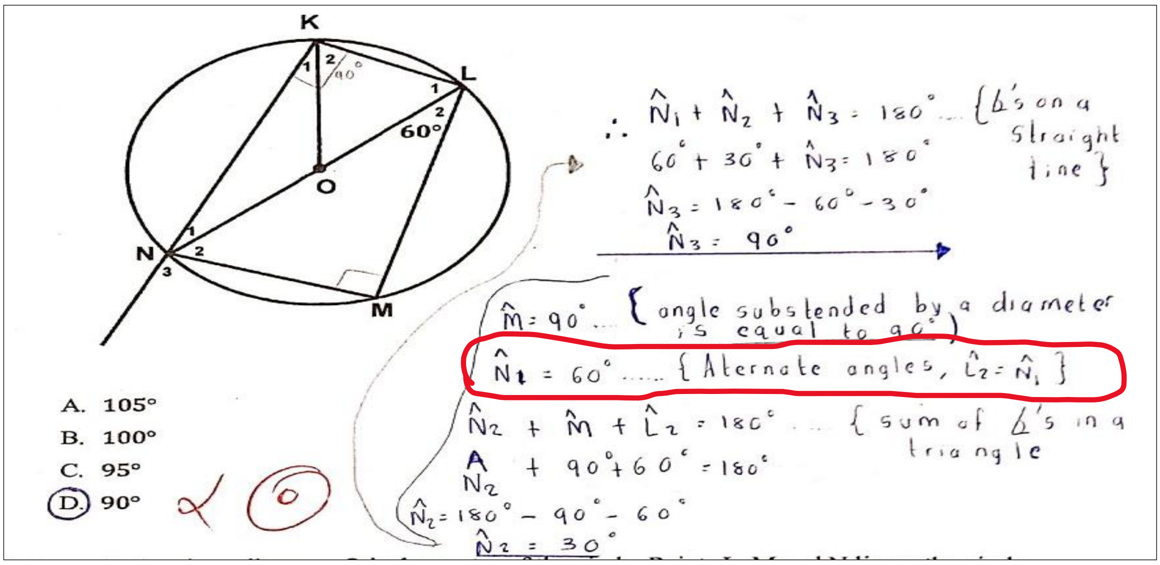


Figure 4.5: LB28 response to item 1.2

A few participants seemed to be aware that the exterior angle of a cyclic quadrilateral is equals to the interior opposite angle but were confused as to which interior angle is opposite to the exterior angle. Instead of noting that $\hat{N}_3 = \hat{L}_1 + \hat{L}_2$, participant LB22 stated that $\hat{N}_3 = \hat{M} = 90°$ (see Figure 4.6 below).

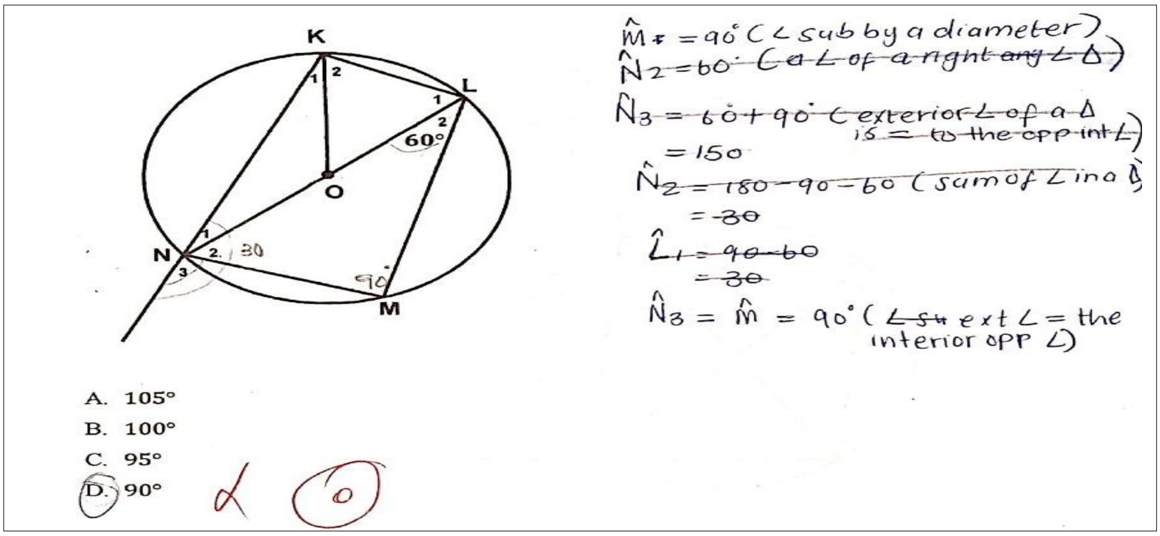


Figure 4.6: LB22 response to item 1.2

The response of LA13 in Figure 4.7 reveals the participant's misconception on the angles in the same segment. The theorem states that an arc or chord of a circle subtends[12] equal angles

12 An angle formed between two lines which come from each of the endpoints of the arc, or a chord is said to be subtended by that arc or chord.

at the circumference of the circle (Phillips et al., 2012). Which means angles subtended by the same arc or chord at the circumference of a circle are equal. It is then said that the angles in the same segment of the circle are equal. In item 1.2 we see that arc/ chord NM subtends angle L_2 at the circumference of the circle, but angle K_1 is not subtended by any arc or chord. Yet the participant stated that angle K_1 is equal to 60^0 with a reason that these are angles in the same segment. Another misconception witnessed in LA13's response is an assumption that a line outside the circle is a tangent. The participant erroneously applied a tangent and chord theorem. A tangent is a line touching the circle at only one point (Phillips et al., 2012). In item 1.2 the line goes through the circle and thus cannot be a tangent. This is an indication of the participant's lack of conceptual understanding of geometric properties. This could be a sign that the participant memorized geometric theorems without understanding them.

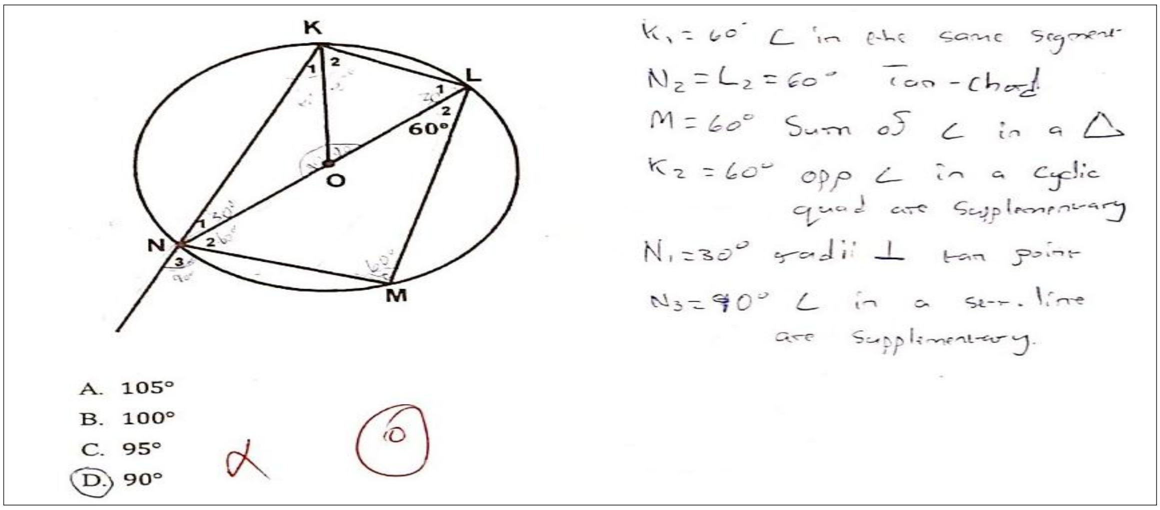
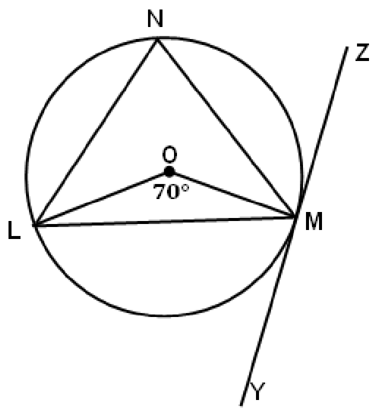


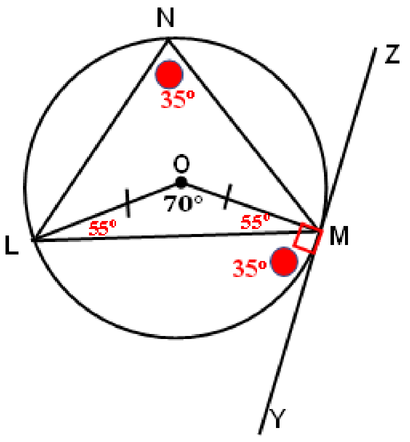
Figure 4.7: LA13 response to item 1.2

4.4.4.3. Item 1.3 (Section 1)



This item is a cognitive Level Two question (see Table 3.4). It required learners to use well known procedures, simple applications and calculations which might involve few steps. The item tested the learner's application of their knowledge of the angle subtended by the arc at the centre and the circumference of a circle and the properties of a tangent to determine the size of an angle. It was stated that O is the centre of the circle. Points L, M and N lie on the circle and YMZ is a tangent to the circle at M. $L\hat{O}M = 70°$. Learners had to calculate the size of $Y\hat{M}L$. Possible options were (A) $110°$ (B) $140°$ (C) $90°$ (D) $35°$.

Solution to item 1.3 (Option D)



$L\hat{N}M = 35°$ $\angle$ at centre = $2\times\angle$ at circumference

$Y\hat{M}L = L\hat{N}M = 35°$ Tan-chord Theorem

OR

$O\hat{M}Y = 90°$ Tan $\perp$ Rad

$O\hat{L}M = O\hat{M}L$ Radii

$O\hat{L}M = O\hat{M}L = 55°$ sum of $\angle s$ in Δ

$Y\hat{M}L = 90° - 55° = 35°$

Table 4.10: Summary of learners' responses to item 1.3

School	A	B	C	D	No response	Total
	Number and percentage of learners' responses per option					
A	4	11	2	**16**	0	33
	12%	33%	6%	**49%**	0%	100%
B	3	9	3	**15**	0	30
	10%	30%	10%	**50%**	0%	100%

Half of the participants in both schools correctly answered this item. There were 16 (49%) of the participants from School A who selected the correct option and 15 (50%) of the participants from School B who selected the correct option. LA19 is one of the participants who displayed sound understanding of the concept tested in this item. The participant correctly divided the angle at the centre by two to obtain the angle at the circumference with the reason that the angle at the centre is twice the angle at the circumference. After, the participant applied the tangent-chord theorem to find the size of the angle between the tangent and the chord (see Figure 4.8 below).

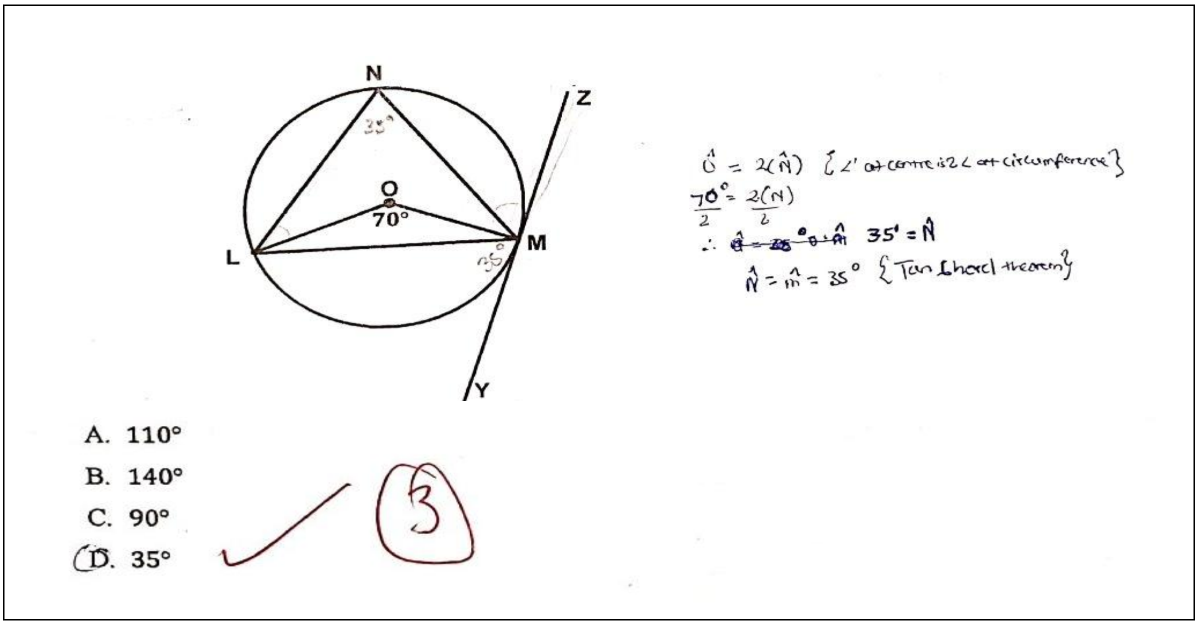


Figure 4.8: LA19 response to item 1.3

About one-third of the participants in both schools selected option B which suggests that angle YML is 140^0. In their responses they stated a valid reason but confused the application of the theorem. The theorem states that the angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle (Phillips et al., 2012). However, the participants made the angle at the circumference to be twice the angle at the centre. This is one of the common error learners commits. In as much as they reasoned that the angle at the centre is twice the angle at the circumference, they displayed an inability to represent the theorem in values. One of the examples of such an error is displayed in the response of LB11 shown in Figure 4.9 below.

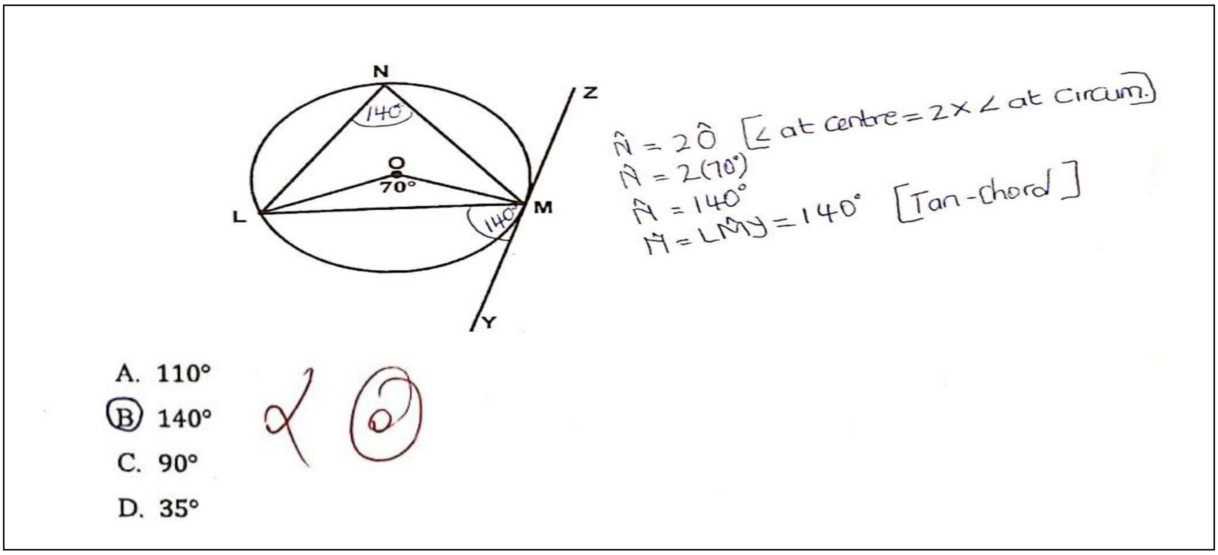
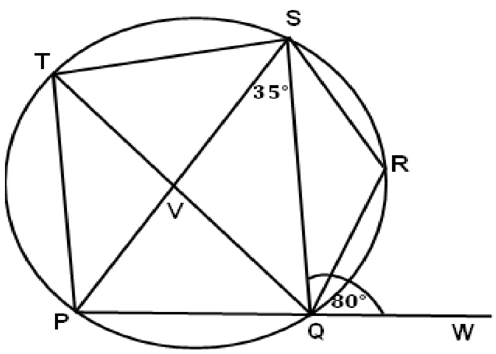


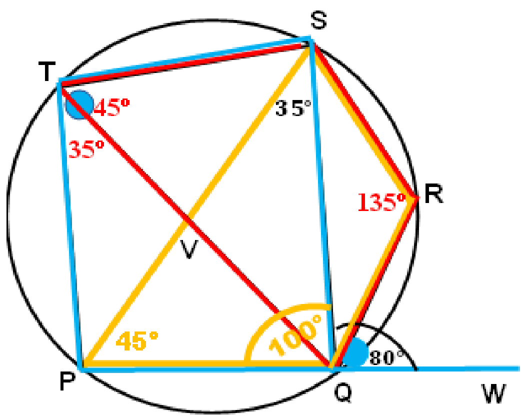
Figure 4.9: LB11 response to item 1.3

4.4.4.4. Item 1.4 (Section 1)



This item is a cognitive Level Four question (see Table 3.4). It required learners to apply higher order reasoning and process to integrate their knowledge. They had to break the problem into pieces to identify what must be solved. By so doing, they should have been able to identify three cyclic quadrilaterals, namely, PQTS, PQRS and QRST. For it was stated that points P, Q, R, S, and T lie on the circumference of a circle. It was given that $S\hat{Q}W = 80°$ and $P\hat{S}Q = 35°$. They had to determine the size of $S\hat{R}Q$. Possible options were (A) $125°$ (B) $100°$ (C) $135°$ (D) $45°$.

Solution to item 1.4 (Option C)



In cyclic quad PQRS

$P\hat{Q}S = 100°$ Adjacent supplementary ∠s

$S\hat{P}Q = 45°$ Sum of $\angle s$ in Δ PQS

$S\hat{P}Q + S\hat{R}Q = 180°$ opp $\angle s$ of cyclic quad

$\therefore S\hat{R}Q = 180° - 45° = 135°$

OR

In cyclic quad PQST

$S\hat{Q}W = P\hat{T}S = 80°$ Ext ∠ of cyclic quad

$Q\hat{T}P = 35°$ $\angle s$ in the same segment

$\therefore Q\hat{T}S = 80° - 35° = 45°$

In cyclic quad QRST

$Q\hat{T}S + S\hat{R}Q = 180°$ opp $\angle s$ of cyclic quad

$\therefore S\hat{R}Q = 180° - 45° = 135°$

Table 4.11: Summary of learners' responses to item 1.4

School	A	B	C	D	No response	Total
	Number and percentage of learners' responses per option					
A	4	9	**8**	10	2	33
	12.1%	27.3%	**24.2%**	30.3%	6.1%	100%
B	2	4	**9**	11	4	30
	7%	13%	**30%**	37%	13%	100%

Less than one-third of the participants in both schools answered this item correctly. There were 8 (24%) of the participants from School A who selected the correct option and 9 (30%) of the participants from School B who selected the correct option. LB30 managed to identify PQRS as a cyclic quadrilateral in which enough information can be found that can link with angle SRQ. However, the participant did not provide reasons for the indicated sizes of angles (see Figure 4.8 below).

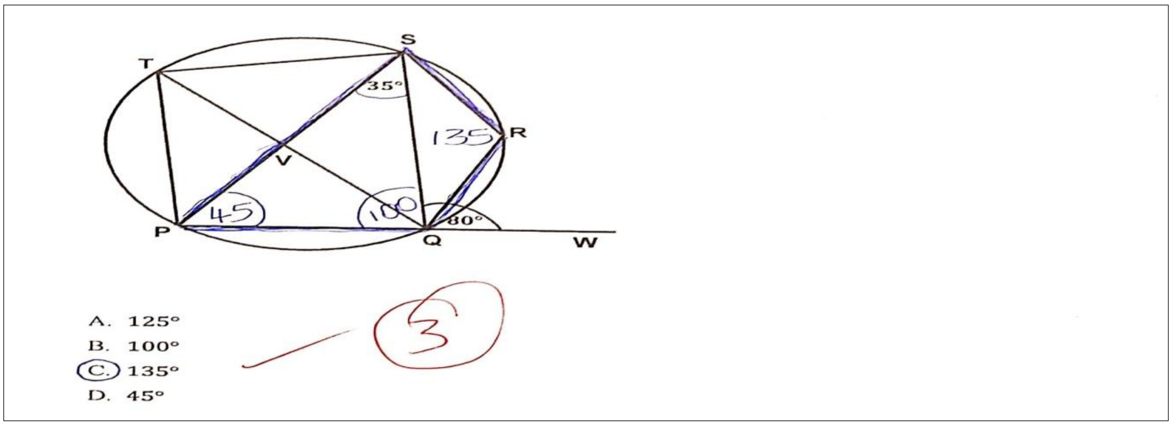


Figure 4.10: LB30 response to item 1.4

It appears that most participants guessed what they thought could be the correct solution to this problem because they did not show their working in this item. Very few of them attempted to work out the solution. While two participants from School A and 4 participants from School B did not respond to the item at all (no option is selected). Most participants failed to identify the other two cyclic quadrilaterals which are directly linked to angle SRQ other than PQST. It appears that many of them have not yet developed spatial perception. Their lack of spatial sense hinders their ability to visualize figures presented to them in unusual forms. Cassim (2006) refers to this shortcoming as a lack of geometric eye. Therefore, they could not logically relate the size of the angle in question with the given information. Subsequently, they forced the solution to the problem. evidenced in the response of LB28 shown in Figure 4.11, the participant first erroneously divided the size of angle SQW into two equal halves. Then assumed the equality of line SR and line RQ from the appearance of the diagram without any proof. From there the participant calculated the sum of angles in triangle QRS. This indicates that the solution approach of the participant to the problem was based on the perceived visual appearance of the diagram.

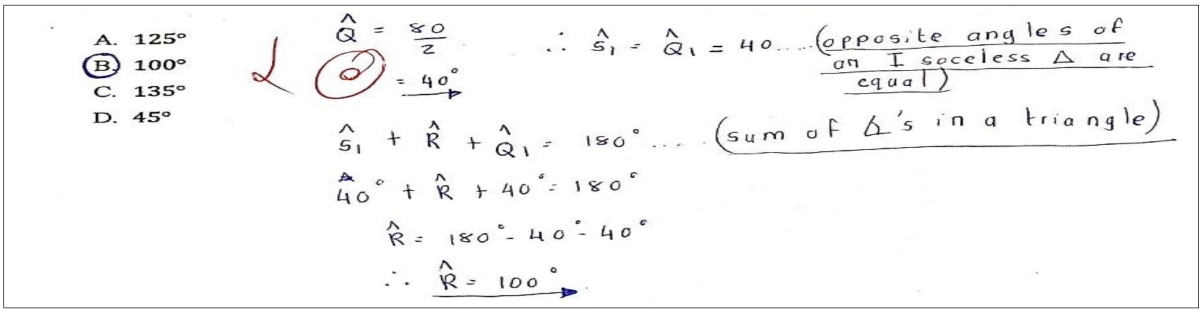
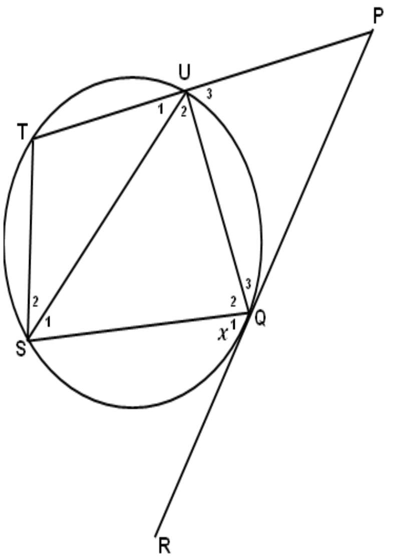


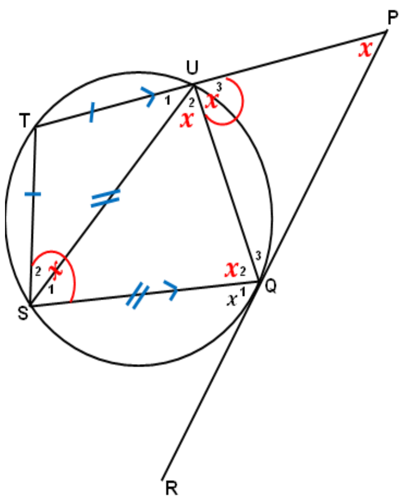
Figure 4.11: LB28 response to item 1.4

4.4.4.5. Item 1.5 (Section 1)



This item is a cognitive Level Three question (see Table 3.4). Learners required the conceptual understanding to solve this problem involving complex or multistep calculations where there was no obvious route to the solution. The item tested the learner's application of their previous and current knowledge in relation to the properties of a cyclic quadrilateral, tangent, parallel lines, and triangles. It was stated that QUTS is a cyclic quadrilateral. PQR is a tangent to the circle at Q. TS = TU; SU = SQ and TP ∥ SQ. it was given that $S\hat{Q}R = x$, learners had to identify an angle in the given options that is **not** equal to x. Possible options were (A) $\hat{P}$ (B) $\hat{Q}_2$ (C) $\hat{U}_1$ (D) $\hat{S}_1 + \hat{S}_2$.

Solution for item 1.5 (Option C)



$\hat{Q}_1 = \hat{P} = x$ Corresp $\angle s$: TP ∥ SQ

$\hat{Q}_1 = \hat{U}_2 = x$ Tan-chord Theorem

$\hat{U}_2 = \hat{Q}_2 = x$ $\angle s$ opp = sides

$\hat{Q}_2 = \hat{U}_3 = x$ Alt $\angle s$: TP ∥ SQ

$\hat{U}_3 = \hat{S}_1 + \hat{S}_2 = x$ Ext $\angle$ of cyclic quad

Table 4.12: Summary of learners' responses to item 1.5

School	A	B	C	D	No response	Total
	Number and percentage of learners' responses per option					
A	10	10	**4**	8	1	33
	30.3%	30.3%	**12.1%**	24.3%	3%	100%
B	7	4	**8**	10	1	30
	23.33%	13.33%	**26.67%**	33.33%	3.33%	100%

This is one of the least correctly answered items in School A with 4 (12%) of the participants who selected the correct option. In School B, it is the second last correctly answered item with 8 (27%) of the participants who selected the correct option. Most participant's diagrams were clean, no markings were made, and this made it difficult for them to follow through to other angles which are equal to x in order to identify an angle in the given options that was not equal to x. It appears that they lack a solution strategy or approach into geometric problems. LA2 simply enlisted equal angles and presumed that they are equal to x without making any connection to the given angle that is equal to x (see Figure 4.12 below).

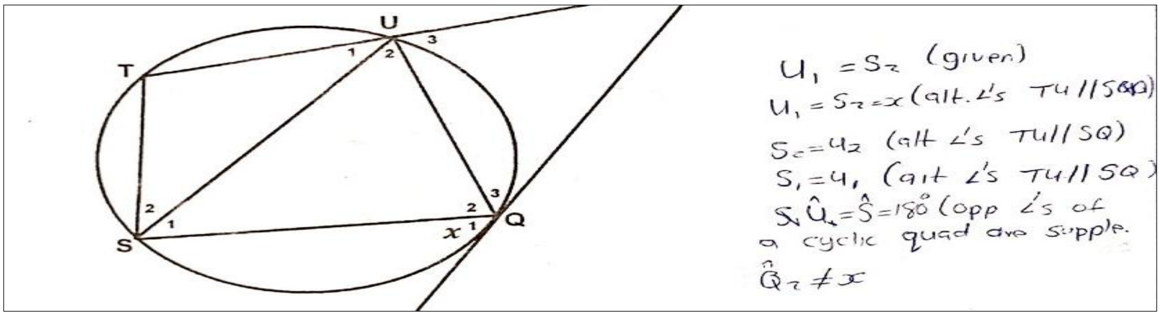
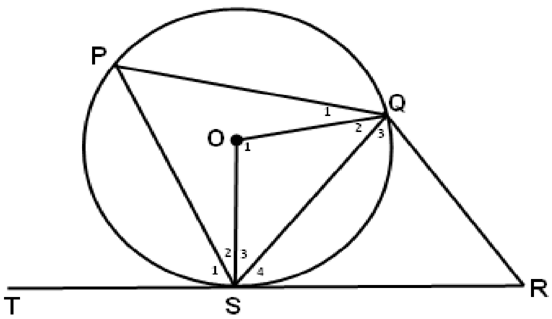


Figure 4.12: LA2 response to item 1.5

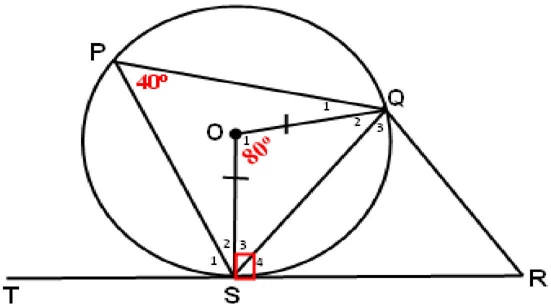
4.4.4.6. Item 2.1 (Section 2)



This item consisted of cognitive Level One and Two questions. Learners were required to recall relevant theorems and use well known procedures, simple applications and calculations which involved few steps. The item tested the learner's application of their previous and current knowledge of the angle subtended by the arc at the centre and the circumference of a circle and the properties of a tangent and triangles to determine the sizes of the requested angles. It was stated that in the given circle SQP, O is the centre of the circle. TSR is a tangent to the circle at S. $Q\hat{O}S = 80°$. Learners had to determine the sizes of (2.1.1) $\hat{P}$, (2.1.2) $\hat{Q}_2$, (2.1.3) $\hat{S}_4$.

4.4.4.6.1. Item 2.1.1 (Section 2)

Solution for item 2.1.1

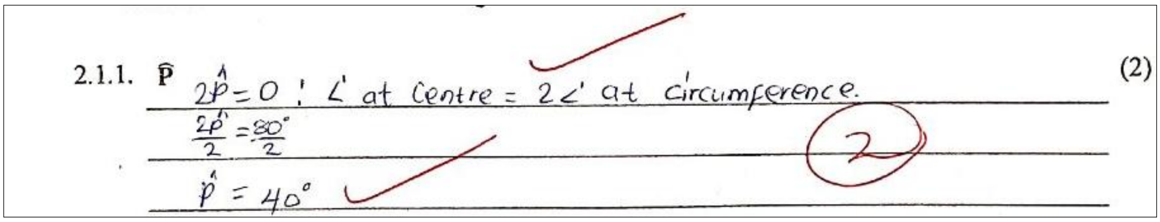


$\hat{P} = 40°$ [$\angle$ at centre $= 2 \times \angle$ at circumference]

Table 4.13: Summary of learners' responses to item 2.1.1

School	Solved problem	Partial solved problem	Un-solved problem	No attempt made	Total
A	21	7	4	1	33
	64%	21%	12%	3%	100%
B	24	4	2	0	30
	80%	13%	7%	0%	100%

This item has the highest number of participants who provided the correct response consisting of the correct statement with a valid reason in Section Two. Most participants appeared to be aware of the theorem which states that the angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle and its application. 21 (64%) of the participants from School A and 24 (80%) of the participants from School B falls within the category of the fully solved problem. Figures 4.13 and 4.14 below are samples of the participant's correct responses.

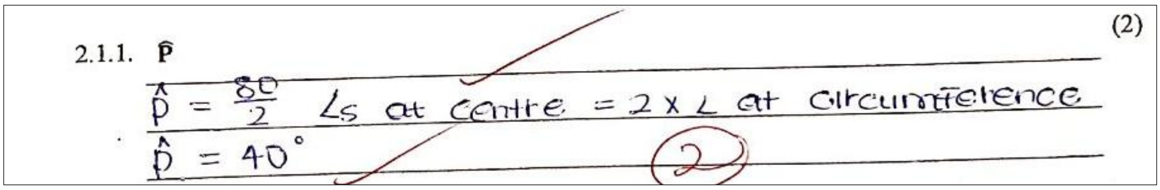


2.1.1. $\hat{P}$ $2\hat{P} = 0$; ∠' at Centre = 2∠' at circumference. (2)

$\frac{2\hat{P}}{2} = \frac{80°}{2}$

$\hat{P} = 40°$

Figure 4.13: LA28 response to item 2.1.1



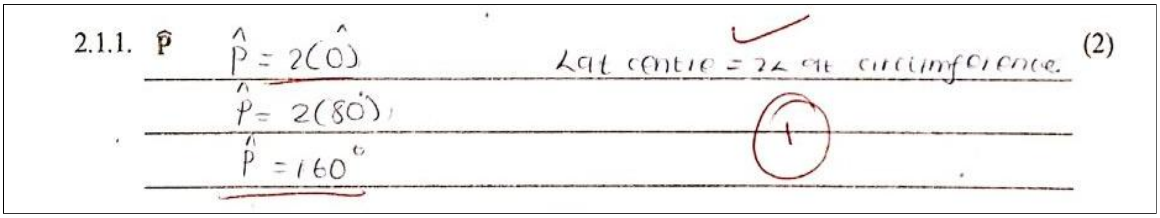
2.1.1. $\hat{P}$ (2)

$\hat{P} = \frac{80}{2}$ ∠s at centre = 2 x ∠ at circumference

$\hat{P} = 40°$

Figure 4.14: LB9 response to item 2.1.1

Some of the participants who fall under the category of partial solved problem stated a valid reason but confused the application of the theorem. Just like in item 1.3 they continued to make the angle at the circumference to be twice the angle at the centre. They display an inability to represent properties in values. Figure 4.15 below is a sample of one of the participant's responses.



2.1.1. $\hat{P}$ $\hat{P} = 2(\hat{O})$ ∠at centre = 2∠ at circumference. (2)

$\hat{P} = 2(80°)$

$\hat{P} = 160°$

Figure 4.15: LA18 response to item 2.1.1

LA27 is an example of participants who were familiar with the procedure to determine the angle at the circumference when given the angle at the circle subtended by the same arc, however, the participant could not state the valid reason. Instead, the participant erroneously assumed parallel lines between line PQ and line TS shown in the statement indicating that angle P and angle S_1 are equal because they alternate according to the participant. Furthermore, the participant misapplied vertically opposite angles between angle S_1 and S_4 (see Figure 4.16).

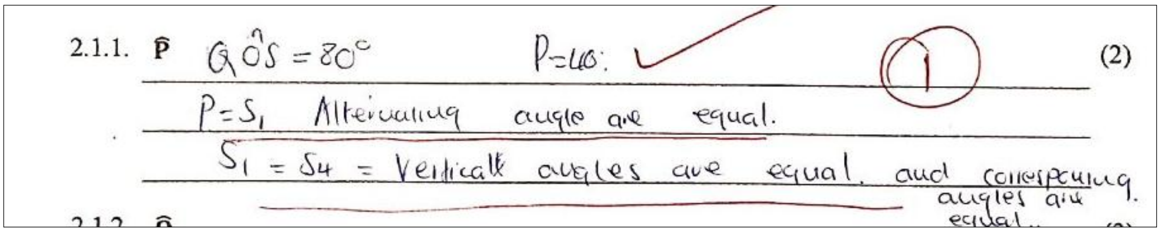
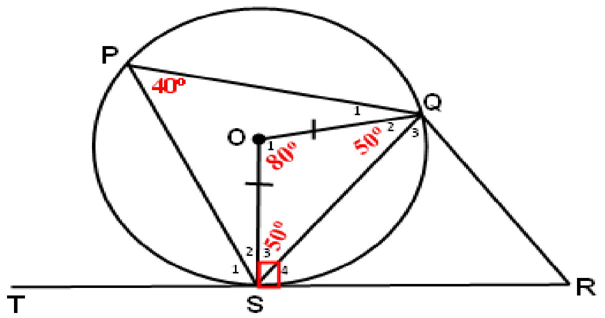


Figure 4.16: LA27 response to item 2.1.1

4.4.4.6.2. Item 2.1.2 (Section 2)

Solution for item 2.1.2



$\hat{Q}_2 = \hat{S}_3$ [$\angle s$ opposite equal sides]

$\therefore \hat{Q}_2 = 50°$ [$\angle s$ of Δ]

Table 4.14: Summary of learners' responses to item 2.1.2

School	Solved problem	Partial solved problem	Un-solved problem	No attempt made	Total
A	12	11	8	2	33
	36.4%	33.3%	24.2%	6.1%	100%
B	18	3	7	2	30
	60%	10%	23%	7%	100%

This item has the second highest number of participants who provided the correct response consisting of the correct statements with valid reasons in Section Two. There were 12 out of 33 (36.4%) of the participants from School A and 18 out of 30 (60%) of the participants from School B who provided correct statements and all valid reasons. These participants were aware that OQ and OS are equal radii though there were no indications on the diagram because the lines are drawn from the centre of the circle to the edge of the circumference. Figure 4.17 is a sample of the participant's response that falls under the category of fully solved problem.

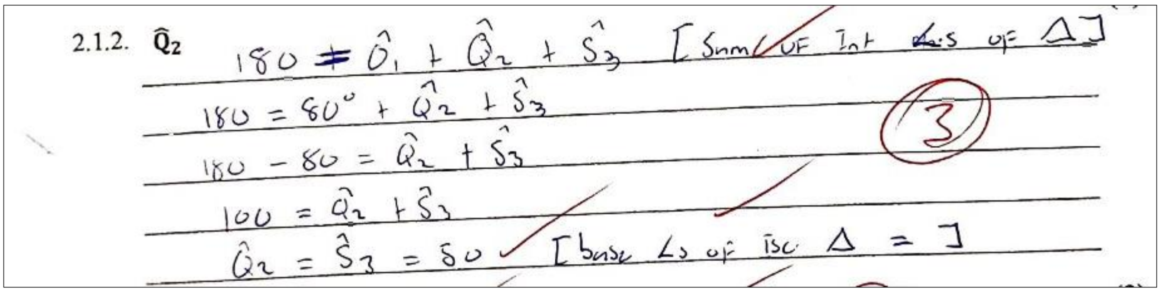


Figure 4.17: LB7 response to item 2.1.2

Most participants who fell under the category of partial solved problem provided the correct statements and omitted the reason(s) (see Figure 4.18 and Figure 4.19).

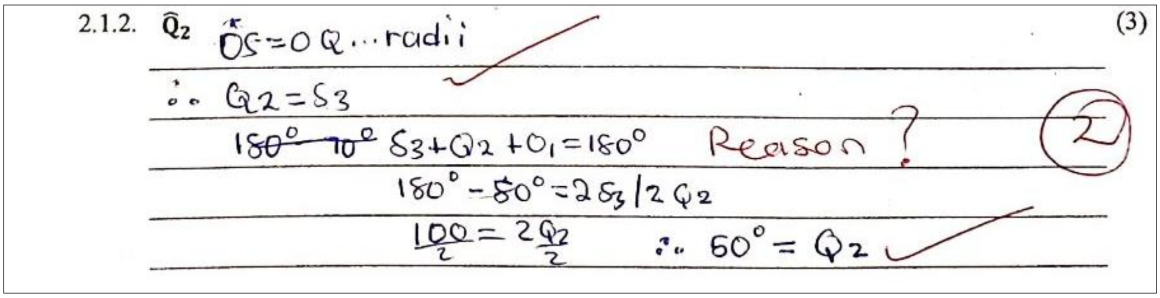


Figure 4.18: LA11 response to item 2.1.2

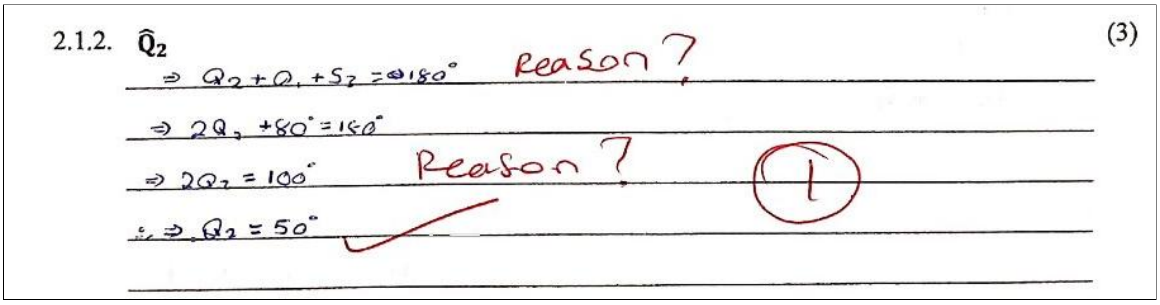


Figure 4.19: LA20 response to item 2.1.2

All participants under the category un-solved problem could not relate angle Q_2 to triangle SQO. Hence, they tried to force other solutions erroneously. LA29 misconceived the angles in the same segment theorem. The theorem states that angles subtended by the same arc or chord at the circumference of a circle are equal and therefore, it is said that the angles in the same segment of the circle are equal (Phillips et al., 2012). The participant however, seemingly thought that angle P and angle Q_2 are angles in the same segment because they are both at the circumference of a circle without considering that they must both be subtended by the same arc or chord (see Figure 4.20 below).

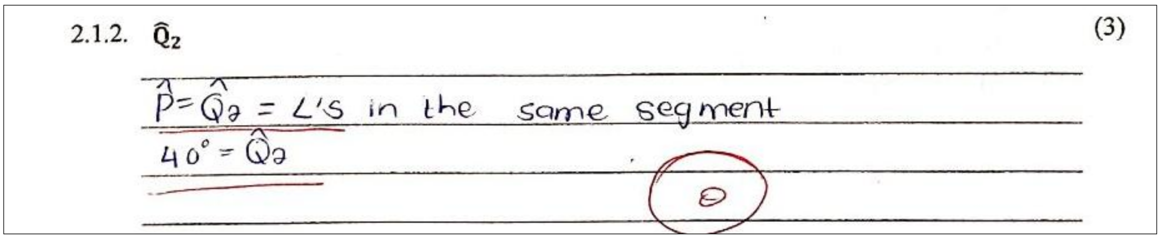
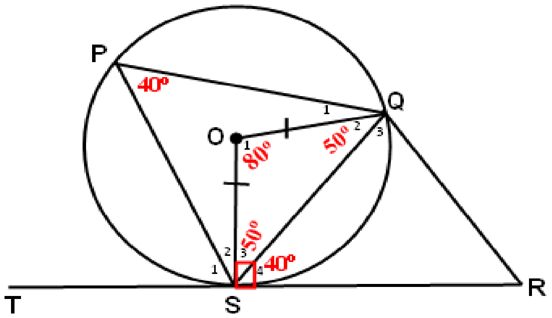


Figure 4.20: LA29 response to item 2.1.2

4.4.4.6.3. Item 2.1.3 (Section 2)

Solution for item 2.1.3



$\hat{S}_4 = \hat{P} = 40°$ [Tan-Chord Theorem]

OR

$\hat{S}_3 + \hat{S}_4 = 90°$ [Tan ⊥ Rad]

$\therefore \hat{S}_4 = 90° - 50° = 40°$

Table 4.15: Summary of learners' responses to item 2.1.3

School	Solved problem	Partial solved problem	Un-solved problem	No attempt made	Total
A	11	5	14	3	33
	33.3%	15.2%	42.4%	9.1%	100%
B	11	1	13	5	30
	37%	3%	43%	17%	100%

Item 2.1.3 is a follow-up question on item 2.1.1 or item 2.1.2. It is seen that 11 participants from both schools managed to provide the correct statement with a valid reason. These numbers made this item to consist of the third highest number of participants who provided the correct response consisting of the correct statement with a valid reason in Section Two. Figure 4.21 and Figure 4.22 are samples of participant's responses in the category of solved problem.

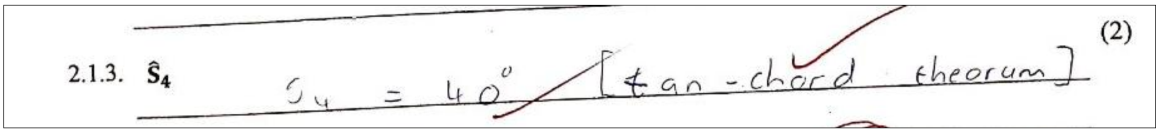


Figure 4.21: LB1 response to item 2.1.3

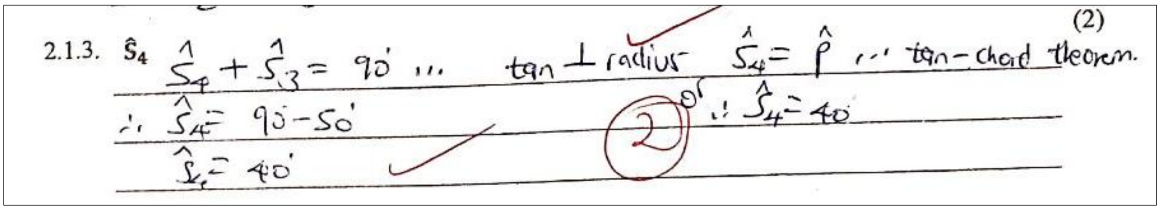


Figure 4.22: LA23 response to item 2.1.3

While the participants who fall under the category of partial solved problem only provided the correct statement and did not include the reason or the given reason was invalid as seen in Figure 4.23, others made no attempt to determine the size of the angle. These participants who left blank spaces appeared to either lack confidence, the necessary skills, or the know-how to do the task. Bransford and Stein (1993) refer to this situation as the “the let-me-out-of-here approach”, as evidenced by LA13 in Figure 4.24.

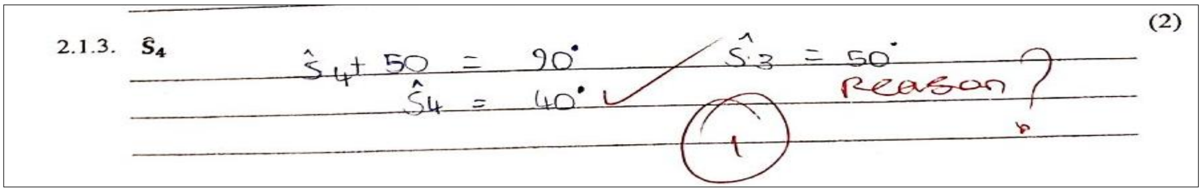


Figure 4.23: LB24 response to item 2.1.3

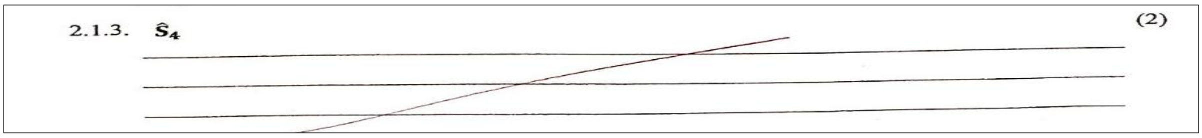


Figure 4.24: LA13 response to item 2.1.3

Moreover, more participants in their responses did not provide any size of the angle in question and others erroneously worked out the size of the angle. LB21 simply related angle S_4 and angle Q_2 as alternate angle just because the angles are founded in a z shape. This is one of the common errors made by the participants. LB21’s response shown in Figure 4.25 below does not even include the parallel lines which resulted in the alternate angles in the reason.

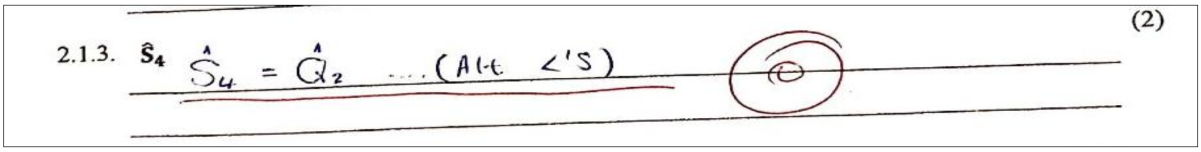


Figure 4.25: LB21 response to item 2.1.3

LB16 seen in Figure 4.26 simply divided angle S into four equal angles along a straight line. The participant was aware that angles lying along a straight line add up to 180^0 (adjacent supplementary angles). However, they misconceived the application of the property. This is a sign of not understanding the concept.

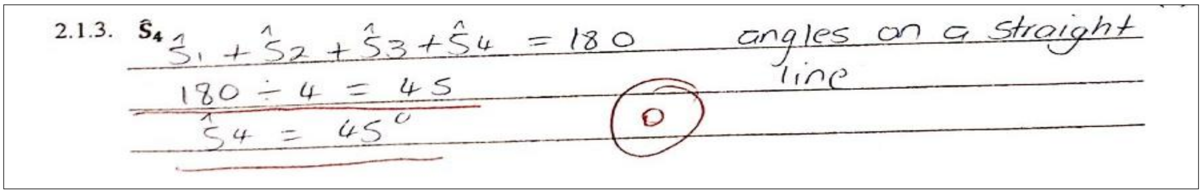
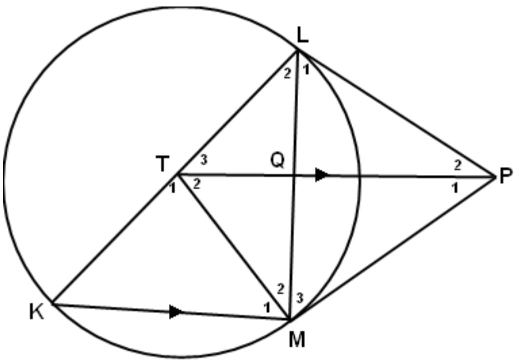


Figure 4.26: LB16 response to item 2.1.3

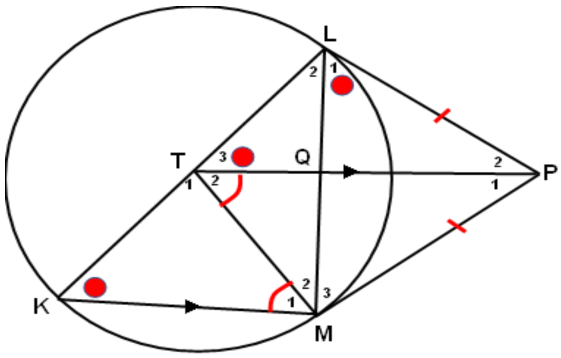
4.4.4.7. Item 2.2 (Section 2)



This item consisted of cognitive Level 3 and 4 questions. In cognitive level three questions learners required conceptual understanding to solve complex problems where there was no obvious route to the solution and cognitive level four questions required learners to apply higher order reasoning and process to integrate their knowledge in linking relevant information to solve the problem. It was stated that the vertices K, L and M of ΔKLM are concyclic. PL and PM are tangents to the circle at L and M, respectively. T is a point on KL such that TP is parallel to KM. LM cuts TP at Q. The item focused on proof type questions. Firstly, in item 2.2.1 learners had to Prove that $\hat{L}_1 = \hat{T}_3$. Then in item 2.2.2 they had to prove that TLPM is a cyclic quadrilateral and lastly, in item 2.2.3 they had to prove that TK = TM.

4.4.4.7.1 Item 2.2.1 (Section 2)

Solution for item 2.2.1



$\hat{L}_1 = \hat{K}$ [Tan-chord theorem]

$\hat{K} = \hat{T}_3$ [Corresp $\angle s: TP \parallel KM$]

$\therefore \hat{L}_1 = \hat{T}_3$

Table 4.16: Summary of learners' responses to item 2.2.1

School	Solved problem	Partial solved problem	Un-solved problem	No attempt made	Total
A	0	3	27	3	33
	0%	9%	82%	9%	100%
B	0	3	18	9	30
	0%	10%	60%	30%	100%

In this item none of the participants from both schools were able to solve the given problem. In the given diagram there was no direct connection between angle L_1 and angle T_3. Therefore, participants had to identify an angle that was equal to both given angles in order to prove that they were equal. However, though there were three participants from each school who established a link between angle L_1 and angle K, none of them realised that angle T_3 and angle K were corresponding angles given that line TP is parallel to line KM. Rather, most participants directly connected angle L_1 and angle T_3 with an application of a tan-chord theorem. The theorem states that the angle between the tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment (Phillips et al., 2012). furthermore, a chord is a line with endpoints on the circumference of a circle, and the angle in the alternate segment must be at the circumference of the circle opposite that same chord. Yet the participants considered line LQ to be a chord and angle T_3 as an angle in the alternate segment (e.g., Figure 4.27 below).

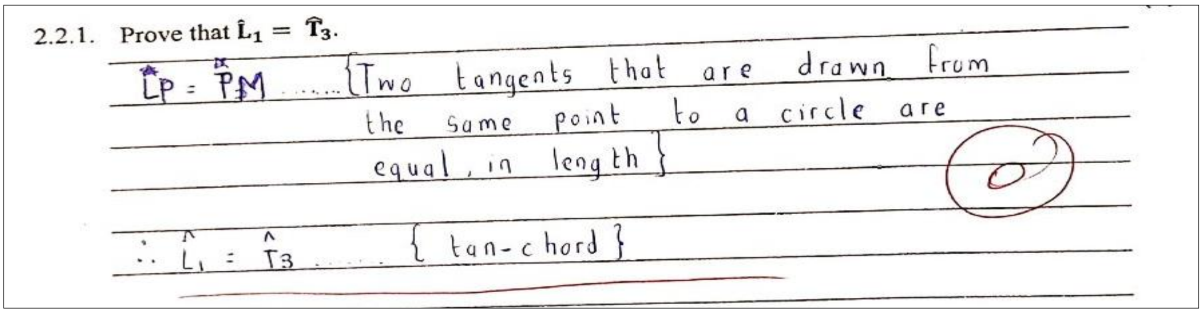


Figure 4.27: LB28 response to item 2.2.1

Another misapplication of the tangent-chord theorem is witnessed in Figure 4.28 evidenced by LA13. It appears that the learners exhibit a misconception of the theorem whenever there are other lines drawn in the triangle. This is an indication of lack of conceptual understanding of tangent-chord theorem. Hence, the resultant misapplication of the theorem. Another thing could be that as Cassim (2006) argued, the learners lack a geometric eye to differentiate between a chord and any other line. Consequently, they are unable to visualize how geometric theorems detach themselves from the given diagram.

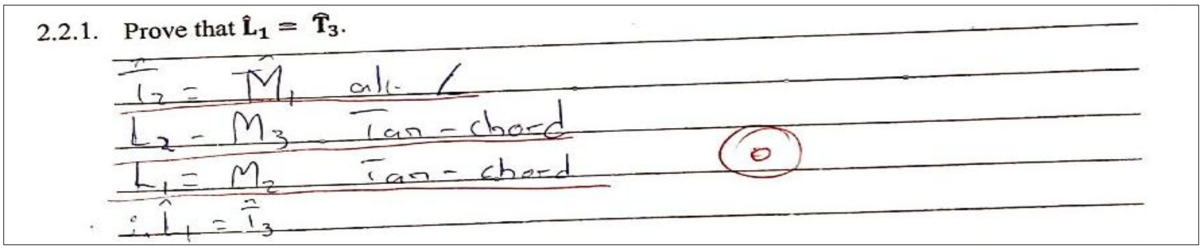
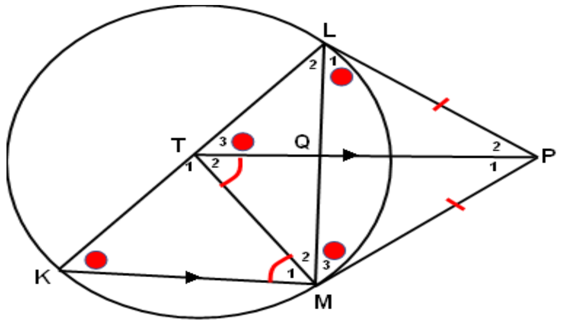


Figure 4.28: LA13 response to item 2.2.1

4.4.4.7.2 Item 2.2.2 (Section 2)

Solution for item 2.2.2



LP = MP [Tangents from same point]

$\hat{L}_1 = \hat{M}_3$ [$\angle s$ opposite equal sides]

$\hat{L}_1 = \hat{T}_3$ [Proved in 2.2.1]

$\therefore \hat{M}_3 = \hat{T}_3$

$\therefore$ TLPM is a cyclic quad [Converse $\angle s$ in the same segment or LP subtends = $\angle s$]

OR

$\hat{T}_3 = \hat{K}$ [Proved in 2.2.1]

$\hat{M}_3 = \hat{K}$ [Tan-Chord Theorem]

$\therefore \hat{M}_3 = \hat{T}_3$

$\therefore$ TLPM is a cyclic quad [Converse $\angle s$ in the same segment or LP subtends = $\angle s$]

Table 4.17: Summary of learners' responses to item 2.2.2

School	Solved problem	Partial solved problem	Un-solved problem	No attempt made	Total
A	0	5	27	1	33
	0%	15%	82%	3%	100%
B	0	5	18	7	30
	0%	17%	60%	23%	100%

In this item the participants had to identify two equal angles which were subtended by the same chord in order to prove that TLPM W a cyclic quadrilateral. However, none of the participants could reason this way. Rather, most of them applied the properties of a cyclic quadrilateral in their attempt to prove that TLPM is cyclic (has all its four vertices on the circumference of a circle). This is one of the major errors committed by learners when responding to proof type questions. The response of LA28 shown in Figure 4.29 is one of the examples indicating the application of properties of what needs to be proved as though it already exists. For instance, the participant equated angles with the reason that they are angles in the same segment, but this theorem can only be applied if a quadrilateral is indicated or proven that it is cyclic.

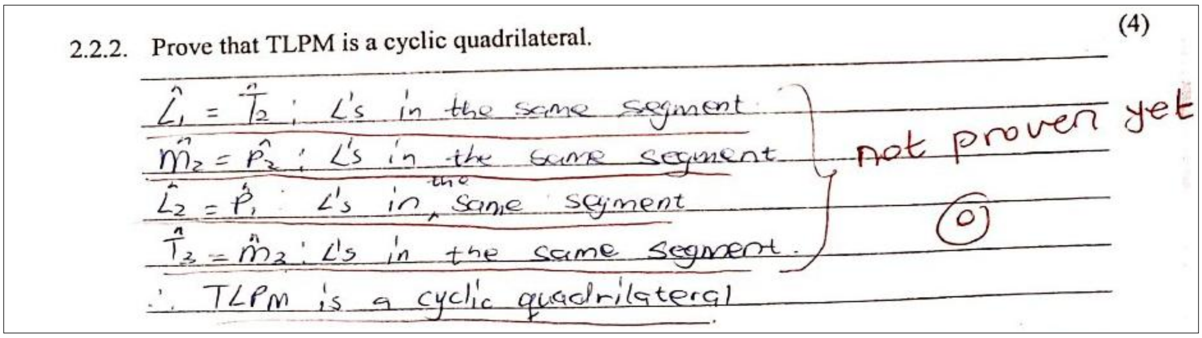


Figure 4.29: LA28 response to item 2.2.2

On the same note, some participants like LB23 shown in Figure 4.30 erroneously stated that opposite angles are supplementary. This is one of the properties that could be used to prove that a quadrilateral is cyclic. In this case one would need to determine the sizes of the opposite angles of a quadrilateral and show that they add up to 180°. in that way it would mean that the quadrilateral is cyclic. However, in item 2.2.2 this was not possible because the diagram is not numeric enabled.

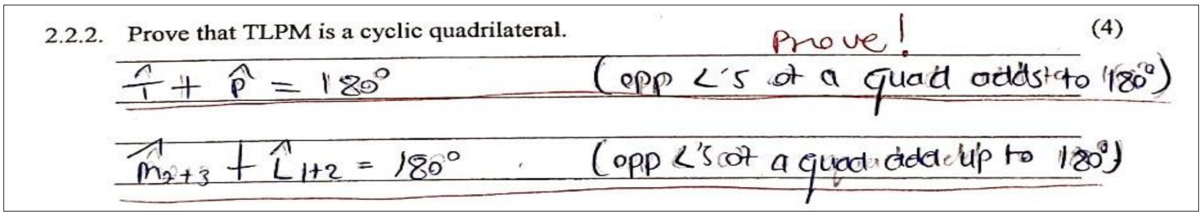


Figure 4.30: LA23 response to item 2.2.2

LA2 decided to apply both above-mentioned methods (see Figure 4.31). It appears that the participants were aware of the properties used to prove that a quadrilateral is cyclic but lacked the know how to go about proving. It appears that the participants have not yet developed logic and competency in dealing with proof type questions.

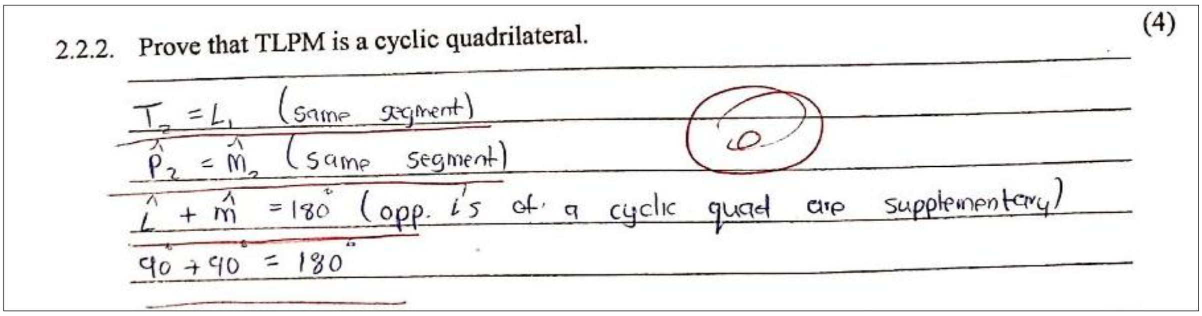
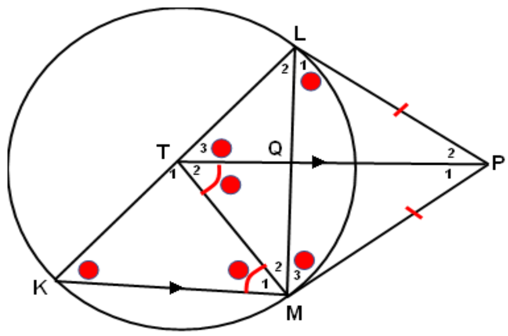


Figure 4.31: LA2 response to item 2.2.2

4.4.4.7.3 Item 2.2.3 (Section 2)

Solution for item 2.2.3



$\hat{L}_1 = \hat{T}_2$ [MP subtends = $\angle s$]

$\hat{T}_2 = \hat{M}_1$ [Alt $\angle s$: $TP \parallel KM$]

$\hat{L}_1 = \hat{K}$ [Proved in 2.2.1]

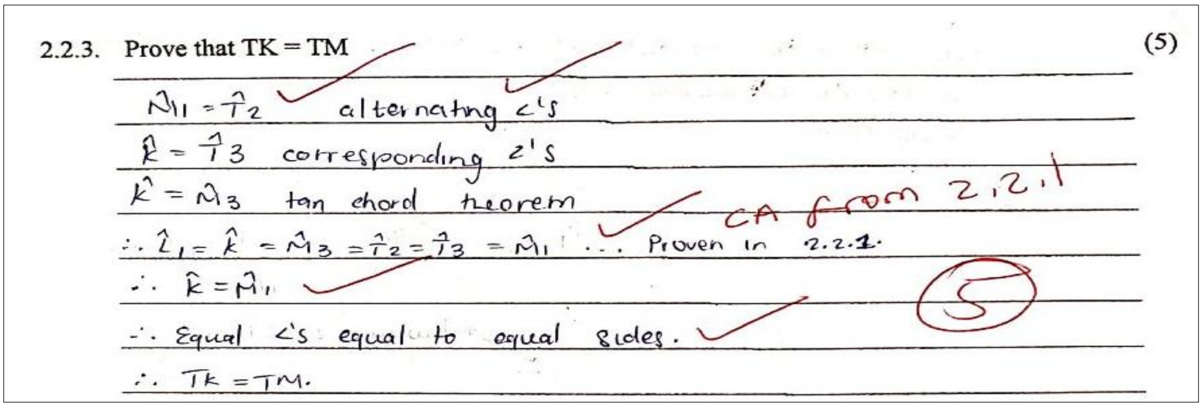
$\therefore \hat{K} = \hat{M}_1$

$\therefore$ TK = TM [$\angle s$ opp = sides]

Table 4.18: Summary of learners' responses to item 2.2.3

School	Solved problem	Partial solved problem	Un-solved problem	No attempt made	Total
A	1	15	15	2	33
	3%	45.5%	45.5%	6%	100%
B	0	8	14	8	30
	0%	26.7%	46.7%	26.7%	100%

In this item only one participant managed to solve this problem. The item was a follow-up question on what has already been established in the two previous items. It required participants to show that angle K is equal to angle M_1 in order to prove that line TK is equal to line TM. LA17 was able to establish a link between angles that led to the connection of angle K and angle M_1 (see Figure 4.32).


2.2.3. Prove that TK = TM (5)

$\hat{M}_1 = \hat{T}_2$ alternating ∠'s

$\hat{K} = \hat{T}_3$ corresponding ∠'s

$\hat{K} = \hat{M}_3$ tan chord theorem

$\therefore \hat{L}_1 = \hat{K} = \hat{M}_3 = \hat{T}_2 = \hat{T}_3 = \hat{M}_1$ ... Proven in 2.2.1.

$\therefore \hat{K} = \hat{M}_1$

∴ Equal ∠'s equal to equal sides.

∴ TK = TM.

CA from 2.2.1

5

Figure 4.32: LA17 response to item 2.2.3

All participant's responses that were categorised under partial-solved problem only managed to link angle T_2 and angle M_1 as alternate angles but could not go further to link angle T_2 and angle L_1. This means the participants failed to recognise the existence of the properties of the proven cyclic quadrilateral from the previous item. LA4's response shown in Figure 4.33 is one of the examples of the participant's responses categorised under partial-solved problem.

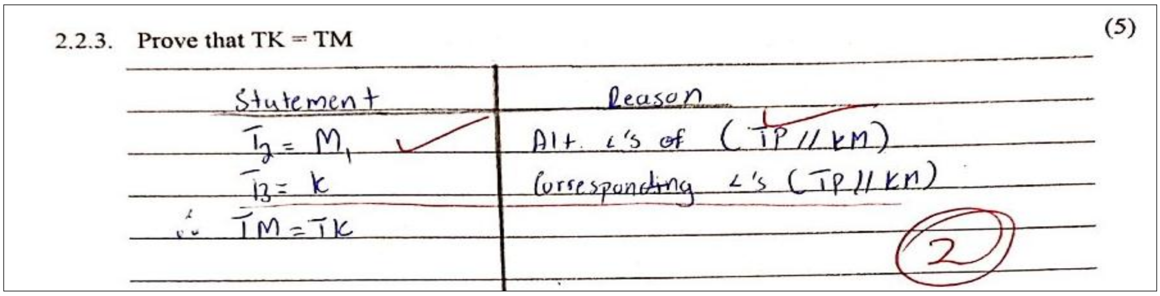


Figure 4.33: LA4 response to item 2.2.3

Point T is assumed to be a centre in most of the participant's response that were categorised under un-solved problem. The participants simply made their assumption based on the appearance of the diagram without any statement or proof that point T is a centre. This is also one of the common error's learners commit. They make assumptions of things not stated or proven yet and apply their properties as though they have already been established. LA30 is one of the participants who believed that point T is a centre and therefore, according to her line TK is equal to line TM (seen by the indication of radii). It is also observed that the participant did not take the mark allocation into consideration (see Figure 4.34).

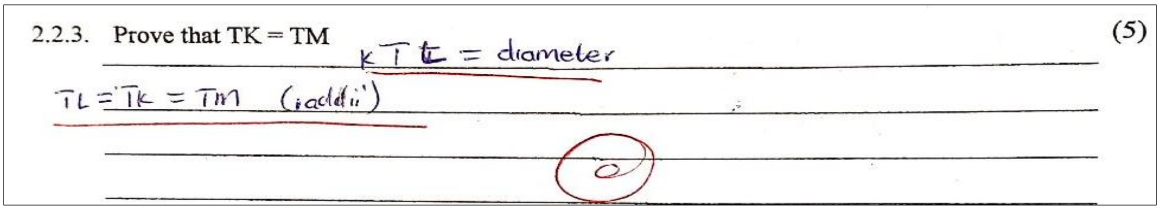


Figure 4.34: LA30 response to item 2.2.3

General findings on item 2.2 (proof type questions) evidenced by most participants were the use of properties of figures randomly without logic or connection or relevance to the situation at hand. They seem to be aware of the properties but apply them incorrectly or were used in incorrect contexts. This means, they would provide correct statements and valid reason which were irrelevant to the immediate question. This is a sign of not understanding the problem and the know-how to go about finding the solution to the problem. Consequently, the observed findings could be the effects of a high cognitive load. Some made unjustified conclusions based on assumptions or incorrect reasons. They show lack of deductive reasoning and logic in solving geometric problems. Moreover, they never look back to check if the questions were answered or the problems were solved.

4.4.4.8 Summary of item-by-item analysis of learners' responses

The following strategies, common practice, errors, and misconceptions were observed from the participants' responses to the given Euclidean geometry task:

1. Assumption of properties based on the visual appearance of a diagram.
2. Incorrect application of geometric properties to solve problems.
3. Random use of theorems without logic, connection, and relevance to the solution of the problem.
4. Never look back to relate their problem solution to the originally given problem.
5. Challenges dealing with proof type questions.

4.4.5. A learner's feedback on the written task

After writing the Euclidean geometry task, the participants were requested to provide feedback by reflecting on the mental effort they invested when performing the task. A participant's feedback tool was provided to every individual where they had to rate the level of difficulty of the task they had written (see Appendix G). Participants were instructed not to write their names on the feedback tool but encouraged to be sincere in their response in rating themselves. Participants' feedback on the written task was of crucial important to the study, as it provided valuable insights into their feelings, views, and perspectives on the task, content, topic, and problem-solving in Euclidean geometry. A self-rating scale of levels of difficulty consisted of a five-point scale. Table 4.19 show the number of participants in each level of choice.

Table 4.19: Participant's feedback on EGT level of difficulty

No.	Level of difficulty	School A	School B
1	Easy	3 (9%)	2 (7%)
2	Fair	4 (12%)	3 (10%)
3	Some easy, but mostly difficult	15 (46%)	12 (40%)
4	Difficult	7 (21%)	8 (27%)
5	Very difficult	4 (12%)	5 (16%)
Total		**33 (100%)**	**30 (100%)**

Table 4.19 show that more participants acknowledged that there were some items in the task that were easy, but they perceived most of the items to be difficult. This category had the highest number of participants from both schools followed by those who perceived the task to be difficult. Some viewed the task as very difficult, while others thought it was fair and a fewer number of participants perceived the task as easy.

4.5. ANALYSIS OF SEMI-STRUCTURED INTERVIEWS

The semi-structured interviews were conducted with a sample of six participants. Three participants from each school were selected. Participants were identified as LA13, LA19, LA28, LB4, LB6 and LB7 with "L" standing for learner, A or B for School A or School B and the numbers corresponds to the sequence of the learners as they appear on the recording mark sheet used to capture their scores for the written task (see Section 3.6, Table 3.5, and Table 4.2). The interview questions (in Appendix C) were articulated to address the objectives of the study which are discussed in detail in the next chapter. In the following sections, the data from the transcribed interviews were grouped based on the questions that were asked during the interview sessions.

4.5.1 Analysis of learners' responses to interview item one

The first question of the semi-structured interview was tapping into the participant's understanding of problem-solving in mathematics. The interview question asked:

In your own understanding, what is problem-solving in mathematics?

The interviewee's responses had a tendency of pointing towards a particular direction. Each participant responded as follows:

LB4: I think problem-solving is finding a solution to a problem given.

LA11: It is just solving the equation or the questions that you are given.

LA13: Is to be given the problem and you try to solve it given the practice or multiple ways that you were taught. But I think the main objective is to further your mind, is to think deeper to let you come up with your way of solving it.

LB6: Is reaching a solution to a maths problem basically.

LB7: umm… I think it is the investigation sort of to try to find a solution to a problem that you would have been given. So, problem-solving is finding the solution to a problem.

LA28: I believe it is when you try to find a solution to something.

The definition of mathematical problem-solving adopted in the study is that it is a cognitive activity where a learner encounters a problem in which there is no readily available solution process. The learner seeking the solution, actively explores a variety of approaches to solve the problem (see Section 2.2.1). Observed from all interviewee's responses is the awareness that in mathematical problem-solving there is a search for a solution to the given problem. While five participant's understanding was limited to finding the solution, **LA13** demonstrated more insight into the concept by indicating the application of the mind in exploring the ways of solving the problem. The learner's understanding of problem-solving in mathematics plays a significant role in their approach when dealing with mathematical tasks.

4.5.2 Analysis of learners' responses to interview item two

The second question of the semi-structured interview was topic specific. It tended to assess the participant's awareness of the basic knowledge required of them to solve Euclidean geometry problems. The interview question was presented in the following format:

What do you think is the basic knowledge required to solve Euclidean geometry problems in Grade 11?

Most interviewees in their responses seem to suggest that the knowledge of the theorems in solving Euclidean geometry is of outmost importance. Their actual word-to-word responses to the above question are as follows:

LB4: I think knowing the theorems first… and knowing where to start.

LB7: I think the required knowledge to work with Euclidean geometry is firstly, most importantly to understand the theorems that work with Euclidean geometry. I think its most important to understand the theorems.

LA11: It is just understanding those… ah the things you learnt in grate 10 and 9, which are those things for understanding parallel lines and other thing.

LA28: I would say you have to know the theorems and then, you have to be able to identify them in a rider.

LB6: It is knowing how to apply the theorems because that is the problem. The thing with Euclidean geometry you can know all the theorems, but then when the diagram is presented to you, it becomes a real problem.

LA13: I think it is really understanding the sheet first to get to know the reasoning because that is what you need. And finally, you learn how to get all those things together in one in order to solve the rider.

LA11 did not seem to have the vocabulary "theorem" in mind, but the word "things" that was used in her response could suggest a reference to the properties of lines and shapes that they learnt in Grade 9 and the theorems they were taught in Grade 10 as revealed in the context of the response by using parallel lines as an example. Together with the knowledge of theorems, LA28 added, "… you have to be able to identify them in a rider". This was a distinguishing factor from other interviewees whose responses were listed ahead. However, LB6 approached their response differently. The participant believed that the basic knowledge required to solve Euclidean geometry was "… knowing how to apply the theorems". This was an interesting view.

It was earlier established in this study that Euclidean geometry is based on properties of lines and shapes, proved axioms and theorems (see Section 1.11.3 and Section 2.3.3) Additionally, the topic requires the genuineness of knowledge in mathematics, spatial perception and understand, and intelligent thoughts for applying the properties and theorems in exceptional circumstances. Most of the interviewees responded in line with these realities, except for LA13. The participant's response suggests that they did not understand the question asked. In the statement "I think it is really understanding the sheet first to get to know the reasoning…", the use of the word "sheet" could be interpreted as statement or diagram or statement and diagram. This means the participant was alluding to how they approach a Euclidean geometry problem.

4.5.3 Analysis of learners' responses to interview item three

The third question of the semi-structured interview related to the participant's actions in tackling a Euclidean geometry task. The question was posed as follows:

> When solving a Euclidean geometry problem, what is your usual approach?

In this question, it was anticipated that the respondents would narrate their approach in line with how they confront Euclidean geometry tasks. For some respondents it was not easy to express themselves properly using the correct mathematics language. Evidenced by LA13 is an insufficient use of relevant vocabulary. The respondent related his usual approach to mean that they first read the given statement and confirms what is stated with the diagram. "Then… I try to solve the picture…before I look at the question". This tended to indicate that participants analysed the diagram to unpack the theorems involved before they could read the question. Below are the respondent's actual words:

LA13: I try to… I read the question because I think the question contains to solve which. So, because the question usually reflects what the image shows. So, yea, I read the question a bit of it and then confirm with the picture. That is what I do. Then when I check the picture, I try to solve the picture without even looking at the question to try what I am required… before I even look at the question. Then I jump to the question.

LB7, LA28 and LA11 also narrated their usual approach when solving a Euclidean geometry problem. However, their focus is mainly on identifying theorems involved in the given diagram or the rider. They did not indicate whether they do read the given statement or not, or whether they do confirm if all the information on the statement is represented on the diagram or not. They seem to be more interested in looking at the diagram or rider to identify the theorems they know without considering the information that is given, unknown and what is required of them. Their actual words are as follows:

LB7: My usual approach would be to first try to identify any particular diagram that would be consisting of single theorems. Go through my theorems that I know and try to locate them within the diagram.

LA28: I would usually start by checking which theorem is involved in that rider. And then, I would identify the angles given and the information from the question.

LA11: I usually check what theorems we are given. Then I… by then, I already know what I am required to prove or what I must do. So, it is just knowing the theorems well.

On the other hand, LB6 did not really answer the question asked. The interviewee simply responded as follows:

LB6: The topic itself it is hard, but I try by all means to practice different questions so that I can know how to apply the theorems more.

Moreover, LB4 unashamedly voiced out her lack of know-how to go about solving a Euclidean geometry problem. when the researcher made a follow-up, the respondent merely said, "I try looking for theorems I know." The following are the respondent's actual words:

LB4: To tell the truth, I normally do not know what to do… I try looking for theorems I know.

This is truly concerning because what the respondent said reflected on the Euclidean geometry task responses. This is the reason why some participants made no attempt in responding to some of the questions. It is evident that the participants who left blank spaces they lacked confidence, the necessary skills and know-how to do the task. The situation reflected the "let-me-out-of-here approach" (Bransford & Stein, 1993). What also emerged from the interviewee's responses is their attitude and self-efficacy in problem-solving.

4.5.4 Analysis of learners' responses to interview item four

The fourth question of the semi-structured interviews addressed an important construct of the study. The researcher probed the participants on problem-solving strategies, if any, that they were using when solving tasks in Euclidean geometry. The question was presented to the respondents in the following format:

> What strategies do you normally use when solving Euclidean geometry problems?

As established earlier that problem-solving is a cognitive activity/ process (see Section 2.2.1), Thus, the researcher was interested in hearing about the learner's problem-solving strategies (plan of action, method, way, or procedure) that they employ when solving tasks in Euclidean geometry. In responding to the question, below are four respondent's actual words who presented their how-to-go-about the solution process:

LB4: I usually try separating the shapes within the circles. Then I try finding if there is a theorem that I know. Then I take it from there. But then, it is hard, … It is hard figuring out which/ what might be exactly the solution when things are mixed up. So, I separate the shapes from the circle.

LA13: Strategies… I first look at the object that I am given, and I look at the question, then the rider. And I try to find the reasons without even looking at the question. Then I approach the question to see what I can do.

LB7: To let us say… I first try to locate in the diagram if there is any alternate angles or supplementary angles or equal angles within the diagram. And then if it is a cyclic quad, I would try to first identify the angle that is supplementary to the other.

LA28: I identify the type of theorem that is involved in the circle and then check the quadrilaterals and triangles in that rider and try to solve it.

The most common strategy or plan of action that emerged from the responses of the respondents is the analysis of the diagram to identify the associated theorems. In analysing the diagram, LB4 alluded, "I usually try separating the shapes within the circles…" LA13 added, "…I try to find the reasons…". However, there are other respondents who attempted to respond to the question but did not provide a way how-they-go-about the solution process. Their actual words are as follows:

LA11: I think what we are given on the rider is what tells me what to do.

LB6: I make sure that I know all the theorems, know how to apply them using different diagrams. Yea, I do that most of the time.

This is an indication of an inability of a learner to come up with a suitable method to solve the mathematical problem. The respondent's lack of plan of action to facilitate a solution process may be the reason for the learner's random use of theorems without logic, connection, and relevance to the solution of the problem at hand. Ideally, in solving Euclidean geometry problems, learners are expected to explore appropriate patterns that will navigate them to the relevant solutions.

4.5.5 Analysis of learners' responses to interview item five

The fifth question of the semi-structured interviews was a follow-up question to the previously presented question to explore the respondent's voiced skill of carrying out the strategies developed. The question was posed as follows:

> May you explain how you use the strategies that you mentioned in the previous question.

The researcher was interested in hearing about how the participants implement their strategies or plans in carrying out their solution process. According to LA13, after identifying the diagram associated theorems, the participant "try to find the real links" between the relevant theorems that will facilitate the solution to the problem. Likewise, LB7 added, "in Euclidean geometry, I think before an equation that you need to solve, there's a specific equation or angle that needs to be found that will lead to the discovery of the calculations to the rest of the angles. So, what I can do is to see which equation or angle that will lead to the rest of the solution to the problem." In the same point of view, LA28 uses the identified theorems to "see if they help me with the questions they asked me." Below are the respondent's words:

LA13: The question will reflect what the rider is all about. Maybe I will be given that O is the centre, and maybe what and what is a tangent. Well, when I am given the tangent, what first comes to my mind is what can I do about it, why am I given the tangent? When I am given the centre, what can I do about the centre? What is it that corresponds to the centre, like theorem one and other theorems or maybe circle … umm, I try to find the real links around a tangent and a centre to see … I try to find their theorems to see if I can use them to get the answer.

LB7: To my understanding, I have noticed that within every problem given to us in the topic, there would be, let us say, an equation that would be required to be solved first before you can solve the next. For example, when finding an equation of a straight line, you need to find the gradient first. So, in Euclidean geometry, I think before an equation that you need to solve, there is a specific equation or angle that needs to be found that will lead to the discovery of the calculations to the rest of the angles. So, what I can do is to see which equation or angle that will lead to the rest of the solution to the problem.

LA28: After I find the theorems and the triangles, I check the angles and then, I make sure that maybe if there are equal angles, I identify them on the rider. And then see if they help me with the questions-they asked me.

In contrast, LB4 maintained her position from the previous question without any further elaboration as to how they carry out their strategy. Similarly, LB6 narrated on how they study or practice the topic. Consequently, they did not really answer the question. Their actual words are as follows:

LB4: Let us say, for instance there were two triangles within a circle. I separate the triangles just to find out if there is an angle which I'm missing or if there's a theorem which I can identify.

LB6: If maybe there are… the theorems, I practice a lot of diagrams based on that particular theorem. And I also do that with other theorems. Maybe at least have five diagrams for each theorem, so that I can know how to apply them using different diagrams.

4.5.6 Analysis of learners' responses to interview item six

The sixth question of the semi-structured interviews probed the respondent's cognizance to interpret the solution according to the problem of origin. The question asked:

When solving a geometry problem, how do you know that you have reached the solution stage?

The researcher observed that some learners would fill the answer sheet with non-relevant information without providing correct responses that were required of them. Subsequently, this question intended to probe the participant's mindfulness of sticking to the specific solution to the problem required of them to solve. Some of the respondents' emerged reasons of how they know that they have reached the solution stage when solving a geometry problem are; namely, confirming the solution using other theorems (LA13 & LB6), doing a reverse back calculation (LB7) and that the answer must consist of the properties of what was required to be proved (LA28). Their actual words are as follows:

LA13: Mathematics can be proven in many ways. What I usually do is, I try to check the answer in another way because I believe there are so many ways to solve one problem. So, what I do is, I confirm, let us say I confirm with a lot of things. Let us say we have an exterior angle of a triangle. I try to see that the sum probably, what I am not given is part of the sum. If it adds up to the exterior angle, then I know that definitely I can confirm that it is the sum.

LB6: I always check for like the answer. Maybe in a triangle, if my angles add up to 180^0, then I can be confident that what I did is right. And if the supplementary angles add up to 180^0 in a cyclic quad, then I can be confident that what I did was right. And even with regards to the one with the angle at the centre is twice the angle at the circumference. I also use that to make sure that everything I did was right. And I apply most of the theorems to confirm that my answer was right.

LB7: Depending on the required solution for the given diagram. If let us say, I was using a cyclic quad theorem, I would know that two opposite angles within that diagram are supplementary-add up to 180^0. So, in my understanding, if I let us say, get the first angle I would minus that angle from 180^0 and get the opposite angle. If those two angles when I add them, they give me 180^0, I would know that I have found the solution. So, if it was the theorem that say angle at the centre is twice the angle at the circumference. I would first identify the angle at the centre and then divide it by two, and then it should give me the angle at the circumference. And if I multiply the angle at the circumference by two, it should give me the angle at the centre. You see what I do after finding the solution to the problem, I would reverse the equation going back. If my solution is correct, it is supposed to take me back to the beginning.

LA28: I believe it is when, if they are looking for an angle, then, if I find that angle using given angles or maybe a theorem as my reason, then I know I have reached my solution. And if they are asking me to prove something, my answer has to have the properties or the requirements for that statement to be true.

Conversely, there are some respondents who could not provide relevant responses to the question asked. LA11 simply said, “I know”, without proper support for the knowing. Moreover, LB4 merely narrated, “I do not know how you know”.

LA11: I know when I have found what I am required to prove or what I’m asked to find. If I found it, I already know that this is the correct thing.

LB4: I do not know how you know.

This could be the reason for the failure of some learners to make a link to solve some problems where the majority have no idea where to even start. In some cases, learners begin with the statement they are trying to prove (see Section 2.3.4).

4.5.7 Analysis of learners’ responses to interview item seven

The seventh question of the semi-structured interviews examined the respondent’s understanding of the concept ‘problem-solving ability’. The following format was used to phrase the question:

> In your own understanding, how would you describe a learner who possesses a problem-solving ability?

Most Respondents provided interesting responses except LA13 who’s response was completely off the question that was asked. They seem to have provided their opinions based on how everyone understood what problem-solving ability is. The following are their actual responses.

LB6: Somebody who always reaches solutions that are correct. Somebody who can apply all the theorems properly and find correct values. Someone who can apply the knowledge that they have acquired on a question paper.

LB7: It is a learner who is able to see or calculate solutions to a given problem with true understanding, not just the memorization of theorems or formulas; but then with true understanding of the topic or formula or what is truly happening within the equation. That is a student with a problem-solving ability.

LA28: I think basically they should just be able to identify what is going on-on that rider. And then, they should know which step to take like how to identify those theorems, how you identify angles using those theorems. Yea, I think that is how you know.

LA11: The learner must be able to identify what you are asked, how you are supposed to do it and not being fearful of that question.

LB4: Based on the topic we are talking about right now; it is knowing where to start. Usually, you do not even know which angle you should look at or which shape you should look at. So, you find it very hard to solve. And mostly, also knowing the theorems. Sometimes you mix up the theorems together.

LA13: I would say it is the fundamental skill required by the world, especially in these times. We really need leaders and scientists to lead us to be part of that. And then, we get all about real life solutions, not only for the world. But, also to your family. It needs you, your best ability to think about the problem and find the best solution.

4.6. ASSESSING PROBLEM SOLVING ABILITIES

The researcher designed a problem-solving scoring rubric to evaluate the learner's problem-solving abilities (see Appendix F). To avoid overtasking the teachers with research related arrangements in administering two different tasks, the same Euclidean geometry task that was administered to the learners was used in this regard. However, due to the nature of the study that focussed on the exploration of learners' problem-solving abilities, only Section 2 item number 2.2 of the task was evaluated. The item consisted of cognitive Level Three and Four questions. In cognitive level three questions, learners required conceptual understanding to solve complex problems where there was no obvious route to the solution and cognitive level four questions required learners to apply higher order reasoning and process to integrate their knowledge in linking relevant information to solve the problem. The rubric in Appendix F evaluated the following areas of problem-solving performance:

1. A learner’s ability to understand the problem.
2. A learner’s ability to develop strategies to solve the problem.
3. A learner’s ability to use the selected strategies to solve the problem.
4. A learner’s ability to interpret the answers according to the given problem.

The next subsections show the results of each problem-solving ability indicator and the overall performance of problem-solving ability per school. The results are categorised in Table 4.20.

Table 4.20: Categories of problem-solving ability

Percentage (%)	Problem solver	level
70 – 100	Proficient	High problem-solving ability
40 – 69	Apprentice	Average problem-solving ability
0 – 39	Novice	Low problem-solving ability

4.6.1 A learner’s ability to understand the problem

For learners to solve a Euclidean geometry task, it is anticipated that they should first understand the given problem. Thus, the participants were expected to be able to identify the elements that are known and those asked for. Subsequently, they were evaluated on their ability to identify relevant data and correctly interpret the identified data using appropriate notations/ symbols and terminology. The results in Table 4.21 were obtained.

Table 4.21: Participant’s ability to understand the problem

Ability to understand the problem	High	Average	Low
Number of participants from School A	1 (3%)	19 (58%)	13 (39%)
Number of participants from School B	0 (0%)	15 (50%)	15 (50%)

Table 4.21 shows that the majority of the participants from School A with 58% are in the average category of problem-solving ability, while 39% of the participants are on low problem-solving ability category. Only 1 (3%) of the participants managed to identify relevant data and used appropriate notations and terminology adequately. The results from School B show that 50% of the participants are on low and average category of problem-solving ability, while none of the participant has reached the high level.

4.6.2 A learner’s ability to develop strategies to solve the problem

The problem-solving process require the problem solver to develop problem-solving strategies that will map out possible steps to the solution. Therefore, in this performance area, the participants were evaluated on their ability to identify relevant properties, axioms or theorems that will facilitate the solution process. The results in Table 4.22 were obtained.

Table 4.22: Participant's ability to develop strategies to solve the problem

Ability to develop strategies to solve the problem	High	Average	Low
Number of participants in School A	1 (3%)	19 (58%)	13 (39%)
Number of participants in School B	0 (0%)	11 (37%)	19 (63%)

Table 4.22 shows that most participants maintained their standpoint from the first problem-solving indicator, except that 4 of the participants from School B moved from the average to the low category. This means though the relevant data was identified, yet it was not fully recalled.

4.6.3 A learner's ability to use the selected strategies to solve the problem

In accordance with the objectives of the study to identify forms of knowledge and skills that learners demonstrated when solving tasks in Euclidean geometry, this problem-solving ability indicator evaluated the learner's knowledge and skills to carry out the selected strategies to solve the problem. On this note, the participants had to efficiently link the information obtained to reach the solution stage of the problem. The results in Table 4.23 were obtained.

Table 4.23: Participant's ability to use the selected strategies to solve the problem

Ability to use the selected strategies to solve the problem	High	Average	Low
Number of participants in School A	1 (3%)	10 (30%)	22 (67%)
Number of participants in School B	0 (0%)	8 (27%)	22 (73%)

Table 4.23 shows that approximately two-thirds of the participants in both schools were in the low problem-solving ability category. This entails that most participants used properties of figures randomly without logic or connection or relevance to the solution of the given problem. This is a result of lack of relevant knowledge and skills to facilitate the problem-solving process. While one-third of the participants were on the average problem-solving ability category; there was only one participant (3%) from School A who demonstrated the ability to carry out the plan to reach the solution stage of the problem.

4.6.4 A learner's ability to interpret the answers according to the given problem

The evaluated items of the administered Euclidean geometry task required the participants to interpret the answers according to the given problem. Thus, after providing the correct statements with valid reasons, the participants had to declare their solution and provide a reason for the given solution. For example, in item 2.2.2 after showing the existence of the properties of a cyclic quadrilateral, the participants had to declare with the reason that TLPM is a cyclic quad [Converse $\angle s$ in the same segment or LP subtends = $\angle s$]. The results in Table 4.24 were obtained.

Table 4.24: Participant's ability to interpret the answers according to the given problem

Ability to interpret the answers according to the given problem	High	Average	Low
Number of participants in School A	1 (3%)	13 (39%)	19 (58%)
Number of participants in School B	0 (0%)	9 (30%)	21 (70%)

Table 4.24 shows that most of the participants were found wanting in this performance area. 19 (58%) of the participants from School A and 21 (70%) of the participants from School B were categorised under low problem-solving ability. Most of these participants provided irrelevant statements and reasons without looking back as to whether the problem has been solved or not. No declaration of the solution was provided, nor the reason for the solution. Some of them made unjustified claims based on incorrect statements and reasons. Only one (3%) of the participants from School A demonstrated this ability.

4.6.5 Overall performance of learners' problem-solving ability

To make an overall judgement of the participant's level of problem-solving ability, the total scores from the rubric (Appendix F) of each individual was converted into a percentage and categorised according to Table 4.20. The results in Table 4.25 were obtained.

Table 4.25: Overall performance of the participant's problem-solving ability

Categories		School		Cumulative
Problem solver	**level**	**A** (No. of learners)	**B** (No. of learners)	**Both A and B** (No. of learners)
Proficient	High problem-solving ability	1 (3%)	0 (0%)	1 (1.6%)
Apprentice	Average problem-solving ability	10 (30%)	8 (27%)	18 (28.6%)
Novice	Low problem-solving ability	22 (67%)	22 (73%)	44 (69.8%)

Table 4.25 shows that just above two-thirds of the participants from both schools are novice problem solvers. They function on low problem-solving ability, with 44 (69.8%) of the participants on this level. About one-third of the participants from both schools are apprentice problem solvers. They function on average problem-solving ability, with 18 (28.6%) of the participants on this level. Only 1 (1.6%) of the participants is a proficient problem solver, functioning on high problem-solving ability.

4.7 CONCLUSION

Chapter Four of the study analysed the data and presented the results from the Euclidean geometry task and the semi-structured interviews. The summary of the findings from this chapter is discussed in more details in the next chapter.

CHAPTER FIVE

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In this chapter, a summary of the results is first presented as a composite of both School A and School B. The findings of the study are then discussed and interpreted in terms of the research questions which feature the study's objectives in line with the theoretical framework. The concluding comments are made from the discussion of the findings. Lastly, the study limitations and recommendations conclude the chapter.

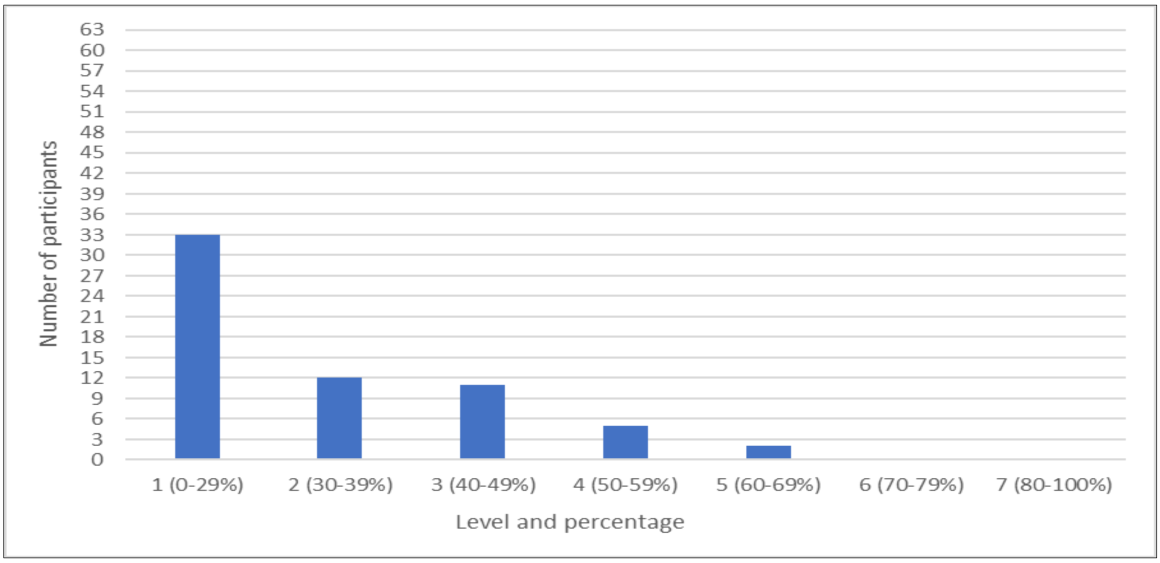
5.2 SUMMARY OF RESULTS

A sample of 63 Grade 11 learners who were enrolled in pure mathematics subject classes was studied (see Table 3.2). The participants were selected from two City, public secondary schools in the Tshwane South district, in Pretoria, Gauteng province in South Africa. The researcher administered a Euclidean geometry task to the participants as a primary data gathering tool. The task was divided into two sections (Section 1 and 2) and had a total mark of 35. To corroborate the results obtained from the task, semi-structured interviews were conducted with six participants which involved three candidates from each school. The results of the data analysis of both schools (School A and School B) were relatively similar. This provided some credibility to the study. Consequently, the researcher referred to the composite results of both schools in the discussion of the findings in this chapter.

The participant's responses were first marked against the prepared marking guideline. A total mark out of 35 was recorded for each learner and converted to a percentage. The results of the participant's statistical performance based of the CAPS grading levels were compiled (see Table 4.3). It emerged that 33 out of 63 participants did not achieve the task. This meant that 52.4% of the participants obtained level one, a percentage that is less than 30%. 12 out of 63 (19%) of the participants attained level two, a percentage between 30 and 39%. 11 out of 63 (17.5%) of the participants attained level three, a percentage between 40 and 49%. 5 out of 63 (7.9%) of the participants attained level four, a percentage between 50 and 59%. While only 2 out of 63 (3.2%) of the participants attained level five, a percentage between 60 and

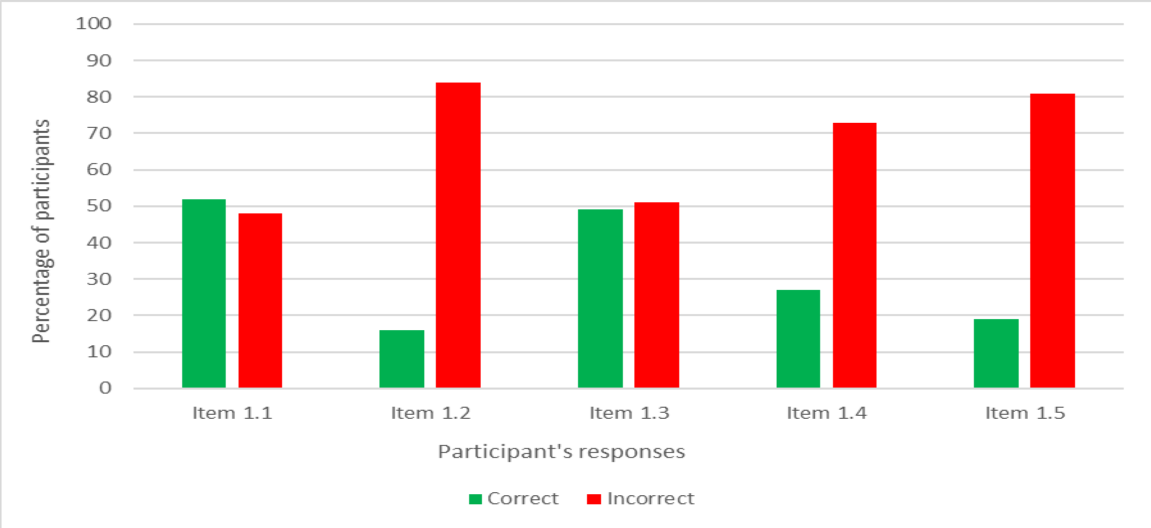
69%. None of the participants attained level six and seven, a percentage between 70 and 100%. A graphic representation of these results is shown in Figure 5.1 below.

Figure 5.1: graphic representation of participant’s grading level performance



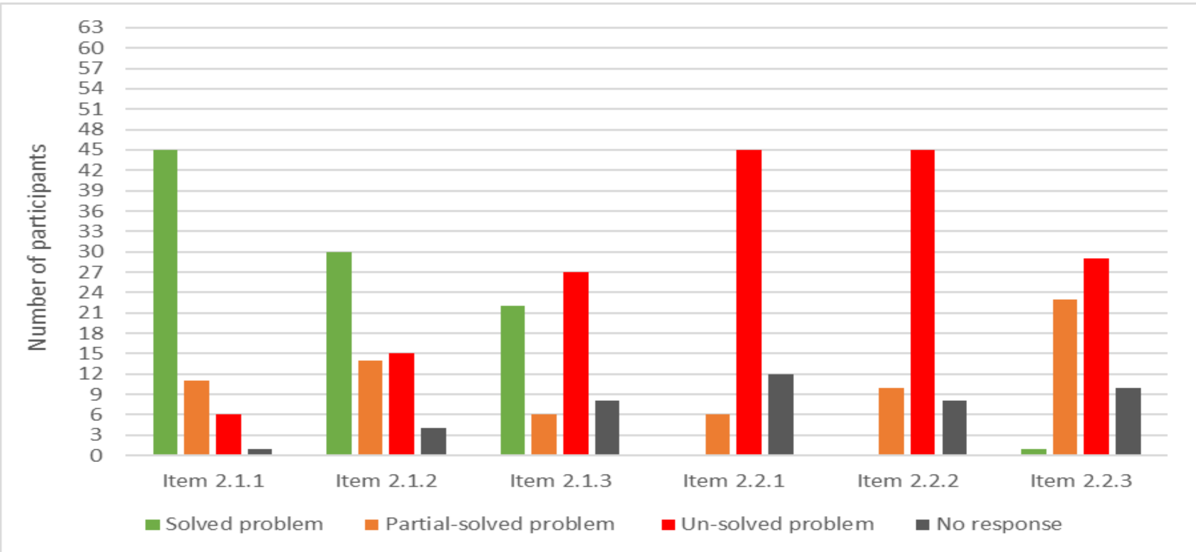
Section one of the participant’s responses to the Euclidean geometry task was analysed by the researcher. The number of participants who selected the correct options in items 1.1 to 1.5 was indicated (see Table 4.4). It emerged that 33 out of 63 (52%) of the participants selected the correct option for item 1.1. This item had the highest number of participants who selected the correct option. Item 1.2 had the least number of participants who selected the correct option with 10 out of 63 (16%) of the participants. Item 1.3 had the second highest number of participants who selected the correct option with 31 out of 63 (49%) of the participants. Item 1.4 had only 17 out of 63 (27%) of the participants who selected the correct option. Item 1.5 had the second least number of participants who selected the correct option with 12 out of 63 (19%) of the participants. Figure 5.2 is an illustration of the percentage of participants who selected the correct options in comparison with the percentage of the participants who selected incorrect options.

Figure 5.2: Graphic representation of participant's responses to Section 1 of the EGT



The researcher also analysed Section Two of the Euclidean geometry task. The formulated classification criteria in Table 4.5 were used to analyse the participant's responses in Table 4.6 and the results were recorded in Table 4.7. It emerged that as the number of items and cognitive levels increased, the number of participants who managed to solve the given problems decreased until the point where none of the participants managed to solve the given problems. An illustration of the composite results of the participant's responses to Section Two of the Euclidean geometry task are shown in Figure 5.3 below:

Figure 5.3: Graphic representation of participant's responses to Section 2 of the EGT



Subsequently, when item-by-item analysis of the participants' responses to the Euclidean geometry task in both Section 1 and 2 was done, the following observations were made regarding the participants' responses to the written task:

1. Assumption of properties based on the visual appearance of a diagram.
2. Incorrect application of geometric properties to solve problems.
3. Random use of theorems without logic, connection, and relevance to the solution of the problem.
4. Never look back to relate their problem solution to the originally given problem.
5. Challenges dealing with proof type questions.

Additionally, Table 5.1 shows the summary of the analysed semi-structured interviews from the previous chapter. Six learners participated in the interviews. The interview questions (in Appendix C) were articulated to address the objectives (interview themes) of the study which are discussed in further detail in Section 5.3.

Table 5.1: Summary of the themes of the respondent's interview questions

Understanding of problem-solving/ content knowledge related questions			
Question	**Understands**	**Average understanding**	**Do not understand**
In your own understanding, what is problem-solving in mathematics?	2 (33%)	4 (67%)	0 (0%)
In your own understanding, how would you describe a learner who possesses a problem-solving ability?	2 (33.3%)	2 (33.3%)	2 (33.3%)
What do you think is the basic knowledge required to solve Euclidean geometry problems in Grade 11?	2 (33%)	3 (50%)	1 (17%)
Study objective 1: Establishing if learners are able to demonstrate understanding of the problem-solving tasks given to them in Euclidean geometry			
Question	**Clear approach**	**Unclear approach**	**No approach**
When solving a Euclidean geometry problem, what is your usual approach (starting point)?	1 (17%)	3 (50%)	2 (33%)
Study objective 2: Examining the mathematical problem-solving strategies, if any, that learners use when solving tasks in Euclidean geometry			

Question	Has a strategy	Strategy not clear	No strategy
What strategies do you normally use when solving Euclidean geometry problems?	1 (17%)	3 (50%)	2 (33%)
Study objective 3: Identifying forms of knowledge and skills that the learners demonstrate when solving tasks in Euclidean geometry			
Question	**Clear knowledge and skill**	**Unclear knowledge and skill**	**No knowledge and skill**
May you explain how you use the strategies that you mentioned in the previous question.	1 (17%)	3 (50%)	2 (33%)
Study objective 4: Determining if learners are able to relate their problem solution to the originally given problem			
Question	**Fully aware**	**aware**	**Not aware**
When solving a geometry problem, how do you know that you have reached the solution stage?	3 (50%)	1 (17%)	2 (33%)

5.3 DISCUSSION OF THE FINDINGS IN TERMS OF THE RESEARCH QUESTIONS

5.3.1 Sub-research question 1:

To discuss the sub-research question, "How do the problem-solving abilities of Grade 11 learners manifest when they solve mathematical tasks in Euclidean geometry?" the researcher unpacked the findings in terms of the study objectives.

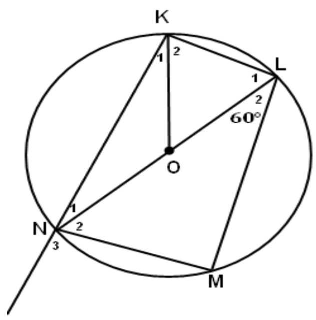
5.3.1.1 Study objective 1: To establish if learners are able to demonstrate understanding of the problem-solving tasks given to them in Euclidean geometry.

The analysis of participants' responses to the written Euclidean geometry task revealed that there were fewer participants who were able to demonstrate understanding of the task given to them and more of those who were unable to demonstrate understanding of the given task. The summary of the results of the participant's responses to Section 1 shown in Figure 5.2 and Section 2 shown in Figure 5.3 testifies to this reality. Given these findings, the researcher went ahead to scrutinise the participant's responses item-by-item and made the following observations:

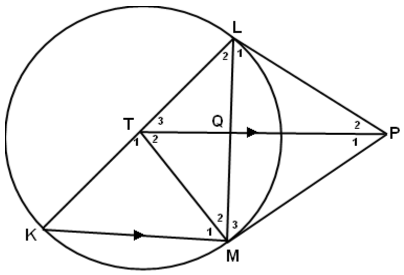
1. Participants based their responses to the Euclidean geometry task questions on the visual appearance of the given diagrams.
2. Some items were not attempted and therefore, no understanding was demonstrated.

Euclidean geometry is one of the branches of mathematics that utilizes representational skills to communicate mathematical problems. Thus, visual representations in the form of diagrams or riders are utilized. Likewise, learners may be required to use their representational abilities to interpret geometric figures or riders. Ideally, they may use symbolic representations (mathematical statements / mathematical notations, numerical / algebraic symbols) and verbal representations (written texts/reasons) to communicate their solutions to the given problems (see Section 2.2.1). Moreover, diagrams are not drawn to scale. This means that a figure is not drawn in its actual size or measure or that the proportions are not necessarily the same. Therefore, specific shapes, sizes of angles, lengths of line segments, parallel lines, perpendicular lines, a centre of a circle cannot be assumed unless stated or proven.

When analysing the Euclidean geometry task written responses of the participants, the researcher found that the error mostly committed by the participants was an erroneous identification of angles as alternate. The error was a result of the participant's incorrect assumption of the existence of parallel lines. This was more evident when responding to item 1.2. In their responses, 46 out of 63 participants selected option D, suggesting that angle N_3 is equal to 90^0. In the diagram alongside, the participants were able to identify angle M as an angle in a semi-circle and determined angle N_2 to be 30^0 from the sum of angles in triangle LNM. But they erroneously identified angle N_1 to be 60^0 because of alternate angles without any stated or proven parallel lines. According to the participants, they believed that line NK and line ML are parallel lines because they are straight lines along the same direction. Moreover, a transverse line is drawn in between them. The researcher observed that in just about every diagram where a Z shape is formed, the participants assumed parallel lines by stating alternate angles.



Another example where the participants made incorrect judgments based on the appearance of a given diagram was evident in item 2.2 questions. Point T was assumed to be a centre in



most of the participants' responses that were categorised under un-solved problem. Participants simply made their assumption based on the appearance of the diagram without any statement or proof that point T is a centre (see diagram alongside). They applied the properties of a circle with a centre as though the centre had already been established.

In terms of the van Hiele model of geometric thought, the identification, naming and grouping of shapes according to their appearance is referred to as visualisation stage, a Level 0 category (see Section 2.3.3). According to CAPS (DBE, 2011) however, learners are anticipated to have surpassed Level 0 ahead of entering the secondary school, to have accomplished Level 2 ahead of entering the Further Education and Training phase and to be operating within Level 3 (formal deduction stage) of the van Hiele levels of reasoning in Grade 11. These results are consistent with Makhubela's (2015) findings in his study conducted to investigate the misconceptions and resulting errors displayed by Grade 11 leaners in the learning of geometry. The author observed that learners often based their responses to questions on the visual appearance of a given diagram. Furthermore, the author found that most Grade 11 learners were operating within Level 1 and 2 of the van Hiele levels of geometric reasoning. On the same note, the diagnostic reports on National Senior Certificate Examination (DBE, 2020, 2019, 2018, 2017, 2016, 2015) noted the very same concerns regarding the matric candidates' responses to the Euclidean geometry questions. An indication is made that candidates incorrectly established the equality of angles that are not necessarily equal, due to an erroneous assumption of properties or theorems which are not applicable in the given diagrams.

Some participants could not demonstrate any understanding of the problem-solving activities given to them in some items of the Euclidean geometry task. They simply made no attempt to provide solutions to some items. These participants who left blank spaces seemed to have lacked confidence, the necessary skills, and know-how to do the task. Table 5.2 shows the number out of 63 participants who made no attempt to provide solutions to the specified items.

Table 5.2: Number of participants who made no attempt to provide solutions per item

	Section 1					Section 2					
Item	1.1	1.2	1.3	1.4	1.5	2.1.1	2.1.2	2.1.3	2.2.1	2.2.2	2.2.3
No. of participants	2	2	0	6	2	1	4	8	12	8	10

Though Section 1 of the Euclidean geometry task consisted of multiple-choice questions, the number of participants indicated on Table 5.2 above did not bother to select any option. Section 2 of the Euclidean geometry task consisted of open-ended questions with item 2.1 based on numeric questions and item 2.2 based on proof type questions. It appears that more participants lacked the know-how to do the proof type questions than other types of questions. During the interviews, when one of the interviewees was asked the question, "When solving a Euclidean geometry problem, what is your usual approach (starting point)?" LB4 unashamedly voiced out her lack of know-how to go about solving a Euclidean geometry problem. The participant simply said, "To tell the truth, I normally do not know what to do." when the researcher made a follow-up, the respondent merely said, "I try looking for theorems I know." This may suggest that when learners lack understanding of geometric concepts or the know-how to do the task, they simply make no attempt to provide the solution to the question.

The ability of learners to understand the problem-solving tasks given to them in Euclidean geometry is largely dependent on their approach to the problems. The interview findings revealed that only 1 out of 6 (17%) of the respondents had a clear approach, while 3 out of 6 (50%) of the respondents could not provide a clear approach, while 2 out of 6 (33%) of the respondents had no approach at all (see Table 5.1). Most participants seem to have committed what Newman in Abdul (2015) coined comprehension errors. According to the author, a comprehension error is an inability of a learner to understand mathematics problems (see Table 2.4).

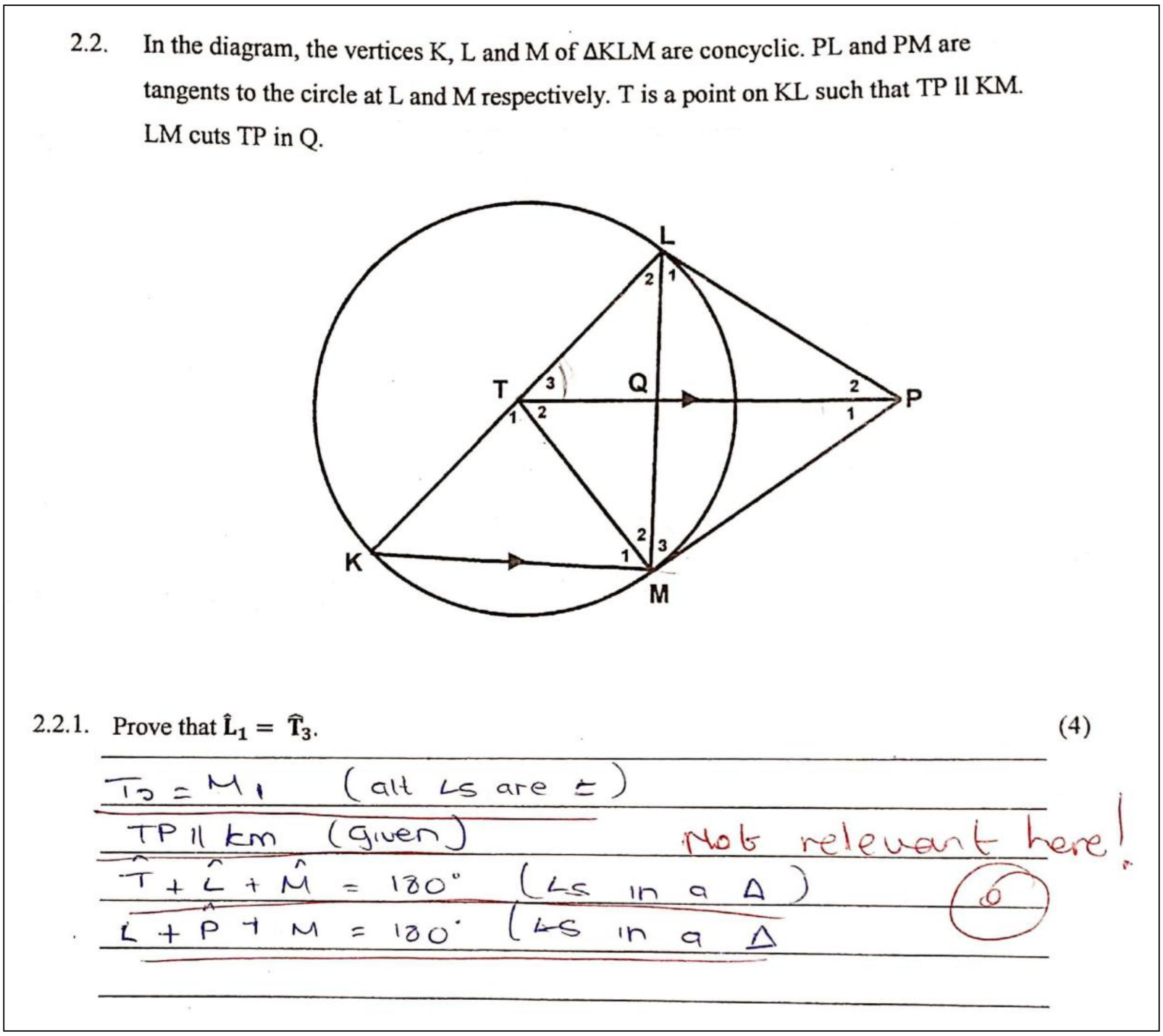
Moreover, the participant's problem-solving abilities were evaluated using the problem-solving scoring rubric (see Section 4.6). When the participant's ability to understand the given problems was evaluated, it emerged that only 1 out of 63 (1.6%) of the participants demonstrated high problem-solving ability, while 34 out of 63 (54%) of the participants demonstrated an average problem-solving ability and 28 out of 63 (44.4%) of the participants demonstrated low problem-solving ability (see Table 4.21).

Given the preceding discussions, it is reasonable to assume that most participants had difficulties understanding the given problem-solving task in Euclidean geometry. Also, evidenced by participants' feedback on the level of difficulty of the written task. There were more participants who felt that the task was difficult than those who felt otherwise. The analysis in Table 4.19 revealed that 51 out of 63 (81%) of the participants felt that the task was difficult to understand and only 12 out of 63 (19%) of the participants felt that the task was fair or easy to understand. A variety of factors could have contributed to these findings, for instance, cognitive processing or cognitive load due to the type of instruction provided to the learners (see the recommendation section).

5.3.1.2 Study objective 2: To examine the mathematical problem-solving strategies, if any, that learners use when solving tasks in Euclidean geometry.

It was earlier established that problem-solving process require the problem solver to develop problem-solving strategies that will map out possible steps to the solution (see Section 2.2.4). On this note, it is in-evitable for the problem solver to determine what is known, asked for and the information required (Kistian & Verawati, 2020). In Euclidean geometry the question for the learner could be, "do you know a theorem that could be useful?". Thereby, the learner is anticipated to identify relevant properties, axioms or theorems that will facilitate the solution process. However, in this study, the researcher discovered that most participants randomly used theorems without logic, connection, and relevance to the solution of the given problems. Given this discovery, there were two categories of presented responses observed by the researcher. (1) Participants knew the geometric concepts, but applied them in incorrect contexts, while others (2) presented illogical statements.

Some of the participants appeared to have been aware of the properties or theorems governing the lines or shapes but applied them in incorrect contexts. This means, they would provide correct statements with valid reasons in accordance with the given diagram, but irrelevant to the immediate question. For instance, the response of LB4 to item 2.2.1 of the Euclidean geometry task displayed in Figure 5.4 shows all correct statements with valid reasons in accordance with the given diagram.



2.2. In the diagram, the vertices K, L and M of ΔKLM are concyclic. PL and PM are tangents to the circle at L and M respectively. T is a point on KL such that TP ll KM. LM cuts TP in Q.

2.2.1. Prove that $\hat{L}_1 = \hat{T}_3$. (4)

$T_2 = M_1$ (alt ∠s are =)

TP ll km (given)

$\hat{T} + \hat{L} + \hat{M} = 180°$ (∠s in a Δ)

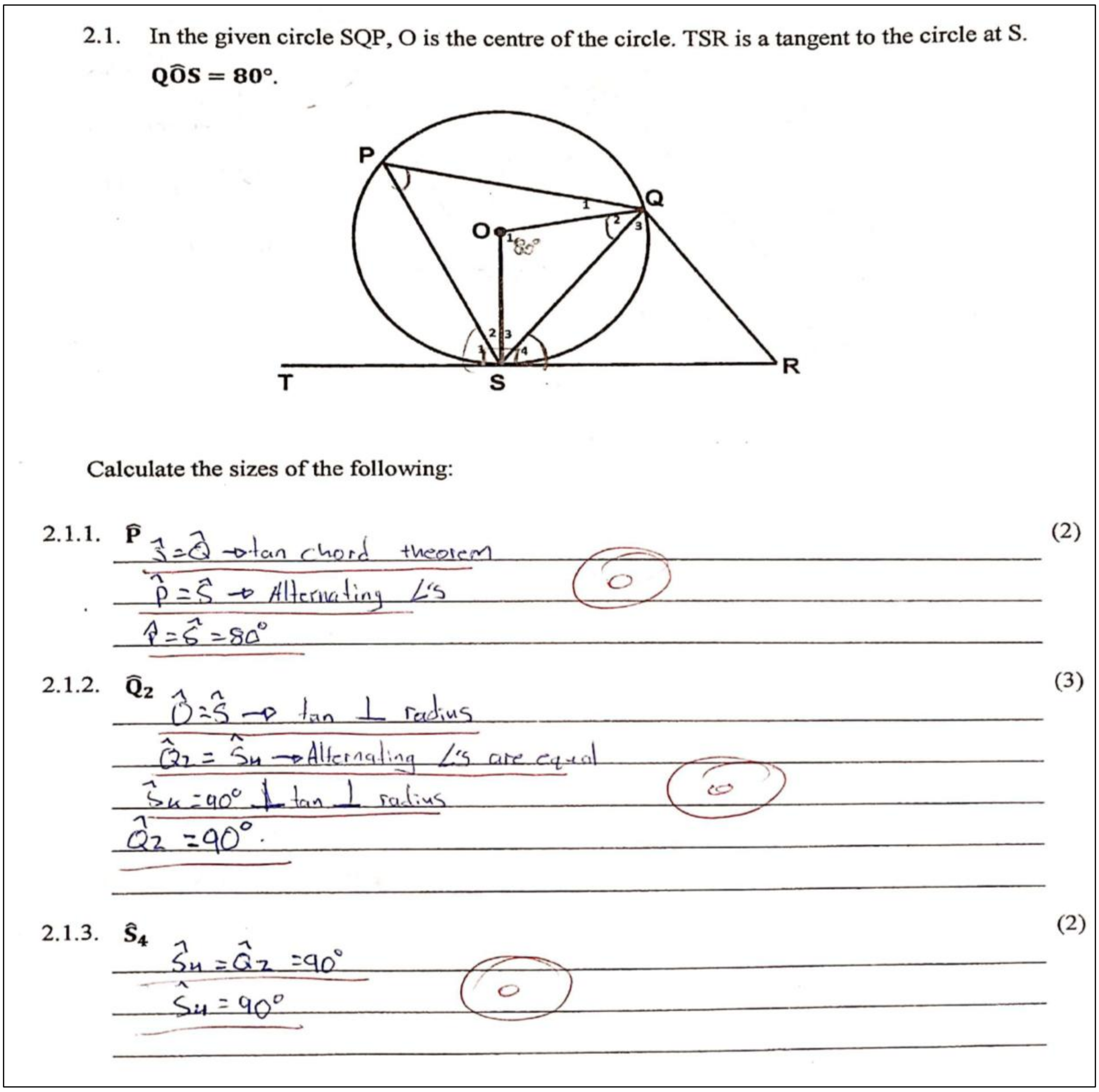
$\hat{L} + \hat{P} + M = 180°$ (∠s in a Δ

Not relevant here!

0

Figure 5.4: LB4's response – an example of irrelevant information

Shown in Figure 5.4, it is true that angle T_2 and angle M_1 are equal because they alternate since line TP and line KM are parallel lines. It is also given that line TP and line KM are parallel lines and that the sum of the interior angles of any triangle is equal to 180^0. However, all these properties were irrelevant in item 2.2.1. Therefore, this category of written responses indicates that this group of participants could not identify relevant properties, axioms, or theorems that could facilitate the solution process even though they had some form of knowledge of geometric concepts. There were many participants who displayed this type of error where they blindly enlisted properties or theorems in incorrect situations to solve geometric problems. This implies that learners might have memorized the theorems and when to apply the information without a conceptual understanding. On the other hand, another category of written responses is characterised with illogical presentation of information. In other words, the presented statements and reasons do not make sense as far as geometry is concerned. For example, see the response of LA8 to item 2.1 on Figure 5.5 below:



2.1. In the given circle SQP, O is the centre of the circle. TSR is a tangent to the circle at S. $Q\hat{O}S = 80°$.

Calculate the sizes of the following:

2.1.1. $\hat{P}$ (2)

$\hat{S} = \hat{Q}$ → tan chord theorem

$\hat{P} = \hat{S}$ → Alternating ∠'s

$\hat{P} = \hat{S} = 80°$

2.1.2. $\hat{Q}_2$ (3)

$\hat{O} = \hat{S}$ → tan ⊥ radius

$\hat{Q}_2 = \hat{S}_4$ → Alternating ∠'s are equal

$\hat{S}_4 = 90°$ ⊥ tan ⊥ radius

$\hat{Q}_2 = 90°$.

2.1.3. $\hat{S}_4$ (2)

$\hat{S}_4 = \hat{Q}_2 = 90°$

$\hat{S}_4 = 90°$

Figure 5.5: LA8's response – an example of illogical information

Figure 5.5 shows that there is no connection between the provided statements with the reasons and the diagram. Not only does the participant display a lack of conceptual understanding of geometric concepts, but it is also clear that the learner could not analyse the diagram. When observing the participant's response to item 2.1.1, the learner could not distinguish between the singularity of angle P and the plurality of angle S and angle Q. It was also not clear how 80^0 was connected to angle S and angle P. This type of geometric reasoning shows the characteristics of a learner who operates either within the pre-recognition level or within the van Hiele Level 0 of geometric thought. It appears that many of them in this category have not yet developed spatial perception. According to Cassim (2006) the learner's lack of spatial sense hinders their ability to visualize how geometric properties detach themselves from the figures. Most of the time, the learner's plan, or strategy to find applicable properties, theorems and proofs involves analysing the given diagram. Unfortunately, most participant's diagrams were clean, no highlights or markings were made, and this made it difficult for them to follow through in the identification of relevant information.

After the administering of the Euclidean geometry task, the researcher was interested in hearing about the participant's voiced-out strategies that they apply whenever they encounter Euclidean geometry tasks. Thus, the sampled interviewees were asked the question, "What strategies do you normally use when solving Euclidean geometry problems?" The interview findings revealed that 2 out of 6 (33%) of the respondents had no strategy at all. LA11 responded, "I think what we are given on the rider is what tells me what to do." And LB6 narrated, "I make sure that I know all the theorems, know how to apply them using different diagrams… Yea, I do that most of the time." However, the question was still not answered. On the other hand, 3 out of 6 (50%) of the participant's voiced-out strategies were not clear. Only 1 out of 6 (17%) of the respondents expressed a clear strategy.

Generally, most of the study participants appeared to have committed what Newman in Abdul (2015) coined transformation errors. According to the author, a transformation error is an inability of a learner to come up with a suitable method of mathematical solution (see Table 2.4). In addition, the participant's problem-solving abilities were evaluated using the problem-solving scoring rubric (see Section 4.6). When the participant's ability to develop strategies to solve the problem was evaluated, it emerged that only 1 out of 63 (1.6%) of the participants demonstrated high problem-solving ability, while 30 out of 63 (47.6%) of the participants demonstrated an average problem-solving ability and 32 out of 63 (50.8%) of the participants demonstrated low problem-solving ability (see Table 4.22).

These findings are consistent with the researcher's observation of learners' responses to Euclidean geometry tasks. These learners' challenges have been observed from Grades 10 to 12 throughout the researcher's five years of mathematics instruction in the FET phase and as a National Senior Certificate Examination marker for two consecutive years. In most cases, the majority of the learners leave out the Euclidean geometry section of the Paper 2 examinations unwritten. Some learners fill the answer sheet with irrelevant information as opposed to what is required of them to write (see Section 1.1).

5.3.1.3 Study objective 3: To identify forms of knowledge and skills that the learners demonstrate when solving tasks in Euclidean geometry

The problem-solving process requires learners to draw on their previous knowledge to develop new problem-solving strategies (Polya,1973) and the use of problem-solving skills in applying these strategies (Son and Ditasona, 2020). When analysing the Euclidean geometry task written responses of the participants to identify forms of learners' knowledge, and skills in carrying out the selected strategies to solve the given problems, the researcher made a concerning discovery. There was a consistency of incorrect application of geometric properties to solve problems demonstrated in several scripts on one item or the other. For instance, learners would state a valid reason but confuse the application of the theorem. In the analysis of item 1.3 and 2.1.1 though some participants reasoned that the angle at the centre of a circle is twice the angle at the circumference of a circle when subtended by the same arc, yet they made the angle at the circumference to be twice the angle at the centre (see Figure 4.9 and Figure 4.15). In these items, the learners displayed an inability to represent properties in values.

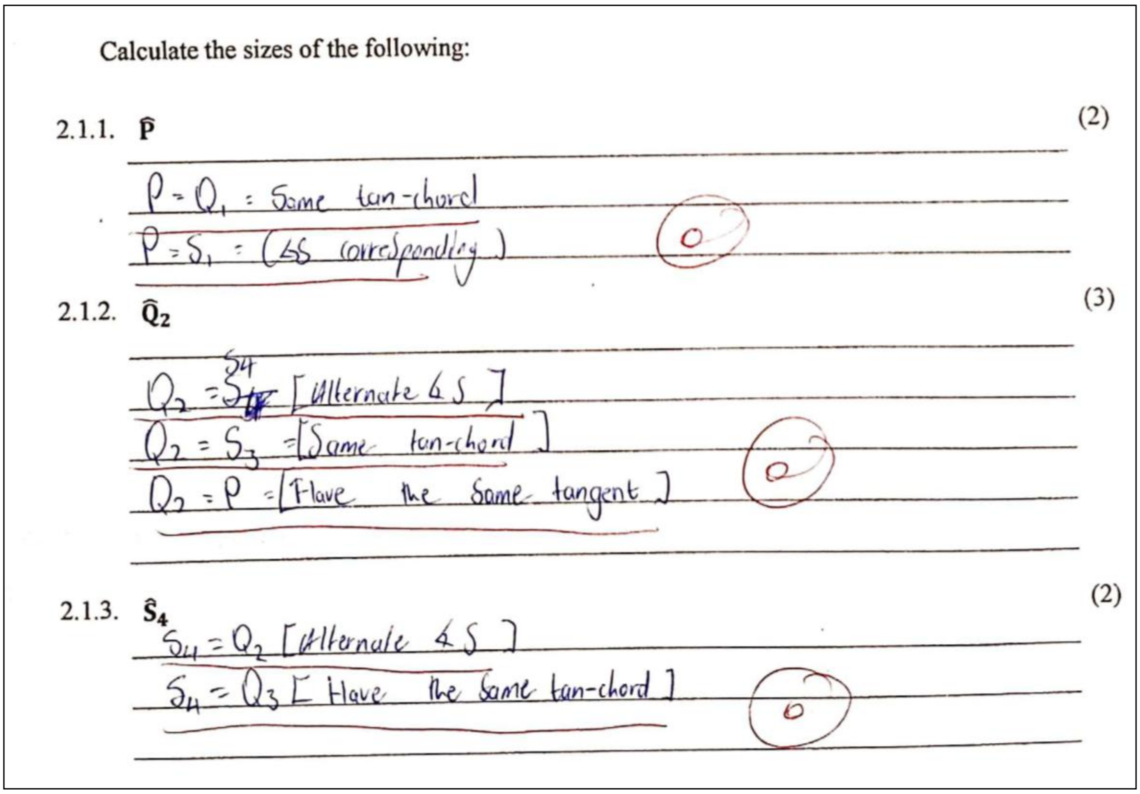
Another example where the majority of participants incorrectly applied geometric properties to solve the given problem was on item 2.2.1. On this item none of the participants from both schools were able to solve the given problem. On the provided diagram there was no direct connection between angle L_1 and angle T_3. Yet, most participants erroneously connected angle L_1 and angle T_3 with an application of a tan-chord theorem. It appeared that in just about every place where learners could not link the information, they forced the solution erroneously. In many cases, there was no logical connection in the presented simplifications. This is an indication of the participant's lack of conceptual understanding of geometric properties. Consequently, the observed findings could be the effects of high cognitive load. It could be that learners may have not developed the appropriate mental schema for geometric concepts.

During the interview session, the researcher posed a follow-up statement requesting the interviewees to explain how they use the strategies that they mentioned in the previous question. On this account, only 1 out of 6 (17%) of the participants demonstrated a clear form of knowledge and skills in carrying out the selected strategies to solve geometric problems, 3 out of 6 (50%) of the participant's knowledge and skills were unclear, and 2 out of 6 (33%) of the participants demonstrated no evidence of knowledge and skills in carrying out the selected strategies to solve geometric problems.

Generally, several study participants seemed to have committed what Newman in Abdul (2015) coined process skill errors. According to the author, a process skill error is an inability of a learner to correctly process the solution to mathematics problems (see Table 2.4). This type of error was mostly evident where the answer required more than one step. Moreover, when the participant's ability to use the selected strategies to solve the given problems were evaluated using the problem-solving scoring rubric (see Section 4.6), it emerged that only 1 out of 63 (1.6%) of the participants demonstrated high problem-solving ability, while 18 out of 63 (28.6%) of the participants demonstrated an average problem-solving ability and 44 out of 63 (69.8%) of the participants demonstrated low problem-solving ability (see Table 4.23). Two-thirds of the participants were found to be in the low problem-solving ability category. This is a result of lack of relevant knowledge and skills to facilitate the problem-solving process.

5.3.1.4 Study objective 4: To determine if learners are able to relate their problem solution to the originally given problem.

The findings of the researcher in item-by-item analysis revealed that most participants never looked back to relate their problem solutions to the originally given problems. For instance, items in Section 2.1 of the Euclidean geometry task required learners to determine the sizes of the angles in question. This meant that the participant's responses were expected to be presented in numeric values with reasons. However, one-third of the participants did not hit to the instruction. Meaning, though the participants provided responses to the questions asked, their responses did not contain any numeric values (sizes of angles). The response of LB15 shown in Figure 5.6 is one of such examples.



Calculate the sizes of the following:

2.1.1. $\hat{P}$ (2)

P = Q1 = Same tan-chord

P = S1 = (∠s corresponding)

2.1.2. $\hat{Q}_2$ (3)

Q2 = S4 [Alternate ∠s]

Q2 = S3 = [Same tan-chord]

Q2 = P = [Have the Same tangent]

2.1.3. $\hat{S}_4$ (2)

S4 = Q2 [Alternate ∠s]

S4 = Q3 [Have the Same tan-chord]

Figure 5.6: LB15's response

Though others did present their responses in numeric values, some responses were not necessarily correct. Participants had to use the given size of an angle to correctly link it to other angles using relevant theorems to perform the calculations. Another example of such an error is majorly evident in the learner's responses to item 2.2.2. The participants were instructed to prove that TLPM on the given diagram is a cyclic quadrilateral. However, no participant presented any correct information (see Table 4.7). Moreover, very few participants provided a conclusion following their statements and reasons indicating that "therefore, TLPM is a cyclic quadrilateral" with a valid reason. This entails that most learners could not interpret their results according to the problem of origin. Figure 5.7 below show an example of LA24's response to item 2.2.2.

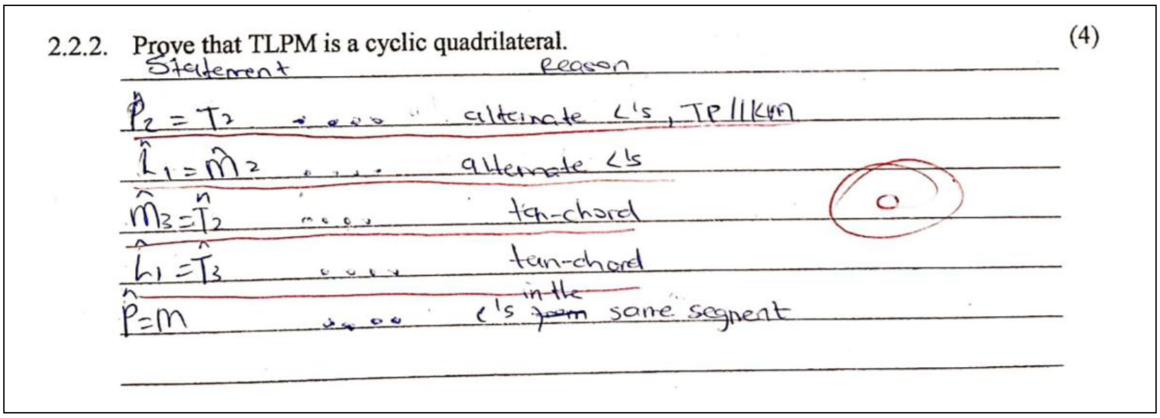


Figure 5.7: LA24's response

During the interview session, the researcher posed the question, "When solving a geometry problem, how do you know that you have reached the solution stage?" in response to the question, 3 out of 6 (50%) of the participants demonstrated full awareness, while 1 out of 6 (17%) of the participants did demonstrate some form of awareness. Some of the respondent's emerged reasons of how they know that they have reached the solution stage when solving a geometry problem are; namely, confirming the solution using other theorems (LA13 & LB6), doing a reverse back calculation (LB7) and that the answer must consist of the properties of what was required to be proved (LA28). However, what was said was inconsistent with the written responses. on the other hand, 2 out of 6 (33%) of the participants demonstrated no awareness at all. LA11 simply said, "I know", without proper support for the knowing. While LB4 merely recounted, "I do not know how you know".

According to Newman in Abdul (2015), several learners committed encoding errors (see Table 2.4). The author describes encoding error as an inability of a learner to interpret the solution according to the given question. In addition, when the participant's ability to interpret the answers according to the given problems were evaluated using the problem-solving scoring rubric (see Section 4.6), it emerged that only 1 out of 63 (1.6%) of the participants demonstrated high problem-solving ability, while 22 out of 63 (34.9%) of the participants demonstrated an average problem-solving ability and 40 out of 63 (63.5%) of the participants demonstrated low problem-solving ability (see Table 4.24). Approximately two-thirds of the participants demonstrated low problem-solving ability. Most of these participants provided irrelevant statements and reasons without looking back as to whether the problem has been solved or not. No declaration of the solution was provided, nor the reason for the solution. Some participants made unjustified claims based on incorrect statements and reasons.

This is a concerning picture of how the problem-solving abilities of Grade 11 learners manifested when they solved the administered Euclidean geometry task. The findings of this study are very consistent with the realities of what is happening in some or most of the South African school classrooms regarding the Euclidean geometry topic in the mathematics subject. The same errors and misconceptions revealed in the current study were reported in the past 5 years in the diagnostic reports on National Senior Certificate Examination (DBE, 2020, 2019, 2018, 2017, 2016, 2015) (see section 2.3.4).

5.3.2 Sub-research question 2: To what extent do learners apply the knowledge and skills required to solve Euclidean geometry tasks?

To answer this question, the discussion of the findings focused on the analysis of the learner's responses to the Euclidean geometry task against the cognitive levels of the items and the existing van Hiele levels of geometric thought.

The summary of the results of Section 1 of the Euclidean geometry task shown in Figure 5.2 revealed that the majority of the participants were more comfortable responding to cognitive Level 1 and 2 questions than they were when responding to cognitive Level 3 and 4 questions. This is because there are more participants who obtained the correct answers in the first two cognitive levels than there are in the last two cognitive levels (see Figure 5.2. and Table 3.4). Similarly in Section 2 of the Euclidean geometry task, we see that there are $\pm$50% of the participants who managed to respond correctly to items under cognitive Level 2 questions and none of the participants succeeded to solve the problems under cognitive Level 3 questions, while only 1.6% of the participants managed to solve a cognitive level four question (see Figure 5.3 and Table 3.4). There was a consistency in the performance of learners in Section 1 and Section 2 in terms of cognitive levels. However, most learners did not show their workings or calculations in Section 1, the multiple-choice questions section of the Euclidean geometry task. Most of them simply circled what they thought could be the correct answer. Consequently, further analysis of learners' responses could not be carried out in most participant's scripts for this section, hence this was discussed in the limitation of the study section. Subsequently, the discussion of the findings in this section is more focused on Section 2 of the Euclidean geometry task.

Section 2 of the Euclidean geometry task consisted of open-ended questions which involved cognitive levels 2 to 4 questions (see Section 3.5.2.1). The cognitive Level 2 questions in Section 2 of the Euclidean geometry task involved numeric questions where the learners had to calculate the sizes of angles. In these cognitive Level 2 questions learners were required to use well known procedures, simple applications and calculations which involved one or few steps. On the other hand, cognitive Level 3 and 4 questions were mainly proof type questions. In cognitive Level 3 questions learners required conceptual understanding to solve problems involving complex or multistep calculations where there was no obvious route to the solution. Cognitive Level 4 questions required learners to apply higher order reasoning and process to integrate their knowledge or break the problem into pieces to identify what must be solved (DBE, 2011). This is intended to facilitate the development and improvement of learners' problem-solving and cognitive skills (DBE, 2011). However, the Grade 11 participants in the current study were found wanting in the area of problem-solving skills.

Participant's challenges in dealing with proof type questions was one of the concerning observations made by the researcher when doing item-by-item analysis of the participant's written responses to the Euclidean geometry task. Item 2.2 questions were all proof type questions, and all the sampled learners could not solve any of the questions, except for LA17 (see Table 4.7 and Section 4.4.4.7). Most of the errors that learners committed have already been discussed in the previous section. However, some challenges were specific to proof type questions. The researcher's findings involve a lack of logic and connections evident in most of the participants' responses. This means that most participants could not apply deductive reasoning and process to integrate their knowledge in linking relevant information to solve the problems. Some of them had no idea where to start, hence the resultant blank spaces left out. In some cases, the learners began with the statement they were trying to prove, while others applied the properties of what they were trying to prove as though they already existed. The latter is shown where learners applied properties of a cyclic quadrilateral in their attempt to prove that a quadrilateral is cyclic (see Section 4.4.4.7.2). When it comes to follow-up questions, the participants failed to recognise the existence of the properties of the proven elements from the previous item(s) (see Section 4.4.4.7.3). Moreover, some made unjustified conclusions based on assumptions or incorrect reasons. Given this background, it is reasonable to conclude that all participants with the exception of LA17, have not yet developed logic and competency in dealing with proof type questions.

These results are consistent with the findings stated in the diagnostic reports on National Senior Certificate Examination (DBE, 2020, 2019, 2018, 2017, 2016, 2015) and the findings of other researchers who expressed the same concerns with learners' difficulties in dealing with proof type questions in Euclidean geometry (Makhubela, 2015; Ngirishi 2015; Cassim, 2006) (see Section 2.3.4). According to the van Hiele model of thinking, the ability to construct and understand proofs is linked to Level 3. Level 3 of the van Hiele model of thinking is called a formal deduction stage. Learners at this level develop proofs using axioms and definitions; where they identify the problem, pose the exploration questions and construct proofs based on their understanding from the informal deduction stage (Usiskin, 1982; Crowley, 1987; van de Walle et al., 2013). At this level, learners work with abstract statements about geometric properties and make conclusions based more on logic than intuition (Makhubela, 2015).

Unfortunately, the literature reveals that learners are less capable of reaching this level of high reasoning, creative thinking, and information because they have not gained mastery at the lower levels of the van Hiele model of geometric thought, (i.e., the visualization, analysis, and informal deduction levels) (Ngirishi, 2015; Makhubela, 2015; Alex & Mammen, 2014). For instance, Alex and Mammen (2014) conducted a study with the Grade 10 mathematics learners using the van Hiele model of thinking. The authors found that most study participants were operating within Level 0 of the van Hiele model. Ngirishi (2015) explored the Grade 10 and 11 learners' understanding on basic geometric concepts using the van Hiele model of thinking. The findings of the study revealed that most participants were operating at Level 0 and 1 with a few participants able to reason at Level 2 of the van Hiele model. Makhubela (2015) in his study conducted to investigate the misconceptions and resulting errors displayed by Grade 11 leaners in the learning of geometry found that most participants were operation within Level 1 and 2 of the van Hiele levels of geometric reasoning.

In the current study, learners in Grade 11 were anticipated to operate within Level 3 (formal deduction stage) and working towards Level 4 (rigor stage) of the van Hiele levels of reasoning (DBE, 2011). However, only 1 out of 63 (1.6%) of the participants were found to be operating within Level 3 of the van Hiele model, while 62 out of 63 (98.4%) of the participants were operation below Level 3 of the van Hiele model. This means most of the study participants were operating below the set standard in geometry (DBE, 2011). The results are proof that learners are progressed to the FET phase with lack of geometric conceptual understanding, knowledge and experience from the lower grades that should serve as a building block for higher order reasoning and creative thinking. Thus, Euclidean geometry poses a threat to most learners. They fail to cope with higher thinking processes in Grade 11 and 12 because of the lack of geometric knowledge and understanding that should have been acquired in the lower grade levels (Masilo, 2018). Unfortunately, the challenges persists because Euclidean geometry instruction in Grade 11 and 12 is pitched at a very formal abstract deduction level despite the content gaps experienced by learners (Alex & Mammen, 2014). This could make the development process of learners' mathematical problem-solving abilities to be slow and inadequate (Kistian & Verawati, 2020).

5.3.3. **Primary Research Question:** What is the level of problem-solving ability at which the Grade 11 learners are operating when solving mathematical tasks in Euclidean geometry?

The participant's problem-solving abilities were evaluated by the researcher (see Section 4.6). A problem-solving ability scoring rubric was utilised to evaluate the participant's responses to the written Euclidean geometry task (see Appendix F). Four problem-solving indicators were used to analyse and interpret the learners' problem-solving abilities that guided them to solve the related problem-solving task (see Section 3.8.1 and Section 4.6). The findings of each problem-solving indicator have already been discussed in detail in response to sub-research question 1 presented in Section 5.3.1. However, this section focused on the discussion of the overall performance of the learner's problem-solving ability scores (see Table 4.25). Figure 5.8 below is a graphic representation of the summary (as a percentage) of the participant's levels of problem-solving ability in terms of low, average, or high ability that they operated within when solving the analysed Euclidean geometry task.

Figure 5.8: graphic illustration of participant's levels of problem-solving ability

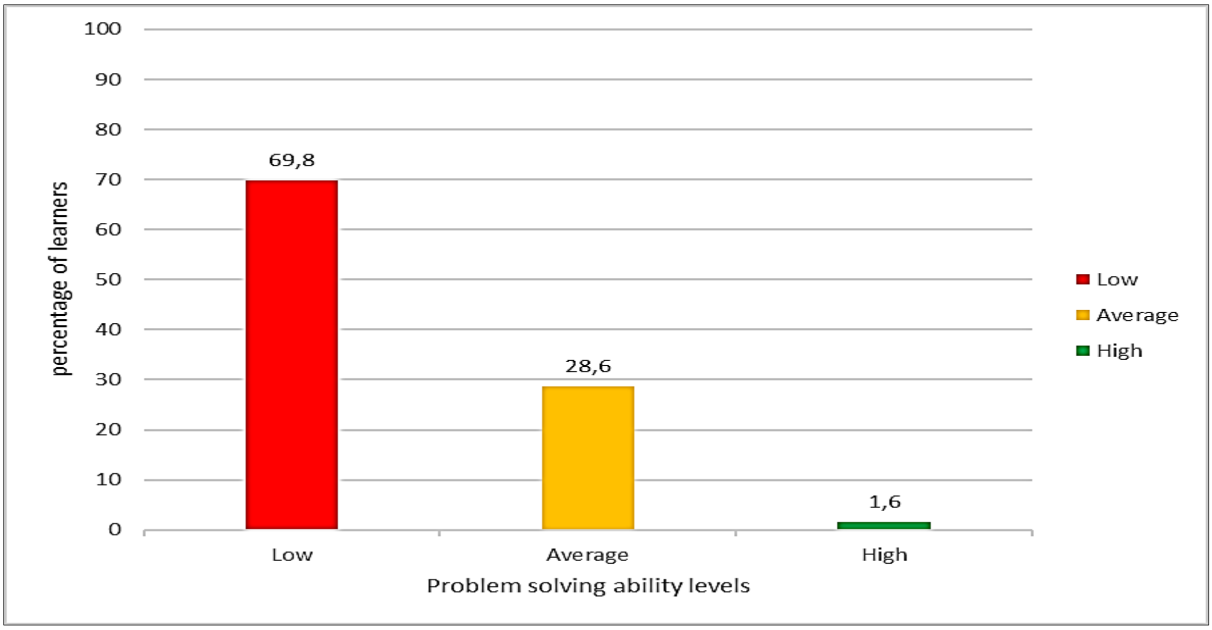


Figure 5.7 shows that 69.8% of the participants were operating within the low level of problem-solving ability, while 28.6% of the participants operated at an average level of problem-solving

ability, whereas only 1.6% of the participants possessed high level of problem-solving ability. This means most study participants were found to be novice problem solvers. They had little or no experience or skill in problem-solving. Many of them had not yet acquired or developed problem-solving abilities. Given the summary of the learner's performance in Euclidean geometry task shown in Figures 5.1 to 5.3, it is reasonable to think that a learner's ability to solve Euclidean geometry problems is influenced by the level of ability in terms of low, average, or high ability that one possesses.

5.4. CONCLUSIONS

The purpose of the study was to explore the Grade 11 learners' problem-solving abilities that are manifest when they solve Euclidean geometry tasks and to evaluate the levels of problem-solving ability in terms of low, average, and high they operate within. In addition, the researcher was able to identify the learner's misconceptions and errors which largely showed themselves in the learner's responses. In this regard, the study objectives were fulfilled, and the primary research question was answered. Therefore, the purpose of the study was achieved. This was hoped that the findings will serve as a reference for teachers and curriculum developers in improving the teaching and learning of Euclidean geometry, thereby contributing towards the improvement of learners' problem-solving skills in Euclidean geometry and mathematics performance entirely.

5.5. LIMITATIONS

1. The study focused on learners only without examining the teachers' pedagogy. It is the researcher's belief that teachers' pedagogy may have influenced the nature of learners' acquired problem-solving abilities. Hence this study would not have managed to study the problem holistically, which could have generated more representative and comprehensive findings towards the stated problem.

2. The Euclidean geometry task included the multiple-choice questions section. However, most learners did not show their workings or calculations in Section 1, the multiple-choice questions section of the Euclidean geometry task. Most of them simply circled what they thought could be the correct answer. Consequently, further analysis of learners' responses could not be carried out in most participant's scripts for this section, hence this was considered to be a limitation for the study.

5.6. RECOMMENDATIONS

The recommendation of the current research emphasises the significance of prioritizing the acquisition and development of learners' problem-solving abilities. The researcher advocates for a strong emphasis on producing skilful problem solvers among mathematics learners, who possess advanced abilities in this critical cognitive domain. To achieve this goal, particularly with respect to Euclidean geometry, the researcher recommends the following points, among others (though not an exhaustive list):

1. Instructional designers or teachers are encouraged to be knowledgeable and consider students' structure of information or cognitive architecture when developing instruction (see Section 2.3.1.2).
2. Teachers are recommended to teach according to the van Hiele model of geometric thought (see Section 2.3.3)
3. Teachers could expose learners to Polya's model of problem-solving (see Section 2.2.2 and Section 2.4.1).
4. Teachers could expose learners to questions across all cognitive levels, such as questions and proof-type queries from cognitive level 3 and 4.
5. Teachers are encouraged to emphasise the following points when teaching Euclidean geometry:
   a) It should be communicated to the learners not to make assumptions based on the visual appearance of the diagram. They should be constantly reminded that diagrams are not drawn to scale. For instance, two lines in the same plane with equal distance apart and which never meet cannot be said to be parallel unless it is indicated on them or stated, or it can be proved that they are parallel. Moreover, it is important that parallel lines be cited in the reason when mentioning corresponding, alternate, and co-interior angles.
   b) The correct terminology should be used during the instruction and class discussions. The meaning of the terms such as tangent, chord, cyclic quadrilateral, etc, should be frequently voiced out so that the learners will be able to use them correctly. In addition, the correct application of the theorems should be demonstrated. This could be done by ensuring that the explanations of the theorems are supplemented by showing the relationship in the diagram.
   c) Learners should be made aware that they will not be simply awarded the correct marks for randomly stating the correct statement and the correct reason, but that the statement and the reason should be related to the question asked or should lead to the solution

of the problem at hand. Therefore, learners should be encouraged to read the given information and analyse the diagram to find applicable properties, theorems and proves. Then use the identified properties, theorems and proves to link only the information that is relevant to what is required.

d) There must be an insistence on learners to always provide a reason for every statement they make. Furthermore, the learners must be encouraged to always verify their solutions to see if the solution relates to the originally given problem.

REFERENCES

Abakah, F. (2019). Exploring mathematics learners' problem-solving skills in circle geometry in South African schools: a case study of a high school in the northern cape province. Unpublished master's thesis, University of South Africa.

Abdul, A.H. (2015). Analysis of problem solving in mathematics using theory of Newman. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(6), 1395-1406.

Akdeniz, C., Bacanli, H., Baysen, E., Cakmak. M., Celikoz, N., Dogruer, N. & Eristi, B. (2016). Learning and teaching theories, approaches, and models. (1st ed.). Ankara, Turkey: Cozum Egitim Yayincilik.

Alex, J.K., & Mammen, K.J. (2014). An assessment of the readiness of Grade 10 students for geometry in the context of curriculum and assessment policy statement (CAPS) expectation. *International Journal of Educational Sciences*, *7*(1), 29-39.

Alex, J.K., & Mammen, K.J. (2018). Student's understanding of geometry terminology through the lens of van Hiele theory. *Open Access Journal of Education*, 39(1). https://pythagoras.org.za/index.php/pythagoras/article/view/376/619

Anderson, L.W., & Krathwohl, D.R. (2001). A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives. Boston, MA: Allyn & Bacon.

Anderson, T., & Dron, J. (2011). Three generations of distance education pedagogy. *The International Review of Research in Open and Distributed Learning*, *12*(3), 80-97.

Anggraini, R.S., & Fauzan, A. (2020). The effect of realistic mathematics education approach on mathematical problem-solving ability. *Journal of Mathematics Education*, *3*(2), 94-102.

Artzt, A.F., & Armour-Thomas, E. (2007). Mathematics teaching as problem-solving: A framework for studying teacher metacognition underlying instructional practice in mathematics. *Instructional Science*, *26*(1-2), 5-25.

Atkinson, R.C., & Shiffrin, R.M. (1968). Human memory: A proposed system and its control processes. In K.W. Spence (Ed.), The Psychology of Learning and Motivation: Advances in Research and Theory (Vol. 2, pp. 89-105). New York, NY: Academic Press.

Ay, Y. (2017). A review of research on the misconceptions in mathematics education. Unpublished BA Thesis, Ege University.

Bandura, A. (1989). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review*, *84*(2), 191-215.

Bankov, K. (2013). Teaching of geometry in Bulgaria*. European Journal of Science and Mathematics Education*, *1*(3), 158. http://www.studymode.com/essay/Geometry-In-Everyday-Life-408843.html

Basson, J., Odendaal, J., Gouws, H., & Nieuwoudt, L. (2020). Mind Action Series Mathematics: Grade 12 Textbook. Sanlamhof, Allcopy Publishers.

Bleeker, C. (2011). The relationship between teachers' instructional practices and learners' levels of geometry thinking. Unpublished MEd Thesis, University of Pretoria.

Bouchrika, I. (2021). What is information processing theory? Stages, Models & Limitations. https://research.com/education/what-is-information-processing-theory.

Bransford, J.D., & Stein, B.S. (1993). The IDEAL problem solver (2nd ed.). New York, NY: Freeman.

Brown, D. (2019). Cognitive Learning Theory. https://www.edapp.com/blog/cognitive-learning-theory/.

Cassim, I. (2006). An exploratory study into Grade 12 learners' understanding of Euclidean Geometry with special emphasis on cyclic quadrilateral and tangent theorems. Unpublished MSc Thesis, University of Witwatersrand.

Chusinkunawut, K., Nugultham, K., Wannagatesiri, T., & Fakcharoenphol, W. (2018). Problem-solving ability assessment based on design for secondary school students. *International Journal of Innovation in Science and Mathematics Education*, *26*(3), 1-20.

Claybourne, A. (2012). Life processes. London, England: Raintree Publishers. https://books.google.com/books/about/Life_Processes.html

Cohen, D. & Crabtree, B. (2006). Qualitative Research Guideline Project. Robert Wood Johnson Foundation, Princeton.

Cohen, L., & Manion, L. (1994). Research methods in education (4th ed.). London, England: Routledge.

Cohen, L., Manion, L., & Morrison, K. (2007). Research methods in education (6th ed.). London, England: Routledge.

Connelly, L.M. (2016). Trustworthiness in qualitative research. *MedSurg Nursing*, *25*(6), 435-436.

Cowan, N. (2008). What are the differences between long-term, short-term, and working memory? *Prog Brain Res.*, 169, 323-338. https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2657600/.

Creswell, J.W. (1994). Research design: Qualitative and quantitative approaches. Thousand Oaks, CA: Sage Publications.

Creswell, J.W. (2003). Research design: Qualitative, quantitative, and mixed methods approaches (2nd ed.). Thousand Oaks, CA: Sage Publications.

Crowley, M.L. (1987). The van Hiele Model of the Development of Geometric Thought. In Mary Montgomery Lindquist (Ed.), Learning and Teaching Geometry. Chicago.

Curry, L.A. & Creswell, J.W. (2013). Achieving integration in mixed methods designs: Principles and practices. *Health Services Research*. https://doi.org/10.1111/1475.6773.12117.

Da Zhou, Du, X., Hau, Luo, H., Feng, P. & Liu, J. (2019) Teacher-student relationship and mathematical problem-solving ability: mediating roles of self-efficacy and mathematical anxiety. *Educational Psychology*, 40(4), 473-489.

Dahar, R.W. (1989). Learning theories. Jakarta: Erlangga.

Daulay, K. R. & Ruhaimah, I. (2019). Polya theory to improve problem-solving skills. *Journal of Physics*, IOP Publishing.

De Villiers, M.D. (1996). The future of secondary school geometry. Slightly adapted version of Plenary presented at the SOSI Geometry Imperfect Conference, 2-4 October, 1996. Pretoria: UNISA.

Department of Basic Education. (2011). Curriculum and Assessment Policy Statement (CAPS): Mathematics. Pretoria: Government printer.

Department of Basic Education. (2011). National Policy Pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grade R-12. Pretoria: Government printer.

Department of Basic Education. (2014). Annual National Assessment. National Diagnostic Report. Pretoria: Government Printer.

Department of Basic Education. (2015; 2016; 2017; 2018; 2019 & 2020). National Senior Certificate Examination: Diagnostic report. Pretoria: Government Printer.

Department of Basic Education. (2019). National Senior Certificate: School subject report. Pretoria: Government Printer.

Department of Education (2006). The National Policy Framework for Teacher Education and Development in South Africa. "More teachers; Better teachers". Pretoria: Government printers.

Department of Education. (2003). National curriculum statement: General. Pretoria. Government printer.

Department of Education. (2006). National Senior Certificate: Mathematics. Pretoria: Government printer.

Dhlamini, J.J. (2012). Investigating the effect of context-based problem-solving instruction on the performance of learners. Unpublished doctoral thesis. University of South Africa.

Elaine, J.H. (2014). What is STEM education? Beta live science. *Science and Technology***,** 2016, *2*(6), 251-256.

Engelbrecht, J.C.V. & Potgieter, M. (2005). Undergraduate students' performance and confidence in procedural and conceptual mathematics. *International Journal of Mathematical Education*. https://doi.org/1080/00207390500271107

Ertmer, P.A. & Newby, T.J. (2013). Behaviourism, cognitivism, constructivism: Comparing critical features from an instructional design perspective. *Performance Improvement Quarterly*, *26*(2), 43-71.

Euclidean geometry. www.britannica.com

Éva Fülöp, (2021). Developing problem-solving abilities by learning problem-solving strategies: An exploration of teaching intervention in authentic mathematics classes. *Scandinavian Journal of Educational Research.*

Fard, H.D. (2012). Research paradigms in public administration. *International Journal of Humanities*, *19*(4), 55-108.

Fazlıogulları , O. (2012). Scientific research paradigms in social sciences. *International Journal of Educational Policies*, *6*(1), 41-55.

Fraenkel, J.R., & Wallen N.E. (2009). How to design and evaluate research in education (7th Ed.). McGraw-Hill: New York.

Fuadi, I., Minarni, A., & Banjarnahor, H. (2017). Analysis of students' mathematical problem solving ability in IX grade at Junior High School Ar-Rahman Percut. *International Journal of Novel Research in Education and Learning*, *4*(2), 153-159.

Gauteng Department of Education. (2003). Examiner's report: Mathematics-Higher grade and Standard grade. Johannesburg: Assessment and Monitoring Unit.

Gauteng Department of Education. (2021). School Based Assessment for Grade 11 Mathematics. Pretoria, Tshwane South District.

Gordon, M. (2009). Towards a pragmatic discourse of constructivism: Reflections on lessons from practice. Educational Studies, 45, 39-58.

Gray, A. (2016). The 10 skills you need to thrive in the Forth Industrial Revolution. World Economic Forum. https://www.weforum.org/agenda/2016/01/the-10-skills-you-need-to-thrive-in-the-fourth-industrial-revolution/

Grider, C. (1993). Foundations of cognitive theory: A concise review. San Francisco, CA: Jossey Bass Inc.

Guba, E.G., & Lincoln, Y.S. (1994). Competing paradigms in qualitative research. In Denzin, N.K., & Lincoln, Y.S. (Eds.), Handbook of qualitative research (pp. 105-117). Sage Publications, Inc.

Gupta, U. & Zheng, R. Z. (2020). Cognitive load in solving mathematics problems: Validating the role of motivation and the interaction among prior knowledge, worked examples, and task difficulty. *European Journal of STEM Education*, *5*(1), 05.

Gweshe, L.G. (2015). The effect of using a Computer Assisted Instruction on teaching Circle Geometry in Grade 11. Unpublished masters' dissertation. University of South Africa.

Harper, D. (2010). Online Etymology Dictionary. http://dictionary.refernce.com/cite.html?qh=geometry&ia=etymon.

Huitt, W. (2003). Classroom instruction. Educational Psychology interactive. Valdosta, GA: Valdosta State University. http://www.edpsycinteractive.org/topics/instruct/instruct.html

In'am, A. (2016). Euclidean geometry's problem-solving based on metacognitive in aspect of awareness. Mathematics Education, 11(4), 961-974. Retrieved from http://creativecommons.org

Instrumentation. www.slideshare.net

International Association for the Evaluation of Education Achievement (IEA) (2007). *International Mathematics and Science Study*. http://timss.bc.edu/timss2007i/TIMSSPDF/BSItems.pdf

Jeroen, J.G., van Merrienboer, & Sweller, J. (2005). Cognitive load theory and complex learning: Recent developments and future directions. *Educational Psychology Review*, *17*(2), 147-178.

Kistian, A., & Verawati, (2020). The effect of problem based learning (pbl) learning models on mathematic problem solving ability students in primary school. *Budapest International Research and Critics Institute-Journal*, *3*(3), 2615-307.

Krauss, S.E. (2005). Research paradigms and meaning making: A primer. *The Qualitative Report*, *10*(4), 758-770.

Krulik, S. & Rudnick, J.A. (1980). *Problem Solving: A Handbook for Teachers*. Newton, MA : Allyn & Bacon.

Kulm, G. (1994). Mathematics assessment: What works in the classroom. San Francisco, CA: Jossey Bass Inc.

Leendertz, V., Blignaut, A.S., Nieuwoudt, H.D., Els, C.J. & Ellis S.M (2013). Technological pedagogical content knowledge in South African mathematics classrooms: A secondary analysis of SITES 2006 data. *Pythagoras*, *34*(2), 96-112. http://dx.doi.org/10.4102/Pythagoras.v34i2.232

Lincoln, Y.S., & Guba, E.G. (1985). Naturalistic inquiry. Beverly Hills, CA: Sage.

Louridas, S.E, & Rassias, M.T (2013). Problem-solving and selected topics in Euclidean geometry. New York: Springer.

Luneta, K. (2008). Error discourse in Fundamental Physics and Mathematics: Perspectives of students' misconceptions. In International Council on Education for Teaching (ICET) 2008 Yearbook (pp. 386-400).

Lynch, M. (2021). The working memory and short-term memory. The EdAdvocate. https://www.theedadadvocate.org/the-working-memory-and-short-term-memory/

Makhubele, Y. E. (2015) Misconceptions and resulting errors displayed by Grade 11 learners in the learning of geometry. Unpublished MEd Thesis, University of Johannesburg.

Mamali, N. R. (2015). Enhancement of learners' performance in geometry at secondary schools in the Vhembe district of the Limpopo province. Unpublished MEd Thesis, University of Venda.

Maree, J., & Pietersen, J. (2007). Surveys and the use of questionnaires. In K. Maree (Ed.), First Steps in Research. Pretoria: van Schaik.

Maree, K. (Ed.). (2007). First steps in Research. Pretoria: van Schaik Publishers.

Masilo, M.M. (2018). 'Implementing inquiry- based learning to enhance Grade 11 students' problem-solving skills in Euclidean geometry.' Unpublished dissertation, University of South Africa.

Mateya, M. (2008). 'Using the van Hiele theory to analyse geometrical conceptualisation in Grade 12 students: a Namibian perspective'. Unpublished MEd Thesis, Rhodes University.

McGinnis, D. (2020). What is the Fourth Industrial Revolution? https://www.salesforce.com/blog/what-is-the-fourth-industrial-revolution-4ir/

McLeod, S.A. (2018). Task-Based Worksheet. https://www.simplypsychology.org

Mcmillan, J.H. & Schumacher, S. (2010). Research in education: Evidence-based inquiry. 8th edition. International edition. Boston: Pearson Education Inc.

McNeese, B. (2021). Problem-solving model. https://www.spcforexcel.com/knowledge/process-improvement/problem-solving-model

Merriam, S. (1991). Case study research in education: A qualitative approach. San Francisco, Jossey-Bass Publishers.

Merriam, S.B (1988) Qualitative research and case study applications in education. San Francisco, CA: Jossey-Bass.

Mestre, J. (1987). Hispanic and Anglo Students' Misconceptions in Mathematics. Eric Clearinghouse on Rural Education and Small Schools Charleston WV, 1989-03-00.

Michael, J. (2001). The significance of misconceptions for teaching and learning. Paper Presented in the 2001-2002 SOTL Schedule of Events, Indiana University. http://www.indiana.edu/sotl/eventsdescr.html

Michela, E. (2018). Cognitivism. In R. Kimmons, The Students' Guide to Learning Design and Research. EdTech Books. https://edtechbooks.org/studentguide/cognitivism

Miller, G. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *The Psychological Review*, *63*, 81-97.

Misconception. www.encarta.com

Moon, K., Brewer, T.D., Januchowski-Hartley, S.R., Adams, V.M. & Blackman, D.A. (2016). A guideline to improve qualitative social science publishing in ecology and conservation journals. *Ecology and Society*, *21*(3), 17-35.

Mosia, M.S. (2016). Improving Teachers Technological Pedagogical Content Knowledge for Teaching Euclidean Geometry Using Integrated Information Communication Technologies Software. Published Master's thesis. University of Free State.

Mphuthi, G.T. (2020). Exploring teachers' mathematical meanings for teaching trigonometric ratios and functions to Grade 10 learners in the Tshwane South District of Gauteng Province. Unpublished Masters' thesis. University of South Africa.

Mullis, I.V.S., Martin, M. O., Foy, P., & Arora, A. (2019). TIMSS 2019 international results in mathematics. TIMSS & PIRLS International Study Centre.

National Council of Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA.

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA.

National Council of Teachers of Mathematics. (2021). Problem-solving. https://www.nctm.org/Research-and-Advocacy/Research-Brief-and-Clips/Problem-Solving/

Ngirishi, H. (2015). An exploration of FET mathematics learners' understanding of geometry. Unpublished MEd Thesis, University of KwaZulu-Natal.

Ngirishi, H., & Bansilal, S. (2019). An exploration of high school learners' understanding of geometric concepts. *Problems of Education in the 21st Century*, *77*(1), 82-96. https://doi.org/10.33225/pec/19.77.82

Nieuwenhuis, J. (2007). Research designs and data gathering techniques. In An educators guide to school management skills. Edited by I. van Deventer & A.G. Kruger. Pretoria: Kagiso.

NSW Department of Education. (2017). Cognitive load theory: Research that teachers really need to understand. Centre for Education Statistics and Evaluation, literature review.

Paas, F., Renkl, A., & Sweller, J (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist*, *38*(1), 1-4.

Paas, F., van Gog, & Sweller, J. (2010). Cognitive load theory: New conceptualizations, specifications, and integrated research perspectives. *Educational Psychology Review*, *22*(1), 115-121.

Pasaribu, K.A. & Suyanto, W. (2020). The effect of STEM-based (Science, Technology, Engineering, and Mathematics) learning model toward the students' mathematical problem-solving ability in SD Muhammadiyah Condongcatur, Yogyakarta. *Journal of Physics*, *9*(2), 256-301.

Pasaribu, R.T. (2021). Efforts to Improve Student's Mathematical problem-solving abilities by Application of the STAD Type Cooperative Learning Model, in E-Learning / Online Learning During the Covid-19 Pandemic. Mathematics Education, State University of Medan.

Peranginangin, S.A. & Surya, E. (2017). An Analysis of Students' Mathematics Problem Solving Ability in VII Grade at SMP Negeri 4 Pancurbatu*. International Journal of Sciences: Basic and Applied Research*, *33*(2), 57-67.

Phillips, M.D., Basson, J. & Botha, C. (2012). Mind Action Series Mathematics: Grade 11 Textbook and workbook. Sanlamhof, Allcopy Publishers.

Polya, G. (1945). How to solve it: A new aspect of mathematical method. Princeton, NJ, US.

Polya, G. (1957). How to solve it: A new aspect of mathematical method. Princeton University Press.

Polya, G. (1973). How to Solve It, 2nd Edition. Princeton University Press, United States.

Polya, G. (1981). Mathematical discovery: On understanding, learning, and teaching problem-solving. New York, NY: Wiley.

Putri, W.K.H.W., & Prabawanto, S. (2019). The analysis of students' self-efficacy in learning mathematics. *Journal of Physics*: *Conference Series*.

Rittle-Johnson, B. & Schneider, M. (2014). Developing conceptual and procedural knowledge of mathematics. In R. Cohen Cadosh & A. Dowker (Eds.), Oxford handbook of numerical cognition. Oxford University Press.

Roh, K.H. (2003). Problem-Based Learning in Mathematics. Eric Digest. https://eric.ed.gov/?id=ED482725

Rosli, R., Goldsby, D. & Capraro, M. M. (2013). Assessing Students' Mathematical Problem-Solving and Problem-Posing Skills. *Asian Social Science*, *9*(16), 54-60.

Ruliani, I.D., Nizaruddin, & Murtianto, Y. H. (2018). Profile Analysis of Mathematical Problem Solving Abilities with Krulik & Rudnick Stages Judging from Medium Visual Representation. *Journal on Mathematics Education*, *7*(1), 22-29. http://journal.unipma.ac.id/index.php/jipm

Ruseffendi. (2006). Introduction to help mentor develop competence in mathematics. Bandung: Tarsito.

Saman, S. & Chin, K.E. (2016). A Model for Mathematics Problem Solving. Faculty of Psychology and Education, University Malaysia Sabah.

Schoenfeld, A.H. (1985). Making sense of "out loud" problem-solving protocols. *Journal of Mathematical Behaviour*, *4*, 171-191.

Schunk, D.H. (2012). Learning Theories: An Educational Perspective. 6th Edition. Boston, MA: Allyn & Bacon.

Schwab, K. (2016). The Fourth Industrial Revolution. Global agenda. Cologny, Switzerland: World Economic Forum. https://www.weforum.org/about/the-fourth-industrial-revolution-by-klaus-schwab

Scotland, J. (2012). Exploring the philosophical underpinning of research: relating ontology, epistemology to the methodology and methods of the scientific, interpretive, and critical research paradigms. English language Teaching, *5*(9).

Serin, H. (2018). Perspectives on the teaching of Geometry: Teaching and Learning Methods. *Journal of Education and Training*, *5*(1), 45-52. http://dx.doi.org/10.5296/jet.vsil.xxx

Simamora, R.E., Saragih, S. & Hasratuddin, (2019). Improving Students' Mathematical Problem Solving Ability and Self-Efficacy through Guided Discovery Learning in Local Culture Context. *International Electronic Journal of Mathematics Education*, *14*(1), 61-72.

Simon, M.A. (2017). Explicating mathematical concept and mathematical conception as theoretical constructs. *Educational Studies in Mathematics*, *94*(2), 36-43.

Siregar, N. (2017). Problem Solving Ability Of Students Mathematics In Problem Based Learning. *Journal of Educational Science and Technology*, *3*(3), 185- 189.

Smith, J.P., Disessa, A.A. & Roschelle, J. (1993). Misconceptions Reconceived: A constructivist analysis of knowledge in transition. *The Journal of Learning Sciences*, *3*(2), 115-163.

Son A.L, & Fatimah, S. (2020) Students' Mathematical Problem-Solving Ability Based On Teaching Models intervention and Cognitive Style. *Journal on Mathematics Education*, *11*(2), 209-222.

Son, A.L. & Ditasona C. (2020). CORE RME learning model on improving students' Mathematical problem-solving ability. *Journal of Physics*, *10*(2), 39-50.

Song, M. (2011). Effects of background context and signaling on comprehension recall and cognitive load: The perspective of cognitive load theory. PhD thesis. Lincoln: University of Nebraska.

South African government. (2021). Disaster Management Act: Regulations to address, prevent and combat the spread of coronavirus COVID-19: Adjusted alert Level 1: Amendment. Government notices.

Steps on Solving Euclidean Geometry Problems. www.theanswer.co.za

Stramel, J. (2020). Mathematics Methods for Early childhood. Great River Learning. Dubuque, IA.

Sutrisno, Rahayuningsih, D. & Purwati, (2020). The impact of cognitive style-based learning models on students' problem-solving abilities. *Journal of Education and Teacher Training*, *5*(2), 297-306.

Swan, B., & Dixon, J. (2006). The effects of mentor-supported technology professional development on middle school mathematics teachers' attitudes and practice. *Contemporary Issues in Technology and Teacher Education*, *6*(1), 67-86.

Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction*, *4*(1), 293-312.

Syaiful, Kamid, Muslim, & Huda, N. (2020). Identifying of problem-solving abilities in Mathematics among Junior High School students. *Journal of Education and Learning*, *14*(2), http://journal.uad.ac.id/index.php/EduLearn

Usiskin, Z. (1982). van Hiele levels and achievement in secondary school geometry. Final report of CDASSG-report. Chicago: University of Chicago.

Usiskin, Z. (1987). Resolving the continuing dilemmas in school geometry. In Lindquist, M.M & Shulte, A.P (Eds), Learning and Teaching geometry, K-12. Reston, VA: NCTM.

van de Walle, J. A., Karp, K. S., Bay-Williams, J. M., & Wray, J. (2013). Elementary and middle school mathematics: Teaching developmentally. 8th edition. Boston, Pearson.

van der Berg, S. (2015). What the Annual National Assessment can tell us about learning deficits over the education system and the school career. *South African Journal of Childhood Education*, *5*(2), 29-43.

van Merrienboer, J. J. G., Kirschner, P. A., & Kester, L. (2003). Taking the load off a learner's mind: Instructional design for complex learning. *Educational Psychologist*, *38*(1), 5-13.

Wahab, W.S., Saragih, S., & Siman, (2020). Development of Learning Tools Based on Realistic Mathematical Approaches to Improve Mathematical Problem Solving Ability of Students in Class V SD Angkasa 2 Lanud Soewondo Medan. Budapest International Research and Critics in Linguistics and Education (BirLE) Journal, 3(1), 365-376.

Wahyuni, D. (2012). The research design maze: Understanding paradigms, cases, methods and methodologies. *Journal of Applied Management Accounting Research*, *10*(1), 69-80.

Wang, Y., & Chiew, V. (2010), On the cognitive process of human problem solving, *Cognitive Systems Research: An International Journal*, *11*(1), 81-92.

Wesslen, L., & Maria, F. (2005). Transformation geometry. *Journal for Mathematics Teaching*, *191*, 27-29.

Western Cape Education Department. (2013). Background to the national quintile system. Media release. www.online.westercape.gov.za

Wiggins, G. (1990). The case for authentic assessment. Retrieved from ERIC database (ED328611).

Willis, J.W. (2007). Foundations of Qualitative Research, Thousand Oaks, CA: Sage.

Yilmaz, K. (2011). The cognitive perspective on learning: its theoretical underpinnings and implications for classroom practices. *European Journal of Education*, *84*(1), 204-212.

Yilmaz, K. (2013). Comparison of quantitative and qualitative research traditions: Epistemological, theoretical, and methodological differences. *European Journal of Education*, *48*(2).

Yin, R.Y. (1984). Case study research: Design and Methods. Newbury Park, CA: Sage.

Zanzali, N.A.A., & Nam, L.L. (2020). Evaluating the levels of problem-solving abilities in mathematics. Faculty of Education, Malaysia University of Technology.

APPENDICES SECTION

APPENDIX A: EUCLIDEAN GEOMETRY TASK

MATHEMATICS TASK

GRADE 11

ALLOCATED MARKS: 35 **TIME: 1 Hour**

Name of learner (Surname first)	
Name of school	
Grade/Class	
Age (in years)	
Gender (Male or Female)	
Date	

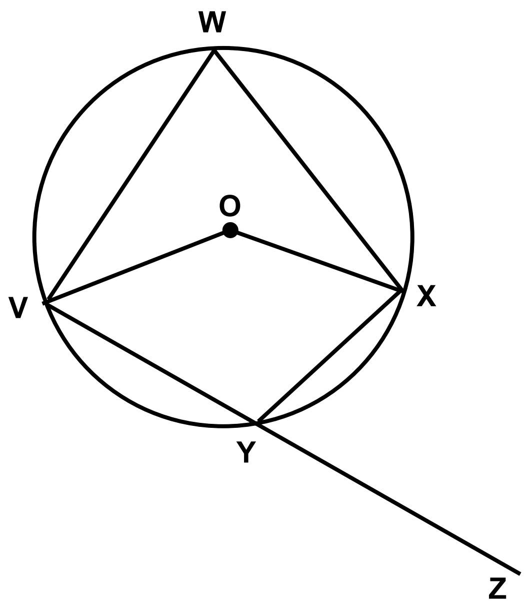
INSTRUCTIONS:

1. This task consists of 8 pages, and it is divided into two sections:
1. Section 1: Multiple choice questions.
2. Section 2: Open-ended questions.
3. Answer all the questions from both Section 1 and Section 2 on the space provided on this question paper.
4. Read each question carefully.
5. Write neatly and legibly.

SECTION 1

- **Each question consists of 4 options. There is only one correct answer. Decide upon the answer you think is correct and CIRCLE the letter corresponding to your answer.**
- **Kindly use the blank space next to the question to work out your answer.**

1.1. VWXY is a cyclic quadrilateral. O is the centre of the circle. Line VY is produced to point Z, outside the circle. (3)



From this diagram, one can prove that $\widehat{W} = X\widehat{Y}Z$. What would you conclude from this proof?

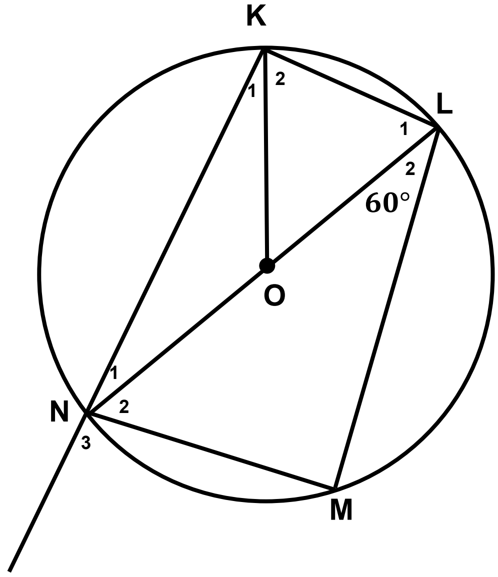
A. Only in this cyclic quadrilateral can we be sure that $\widehat{W} = X\widehat{Y}Z$.

B. Given any cyclic quadrilateral VWXY with VY produced to Z, then $\widehat{W} = X\widehat{Y}Z$.

C. Only when the quadrilateral VWXY looks like a kite can we be sure that $\widehat{W} = X\widehat{Y}Z$.

D. Given any quadrilateral, VWXY with VY produced to Z, then $\widehat{W} = X\widehat{Y}Z$.

1.2. KLMN is a cyclic quadrilateral of a circle with centre O. Line OK bisects $N\hat{K}L$ and NL is a diameter. $\hat{L}_2 = 60°$. Therefore, $\hat{N}_3 =$……….. (3)



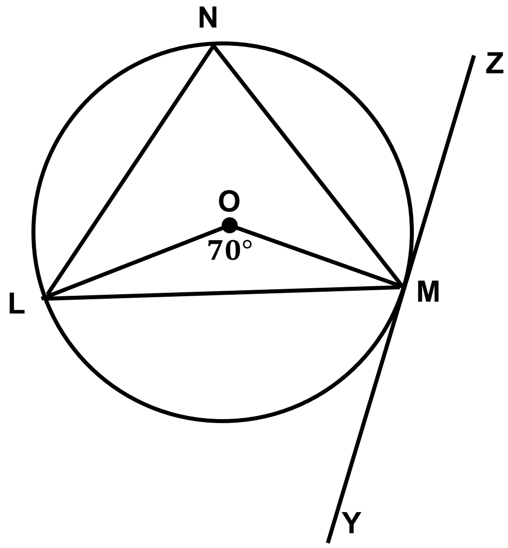
A. 105°

B. 100°

C. 95°

D. 90°

1.3. In the given diagram, O is the centre of the circle. Points L, M and N lie on the circle and YMZ is a tangent to the circle at M. $L\hat{O}M = 70°$ and $Y\hat{M}L =$……… (3)



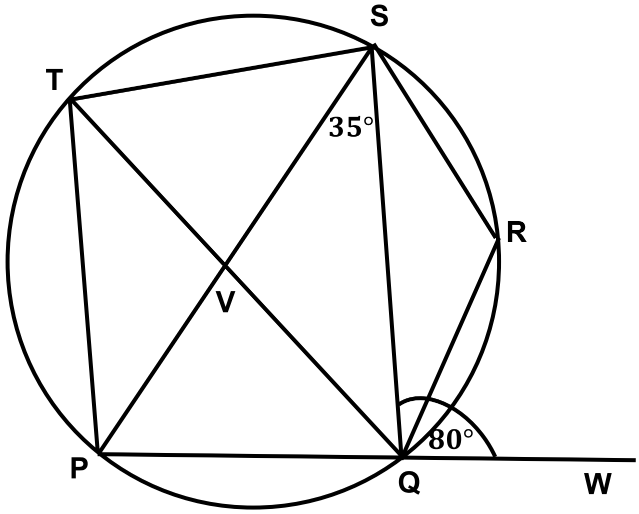
A. $110°$

B. $140°$

C. $90°$

D. $35°$

1.4. Points P, Q, R, S, and T lie on the circumference of a circle. $S\hat{Q}W = 80°$ and $P\hat{S}Q = 35°$. The size of $S\hat{R}Q =$………… (3)



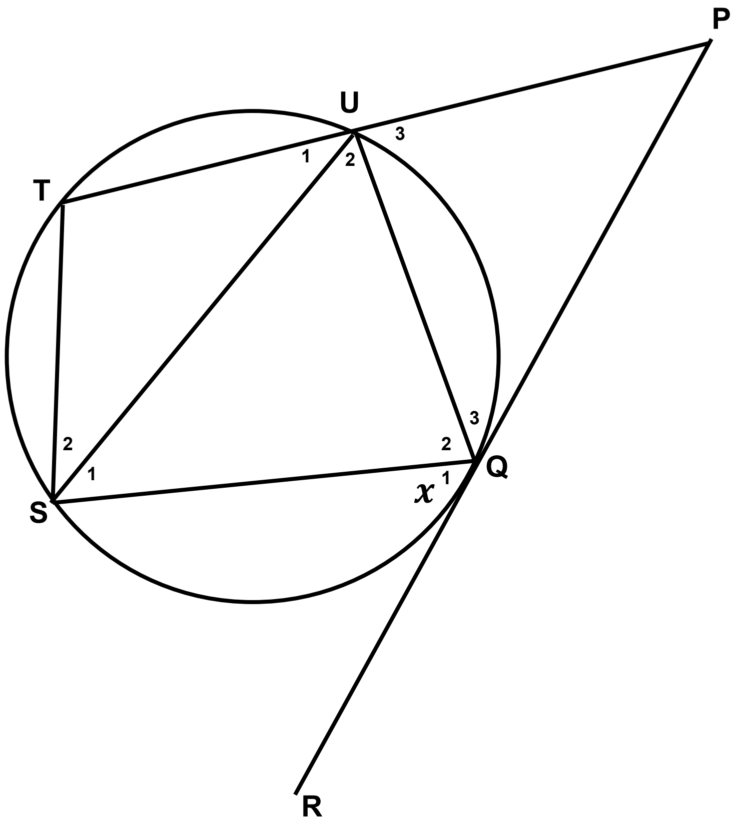
A. $125°$

B. $100°$

C. $135°$

D. $45°$

1.5. In the given diagram, QUTS is a cyclic quadrilateral. PQR is a tangent to the circle at Q. TS = TU; SU = SQ and TP ∥ SQ. If $S\hat{Q}R = x$, which angle in the given options is **not** equal to x? (3)



A. $\hat{P}$

B. $\hat{Q}_2$

C. $\hat{U}_1$

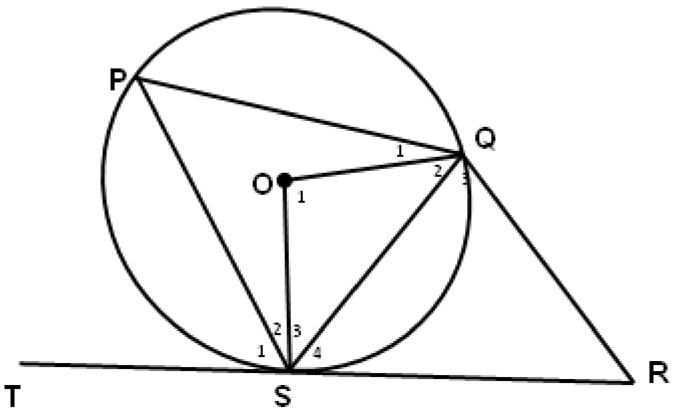
D. $\hat{S}_1 + \hat{S}_2$

TOTAL SECTION* 1: 15 *MARKS

SECTION 2

- **Show all your calculations on the space provided.**
- **Give reasons for your statements and calculations.**

2.1. In the given circle SQP, O is the centre of the circle. TSR is a tangent to the circle at S. $\boldsymbol{Q\hat{O}S = 80°}$.



Calculate the sizes of the following:

2.1.1. $\hat{\mathbf{P}}$ (2)

____

____

____

2.1.2. $\hat{\mathbf{Q}}_2$ (3)

____

____

____

____

____

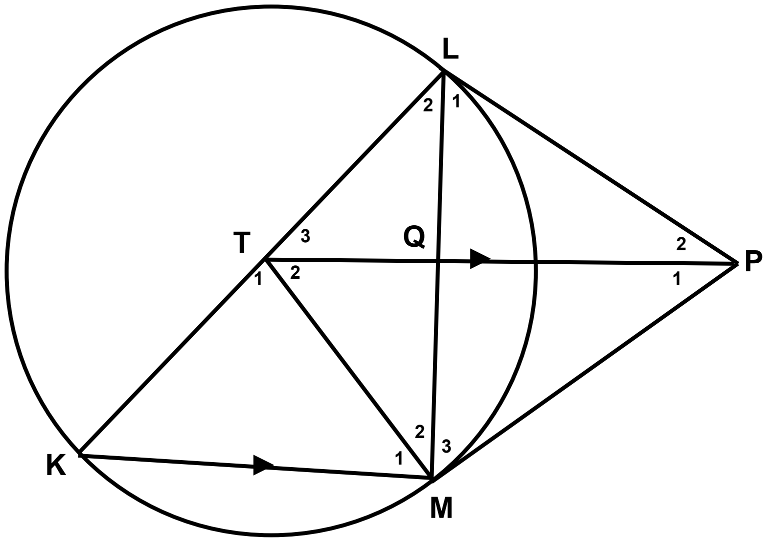
2.1.3. $\hat{\mathbf{S}}_4$ (2)

____

____

____

2.2. In the diagram, the vertices K, L and M of ΔKLM are concyclic. PL and PM are tangents to the circle at L and M, respectively. T is a point on KL such that TP ll KM. LM cuts TP in Q.



2.2.1. Prove that $\hat{\mathbf{L}}_1 = \hat{\mathbf{T}}_3$. (4)

2.2.2. Prove that TLPM is a cyclic quadrilateral. (4)

2.2.3. Prove that TK = TM (5)

TOTAL SECTION 2: 20 MARKS

TOTAL: 35 MARKS

APPENDIX B: EUCLIDEAN GEOMETRY TASK MARKING GUIDELINE

MATHEMATICS TASK

GRADE 11

Allocated Marks: 35

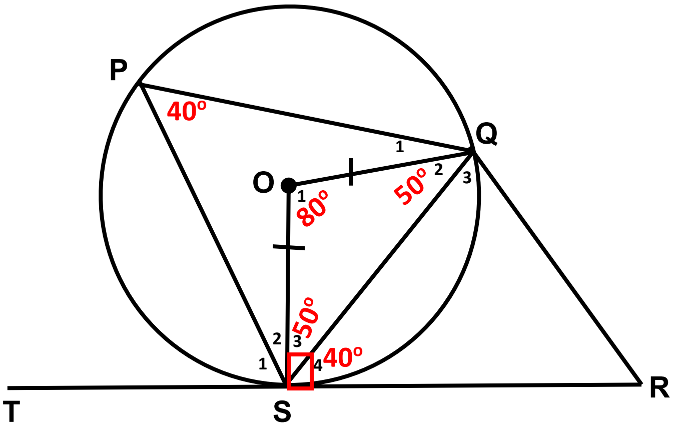
MARKING GUIDELINE

SECTION 1: Multiple choice questions

1.1.	**B**	**(3)**
1.2.	**A**	**(3)**
1.3.	**D**	**(3)**
1.4.	**C**	**(3)**
1.5.	**C**	**(3)**

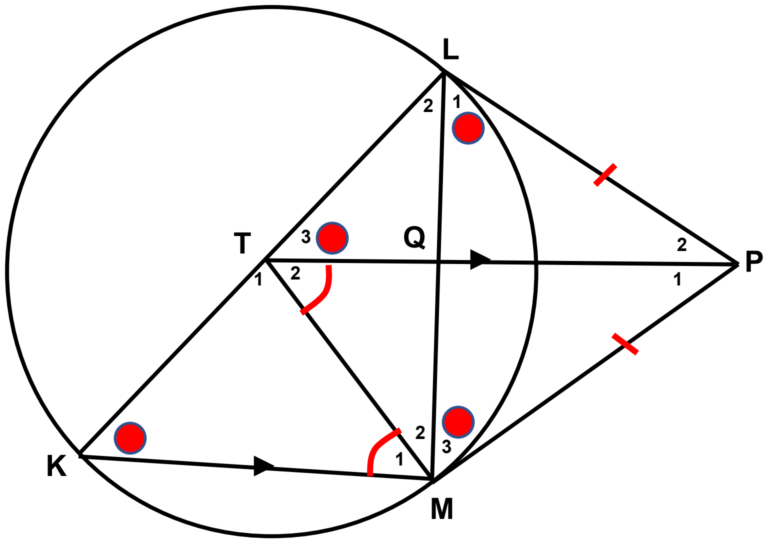
SECTION 1: 15 MARKS

SECTION 2: Open-ended questions



Given: $\boldsymbol{Q\hat{O}S} = \mathbf{80°}$.

2.1.1.	$\hat{P} = 40°$ [∠ at centre = 2 × ∠ at circumference]	✓S ✓R (2)
2.1.2.	$\hat{Q}_2 = \hat{S}_3$ [∠s opposite equal sides] ∴ $\hat{Q}_2 = 50°$ [∠s of Δ]	✓ S & R ✓S ✓R (3)
2.1.3.	$\hat{S}_4 = \hat{P} = 40°$ [Tan-Chord Theorem] OR $\hat{S}_3 + \hat{S}_4 = 90°$ [Tan ⊥ Rad] ∴ $\hat{S}_4 = 90° - 50° = 40°$	✓S ✓R (2)
		[7]



2.2.1.	$\hat{L}_1 = \hat{K}$ [Tan-chord theorem] $\hat{K} = \hat{T}_3$ [Corresp ∠s: TP ∥ KM] $\therefore \hat{L}_1 = \hat{T}_3$	✓S ✓R ✓S ✓R (4)
2.2.2.	LP = MP [Tangents from same point] $\hat{L}_1 = \hat{M}_3$ [∠s opposite equal sides] $\hat{L}_1 = \hat{T}_3$ [Proved] $\therefore \hat{M}_3 = \hat{T}_3$ ∴ TLPM is a cyclic quad [Converse ∠s in the same segment or LP subtends = ∠s] OR $\hat{T}_3 = \hat{K}$ [Proved in 2.2.1] $\hat{M}_3 = \hat{K}$ [Tan-Chord Theorem] $\therefore \hat{M}_3 = \hat{T}_3$ ∴ TLPM is a cyclic quad [Converse ∠s in the same segment or LP subtends = ∠s]	✓R ✓S ✓S ✓R (4) ✓S ✓R ✓S ✓R

2.2.3.	$\hat{M}_1 = \hat{T}_2$ [Alt ∠s: TP ∥ KM] $\hat{T}_2 = \hat{L}_1$ [MP subtends = ∠s] $\hat{L}_1 = \hat{K}$ [Proved in 2.2.1] ∴ $\hat{K} = \hat{M}_1$ ∴ TK = TM [∠s opp = sides]	✓S ✓R ✓S & R ✓S ✓R
		[13]

SECTION 2: 20 MARKS

TOTAL MARKS: 35

APPENDIX C: INTERVIEW SCHEDULE

INTERVIEW SCHEDULE FOR LEARNERS

1. In your own understanding, what is problem-solving in mathematics?

2. What do you think is the basic knowledge required to solve Euclidean geometry problems in Grade 11?

3. When solving a Euclidean geometry problem, what is your usual approach (starting point)?

4. What strategies do you normally use when solving Euclidean geometry problems?

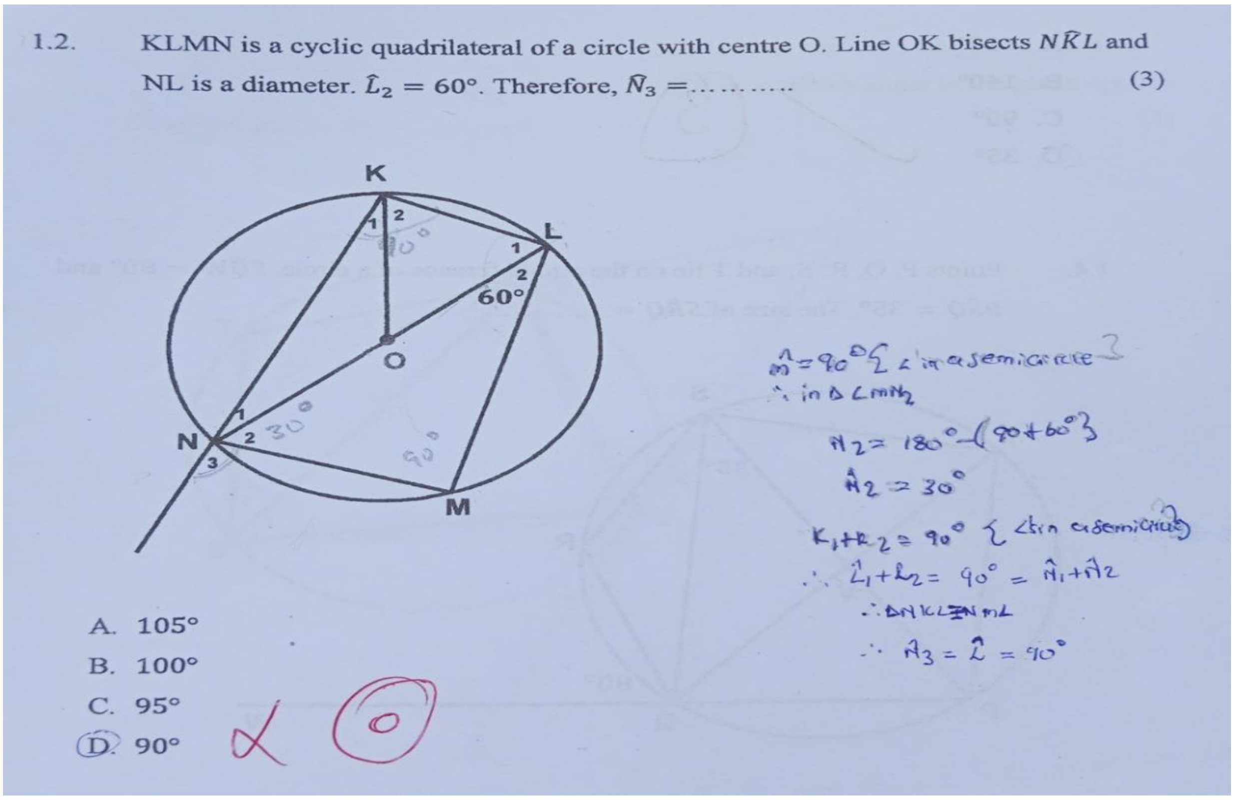
5. May you explain how you use the strategies that you mentioned in the previous question.

6. When solving a geometry problem, how do you know that you have reached the solution stage?

7. In your own understanding, how would you describe a learner who possesses a problem-solving ability?

APPENDIX D: SAMPLE OF LEARNERS' TASK RESPONSES

LA19's response to item 1.2

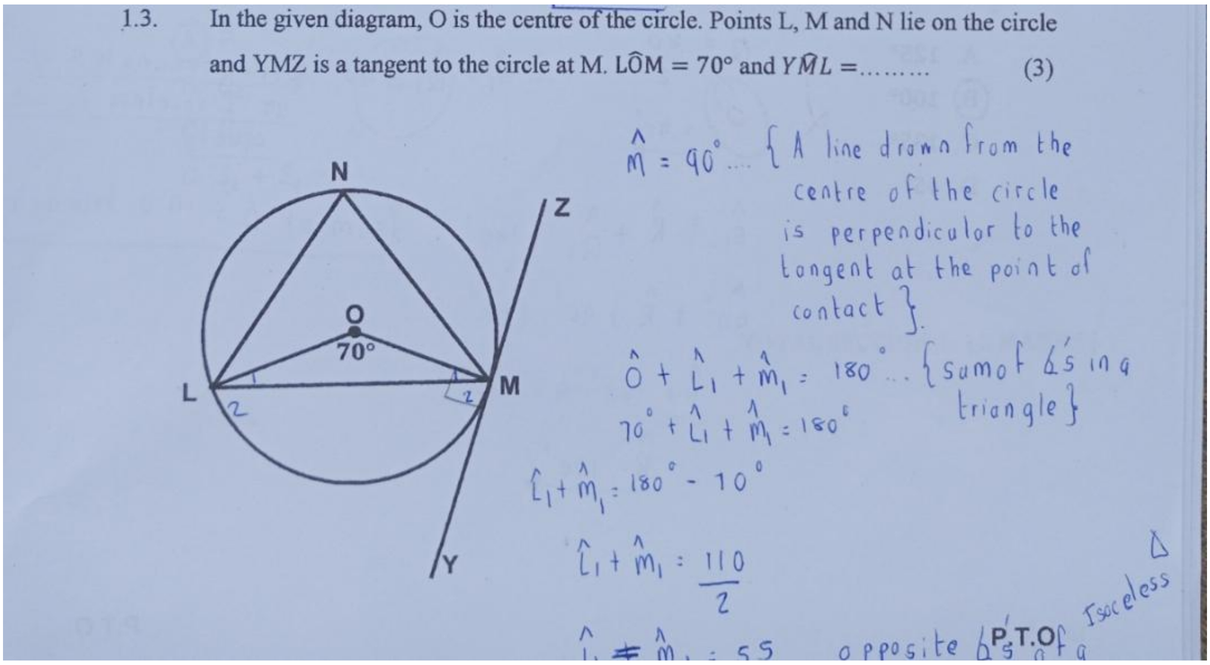


1.2. KLMN is a cyclic quadrilateral of a circle with centre O. Line OK bisects $N\hat{K}L$ and NL is a diameter. $\hat{L}_2 = 60°$. Therefore, $\hat{N}_3 =$……….. (3)

$\hat{M} = 90°$ {∠ in a semicircle}
∴ in Δ LMN₂
$\hat{N}_2 = 180° - (90 + 60°)$
$\hat{N}_2 = 30°$
$K_1 + K_2 = 90°$ {∠ in a semicircle}
∴ $\hat{L}_1 + \hat{L}_2 = 90° = \hat{N}_1 + \hat{N}_2$
∴ ΔNKL ≡ NML
∴ $\hat{N}_3 = \hat{L} = 90°$

A. 105°
B. 100°
C. 95°
D. 90°

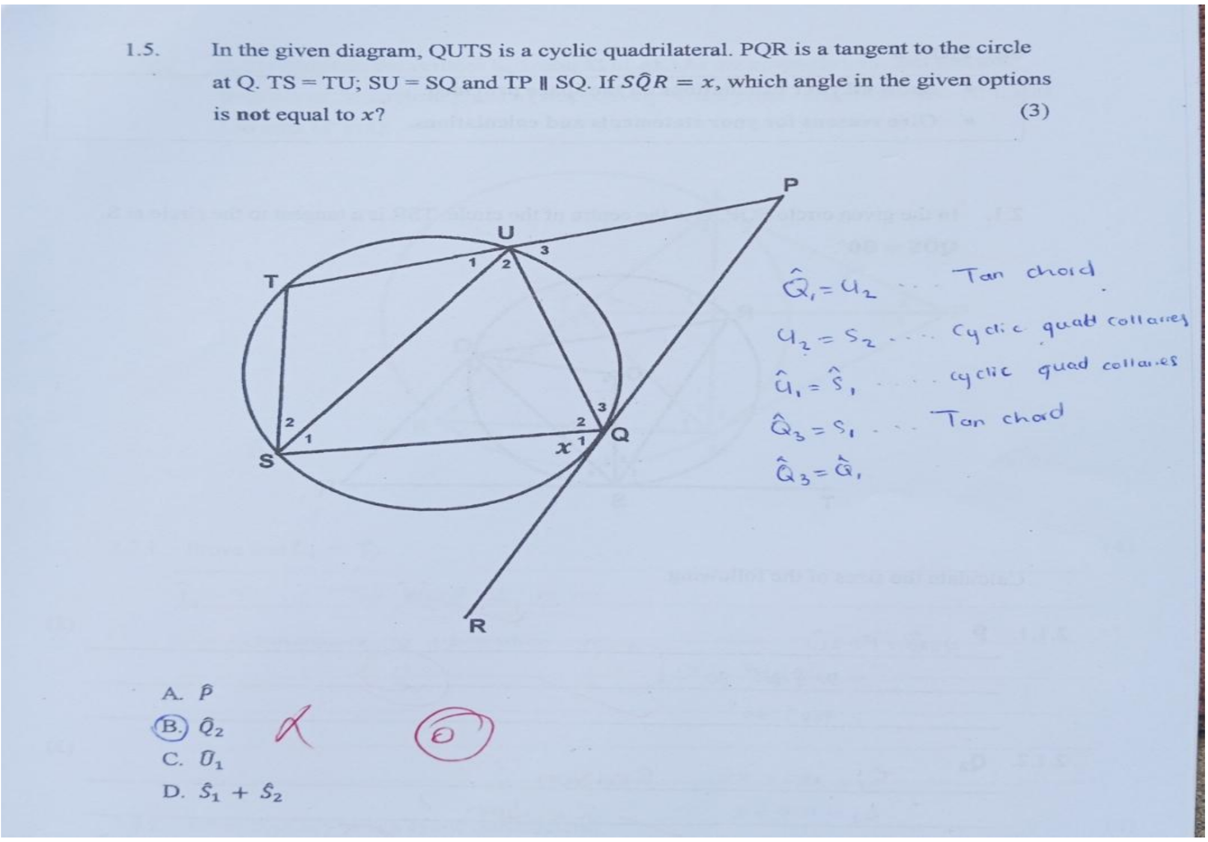
LB28's response to item 1.3



1.3. In the given diagram, O is the centre of the circle. Points L, M and N lie on the circle and YMZ is a tangent to the circle at M. $L\hat{O}M = 70°$ and $Y\hat{M}L =$……… (3)

$\hat{M} = 90°$… {A line drawn from the centre of the circle is perpendicular to the tangent at the point of contact}.
$\hat{O} + \hat{L}_1 + \hat{M}_1 = 180°$… {sum of ∠'s in a triangle}
$70° + \hat{L}_1 + \hat{M}_1 = 180°$
$\hat{L}_1 + \hat{M}_1 = 180° - 70°$
$\hat{L}_1 + \hat{M}_1 = \frac{110}{2}$
$\hat{L}_1 = \hat{M}_1 = 55$ opposite ∠'s of a Isosceles Δ
P.T.O

LA33's response to item 1.5


1.5. In the given diagram, QUTS is a cyclic quadrilateral. PQR is a tangent to the circle at Q. TS = TU; SU = SQ and TP ∥ SQ. If $S\hat{Q}R = x$, which angle in the given options is **not** equal to x? (3)

$\hat{Q}_1 = U_2$ ... Tan chord

$U_2 = S_2$ ... Cyclic quad collaries

$\hat{U}_1 = \hat{S}_1$ ... cyclic quad collaries

$\hat{Q}_3 = S_1$ ... Tan chord

$\hat{Q}_3 = \hat{Q}_1$

A. $\hat{P}$

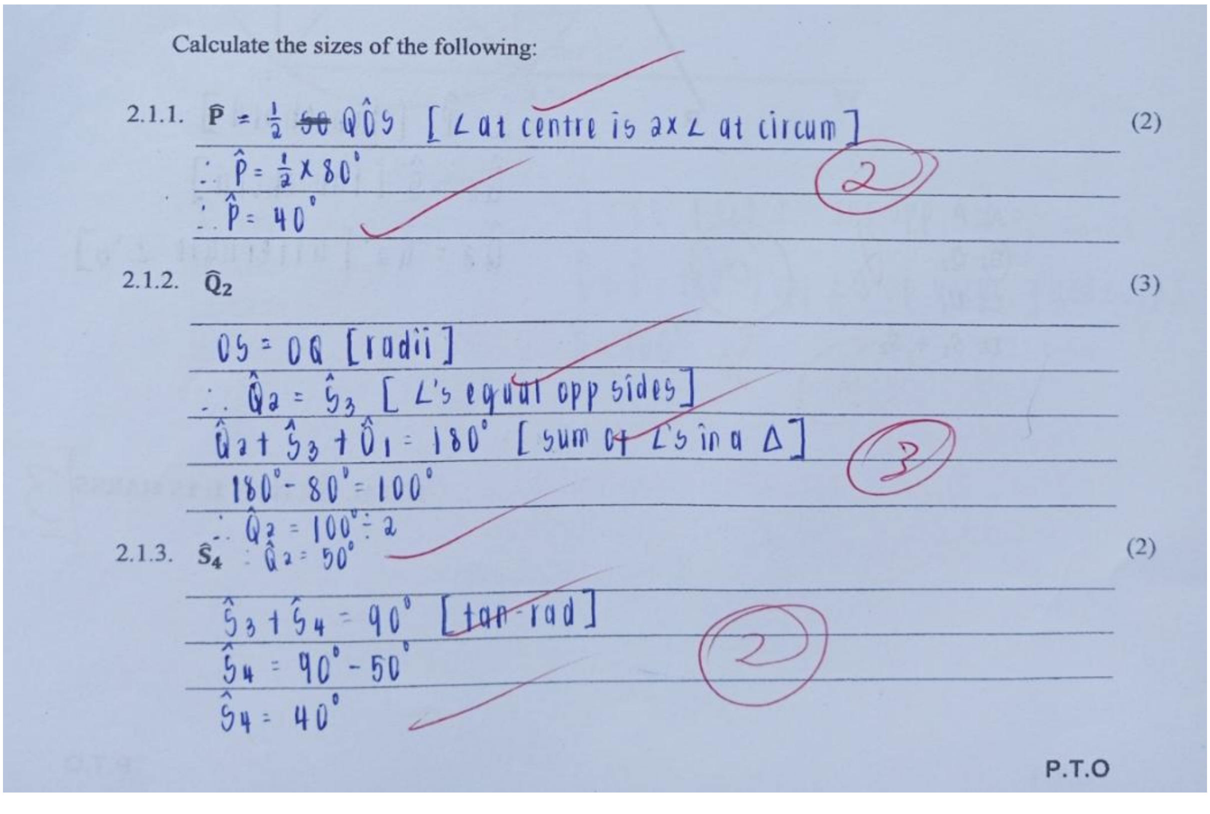
(B.) $\hat{Q}_2$

C. $\hat{U}_1$

D. $\hat{S}_1 + \hat{S}_2$

0

LB6's response to item 2.1


Calculate the sizes of the following:

2.1.1. $\hat{P}$ (2)

$\hat{P} = \frac{1}{2}$ ~~90~~ $Q\hat{O}S$ [∠ at centre is 2× ∠ at circum]

$\therefore \hat{P} = \frac{1}{2} \times 80°$

$\therefore \hat{P} = 40°$

2

2.1.2. $\hat{Q}_2$ (3)

OS = OQ [radii]

$\therefore \hat{Q}_2 = \hat{S}_3$ [∠'s equal opp sides]

$\hat{Q}_2 + \hat{S}_3 + \hat{O}_1 = 180°$ [sum of ∠'s in a Δ]

$180° - 80° = 100°$

$\therefore \hat{Q}_2 = 100° \div 2$

$\therefore \hat{Q}_2 = 50°$

3

2.1.3. $\hat{S}_4$ (2)

$\hat{S}_3 + \hat{S}_4 = 90°$ [tan-rad]

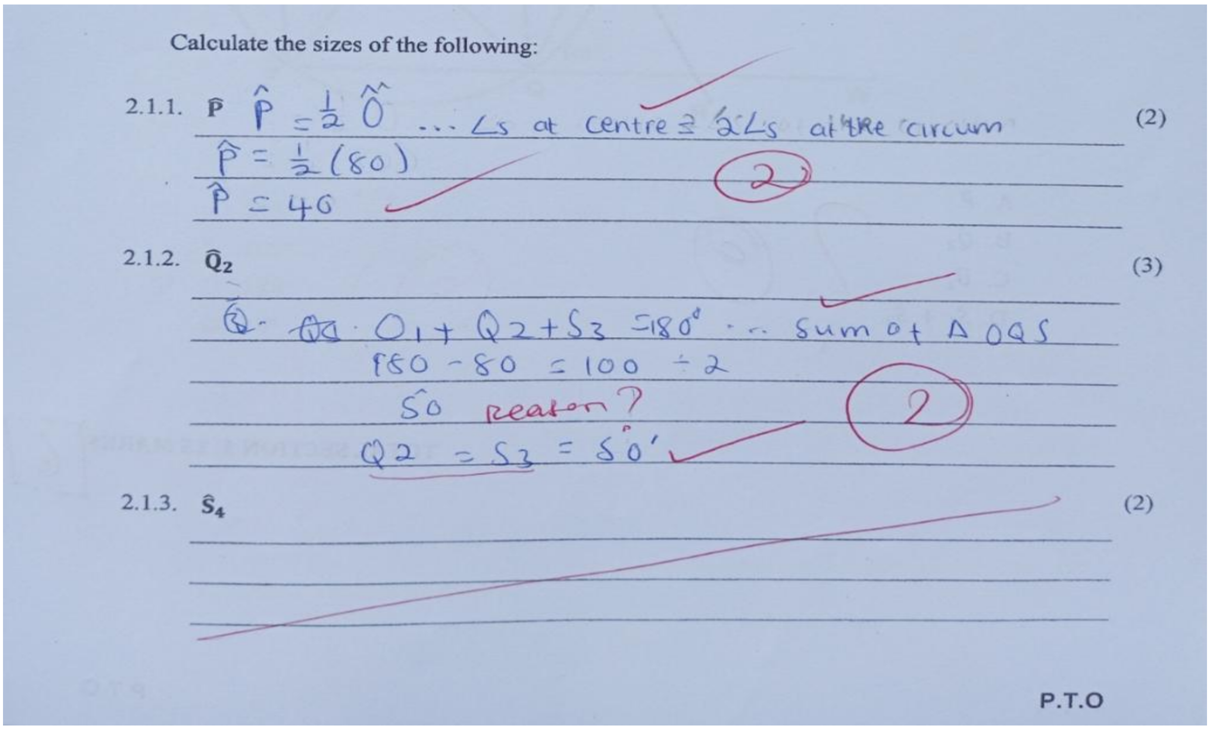
$\hat{S}_4 = 90° - 50°$

$\hat{S}_4 = 40°$

2

P.T.O

LB19's response to item 2.1



Calculate the sizes of the following:

2.1.1. $\hat{P}$ (2)

$\hat{P} = \frac{1}{2}\hat{O}$ ... ∠s at centre = 2∠s at the circum

$\hat{P} = \frac{1}{2}(80)$

$\hat{P} = 40$

(2)

2.1.2. $\hat{Q}_2$ (3)

$O_1 + Q_2 + S_3 = 180°$ ... sum of Δ OQS

180 − 80 = 100 ÷ 2

So reason?

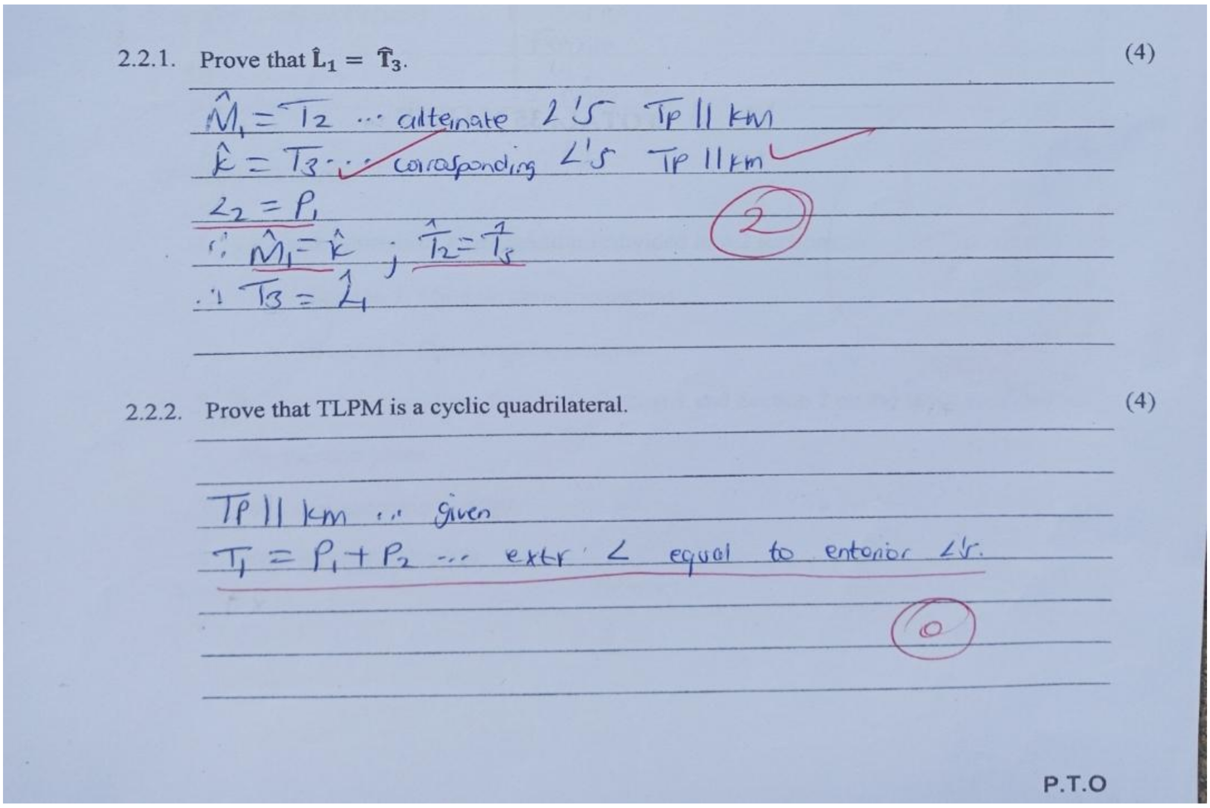
$Q_2 = S_3 = 50°$

(2)

2.1.3. $\hat{S}_4$ (2)

P.T.O

LA23's response to item 2.2



2.2.1. Prove that $\hat{L}_1 = \hat{T}_3$. (4)

$\hat{M}_1 = T_2$ ... alternate ∠'s TP ∥ KM

$\hat{K} = T_3$ ... corresponding ∠'s TP ∥ KM

$L_2 = P_1$

∴ $\hat{M}_1 = \hat{K}$; $\hat{T}_2 = \hat{T}_3$

∴ $T_3 = \hat{L}_1$

(2)

2.2.2. Prove that TLPM is a cyclic quadrilateral. (4)

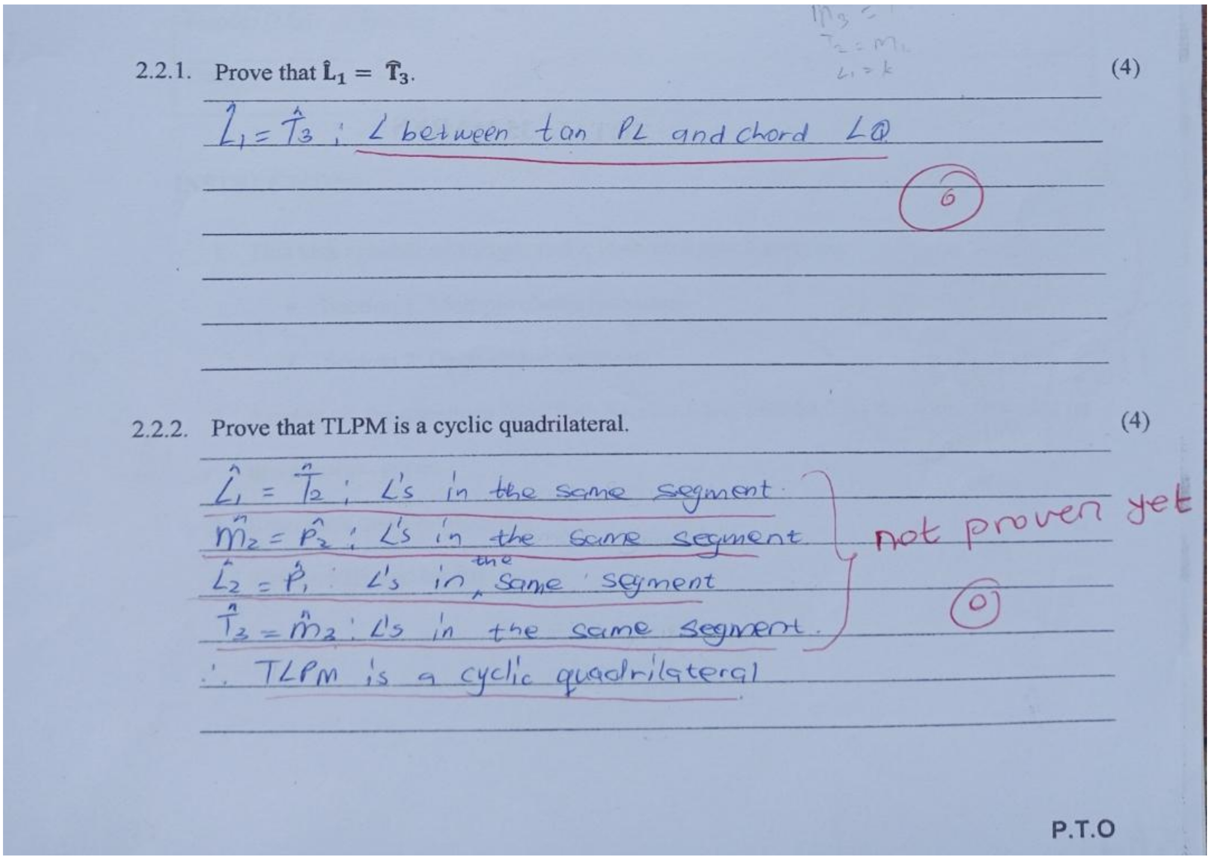
TP ∥ KM ... given

$T_1 = P_1 + P_2$ ... extr ∠ equal to enterior ∠'s.

(0)

P.T.O

LA28's response to item 2.2



2.2.1. Prove that $\hat{L}_1 = \hat{T}_3$. (4)

$\hat{L}_1 = \hat{T}_3$; ∠ between tan PL and chord LQ

2.2.2. Prove that TLPM is a cyclic quadrilateral. (4)

$\hat{L}_1 = \hat{T}_2$; ∠'s in the same segment

$\hat{M}_2 = \hat{P}_2$; ∠'s in the same segment

$\hat{L}_2 = \hat{P}_1$ ∠'s in the same segment

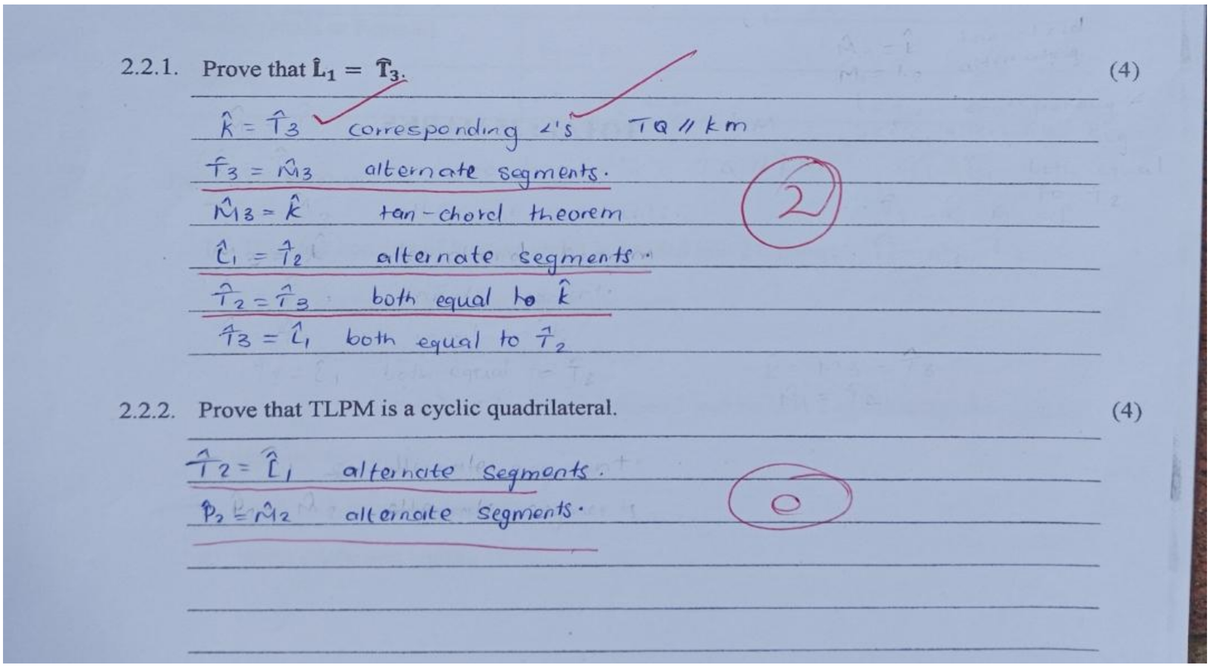
$\hat{T}_3 = \hat{M}_3$; ∠'s in the same segment.

∴ TLPM is a cyclic quadrilateral

not proven yet

P.T.O

LA17's response to item 2.2



2.2.1. Prove that $\hat{L}_1 = \hat{T}_3$. (4)

$\hat{K} = \hat{T}_3$ corresponding ∠'s TQ // km

$\hat{T}_3 = \hat{M}_3$ alternate segments.

$\hat{M}_3 = \hat{K}$ tan-chord theorem

$\hat{L}_1 = \hat{T}_2$ alternate segments.

$\hat{T}_2 = \hat{T}_3$ both equal to $\hat{K}$

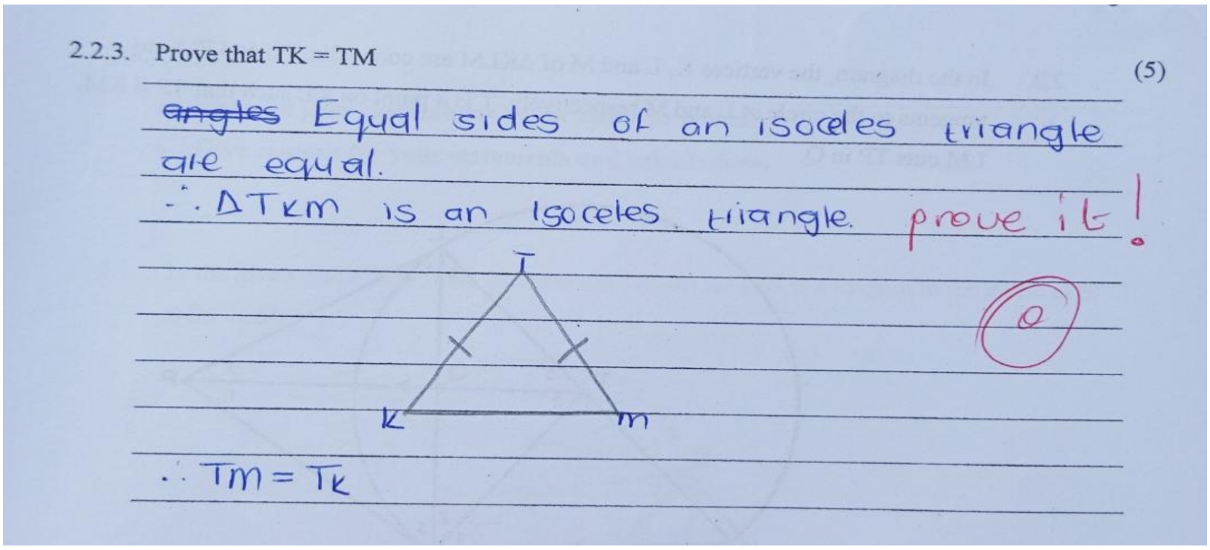
$\hat{T}_3 = \hat{L}_1$ both equal to $\hat{T}_2$

2.2.2. Prove that TLPM is a cyclic quadrilateral. (4)

$\hat{T}_2 = \hat{L}_1$ alternate segments.

$\hat{P}_2 = \hat{M}_2$ alternate segments.

LA29's response to item 2.2.3



2.2.3. Prove that TK = TM (5)

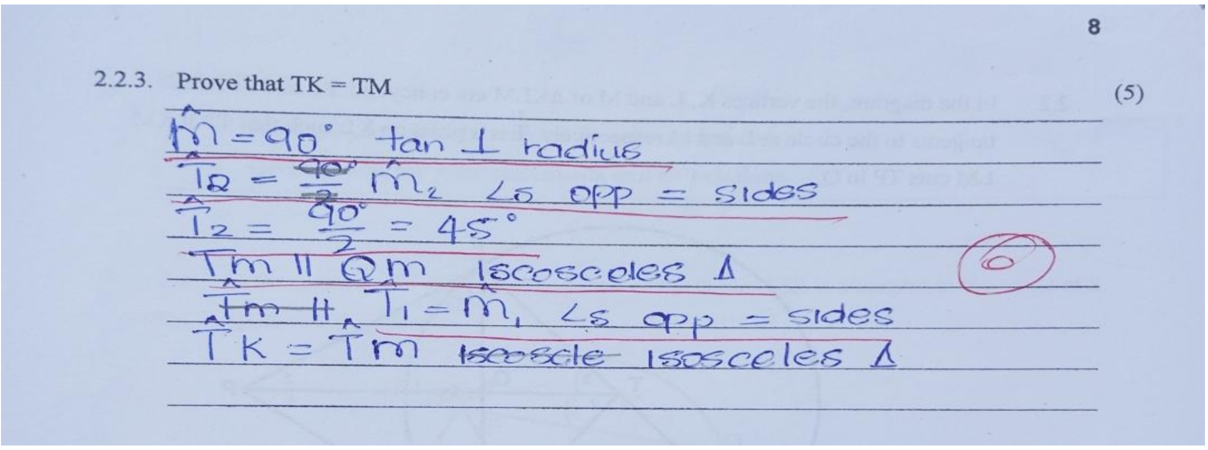
~~angles~~ Equal sides of an isoceles triangle are equal.

∴ ΔTKM is an Isoceles triangle. prove it!

∴ TM = TK

0

LB9's response to item 2.2.3



8

2.2.3. Prove that TK = TM (5)

$\hat{M} = 90°$ tan ⊥ radius

$\hat{T}_2 = \frac{90°}{2}$ $\hat{M}_2$ ∠s opp = sides

$\hat{T}_2 = \frac{90°}{2} = 45°$

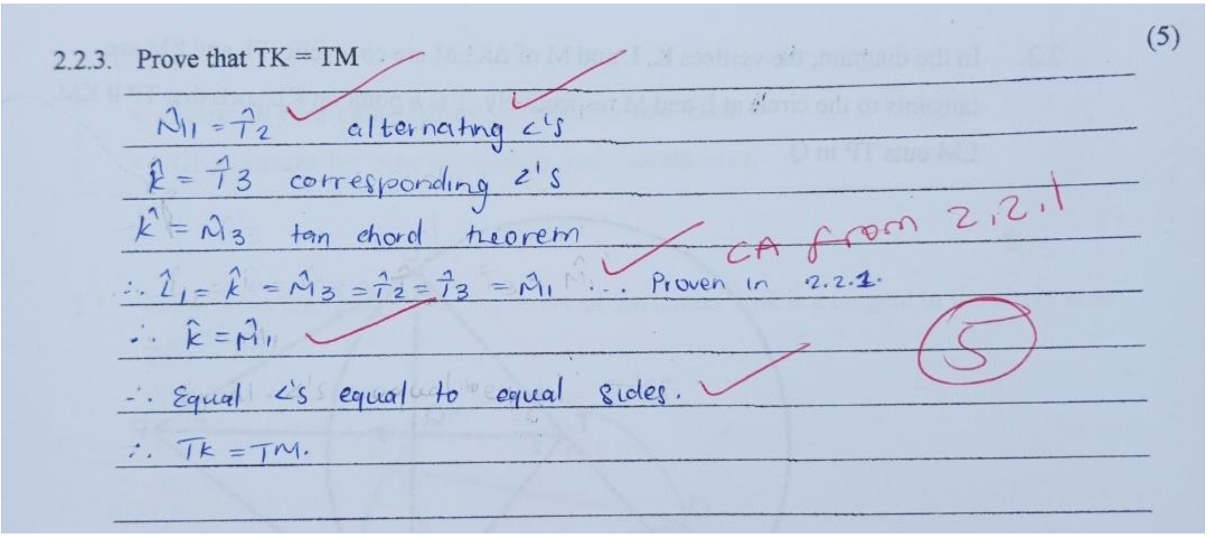
TM ∥ QM isocosceles Δ

~~TM ∥~~ $\hat{T}_1 = \hat{M}_1$ ∠s opp = sides

$\hat{T}K = \hat{T}M$ ~~isoosele~~ isosceles Δ

0

LA17's response to item 2.2.3



2.2.3. Prove that TK = TM (5)

$\hat{M}_1 = \hat{T}_2$ alternating ∠'s

$\hat{K} = \hat{T}_3$ corresponding ∠'s

$\hat{K} = \hat{M}_3$ tan chord theorem

CA from 2.2.1

∴ $\hat{L}_1 = \hat{K} = \hat{M}_3 = \hat{T}_2 = \hat{T}_3 = \hat{M}_1$ ... Proven in 2.2.1.

∴ $\hat{K} = \hat{M}_1$

∴ Equal ∠'s equal to equal sides.

∴ TK = TM.

5

APPENDIX E: TASK VERIFICATION TOOL

TASK VERIFICATION FORM	
Details of the researcher	**Details of the verifier**
Name: Catherine Lindiwe Mahlangu **Institution:** University of South Africa	**Name:**___________________________________ **Institution/Organisation:**____________________
The following questions are based on the Euclidean Geometry Task to be administered to a group of Grade 11 mathematics learners. You are kindly requested to provide feedback about the validity of the Task by answering the questions below. You can provide your feedback by inserting a cross (X) in the appropriate spaces and make a comment if necessary. Your feedback to the questions will be highly valued for the success of this research.	

QUESTIONS	YES	NO	COMMENTS
Is the duration of the task indicated, and the time allocated for the completion of the task adequate?			
Is the mark allocation commensurate with the level of difficulty of the questions and the time allocated for completion of the task?			
Are the instructions clear and unambiguous?			
Are the marks appropriately allocated and distributed in accordance with CAPS Document?			
Does the paper cater for a variety of questions?			
Does the task incorporate the different cognitive levels (Blooms' Taxonomy)? (Refer to the weighting grid of the test)			
Is the language and terminology used appropriate and relevant?			
Is the quality of the illustrations, graphs, or tables clear, relevant, and user-friendly?			
Are the items in the task representative of the concepts covered in Euclidean geometry in Grade 11 mathematics?			
Are the items in the task pitched at the level of understanding of the learners in the Grade 11 mathematics class?			

Has the task considered different abilities of learners?			
Does the task provide opportunities for the learners to demonstrate their problem-solving skills?			
Is the task appropriate and relevant to assess a learner's problem-solving abilities in mathematics?			
Is the marking guideline relevant and appropriate for marking of the set task?			
Does the marking guideline allow for alternative responses?			

Please provide comments, if necessary, on the strengths and weaknesses of the paper.

____

____

Signature:______________________________ **Date:** ________________________

OFFICIAL STAMP

APPENDIX F: PROBLEM SOLVING SCORING RUBRIC

Problem-solving ability indicators	5	4	3	2	1	0	Score
Understanding the problem (Identify the elements that are known and those asked for)	The learner identified relevant data and used appropriate notations/ symbols and terminology adequately.	The learner identified relevant data and used appropriate notations/ symbols and terminology partially.	The learner identified limited amount of relevant data and used appropriate notations/ symbols and terminology ineffectively.	The learner identified very limited amount of relevant data and used appropriate notations/ symbols and terminology ineffectively.	The learner could not identify relevant data and used inappropriate notations/ symbols and terminology.	The learner did not make any attempt to solve the problem	
Devising a plan (Develop a problem-solving strategy)	The learner was able to identify the appropriate properties, axioms, or theorems adequately.	The learner was able to identify some of the appropriate properties, axioms, or theorems.	The learner was able to partially identify some of the appropriate properties, axioms, or theorems.	The learner inadequately identified the properties, axioms, or theorems, but mixed them up.	The learner could not remember the appropriate properties, axioms, or theorems.	The learner did not make any attempt to solve the problem	
Carry out the plan (Implement the strategy to solve the problems)	The learner efficiently linked the information obtained to reach the solution stage. The procedures and skills that the learner used demonstrates critical thinking.	The learner linked some of the information obtained to reach the solution stage. The procedures and skills that the learner used demonstrates some critical thinking.	The learner recognizes the relationship between the obtained information but expands it incorrectly.	The learner has not fully explored and executed the strategies.	Only irrelevant unconnected information is provided. No evidence of understanding.	The learner did not make any attempt to solve the problem	
Looking back (Interpret the results according to the problem of origin)	The learner's solution to the problem is complete with correct statements and valid reasons. Conclusion provided with a reason.	The learner's solution to the problem is almost complete with most correct statements and valid reasons. Conclusion provided with a reason.	The learner's solution to the problem is incomplete with some correct statements and valid reasons. Conclusion provided with no reason.	The learner's solution to the problem is irrelevant with correct statements and valid reasons. No conclusion provided.	The learner's solution to the problem is irrelevant with incorrect statements and invalid reasons. No conclusion provided.	The learner did not make any attempt to solve the problem	
						TOTAL (20)	

APPENDIX G: PARTICIPANT'S EUCLIDEAN GEOMETRY TASK FEEDBACK TOOL

No name required.

After completing the task, you are requested to complete the questionnaire below to report the amount of mental effort which you invested in performing the problem-solving activities in the task. You are required to choose only one of the provided choices from 1 to 5.

No.	Level of difficulty	Mark (X)
1	Easy	
2	Fair	
3	Some easy, but mostly difficult	
4	Difficult	
5	Very difficult	

APPENDIX H: CONSENT LETTERS

Letter of permission application to the school principal

3040 Block VV Ext 3
Soshanguve
0152
Date:________________

Dear Principal

Request for permission to conduct research at your school

My name is Catherine Lindiwe Mahlangu. I am currently doing Master of Education degree in Mathematics Education at the University of South Africa. The title of my research is “Learners’ problem-solving abilities in Grade 11 Euclidean geometry tasks”. As part of the research, I need to collect data from schools. I therefore ask for your permission to allow me to use your school as a site for this research to be carried out.

The study aims to probe learners’ problem-solving abilities that are manifest when solving problems in Grade 11 Euclidean geometry tasks. It is being carried out in the hope that it will contribute to the improvement of learners’ problem-solving abilities when doing Euclidean geometry tasks which is hoped to result in enhancing learner performance in mathematics. The researcher will: (1) administer a task-based worksheet to the Grade 11 Mathematics learners; and (2) conduct semi-structured interviews with a purposive sample of Grade 11 learners telephonically. The interviews will be audio recorded for verbal transcription.

The benefit of participating in this research will be to assist in the identification of areas requiring improvement in the teaching and learning of geometry concepts. There are no potential risks to participants. There will be no reimbursement or any incentives for participation in the research as participation will be voluntarily. Kindly note that the name of the school and the learners will not be exposed; the school and participants will be referred to by a pseudonym. Furthermore, the final report of the findings of the research will be availed to the school.

Should you require further information, please do not hesitate to contact me by email at 54522617@mylife.unisa.ac.za.or contact 076 499 6222. I have enclosed a form which you may use to respond to this request.

Yours sincerely

Catherine Lindiwe Mahlangu

INFORMED CONSENT FORM FOR THE PRINCIPAL'S RESPONSE

Dear Ms C. Lindiwe Mahlangu

I,____, the principal of, ____high/ secondary school, acknowledge the receipt of the letter of request for my school to participate in your research. I have read and understood the content of the letter that explains your research, which is entitled, Learners' problem-solving abilities in Grade 11 Euclidean geometry tasks. I have also understood the aim of your research.

I, therefore, **give consent**/ **do not give consent** that my school will take part in your research.

Principal signature : ________________________________

Date : ________________________________

Researcher signature : ________________________________

Date : ________________________________

SCHOOL STAMP

LETTER REQUESTING PARENTAL CONSENT

Dear Parent

Your child is invited to participate in a study entitled Learners' Problem-solving Abilities in Grade 11 Euclidean Geometry Tasks.

I am undertaking this study as part of my master's research at the University of South Africa. This research study is being carried out in the hope that it will contribute to the improvement of learners' problem-solving abilities when doing Euclidean geometry tasks which is hoped to result in enhancing learner performance in mathematics. I am asking permission to include your child in this study because he/she is a Grade 11 mathematics learner. I expect to have about 30 other children participating in the study in the same school.

If you allow your child to participate, I shall request him/her to:

1. Complete a task-based worksheet during a specially arranged mathematics lesson.
2. Take part in a telephonic interview that will be audio recorded if selected.

Any information that is obtained in connection with this study and can be identified with your child will remain confidential and may only be disclosed with your permission. His/her responses will not be linked to his/her name or your name or the school's name in any written or verbal report based on this study. Such a report will be used for research purposes only.

There are no foreseeable risks to your child by participating in the study. The possible benefits to education is to assist in the identification of areas requiring improvement in the teaching and learning of geometry concepts which may lead to the identification of relevant interventions which will enhance learner performance in mathematics.

Please kindly note that neither you nor your child will receive any type of payment for participating in this study. Your child's participation in this study is voluntary. Your child may decline to participate or withdraw from participation at any time. Withdrawal or refusal to participate will not affect him/her in any way. Similarly, you can agree to allow your child to be in the study now and change your mind later without any penalty.

The study will take place during regular classroom activities with the prior approval of the school and your child's teacher. In addition to your permission, your child must agree to participate in the study and you and your child will also be asked to sign the assent form which accompanies this letter. The information gathered from the study and your child's participation in the study will be stored securely on a password locked computer in my locked office for five years after the study. Thereafter, records will be erased.

There are no potential risks in this research study. There will be no reimbursement or any incentives for participation in the research.

If you have questions about this study, please ask me (Ms. Mahlangu CL) or my study supervisor, Dr Dhlamini, Department of Mathematics, College of Education, University of South Africa. My contact number is 0764996222 and my e-mail is 54522617@mylife.unisa.ac.za. The e-mail of my supervisor is dhlamjj@unisa.ac.za. The Ethics Committee of the College of Education, UNISA, and the Gauteng Department of Education have already given permission for the study to be conducted.

You are making a decision about allowing your child to participate in this study. Your signature below indicates that you have read the information provided above and have decided to allow him or her to participate in the study. You may keep a copy of this letter.

Name of child:___________________________________

_________________________	_________________________	______________
Parent/guardian's name (print)	Parent/guardian's signature:	Date:
_________________________	_________________________	______________
Researcher's name (print)	Researcher's signature	Date:

A LETTER REQUESTING ASSENT FROM LEARNERS IN A SECONDARY SCHOOL TO PARTICIPATE IN A RESEARCH PROJECT

Title of my research is Learners' Problem-Solving Abilities In Grade 11 Euclidean Geometry Tasks.

Dear Learner

I am doing a study on Master of Mathematics Education as part of my studies at the University of South Africa. Your principal has given me permission to do this study in your school. I would like to invite you to be a very special part of my study. I am doing this study so that we can shed light on the challenges you encounter when solving Euclidean geometric problems. This may help you and many other learners of your age in different schools.

This letter explains what I would like you to do. There may be some words you do not know in this letter. You may ask me or any other adult to explain any of these words that you do not know or understand. You may take a copy of this letter home to think about my invitation and talk to your parents about this before you decide if you want to be in this study.

I would like to ask you to complete a task-based worksheet on Euclidean geometry topic and to take part in a telephonic interview that will be audio recorded if selected. Answering the interview questions will not take more than 30 minutes.

I will write a report on the study, but I will not use your name in the report or say anything that will let other people know who you are. Participation is voluntary and you do not have to be part of this study if you do not want to take part. If you choose to be in the study, you may stop taking part at any time without penalty. You may tell me if you do not wish to answer any of my questions. No one will blame or criticise you. When I have completed the study, I will return to your school to give a presentation about some of the helpful and interesting things I found out in my study.

The benefits of this study are to help you to improve your abilities/skills when solving problems in Euclidean geometry tasks. There are no potential risks in this study. You will not be reimbursed or receive any incentives for your participation in the research.

If you decide to be part of my study, you will be asked to sign the form provided. If you have any other questions about this study, you can talk to me or you can have your parent or another adult call me at 0764996222 / 0814044262. Do not sign the form until you have all your questions answered and understand what I would like you to do.

WRITTEN ASSENT

I have read this letter which asks me to be part of a study at my school. I have understood the information about the study, and I know what I will be asked to do. I am willing to participate in the study.

______________________	______________________	______________________
Learner's name (print):	Learner's signature:	Date:
______________________	______________________	______________________
Witness's name (print)	Witness's signature	Date:

(The witness must be over 18 years old and present when signed.)

______________________	______________________	______________________
Parent/guardian's name (print)	Parent/guardian's signature:	Date:
______________________	______________________	______________________
Researcher's name (print)	Researcher's signature:	Date:

APPENDIX I: GDE RESEARCH APPROVAL LETTER

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**13 April 2021**
Validity of Research Approval:	**08 February 2021– 30 September 2021** **2021/87**
Name of Researcher:	**Mahlangu CL**
Address of Researcher:	**3040 Kgwerano Sreet**
	Soshanguve East Ext 3
Telephone Number:	**076 499 6222**
Email address:	**Mlindiwe90@gmail.com**
Research Topic:	**Learners problem solving abilities in grade 11 Euclidean geometry tasks**
Type of qualification	**Masters of Education**
Number and type of schools:	**2 Secondary Schools**
District/s/HO	**Tshwane South**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

13/04/2021

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. *Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

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2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. ***Because of COVID 19 pandemic researchers can ONLY collect data online, telephonically or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate. The approval letter will then indicate the type of arrangements that have been made with the school.***
4. ***The Researchers are advised to make arrangements with the schools via Fax, email or telephonically with the Principal.***
5. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
6. *A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
7. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
8. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
9. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
10. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
11. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
12. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
13. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
14. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
15. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
16. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

....

Mr Gumani Mukatuni
Acting CES: Education Research and Knowledge Management

DATE: 13\04\2021

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

APPENDIX J: TSHWANE SOUTH DISTRICT RESEARCH SUPPORT LETTER

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

Enquiries: Lucky Rapudi
Tel: (012) 401 6330
Fax: 0866 522 388
Email: Lucky.Rapudi@gauteng.gov.za

TO: **The Principal**
TS District Schools

FROM: **Mrs. Paula Galego**
District Director: Tshwane South

DATE: **20th April 2021**

SUBJECT : PERMISSION TO CONDUCT RESEARCH AT AN EDUCATION INSTITUTION

Dear Sir/ Madam

Permission is hereby granted to **C.L. Mahlangu** to conduct an academic research at your institution.

The researcher shall make arrangements for research with the school management. The school staff, learners and SGB are requested to co-operate with and give support to the researcher. Research findings and recommendations are critical for policy review in public education sector.

The researcher may however not disrupt the normal school programme in the course of research. The research may only take place between the months of February and September. Attached are other conditions to be observed by the researcher.
Covid-19 safety protocols must be strictly adhered to at all times.

The school may request for the research outcome presentation directly from the researcher or obtain research document from Research & Knowledge Management Directorate at GDE Head Office.

Regards

Mrs P. Galego
District Director: Tshwane South
Date: 21/4/2021

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Office of the District Director: Tshwane South
(Mamelodi/Eersterust/Pretoria East/Pretoria South/Atteridgeville/Laudium)
President Towers building, 265 Pretorius Street, Pretoria, 0002
Private Bag X198, Pretoria, 0001 Tel: (012) 401 6317; Fax: (012) 401 6318
Website: www.education.gpg.gov.za

APPENDIX K: ETHICS CLEARANCE CERTIFICATE

UNISA | university of south africa

UNISA COLLEGE OF EDUCATION ETHICS REVIEW COMMITTEE

Date: 2021/03/10

Dear Ms CL Mahlangu

Decision: Ethics Approval from 2021/03/10 to 2024/03/10

Ref: **2021/03/10/54522617/27/AM**
Name: Ms CL Mahlangu
Student No.: 54522617

Researcher(s): Name: Ms CL Mahlangu
E-mail address: 54522617@mylife.unisa.ac.za
Telephone: 0764996222

Supervisor(s): Name: Dr JJ Dhlamini
E-mail address: dhlamjj@unisa.ac.za
Telephone: 0124292023

Title of research:

Learner's problem solving abilities in grade 11 Euclidean geometry tasks

Qualification: MEd Mathematics Education

Thank you for the application for research ethics clearance by the UNISA College of Education Ethics Review Committee for the above mentioned research. Ethics approval is granted for the period 2021/03/10 to 2024/03/10.

The ***medium risk*** *application was reviewed by the Ethics Review Committee on 2021/03/10 in compliance with the UNISA Policy on Research Ethics and the Standard Operating Procedure on Research Ethics Risk Assessment.*

The proposed research may now commence with the provisions that:

1. The researcher will ensure that the research project adheres to the relevant guidelines set out in the Unisa Covid-19 position statement on research ethics attached.
2. The researcher(s) will ensure that the research project adheres to the values and principles expressed in the UNISA Policy on Research Ethics.

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University of South Africa
Preller Street, Muckleneuk Ridge, City of Tshwane
PO Box 392 UNISA 0003 South Africa
Telephone: +27 12 429 3111 Facsimile: +27 12 429 4150
www.unisa.ac.za

3. Any adverse circumstance arising in the undertaking of the research project that is relevant to the ethicality of the study should be communicated in writing to the UNISA College of Education Ethics Review Committee.
4. The researcher(s) will conduct the study according to the methods and procedures set out in the approved application.
5. Any changes that can affect the study-related risks for the research participants, particularly in terms of assurances made with regards to the protection of participants' privacy and the confidentiality of the data, should be reported to the Committee in writing.
6. The researcher will ensure that the research project adheres to any applicable national legislation, professional codes of conduct, institutional guidelines and scientific standards relevant to the specific field of study. Adherence to the following South African legislation is important, if applicable: Protection of Personal Information Act, no 4 of 2013; Children's act no 38 of 2005 and the National Health Act, no 61 of 2003.
7. Only de-identified research data may be used for secondary research purposes in future on condition that the research objectives are similar to those of the original research. Secondary use of identifiable human research data requires additional ethics clearance.
8. No field work activities may continue after the expiry date **2024/03/10**. Submission of a completed research ethics progress report will constitute an application for renewal of Ethics Research Committee approval.

Note:

The reference number **2021/03/10/54522617/27/AM** *should be clearly indicated on all forms of communication with the intended research participants, as well as with the Committee.*

Kind regards,

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APPENDIX L: LANGUAGE EDITING CERTIFICATE

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APPENDIX M: TURNITIN RECEIPT

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LEARNER'S PROBLEM SOLVING ABILITIES IN GRADE 11 EUCLIDEAN GEOMETRY TASKS

by

CATHERINE LINDIWE MAHLANGU

submitted in accordance with the requirements for the degree of

MASTER OF EDUCATION

in the subject

MATHEMATICS EDUCATION

at the

UNIVERSITY OF SOUTH AFRICA

SUPERVISOR: PROF JJ DHLAMINI

FEBRUARY 2023